Chapter 13
A New Model and Method for Order Selection Problems in Flow-Shop Production

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Abstract As the economic growth of China gradually slows down in recent years, the flow-shop production enterprises pay more and more attention to the production capacity planning problem. The order selection problem plays a central role in the production capacity planning of flow-shop production enterprises. Traditional order selection models separate the processes of production scheduling and order selection. The performance of the order selection depends entirely on production scheduling. In this paper we study the relationship between the processes of order selection and production scheduling, and propose a new nonlinear 0–1 programming model aiming at profit maximization. Our new model considers simultaneously order selection and production scheduling and we will demonstrate that our new model generates a production schedule that is much better than that from traditional models. We solved the new model using Lingo 11.0 and numerical results show that the optimal solution can be obtained within an hour on a personal computer when the order size is less than 16.

Keywords Flow-shop production • Order selection • Production scheduling • Nonlinear 0–1 programming

13.1 Introduction

As the economic growth of China gradually slows down in recent years, the economy of China has entered a "new normal" era, where the economy has shifted gear from the previous high speed to a medium-to-high speed growth and the economic structure is constantly improved and upgraded. Along with the new

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economic condition, production enterprises, especially the flow-shop production enterprises, pay more and more attention to the process of production capacity planning, to which the order selection problem is most crucial.

Many flow-shop production enterprises, such as those from the chemical industry, the metallurgical industry, and the feed industry, have a very high transition cost from one order to another due to their special production specification. This means that once an order is put on production, it will be finished before another order is put on production. Also, under the capacity planning framework, a limited capacity is assumed. Delaying an order fulfillment entails additional costs or penalties, while finishing an order too early can lead to an excess inventory cost. Because of these complications, the processes of order selection and production scheduling are of great importance.

The order selection problem has been actively studied for the last 20 years. In the usually conventional approach, before the order selection, all the orders are sorted according to a certain ranking scheme, such as first-come-first-serve (FCFS), earliest-due-date (EDD), shortest-processing-time (SPT), or just sorted by profits from different orders. Then order selection is conducted on the sorted order list. In the literature, Wester et al. (1992) proposed an order acceptance strategy under a single-machine environment with setup times. Slotnick and Morton (2007) considered the situation with a limited capacity and delay cost and studied how the orders such that can be selected the overall profit is maximized. Song and Ma (2007) studied the inter relationship of the flow line balancing, production scheduling, and the makespan of flow line. They proposed a co-optimizing genetic algorithm to optimize the makespan of the mixed-model assembly flow line. Liao et al. (2011) studied the order selection strategy by the analytic hierarchy process (AHP) and fuzzy comprehensive evaluation approaches. They chose indicators from the aspects of delivery time, order size, the importance of customer, etc., and then used AHP to determine the weights. With a comment rating scale, they defined the fuzzy evaluation matrix. Finally the scores of each grading level are summed up to form the basis for order priority. Li and Wang (2014) incorporated the factors of out-sourcing and reputation cost into their order selection model and studied the order acceptance problem with an aim of maximizing overall profit of the production enterprise.

Traditional order selection models separate the processes of production scheduling and order selection. The performance of the order selection depends entirely on production scheduling. However, in flow-shop production, production scheduling has a direct impact on the decision of order selection. On the other hand, the outcome of order selection can also change the final production schedule. Xu et al. (2014) studied the order selection problem in a flow-shop environment and proposed a mixed-integer programming model to simultaneously optimize the production scheduling and order selection. Their model would allocate those orders that are not selected into a delayed group, and the delay cost and the machine idle cost are incorporated into the objective of the model.

In this paper we study the relationship between the processes of order selection and production scheduling and propose a new nonlinear 0–1 programming model aiming at profit maximization. Our new model is a generalization and improvement of Slotnick-and-Morton's model (Slotnick and Morton 2007) as we consider simultaneously the optimal order selection and production scheduling.

13.2 Slotnick's Order Selection Model

The model proposed by Slotnick and Morton (2007) has the following assumptions:

- The production capacity is limited.
- Only a part of the orders to fulfill is selected in order to maximize the overall profit.
- The set of orders to be selected from is given at time zero, with the complete specification such as processing time, delivery due date, delay cost, and profit for each order.
- Linear delay cost.

For the i-th order, let q_i be its profit if it is finished on time; p_i be the processing time; d_i be the delivery due date; w_i be the delay cost for the i-th order. Then Slotnick-and-Morton's order selection model is formulated as follows:

$$\max \quad \sum_{i=1}^{n} x_i \left[q_i - w_i (c_i - d_i)^+ \right]$$

$$\text{s.t.} \quad c_i = \sum_{j=1}^{i} x_j p_j, \ x_i \in \{0, 1\}, \ i = 1, \ldots, n,$$

where the decision variable c_i is the actual delivery date of the i-th order and the decision variable x_i is binary: $x_i = 1$ or 0 means i-th order is accepted or not. This model aims at maximizing the overall profit and if a selected order is finished earlier than the delivery due date, there would be no additional profit to the production enterprise, i.e., it is of no use finishing an order too early.

One key issue in Slotnick-and-Morton's model is how to rank the n orders before the order selection. Given different ranking schemes, the optimal solutions for the order selection problem would be different. This fact is demonstrated in the following example.

Example 1 In this example, there are four orders to be selected from. Their specifications are in Table 13.1. If the orders are ranked using the FCFS scheme, the ranking would be $(1, 2, 3, 4)$ and the optimal solution of Slotnick-and-Morton's model is to select orders $1, 2, 3$ and forego order 4, resulting a profit of 60. If the orders are ranked using the SPT scheme, the ranking would be $(2, 3, 1, 4)$ and the optimal solution of Slotnick-and-Morton's model is to select orders $2, 3, 1$ and forego order 4, resulting a profit of 64.

Table 13.1 The data of Example 1

Order i	Processing time p_i	Due date d_i	Delay cost coefficient w_i	Profit q_i
1	8	22	5	19
2	5	20	3	28
3	7	19	4	17
4	9	15	6	32

Slotnick-and-Morton's order selection model separates the processes of production scheduling and order selection. Under different order ranking schemes, the optimal solutions could be different. This has been shown in Example 1. Thus the true global optimal solution cannot be guaranteed by solving this model. This motivates us to find a global optimization model that integrates the processes of production scheduling and order selection together.

13.3 New Order Selection Model

In flow-shop production, production scheduling has a direct impact on the decision of order selection. On the other hand, the outcome of order selection can also change the final production schedule. In this section, to generalize and improve Slotnick-and-Morton's model, we study the relationship between the processes of order selection and production scheduling and propose a new order selection model that integrates these two processes.

13.3.1 Basic Assumptions

Our new model takes the following additional assumptions except for those of Slotnick-and-Morton's model:

- The order selection is conducted for a single planning period.
- The n orders to be selected have been ranked from 1 to n so that the i-th order must be fulfilled before the $(i+1)$-th order.
- No inventory cost.

13.3.2 Model Description

Index the orders from $j = 1, \ldots, n$, where n is the total number of orders to be selected from. For order j, let q_j be its profit if it is finished on time; p_j be the processing time; d_j be the delivery due date; w_j be the delay cost coefficient for the

i-th order. Index the position in production schedule from $i = 1 \cdots, n$. Then our new order selection model is formulated as follows:

$$\max \quad \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij}\left[q_j - w_j(c_j - d_j)^+\right]$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{ij} \leq 1,\ j = 1, \ldots, n, \tag{3.1}$$

$$\sum_{j=1}^{n} x_{ij} \leq 1,\ i = 1, \ldots, n, \tag{3.2}$$

$$s_0 = 0,\ s_i = s_{i-1} + \sum_{j=1}^{n} p_j x_{ij},\ i = 1, \ldots, n, \tag{3.3}$$

$$c_j = \sum_{i=1}^{n} s_i x_{ij},\ j = 1, \ldots, n, \tag{3.4}$$

$$x_{ij} \in \{0, 1\},\ i, j = 1, \ldots, n, \tag{3.5}$$

where the decision variables are explained as follows:

- When x_{ij} is set to 1, then order j is selected and it is the i-th order to be fulfilled. When x_{ij} is set to 0, then order j is given up.
- s_i is the finished time for the i-th order to be fulfilled.
- c_j is the actual delivery date of order j.

Constraint (3.1) ensures that each order is fulfilled at most once. When it is not binding, some order is not selected and thus given up. Since the production enterprise has a limited capacity, in order to maximize the profit, some of the orders will be foregone. Constraint (3.2) ensures that the i-th order to be fulfilled is unique, because we do not allow for fulfilling two orders at the same time. Constraint (3.3) ensures that s_i is the finished time for the i-th order to be fulfilled. Constraint (3.4) ensures that c_j is the actual delivery date of order j.

Our new model still aims at maximizing the overall profit. But we have integrated the processes of order selection and production scheduling, and thus no prior ranking scheme is needed. In the following example, we continue to use the problem in Example 1 to demonstrate the effectiveness of our new model.

Example 2 To tackle the problem in Example 1, we use Lingo 11.0 to solve our new model. The resulting optimal solution is displayed in Table 13.2, This table shows that the optimal solution for our new model is to fulfill orders $4, 2, 1$ sequentially. And the overall profit for this optimal solution is 79. We can show that this is indeed the globally optimal solution by enumerating all the combinations.

Table 13.2 The optimal solution of Example 2

	x_{ij}	j 1	2	3	4
i	1	0	0	0	1
	2	0	1	0	0
	3	1	0	0	0
	4	0	0	0	0

Table 13.3 The computational results of the new model

Number of orders	Worse case	Best case	Average computational time
4	14	1	8.6
8	385	9	178.2
12	2154	47	900.6
16	3593	232	1842.8

13.3.3 Model Complexity

Because our new model is a nonlinear mixed-integer programming problem, its computational complexity grows exponentially as the number of orders increases. To test the computational effectiveness of our model, we randomly generate four group instances with $n = 4, 8, 12, 16$. Each group contains 10 instances. Each instance is solved by Lingo 11.0 on a 2.20 Ghz thread with 2G memory. The computational time (in seconds) is summarized in Table 13.3. The above table shows that when the number of orders is less than 16, the order selection problem can be solved to optimality within an hour by our new model.

13.4 Conclusion

Traditional order selection models separate the processes of production scheduling and order selection. The performance of the order selection depends entirely on production scheduling. In this paper we have proposed a new nonlinear 0–1 programming model that integrates the processes of order selection and production scheduling. Our new model can find better solutions than traditional models in terms of profit maximization. We have solved the new model using Lingo 11.0 and numerical results show that the optimal solution can be obtained within an hour on a personal computer when the order size is less than 16. Because the problem is NP-hard in general, when the number of orders increases, the computational complexity grows exponentially. In future research, we will apply the exact methods (Wang et al. 2007; Duan Li et al. 2007) or develop heuristics for our new model targeting larger problem sizes, which would be critical to effectively solve the problem in the big data era when the amounts of orders are massive. On the other hand, the big data technology could provide a possible way to deal with the estimation errors of parameter in the new model.

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