Delineation of Rectangular Management Zones and Crop Planning Under Uncertainty in the Soil Properties

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Abstract. In this article we cover two problems that often farmers have to face. The first one is to generate a partition of an agricultural field into rectangular and homogeneous management zones according to a given soil property, which has variability in time that is presented by a set of possible scenarios. The second problem assigns the correct crop rotation for those management zones defined before. These problems combine aspects of precision agriculture and optimization with the purpose of achieving a site and time specific management of the field that is consistent and effective in time for a medium term horizon. Thus, we propose a two-stage stochastic integer programming model with recourse that solves the delineation problem facing a finite number of possible scenarios, after this we propose a deterministic crop planning model, and then we combine them into a new two-stage stochastic program that can solve both problems under ucertainty conditions simultaneously. We describe the proposed methodology and the results achieved in this research.

Keywords: OR in agriculture · Stochastic programming · Management zones · Crop planning · Precision agriculture

1 Introduction

In agriculture, spatial variability of the soil properties is a key aspect in yield and quality of crops. In fact, one of the problems in precision agriculture consists in dividing the field into site specific management zones, which based on a soil property such as: pH, organic matter, phosphorus, nitrogen, crop yield, etc. Delineating rectangular zones into zones relatively homogeneous allows better agricultural machines performance and eases the design of irrigation systems, being also important to consider the zones size and the total amount of management zones from the field partition.

The problem of defining management zones in presence of site specific variability has been studied in [6], where an integer programming model for determining rectangular zones is defined, this problem considers spatial variability of an specific soil property and choose the best field partition. The main idea

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is to define homogeneous management zones to optimize the use of inputs for crops. The model is solved by the complete enumeration of the variables, thus it is possible only to solve small and medium size instances due to the problem is NP-hard. To deal with this problem, a column generation algorithm was proposed in [3] which allows to efficiently solve large instances of the problem.

Recently, this previous problem has been applied for irrigation systems design, see [9] where linear programming is used as one of the methods for delineating management zones. Other methods for delineation are classified as clustering methods, see [12,13,17], but their major drawback is the resulting fragmentation of the zones, because these methods generate oval shaped and disjoint zones.

On the other hand, there is an important problem related to choose the correct crop planning in these management zones previously defined. This problem has been studied in [2], where a hierarchical scheme is shown, the first step is to define de rectangular zones and the second step is to define the correct crop to be cultivated in each zone with a single-objective that maximize profits. Other authors have studied this crop planning problem with multi-objective models like in [16] where the objective is to maximize profits and minimize a monthly irrigation planning. Within this context, the crop rotation problem rises, in these kind of models the objective is to find an optimal set of crops for a temporal horizon, it is also possible to change crops in each period. This problem is studied in [7] where a linear problem is developed to define the optimal crop rotation plan subject to certain ecologically-based constraints and considering that harvested crops can be stocked but only for a limited period of time.

Although the problem of defining management zones in presence of site specific variability has been studied in previous works, to the best of our knowledge, an important characteristic that has not been considered yet is the variability in time of the chosen soil property. Based on cited works, first we propose a two-stage stochastic programming model with recourse that solves field partition problem considering the chosen soil property as a random variable which can be modeled by a finite number of scenarios. Then we extend the previous model by combining the field partition problem with the crop planning problem into a new two-stage stochastic program. This is a new proposal based on the fact that the crop yield depends directly of the soil propierties where it is cultived.

Stochastic programming is chosen in these situations because deterministic models are not capable of adding the effect of uncertainty to the solutions. Stochastic programming is based on considering random variables that are described by a number of possible scenarios; see e.g. [4,19,20].

In the last few years, stochastic programming has been used more often in a wide variety of applications due to its capacity of solving problems increasingly large, thus more realistic models, see e.g. [8,22] for general applications.

In agriculture, stochastic programming has been used to solve different problems related with situations where uncertainty is a key aspect in the decision making process. Besides delineation decision there are other important decisions to make, as crop planning, water planning, food supply chain and agricultural raw materials supply planning, among others. Crop planning is a decision where a crop pattern must be chosen for each management zone, this pattern last a specific number of crop cycles and thus must face future weather scenarios and prices, see [11,14,24]. Water planning is important because the need for more agricultural production requires large amounts of water for irrigation purposes, making water resources scarce, thus surface water resources must be allocated among farmers and also a plan must be made for the use of this water, see [5] and [15]. Stochastic programming is also applied in agricultural supply chain problems, as food supply chain where a growing and distribution plan must be made, and raw materials supply where a raw material acquisition plan must be made considering that some raw materials are seasonal, in these problems variability appears in the form of weather conditions and product demands, see [1,23].

Within stochastic programming models exists the two-stage models with recourse. These models recognize two types of decisions that must be made sequentially. First stage decision or here-and-now must be made previously to the performance of the random variables. Then, second stage decision or wait-and-see, which must compensate the effects of the first stage decisions once the performance of the random variables are known, due to this, the variables in this stage are denoted as recourse variables. The goal of these models consists in finding the optimal first stage decision that minimize total costs, defined by the sum of the first stage decision costs and the expected costs of the second stage decisions; see e.g. [10].

For example, in the proposed model for generating a partition, the first stage decision chooses a field partition that minimizes the number of management zones; these zones must satisfy certain homogeneity level that depends on the performance value of the sample points which are the random variables in this case. On the other hand, second stage decision uses looseness variables that relax homogeneity constraints in exchange of a penalty. This penalty helps to achieve management zones homogeneity goal while minimizes the use of the looseness variables.

In this article, problem formulation needs the generation of the total number of potential management zones; in other words, problem solving considers the complete enumeration of zones is known. This is feasible for small and medium size instances as the ones used in this work, which represents a good starting point to approach to this problem. Although, proposed formulation can be extended to large instances by the application of a column generation algorithm, but its use exceeds the purpose of the present research, see [3].

In following sections, the article is organized as follows. Section 2 details the proposed models to solve the delineation problem, the crop planning problem and the combined problem, from data collection to the solving process itself. After this, in Sect. 3, results obtained by the application of proposed methodology are presented. Finally in Sect. 4, main conclusions and future works from the application of the model are presented.

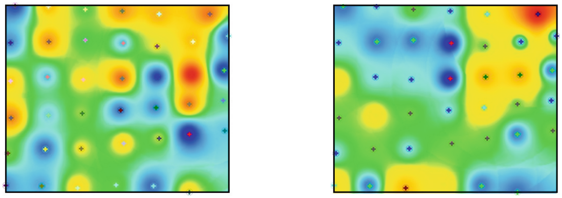
2 Materials and Methods

As we mentioned before, this work consists in generating a field partition composed by a group of management zones based on a chosen soil property which has variability in space and time, and also choosing the optimal crop plan that minimizes the cost involved in the production horizon period. The proposed methodology has two steps. First, the task is to model the soil property space variability by taking samples on the field, this process must be done several times in different periods to measure variability in time, with this data, instances are generated. And the second step, consists on solving the problems using two different approaches: The first one consists in a hierarchical scheme where we define a two-stage stochastic integer programming model for the delineation problem that minimizes the number of management zones in its first stage and minimizes noncompliance of the homogeneity level in the second stage, and a deterministic crop planning model that selects the correct crop pattern that should be cultivated in the previously defined management zones in a certain period of time. The second approach consists in defining a combined two-stage stochastic model that covers both problems with the same uncertainty condition that were considered in the stochastic delineation model.

2.1 Instance Generation

In the first step, we generate instances that will be solved by the models. To achieve this is necessary to use specialized software as MapInfo; this software creates thematic maps of the field that summarizes and shows spatial variability of the soil properties measured from the sample points. This includes sample coordinates, pH level, organic matter index, phosphorus, base sum, crop yield, etc. As an example, Fig. 1 shows two thematic maps from the same field, one with organic matter (OM) and the other with phosphorus (P). In OM case, green zones represent reference levels of OM, while sky blue and blue zones represent zones with 34.8% and 3.97% above normal values of OM, also red and yellow zones presents values with 6.06% and 3.97% under normal OM values. On the other hand, in P case sky blue and blue zones are 27.31% and 10.34% above normal, and red and yellow zones are 13.79% and 48.27%, respectively. Both maps show spatial variability of these indexes in a field, this proves the importance of dividing the field into management zones with uniform characteristics, to apply inputs needed in each zone through site specific farming.

Also, we need to include variability in time of the measured indexes. For that, we use thematic map data sets from the same field for several time periods; these



Organic Matter (OM) Phosphorus (P)

Fig. 1. Organic matter and phosphorus map. (Color figure online)

will be used either to generate the probability distribution function of the soil property or to create different scenarios with each one of these instances. A possible value of the random variable consist in assign a specific value to each of the sample points on the field, i.e., the random variable is represented by a vector that includes each one of the sample points; this vector has a finite number of possible values. Scenario probabilities are assigned depending on the number of instances and the time between each sampling process. It is important to notice that a field partition is a medium term decision, i.e., this partition will last a specific number of years and after that horizon is reached, another partition must be set, thus the model must take into account possible changes in soil properties during this time. This article uses only historical data for scenario creation, but it is also valid to consider forecasts for future periods in the scenario creation step, but this exceeds the purpose of this article.

Finally, potential management zones are generated (Z set) through an algorithm that uses all sample points (S set) as inputs. As an example, in Fig. 2 there is an instance with 42 sample point field (6 rows and 7 columns) and three potential management zones from a total of 588, each one of them has rectangular form and includes at least one sample point.

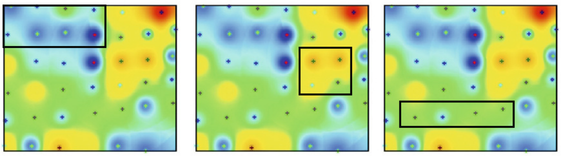


Fig. 2. Potential management zones example.

A relationship matrix $C = (c_{sz})$ is created from potential zones generation, where $c_{sz} = 1$ means that potential zone z includes sample point s, and $c_{sz} = 0$ otherwise, for every $z \in Z$, $s \in S$. Besides, index variance $\sigma^2_{z\omega}$ is obtained for each potential quarter z and each scenario $\omega \in \Omega$, where Ω is the set of possible scenarios. Both parameters are used in the model presented in the following section.

2.2 Optimization Model for the Delineation Problem

Proposed model consist in a two-stage stochastic integer programming model with recourse. In the first stage, the problem minimizes the number of management zones that cover the entire field. In the second stage, the problem minimizes noncompliance of the homogeneity level using looseness variables for each scenario but with a penalty cost for using them. This second stage is necessary because field partition must be chosen before knowing random variables performance, and it must satisfy the homogeneity constraint for any scenario, this is achieved by minimizing the expected value of the penalty for the noncompliance of the homogeneity level.

Sets, parameters and variables used in the model are described below:

Sets:

Z: set of potential management zones, with $z \in Z$.
S: set of sample points of the field, with $s \in S$.
Ω: set of possible scenarios, with $\omega \in \Omega$.

Parameters:

c_{sz}: Coefficient that represents if quarter z covers sample point s or not.
M_ω: Penalty cost per unit for noncompliance of the required homogeneity level.
n_z: Number of sample points in quarter or management zone z.
p_w: Probability of scenario ω.
$\sigma^2_{z\omega}$: Quarter variance z calculated from the soil property in scenario ω.
$\sigma^2_{T\omega}$: Total variance of the field calculated from the soil property data in scenario ω.
N: Total number of sample points.
UB: Upper bound for the number of management zones chosen.
α: Required homogeneity level.

Decision variables:

$$q_z = \begin{cases} 1, & \text{if quarter z is assigned to field partition} \\ 0, & \text{otherwise}, z \in Z \end{cases}$$

h_ω: Looseness for the homogeneity level in scenario $\omega \in \Omega$.

The two-stage stochastic model with recourse is presented now:

$$Min \sum_{z \in Z} q_z + \sum_{\omega \in \Omega} p_\omega Q(q, h_\omega) \tag{1}$$

$$s.t.$$

$$\sum_{z \in Z} c_{sz} q_z = 1 \quad \forall s \in S \tag{2}$$

$$\sum_{z \in Z} q_z \leqslant UB \tag{3}$$

$$q_z \in \{0, 1\} \quad \forall z \in Z \tag{4}$$

$$Where \ Q(q, h_\omega) = Min M_\omega h_\omega \tag{5}$$

$$s.t.$$

$$h_\omega \geqslant \sum_{z \in Z} [(n_z - k)\sigma^2_{z\omega} + (1 - \alpha)\sigma^2_{T\omega}] q_z - (1 - \alpha)\sigma^2_{T\omega} N \tag{6}$$

$$h_\omega \geqslant 0 \tag{7}$$

Problems (1)–(4) correspond to the first stage decision, while (5)–(7) correspond to the second stage decision. Objective function (1) minimizes the sum of management zones chosen and minimizes the expected value of the penalty cost for noncompliance of the required homogeneity level, these are first and second stage objective functions respectively. Constraint (2) is typical for set partition models, guarantees that each sample point on the field is assigned only to one quarter. Constraints (3) establishes an upper bound to the number of management zones chosen to divide the field. Constraint (4) defines that quarter variables must be binary. Objective function (5) represents second stage decision for each scenario. Constraint (6) states that a required homogeneity level must be accomplished; this constraint is made from the linear version of the relative variance concept and a looseness variable for each scenario. Finally, constraint (7) states nature of second stage variables.

It is important to notice that this model, as in [6], uses an equivalent linear version of the constraint related to the relative variance concept. However in this case, as we have different possible scenarios, we must meet homogeneity level in each one of these scenarios, thus we will have a relative variance constraint for each scenario. As we have to choose only one field partition we need a way to deal with uncertainty because otherwise we will have to choose the best field partition for worst possible scenario in terms of relative variance. We propose to add new variables named as looseness variables as part of the second stage decision to get a solution that considers all possible scenarios, meeting the required homogeneity level in each one of these, and without being forced to solve the problem for the worst scenario.

Constraint (6) is created from the following non-linear constraint used in [6]:

$$1 - \frac{\sum_{z \in Z} (n_z - k)\sigma_z^2 q_z}{\sigma_T^2 [N - \sum_{z \in Z} q_z]} \geqslant \alpha \tag{8}$$

This constraint uses relative variance concept, presented in [18], is a widely used criteria to measure effectiveness of chosen management zones and it must be equal or higher to a given α value, which is the required homogeneity level, that should be at least 0.5 to validate an ANOVA test hypothesis assuming k degrees of freedom. To create constraint (6) first we need to linearize Eq. (8) obtaining the following expression:

$$(1-\alpha)\sigma_T^2 [N - \sum_{z \in Z} q_z] \geqslant \sum_{z \in Z} (n_z - k)\sigma_z^2 q_z \tag{9}$$

Then if we reorder Eq. (9) we obtain:

$$\sum_{z \in Z} [(n_z - k)\sigma_z^2 + (1-\alpha)\sigma_T^2] q_z \leqslant (1-\alpha)\sigma_T^2 N \tag{10}$$

As we have a number of possible scenarios we define a relative variance constraint for each one of these, and also different parameters for each scenario ω:

$$\sum_{z \in Z} [(n_z - k)\sigma_{z\omega}^2 + (1-\alpha)\sigma_{T\omega}^2] q_z \leqslant (1-\alpha)\sigma_{T\omega}^2 N \tag{11}$$

Here is when we add the looseness variables h_ω to the right side of Eq. (11):

$$\sum_{z \in Z} [(n_z - k)\sigma_{z\omega}^2 + (1-\alpha)\sigma_{T\omega}^2] q_z \leqslant (1-\alpha)\sigma_{T\omega}^2 N + h_\omega \tag{12}$$

These variables allow the problem to choose a field partition that considers all possible scenarios and meet all relative variance constraints by relaxing the right side of Eq. (11) for each scenario, thus finally obtaining constraint (6). It is important to notice that looseness variables are added to the linear version of this constraint to have only linear constraints in the model.

2.3 Optimization Model for the Crop Planning Problem

The model presented in the previous section is capable of determine the optimal delineation of the field considering variability in time, however, we are going to cover a second important problem, as was mentioned before, it consists in define what kind of crop should be cultivated for a period of time. This problem is known as Crop Planning Problem. The proposed model is based on (Santos et al. 2011), this model guarantees that the demand for a specific period of time is supplied, defining a limit for the area to be used.

The sets, parameters and variables are described below:

Sets:

K: Set of potential crop rotation plans, with $k \in K$.
T: Set of periods of time, with $t \in T$.
I: Set of crops, with $i \in I$.

Parameters:

L_{kz}: Cost for cultivating the rotation k in the management zone
D_{it}: Demand for the crop i in the period t
A_{it}^{k}: Amount of crop i harvested in period t in crop rotation plan k.

Variable:

$$x_{kz} = \begin{cases} 1, & \text{if quarter z with crop rotation k is assigned to field partition} \\ 0, & \text{otherwise}, k \in K, z \in Z \end{cases}$$

The new model is as follows:

$$Min \sum_{z \in Z} \sum_{k \in K} L_{kz} x_{kz} \tag{13}$$

$$s.t.$$

$$\sum_{z \in Z} \sum_{k \in K} c_{sz} x_{kz} = 1 \quad \forall s \in S \tag{14}$$

$$\sum_{z \in Z} \sum_{k \in K} x_{kz} \leqslant UB \tag{15}$$

$$\sum_{z \in Z} \sum_{k \in K} A_{it}^{k} x_{kz} \geqslant D_{it}, \quad \forall i \in I, t \in T \tag{16}$$

$$\sum_{z \in Z} \sum_{k \in K} [(n_z - k)\sigma_z^2 + (1-\alpha)\sigma_T^2] x_{kz} \leqslant (1-\alpha)\sigma_T^2 N \tag{17}$$

$$x_{kz} \in \{0, 1\} \quad \forall z \in Z, k \in K \tag{18}$$

Objective function (13) minimizes the cost of cultivate the crop rotation k in management zones chosen. Constraint (14) guarantees that each sample point on the field is assigned only to one quarter and one crop rotation plan. Constraint (15) establishes an upper bound to the number of management zones chosen to divide the field. Constraint (16) ensures that a specific demand in each period must be achieved for each crop. Contraint (17) assumes a minimum level for the homogeneity level. Constraint (18) defines that management zone variables with a crop rotation plan assigned must be binary.

2.4 Optimization Model for Field Delineation and Crop Planning Problem

The field delineation problem was defined in the model of the Sect. 2.2, and the deterministic crop rotation problem was proposed in the Sect. 2.3. With these models is possible to get an answer to these two important problems separately in a hierarchical approach. The second approach considers a new two-stage stochastic programming model that determines simultaneously what is the best field delineation and what to cultivate there, all of this under uncertainty conditions due to the variability in time of the chosen soil property. Thus, using the previous notation, we propose the combined two-stage stochastic program. The proposed model is as follows:

$$Min \sum_{z\in Z}\sum_{k\in K} L_{kz}x_{kz} + \sum_{\omega\in\Omega} p_\omega Q(x,h_\omega) \tag{19}$$

$$s.t.$$

$$\sum_{z\in Z}\sum_{k\in K} c_{sz}x_{kz} = 1 \quad \forall s \in S \tag{20}$$

$$\sum_{z\in Z}\sum_{k\in K} x_{kz} \leqslant UB \tag{21}$$

$$\sum_{z\in Z}\sum_{k\in K} A^k_{it}x_{kz} \geqslant D_{it}, \quad \forall i \in I, t \in T \tag{22}$$

$$x_{kz} \in \{0,1\} \quad \forall z \in Z, k \in K \tag{23}$$

$$Where \;\; Q(x,h_\omega) = Min M_\omega h_\omega \tag{24}$$

$$s.t.$$

$$h_\omega \geqslant \sum_{z\in Z}\sum_{k\in K}[(n_z-k)\sigma^2_{z\omega} + (1-\alpha)\sigma^2_{T\omega}]x_{kz} - (1-\alpha)\sigma^2_{T\omega}N \tag{25}$$

$$h_\omega \geqslant 0 \tag{26}$$

Problems (19)–(23) corresponds to the first stage model, while (24)–(26) are the second stage model. Objective function (19) minimizes the cost of cultivate the crop rotation k in management zones chosen and minimizes the expected value of the penalty cost for noncompliance of the required homogeneity level, these are first and second stage objective functions respectively. Constraints

(20)–(23) are the same as constraints (14)–(17). Objective function (24) represents second stage decision for each scenario. Constraint (25)–(26) states the same condition presented in constraints (6)–(7).

3 Results

To analyze the delineation and crop planning models behavior we used one instance for the problem, using crop yield as soil property because this index has strong variability in time. In this instance, there are six possible scenarios, all of them with similar probabilities, where the two latest scenarios are more likely to occur. Chosen parameter values are:

$M_\omega = 1.5 \quad \forall \omega \in \Omega$
$UB = 40$
$\alpha = 0.9$
$p_\omega = 0.15 \quad \omega \in \{1, ..., 4\}$
$p_\omega = 0.2 \quad \omega \in \{5, 6\}$

We worked with one instance with 42 sample points, that generates 588 potential management zones. The number of potential management zones is obtained by the formula $\frac{((n+1)n(m+1)m)}{4}$ presented in [3], where $n = 6$ is the number of sample points in length and $m = 7$ is the number of sample points in width. The cultivating cost is estimated according to data sheets from analysis in the south of Chile. The rest of the parameters are calculated from crop yield data for each scenario.

The different models were solved with a Lenovo Thinkpad with processor Intel Core i3-2310M 2.10 GHz with 4 Gb RAM memory by using AMPL and CPLEX 12.4.

3.1 Instance Solving

In this Section, the results obtained from this instance are compared using the two approaches described before.

In the hierarchical approach, first we solved two-stage model (1)–(7). The optimal solution can be seen in Fig. 3.

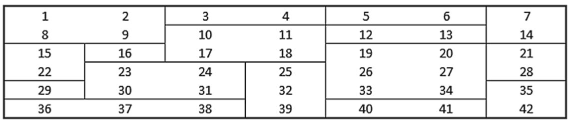


Fig. 3. Optimal solution for stochastic delineation model.

Figure 3 shows the entire field and every rectangle represents a single rectangular management zone. The numbers on it are the sample points. For this instance, the optimal solution is a plot with only 16 management zones and a penalty value of 1.83.

Once the delineation problem is solved, the farmer can choose which crop rotation plan is the best choice to cultivate, using the model (13)–(18), this is how a hierachical approach works. The results are shown as follows in Fig. 4.

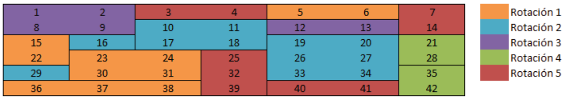


Fig. 4. Crop planning decision over optimal solution for stochastic model.

Now, it would be interesting to compare this solution with the result that can be obtained after applying the combined two-stage stochastic model (19)–(26). That model chooses a different delineation with another crop rotation plan in each management zone, this can be seen in Fig. 5.

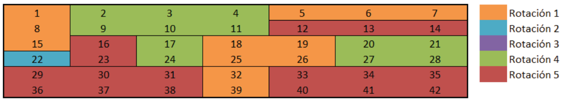


Fig. 5. Optimal solution for simultaneous model.

In order to compare with the previous results, this problem can also be solved for its average scenario by using a deterministic hierarchical approach that consists in obtaining a field delineation for the average scenario and then evaluate this solution with the stochastic model so we can measure the penalty cost for the noncompliance of the required homogeneity level in each one of the scenarios, after this the crop planning model is used to obtain the crop rotation plan for this field delineation.

The results for the delineation problem are shown in Fig. 6.

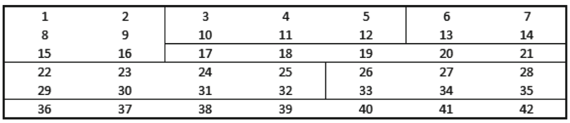


Fig. 6. Optimal solution for average scenario.

In this case, the optimal solution considers less management zones than the previous results, creating a field partition with only seven management zones

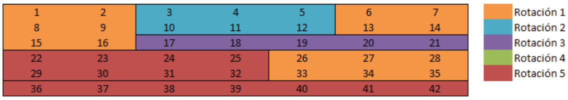


Fig. 7. Crop Planning decision over optimal solution for average model.

Table 1. Results from combined model and hierarchical models.

Instances models	Handling cost	Penalty	O.F.
Hierarchical stochastic approach	340	1,831	341,831
Combined stochastic approach	37	14.562	51.562
Deterministic hierarchical approach	116	71,558	187,558

with a penalty cost equal to 0,61. Once the delineation problem is solved, we obtain an optimal crop planning using model (13)–(18), this is shown in Fig. 7. Finally the results are summarized in Table 1.

This table shows 4 columns, the first column shows the specific approach used to run the instance. The second column shows the handling cost for cultivating a rotation plan in a management zone, in other words, this represents the first stage decision function. The third column represents the second stage decision related to the penalty cost for the noncompliance of the required homogeneity level using looseness variables for each scenario. Finally, the fourth columns shows the total value for the objective function.

In this table it is importante to notice that costs and penalties in the objective function have a high increase when we use a hierarchical approach, reaching an increase over 200%. Also, there are two importants results: the first one is that the best choice for reducing total cost is the combined stochastic model, and the second one is that the best approach that reduces the noncompliance penalty cost is the hierarchical approach using the stochastic delineation decision. This table also shows that the worst result related to the homogeneity requirements is obtained by the hierarchical deterministic approach, this is because the average solution doesn't consider the homogeneity requirements of each scenario separately. In this instance was better to cultivate in the seven management zones, proposed for average scenario delineation, than in the seventeen management zones proposed for the stochastic model, this is because the handling cost increase has a higher impact on the objective function than the penalty cost increase. This could change depending on the parameter values chosen. However the best option is to solve both problems with the combined two-stage stochastic model, this model shows significantly better results than the other approaches. For this reason, it is important to make a decision that considers the field delineation and the crop rotation plan simultaneously. Thus, with this model is possible to achieve a good solution that ensures high savings and that will improve handling work.

4 Conclusions and Further Research

This work presents two different approaches, the first one is hierarchical approach that considers a two-stage stochastic integer programming model to solve the field delineation problem facing uncertainty conditions represented by a soil property and a deterministic crop planning model with the objective to solve what was the best option to cultivate in the management zones defined before. The second approach consists in a combined two-stage stochastic model that minimizes the cost related to cultivate a specific crop rotation at the same time that chooses the field delineation of management zone, also considering the variability in time of the soil property. In Both approaches, models solutions define an optimal field partition with a recourse function that considers looseness variables that help to achieve the required homogeneity level. These approaches were applied to a real instance with 42 sample points, and it showed that the combined stochastic approach is a better choice than a hierarchical approach. The combined model minimizes handling costs related to cultivate a specific crop rotation plan in each management zone, under uncertainy condition of the soil properties, and the results were at least 200% more cheap than the hierarchical approaches.

This methodology covers small size instance solving by the complete enumeration of all potential management zones, this also needs computation of parameters described in Sect. 2.1 for each potential management zone. This is not feasible for large instances due to the problem is NP-hard and the number of variables increases at an exponential rate when number of sample points grows, thus we need more computational effort to calculate all the parameters for each variable. To deal with this issue, we propose to design a decomposition method for the combined two-stage stochastic model based in column generation to solve large instances without using all the problem variables. This will be developed based on the decomposition of the deterministic version of the delineation model presented in this article, see [3], because structure is similar, and management zones can be added as columns in the algorithm as well. Also, it would be interesting to include uncertainty in the handling costs for the crop rotation decision.

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