# Chapter 5 Complex Systems Modeling

Abstract Modeling is a necessary mechanism for understanding complex phenomena such as the messes this book is designed to help with. This chapter compares methods available for complex systems modeling. A method is then recommended for use in addressing messes. A framework for the development and use of such a model and an accompanying simulation is then presented. This framework is demonstrated on an example problem, with an eye toward using this approach to first think about, then act on, and finally observe our mess systemically.

## 5.1 Introduction

We use models to gain understanding about complex phenomena; indeed, modeling is a "purposeful abstraction of reality" (Hester & Tolk, [2010,](#page-24-0) p. 18). Maria [\(1997](#page-24-0)) offers, "a model should be a close approximation to the real system and incorporate most of its salient features. On the other hand, it should not be so complex that it is impossible to understand and experiment with it. A good model is a judicious tradeoff between realism and simplicity" (p. 7). It is a necessary simplification of the real-world system it models.

Figure [5.1](#page-1-0) illustrates the methods available for modeling a system. If the option is available and it is feasible (i.e., it is not too dangerous, timely, or costly), we would prefer to experiment with the actual system to improve our understanding. Given the messes this book is intended to address, this is not realistic. These systems are too complex and unwieldy for full-scale experimentation to be undertaken (i.e., imagine experimenting with a nuclear missile attack or catastrophic flood in order to test potential mitigation strategies). For similar scale-driven reasons, a physical model is unobtainable for experimentation purposes. The underlying complexity and divergent perspectives associated with the associated systems make closed-form analytical solutions problematic as well. This leaves us with a simulation in order to gain understanding about our mess.

Mechanics regarding the simulation of a real-world system are not trivial. The decision to create a simulation carries with it the burden of choosing an appropriate

<span id="page-1-0"></span>

mathematical framework on which to build it. This chapter compares the methods available for complex systems modeling, it outlines the choice of a method considering mess characteristics discussed in previous chapters, and it presents a framework for developing such a model and an accompanying simulation. This framework is then demonstrated on an example problem, with an eye toward using this approach to first think about, then act on, and finally observe our mess systemically.

## 5.2 The Role of Modeling

While it may be natural to think of first observing the world before we do anything else, the reality is that all of our observations are biased (a topic we will return to in Chap. 15) and we cannot conduct true observation without first thinking about the world (i.e., developing a model). "The first step in the scientific process is not observation but the generation of a hypothesis which may then be tested critically by observations and experiments" (Banerjee, Chitnis, Jadhav, Bhawalkar, & Chaudhury, [2009](#page-23-0), p. 127). This stresses the importance of a theory (and accompanying model) before observation. Thus, we must think before we observe (as outlined in our model of systemic decision making discussed in Chap. 3). We create a hypothesis. The reason for this lies in the very nature of scientific inquiry; the goal of a scientist is not to prove a hypothesis (or a model) correct, but rather to falsify it. Even in circumstances in which we have not disproven a hypothesis, we do not say it has been proven; rather, we say it has not yet been disproven. This notion is the essence of modeling and one we will return to many times throughout this text.

As the goal of this text is to help the reader make better decisions in a complex environment, it helps us to understand the role of modeling in the systemic decision making process and the overarching purpose of modeling. When we speak of the



Fig. 5.2 Systemic decision making with modeling

process of modeling as a purposeful abstraction of reality, we are specifically referring to the process that incorporates the structuring and thinking phases of systemic decision making. Action and observation with the real system are beyond the scope of modeling as they extend beyond the conceptual world and into the real world. This notion is captured in Fig. 5.2.

Building a model and experimenting with that model help us to understand the behavior of the real-world phenomena being modeled. We model the structure of the system (i.e., its relationships) and then observe its behavior over time (i.e., we simulate it). Ultimately, we are trying to understand the response of a given system to a proposed stimulus. Complex problems are more difficult to predict and require a more concerted modeling effort due to their emergent behaviors. So, which method is appropriate for modeling a complex problem? This is the question we now turn our attention to.

### 5.3 Method Comparison

Simulations (a time-evolving realization of a model) first require an underlying model to evaluate. Thus, it is necessary to review available methods and select an appropriate modeling paradigm before developing a simulation. It is desirable to have a method that can help with both problem structuring and problem assessment to span the thinking, acting, and observing phases of mess understanding. We can draw from available methods in both the problem structuring and modeling and simulation literature. Ackermann  $(2012)$  $(2012)$  and Mingers  $(2011)$  $(2011)$  identify the most prominent problem structuring methods as soft systems methodology (SSM) (Checkland, [1999;](#page-23-0) Checkland & Scholes, [1999](#page-23-0)), strategic options development and analysis (SODA) (Eden & Ackermann, [1998\)](#page-23-0), and strategic choice approach (SCA) (Friend & Hickling, [1987;](#page-23-0) Friend & Jessop, [1977](#page-23-0)). The underlying models for each of these methodologies are described by Mingers [\(2011](#page-24-0)) as rich pictures (for SSM), cognitive mapping (for SODA), and soft decision analysis (for SCA). Hester and Tolk ([2010\)](#page-24-0) identify the most well-known general modeling paradigms as system dynamics (Forrester, [1961](#page-23-0); Sterman, [2000\)](#page-24-0), discrete event simulation (Zeigler, Praehofer, & Kim, [2000\)](#page-24-0), and agent-based simulation (Yilmaz & Ören, [2009\)](#page-24-0). These six methods are contrasted with three questions of interest for a

<span id="page-3-0"></span>comprehensive modeling technique that is applicable to both problem structuring and assessment:

- (1) Does it provide a visual representation of the scenario? Visualization is necessary in complex scenarios so that stakeholders can view a holistic articulation of a scenario and communicate across disciplinary boundaries. As the saying goes, a picture is worth 1,000 words. Complex scenarios are better understood when accompanied with graphics.
- (2) Does it support simulation? Simulation is necessary to account for emergent behavior. The inability of humans to predict emergent behavior requires a mechanism such as simulation to explore a myriad of potential scenarios in order to understand what potential outcomes may occur in a given mess.
- (3) Does it support qualitative assessment? Complex problems have both qualitative and quantitative elements. In many cases, qualitative, or soft, elements dominate problem structuring and assessment. Thus, any method chosen for modeling complex problems must be able to account for qualitative, softer elements of representation and not be strict in its mathematical requirements.

A comparison of the potential methods across these criteria is shown in Table 5.1.

Arguably, the most well-known modeling approach of those listed in Table 5.1 is system dynamics. System dynamics, however, requires significant empirical data, typically unavailable in messes for all but a few of the relevant entities of interest. Thus, system dynamics may be useful, but "since numerical data may be uncertain or hard to come by, and the formulation of a mathematical model may be difficult, costly or even impossible, then efforts to introduce knowledge on these systems should rely on natural language arguments in the absence of formal models" (Carvalho & Tome, [2000](#page-23-0), p. 407). The same criticism can be levied on agent-based simulation. Both techniques are useful for visually representing scenarios, as well as for simulating those scenarios, but they are too rigorous in their mathematical requirements to be of use for systems age messes. Discrete event simulation

Paradigm/technique	Visual representation of scenario?	<b>Supports</b> simulation?	Supports qualitative assessment?
Rich picture	<b>Yes</b>	No	<b>Yes</b>
Cognitive mapping	Yes	Yes	$Yes*$
Soft decision analysis	Yes	N <sub>0</sub>	Yes
System dynamics	Yes	Yes	N <sub>0</sub>
Discrete event modeling	N <sub>0</sub>	Yes	N <sub>0</sub>
Agent-based simulation	Yes	Yes	N <sub>0</sub>

Table 5.1 Comparison of modeling paradigms/techniques

\* Denotes fuzzy cognitive mapping

supports a simulation environment, but it does not provide a useful visual representation of a scenario, and it is also too rigorous in its mathematical specification requirements. Rich pictures and soft decision analysis are useful for representing scenarios graphically, as well as incorporating qualitative assessment of stakeholders, but they lack in their ability to support simulation. Simulation is a necessary element of any complex problem assessment, and its absence prevents whatif scenarios from being explored. This leaves cognitive mapping as the remaining method that meets all of the specified requirements. Regarding cognitive mapping, Heyer ([2004](#page-24-0)) offers the following:

Cognitive mapping, a form of influence diagram, is a technique that has been used by a variety of researchers in a variety of settings. Cognitive maps provide a holistic picture of an individual's overall perspective, without the loss of any detail; enabling researchers to move beyond the assumption of internal consistency to the detailed assessment of specific concepts within the map. For OR, this means gaining a better understanding of the clients perception of a problem which is vital for a successful OR study. In cognitive mapping, self-defined constructs represent the causal knowledge of a decision maker in the form of a map of their own subjective world. Cognitive maps can be seen as a model of action-orientated thinking about a situation where arrows signify influences in a line of argument linking cause and effect (Eden, [1992](#page-23-0)). Cognitive maps can be analysed through interpretative coding (where individual concepts are interpreted); in terms of their content (the meanings they contain); and in terms of the complexity of configuration of the maps (for example, link to node ratio, cluster analyses). (p. 9)

Cognitive mapping's entry in Table [5.1](#page-3-0) as it concerns the qualitative assessment criteria, is marked with an asterisk, however, as fuzzy cognitive mapping, a special variant of cognitive mapping (FCM), is required to fully support qualitative assessment. Bueno and Salmeron ([2009\)](#page-23-0) discuss the distinction between the two:

Cognitive maps possess, as their main limitation, the impossibility of quantifying relationships among variables. With the purpose of offering a solution to this weakness and enhancing cognitive maps, fuzzy numbers have been conjugated with cognitive maps… FCM substitute the signs (+) and (-) for a fuzzy value between  $-1$  and 1. The zero value indicates the absence of weight. (p. 5222)

Further, in direct comparison with other methods (including decision analysis and system dynamics), Özesmi and Özesmi ([2004](#page-24-0)), in Table 5.2, offer an assessment of FCM.

Method	Advantages	Disadvantages	FCM comparison
Multiattribute decision theory	Useful for ranking a finite number of alternatives with conflicting criteria; can aggregate qualitative and quantitative data	Does not allow for feedback loops; alternatives must be prespecified	FCM can suggest alternatives through exploratory analysis
System dynamics	Use differential or difference equations; dynamic models	Require significant empirical data	FCM <sub>s</sub> are not dynamic models, but they are useful for data-poor situations

Table 5.2 FCM compared to other methods

"During the past decade, FCMs played a vital role in the applications of diverse scientific areas, such as social and political sciences, engineering, information technology, robotics, expert systems, medicine, education, prediction, environment, and so on" (Papageorgiou & Salmeron, [2013](#page-24-0), p. 67). Özesmi and Özesmi  $(2004,$  $(2004,$ pp. 46–47) discuss the choice of FCM in the context of modeling preferences:

Why choose FCM over other modeling methods? To answer this question, we must consider the issues of model complexity and the reason for the model. Obviously it is important to have a model that is complex enough for the problem to be solved; however data poor situations limit model complexity. Data is costly and often not available, especially in developing countries, where conservation efforts and management are important but not resolved. The…approach…is not obtained from empirical data but can be used for modeling perception and therefore social ideas of how systems work. This is essential…where the support of many stakeholders is necessary. It is also useful for extension activities to educate stakeholders, if there are any misperceptions.

The main advantage of the multi-step FCM approach is that it is easy to build and gives qualitative results. It does not require expert knowledge in every field but can be constructed based on simple observations by anybody…It does not make quantitative predictions but rather shows what will happen to the system in simulations under given conditions of relationships. The model provides a better summary of relationships between variables instead of articulating how that relationship is in detail.

With FCMs the strengths and signs of relationships can be easily changed and simulations run easily and quickly. Thus, they are ideal tools for theory development, hypothesis formation, and data evaluation. However, FCMs are not substitutes for statistical techniques; they do not provide real-value parameter estimations or inferential statistical tests.

Jetter ([2006,](#page-24-0) p. 511) further discusses the appropriate use of FCMs:

Adoption of FCMs can furthermore be improved through a better choice of applications: In the past, FCMs have been used for all kinds of problems and in some cases, the reason for choosing FCMs over other modeling techniques (e.g. System Dynamics or Bayesian networks) is all but clear. Future FCM research should focus on problems that FCMs are "good at": they are a powerful means to represent knowledge domains that are characterized by high complexity, by widespread knowledge sources that usually only have partial knowledge, by qualitative information that frequently changes, and by a lack of a commonly accepted "theory" or "truth". They can thus be useful for the analysis of business ecosystems, scenario planning, and the forecasting of market or technology trends and should be increasing applied in these areas.

Like many other models, e.g. System Dynamics models, they can help decision-makers to reflect upon their worldviews and to improve their understanding of the dynamic systems and decision alternative they encounter. Unlike these models, they can handle qualitative concepts with no dimensions and linguistic imprecision and so (relatively) simple to understand that they allow for a strong involvement of the decision-maker in modeling, simulation and interpretation of results.

Amer, Jetter, and Daim ([2011\)](#page-23-0) have additional comments about the utility of fuzzy cognitive mapping:

Cognitive maps are mainly used to analyze and aid the decision-making process by investigating causal links among relevant concepts…The mapping process fosters system thinking and allows experts to better assess their own mental models…The visual nature of

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concept maps facilitates understanding of existing dependencies and contingencies between various concepts. (p. 567)

Additionally, FCMs are scalable, in terms of the maps themselves and the number of participants. "With FCMs you can have as many knowledge sources as wanted with diverse knowledge and different degrees of expertise. These knowledge sources can all be easily combined into one FCM. There is no restriction on the number of experts or on the number of concepts" (Özesmi  $\&$  Özesmi, [2004](#page-24-0), p. 45).

Given its advantages over alternative methods, fuzzy cognitive mapping is advised as a method for modeling and accompanying simulation for messes and their constituent problems. This method is used as part of a larger multimethodology which will incorporate numerous other techniques for populating a cognitive map, discussed in subsequent chapters. We now turn to details regarding the use of fuzzy cognitive maps.

## 5.4 Fuzzy Cognitive Mapping

Fuzzy cognitive mapping was introduced by Kosko ([1986\)](#page-24-0), based on the foundational causal map work of Axelrod ([1976\)](#page-23-0) as a way to visually and logically capture the relationships between elements in a problem. Fuzzy cognitive maps (FCMs) are network-based collections of concepts (represented as nodes) and causal relationships (represented as arcs between the concepts). Arcs have weights that indicate both the strength and direction of a causal relationship; thus, a given relationship can be increasing or decreasing (i.e., A increases B or A decreases B). Arc weights are typically defined on [−1,1] to represent direction (positive weights are reinforcing; negative are decreasing) and magnitude of influence (a weight of one means complete influence of one concept over another, whereas a weight of zero indicates no connection between two concepts).

Concepts can represent variables in a system (see, e.g., Tsadiras, [2008](#page-24-0), pp. 3881–3882), with concept values defined on [0,1] as their value relative to the defined range for the given variable (i.e., 50% for a valve status means 50% open). Concepts can also represent system performance measures (see, e.g., Tsadiras, [2008,](#page-24-0) pp. 3882–3883), where causal relationships show the effect of increasing or decreasing a given performance measure on others. Carvalho [\(2013](#page-23-0)) elaborates on the meaning of a concept:

Each concept represents the actors, entities and social, political, economic or abstract concepts that compose the system. Examples of concepts might be Inflation, the actions of an influent Politic, a Revolution, the Wealth of an individual or a nation, the Welfare of population, Road conditions, etc. Each concept is characterized by a value usually ranging from  $[0...1]$  or  $[-1...$ 1] representing a normalized transformation from its real world value. (p. 8)

Mathematically, the concepts within a FCM can be represented by a matrix  $C$ , the relative activation level of each concept can be represented by the matrix  $A$ , and

#### Fig. 5.3 Example FCM





W can represent the matrix of weights where each element  $w_{ij}$  represents the influence of concept  $i$  on  $j$ . An example of a three concept FCM is shown in Fig. 5.3.

In this example, there is positive causality from B to both A and C, and negative causality from C to both A and B. This same example is presented as an adjacency matrix in Table 5.3. Note that the row element imparts a causality on the column element (e.g., B imparts a positive causal relationship to A).

"An FCM can be considered as a type of recurrent artificial neural network" (Tsadiras, [2008](#page-24-0), p. 3884). Thus, FCMs evolve over time (i.e., are dynamic) and can be analyzed relative to this evolution. We can use the matrix form of the example FCM (shown in Table 5.3) to study the evolution of this problem over time. Time step  $t + 1$  can be evaluated using information from the previous time step,  $t$ , as shown in Eq.  $5.1$ :

$$
\mathbf{A}^{t+1} = f(\mathbf{A}^t \mathbf{W}) \tag{5.1}
$$

where  $f$  is known as a transfer function used to evolve the FCM from one time stamp to the next. This transfer function typically takes on one of the following three forms (Tsadiras, [2008](#page-24-0)): (1) binary, (2) trivalent, or (3) sigmoid.

The binary function is shown in Eq. 5.2:

$$
f_{\text{bi}}(\mathbf{x}) = \begin{cases} 1, & x > 0, \\ 0, & x \le 0. \end{cases} \tag{5.2}
$$

This creates a two-state FCM. When the activation level of a concept,  $C_i$ , is 1, the concept is said to be activated (i.e., on), whereas a value of 0 indicates the concept is not activated (i.e., off).

The trivalent function is shown in Eq. 5.3:

$$
f_{\text{tri}}(\mathbf{x}) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \tag{5.3}
$$

This creates a three-state FCM. When the activation level of concept  $C_i$  equals 1, the concept is increasing, when the activation level equals −1, the concept is decreasing, and when the activation level equals 0, there is no change in the concept.

Finally, there is a sigmoid function in Eq. 5.4, with limits of  $[-1,1]$ :

$$
f_{\rm sig}(\mathbf{x}) = \tanh(\lambda x) = \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\lambda x} + e^{-\lambda x}} \tag{5.4}
$$

The activation level of a given concept can take any value over [−1,1]. Thus, a continuous FCM is created. In this formulation,  $\lambda$  represents a tuning parameter, "a constant value that indicates the function slope (degree of normalization) of the sigmoid functions. Each FCM designer specifies the value of  $\lambda$ " (Bueno & Salmeron, [2009,](#page-23-0) p. 5223). As  $\lambda$  increases to 10 or more, it approximates a trivalent function. For a small value of  $\lambda$  (e.g., 1 or 2), the function is near linear. Bueno and Salmeron [\(2009](#page-23-0)) advocate the use of a  $\lambda$  value of 5.

Tsadiras ([2008\)](#page-24-0) provides additional details on each of the three transfer functions as well as the following guidance on the appropriateness and implementation of each:

- (1) Binary FCMs are suitable for highly qualitative problems where only representation of increase or stability of a concept is required.
- (2) Trivalent FCMs are suitable for qualitative problems where representation of increase, decrease, or stability of a concept is required.
- (3) Sigmoid FCMs are suitable for qualitative and quantitative problems where representation of a degree of increase, a degree of decrease, or stability of a concept is required and strategic planning scenarios are going to be introduced (p. 3894).

Additional guidance on transfer function choice is found in Bueno and Salmeron [\(2009](#page-23-0)). As a general rule, for all but the simplest of problems, binary FCMs do not offer a great deal of insight (although, as it will be shown later in this chapter, they are useful for FCM validation purposes). Trivalent and sigmoid FCMs are more useful. Sigmoid functions yield more detailed results but require more detailed input (i.e., nonbinary weights). Sigmoid functions were observed empirically by Bueno and Salmeron [\(2009](#page-23-0)) to be:

A useful tool for decisional processes…the sigmoid function can be considered an excellent decision support tool within any scope. In a complex decisional environment the sigmoid function offers the possibility of attaining easily comparative analyses between scenarios that define decisional situations. These scenarios allow the obtaining of a future vision of the more suitable alternatives, and, therefore, the reaching of successful decisions. Therefore, the sigmoid function can define decisional scenarios for decision-makers of any decisional environment.

Nevertheless, it has some disadvantages if it is compared with the other functions analyzed. First, decision-makers need an extensive number of interactions with the sigmoid function to reach the stable scenario, whereas with the other functions few interactions are needed. (p. 5228)

Thus, the utility of each function is dependent on the scenario and the desired output to be gained from the use of an FCM. It is worth making a few comments regarding the potential end states of each method. A binary FCM composed of n concepts has  $2^n$  potential end states, representing each of the possible final configurations each FCM may exhibit. A trivalent FCM has  $3<sup>n</sup>$  potential end states, and a sigmoid function, with continuous variables, has an infinite number of final states.

Behavior of a FCM is dynamic, but deterministic. Dynamic behavior means that we must study the evolution of an FCM over time to gain the most meaningful insights. "Such systems are composed of a number of dynamic qualitative concepts interrelated in complex ways, usually including feedback links that propagate influences in complicated chains, that make reaching conclusions by simple structural analysis an utterly impossible task" (Carvalho, [2013,](#page-23-0) p. 6). Indeed, it is in this dynamic behavior where the true insight from a FCM lies:

…one might question what is the added value of FCM in what concerns causality. The answer is quite simple: it resides in the fact that even if the direct relations between concepts is certain, the propagation of the effects through time is hard to understand or predict due to the feedback loops. Hence, in FCM one jumps the focus from the problem of finding and representing causality, to the problem of system dynamics and scenario simulation. (Carvalho, [2013](#page-23-0), p. 11)

"System behavior is a function of both the system itself (which provides the internal structure that can allow for complex behaviors) and the environment (i.e., initial conditions) that the system is placed in" (Hester, [2016\)](#page-24-0). A given system can exhibit simple, complicated, complex, and chaotic behavior when initialized with a different initial state vector,  $A^0$ . This initial state vector can either represent the stakeholders' estimates of the values of the current system concepts or be based on a specific what-if scenario that we wish to explore. "After that, the concepts are free to interact. The activation level of each concept influences the other concepts according to the weight connections that exist between them" (Tsadiras, [2008](#page-24-0), p. 3885).

Then, one of the following end states is reached as discussed by Hester ([2016\)](#page-24-0):

(1) Equilibrium is reached. Equilibrium is defined as where  $A^{t+1} = A^t$  for all concepts. This behavior can be described as complicated. It is dynamic and more difficult to predict than simple behavior, which is a special case of equilibrium where  $A^t = A^1 = A^0$ . In other words, simple behavior is static and

does not change after an initial disturbance. It is obvious and easily predicted by an analyst.

- (2) Cyclic behavior is reached. This is described as complex behavior. This is defined as where  $A^{t+ \Delta t} = A^t$  for all concepts. "The system exhibits a periodic behavior where after a certain number of time steps, that is equal to the period  $[\Delta t]$  of the system, the system reaches the same state" (Tsadiras, [2008](#page-24-0), p. 3885). It is dynamic and more difficult to predict than complicated behavior.
- (3) The system exhibits chaotic behavior. In this case, there is no equilibrium reached and the scenario never demonstrates periodic behavior. By its very definition, truly chaotic behavior is not predictable.

The deterministic nature of an FCM is also important to note. Determinism means that for a given initial state vector,  $A^0$ , the FCM will always produce the same end state. If a FCM enters a state it has encountered previously, "the system will enter a closed orbit which will always repeat itself" (Tsadiras, [2008,](#page-24-0) p. 3885). Given the finite number of end states in both binary and trivalent cases, they cannot exhibit chaotic behavior, but only either reach equilibrium or exhibit cyclic behavior with a periodicity of at most  $2<sup>n</sup>$  states (in the case of binary functions) or  $3<sup>n</sup>$  states (in the case of trivalent functions). Continuous FCMs, however, due to their infinite number of potential end states, can exhibit chaotic behavior. Both the equilibrium point and the limit cycle behavior reveal hidden patterns encoded in the FCM (Kosko, [1988](#page-24-0)). Tsadiras [\(2008](#page-24-0)) suggests that encoding these patterns in the underlying structure of an FCM remains an open research question.

It is worth noting, finally, that given their deterministic and finite nature, the end states of binary and trivalent functions could be exhaustively enumerated, certainly with the assistance of a computer algorithm. This enumeration may lead to additional insight regarding the behaviors of the underlying system. Sigmoid functions are unable to enumerate completely, due to their continuous nature, but they can be probabilistically approximated.

## 5.5 A Framework for FCM Development

While there may be multiple manners in which a FCM can be constructed, tested, and utilized, the authors advocate the use the following six-step framework developed by Jetter and Kok [\(2014](#page-24-0)) for its simplicity and ease of deployment:

- (1) Clarification of project objectives and information needs (Step 1),
- (2) Plans for knowledge elicitation (Step 2),
- (3) Knowledge capture (Step 3),
- (4) FCM calibration (Step 4) and testing (Step 5), and
- (5) Model use and interpretation (Step 6).

This general framework provides a straightforward and robust approach to utilizing FCM to increase the understanding of a mess. The following subsections detail the mechanics regarding each of these steps.

## 5.5.1 Step 1: Clarification of Project Objectives and Information Needs

Step 1 is, fundamentally, a problem structuring effort. What is the purpose of the model you are constructing? A model is built with a purpose, and its construction and use should reflect this purpose. Sterman [\(2000](#page-24-0)) offers the following insights regarding problem articulation as it relates to model formulation:

The most important step in modeling is problem articulation. What is the issue the clients are most concerned with? What problem are they trying to address? What is the real problem, not just the symptom of difficulty? What is the purpose of the model?…Beware the analyst who proposes to model an entire business or social system rather than a problem. Every model is a representation of a system-a group of functionally interrelated elements forming a complex whole. But for a model to be useful, it must address a specific problem and must simplify rather than attempt to mirror an entire system in detail. (p. 89)

More succinctly, he offers, "Always model a problem. Never model a system" (Sterman, [2000](#page-24-0), p. 90). The intent of his remarks are to clarify what was said at the outset of the chapter regarding models as a "purposeful abstraction of reality." Models exist for a specific reason and that reason is to explore a particular problem in a manner that is a necessary simplification of the real-world system it is representing.

Timing is an important element to consider when examining the purpose of a FCM. What is the intended model time frame (i.e., 6 months, 10 years)? Carvalho [\(2013](#page-23-0)) discusses the importance of time:

It is important to notice that "time" should be considered essential when modeling a FCM, since the rate of change on a social system (or in fact in most real world systems) cannot be infinite; i.e., when simulating a FCM, one cannot assume that the value of a concept can change from its minimum to its maximum value on a single iteration unless this iteration represents a large enough amount of time. (pp. 8–9)

Carvalho and Tome ([2000\)](#page-23-0) provide further guidance on the selection of a time interval for a FCM:

It is important to choose a base time interval (btime) to represent each iteration (1 day, 2 days, 1 week, 1 month, etc.). When defining the relations, btime must always be implicitly present. The rules that represent causal effects are tightly dependent on btime: If btime is 1 day, then rules expressing the effect of a Level in Inflation would most certainly indicate a very small change. If however btime is 1 year then the rules would have to indicate a larger variation. Btime is obviously dependent on the system we are representing and on the time gap we are hoping to analyze. However smaller btimes usually need more detailed and complex rule bases. (p. 411)

Proper problem articulation provides a purpose for each FCM. The intent of a problem-level FCM is to address the stated problem, whereas the intent of a messlevel FCM is to model the interaction effects among the problems in a holistic manner.

### 5.5.2 Step 2: Plans for Knowledge Elicitation

Since FCMs are primarily qualitative in nature, the knowledge used to construct them comes from expert elicitation. There are three options for this step (Jetter  $\&$ Kok, [2014\)](#page-24-0):

- 1. The modeler is the expert. This is the case with most academic literature but may be unrealistic for a real-world scenario, where multiple stakeholders are necessary. While this may be a necessary approach due to a lack of expert availability, it is advised against due to its reliance on a unitary perspective.
- 2. The modeler surveys experts. This can be done either on an individual basis or in a group setting. This is the preferred method of knowledge elicitation when available. Although it requires stakeholder commitment to complete, it will result in a representation of the problem that has the greatest degree of buy-in as the stakeholders were an integral part of its development. Time or resource constraints may prevent this method from being deployed.
- 3. The modeler analyzes documents. This involves the use of content analysis to infer cognitive maps from relevant documentation, i.e., scientific publications, technical reports, newspaper articles, and textbooks. This is the second-best option in the absence of the expert availability.

Additionally, these methods can be combined. For example, the modeler can develop an initial articulation of the problem and then present it to interested stakeholders for refinement. This can, if done correctly, reduce the required stakeholder involvement as it eliminates the need for all stakeholders to begin from a blank slate. It can also, however, bias them toward a singular representation of the problem, which may be problematic.

It is worth noting that the plan for knowledge elicitation may influence problem articulation. For example, a group of experts should be surveyed regarding their agreement with project objectives from Step 1. Failure to do so puts individuals at risk of committing the Type III error that was described in Chap. 1.

## 5.5.3 Step 3: Knowledge Capture

Once the problem has been agreed upon and the participants selected, the process of knowledge capture can begin. This can be either face to face or via written

instruction. Coached map development may lead to imposition of facilitator worldviews, but may also preclude questions from experts. At a minimum, experts should be reminded that the maps they are creating are *causal* maps and not correlation maps. Thus, in an effort to avoid the Type VI error of an unsubstantiated inference (described in Chap. 1), experts should not overly prescribe causality among concepts. Asking the simple question, *does a change in A cause a change in* B to occur?, will assist in the development of appropriate causal linkages.

Information can be collected individually or in a group setting. Group elicitation may lead to groupthink, but individuals may also benefit from the presence of one another. "Group cognitive mapping furthermore has practical advantages: it requires fewer contact hours between interviewers and respondents and directly results in the integrated map needed for most FCMs…To balance the advantages and drawbacks of individual cognitive mapping and group meetings approaches can be mixed" (Jetter & Kok, [2014,](#page-24-0) pp. 50–51).

Combination of individual maps can be done in a straightforward manner by computing the average map (adding all stakeholder maps together and dividing by the number of stakeholders) (Kosko, [1988\)](#page-24-0). Additionally, credibility weights can be assigned to experts (Taber, [1991](#page-24-0); Taber & Siegel, [1987](#page-24-0)), or more advanced combination methods can be invoked (Kosko, [1988;](#page-24-0) Özesmi & Özesmi, [2004\)](#page-24-0). "In participatory studies, however, that equally value the input of all respondents, it is not applied" (Jetter & Kok, [2014,](#page-24-0) p. 51). Given the focus in systemic decision making on the value of complementarity, it is advantageous that all opinions are equally considered. It is worth noting, however, that we do not advocate consensus as a necessary requirement for FCM development. Consensus is defined by Susskind [\(1999](#page-24-0), p. 6) as follows:

… reached when everyone agrees they can live with whatever is proposed after effort has been made to meet the interests of all stakeholding parties….Participants in a consensus building process have both the right to expect that no one will ask them to undermine their interests and the responsibility to propose solutions that will meet everyone else's interests as well as their own.

Consensus is an unnecessarily restrictive requirement and problematic to achieve. Bueno and Salmeron ([2009\)](#page-23-0) elaborate on the topic within the framework of FCM development:

The difficulty is in reaching a consented value not only for the causal weight but for the sign between the cognitive map relationships as well. This measure will be distinct with respect to the experts who assign the fuzzy numbers to each of the relationships. (p. 5222)

Integration of maps requires that concept names and definitions are standardized. "Most workshops devote about half of the total time on finalizing a commonly agreed upon list of concepts" (Jetter & Kok,  $2014$ , p. 51). It should be recognized that some details may be lost to aggregation. For example, if two respondents assign equal weight to the same connection, one with  $a + sign$  and one with  $a - sign$ , the two will cancel one another out, thereby losing the important detail that both

Approach	Facilitator aggregation of individual models	Group FCM construction
<b>FCM</b> appropriation	Capturing knowledge	Stakeholder learning
Emphasis	Models as "artifacts for decision" making or enhancing systemic understanding"	"Model and modeling process as a tool for social learning"
Research purpose	"Combining representations of stakeholder/expert mental models to (1) reduce uncertainty about a system and (2) compare mental models across groups"	"Community generated" representation of knowledge used for planning and learning, often through participatory scenario development"

Table 5.4 Facilitator FCM aggregation versus group FCM construction

participants felt a connection was present. "Given the limited state-of-the-art, mathematical aggregation of individual maps, modeler-generated integration of all maps, and group generated cognitive maps all seem viable approaches to pooling the knowledge of individual respondents" (Jetter & Kok, [2014,](#page-24-0) p. 51). It may not always be advantageous to combine the perspectives, i.e., if the aim is to inform discussion. Gray, Zanre, and Gray [\(2014](#page-23-0), p. 42) elaborate on the differences between facilitator aggregation and group FCM construction in Table 5.4.

Once concepts are agreed upon (if desired), then linkages between concepts are identified. Respondents then add signs  $(+ or -)$  to indicate the direction of causality. Once direction is specified, magnitude of weight can also be assigned. This can use a qualitative scale such as the Likert ([1932\)](#page-24-0)-type scale as shown in Table 5.5 to indicate the strength of connection, a simple positive or negative weight, or respondents can assign numerical weights in the range of  $[-1,1]$ . It is worth noting that FCM values are taken to be relative to one another, rather than absolute, so it is advisable that respondents use linguistic scales to avoid subconsciously quantifying relationships.

These maps are then translated into adjacency matrices for computation as FCMs. Several adjustments may need to be made (Jetter & Kok, [2014](#page-24-0)):



Table 5.5 Sample weight

- Elimination of causal links that exist only for definitional and not causal purposes.
- Removal of concepts with no "Out"-arrows, unless they are a target concept, that is, one that is a focus of the analysis. These links typically indicate poor map construction.
- Indication of conditional causality using activation function thresholds or using a nested FCM.
- Synchronization of time steps using intermediate concepts to break up long-term concepts. This may lead to redefinition of concepts.

This stage, as well as subsequent ones, may be aided by the use of software packages. Mental Modeler (Gray, Gray, Cox, & Henly-Shepard, [2013\)](#page-23-0) and FCMapper (www.fcmappers.net) are two packages designed specifically for FCM modeling and assessment, although the authors advocate the use of Mental Modeler due to its graphical capabilities and ease of use.

## 5.5.4 Step 4: FCM Calibration and Step 5: Testing (Step 5)

Once a FCM has been constructed, it is necessary to calibrate it. It is extremely important to note that the goal of FCM development is not to create an objectively "correct" model, "but a useful and formal description of the perception of a group of people, such as subject matter experts or stakeholders, of the problem at hand" (Jetter & Kok,  $2014$ , p. 54). In the absence of historical data against which to benchmark a model, three steps should be taken to calibrate and test it (Jetter  $\&$ Kok, [2014\)](#page-24-0):

- (1) Calibrate the FCM with binary nodes and trivalent edges (Kosko, [1988\)](#page-24-0). Test it against known or expected cases or generally understood phenomena. If the FCM does not perform as expected, it should be investigated for mistakes.
- (2) Once it is calibrated, more sophisticated causal weights and activation functions can be used to gain more insight. Remember that FCMs are constructed to represent scenarios where there is a lack of concrete information, so trying to overspecify them may defeat their purpose.
- (3) The model can then be tested against more complex scenarios. Disagreements may arise due to a dichotomy between expected and observed behaviors. This is an opportunity for insight into the expert's mental models and assumptions, or an opportunity to revise the model if necessary. The key is to have a discussion with experts as to the meaning of the results. Additional model tests can be undertaken to assess their validity.

It is noted that "FCM researchers have few approaches, other than trial-and-error, to know if their model will reach a stable state, how many stable states it has, how to select [transfer] functions, and how to deal with temporal aspects" (Jetter & Kok, [2014](#page-24-0), p. 56). Sterman ([2000\)](#page-24-0) elaborates on this idea, specifically as it pertains to model validation:

The word validation should be struck from the vocabulary of modelers. All models are wrong, so no models are valid or verifiable in the sense of establishing their truth. The question facing clients and modelers is never whether a model is true but whether it is useful. The choice is never whether to use a model. The only choice is which model to use. Selecting the most appropriate model is always a value judgment to be made by reference to the purpose. Without a clear understanding of the purpose for which the model is to be used, it is impossible to determine whether you should use it as a basis for action. (p. 890)

The goal, then, is to determine whether or not the perception is that the model is acting as expected. This, ultimately, is a subjective evaluation, as the model inputs are subjective and it is very qualitative in nature.

## 5.5.5 Step 6: Model Use and Interpretation

The model should spark debate and discussion and lead to further narratives being developed regarding expected system behaviors. Given the explored scenarios, they could also feed into decisions about future actions. Given the results of actions, models may be revised as appropriate. The notion of using FCMs as a decision support mechanism will be explored later in the book in Part III.

At a minimum, the model can be used to explore speculative, what-if scenarios. An initial scenario can be suggested, and the equilibrium resulting from this initial scenario can be explored. This may take the form, for example, of a temporary rise of a concept such as S&P 500 Value in a macroeconomic model of the economy. The interest of exploring such an initial perturbation may be to investigate what effects this change would have on the overall economy. This may lead to insight regarding emergent behavior and unexpected model outputs. Alternatively, a concept may be clamped or permanently set to a particular value. This may represent a scenario such as a permanent policy change. "The calculation is slightly different, if activation of concept  $C_1$  is not a one-time impulse (e.g. an election, a natural disaster), but a change that lasts over extended periods of time (e.g. new tax laws). In this case, the concept is 'clamped' and always set back to its initial activation level…" (Jetter & Schweinfort, [2011,](#page-24-0) p. 55). Either scenario (an initial perturbation or a clamped variable), or a combination of them, will yield insight into the model's behaviors.

## <span id="page-17-0"></span>5.6 Example FCM Application

This example expands on the one developed in Hester, Akpinar-Elci, Shaeffer, and Shaeffer [\(2016](#page-24-0)). It centers on examination of the Ebola virus disease. The identified problem is how to reduce the incidence of the Ebola virus disease, viewed through a global health perspective. The developed FCM included economic, health care, and political ramifications, among others. Although this example focuses on the Ebola virus disease, the specific disease is not important; rather, the general case of communicable diseases is the intent of analysis. For contextual understanding, we may define communicable diseases and those that can be transmitted "from person to person by either direct or indirect methods. Direct transmission is either by direct physical contact or by means of droplet spread, such as by coughing or sneezing. Indirect transmission of an infectious agent is accomplished by some intermediary mechanism, such as transmission in contaminated water or by means of insects. A few communicable diseases are primarily diseases of animals and are transmitted to humans only incidentally" (Crowley, [2007,](#page-23-0) p. 149). Figure 5.4 shows a depiction of the Ebola virus disease mess.

Given that the focus of analysis is to determine ways in which we may reduce the incidence rate of Ebola virus disease, this becomes the central focus of the scenarios that we may choose to explore. For an initial inquiry, it may be worthwhile to simply explore the question, What if we could reduce the Incidence rate of Ebola virus disease? Thus, absent a mechanism for doing so, we wish to explore what the effects would be on the overall FCM if we initially set the concept



Fig. 5.4 Ebola virus disease depiction (adapted from Hester et al., [2016](#page-24-0))



Fig. 5.5 Effect of initial reduction in incidence rate of Ebola

Incidence rate of Ebola virus disease to  $-1$  (understanding that if we were able to take such an action, we would not have a mess to begin with). This result is shown in Fig. 5.5. This scenario exhibits complex behavior. Despite initially reducing the concept, Incidence rate of Ebola exhibits periodic behavior, oscillating between concept reduction and no change. This tells us that an initial reduction of the incidence rate of Ebola would not be a sustainable solution and would require additional intervention for longevity.

The next experiment results from a qualitative investigation of Fig. [5.4](#page-17-0). In it, the World Health Organization's involvement appears to be a driving concept, with many other concepts linked to it. Practically, influence of this concept also seems achievable as getting the World Health Organization (WHO) more involved simply requires their buy-in and minimal resources to be committed, which is much simpler than, for example, improving *Vaccine efficacy* or *Clean water availability*. Figure [5.6](#page-19-0) shows the effects on Incidence rate of Ebola virus disease from initially setting World Health Organization's involvement to  $+1$ .

This scenario appears to exhibit complicated behavior. While the results of Fig. [5.6](#page-19-0) appear promising, an examination of the effects of this scenario on all of

<span id="page-19-0"></span>

Fig. 5.6 Effect of initial reduction in World Health Organization's involvement on incidence rate of Ebola

the concepts reveals additional insights, as shown in Fig. [5.7,](#page-20-0) which reveals complex behavior. This reveals that both *media bias* and WHO involvement are cyclical. WHO involvement increases (to  $+1$ ), followed by a drop in *media bias* (to −1) and WHO involvement (to 0) the following period, followed by an increase in WHO involvement increases (to  $+1$ ) and *media bias* (to 0). The pattern repeats with a periodicity of 2.

If the scenario depicted in Fig.  $5.7$  is acceptable, then we have found an acceptable mechanism for reducing the *Incidence rate of Ebola*. If, however, a more stable scenario is desired, then we can explore additional options. Given that *media* bias appears to be a trigger for the oscillating behavior, we should explore an initial increase of WHO involvement and an initial decrease of media bias. This scenario is shown in Fig. [5.8.](#page-21-0)

The results of this scenario are complicated; the scenario shown in Fig. [5.8](#page-21-0) is stable for all concepts after period 2. It results in an increase in the concepts of WHO involvement, Education, Insurance company involvement, Medical intervention, Strength of infected individuals' immune systems, and Healthcare availability. The scenario also results in the reduction of the concepts of Media bias,

<span id="page-20-0"></span>

Fig. 5.7 Effect of initial increase in World Health Organization's involvement on all concepts

Mortality rate, Morbidity rate, and, most importantly, Incidence rate of Ebola. The remaining concepts had no change. If we believe that we have no control over the media and thus cannot influence Media bias, then we can explore an additional scenario, that of clamping WHO involvement at  $+1$ . That is, we can secure a long-term commitment from the WHO for involvement in the fight against the Ebola virus disease. This result is shown in Fig. [5.9](#page-22-0).

<span id="page-21-0"></span>

Fig. 5.8 Effect of initial reduction in World Health Organization's involvement and initial decrease in media bias on all concepts

The scenario shown in Fig. [5.9](#page-22-0) is complicated as well, but it may be characterized as the most desirable. It results in the same equilibrium as the one depicted in Fig. 5.8, yet it requires no initial change in Media bias, which may be problematic to achieve. Thus, with this simple set of experiments, this section has demonstrated the utility of fuzzy cognitive maps in assessing a complex system.

<span id="page-22-0"></span>

Fig. 5.9 Effect of clamped increase in World Health Organization's involvement on all concepts

## 5.7 Summary

This chapter introduced the reader to complex systems modeling methods. Specifically, a comparison of available methods was presented which outlined the appropriateness of fuzzy cognitive mapping for understanding and investigating complex systems. Finally, a framework was presented for the development and use of a fuzzy cognitive map to explore these problems. Efficacy of FCM modeling was

<span id="page-23-0"></span>demonstrated on an example problem. While it may be unclear at this point to the reader, ensuing chapters will provide additional details regarding the development and use of FCMs, as a mechanism for thinking about, acting on, and observing messes.

After reading this chapter, the reader should

- 1. Be able to identify methods available for complex systems modeling;
- 2. Understand the appropriateness of fuzzy cognitive mapping for representing complex systems; and
- 3. Understand how to construct and use a fuzzy cognitive map.

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