



# Probabilistic Analysis of Shallow Landslide Susceptibility Using Physically Based Model and Fuzzy Point Estimate Method

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## Abstract

The geomechanical parameters of soils used in physically based model for landslide susceptibility analyses are uncertain due to the inherent uncertainty and variability. In addition, limited sampling is another source of the uncertainty since the input parameters were obtained from very wide study area. Therefore, the analysis of rainfall-induced shallow landslides susceptibility using physically based model requires accounting for the uncertainty. Subsequently, the probability theory has been used to quantify the uncertainty. However, some uncertainties, caused by incomplete information, cannot be managed satisfactorily by probability theory, so fuzzy set theory is more appropriate in the case. In this study, the uncertain parameters in landslide susceptibility analysis were expressed as fuzzy numbers and fuzzy set theory was employed. In order to take into account the fuzzy uncertainties in the evaluation of the probability of failure, point estimate method was applied with fuzzy set theory. This proposed process was performed in GIS based environments since GIS has strong spatial data processing capacity. In order to check the feasibility of the proposed approaches, the proposed methods were applied to a practical example. To evaluate the performance of the model, the results of the landslide susceptibility assessment were compared with the landslide inventories using ROC graph. Based on the results of the practical application, it was concluded that the application of fuzzy set theory shows consistent analysis results and can obtain reasonable results.

## Keywords

GIS • Uncertainty • Fuzzy point estimate method

## Introduction

Since landslide is one of the repeated geological hazards and causes a terrible loss of life and properties, various researches have been carried out to evaluate the susceptibility of

landslide, and can be divided into qualitative approach (inventory based and knowledge driven methods) and quantitative approach (data driven methods and physically based models). Recently, the physically based model approaches were widely used because they have a higher predictive capability and are the most suitable for quantitatively assessing the influences of individual parameters that contribute to landslide initiation (Corominas et al. 2014). The advantage of the approach is that they are based on slope stability models, allowing the calculation of quantitative values of stability. Therefore, the infinite slope model, one of the most widely used slope stability models, has been used to evaluate the factor of safety as quantitative index of stability for rainfall-induced shallow landslide. However, the

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drawback of this method is that the large amounts of reliable input data are needed but it is difficult to obtain large spatial data set from broad area. Therefore, due to limited amount of data, the uncertainties were inevitably involved in this approach.

A large amount of uncertainties, involved in the analysis of physically based model, such as spatial variability and uncertainties in input parameters are the reason for the discrepancy between field response and results of theoretical model (Zhang et al. 2016). Consequently, the probabilistic approach has been used to account for the uncertainties in slope stability analysis (Shou and Chen 2005; Shou et al. 2009; Santoso et al. 2011; Park et al. 2013; Zhang et al. 2014). However, the probabilistic analysis is associated with difficulties due to the lack of information typical in landslide studies. The data needed for utilization of mathematical statistics are frequently not available to a sufficient extent and quality. In addition, some uncertainties connected to measured geotechnical parameters may be non-stochastic (Juang et al. 1998). This is because some uncertainties, especially those based on incomplete information, are due to cognitive sources (Zimmermann 2001). Under such conditions of limited information, it appears reasonable to base estimation on the concepts of fuzzy set (Dodagoudar and Venkatachalam 2000). Fuzzy set theory has been effective and suitable for modeling uncertainty in geotechnical parameters when data are insufficient to fully define a probability distribution (Luo et al. 2011). Consequently, the fuzzy set theory has been used in several single slope stability analysis (Lee and Juang 1992; Davis and Keller 1997; Dodagoudar and Venkatachalan 2000; Giasi et al. 2003; Li and Mei 2004; Park et al. 2012). In the case of landslide susceptibility analysis, the site specific input data used in physically based model are often limited and consequently, the uncertain parameter should be considered as fuzzy numbers. Therefore, in this study, the fuzzy set theory was employed to evaluate the landslide susceptibility over broad area. Then the results of fuzzy based analysis were compared with the results of the probabilistic analysis.

## Fuzzy Point Estimate Method

### Fuzzy Set Theory

In classical set theory, an element either belongs or does not belong to the set. A set can be defined by membership function that declares which elements of  $x$  are members of the set and which are not.

$$\mu_A(A) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases} \quad (1)$$

For each  $x \in A$ , when  $\mu_A(x) = 1$ ,  $x$  is declared to be a member of  $A$ . When  $\mu_A(x) = 0$ ,  $x$  is declared to be a non-member of  $A$ .

However, in fuzzy sets, more flexible sense of membership is possible (Zadeh 1965). That is, the membership function can be generalized such that the values assigned to the elements fall within a specified range. In fuzzy set, the degree of membership to a set is indicated by a number of between 0 and 1.

In fuzzy set theory, each fuzzy set is defined by a membership function. Since an element's membership function in a fuzzy set may admit some uncertainty, its membership is a matter of degree. The membership function can be manifested by many different types of function and different shapes of their graphs. Triangular and trapezoidal shapes are most common types in the membership function. Figure 1 shows the concept for support, core and height in a trapezoidal shaped fuzzy set. The support is the set of all elements of set  $x$  that have nonzero membership in  $A$ . In addition, core is the set of all elements of  $x$  for which the degree of membership in  $A$  is 1. The height of a fuzzy set  $A$  may be defined as the largest membership grade obtained by an element in that set. If the height of a fuzzy set  $A$  is 1, set  $A$  is called normal and otherwise, it is called subnormal.

### Alpha ( $\alpha$ )—Cuts for Fuzzy Sets

The  $\alpha$ -cut of a fuzzy set  $A$  can be defined as:

$$A = \{x \in X | \mu_A(x) \geq \mu\} \quad (2)$$

for  $\mu \in [0, 1]$

The  $\alpha$ -cut of a fuzzy set  $A$  is the crisp set  $A_\alpha$  that contains all the elements whose membership grades in  $A$  are greater than or equal to (or only greater than) the specified value of  $\alpha$ . The  $\alpha$ -cut concept means the discretizing of a fuzzy number into a group of  $\alpha$ -cut intervals. For each of the

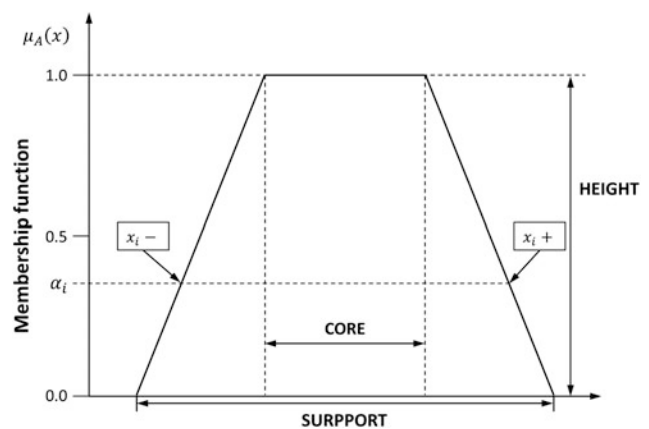


Fig. 1 Concept of fuzzy membership function

uncertain parameters, the  $\alpha$ -cut of a fuzzy set will give an interval having two points, i.e. upper and lower bound values for a particular  $\alpha$ -cut (Dodagoudar and Venkatachalam 2000).

### Fuzzy Based Point Estimate Method

According to extension principle of Zadeh (1965), algebraic operation on real numbers can be extended to fuzzy numbers. But the implementation of the computation is not trivial. Therefore, several authors proposed solution procedure but it is complicated and does not guarantee solution. A simple way is to use the discretization technique and consequently, the vertex method, which is based  $\alpha$ -cut concept and interval analysis (Dong and Shah 1987) was proposed. In the vertex method, uncertainty parameters can be expressed as an interval, involving an estimate of the lower and upper bounds, using  $\alpha$ -cut concept. By replacing fuzzy numbers with intervals, the point estimate method, which was proposed by Rosenblueth (1975) can be used to evaluate the uncertain parameters in performance function such as infinite slope model. The point estimate method can evaluate the mean and standard deviation of the performance function using only two point estimates of uncertain parameters. Therefore, the vertex method in fuzzy number calculation can be coupled with the point estimate method. Consequently, the fuzzy based point estimate method can obtain the mean and standard deviation for factor of safety (FS) in slope stability model. Then using the mean and standard deviation of FS, the reliability index and the probability of failure can be obtained (Baecher and Christian 2003). However, in order to represent the overall variability in FS value and to determine the expected FS, all the nine  $\alpha$ -levels from 0.1 to 0.9 are considered in the evaluation of expected value and standard deviation of FS for fuzzy based point estimate method.

If the performance function for the evaluation of FS is  $FS(x)$ , the sum of the upper and lower bound value of FS is calculated at each  $\alpha$ -level using Eq. 3.

$$w_{\alpha i}^r = p_+ FS^r(x_{\alpha i +}) + p_- FS^r(x_{\alpha i -}) \quad (3)$$

The expected value of FS and standard deviation of FS are estimated using Eq. 4.

$$E[W^r] = \frac{\sum_{i=1}^N w_{\alpha i}^r}{N} \quad (4)$$

Therefore, the expected value and standard deviation of FS can be evaluated from the above equations using fuzzy based point estimate method.

In this study, infinite slope model coupled with TRIGRS was used for the slope stability model in the evaluation of

safety factor for the physically based model approach (Baum et al. 2002, 2008; Savage et al. 2004).

### Study Area and Database Construction

The Inje area, which is located in the Gangwon Province, was selected as the study area for this study to assess landslide susceptibility using the proposed analysis method. This area was experienced a large number of landslides in July 2006, due to Typhoon Ewiniar and heavy rainstorm. On 15–16 July 2006, this area experienced heavy rainfall of 332.5 mm and approximately 800 landslides were reported.

The geographical coordinates of the area are longitudes between  $128^\circ 11' 44.81''$  and  $128^\circ 18' 8.99''$  and latitudes between  $38^\circ 3' 3.93''$  and  $38^\circ 15' 58.55''$ . The total study area is  $31.65 \text{ km}^2$ . The altitude of this area ranges from 215 to 1220 m, with an average altitude of 660 m.

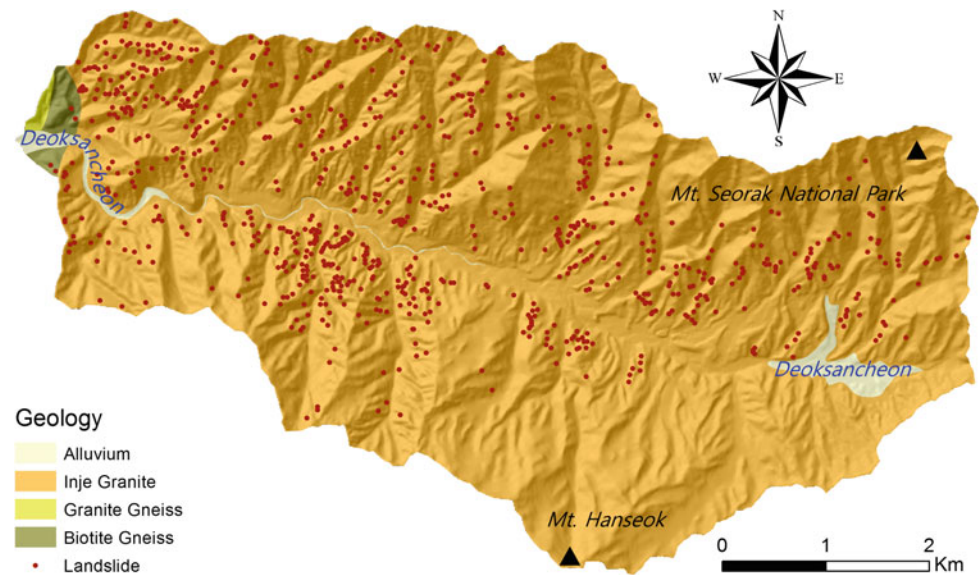
As seen in Fig. 2, the area is located in a high altitude region with Mt. Seorak in the northeast and Mt. Hanseok in the south with the Deoksancheon stream in the center flowing from southeast to northwest. The eastern region has a higher altitude than the west while the slopes are steeper in the lower lying west. Geologically, this area is composed of mainly the Mesozoic Inje granite and partly the Precambrian biotite gneiss and the limited deposits of alluvium along the lower end of the stream.

The landslide inventory is one of the most important factors in landslide susceptibility analysis because the accuracy of the landslide prediction models can be evaluated. In this study, landslide inventory for susceptibility analysis was acquired through aerial photographs and confirmed by field surveys. Aerial photographs with a ground resolution of 0.5 m were obtained from Samah Aerial Survey Co. Ltd before and after the landslide occurrence. A landslide inventory map was constructed from 877 landslide locations (Fig. 2).

The geomorphological characteristics such as slope angles and elevation were extracted from a scale of 1:5000 digital topographic maps, provided by National Geographic Information Institute of Korea, and a digital elevation model (DEM) with a 10 m resolution was constructed. The DEM was used to calculate the thematic maps, which related to slope stability factors, such as slope angle and elevation. Further, the soil thickness in the study area was acquired from 1:25,000 scale digital soil maps produced by the National Institute of Agricultural Science. The soil thickness was evaluated from the depth to bedrock and used as soil depth in the infinite slope model. Applying the Z-model (Saulnier et al. 1997) to the altitude thematic map, soil thickness in the area was mapped into the thematic map.

In order to evaluate the landslide susceptibility using physically based model, the geotechnical input parameters

**Fig. 2** Geological map and landslide inventory



such as cohesion and friction angle for the soils should be obtained. In this study, the requisite input parameters were obtained from laboratory tests for the study area. Soil samples were collected from the landslide occurrence locations in each geological unit. For each sampling location, six to nine soil samples were obtained for the laboratory tests and direct shear tests were performed to obtain shear strength parameters for each soil type. The unit weights of each soil type were also obtained from laboratory tests. However, as mentioned in previous works (Xie et al. 2004; Shou and Chen 2005; Huang et al. 2006; Zolfaghari and Heath 2008; Griffiths et al. 2011), cohesion and friction angle of slope materials were considered to be the major sources of uncertainty because of spatial variability and limited sampling. Thus, cohesion and friction angle were considered as uncertain variables. That is, the cohesion and friction angle were considered as fuzzy numbers in this proposed analysis approach.

In this study, in order to compare the proposed method with the probabilistic analysis, the uncertain parameters were considered as random variables and the Monte Carlo simulation was used to evaluate the probability of failure. The random input parameters used in Monte Carlo simulation were as considered as normally distributed variable as many previous researches suggested (Liu and Wu 2008; Zolfaghari and Heath 2008; Wang et al. 2010; Melchiorre and Frattini 2012; Park et al. 2012, 2013).

The precipitation input for the physically based model, such as rainfall intensity, is one of the most important parameters to be obtained. Rainfall intensity values were obtained from Inje AWS (automatic weather system) the closest and most representative rain gauge in the study area. The rainfall intensity for the study area was obtained from the hourly rainfall records during the rainstorm on 15–16

July 2006. At that time, the rainfall intensity was an average 10.08 mm/h during the 37 h, and maximum amount of rainfall recorded 66 mm/h.

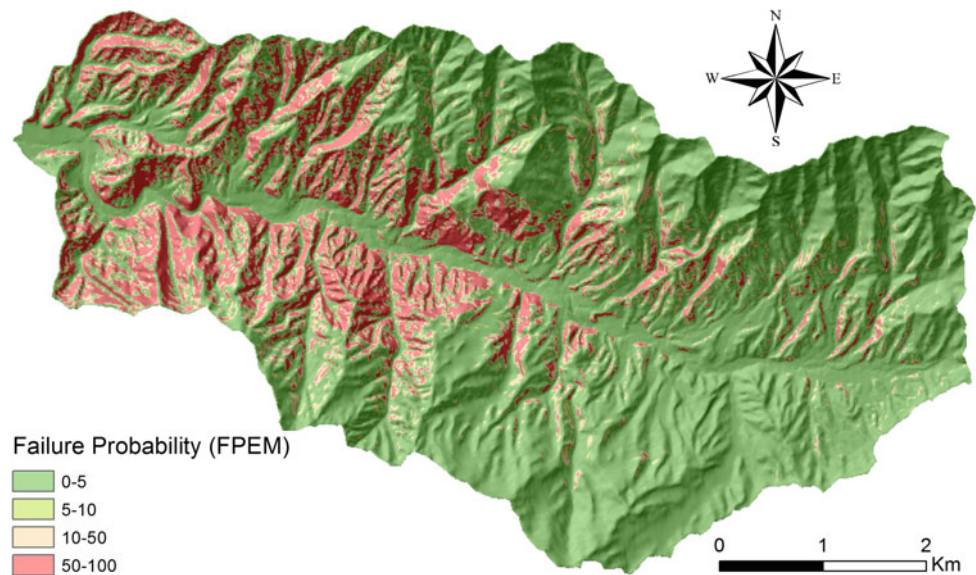
## Results and Conclusions

In this study, the proposed analysis was conducted following the procedure using point estimate method coupled with fuzzy membership function of the input parameters. First, uncertainties of the geomechanical parameters, such as cohesion and friction angle, were quantified as fuzzy numbers, into a group of  $\alpha$ -level intervals. Each of the fuzzy variables is to be discretized into appropriate  $\alpha$ -level intervals. At each selected  $\alpha$ -level, an interval is obtained for each of the two uncertain parameters. Totally nine  $\alpha$ -levels are considered, i.e. 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9, in order to represent the possible variability in the input parameters. The expected value and standard deviation of FS are obtained using fuzzy based point estimate method and then, the reliability index  $\beta$  and subsequently, the probability of failure is evaluated. This procedure repeated through all the pixels in the study area in the GIS environments.

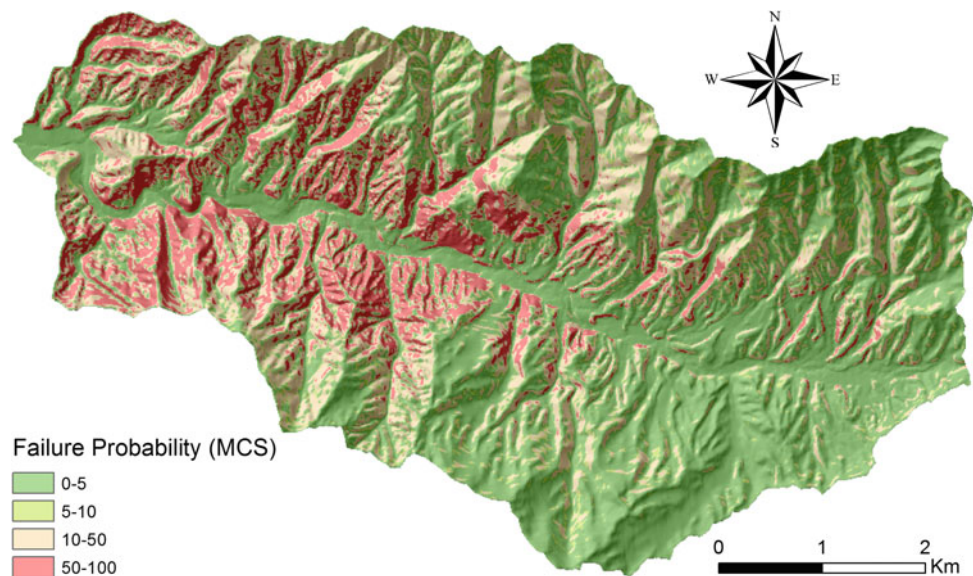
Figure 3 shows the spatial distribution map for the probability of failure calculated using the coupled infinite slope model with the fuzzy point estimate method. As can be seen in Fig. 3, 68.3% of actual landslides (or the mapped landslides in the inventory map) were evaluated as unstable, meaning that the TPR, which is the ratio of the number of correctly predicted landslide grids (true positives) to the total number of landslide occurrence grids (positives), was calculated as 0.683. In addition, 27.0% of non-landslide grids were predicted as unstable,



**Fig. 3** Probability of failure evaluated using fuzzy point estimate method



**Fig. 4** Probability of failure evaluated using Monte Carlo simulations

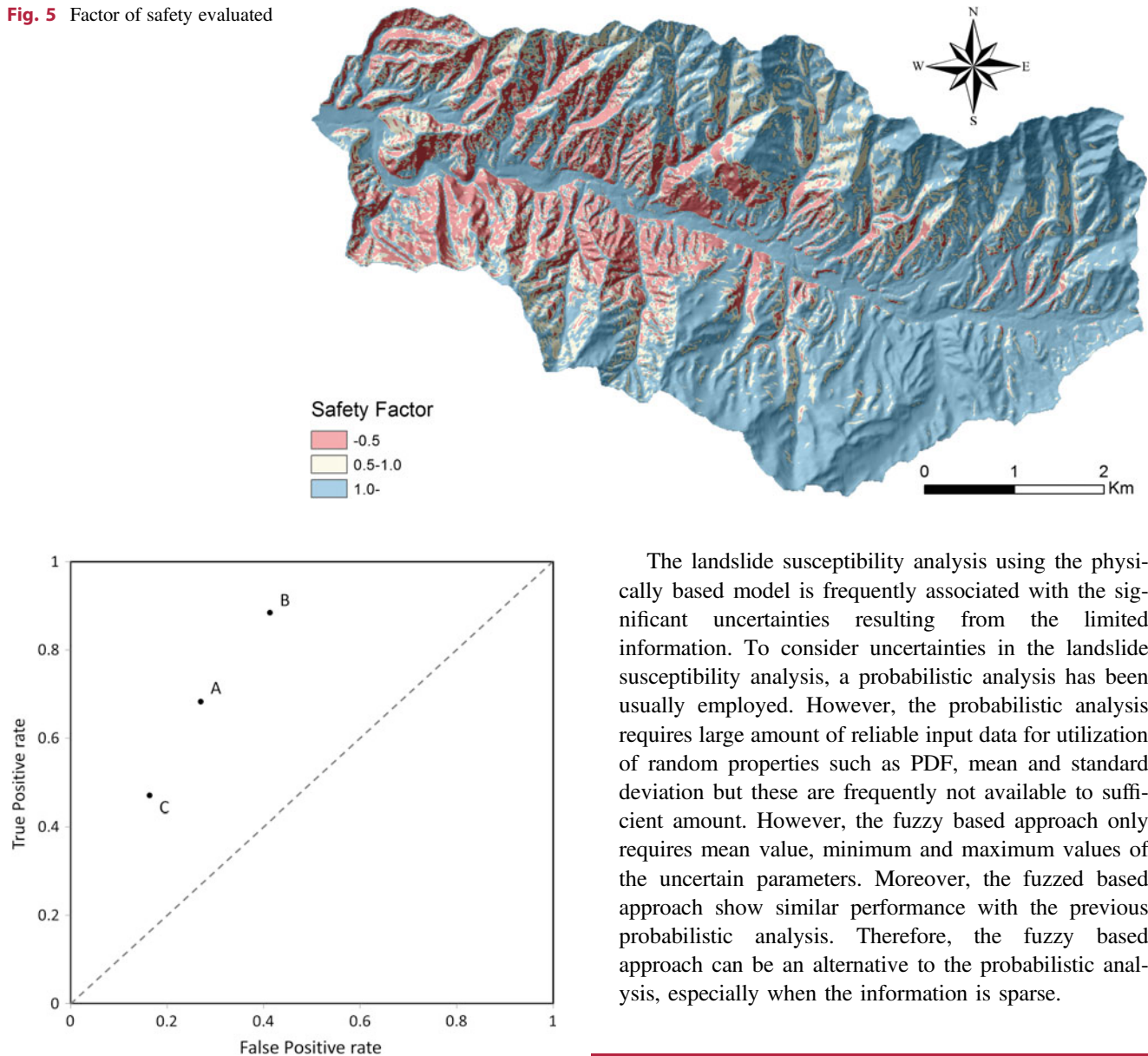


which means that the FPR was 0.270. Thus, the evaluated AUC from ROC graph in Fig. 6 was 0.707, and the TPR/FPR ratio was 2.534.

To compare the results of fuzzy point estimate method with the results of the probabilistic analyses, the Monte Carlo simulation was performed as the probabilistic analysis (Fig. 4). In Monte Carlo simulation (MCS), the random properties of input parameters (mean, standard deviation) were used for cohesion and friction angle. In order to obtain these, the mean and standard deviation was calculated from laboratory test results in this study, while probability density function was assumed to be normal distribution, in reference to earlier studies. TPR and FPR values were evaluated as 0.885 and 0.413,

respectively. Then AUC was evaluated as 0.736, and TPR/FPR ratio was 2.144.

Further, the deterministic analysis was also conducted on the basis of the factor of safety concept in order to compare the fuzzy based approach and the probabilistic analysis with the deterministic analysis. In the deterministic analysis, the infinite slope model with mean values of the random variable (such as, cohesion and friction angle) as the representative single values for the deterministic analysis were used. Figure 5 shows the spatial distribution map for the factor of safety calculated. TPR and FPR values were evaluated as 0.471 and 0.163, respectively. Then AUC and TPR/FPR ratio were evaluated as 0.654 and 2.884.

**Fig. 5** Factor of safety evaluated**Fig. 6** ROC graph comparing with the probabilistic analysis using **a** FPEM, **b** MCS and **c** the deterministic analysis

As can be seen in Fig. 6, the AUCs of the FPEM analysis (0.707) and the probabilistic analysis (0.736) showed reasonable performance and higher than the AUC of the deterministic analysis (0.654), which means the FPEM and MCS showed superior performance than the deterministic analysis. Even if the AUC value of FPEM is lower than the AUC of the MCS, another value to compare the performance, TPR/FPR ratio shows that the performance of the FPEM is better than the performance of the MCS. Consequently, the FPEM and MCS show similar performance.

The landslide susceptibility analysis using the physically based model is frequently associated with the significant uncertainties resulting from the limited information. To consider uncertainties in the landslide susceptibility analysis, a probabilistic analysis has been usually employed. However, the probabilistic analysis requires large amount of reliable input data for utilization of random properties such as PDF, mean and standard deviation but these are frequently not available to sufficient amount. However, the fuzzy based approach only requires mean value, minimum and maximum values of the uncertain parameters. Moreover, the fuzzed based approach show similar performance with the previous probabilistic analysis. Therefore, the fuzzy based approach can be an alternative to the probabilistic analysis, especially when the information is sparse.

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