Chapter 19
Methods for Component Mode Synthesis Model Generation for Uncertainty Quantification

A.R. Brink, D.G. Tipton, J.E. Freymiller, and B.L. Stevens

Abstract Component mode synthesis (CMS) is a widely employed model reduction technique used to reduce the computational cost associated with the dynamic analysis of complex engineering structures. To generate CMS models, specifically the formulation of Craig and Bampton, both normal fixed-interface modes and constraint modes of the component's structure are calculated. These modes are used in conjunction with the component level mass and stiffness matrices to generate reduced mass and stiffness matrices used in the final analyses. For some component models, the most computationally expensive part of this procedure is calculating the component normal modes information. Several different approaches are utilized to investigate the sensitivity of system level responses to variations in several aspects of the CMS models. One approach evaluates changes due to modifications of the reduced mass and stiffness matrices assuming that the mode shapes do not change. The second approach assumes that the mode shapes change but the reduced mass and stiffness matrices do not change. An example is presented to show the influence of these two approaches.

Keywords Component mode synthesis • CMS • Craig-Bampton • Uncertainty quantification • UQ

19.1 Introduction

As engineering structures increase in complexity, cost and importance, the level of rigor in the computational analyses of these structures must increase proportionally. These computational analyses must include quantification of the uncertainties (UQ) associated with the structure and capture the response with a high level of accuracy. One common method for determining the effect that model uncertainty has on the predicted structural response is to define ranges for the uncertain parameters, then use Monte Carlo, Latin Hypercube, or other similar sampling methods to explore the parameter space [1]. All of these sampling techniques require the numerical model to be rebuilt with each new parameter set assignment and the full analyses suite re-calculated. This method is often extremely computationally expensive for large scale numerical models. To alleviate some of the computational cost, model reduction techniques, such as component mode synthesis, are employed. Component mode synthesis (CMS) has been a heavily utilized model reduction tool since its inception in the early 1960s. It is useful not only for its ability to reduce computational cost during the analysis of complex structures but also in facilitating the sharing of major component assemblies among multiple design agencies. Currently, the most heavily used derivation of CMS is that proposed by Craig and Bampton, which relies on fixed interface modes. A review of the Craig-Bampton method is provided in Sect. 19.2. Since the use of CMS models is required to reduce the computational burden while maintaining high response accuracy, they must also be subjected to a UQ with similar rigor as that applied to the full structure. Recreating the Craig-Bampton reduction for each new parameter set introduces a new computational cost associated with generating the fixed-interface modes of the structure. Many researchers are working to investigate how to handle this UQ while not introducing a large computational expense using parameterized reduced order models (PROMs) [2–7]. According to [2], PROMs include the derivative of parametric terms while building the CMS model. By including these derivatives, uncertain parameters are directly included in the CMS formulation, thus it is not required to rebuild the model with each new parameter set. While these PROM techniques are mathematically elegant and more exact than what is presented here, a method is sought that is easily implemented into existing FE codes. This paper explores two techniques to recreate the Craig-Bampton reduction *without* recomputing the mode shapes each time. One method, developed by Tipton

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and Brink, calculates the mode shapes for the nominal condition, then uses the same mode shapes for all realizations of the structure. The mass and stiffness matrices are updated to reflect the new parameter set. The second method, developed by ATA Engineering, instead retains the mass and stiffness matrices of the nominal structure and perturbs the mode shape matrix with a cross-orthogonality matrix, corresponding to the expected change in mode shapes resulting from the new parameter set. Both of these methods are developed in Sects. 19.3.1 and 19.3.2, respectively. They are then applied to a prototypical structure, and their results compared for validity to full Craig-Bampton reductions, which include recalculation of the modes for each parameter set.

19.2 A Brief Review of Craig-Bampton Reduced Order Models

Craig and Bampton's landmark work [8], first published in 1968, is a simplification of work done by Hurty [9] earlier in the same decade. This simplification involves the way in which interface degrees of freedom are handled. The procedure for generating a Craig-Bampton reduced order model is as follows [10]:

1. Generate a detailed numerical model (finite element or otherwise) of the system or component that is to be reduced.
2. Identify all interface and internal degrees of freedom.
3. Calculate the *fixed-interface modes* modal matrix for the system with all interface degrees of freedom fixed. Engineering judgment and convergence studies dictate how many of these modes to retain.
4. Calculate the *constraint modes* of the interface degrees of freedom.
5. Assemble the *Craig-Bampton transformation matrix*.
6. Calculate the reduced mass and stiffness matrices.
7. Assemble the reduced order model into the system numerical model.

19.2.1 Model Generation and DOF Identification

Generating the numerical model to be reduced is often time consuming. The analyst must build a full model capable of capturing all of the relevant dynamics. For baseline Craig-Bampton reduced order models, the model is completely linear, but can still be quite complex. To perform UQ analysis involving geometric parameters, this model needs to be rebuilt for each new parameter set. However, if changes are small, such as those attributed to geometric tolerances, then the original model can be tweaked without major effort. Identifying the interface and internal degrees of freedom is straight forward. Any degree of freedom that couples to the main structure of interest is considered an interface degree of freedom. All remaining degrees of freedom are considered internal.

19.2.2 Fixed-Interface Modes

In the Craig-Bampton formulation, fixed-interface modes are defined as the normal modes of the internal degrees of freedom with all interface degrees of freedom fixed. This requires a straight forward Eigen analysis. The modal matrix is stored as $\mathbf{\Phi}$. As stated earlier, the analyst must decide the appropriate number of modes to retain to achieve an acceptable level of accuracy.

19.2.3 Constraint Modes

Constraint modes are calculated by statically applying a unit displacement to one interface degree of freedom at a time, while fixing all other interface degrees of freedom. This not only helps to define the proper interface stiffness, but also helps to enforce inter-component compatibility during assembly into the complete structure. The constraint modes of the substructure, are calculated with

$$[\boldsymbol{\Psi}] = -[\mathbf{K_{nn}}]^{-1}[\mathbf{K_{na}}], \tag{19.1}$$

where a represents interface degrees of freedom and n the interior degrees of freedom.

19.2.4 Craig-Bampton Transformation Matrix

The Craig-Bampton transformation matrix, **W** is assembled using the component and constraint modes. This matrix transforms the generalized degrees of freedom used to synthesize the reduced order model back into the original system's degrees of freedom. This takes the form

$$\begin{Bmatrix} \mathbf{u_n} \\ \mathbf{u_a} \end{Bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\Psi} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}}_{\mathbf{W}} \begin{Bmatrix} \mathbf{p_k} \\ \mathbf{u_a} \end{Bmatrix}, \tag{19.2}$$

where $\mathbf{p_k}$ is the vector of modal coordinates, and $\mathbf{u_n}$ and $\mathbf{u_a}$ are displacements in generalized and physical coordinates, respectively.

19.2.5 Reduced Stiffness and Mass Matrices

Once the Craig-Bampton transformation matrix is assembled, it is used to calculate the reduced mass and stiffness matrices for the component. These matrices are then used to assemble the reduced order model back into the main system and include both interface as well as modal degrees of freedom. They are calculated as

$$\left[\hat{\mathbf{M}}_{\mathbf{CB}}\right] = [\mathbf{W}]^T[\mathbf{M}][\mathbf{W}] \tag{19.3}$$

$$\left[\hat{\mathbf{K}}_{\mathbf{CB}}\right] = [\mathbf{W}]^T[\mathbf{K}][\mathbf{W}]. \tag{19.4}$$

19.3 Craig-Bampton Generation for UQ Studies

As was mentioned in Sect. 19.1, there is significant computational cost associated with calculating the fixed-interface modes, which is outlined in Sect. 19.2.2. It is also noted that for extremely large component models, there can be significant cost associated with inverting the stiffness matrix in Eq. (19.1). However, for this study it is assumed that the component model is a small enough size such that the fixed-interface mode calculation dominates. For a UQ study involving hundreds or thousands of parameter sets, re-calculating the fixed-interface modes for each set would put too much computational burden on the analysis. Presented in the next two subsections are methods which perturb the Craig-Bampton model to account for uncertain parameter sets without recalculating the fixed-interface modes.

19.3.1 REMAP Technique

The first technique presented is further separated into two sub techniques, REMAP1 and REMAP2. Both of these technique use the nominal fixed-interface modes when calculating the Craig Bampton transformation matrix. The assumption is that the perturbed state, due to the uncertain parameter set, is small enough that major changes to the fixed-interface modal matrix do not occur. For REMAP1, both the constraint modes, $\boldsymbol{\Psi}$, as well as the component mass and stiffness matrices are updated to reflect the UQ parameters. The Craig Bampton transformation matrix and reduced mass and stiffness matrices are shown in Eqs. (19.5), (19.7) and (19.8), respectively. REMAP2 uses all nominal entries in the Craig Bampton transformation matrix, and only updates the component mass and stiffness matrices, as shown in Eqs. (19.7) and (19.8). Since the old modes are used to map the updated mass and stiffness matrices to the old basis, the technique is call REMAP. This is also short for *RE*duced *MA*trix *P*erturbation.

$$\mathbf{W}_{updated} = \begin{bmatrix} \boldsymbol{\Phi}_{nominal} & \boldsymbol{\Psi}_{updated} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{19.5}$$

where

$$[\boldsymbol{\Psi}]_{updated} = -[\mathbf{K_{nn}}]^{-1}_{updated}[\mathbf{K_{na}}]_{updated}. \tag{19.6}$$

The stiffness terms in Eq. (19.6) are based on the perturbed parameter sets that are generated during the UQ, as are the mass and stiffness matrices used in Eqs. (19.7) and (19.8), respectively.

$$\left[\hat{\mathbf{M}}_{\mathbf{CB}}\right]_{updated} = [\mathbf{W}]^T_{updated}[\mathbf{M}]_{updated}[\mathbf{W}]_{updated} \tag{19.7}$$

$$\left[\hat{\mathbf{K}}_{\mathbf{CB}}\right]_{updated} = [\mathbf{W}]^T_{updated}[\mathbf{K}]_{updated}[\mathbf{W}]_{updated}. \tag{19.8}$$

Since this method requires knowledge of the unreduced structure to generate the mass and stiffness matrices, it is ideal for UQ analysis within the same design agency. If the Craig-Bampton reduced order model is transferred to another design agency for implementation into a larger system model, then this technique is not applicable or requires multiple realizations to be transferred. An analysis on a prototypical structure using this technique is presented in Sect. 19.4.

19.3.2 COMP Technique

Where the REMAP technique perturbs the mass and stiffness matrices of the reduced order model, the COMP technique instead perturbs the modal matrix, and leaves the mass and stiffness matrices in their nominal condition. To perturb the modal matrix, it is pre-multiplied by a cross-orthogonal modal matrix. Hence the acronym *C*ross-*O*rthogonal *M*odal *P*erturbation. Consider if the component modal matrices were known for both the nominal state and the state that corresponds to an updated parameter set from UQ. Then, the cross-orthogonality matrix of the two modal matrices is

$$[\boldsymbol{c}_{\boldsymbol{\Phi}}] = [\boldsymbol{\Phi}]^T_{nominal}\,[\boldsymbol{M}]_{nominal}\,[\boldsymbol{\Phi}]_{updated}. \tag{19.9}$$

If both modal matrices are identical, then $[\boldsymbol{c}_{\boldsymbol{\Phi}}]$ is the identity matrix. The Craig-Bampton transformation matrix used in the COMP technique is then

$$\mathbf{W}_{updated} = \begin{bmatrix} \mathbf{c}_{\Phi}\,\boldsymbol{\Phi}_{nominal} & \boldsymbol{\Psi}_{nominal} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}. \tag{19.10}$$

For the example problem shown in Sect. 19.4, the cross-orthogonality matrix is generated with a priori knowledge of how the UQ parameter set changes the modal matrix. If the matrix is to be perturbed without this knowledge, care must be taken by the analyst to ensure realistic mode shapes are generated by the perturbation. In addition to perturbing the modal matrix, it is also possible using this technique to perturb the Eigen vector, although this is not considered in this paper.

19.4 Application of REMAP and COMP Techniques

In this section the REMAP1, REMAP2 and COMP techniques are applied to the structure shown in Fig. 19.1. The nominal parameters of the structure are given in Table 19.1. In addition to the linear beams that make up the structure, a point mass/inertia is added to the center of each ‘X’ structure. The nominal mass is 2.863 kg and the nominal inertia is

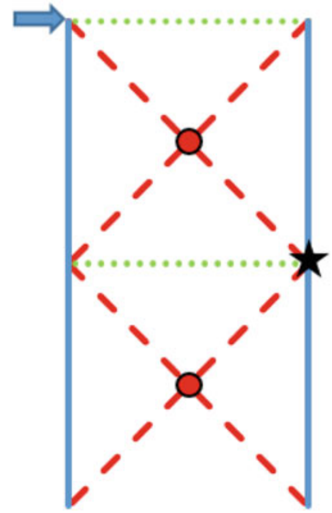

Fig. 19.1 Structure under consideration. The two 'X' shaped structures, assembled with the *red dashed lines*, are the reduced substructures. The *solid blue/green frame* is not reduced

Table 19.1 Nominal beam properties for the example structure shown in Fig. 19.1

Beam type	D (mm)	L (mm)	E (Pa)	ρ (kg/m^3)
Frame (blue/solid)	12.50	300.00	69×10^9	2700.0
Frame (green/dotted)	12.50	300.00	69×10^4	2700.0
'X' (red/dashed)	6.25	212.13	69×10^9	2700.0

0.8052 kg·m^2. Each node of the beams has two translational and one rotational degrees of freedom. The two red/dashed 'Xs' are reduced using a full Craig-Bampton reduction (i.e. full recalculation of the Craig-Bampton transformation matrix), as well as the REMAP1, REMAP2 and COMP techniques. To simulate a UQ study, each parameter listed in Table 19.1, as well as the point mass/inertia and the angle of the beams, are perturbed to form a Gaussian distribution, as shown in Fig. 19.2. This parameter perturbation only occurs on the 'X' structure being reduced. The remainder of the frame is left in its nominal state.

To conduct the simulation, a random parameter set is generated. This same parameter set is then propagated through the full, REMAP1, REMAP2 and COMP Craig-Bampton reduction techniques. Each realization is then assembled into the frame structure, from which modal information as well as frequency response functions (FRFs) are extracted. For this study, 300 unique parameter sets are analyzed. All fixed-interface modes are retained for this study and a structural damping factor of 3% is used for all post-processing. Looking first at the FRFs generated, Fig. 19.3 shows the upper and lower bounds of the FRF computed using each reduction technique. Figure 19.4 magnifies two of the FRF peaks for clarity. These figures show that the REMAP1 technique produces nearly identical bounds on the problem as the full Craig-Bampton reduction does. REMAP2 tracks the true envelope closely for lower frequencies, then tends to collapse on the COMP technique plot. This indicates that the constraint modes are important in this frequency range. The FRF bounds computed by the COMP method are collapsed nearly identically to the nominal parameter set. This indicates that the FRF is insensitive to the fixed-interface modes and instead relies mainly on the constraint modes to produce the response. The drive point FRF is shown in Fig. 19.5 and a magnification of the two regions of interest in Fig. 19.6. This figure shows that up to 450 Hz, the constraint modes dominate, between 450 Hz and 950 Hz, there is a transition to reliance on the fixed-interface modes. This is evident in the collapse of the full Craig-Bampton reduction away from the REMAP1 and REMAP2 solutions and onto the COMP solution. For this drive point FRF, the REMAP1 and REMAP2 techniques produce nearly identical bounds on the problem.

To compare the modal data, the cross-orthogonality of the modes calculated by the REMAP and COMP techniques are compared to the full Craig-Bampton reduction. Figure 19.7 shows the statistics for the diagonal terms in the cross-orthogonality matrix. Figure 19.8 shows the maximum off diagonal term in the matrix. If the modes are exactly orthogonal, then the diagonal term is unity and the off diagonal terms are zero. The red horizontal line indicates the mean value, the blue bounding box shows the 25th and 75th percentile bounds, and the end of the whiskers show the maximum and minimum values, excluding statistical outliers. These plots show that the each technique calculates certain modes properly and others not as well. Which modes it calculates correctly is problem dependent. Note that REMAP1 and REMAP2 produce nearly identical graphs, for this reason, only the REMAP1 plot it shown.

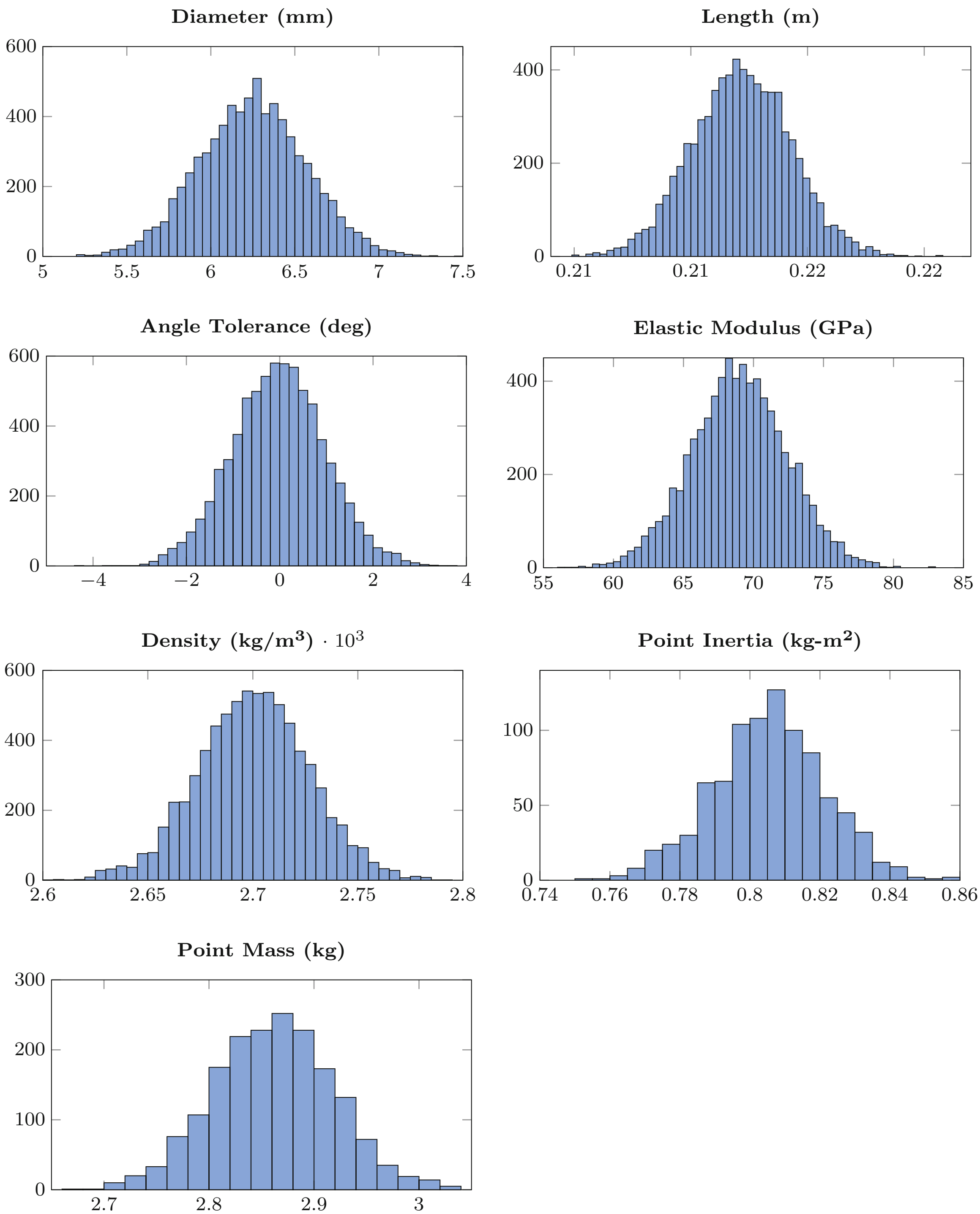

Fig. 19.2 Distribution of parameters

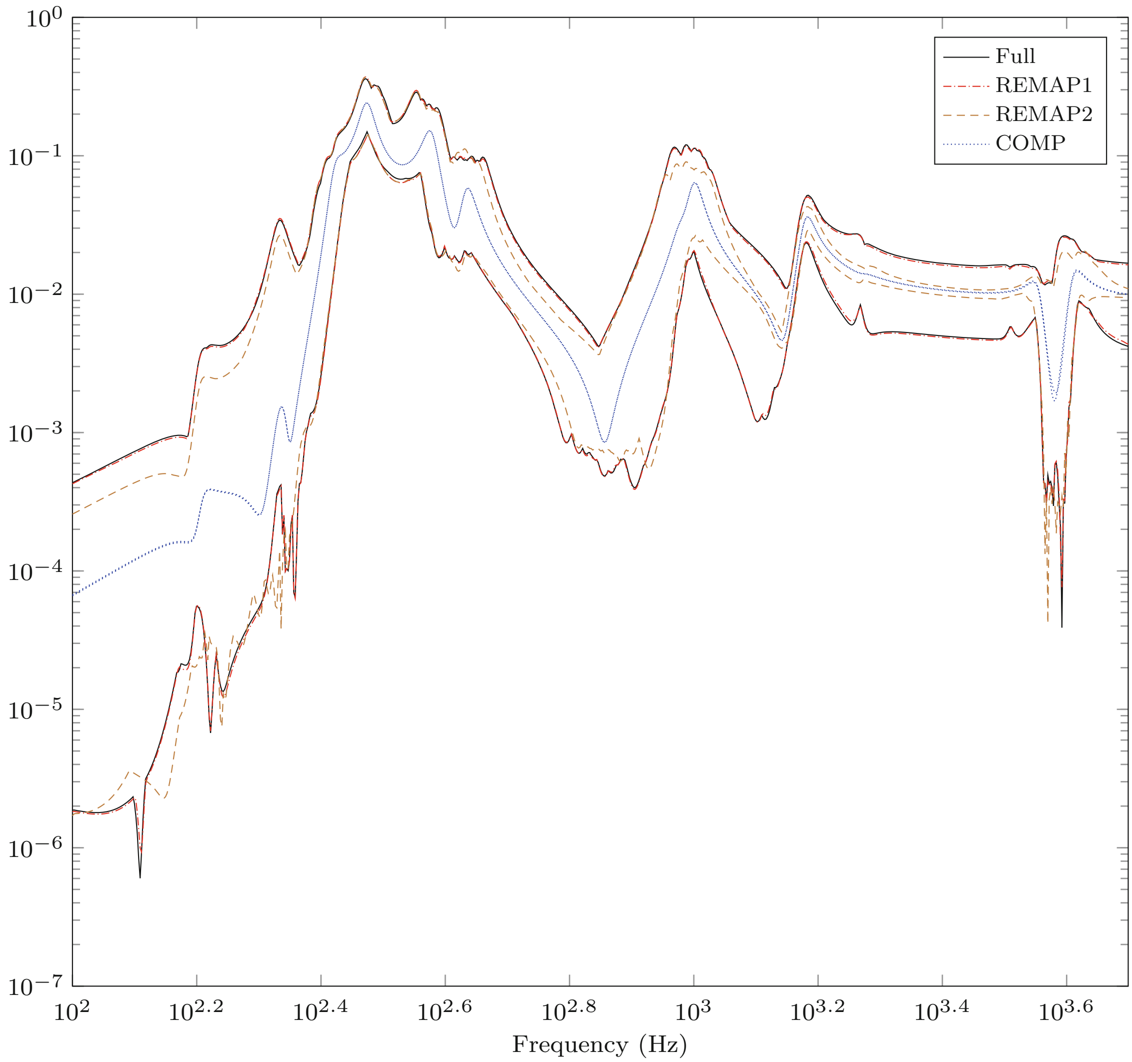

Fig. 19.3 Referencing Fig. 19.1, FRF envelopes for force in at the arrow and acceleration out at the star

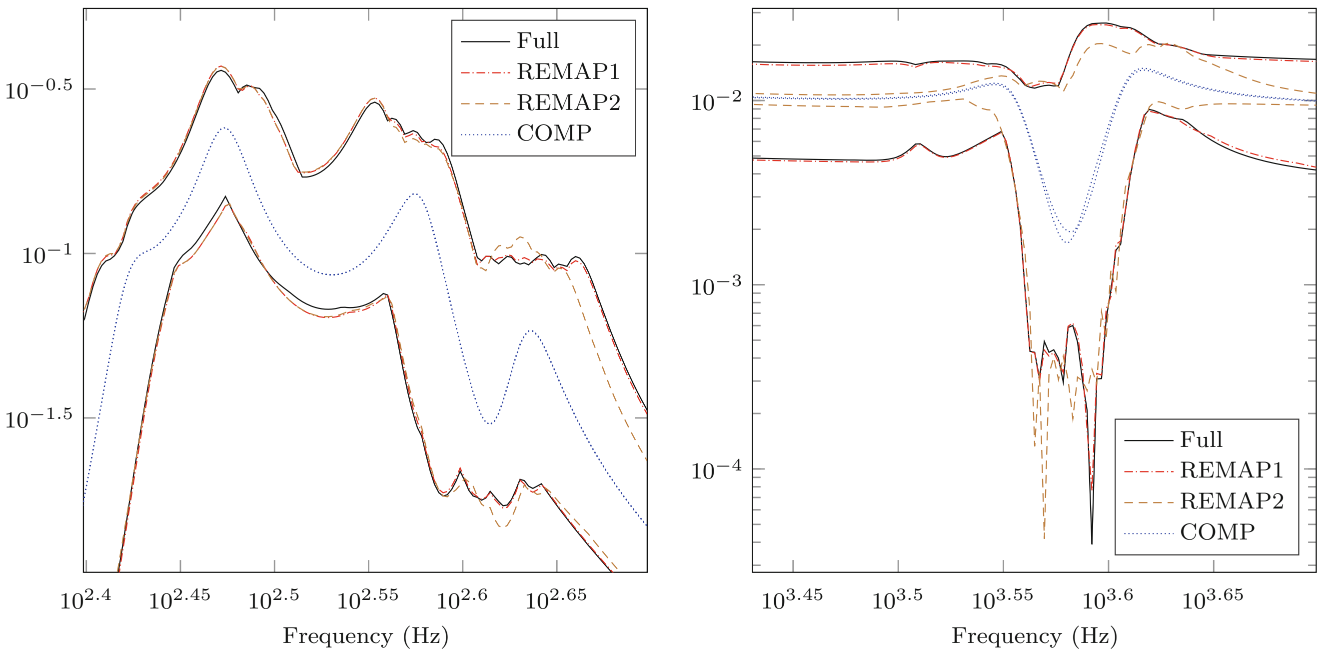

Fig. 19.4 Magnification of two regions of interest from the FRF in Fig. 19.3

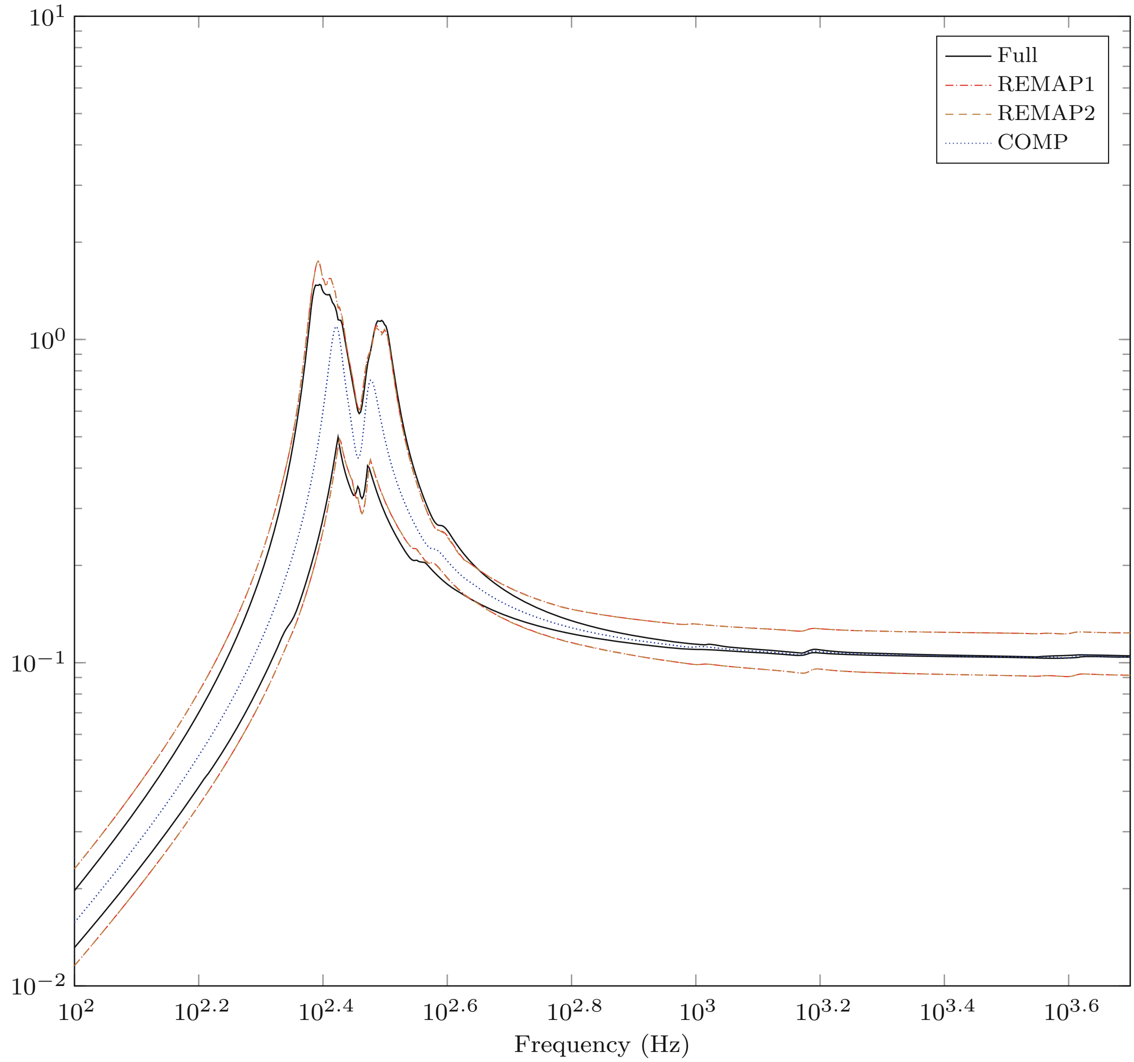

Fig. 19.5 Drive point FRF

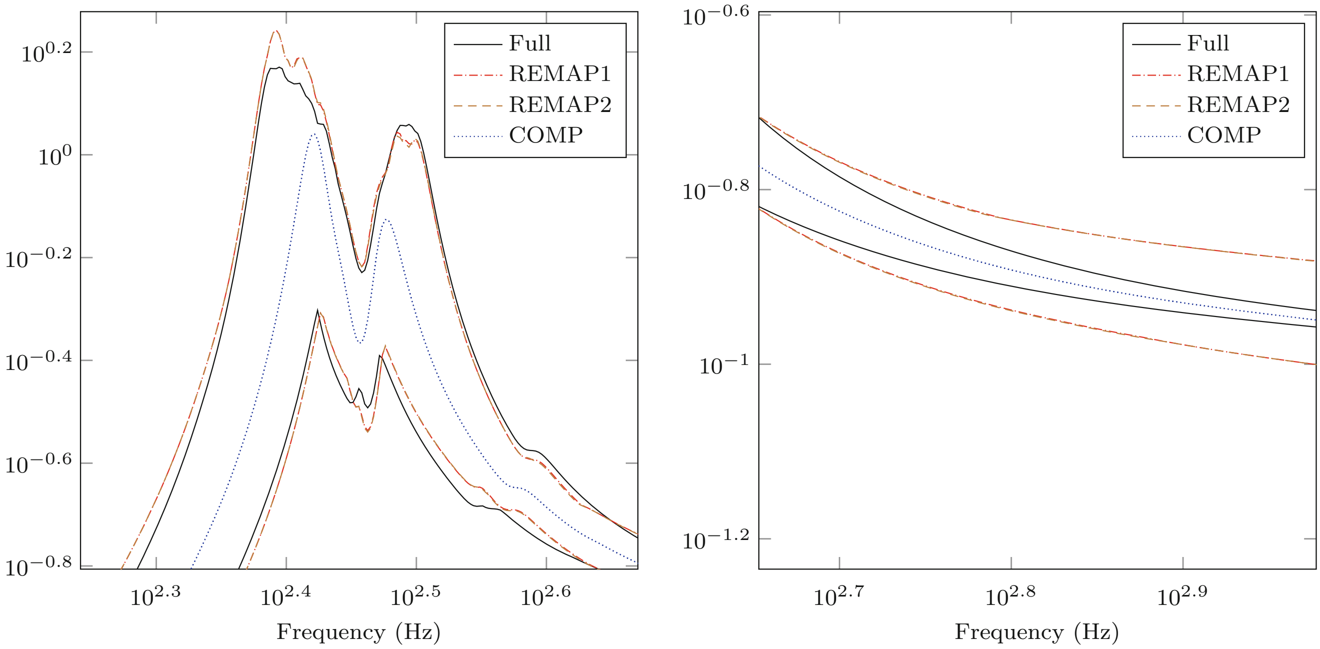

Fig. 19.6 Magnification of drive point FRF in frequency ranges of interest

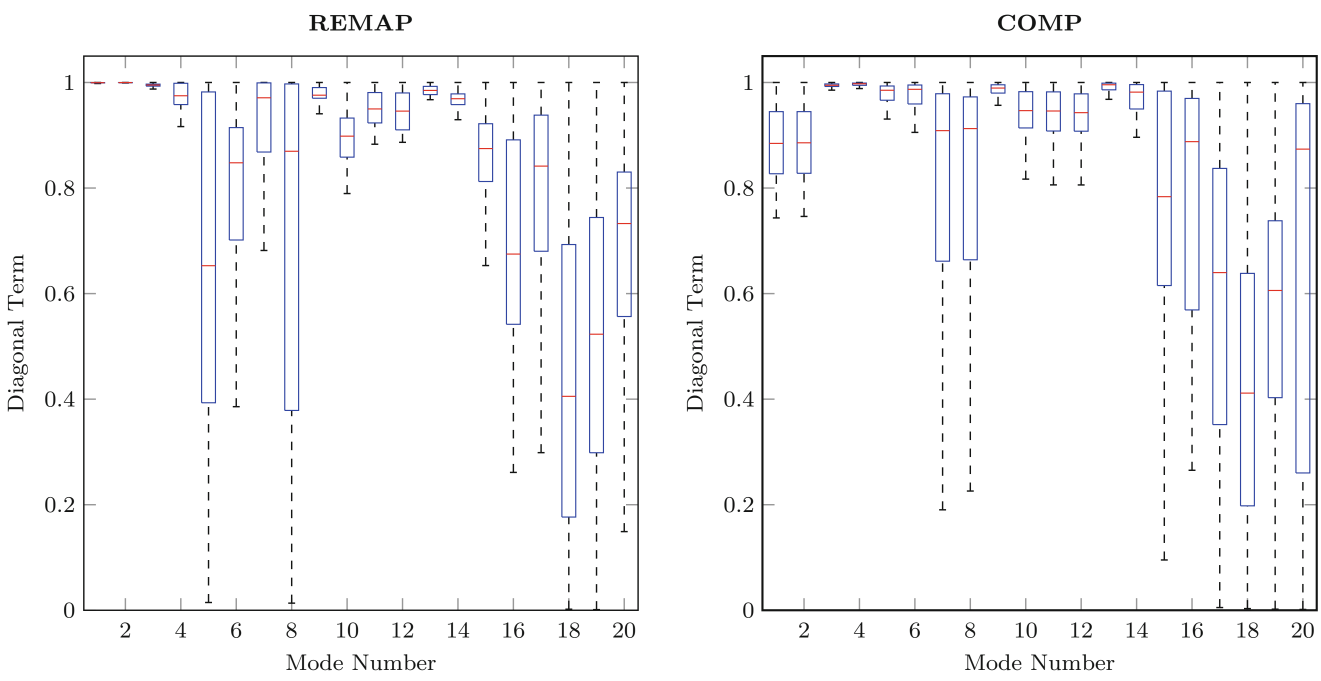

Fig. 19.7 Statistical analysis of the diagonal terms from the cross-orthogonality check between the REMAMP and COMP techniques compared to the full Craig-Bampton reduction

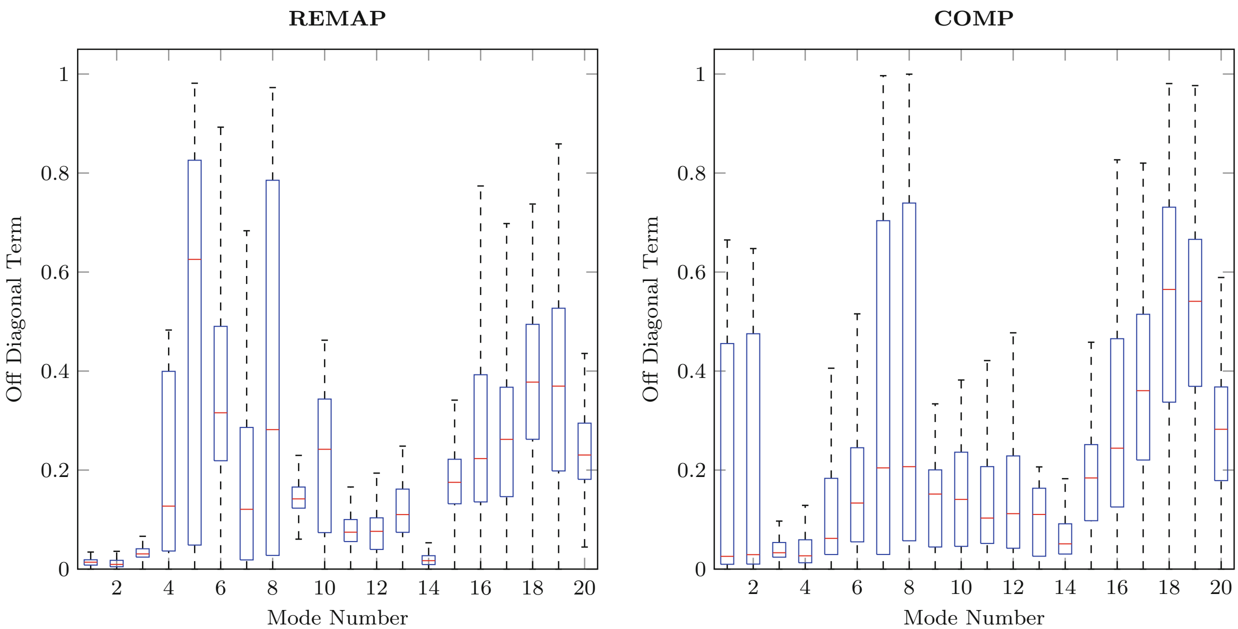

Fig. 19.8 Statistical analysis of the maximum off diagonal terms from the cross-orthogonality check between the REMAMP and COMP techniques compared to the full Craig-Bampton reduction

19.5 Conclusions

This paper presents three techniques to increase the computational efficiency when creating many Craig-Bampton reductions of the same component for UQ studies. The REMAP1 technique, which retains nominal fixed-interface modes and perturbs the constraint modes, the REMAP2 technique, which uses a fully nominal Craig-Bampton transformation matrix and only updates the component mass and stiffness matrices, and the COMP technique, which retains the nominal constraint modes and perturbs the fixed-interface modes. An example problem, which compared FRFs generated for a sample frame structure, shows that the REMAP1 technique accurately bounds the UQ problem, where as the REMAP2 and COMP techniques do not. A major concern for the COMP technique is how to properly perturb the component modal matrix such that permissible mode shapes are calculated and that also bound the UQ space. Both of these techniques require access to portions of the Craig-Bampton transformation matrix, which excludes them from being used if only the reduced mass and stiffness matrices are known. The choice of perturbing either the component or constraint modes is likely a problem specific choice that must be explored by the analyst prior to committing to a particular technique. It is suggested that a sensitivity study be carried out to determine the best technique for the system/problem of interest. Future work will explore these techniques on additional and more complex systems.

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References

1. Haldar, A., Mahadevan, S.: Probability, Reliability and Statistical Methods in Engineering Design. Wiley, Hoboken, NJ (2000)
2. Brake, M., Fike, J., Topping, S.: Parameterized reduced order models from a single mesh using hyper-dual numbers. J. Sound Vib. **371**, 370–392 (2016)
3. Hong, S., Epureanu, B., Castanier, M.: Joining of components of complex structures for improved dynamic response. J. Sound Vib. **331**, 4285–4298 (2012)
4. Hong, S., Epureanu, B., Castanier, M.: Next-generation parametric reduced-order models. Mech. Syst. Signal Process. **37**, 403–421 (2013)
5. Kammer, D., Krattiger, D.: Propagation of uncertainty in substructured spacecraft using frequency response. AIAA J. **52**, 353–361 (2013)

6. Kammer, D., Nimityongskul, S.: Propogation of uncertainty in test-analysis correlation of substructured spacecraft. J. Sound Vib. **330**, 1211–1224 (2011)
7. Momin, F., Millwater, H., Osborn, R., Enright, M.: A non-intrusive method to add finite element-based random variables to a probabilistic design code. Finite Elem. Anal. Des. **46**, 280–287 (2010)
8. Bampton, M.C., Craig Jr., R.R.: Coupling of substructures for dynamic analyses. AIAA J. **6**(7), 1313–1319 (1968)
9. Hurty, W.C.: Dynamic analysis of structural systems using component modes. AIAA J. **3**(4), 678–685 (1965)
10. Craig Jr., R.R., Kurdila, A.J.: Fundamentals of Structural Dynamics, 2nd edn. Wiley, Hoboken, NJ (2006)