Chapter 11
Improved Controller Design for Positive Systems and Its Application to Positive Switched Systems

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Abstract This chapter will address a new controller design approach for positive systems. First, we decompose the feedback gain matrix $K_{m\times n}$ into $m \times n$ nonnegative components and $m \times n$ non-positive components. For the nonnegative components, each component contains only one positive element and the other ones are zero. Similarly, each non-positive component contains only one negative element and the other ones are zero. Then, a simple but effective controller design of positive systems is proposed by incorporating the decomposed feedback gain matrix into the resulting closed-loop systems. The present approach is thus applied to positive switched systems. It is shown that the designed controller for positive switched systems is less conservative than those ones in the literature.

Keywords Positive systems · Controller design · Linear programming · Positive switched systems.

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11.1 Introduction

Positive systems are a special class of control systems [1]. Over past two decades, positive systems have gained increasing interests due to their extensive applications in practice and theoretical complexes in control theory [2–9]. Compared with general systems, positive systems do not receive much attention until this century. This leads to that many issues of positive systems are open.

As general systems, stabilization is also a fundamental issue of positive systems. There have been some significant results on the stabilization of positive systems. A linear programming approach to controller design of positive systems was proposed in [10, 11]. The output-feedback controller of positive systems [12] was proposed by using the approach in [10, 11]. The problem of ℓ_1-induced state-feedback controller design for positive systems was investigated by using a linear copositive Lyapunov function in [13]. In [14], a static output-feedback controller design was presented, where an iterative linear matrix inequality algorithm was provided to compute the feedback gain matrix. In [15], the output-feedback controller was designed by virtue of an iterative convex optimization algorithm. More results on positive systems can refer to [16–23].

As far as the stabilization of positive systems is concerned, it is clear that there is still much room for improvements in the above mentioned works. This motivates us to carry out the present work. This chapter will further provide a new controller design approach to remove some restrictions in the heavy computational burden, the controller gain matrix, and the unreliability algorithms in the literature. By decomposing the feedback gain matrix into parts, the new approach removes those restrictions in the literature. Our developed design approach is very efficient in solving the control synthesis problems of positive systems. An application to positive switched systems is also given to show the efficiency of the proposed approach. The rest of the chapter is organized as follows: Sect. 11.2 provides the problem statements; Sect. 11.3 gives main results; Sect. 11.4 concludes the chapter.

Notations Let $\Re$, $\Re^n$, $\Re^{n\times n}$ be the sets of real numbers, n-dimensional vectors and $n\times n$ matrices, respectively. Denote by $\mathbb{N}$, $\mathbb{N}^+$ the sets of nonnegative and positive integers. For a vector $x=(x_1,\ldots,x_n)^T$, $x\succeq 0$ ($\succ 0$) means that $x_i\geq 0$ ($x_i>0$) $\forall i=1,\ldots,n$. Similarly, $x\preceq 0$ ($\prec 0$) means that $x_i\leq 0$ ($x_i<0$) $\forall i=1,\ldots,n$. For a matrix $A=[a_{ij}]\in\Re^{n\times n}$, $A\succeq 0$ ($\succ 0$) means that $a_{ij}\geq 0$ ($a_{ij}>0$) $\forall i,j=1,\ldots,n$. Similarly, $A\preceq 0$ ($\prec 0$) means that $a_{ij}\leq 0$ ($a_{ij}<0$) $\forall i,j=1,\ldots,n$. A matrix A is called as Metzler if all its non-diagonal elements are nonnegative. I is the identical matrix with proper dimension. $\Re^n_+\triangleq\{x|x\in\Re^n, x\succeq 0\}$. Let $\mathbf{1}_n=(\underbrace{1,\ldots,1}_{n})^T$ and $\mathbf{1}_n^{(i)}=(\underbrace{0,\ldots,0}_{i-1},1,\underbrace{0,\ldots,0}_{n-i})^T$. Throughout the chapter, the dimensions of vectors and matrices are assumed to be compatible if not stated.

11.2 Problem Formulation

Consider the following system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{11.1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$ and $y(t) \in \Re^r$ are system state, control input, and output, respectively. Assume that $A \in \Re^{n\times n}$ is a Metzler matrix, $B \succeq 0$ with $B \in \Re^{n\times m}$, and $C \succeq 0$ with $C \in \Re^{r\times n}$.

The following preliminaries are first introduced for later use.

Definition 11.1 [3, 6] System (11.1) is positive if its state and output are nonnegative for all time t whenever the initial condition $x(t_0)$ and control input $u(t)$ are nonnegative.

Lemma 11.1 [3, 6] *System (11.1) is positive if and only if A is a Metzler matrix, $B \succeq 0$ and $C \succeq 0$.*

Noting the assumptions for system (11.1), it follows that system (11.1) is positive by Lemma 11.1.

Lemma 11.2 *A matrix M is Metzler if and only if there exists a positive constant ς such that $M + \varsigma I \succeq 0$.*

11.3 Main Results

In this section, we will address the stabilization of positive systems and positive switched systems (PSSs). The objective of the stabilization is to design a controller such that the resulting closed-loop system is positive and stable.

11.3.1 Stabilization of Positive Systems

We first consider the stabilization of system (11.1).

Theorem 11.1 *If there exist constants $\varsigma > 0$, $k_{ij}^+ > 0$, $k^+ > 0$, $k_{ij}^- < 0$, $k^- < 0$ and vectors $v \succ 0$ with $v \in \Re^n$ such that*

$$A^T v + \zeta^+ + \zeta^- \prec 0, \tag{11.2a}$$

$$\begin{aligned} A\mathbf{1}_m^T B^T v + B \textstyle\sum_{i=1}^m \sum_{j=1}^r \mathbf{1}_m^{(i)} (\zeta_{ij}^+ \\ + \zeta_{ij}^-)^T + \varsigma I \succeq 0, \end{aligned} \tag{11.2b}$$

$$k_{ij}^{+} < k^{+}, \tag{11.2c}$$

$$k_{ij}^{-} < k^{-}, \tag{11.2d}$$

hold for $i = 1, \dots, m, \;\; j = 1, \dots, n,$ *where* $\zeta_{ij}^{\pm} = (\underbrace{0, \dots, 0}_{j-1}, k_{ij}^{\pm}, \underbrace{0, \dots, 0}_{n-j})^T \in \Re^n$ *and* $\zeta^{\pm} = (k^{\pm}, \dots, k^{\pm})^T \in \Re^n$*, then under the state-feedback control law*

$$\begin{aligned} u(t) &= Kx(t) \\ &= \frac{\Sigma_{i=1}^{m} \Sigma_{j=1}^{n} \mathbf{1}_m^{(i)} (\zeta_{ij}^{+} + \zeta_{ij}^{-})^T}{\mathbf{1}_m^T B^T v} x(t) \end{aligned} \tag{11.3}$$

the resulting closed-loop system (11.1) is positive and asymptotically stable.

Proof By $\mathbf{1}_m \succ 0$ with $\mathbf{1}_m \in \Re^m$, $B \succeq 0$ with $B \in \Re^{n \times m}$, and $v \succ 0$ with $v \in \Re^n$, we have $\mathbf{1}_r^T B^T v > 0$. This together with (11.2b) gives that

$$\begin{aligned} A + B &\frac{\Sigma_{i=1}^{m} \Sigma_{j=1}^{n} \mathbf{1}_m^{(i)} (\zeta_{ij}^{+} + \zeta_{ij}^{-})^T}{\mathbf{1}_m^T B^T v} \\ &+ \frac{\varsigma}{\mathbf{1}_m^T B^T v} I \succeq 0. \end{aligned} \tag{11.4}$$

Using (11.3), it follows that

$$A + BK + \frac{\varsigma}{\mathbf{1}_m^T B^T v} I \succeq 0 \tag{11.5}$$

By Lemma 11.2, $A + BK$ is a Metzler matrix. Then, the closed-loop system (11.1) is positive by Lemma 11.1, that is, $x(t) \succeq 0 \; \forall t \geq 0$.

Choose a linear copositive Lyapunov function candidate $V(x(t)) = x(t)^T v$. Then

$$\dot{V}(x(t)) = x(t)^T (A^T v + K^T B^T v). \tag{11.6}$$

By (11.2c) and (11.2d), we get

$$\begin{aligned} &\textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} \mathbf{1}_m^{(i)} (\zeta_{ij}^{+} + \zeta_{ij}^{-})^T \\ &= \textstyle\sum_{i=1}^{m} \mathbf{1}_m^{(i)} \sum_{j=1}^{n} (\zeta_{ij}^{+} + \zeta_{ij}^{-})^T \\ &\preceq \textstyle\sum_{i=1}^{m} \mathbf{1}_m^{(i)} (\zeta^{+} + \zeta^{-})^T \\ &= \mathbf{1}_m (\zeta^{+} + \zeta^{-})^T. \end{aligned} \tag{11.7}$$

Furthermore,

$$\begin{aligned} &K^T B^T v \\ &= \frac{\Sigma_{i=1}^{m}\Sigma_{j=1}^{n}(\zeta_{ij}^{+}+\zeta_{ij}^{-})\mathbf{1}_m^{(i)T}B^T v}{\mathbf{1}_m^T B^T v} \\ &\preceq \frac{(\zeta^{+}+\zeta^{-})\mathbf{1}_m^T B^T v}{\mathbf{1}_m^T B^T v} \\ &= \zeta^{+}+\zeta^{-}. \end{aligned} \tag{11.8}$$

With the fact $x(t) \succeq 0$ in mind, one can obtain from (11.6) that

$$\dot{V}(x(t)) \le x(t)^T(A^T v + \zeta^{+} + \zeta^{-}). \tag{11.9}$$

By (11.2a), we have $\dot{V}(x(t)) < 0$. This completes the proof. □

Remark 11.1 In Theorem 11.1, the gain matrix K is decomposed into

$$\begin{aligned} K = &\frac{1}{\mathbf{1}_m^T B^T v}\begin{pmatrix} k_{11}^{+} & k_{12}^{+} & \cdots & k_{1n}^{+} \\ k_{21}^{+} & k_{22}^{+} & \cdots & k_{2n}^{+} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1}^{+} & k_{m2}^{+} & \cdots & k_{mn}^{+} \end{pmatrix} \\ &+ \frac{1}{\mathbf{1}_m^T B^T v}\begin{pmatrix} k_{11}^{-} & k_{12}^{-} & \cdots & k_{1n}^{-} \\ k_{21}^{-} & k_{22}^{-} & \cdots & k_{2n}^{-} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1}^{-} & k_{m2}^{-} & \cdots & k_{mn}^{-} \end{pmatrix} \\ &= \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\mathbf{1}_m^{(i)}(\zeta_{ij}^{+}+\zeta_{ij}^{-})^T}{\mathbf{1}_m^T B^T v}. \end{aligned} \tag{11.10}$$

Thus, the term $K^T B^T v$ is transformed into the linear programming form. It should be pointed out that the rank of the gain matrix K is general without any restrictions. The condition (2) is solvable by using the linear programming technique.

Remark 11.2 (i) Theorem 11.1 gives the sufficient condition for the existence of feedback controller of positive systems whereas in the literature [10–15] some necessary and sufficient conditions were established. Then, Theorem 11.1 is more conservative than those results in the literature.

(ii) In [13–15], some iterative algorithms were addressed to compute the controller gain matrix. These algorithms contain some complexities and unreliability such as the introduction of some additional parameters and an initial controller gain. The design in [10–12] is nice if one only considers the stabilization of positive systems. In our opinion, the design in [10–12] seems to be restricted if applying it to hybrid positive systems.

(iii) Aiming to these restrictions in those literature, Theorem 11.1 is presented. The advantages of Theorem 11.1 lie in: (a) the implemental algorithm is easy, (b) the restriction in the gain matrix is removed, and (c) it can be easily applied to other control issues of hybrid positive systems.

The following corollary gives the output-feedback controller design of positive systems and its proof is omitted.

Corollary 11.1 *If there exist constants* $\varsigma > 0, k_{ij}^+ > 0, k^+ > 0, k_{ij}^- < 0, k^- < 0$ *and vectors* $v \succ 0$ *with* $v \in \Re^n$ *such that*

$$\begin{aligned} &A^T v + C^T \zeta^+ + C^T \zeta^- \prec 0, \\ &A\mathbf{1}_m^T B^T v + B \sum_{i=1}^m \sum_{j=1}^r \mathbf{1}_m^{(i)} (\zeta_{ij}^+ \\ &\qquad + \zeta_{ij}^-)^T C + \varsigma I \succeq 0, \\ &k_{ij}^+ < k^+, \\ &k_{ij}^- < k^-, \end{aligned} \tag{11.11}$$

hold for $i = 1, \ldots, m,\ \ j = 1, \ldots, r,$ *where* $\zeta_{ij}^{\pm} = (\underbrace{0, \ldots, 0}_{j-1}, k_{ij}^{\pm}, \underbrace{0, \ldots, 0}_{r-j})^T \in \Re^r$ *and* $\zeta^{\pm} = (k^{\pm}, \ldots, k^{\pm})^T \in \Re^r$*, then under the output-feedback control law*

$$\begin{aligned} u(t) &= Ky(t) \\ &= \frac{\Sigma_{i=1}^m \Sigma_{j=1}^r \mathbf{1}_m^{(i)} (\zeta_{ij}^+ + \zeta_{ij}^-)^T}{\mathbf{1}_m^T B^T v} y(t) \end{aligned} \tag{11.12}$$

the resulting closed-loop system (11.1) is positive and asymptotically stable.

11.3.2 Stabilization of PSSs

In this subsection, we propose the feedback controller design of PSSs by applying the present approach in Theorem 11.1. Consider the switched system:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t), \\ y(t) &= C_{\sigma(t)} x(t), \end{aligned} \tag{11.13}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, and $y(t) \in \Re^r$ are system state, control input, and output, respectively. The function $\sigma(t)$ represents the switching law, which is right continuous takes values in a finite set $S = \{1, 2, \ldots, J\}$, $J \in \mathbb{N}^+$. The $\sigma(t_i)$th subsystem is active for $t \in [t_i, t_{i+1})$, $i \in \mathbb{N}$, where t_i and t_{i+1} are the switching time instants. The states of system (11.1) are continuous and do not jump in the switching time instants. For system (11.1), assume that $A_p \in \Re^{n\times n}$ is a Metzler matrix and $B_p \succeq 0$ with $B_p \in \Re^{n\times m}$, $C_p \succeq 0$ with $C_p \in \Re^{r\times n}$ for each $p \in S$.

Theorem 11.2 *If there exist constants* $\varsigma_p > 0, k_{pij}^{+} > 0, k_p^{+} > 0, k_{pij}^{-} < 0, k_p^{-} < 0$ *and vectors* $\upsilon_p \succ 0$ *with* $\upsilon_p \in \Re^n$ *such that*

$$A_p^T \upsilon_p + \zeta_p^+ + \zeta_p^- + \mu \upsilon_p \prec 0, \tag{11.14a}$$

$$\begin{aligned} A_p \mathbf{1}_m^T B_p^T \upsilon_p + B_p \textstyle\sum_{i=1}^m \sum_{j=1}^n \mathbf{1}_m^{(i)} (\zeta_{pij}^+ \\ + \zeta_{pij}^-)^T + \varsigma_p I \succeq 0, \end{aligned} \tag{11.14b}$$

$$k_{pij}^+ < k_p^+, \tag{11.14c}$$

$$k_{pij}^- < k_p^-, \tag{11.14d}$$

$$\upsilon_p \prec \lambda \upsilon_q, \tag{11.14e}$$

hold for $i = 1, \ldots, m,\ j = 1, \ldots, n$, *where* $\zeta_{pij}^{\pm} = (\underbrace{0, \ldots, 0}_{j-1}, k_{pij}^{\pm}, \underbrace{0, \ldots, 0}_{n-j})^T \in R^n$ *and* $\zeta_p^{\pm} = (k_p^{\pm}, \ldots, k_p^{\pm})^T \in R^n$, *then under the state-feedback control law*

$$\begin{aligned} u(t) &= K_p x(t) \\ &= \frac{\Sigma_{i=1}^m \Sigma_{j=1}^n \mathbf{1}_m^{(i)} (\zeta_{pij}^+ + \zeta_{pij}^-)^T}{\mathbf{1}_m^T B_p^T \upsilon_p} x(t) \end{aligned} \tag{11.15}$$

the resulting closed-loop system (11.13) is positive and asymptotically stable with the average dwell time satisfying

$$\tau > \tfrac{\ln \lambda}{\mu}. \tag{11.16}$$

Sketch of Proof From the proof of Theorem 11.1, we can get that, for each $p \in S$, the subsystem is positive and asymptotically stable under the state-feedback control law (11.15). Choose multiple linear copositive Lyapunov functions $V(x(t)) = x(t)^T \upsilon_{\sigma(t)}$, then

$$\begin{aligned} \dot{V}(x(t)) = x(t)^T \big(& A_{\sigma(t_\iota)}^T \upsilon_{(\sigma(t_\iota))} \\ & + K_{\sigma(t_\iota)}^T B_{\sigma(t_\iota)}^T \upsilon_{(\sigma(t_\iota))} \big) \end{aligned} \tag{11.17}$$

for $t \in [t_\iota, t_{\iota+1})$. From (11.14c), (11.14d), and (11.15), we can have

$$K_{\sigma(t_\iota)}^T B_{\sigma(t_\iota)}^T \upsilon_{(\sigma(t_\iota))} \preceq \zeta_{\sigma(t_\iota)}^+ + \zeta_{\sigma(t_\iota)}^-. \tag{11.18}$$

With $x(t) \succeq 0$ in mind, substituting (11.18) into (11.17) gives

$$\begin{aligned} \dot{V}(x(t)) \le x(t)^T \big(& A_{\sigma(t_\iota)}^T \upsilon_{(\sigma(t_\iota))} \\ & + \zeta_{\sigma(t_\iota)}^+ + \zeta_{\sigma(t_\iota)}^- \big). \end{aligned} \tag{11.19}$$

This together with (11.14a) yields

$$\dot{V}(x(t)) \leq -\mu V(x(t)) \tag{11.20}$$

for $t \in [t_\iota, t_{\iota+1})$. Then,

$$V(x(t)) \leq e^{-\mu(t-t_\iota)} V(x(t_\iota)) \tag{11.21}$$

for $t \in [t_\iota, t_{\iota+1})$. By (11.14e), it follows that

$$V(x(t)) \leq \lambda e^{-\mu(t-t_\iota)} V(x(t_\iota^-)). \tag{11.22}$$

By recursive deduction, we get

$$\begin{aligned} V(x(t)) &\leq \lambda^2 e^{-\mu(t-t_{\iota-1})} V(x(t_{\iota-2})) \\ &\leq \cdots \\ &\leq \lambda^{N_{\sigma(t_0,t)}} e^{-\mu(t-t_0)} V(x(t_0)), \end{aligned} \tag{11.23}$$

where $N_{\sigma(t_0,t)}$ is the number of the switching in $[t_0, t]$. Noting $\lambda > 1$, (11.23) is transformed into

$$\begin{aligned} V(x(t)) &\leq \lambda^{N_0 + \frac{t-t_0}{\tau}} e^{-\mu(t-t_0)} V(x(t_0)) \\ &= \lambda^{N_0} e^{(\frac{\ln\lambda}{\tau} - \mu)(t-t_0)} V(x(t_0)), \end{aligned} \tag{11.24}$$

where N_0 is the chatter bound. Then

$$||x(t)||_1 \leq \frac{\varrho_2 \lambda^{N_0}}{\varrho_1} e^{(\frac{\ln\lambda}{\tau} - \mu)(t-t_0)} ||x(t_0)||_1, \tag{11.25}$$

where ϱ_1 and ϱ_2 are the minimal and maximal elements of v_p $\forall p \in S$. By (11.15), $\frac{\ln\lambda}{\tau} - \mu < 0$. In addition, $\frac{\varrho_2 \lambda^{N_0}}{\varrho_1} > 0$ is obvious. So, the resulting closed-loop system (11.13) is positive and exponentially stable. □

Remark 11.3 In [24, 25], the state-feedback controllers of PSSs and nonlinear PSSs were proposed. In should be pointed out that the controller gain matrices contain the restriction on the rank. In [26], we remove the restriction in [24, 25]. However, the method in [26] contain a new restriction on average dwell time. Theorem 11.2 has removed the restrictions in [24–26].

Remark 11.4 It is also worthy noting that the approach in Theorem 11.2 can be applied to positive time-delay systems [27] and thus the restriction in [27] can be removed. Up to now, there have been many interesting results on hybrid positive systems referring to positive Markovian jump systems and positive T-S fuzzy systems. We notice that, when considering the issues of hybrid positive systems, a common restriction is just the one stated in Remark 11.3. Therefore, Theorem 11.2 can be further extended for those issues.

Corollary 11.2 *If there exist constants* $\varsigma_p > 0, k_{pij}^+ > 0, k_p^+ > 0, k_{pij}^- < 0, k_p^- < 0$ *and vectors* $\upsilon_p \succ 0$ *with* $\upsilon_p \in \Re^n$ *such that*

$$\begin{array}{l} A_p^T \upsilon_p + C_p^T \zeta_p^+ + C_p^T \zeta_p^- + \mu \upsilon_p \prec 0, \\ A_p \mathbf{1}_m^T B_p^T \upsilon_p + B_p \sum_{i=1}^m \sum_{j=1}^r \mathbf{1}_m^{(i)} (\zeta_{pij}^+ \\ \qquad\qquad + \zeta_{pij}^-)^T C_p + \varsigma_p I \succeq 0, \\ k_{pij}^+ < k_p^+, \\ k_{pij}^- < k_p^-, \\ \upsilon_p \prec \lambda \upsilon_q, \end{array} \tag{11.26}$$

hold for $i = 1, \ldots, m,\ j = 1, \ldots, r$, *where* $\zeta_{pij}^{\pm} = (\underbrace{0, \ldots, 0}_{j-1}, k_{pij}^{\pm}, \underbrace{0, \ldots, 0}_{n-j})^T \in \Re^r$ *and* $\zeta_p^{\pm} = (k_p^{\pm}, \ldots, k_p^{\pm})^T \in \Re^r$, *then under the output-feedback control law*

$$\begin{aligned} u(t) &= K_p y(t) \\ &= \frac{\Sigma_{i=1}^m \Sigma_{j=1}^r \mathbf{1}_m^{(i)} (\zeta_{pij}^+ + \zeta_{pij}^-)^T}{\mathbf{1}_m^T B_p^T \upsilon_p} y(t) \end{aligned} \tag{11.27}$$

the resulting closed-loop system (11.13) is positive and asymptotically stable with the average dwell time satisfying (11.15).

11.4 Conclusions and Future Work

This chapter has addressed a new approach to control synthesis of positive systems. Sufficient conditions for the feedback controller of positive systems are established by using a linear copositive Lyapunov function associated with linear programming technique. Then, the approach is applied to the controller design of PSSs. It is shown that the restrictions in the literature are removed.

Further work refers to two aspects. On one hand, some extension of the approach in the chapter can be proceeded. On the other hand, necessary and sufficient conditions for the approach are expected.

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