Hans Gersbach

Redesigning Democracy

More Ideas for Better Rules

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Preface

To those who live under tyranny, democracy is an ideal worth fighting for. Many people strive to establish democracy in their own country; equal voting and agenda-setting rights, separation of powers, civic liberties such as freedom of speech and the right to form political parties, appeal to an elementary sense of justice. Democracy distributes decision power equally among citizens, regardless of wealth or education, through the right to vote. As a matter of fact, democracy is the only form of government that can make citizens the owners of their state and can sustain high standards of living.

Yet, democracy-born citizens, who are used to its benefits, sometimes barely acknowledge its existence and do not exercise their right to vote. Although many are putting their life at risk to obtain this right, those who live in a democracy might —if asked—even underline its failures rather than its benefits. These inefficiencies are worth exploring, and have been studied for many decades now.

In this successor of *Designing Democracy*, which was published in 2005, we follow an unexplored route: Are there new forms of democracy that can overcome current shortcomings and achieve higher welfare than the ones of existing democracies? We will present a set of improvements for democracy that have the potential to foster the voters' trust in their own power of decision, and ultimately, in democracy itself. This trust, in turn, could revive the citizens' interest and might improve welfare.

This book is divided into two parts: In the first part, *Contractual Democracy*, we assess those inefficiencies of democracy that depend on the politicians' behavior after their election to office, and suggest to control this behavior through contracts that define rewards and punishments for the office-holders' actions and foster the selection of able office-holders. We show that a judicious linking of such contracts to elections may alleviate a wide range of inefficiencies, while complying with the fundamental principles of democracy.

In the second part, we address the decision process itself, assess possible inefficiencies and present *New Rules for Decision-making and Agenda-setting* that have the potential to yield socially desirable outcomes. Among other rules, we examine flexible majority rules, according to which the size of the majority required to make a decision depends on the contents of the proposal. Another rule we examine is the minority-voting rule that requires that only losers from a first vote on a project decision can determine its financing scheme. Moreover, we explore how proposal-making can be channeled in such a way that it yields socially optimal proposals. Finally, we briefly describe new ideas for voting rules, such as *History-bound Reelections*, *Assessment Voting*, and *Co-voting Democracy*.

When *Designing Democracy* was published, the chances and limits of our research could not be fully estimated yet. But since 2005, our ideas have been widely discussed—a sign that democracy is alive and that its improvement is not only possible, but possibly desirable for society. Thus, we have expanded the scope of our work to include unpublished working papers in this second book on democracy. They complement and support the issues we discussed in *Designing Democracy*, so that *Redesigning Democracy* now offers insights into more than a decade of policy research.

I take great pleasure in expressing my thanks to many friends and colleagues who have helped me critically assess the ideas. Peter Bernholz, Ulrich Erlenmaier, Lars Feld, Volker Hahn, Hans Haller, Matthew Jackson, Verena Liessem, Maik Schneider, and Joel Sobel challenged and improved the models and arguments presented in this book. I benefited from discussions at the annual meetings of the European Public Choice Society, the Econometric and European Economic Association, the German Economic Association, at seminars on my tour of California (Universities of Los Angeles, Davis, Irvine, and San Diego) and at seminars in Basel, Bern, Cologne, Heidelberg, Leuven, Mannheim, Rotterdam, St. Gallen, Tilburg and Zürich. I am deeply grateful for various comments and help along the way from Clive Bell, Robert Dur, Jürgen Eichberger, Sylvester Eijffinger, Christoph Engel, Theresa Fahrenberger, Marc Fleurbaey, Peter Funk, Joao Gata, Amihai Glazer, Hans-Peter Grüner, Martin Hellwig, Stephan Imhof, Susanne Lohmann, Mark Machina, Wolfgang Merkel, Roger Myerson, Thomas Petersen, Till Requate, Annette Schiller, Manfred Schmidt, Armin Schmutzler, Robert Solow, Otto Swank, Christian Schultz, Eva Terberger, Jean Tirole, and Heinrich Ursprung. Finally, I would also like to thank Tettje Halbertsma and Andrew Jenkins for their excellent research assistance, as well as Markus Müller and Johannes Becker for their outstanding cooperation.

Zürich, Switzerland Hans Gersbach
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Chapter 1
Introduction to Part I

1.1 A Metaphor?

One might reason that political offices function like a private labor market and that a political office-holder is the voters' employee. Every applicant for an open position advertises his ability to fill this position successfully. The prospective employer chooses the applicant that seems the most able and the best match, and after a pre-defined period, he assesses his employee's performance and either keeps him or has him fired. The details of employee's and employer's rights and duties are put on record in a contract.

Quite similarly, a political candidate proposes a work agenda, advertises his ability to implement it and if elected, is appointed office-holder for a certain period. During his term in office, he receives a "salary", i.e. a reward package consisting of money, perks, and honor, and is given the authority and resources to put the promised agenda into effect. Towards the end of his term, his performance is assessed, and like any employee who will be kept for good work and fired for poor performance, the office-holder will be reelected if he has performed well and deselected if he fails to do so—theoretically.

The relationship between voters and members of the legislature or of executive boards, however, differs from the relationship between employer and employee in some fundamental aspects. First of all, "equal voting rights" suggests that voting has to be the sole device used to appoint office-holders. Second, voters disagree about many policy issues, rendering it difficult to determine what a good performance is, i.e. which policy measures will improve social welfare. Different answers can be optimal when it comes to the question how much a society should redistribute, for example.

Another important issue is that several aspects, such as separation of powers in the state, parliamentary procedures and referenda, as well as the state's monopoly of power, the protection of basic rights and elementary functions of the state, determine and constrain an office-holder's activities: Performance does not solely depend on ability and effort.

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In addition, elections and reelections seem to be a rather crude device to select and to motivate office-holders, and they encounter several difficulties. Reelection, for instance, does not seem to be linked as directly to the office-holders' performance as one might think. An office-holder could be reelected simply because he is better-known than his challengers thanks to his first office-term, or he might benefit from favorable shocks that affect his jurisdiction. He might also be reelected because the voters do not expect his challengers to perform better and prefer to retain the office-holder, possibly hoping for a certain "learn-by-doing"-effect. Furthermore, an office-holder might pursue policies that are favorable to particular interest groups, which, in turn, might provide stronger support for his reelection bid.

Up to now, deselection seems to be the voters' sole weapon against lack of effort and ability, and this weapon cannot warrant optimal effort, let alone good performance—even if applied consistently.

The length of the office-term is another problem. One full term being much longer than an employee's trial period, a bad performer is costly, even if he is deselected after his first term. On the other hand, this office period can prove too short as well: Despite the fact that during his electoral campaign, the candidate promised to implement long-term projects, an office-holder might give priority to short-time results that will foster his reelection, and neglect those projects that take longer than one term.

Such complications might suggest that new incentive and selection devices cannot be used in democracy to mitigate performance problems. One might even argue that they are not necessary: Good performance and farsighted policy are among the candidates' campaign promises. Yet, many office-holders' actual performance denotes a shift in priorities after election—towards less socially desirable. The other way around, candidates make promises to pander to the public's opinion, whose fulfillment is not socially desirable in the long run. We still end up with the same basic question: How can we foster good performance in democracy?

1.2 Political Contract: Definition

What we need is a device that can complement elections and can be integrated in a democracy. One of our major ideas is that office-holders should be rewarded for promises kept and punished for promises broken. Then, the candidates should promise more realistically and, once elected, invest more effort into keeping their word. But how to link such promises to rewards and punishment? This is where the employee metaphor is helpful: Through a contract.

Yet, the employee metaphor can only start reflection, and our notion of contract must be adapted if we want to use it for political office-holders: Democracy requires new ways of contracting. In particular, a "political" contract would not be a contract of the usual type signed by two parties, but a declaration signed by the candidate and certified by an independent authority. We call these certified declarations "Political

Contracts".[1] They contain the performance a candidate will have to deliver if elected, together with the reward due if he fulfills these promises, and/or the corresponding punishment if he fails to do so. To certify the contract, to evaluate performance, and to award rewards and punishments, an "Independent Authority" is needed. One could imagine a newly-created institution or an already-existing authority being entrusted with this new set of tasks.[2]

Thus, Political Contracts differ from a contract of employment signed in every-day life.[3] A judicious combination of Political Contracts and elections is characterized by the following chronology:

1. Candidacy
   Campaign promises are tied into a Political Contract
   (Performance – Reward – Punishment)
   Certification – Publication
2. Election
3. First office-period
   (a) Implementation of campaign promises
   (b) Assessment of performance
   (c) Independent Authority declares whether the Political Contract was kept or not
   (d) Reward/punishment is declared due
   (e) Reward/punishment is implemented
4. Second Candidacy
5. Reelection/deselection

Note: *While both reward and punishment could take place at any given time after performance assessment, they have to be put on record at Stage (1). Stage (3d) only consists of a declaration by the Independent Authority, stating which of the two is due and will be awarded. The implementation date itself depends on the type of reward or punishment that was chosen for the Political Contract, as we will see in what follows.*

The candidates would not be forced to sign Political Contracts, but as such a contract enhances credibility, competing candidates are likely to resort to such contracts during electoral campaigns. Of course, Political Contracts only become effective if the candidate is elected.

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[1] A more detailed survey of the current state of affairs on Political Contracts can be found in Gersbach (2012).

[2] In Germany, it could be the Federal President (see Gersbach and Schneider 2012b).

[3] As they are certifiable and enforceable, they cannot be compared to the theoretical "social contract" that free men contract with each other to establish civil society, as analyzed in Hobbes' *Leviathan*, for instance. A social contract can be implemented by a set of constitutional rules (see Aghion and Bolton 2003 and Gersbach 2009a, b).

Let us now address the main issues arising from this setting, and start with a crucial point: Whatever is settled in a Political Contract must comply with the fundamental rules of democracy.[4]

1.3 Does Any Campaign Promise Qualify as Contract Matter?

To qualify for inclusion in a Political Contract, a performance has to be definable and measurable with a sufficient degree of precision. Ideally, it consists of a single figure such as the GDP, the unemployment rate, a particular crime index, the level of CO_2 emissions or a debt level to be reached. If the outcome of a particular policy is measurable by social, macroeconomic or environmental indicators, it can be tied into a Political Contract. The contract must contain a precise description of the performance indicator to be used for evaluation. A neutral third party to be entrusted with the collection and verification of the relevant data is also necessary. This third party could either be appointed to verify all contracts that are offered in a particular election, or appointed from contract to contract, depending of the type of assessment needed, in which case its name and duties should be specified in the corresponding Political Contract.

There are cases in which a simple yes/no answer is possible when asking whether the performance was achieved. The building of a bridge, the abolishment of a law, or the raising of retirement age, for instance, do not require a specific performance index, as the completion of the task is evident.

Greater, more complex projects such as the reform of health care, for instance, might not qualify so easily for a Political Contract, or might require the implementation of alternative measuring procedures for performance assessment. One way to assess performance might be to divide the project into stages (sub-projects), and define in the Political Contract which sub-project is to be reached by a certain deadline. One could imagine a series of deadlines by which each stage of the project has to be completed. One might also restrict the measuring of a project outcome to those parts of the project that are precisely measurable, leaving the non-measurable parts out of the Political Contract.

Such necessary precision precludes certain parts of a candidate's agenda from being inserted into a Political Contract, namely rather "ideological" performances that escape every precise assessment, such as "more social justice". Hence, a number of vague, not-verifiable or even demagogic promises would not qualify for being put into a Political Contract—and would, as a rule, have less credibility. It would be a beneficial side-effect of Political Contracts that their introduction would separate campaign promises into verifiable, credible promises—the ones that can be included

[4] See Gersbach (2012).

in Political Contracts—and those vague, not measurable promises that cannot be part of a Political Contract, and thus tend to be less credible.[5]

Extension – Contracts Offered by Parties

Our main suggestion is to allow every candidate for executive or legislative office to sign Political Contracts, but the signatory does not necessarily have to be a person. A party could also offer a contract, such as a tax contract specifying a range of tax rates to which it will be committed if in government (see Gersbach and Schneider 2012a, b). One can also imagine that parties offer Political Contracts in which they define a list of partners with whom they are willing—or unwilling—to enter a government coalition. If one party has excluded another from its list, it cannot form a government coalition with it after elections. A more detailed assessment of the use of Political Contracts by parties is given in Gersbach (2012), and the particular variant of a Political Contract in which a party precludes another, or others, from any government coalition is analyzed in Gersbach et al. (2014).

1.4 Punishment and Rewards

To perform efficiently as incentives, rewards should please and punishments should hurt. Let us focus on wages, pensions, perks, and immaterial benefits that an office-holder is awarded in exchange for his work.

Punishment

- **Money, perks, and honor**

  Our first idea to foster performance is to tie it to the material goods the office-holder receives, be it money or perks. If an office-holder performs badly, one could reduce his wages, for instance. Gersbach (2003, 2004) outline how such a material punishment might be designed and whether Political Contracts really are offered by candidates during their campaigns. The Political Contract could tie the office-holder's salary to his performance, so that this salary could vary during the office-period. If the office-holder does not achieve the performance defined in his Political Contract, he will earn less. Depending on the kind of performance to be achieved—budget goals, debt levels, growth of GDP or project implementation, for example—, the wages can be adapted yearly. But also an adaption over several office-terms is imaginable, as well as wages divided into a basic, regularly-paid part and a performance-dependent part that is to be paid or refused after performance evaluation.

  One might also link the office-holder's performance to the pension he will receive after his term in office. This would allow to judge performance over a longer time

[5] See Gersbach (2012).

span, which, in turn, would foster the implementation of long-term projects. Ideally, one could take the entire time in office as a basis for performance-assessment (see Gersbach and Müller 2010 and Gersbach and Ponta 2017).

As for the perks and honor that are part of an office-holder's salary, it is not easy to draw the line between material and immaterial rewards. The right to use an official car driven by a chauffeur and fixed allowances are examples of material perks. If the car is of first-grade type, its use could be perceived as an honor or as a manifestation of power, a so-called "ego rent". This might be the case for all advantages an office-holder can make use of, from state airplanes to priority seating at sports events.

All these benefits might be used as punishment tools. By reducing the material reward, a simultaneous reduction of the immaterial reward is achieved automatically.

- **No second term – Term limitation – Higher bar for incumbents**

  An alternative way to punish office-holders is to make it more difficult to obtain all of the material and immaterial benefits of holding-office in future terms. This could be particularly powerful as in a variety of cases, money might not be very efficient as an incentive for good performance. The punishment for bad performance could be very simple:

  No right to second candidacy. One sanction for bad performance could be to repeal the right to candidacy for reelection. If a politician promises to renounce candidacy for a second term if he fails to reach a certain performance threshold, he has a powerful incentive to reach this same threshold. And as he would be the one who *offers* such a sanction, this renouncement would be voluntary and would not challenge every citizen's democratic right to candidacy.[6] This incentive is bound to be inefficient during a last term in office—be it because the constitution limits the number office-terms or because the office-holder knows he will not be a candidate anymore. Thus, one should find a replacement incentive for such particular last-term situations, by which office-holders would be more motivated to excel or even undertake socially desirable long-term projects that might be unpopular in the short-term.[7]

  Premature term ending. One might also imagine a premature ending of the office-term in case of bad performance. Yet, the legal implications are intricate, and such punishments are only imaginable for office-holders behaving like criminals or evidently neglecting the state's core interests.

  Higher bars for incumbents. A particular way to assess performance might be to use the vote-share that an office-holder receives on reelection day as an indicator. This vote-share can be used for a Political Contract, in much the same way one would use a pre-defined performance level. Instead of promising a certain performance,

[6]The chances and drawbacks of contracts tying reelection to a certain performance threshold are analyzed thoroughly in Gersbach and Liessem (2008a, b).

[7]For incentive schemes that can overcome this difficulty, see Gersbach and Müller (2010) and Gersbach and Ponta (2017).

the candidate can promise to work so well during his first term in office that his reelection vote-share will reach a certain percentage. In a traditional two-candidate race, the candidate who obtains 50% of the votes or more is elected. The reelection percentage should be above the one needed for first-term election. If the office-holder fails to work well enough and does not reach this pre-defined reelection threshold,[8] he will not take office for a second term, although his vote-share would have been sufficient in a first-term election. Such a threshold is offered by a candidate during his election campaign, and would be tied into a Political Contract.[9]

Such a higher vote-percentage—possibly making the incumbent's reelection more difficult than a candidate's first-term election—counterbalances the incumbency advantage of the office-holder. Be it because he is better known than his challenger, has access to more campaign funds and support of interest groups, or because he is simply perceived as a "safer bet" by the public, it is usually less difficult for an office-holder to be reelected than for his challenger to obtain office. Knowing this, the office-holder might be tempted to put less than the socially optimal amount of effort into his first-term policy and might be reelected even if his ability is below his challengers'. If a Political Contract stipulates that he has to obtain a higher vote-share for reelection than for election, the office-holder will have to invest more effort during his first term in office to earn the extra votes he needs, and the average effort of office-holders would improve. However, higher bars for incumbents may also cause the deselection of candidates having only average ability. Our latest research on this idea and an analysis of its social desirability are presented in Part I, Chaps. 3 and 4.

- **Loss of public funding or of parties' perks**

  Finally, any kind of benefit an office-holder or his party receives from the public treasury can be reduced—which makes it a tool suitable for Political Contract purposes. One could link the office-holder's performance to the public funds his party receives, and pause the payments for a certain time in case of bad performance. This may be particularly useful if Political Contracts are offered by an entire party.

Rewards

Let us now turn to rewards—the flip-side of punishments. We must emphasize that the importance of a particular reward may vary among candidates: A wealthy candidate might not desire higher wages nor suffer much from a salary cut. And candidates typically differ with regard to their intrinsically-desired mix of monetary and non-monetary benefits. This has to be kept in mind if such schemes are introduced.[10]

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[8]The terms "reelection vote-share" and "reelection threshold" are used synonymously for this scheme, the result being the same. A Political Contract that stipulates a certain pre-defined vote-share for reelection is called a "Vote-share Contract".

[9]Alternatively, the public could set higher reelection thresholds.

[10]See Gersbach (2012) for a detailed discussion on who should set the boundaries for material and immaterial rewards and punishments for office-holders.

- **Money, perks, and honor**

  A raise in salary or suitably-designed rewards—material or immaterial—may be used as incentives for good performance. One could make the payment of a certain percentage of the office-holder's salary dependent on the reaching of particular performance levels. If immaterial rewards are preferred, those politicians who have done particularly well in office could be awarded an honorific title such as "Father of the State", or they could be granted a seat in a particular "State Advisory Board" (see Gersbach 2012).

- **Longer term granted**

  One can imagine that the current office-term is prolonged in case of good performance. If the office-holder has fulfilled his Political Contract by the end of his term, he could remain in office for one more year, for example (see Gersbach 2012).

- **Rewards from the future – RSRs**

  Up to now, our examples dealt with candidates acting on behalf of current voters who are the beneficiaries of "their" office-holder's performance. If they are satisfied with these benefits, they will reelect him. Yet, this might prevent candidates from suggesting long-term policies that will only be beneficial to *future generations*, as they might not win elections with such an agenda. As they do want to be elected, they might only suggest policies beneficial to the *current* electorate.[11]
  To foster the implementation of policies that will benefit younger generations, special types of Political Contracts are needed, as an office-holder undertaking such policies will endanger his reelection. For such a case, a possible Political Contract would define a "Rejection-Support-Reward" (RSR), which would be awarded to this office-holder if he is deselected, but has received the majority of votes from the younger generation.
  As most office-holders want to be reelected, few are likely to implement policies that will threaten their reelection, so that they are unlikely to offer this particular type of Political Contract voluntarily. Thus, Political Contracts that should foster the implementation of projects detrimental to reelection would have to be designed by the public.[12] However, as the public itself may have little interest in introducing RSRs, special procedures allowing their delayed introduction might prove necessary.

Alternative Performance Assessment

As we have seen, performance is easily gauged if there exists some kind of measuring tool for it. A figure such as the percentage of unemployed workers cannot be manipulated easily at short-term notice—at least not if it has been defined precisely and applied consistently—, which makes it suitable for our purpose. If an office-holder has managed to achieve a pre-defined figure, he has fulfilled his Political Contract.

[11] Of course, this does not apply to all circumstances. In particular, statesmen may be willing to undertake policies that are unpopular in the current electorate (see Gersbach 1999).

[12] See Gersbach and Kleinschmidt (2009) for details.

Yet, things might prove complex in many cases. Thus, to evaluate an office-holder's achievements, one might also use an information market to produce a "price"—a figure that can be incorporated into a Political Contract. A first analysis of this idea is outlined in Gersbach and Müller (2010), and our latest research findings on that subject are presented in Part I, Chap. 5.

1.5 Renegotiation – Negative Effects

Should Political Contracts Be Renegotiable?

Of course, circumstances can change in the course of an office-term, so that the enforcement of a contract can become unreasonable, illogical or even damaging to society. A goal that was within reach when the corresponding Political Contract was signed can become impossible to achieve due to unforeseen events. Changing circumstances might also render a goal undesirable, despite the fact that it was considered to be socially beneficial at contracting time. If national security is threatened, for instance, balancing the budget might lose priority.

In such cases, it is necessary to include the possibility to cancel, renegotiate or replace a Political Contract in the contract itself. All three amendments should be subject to a particular form a parliamentary approval such as a super-majority (see Gersbach 2012).

Can Political Contracts Be Counterproductive?

As is the case for most institutional devices, there might be circumstances under which particular Political Contracts may not improve welfare, or might even reduce it. If candidates can offer Political Contracts containing fixed salaries that compete with each other, it could happen that large ability differences allow high-ability candidates to obtain large rents, because the voters might prefer an able candidate, even if he will cost substantially more. As a consequence, competitive wage offers made by candidates can yield lower welfare than remunerations set by the public. Competition with wages may lead to higher costs for the voters, or to the election of less able candidates, if the voters prefer a "cheaper", less able office-holder. Our findings on competition for wages and office are presented in Part I, Chap. 6.

Competition with Political Contracts may also invite candidates to make binding promises just to be elected for one term. Such promises might be attractive for the electorate, but could prove difficult to fulfil, once in office. The office-holder might give up the realization of such promises and settle on one term in office.

1.6 Retrospect and New Developments

Despite possible drawbacks in particular circumstances, Political Contracts have the potential to mitigate some of the problems inherent to democracy. Beside a fresh look at democratic institutions and the willingness to allow for a certain experimentation period, the implementation of Political Contracts would require changes at the constitutional and legislative level, a careful assessment in which areas they might be implemented, and a thorough planning and monitoring of all processes connected. This was addressed in more detail in Gersbach (2012). Yet, our latest research shows that such a challenge is worth accepting. Using the basic structure of Political Contracts outlined above, we want to present our most recent research on that subject, after a brief look backwards, to where it all began.

The Start: Competition of Politicians for Incentive Contracts and Elections

In the late 1990s, we developed our basic ideas on Political Contracts. A first publication on the subject, was the chapter "Incentive contracts and elections for politicians and the down-up problem". Edited by Murat R. Sertel and Semih Koray (see Gersbach 2003), it was a contribution to *Advances in Economic Design* and included as Chap. 2 in *Designing Democracy*. Our early research on competition through incentive contracts followed. It was first published in *Public Choice* and included with minor amendments as Chap. 3 in *Designing Democracy* under the title "Short-termism and competition for incentive contracts" (see Gersbach 2004, 2005).

As this particular contribution sets the basis for all our research on democracy issues, we include the original article in this volume. The insights gained such a long time ago are still crucial for our current work: They fructified into a world of insights and we are still harvesting!

Vote-share Contracts Without Knowledge About Ability

Chapters 3 and 4 in Part I of this book were originally developed as companion papers. Both deal with higher vote-thresholds for incumbents, yet with a slight, but important, difference: The knowledge an office-holder has, or doesn't have, about his own ability when he offers a Political Contract containing a higher vote-share for reelection.

In Chap. 3, we will examine a setting in which a candidate has *no knowledge about his ability* at the time of his candidacy for office, and analyze a Political Contract that uses an incumbent's reelection vote-share to assess performance. We will use the terms *Vote-share Contract* and *vote-share thresholds* indiscriminately to denominate a contract that stipulates a certain vote-share to be attained for reelection.

Vote-share Thresholds with Some Knowledge About Ability

In the companion paper presented in Part I, Chap. 4, we will examine the same setting, yet with the candidates having *some knowledge about their own ability* before they offer a Political Contract containing a vote-threshold. As this private information is crucial for the candidate's accuracy when setting the reelection vote-threshold that is optimal for him—i.e., which ensures his reelection—, the comparison of our findings with those from the previous chapter will yield significant new results, which we present in Part I, Chap. 4.

Information Markets and Political Contracts

One of the most important issues with regard to Political Contracts is the assessment of performance. This assessment may be very simple if performance can be summarized in one figure or in a single event. If it cannot be measured and/or described with a certain degree of precision, alternative ways to assess output have to be found, sometimes tailored to the needs of a specific project or policy. An interesting new way to generate performance measures is the use of information markets. We will illustrate performance assessment methods in the context of long-term projects, where it might prove particularly difficult to measure performance, as these projects will be completed only long after the office-term in which they were started.

An information market that predicts the long-term performance of a policy might be used to produce information for a Political Contract. On such a market, an office-holder should reach a certain "price" to be reelected, which is serving as a performance indicator. The price he has to reach could be tied into a Political Contract, and an office-holder that fails to reach "his" pre-defined price—on average, over a certain time-span during his office-term—would lose the right to stand for reelection. We will examine the chances and drawbacks of Political Contracts that are based on such information market prices in Part I, Chap. 5.

Competition of Politicians for Wages and Office: Limits of Political Contracts

In electoral competition, the candidates might offer a Political Contract on their remuneration. At first sight, this seems a good way to save tax money. We will analyze a model in which two politicians compete for office and for wages, and show that surprisingly, competitive wage offers from the candidates can yield lower welfare than remunerations determined by the public. Part I, Chap. 6 will present our latest finding on these limitations and close the first part of this book.

1.7 Background

Our suggestion to establish a "Contractual Democracy" via Political Contracts originates from the firm conviction that better democracies are possible, and that scientists should play a key role in their development. Our current work stems from the observation that Political Contracts are not permitted in democracies.[13] Of course, government officials in the executive and legislative branch are accountable to the

[13]There is an important body of literature on incentive contracts for non-elected public authorities like central bankers, initiated by Walsh (1995a). The government imposes a penalty if it can verify that the central bank has not attempted to meet its target level. For the theory of the enforcement of such arrangements and the nature of penalties, see further Persson and Tabellini (1993), Walsh (1995b), Lockwood (1997), Svensson (1997), and Jensen (1997).

general public. The central manifestation is accountability of officials via elections or accountability to elected intermediaries.[14] Political Contracts would significantly enlarge the menu of manifestations how accountability of government officials can work. It is thus unclear why the exclusion of Political Contracts should be carved in stone—both from a scientific perspective and from the citizen's point of view.[15] We combine democratic decision-making with Political Contracts.

The perspective on democracy put forward in this book significantly differs from traditional political-economic, public-choice, and social-choice approaches. Nevertheless, the knowledge embodied in these areas was instrumental for the development of our ideas and models. The pioneer work of Arrow (1951), Black (1958), Downs (1957), Buchanan and Tullock (1962), Sen (1970), Olson (1965), and Niskanen (1971), together with a large body of literature surveyed in leading textbooks such as Müller (1989), Bernholz and Breyer (1993/94), Drazen (2000), and Persson and Tabellini (2000) is the groundwork on which our research was built. Furthermore, the work of Fleurbaey and Maniquet (2011), as well as the theory of private complete and incomplete contracts[16] (see Hart 1995 and Bolton and Dewatripont 2005) have helped to look beyond utilitarian considerations and to incorporate fairness, equity, and power in the discussion on Political Contracts.

Finally, the suggestions put forward in the first part of this book aim at improving the functioning of democracy. We do not address the appropriate boundaries of collective decision-making and their relation with individual liberties. This set of issues is an enduring theme of democracy research (see Buchanan 1975, Hayek 2002, Samet and Schmeidler 2003 and the assessments by Plattner 1998, for instance). Political Contracts may well help craft new and more efficient boundaries between individual liberties and democracy.

[14]There is a voluminous conceptual (and even larger empirical) literature on the role of electoral accountability that can be traced back to Downs (1957) and the classic work of Barro (1973) and Ferejohn (1986) on how elections may punish poor performance of officials. Theoretical work that identifies the role of elections as a screening device for officials has been triggered by Banks and Sundaram (1993), Samuelson and Fearon (1999). A survey on the potential and limits of electoral accountability can be found in Asworth (2012). The role of finite versus infinite horizons for accountability is surveyed in Duggan and Martinelli (2015). Accountability through elected intermediaries is developed in Vlaicu and Whalley (2015).

[15]Historically, contracts were used to limit the power of rulers. In the Middle Ages, specific contracts for rulers were a step for the development of constitutions or a mean to commit them to pursuing certain policies (see, e.g. Kleinheyer 1968, Vierhaus 1977, Pozza 1997 and Lottes 2000). Moreover, in ancient Athens, the officials were liable with their personal funds. In some cases, officials were even executed if the citizens' assembly was not satisfied with their performance (see, e.g., Bleicken 1991).

[16]With incomplete contracts, allocation of residual control rights—i.e. power—is central. This was examined, e.g., by Persson et al. (1997) and Persson and Tabellini (2000) to rationalize the separation of powers and of the checks and balances of various branches of government. We take an allocation of power as given and examine how Political Contracts can help mitigate abuse of power, and can foster the alignment of the politicians' incentives with the voters' interests.

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Chapter 2
Retrospect – Competition of Politicians for Incentive Contracts

2.1 Background

This chapter provided the groundwork for our research on Contractual Democracy. We started in the 1990s, when we were working on deficiencies of democracy and trying to find ways to overcome them. Soon, first inventions emerged, which started with Gersbach (2003) and with this chapter.

We started by focusing on the difficulty to motivate office-holders to undertake socially desirable long-term projects. Long-term issues such as unemployment problems appear to be difficult for politicians to solve in a limited period of time.

The fact that an office is held for a given period only, as typical for democratic systems, is essential. Without this limitation, the system would not be democratic. However, the shorter the term, the more challenging the fact that the implementation of many projects takes longer than one term. Thus, an office-holder might not have the opportunity to complete long-term projects, although they might be beneficial to society. At the end of his first term, the office-holder could try to be reelected to pursue his long-term projects in his second term. But to earn the votes required for reelection, he needs short-term results—a seemingly inextricable situation.

Our goal was to suggest a way to mitigate this drawback without endangering the basic structure of democracy. We developed a hierarchical structure of incentive contracts and elections and examined the consequences of allowing politicians to compete with them. The contracts stipulate that the office-holder's utility or income in his second office period will depend on a given, verifiable long-term achievement or result. Typically, this corresponds to a long-term project initiated during the first term in office, which yields results during the second term. If they wish, the candidates

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can offer such contracts before the *first election*, although the contracts concern the *second period in office*, and only come into force if an office-holder is reelected.[1]

2.2 Introduction

In a simple model, we examine how competition between politicians for incentive contracts and elections can motivate them to undertake socially desirable long-term projects, while preserving the democratic legitimation of politicians.

Two candidates compete for office in an initial election period and for subsequent reelection. Candidates are motivated by the offices they hold and by the policies they undertake. Once a candidate is elected he can undertake socially desirable long-term projects, opt for inefficient short-term projects or stick to the status quo. Returns from long-term projects only accrue to voters in a second election period. The problem for the public is that the politicians might discount the future more than citizens do, and/or reelection prospects are uncertain and only loosely connected to policy results. In such cases, politicians behave short-term oriented and the public cannot sufficiently motivate a politician to invest in long-term projects. This holds even if the public could commit itself to reelection.

To alleviate these inefficiencies we suggest the electorate to use a hierarchy of incentive contracts and elections. Candidates are given the possibility of offering incentive contracts when campaigning for office for the first election period. The incentive contract stipulates that in the event of reelection the politician's utility or income in the second election period depends on policy returns such as the level of unemployment. Incentive contracts become binding as soon as the politician decides to stand for reelection and is actually reelected. Candidates are free to offer empty contracts or contracts making their income depend on long-term returns.

Our findings are as follows: First, if the politician's discount factor is below a certain threshold, the public cannot motivate him to undertake long-term projects by election alone. This also holds if the public commits itself to a reelection scheme. If reelection prospects are sufficiently uncertain, politicians may not be motivated to undertake long-term policies even if they do not discount the future at all.

Second, when politicians can offer incentive contracts and the public commits itself to a reelection scheme, the result is a unique equilibrium. Both politicians offer the same contract. The equilibrium contract stipulates future transfers ensuring that the politician with the lower discount factor will be indifferent about choosing the long-term project or the short-term project. Transfers are interpreted in a wide sense. For instance, a politician may receive special monetary remunerations in the future or

[1] This chapter was first published as a paper in *Public Choice* and included as a short version as Chap. 3 in *Designing Democracy* under the title 'Short-termism and competition for incentive contracts' (Gersbach 2004, 2005). In the present edition, we include the original paper, with minor amendments.

he may become a special honorary citizen if his policies generate long-term benefits for society. The politician with the larger discount factor is elected; his prospects of reelection are sure-fire and he will take the socially efficient long-term decision.

In the following, we relax two of the assumptions upon which the previous findings have built. Our third result shows that the hierarchy of elections and incentive contracts will still induce politicians to undertake socially beneficial long-term projects, even if the public cannot commit itself to any future reelection behavior. We consider two reasons why current voters may not be able to commit themselves to a certain future voting behavior: The democratic requirement for unconstrained voting in every election and incentives to reject the incumbent in order to economize on his future remunerations.[2] In the first case, future transfers to an elected politician undertaking the long-term project must be higher in equilibrium. In the second case, incentive contracts must include a golden parachute clause guaranteeing a future bonus to a politician, even when he is no longer in office.

In our fourth result we allow for the case where the public does not know the discount factors of politicians competing for office. In the corresponding game between politicians and the public under asymmetric information, there exists a Bayesian Nash equilibrium in which all types of politicians will undertake beneficial long-term projects. Under uncertainty about the politician's discount factor, the public will have to grant benefits to the politician corresponding to the benefits under certainty with the lowest possible realization of the discount factor.

To sum up, competition among politicians for the hierarchy of incentive contracts and elections, appears to be a reasonably robust mechanism to overcome short-termism. Since the contracts suggested in this chapter have no counterpart in reality there are a number of practical issues regarding the application of the hierarchy of incentive contracts and elections, which we will address in the final section.

This chapter is related to the literature about electoral accountability which was initiated by Barro (1973), Ferejohn (1986) and recently extended by Persson et al. (1997) (see Persson and Tabellini 2000 for a survey). Politicians and voters are assumed to have divergent interests, and elections are means by which voters control politician misbehavior, since the possibility of reelection induces self-interested politicians to act on behalf of the interests of the electorate. In this chapter, we introduce competition of politicians for incentive contracts and elections as a novel element in politics. We combine contractual and electoral accountability while at the same time preserving the democratic legitimation of politicians.

Incentive elements in politics other than elections have been discussed in Gersbach (2003). He examines how the public can make the value of holding office in a second term dependent on the realization of macroeconomic variables, such that the incentive for politicians increases to undertake socially desirable policies with long-term consequences in the first term. In this chapter, we introduce competition

[2]The second reason is less important since the remuneration of a politician creates only a negligible burden per capita for the public.

of politicians for incentive contracts and elections in democracies with periodic, free and anonymous elections.[3]

For simplicity, we consider a political economy model where politicians and voters differ with respect to their relative valuation of future and current utilities. This is a tractable model for the analysis of how competition for incentive contracts and elections may alleviate inefficiencies in democracies. In practice, as is discussed in the concluding section, democracies may produce inefficiencies for a wide variety of reasons and it is not clear whether the source of inefficiency we are focusing on is the most important one. However, the ideas presented in this chapter may be useful when applied to other kinds of inefficiencies in political processes.

The chapter is organized as follows: In the next section, we outline the model and our assumptions. In Sect. 2.4, we consider the potentialities and limitations of the election mechanism for achieving optimal decisions. In Sect. 2.5, we show that competition among politicians for incentive contracts and election induces socially optimal decisions. In Sect. 2.6, we extend our analysis to the non-commitment case. In Sect. 2.7, we discuss asymmetric information. Section 2.8 presents our conclusions.

2.3 Model and Assumptions

The game we are analyzing is a dynamic game with two periods. We assume that the politician (or agent) is risk-neutral, and "the public" represents the voters. Returns from projects are denoted by V. V^1 and V^2 are the returns in period 1 and period 2, respectively. Specific realizations will be indexed according to the type of project and the period involved. The game is given as follows:

Stage 1: At the beginning of period 1 two politicians, denoted by $i = 1, 2$, simultaneously offer incentive contracts $C_1(\beta_1 V^2)$ and $C_2(\beta_2 V^2)$ with the following interpretation: if politician i gets reelected in period 2, he receives a net transfer $\beta_i V^2$ if $V^2 > 0$ and has to pay $|\beta_i V^2|$ if $V^2 < 0$, where $\beta_i \in [0, 1]$.[4]

Stage 2: The public decides whether the politician gets elected. We use $p_i \in [0, 1]$ to denote the probability that politician i will be elected, so that $p_1 + p_2 = 1$.

Stage 3: The elected agent must decide whether to undertake certain projects. He has three options. He can undertake a short-term policy (*STP*) generating a positive return $V_S^1 > 0$ in this period, but a negative return $V_S^2 < 0$ next period. The second option is a long-term policy (*LTP*). For simplicity

[3]While there is no further literature on competition for incentive contracts by politicians, there is a rapidly growing literature on incentive contracts for central bankers where democratic requirements play no role initiated by Walsh (1995a, b) and developed by Persson and Tabellini (1993), Lockwood (1997), Svensson (1997) and Jensen (1997).

[4]These payments affect the utility of the voters accordingly.

of presentation the long-term policy is assumed to have no short-term consequences, i.e. $V_L^1 = 0$, but *LTP* does generate positive payoffs $V_L^2 > 0$ in the next period. The last option for the policy-maker is to continue the status quo and to do nothing (*NOT*). Payoffs in this case are $V_N^1 = 0$ and $V_N^2 = 0$, respectively, in the two periods.
To sum up, the elected politician decides among his options in $\{STP, LTP, NOT\}$.

Stage 4: The returns from the first period are made public. The elected politician decides whether he wants to run for office again. The public decides on the reelection of the politician. The probability that politician i is reelected is denoted by $q_i \in [0, 1]$.

All costs and benefits are measured in dollars. The social returns from the status quo have been normalized to zero. There are many examples of *LTP* projects versus *STP* or *NOT* projects. For instance, labor market reforms or transition processes of centrally planned market economies towards market economies may imply no welfare improvements in the short-term,[5] but may generate benefits in the long term. Other examples are political business cycles where politicians adopt short-term policies instead of long-term policies before elections, thus leading to upturns before and downturns after elections,[6] or investments in infrastructure requiring a temporary cut-down on consumption but producing positive returns at a later stage.

We assume that contracts can be conditioned on social returns measured for instance by GDP growth or criminal statistics.[7] However, we assume that contracts cannot be conditioned on the policy choice itself. The latter assumption follows the reasoning in the incomplete contract literature (see the survey of Hart 1995). In politics complete contracts would require to write all conceivable laws into contracts before they are initiated in Parliament, which appears to be impossible.

We assume that a politician can generate private returns if he realizes social returns larger than the returns of the status quo and as long as he is in power. The social returns from the status quo have been normalized to zero. If politician i is in power and realizes a social project return V^1 in period 1 or V^2 in period 2, we assume that his private benefits are:

$$R_i^1 = \alpha V^1 or\ R_i^2 = \alpha V^2, \tag{2.1}$$

respectively, where α is some number, with $0 < \alpha < 1$. The above assumption is justified by the observation that high returns enable the agent to channel some returns to interest groups that support him, as is suggested by the large literature in public

[5]In some cases, short term consequences of *LTP* can even be negative, but this can easily be integrated into our framework.

[6]The literature on political business cycles started with Nordhaus (1975), Ben-Porath (1975) and was expanded to ideological business cycles by Hibbs (1977). In Rogoff (1990), Cuckierman and Meltzer (1986), Hibbs (1992) and Persson and Tabellini (1993), the theory has been adapted to incorporate rational expectations and information asymmetries.

[7]For simplicity of exposition contracts are assumed to be linear in social returns. Since returns in the second period can only take three values, this assumption could easily be relaxed.

choice (see e.g. Mueller 1989).[8] Alternatively, the politician is genuinely concerned about the social returns he generates as long as the outcomes of policies occur while he is in office. We follow the latter interpretation, which simplifies the analysis.[9]

We concentrate on the agent's expected utility in period 1, when politicians stand for election for the first time. We assume that the utility of politician i increases both in the private benefits from holding office, given by $B > 0$, and from the private benefits of investment projects. In particular, we assume that the expected utility of agent i is given by

$$U_i = p_i \left[(1-m)B + mR_i^1 + \delta_i q_i \left((1-m)B + m(R_i^2 + \beta V^2)\right)\right],$$

where $R_i^1 = \alpha V^1$ and $R_i^2 = \alpha V^2$ are the private returns in period 1 and 2, respectively. The $\delta_i \in [0, 1]$ denotes the discount factor of politician $i \in \{1, 2\}$, and reflects the impatience of the politician. The parameter m, with $0 < m < 1$, is the significance the agent assigns to private returns from projects and $1 - m$ is the significance of benefits from holding office. The parameter m is assumed to be the same for both politicians. A significance m close to 1 means that the agent is mainly motivated by the policies he implements. A low value for m corresponds to an agent being mainly concerned to hold office. The utility of outside options is normalized to zero. Throughout the chapter, we assume that $(1-m)B$ is sufficiently large, to ensure that net utilities of politicians in the second period are always non-negative.

To simplify the exposition we use[10]

- $U_i^L(\beta_i, RE)$ to denote the utility of an elected politician i if he has offered the contract $C_i(\beta_i V^2)$, undertakes LTP and is reelected:

$$U_i^L(\beta_i, RE) = (1-m)B + \delta_i \left\{(1-m)B + mV_L^2(\alpha + \beta_i)\right\} \tag{2.2}$$

- $U_i^S(\beta_i, RE)$ to denote the utility of an elected politician i if he has offered $C_i(\beta_i V^2)$, undertakes STP and is reelected:

$$U_i^S(\beta_i, RE) = (1-m)B + m\alpha V_S^1 + \delta_i \left\{(1-m)B + mV_S^2(\alpha + \beta_i)\right\} \tag{2.3}$$

- $U_i^S(\beta_i, NRE)$ to denote the utility of an elected politician i if he has offered $C_i(\beta_i V^2)$, undertakes STP and does not stand for reelection:

$$U_i^S(\beta_i, NRE) = (1-m)B + m\alpha V_S^1 \tag{2.4}$$

[8] An alternative assumption about private returns developed by Coate and Morris (1995) would be $R_i = \max[\alpha V, 0]$. This assumption would strengthen the need to use incentive contracts because STP becomes more attractive.

[9] The first interpretation yields the same qualitative conclusions, but the public needs to take into account that some returns from projects are channeled to the politician or the interest group supporting him.

[10] We can neglect the case where the office-holder selects LTP and is deselected, because in such cases, he is always better off choosing STP.

We allow for the fact that politicians may differ in their discount factor $\delta_i \in [0, 1]$, $i = 1, 2$. In many cases such differences are known to the public. Consider for example the election race between the incumbent, Kohl, and the challenger, Schröder, in 1998 in Germany. It was well known that Kohl was competing for a final term whereas Schröder wanted to start his era as chancellor. Therefore, we assume in the following that δ_1 and δ_2 are known to the public and we label candidates such that $\delta_1 \leq \delta_2$. Later we will relax the informational assumptions about discount factors.[11]

We denote the returns to the public from the options *STP*, *LTP* and *NOT* over the lifetime of the project, by EV_S, EV_L, and EV_N, respectively.[12] Thus, we have

$$\begin{aligned} EV_S &= V_S^1 + \overline{\delta}\, V_S^2, \\ EV_L &= \overline{\delta}\, V_L^2, \text{ and} \\ EV_N &= V_N^1 + \overline{\delta}\, V_N^2 = 0, \end{aligned}$$

where $\overline{\delta}$ is the discount factor of the public $(0 < \overline{\delta} \leq 1)$. The social discount factor may be higher or lower than that of the politicians. Note that $EV_L > EV_N$. We further assume that

$$\begin{aligned} V_S^1 &> EV_L > 0, \\ 0 = EV_N &> EV_S. \end{aligned}$$

The preceding assumption immediately implies that in social terms the optimal policy is *LTP*. To simplify the presentation, we employ three tie-breaking rules. First, if two politicians generate the same social welfare, the public will elect the politician with the higher discount factor. Second, if both politicians are equally good in terms of social welfare and are identical in terms of the discount factor, both politicians have the same chance $p_1 = p_2 = \frac{1}{2}$ of being elected. Third, if a politician is indifferent as to two types of policies, he will select the one that yields higher social welfare. These tie-breaking rules simplify the exposition but are not essential for the results.

2.4 Elections

In this section we discuss how the public can motivate politicians to undertake *LTP* if the only instrument available is the election mechanism. We assume that the public can commit itself in stage 1 to its reelection scheme for stage 4, with the two reelection probabilities $q(V_S^1)$ respectively $q(0)$. The first applies when *STP* is chosen and the

[11] Our main results can easily be extended to more than two politicians and to discount factors picked from a continuous set. For instance, in the case of three or more politicians only those two politicians with the highest discount factors matter for the Propositions 2.3, 2.4 and 2.5 and the corresponding corollaries.

[12] These returns may be further affected by transfers between the public and the office-holder.

second applies when *LTP* or *NOT* are chosen. This gives the best chance of elections inducing elected politicians to choose *LTP*. However, no incentive contracts can be offered. We obtain:

Proposition 2.1 *Suppose the public can commit to a reelection scheme and that* $\delta_i \leq \delta(m)$*, with*

$$\delta(m) = \frac{m\alpha V_S^1}{(1-m)B + m\alpha V_L^2}. \tag{2.5}$$

Then, the politicians cannot be motivated by elections to adopt LTP.

Proof of Proposition 2.1
It is obvious that the politician will never choose *NOT* under any reelection scheme, because he benefits equally or more from *LTP* or *STP*. Additionally, it is obvious that the optimal reelection scheme for voters is $q(0) = 1$ and $q(V_S^1) = 0$, which is the maximum spread to deter the politician from choosing *STP*. The critical discount factor is then determined by setting $U_i^L(0, RE) = U_i^S(0, NRE)$, which yields:

$$\delta(m) = \frac{m\alpha V_S^1}{(1-m)B + m\alpha V_L^2}.$$

If $\delta(m) < 1$, a politician with $\delta_i \in (\delta(m), 1]$, will choose *LTP* under the reelection scheme $q(0) = 1$ and $q(V_S^1) = 0$, and *STP* otherwise. □

From here, we immediately obtain $\delta(0) = 0$ and

$$\frac{\partial \delta(m)}{\partial m} = \frac{\alpha V_S^1 B}{\left\{(1-m)B + m\alpha V_L^2\right\}^2} > 0. \tag{2.6}$$

Therefore, since $m > 0$, we have a range for the discount factor at which politicians will not choose the socially efficient policy. Note that voters are assumed to be fully rational and infer negative future returns from the positive returns of short-term projects in the first election period.

The underinvestment problem becomes more pronounced when the public cannot commit to a reelection scheme, which is the natural assumption for democratic decision-making. As an example for the severity of the underinvestment problem in such cases, suppose that the public votes prospectively and that past policy performance does not influence reelection chances.[13] In particularly, suppose that $q(0) = q(V_S^1) = \frac{1}{2}$ and thus, from the perspective of the beginning of the first term the incumbent is reelected with probability $\frac{1}{2}$ and thus independently of the adopted policy. Then we obtain:

Proposition 2.2 *Suppose that the public cannot commit to a reelection scheme. Furthermore, suppose* $q(0) = q(V_S^1) = \frac{1}{2}$ *and that* $\delta_i \leq \hat{\delta}(m)$*, with*

[13] This is an extreme assumption and solely made for expositional purposes.

$$\hat{\delta}(m) = \max\left\{\frac{2m\,\alpha\,V_S^1}{(1-m)\,B + m\,\alpha\,V_L^2}, \frac{2V_S^1}{V_L^2 - V_S^2}\right\}.$$

Then the politician cannot be motivated by elections to undertake LTP.

Proof of Proposition 2.2
The proof is similar to the proof of Proposition 2.1. The utilities are now calculated using the reelection probability $\frac{1}{2}$. Moreover, the politician has a chance to be reelected if he selects *STP*. The corresponding comparisons yield:

$$\delta_i = \frac{2V_S^1}{V_L^2 - V_S^2} \quad \text{and}$$

$$\delta_i = \frac{2m\,\alpha\,V_S^1}{(1-m)B + m\,\alpha\,V_L^2},$$

which establishes the proposition. □

The preceding proposition illustrates that the underinvestment problem is severe if reelection prospects are not (or only loosely) connected with policies undertaken in the past. In such cases, as the following corollary illustrates, there are circumstances when no politician invests in *LTP*, independent of his discount factor.

Corollary 2.1 *Suppose that the public cannot commit to a reelection scheme. Suppose that* $q(0) = q(V_S^1) = \frac{1}{2}$ *and* $V_S^1 + \frac{1}{2}V_S^2 > \frac{1}{2}V_L^2$. *Then, no politician can be motivated by elections to adopt LTP.*

The corollary immediately follows from Proposition 2.2. Under $V_S^1 + \frac{1}{2}V_S^2 > \frac{1}{2}V_L^2$, the critical discount factor becomes larger than 1 and thus politicians will choose *STP*, no matter how large their discount factors are. Intuitively, if the short-term project is not too bad, the low probability of reelection induces politicians to adopt the STP, since they can benefit with certainty from returns in the first term and they have no influence on their reelection chances. Note that the weight m on policy in the objective function of the politician is irrelevant in Corollary 2.1 since the politician expects the same private benefits from holding office under *LTP* and *STP*. In the next section we begin to address how incentive contracts can overcome the inefficiencies identified in this section.

2.5 Competition for the Incentive Contracts

In this section we consider the whole game and allow politicians to offer incentive contracts before the first election takes place. We assume that voters can commit themselves to a reelection scheme in stage 1, so that we can compare the competition for incentive contracts and elections with the previous section. We obtain:

Proposition 2.3 *Suppose $\delta_1 < \delta_2 \leq \delta(m)$. Then there exists a unique subgame perfect equilibrium*

$$\left\{C_1(\beta_1 V^2), C_2(\beta_2 V^2), p_1 = 0, p_2 = 1, q_1(0) = 1, q_2(0) = 1, q_1(V_S^1) = 0, q_2(V_S^1) = 0\right\},$$

with

$$\beta_1 = \beta_2 = \overline{\beta} = \frac{m\alpha V_S^1 - \delta_1 \left\{(1-m)B + m\alpha V_L^2\right\}}{m\delta_1 V_L^2}, \tag{2.7}$$

if

$$\overline{\delta} \cdot \overline{\beta} V_L^2 < EV_L - EV_S. \tag{2.8}$$

The proof is given in the Appendix. Proposition 2.3 shows that the hierarchy of elections and incentive contracts eliminates inefficient decision-making in politics at the cost of future transfers to the elected politician. Both politicians offer the same contract. The equilibrium contract stipulates future transfers ensuring that the politician with the lower discount factor will be indifferent about choosing the long-term project or the short-term project. The politician with the larger discount factor is elected; his prospects of reelection are certain and he will take the socially efficient long-term decision. Note that if one discount factor is higher than $\delta(m)$, the transfer may be zero.

In the following, we relax the assumptions upon which the previous result has built. In Proposition 2.3 voters were assumed to commit themselves to a state-dependent reelection scheme. But, competition for incentive contracts and election can still work if the public can only commit itself to a fixed reelection probability, as is illustrated in the following corollary:

Corollary 2.2 *Suppose the public could only commit itself to a fixed reelection probability. Then the subgame perfect equilibrium denoted in Proposition 2.3, with $\overline{\beta}$ as in (2.7), still holds correspondingly with $q_1 = q_2 = 1$, if*

$$(1-m)B + mV_S^2(\alpha + \overline{\beta}) < 0. \tag{2.9}$$

The proof is analogous to the proof of Proposition 2.3, because Condition (2.9) directly implies $U_i^S(NRE) > U_i^S(\overline{\beta}, RE)$, and therefore, with incentive contracts $C(\overline{\beta}V^2)$, neither politician has an incentive to adopt *STP* and to stand for reelection. To examine the case of non-commitment in the next section, we denote the equilibrium value for $\bar{\beta}$ in the commitment case by $\overline{\beta}^C$. Note that $\overline{\beta}^C$ in Eq. (2.7) depends negatively on δ_1. A large δ_1 decreases the costs of transfers to the politician and harms the elected politician 2. With the appropriate modifications in the proof, Proposition 2.3 can be extended to the case where politicians are identical:

Corollary 2.3 *Suppose $\delta_1 = \delta_2 \leq \delta(m)$. Then there exists a unique subgame perfect equilibrium*

$$\left\{C_1(\beta_1 V^2), C_2(\beta_2 V^2), p_1 = \frac{1}{2}, p_2 = \frac{1}{2}, q_1(0) = 1, q_2(0) = 1, q_1(V_S^1) = 0, q_2(V_S^1) = 0\right\},$$

with

$$\beta_1 = \beta_2 = \overline{\beta}^C = \frac{m\alpha V_S^1 - \delta_1 \left\{(1-m)B + m\alpha V_L^2\right\}}{m\delta_1 V_L^2}, \tag{2.10}$$

if

$$\overline{\delta}\,\overline{\beta}^C V_L^2 < EV_L - EV_S. \tag{2.11}$$

2.6 Competition Without Commitment

The assumption that voters can commit themselves to a reelection scheme has mainly been made in order to give the election mechanism the best chance to motivate political leaders to invest in long-term, efficient projects. However, from a strictly democratic point of view, voters are unable to commit future citizens to adhere to a particular voting behavior. The contracting problem is rooted in the uncertainty about future electoral interests and the liberal principle of democracies to allow for free and anonymous voting behavior in elections.

The impossibility of commitment to future voting behavior represents another source of inefficiency outlined in Glazer (1989), Gersbach (1993), Besley and Coate (1998) and in related work by Alesina and Tabellini (1990) and Persson and Svensson (1989). We can integrate the impossibility of commitment into our model. There are two non-commitment problems: Incentives of voters to reject an incumbent so as to economize on his future remunerations, and the democratic requirement for unconstrained voting in every election. We deal with the latter case first. Suppose there is complete uncertainty about the voting behavior of future generations, so that an elected politician today has an a priori probability of reelection of $q_i = \frac{1}{2}$ independent of his actions in the past.[14] This is an opposite pole to the commitment case where q_i is 1 if the choice of *LTP* is expected and 0 otherwise. Though we think that intermediate cases are the most plausible, it is instructive to compare these polar opposites. For the non-commitment case we obtain:

Proposition 2.4 *Suppose $\delta_1 \leq \delta_2 \leq \delta(m)$. Then there exists a unique subgame perfect equilibrium*

$$\left\{C_1(\beta_1 V^2), C_2(\beta_2 V^2), p_1 = 0, p_2 = 1, q_1(0) = \frac{1}{2}, q_2(0) = \frac{1}{2}, q_1(V_S^1) = \frac{1}{2}, q_2(V_S^1) = \frac{1}{2}\right\},$$

with

[14]This is equivalent to the notion of unconstrained voting in the context of retrospective voting (Ferejohn 1986).

$$\beta_1 = \beta_2 = \overline{\beta}^{NC} = \max\left\{ \frac{2m\alpha V_S^1 - \delta_1\left\{(1-m)B + m\alpha V_L^2\right\}}{m\delta_1 V_L^2}, \frac{2\alpha V_S^1 - \delta_1\alpha(V_L^2 - V_S^2)}{\delta_1(V_L^2 - V_S^2)} \right\}, \tag{2.12}$$

if [15]

$$\overline{\delta}\,\overline{\beta}^{NC} V_L^2 < EV_L - EV_S. \tag{2.13}$$

The proof is similar to the commitment case. But now we have to compare $U_i^L(\overline{\beta}, RE)$ with $U_i^S(\overline{\beta}, RE)$ and $U_i^S(NRE)$, and the utility in the second period must be evaluated with $q_1 = q_2 = \frac{1}{2}$ instead of certain reelection. An immediate consequence is:

Corollary 2.4 *In the equilibrium above, it holds that*

$$\overline{\beta}^{NC} > \overline{\beta}^{C}. \tag{2.14}$$

It is obvious that under non-commitment it requires a higher future transfer to make the politician with the lower discount factor indifferent as to *LTP* and *STP*. The impossibility of the present generation of voters to commit future voters to a particular election choice entails the larger transfer a reelected politician must receive if he undertakes *LTP*.

There might be a second and even more extreme case of non-commitment if voters at the reelection date definitely reject the incumbent, in order to economize on future remunerations for the politician. In this case the nature of the incentive contracts can be amended in the following way. The incentive contract becomes effective when the politician stands for reelection, independently of whether he is reelected. Thus, he can receive future benefits from *LTP*, even if he is not in office anymore. We call such incentive contracts golden parachute contracts; they are denoted by C_1^{Pa} and C_2^{Pa}, respectively. The expected utility for a politician i who has offered $C_i^{Pa}(\beta_i V^2)$ and is not reelected is denoted by $U_i^{Pa}(\beta_i, NRE)$ and given by

$$U_i^{Pa}(\beta_i, NRE) = p_i\left\{(1-m)B + m(R_i^1 + \delta_i\beta_i V^2)\right\}, \tag{2.15}$$

where R_i^1 is either αV_S^1 or 0. We immediately obtain:

Proposition 2.5 *Suppose that $\delta_1 < \delta_2 < \delta(m)$ and politicians can offer golden parachute contracts and the politician elected in period 1 is never reelected. There then exists a unique subgame perfect equilibrium in which politicians offer golden parachute contracts, given by*

$$\left\{C_1^{Pa}(\beta_1 V^2), C_2^{Pa}(\beta_2 V^2), p_1 = 0, p_2 = 1\right\}, \tag{2.16}$$

[15] Condition (2.13) is sufficient but since politicians selecting *STP* have a chance to be reelected the proposition also holds on weaker conditions.

with

$$\beta_1 = \beta_2 = \overline{\beta}^{NCPa} = \frac{\alpha V_S^1}{\delta_1 V_L^2}, \tag{2.17}$$

if

$$\overline{\delta} \cdot \overline{\beta}^{NCPa} V_L^2 < EV_L - EV_S. \tag{2.18}$$

The proof is analogous to the previous proposition. Note that $\overline{\beta}^{NCPa}$ is determined by setting $U_1^{Pa,L}\left(\overline{\beta}^{NCPa}, NRE\right) = U_1^S(NRE)$ because a politician is not forced to offer a parachute contract. The left-hand side is the utility when the politician chooses *LTP*. Note also that we apply the tie-breaking rule that candidate 2 is elected if the public is indifferent between the two candidates. While we have assumed an extreme case of non-commitment in Proposition 2.5, it is obvious that the option to offer golden parachute contracts also works for intermediate values of positive reelection probabilities when standard contracts cannot induce *LTP* with lower costs for the public.

2.7 Asymmetric Information

While politicians' discount factors may be well known in some circumstances, there may be more uncertainty in other cases. For instance, when two politicians are competing for office for the first time, the public may be uncertain about the preferences of the politicians and in particular about their discount factors. To explore how asymmetric information affects the functioning of the dual mechanism - incentive contracts and elections - we assume that the public knows that both politicians competing for office have discount factors δ^H with probability w and $\delta^L < \delta^H$ with probability $1 - w$. We assume that politicians know the discount factor of their opponent.[16] We further use b_i $(i = 1, 2)$ to denote the beliefs of the public that politician i has discount factor δ^H after incentive contracts $C_1(\beta_1 V^2)$ and $C_2(\beta_2 V^2)$ have been offered. Then we look for perfect Bayesian equilibria of the election and the incentive contract game. We consider the case where the public can commit to a reelection scheme and obtain:

Proposition 2.6 *There exists a perfect Bayesian Nash equilibrium*[17]

$$\left\{C_1(\beta_1^* V^2), C_2(\beta_2^* V^2), p_1^*, p_2^*, q_1^*(0), q_2^*(0), q_1^*(V_S^1), q_2^*(V_S^1), b_1^*, b_2^*\right\},$$

---

[16]The assumption appears to be plausible because of the superior knowledge politicians have about each other through their daily interaction.

[17]Other equilibria exist. For instance, lower values of $\overline{\beta}^{AI}$ can be supported as equilibria as well. Moreover, one can apply refinements to the Bayesian equilibrium notion to support particular values of β_1 and β_2 in equilibrium. Details are available upon request.

if

$$\overline{\delta} \cdot \overline{\beta}^{AI} V_L^2 < EV_L - EV_S, \tag{2.19}$$

where

(i)

$$\beta_1^* = \beta_2^* = \overline{\beta}^{AI} = \frac{m\alpha V_S^1 - \delta_L \left\{(1-m)B + m\alpha V_L^2\right\}}{m\delta_L V_L^2}. \tag{2.20}$$

(ii) An elected politician chooses LTP in equilibrium.

(iii)

$$b_1^*(\beta_1, \beta_2) = \begin{cases} w \ \textit{if} \ \ \beta_1 = \overline{\beta}^{AI} \\ 0 \ \ \textit{otherwise}; \end{cases} \tag{2.21}$$

$$b_2^*(\beta_1, \beta_2) = \begin{cases} w \ \textit{if} \ \ \beta_2 = \overline{\beta}^{AI} \\ 0 \ \ \textit{otherwise}. \end{cases} \tag{2.22}$$

(iv)

$$p_1^*(\beta_1, \beta_2) = \begin{cases} \frac{1}{2} \ \textit{if} \ \ \beta_1 = \beta_2 \\ \frac{1}{2} \ \textit{if} \ \ \overline{\beta}^{AI} > \beta_1 > \beta_2 \ \textit{or} \ \ \overline{\beta}^{AI} > \beta_2 > \beta_1 \\ 1 \ \textit{if} \ \ \beta_1 = \overline{\beta}^{AI} \qquad \textit{and} \ \beta_2 \neq \overline{\beta}^{AI} \\ 1 \ \textit{if} \ \ \beta_1 > \overline{\beta}^{AI} > \beta_2 \ \textit{or} \ \ \overline{\beta}^{AI} < \beta_1 < \beta_2 \\ 0 \quad \textit{otherwise}; \end{cases} \tag{2.23}$$

$$p_2^*(\beta_1, \beta_2) = \begin{cases} \frac{1}{2} \ \textit{if} \ \ \beta_1 = \beta_2 \\ \frac{1}{2} \ \textit{if} \ \ \overline{\beta}^{AI} > \beta_1 > \beta_2 \ \textit{or} \ \ \overline{\beta}^{AI} > \beta_2 > \beta_1 \\ 1 \ \textit{if} \ \ \beta_2 = \overline{\beta}^{AI} \qquad \textit{and} \ \beta_1 \neq \overline{\beta}^{AI} \\ 1 \ \textit{if} \ \ \beta_2 > \overline{\beta}^{AI} > \beta_1 \ \textit{or} \ \ \overline{\beta}^{AI} < \beta_2 < \beta_1 \\ 0 \quad \textit{otherwise}. \end{cases} \tag{2.24}$$

(v)

$$\begin{aligned} q_1^*(0) \ \ &= q_2^*(0) \ \ = 1, \\ q_1^*(V_S^1) &= q_2^*(V_S^1) = 0. \end{aligned} \tag{2.25}$$

The proof of Proposition 2.6 is given in the Appendix. Proposition 2.6 shows that the hierarchy of incentive contracts and elections also works under incomplete information. But, $\overline{\beta}^{AI}$ is evaluated at the lower discount factor. Therefore, the public is forced to accept transfers to the politician, that are higher than the transfers expected when δ was either δ^L or δ^H. The expected transfer in the latter case would amount to

$$w\overline{\beta}^{AI} V_L^2 + (1-w)\overline{\beta}(\delta_H) V_L^2. \tag{2.26}$$

2.8 Discussion and Conclusion

Our simple analysis suggests that the dual mechanism of competition for elections and incentive contracts might alleviate some of the inefficiencies in democratic decision-making. However, there are many issues still waiting to be examined. There are practical issues; for instance regarding which quantitative measures should be used for the incentive contract. This seems fairly obvious in the case of European unemployment, because the incentive contract can be based on the average unemployment rate. But a definition problem remains as the unemployment rate can be defined in many different ways. Hence, there is a need to agree upon a definition that cannot be changed or manipulated once it has been adopted.

Moreover, it is often hard to measure social welfare beyond macroeconomic indicators and politicians usually face multi-task problems. Politicians in the executive and legislative branch are typically concerned with many different issues. Whereas issues such as unemployment or crime can be quantified with sufficient precision, this is not the case for other issues such as reforming health care or the judicial system. Therefore, performance in a significant part of their activities cannot be measured with any real degree of precision. As we know from the theory of multi-task incentive problems, outlined in Holmström and Milgrom (1991), severe measurement constraints can make it impossible to use task-specific performance schemes or aggregate performance measures. For instance, if politicians are only judged by their employment performance, they may simply inflate the public sector to meet the required standard and neglect other important issues.

Nevertheless, the multi-task and the measurement problem might be alleviated by the hierarchical incentive mechanism proposed in this chapter. A politician can only stand for reelection if he is willing to base his future income or the right for future reelection on the performance on one issue, say unemployment. If he accepts the incentive component, he can stand for reelection and voters can judge his performance on the remaining issues. If he has accepted the incentive contract, but only worked to reduce unemployment, voters may not reelect him because he has a bad record on other important issues. Therefore, the hierarchical incentive scheme might cause the politician to choose the socially desirable policy for one dimension without neglecting other issues.[18]

The literature has identified a number of further important inefficiencies in the political system (see the surveys and contributions by Bernholz and Breyer 1993, Mueller 1989, Dixit 1998, Drazen 2000, Frey 1983, Hillman 1989, Niskanen 1971, Olson 1965 and 1982, Stiglitz 1989, Persson and Tabellini 2000 and Tollison 1982 as well as the seminal work on constitutional design by Buchanan and Tullock 1962). How the dual mechanism can be applied for these kinds of inefficiencies and for more sophisticated political-economic models, constitutes a complete research program.

While the actual reach of the dual mechanism can only be judged after these avenues have been explored and a number of obvious practical issues have been

[18] There are a number of further practical issues, for instance enforcing the incentive contract will require a special court.

addressed, we think that well-designed incentive elements could complement the reelection mechanism in motivating politicians to invest in socially desirable policies.

Appendix

Proof of Proposition 2.3
Condition (2.8) ensures that the public is better off committing itself to reelection and accepting a politician with $C_1(\overline{\beta}V^2)$, who implements *LTP*, than setting $q_1(0) = q_2(0) = 0$, which avoids the transfer $\overline{\beta}V_L^2$ but implies *STP*. The public sets $q_i(V_S^1) = 0$ because they will receive negative returns, when a politician undertakes *STP*. The value of $\overline{\beta}$ is calculated such that the first candidate is indifferent as to *STP* and *LTP* if elected. Hence $\overline{\beta}$ is determined by

$$U_1^L(\overline{\beta}, RE) = U_1^S(NRE), \tag{2.27}$$

which gives Eq. (2.7). Since the incentive contract is irrelevant if a candidate does not want to stand for reelection we have

$$U_1^S(NRE) = U_2^S(NRE). \tag{2.28}$$

Because of $\delta_1 < \delta_2$ we have

$$U_2^L(\overline{\beta}, RE) > U_1^L(\overline{\beta}, RE). \tag{2.29}$$

Candidate 2 has a strict preference for *LTP* if elected, in contrast to the indifference as to *LTP* and *STP* of candidate 1 if elected.

To establish equilibrium, we consider four possible deviations from the equilibrium described in Proposition 2.3.

First, suppose that candidate 2 deviates and offers $C_2(\beta_2 V^2)$ with $\beta_2 > \overline{\beta}$. The deviation is not profitable if candidate 2 is not elected; this, in turn, is only a best response for voters if candidate 1 chooses *LTP* when elected and reelected. This requires that the following inequality holds:

$$U_1^L(\overline{\beta}, RE) \geq U_1^S(NRE). \tag{2.30}$$

By construction $U_1^L(\overline{\beta}, RE) = U_1^S(NRE)$. Thus politician 2 will not be elected although he chooses *LTP* because candidate 1 demands less transfer and chooses *LTP* in accordance with our tie-breaking rule. Thus, deviating is not profitable.

Second, suppose candidate 1 deviates to $C_1(\beta_1 V^2)$ with $\beta_1 < \overline{\beta}$. Such a deviation is only profitable if the public finds it in its best interests to elect and reelect him. Voters want to elect a candidate only if the candidate selects *LTP* once in office. Candidate 1 would choose *LTP* if the following inequalities hold:

$$U_1^L(\beta_1, RE) \geq U_1^S(NRE). \tag{2.31}$$

But $\beta_1 < \overline{\beta}$ implies directly $U_1^L(\beta_1, RE) < U_1^S(NRE)$, so candidate 1 will implement STP and the public will elect candidate 2 because he does undertake *LTP*.

Third, suppose candidate 1 deviates to $C_1(\beta_1 V^2)$ with $\beta_1 > \overline{\beta}$. Then the public will not elect politician 1, even if he were to undertake *LTP*, because for voters the payments to the politician are lower when the second candidate is elected. Therefore the deviation is not profitable.

Finally, it is obvious that the second candidate has no incentive to offer a contract $C_2(\beta_2 V^2)$ with $\beta_2 < \overline{\beta}$, because he would receive lower transfers in the second period and in equilibrium can be sure of being elected anyhow.

Uniqueness follows in a similar way. For any offer constellation $C_1(\beta_1 V^2)$, $C_2(\beta_2 V^2)$ with $\beta_i \neq \overline{\beta}$ for at least one candidate, one of the politicians has an incentive to deviate by offering $C_i(\overline{\beta} V^2)$, or by offering an incentive contract that requires slightly fewer transfers from the public.[19] $\square$

Proof of Proposition 2.6

We first observe that for $\beta_i = \overline{\beta}^{AI}$ both types of politicians choose *LTP*, i.e., independently of whether they have high or low discount factors. Thus, in equilibrium politicians choose *LTP* which validates (*ii*).

Given the equilibrium and out-of-equilibrium beliefs, $\beta_1^* = \beta_2^* = \overline{\beta}^{AI}$ are best responses for politicians. Given the equilibrium strategy of other politicians, any choice $\beta_i \neq \overline{\beta}^{AI}$ would result in zero probability of election.

Furthermore, we observe that proposed equilibrium beliefs obey Bayes' law. Finally, we have to check the election strategy of voters. Equilibrium election and reelection strategies are optimal since both politicians are identical and will choose *LTP*. According to our assumptions, the public is better off by *LTP* and paying transfers to an elected politician than by inducing *STP*.

Suppose that voters observe a pair (β_1, β_2) which is different from the equilibrium strategies. The following cases can occur:

- $\beta_1 = \beta_2$;
  Since the politicians offer the same contract and are ex ante identical, they are elected with probability $\frac{1}{2}$.
- $\overline{\beta}^{AI} > \beta_1 > \beta_2$;
  Both politicians if elected would choose *STP*. Since they will not get reelected the public receives no transfers. So both politicians are elected with probability $\frac{1}{2}$.

[19] We omit the tedious but easy description of all possible cases.

- $\overline{\beta}^{AI} > \beta_2 > \beta_1$;
  Both politicians if elected would choose *STP*. Since they will not get reelected the public receives no transfers. So both politicians are elected with probability $\frac{1}{2}$.
- $\beta_1 = \overline{\beta}^{AI}, \beta_2 < \overline{\beta}^{AI}$;
  The first politician chooses *LTP* while the second would select *STP*. According to our assumption the public is better off by electing the first candidate.
- $\beta_1 = \overline{\beta}^{AI}, \beta_2 > \overline{\beta}^{AI}$;
  Both politicians select *LTP*. It is cheaper to elect the first politician.
- $\overline{\beta}^{AI} < \beta_1 < \beta_2$;
  Both politicians choose *LTP*. The first politician is elected since he requires lower transfers from the public.
- $\beta_1 > \overline{\beta}^{AI} > \beta_2$;
  The first politician chooses *LTP*, while the second selects *STP*. The public is better off by *LTP* and paying transfers to an elected politician than by inducing *STP* because a politician who has undertaken *STP* is not reelected and so the public receives no transfers.
- In all other cases, the voters' utility associated with the election of the second candidate is always higher than that of electing the first candidate.

Hence the election and reelection strategies described in (iv) and (v) are indeed optimal. □

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Chapter 3
Vote-share Contracts Without Signaling of Competence

3.1 Background

We started our research on the topic of this chapter in 2005 and it has been on our workbench ever since. Our objective was to strengthen an office-holder's involvement in his tasks, yielding the best-possible output for society. We suggest to connect the office-holder's effort to his reelection, taking into account the fact that an incumbent usually finds it easier to be reelected than to be elected. Our idea was to circumvent a drawback of this incumbency advantage: An incumbent may have fewer incentives to work as hard as possible once in office, and office-holders whose ability is below average might still be reelected. To prevent office-holders from slacking off, we suggested to render reelection more difficult than election. This could be achieved by requiring a higher vote percentage for reelection, which could simply be imposed.

Instead of imposing a fixed impediment to reelection, which would require information about the costs and benefits for the public of these higher vote percentages, we suggest to *allow* candidates for office to offer this hindrance, i.e. *higher hurdles for reelection*, themselves. During his first campaign, a candidate would promise to do so well during his first term, that the percentage of votes necessary for his reelection could be set higher than the one needed for his first election. If 50% of the votes are needed for election, for instance, the 50% for reelection can often be reached easily, due to the incumbency advantage. Before election, a candidate could commit to accepting reelection only if at the end of his first term in office, he reaches a certain vote percentage barrier, that is higher than 50%. This would countermand any incumbency advantage he might have when in office, the extra-votes needed for reelection being due to particular effort. As a matter of fact, a candidate offering such a commitment during his first campaign would be saying: "I promise to work so well during my first term that you will give me a higher vote-share for reelection than for election", thus partly—or entirely—waiving his incumbency advantage.

If the candidate commits to a precise vote percentage in a certified document—i.e. a so-called "Vote-share Contract"—and pledges himself not to accept reelection if he

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does not reach this figure, he will enhance his credibility and raise both his chances to be elected and society's chances to have a well-performing office-holder.

Yet, a particular feature of the setting described in this chapter is that the Vote-share Contract is offered by a candidate when he has *no knowledge about his real competence*. In the next chapter, we will deal with a similar contract, offered by candidate who has *some knowledge about his competence*, at this time.[1]

3.2 Introduction

We illustrate the working of Vote-share Contracts in a simple two-period model. At the start of each period, two candidates compete for office. In each period, the office-holder can undertake a public project whose output is determined by the policy-maker's effort and ability. These are not observable by voters. In each period, the office-holder also chooses an ideological policy or a redistribution policy that affects each voter differently.

Vote-share thresholds have two effects. First, a higher threshold stimulates greater effort, as the marginal gain from higher effort increases in terms of improved reelection prospects. This is socially desirable. Second, a higher vote-share threshold raises the lowest possible ability of those of the incumbents that are reelected, as only such incumbents will be able to garner enough votes. This is socially desirable as long as incumbents with above-average ability are reelected. If the threshold is too high, even incumbents with above-average qualities will be deselected, which is socially undesirable. A socially optimal vote-share threshold for incumbents balances these effects. We show that the socially optimal vote-share threshold for incumbents is typically larger than one half.

A vote-share threshold could be set by the public. More interestingly, we allow that candidates compete with Vote-share Contracts. We show that the majority of voters will elect the candidate who commits to a vote-share threshold that is closer to the socially optimal threshold. As a result, both candidates will commit to the socially optimal vote-share threshold.

We discuss several extensions of the model, alternative voting procedures and further applications of the basic principle. In particular, we suggest that increasing vote-share thresholds can also be used to constrain government debt accumulation, with the yearly roll-over of new debt requiring increasing vote-share thresholds.

The chapter is organized as follows: In the next section we introduce the model. Section 3.4 discusses the benchmark case where there are only standard elections. In Sect. 3.5 we introduce Vote-share Contracts and derive their welfare properties. In Sect. 3.6 we discuss various extensions of the model and alternative election procedures. Section 3.7 concludes.

[1] A first working paper on the subject was published as CEPR DP 6497 in 2007, and the basic idea was outlined on www.voxeu.org, also in 2007 (see Gersbach 2007a, b). This chapter is an updated version.

3.3 The Model

3.3.1 Agents

We consider a society that decides democratically to whom it should delegate policy-making. At the beginning of each of two periods, $t = 1$ and $t = 2$, voters must elect a politician. At both election dates, the same two candidates are competing for office. Candidates are denoted by k or $k' \in \{R, L\}$. Candidate R (L) is the right-wing (left-wing) candidate. There is a continuum of voters. Each individual voter is indexed by $i \in [0, 1]$.

3.3.2 Policies

There are two types of policy problems the policy-maker faces.

- Public Project: P
  In each period, the office-holder can undertake a public project. The result is determined by both the effort invested by the policy-maker and his ability. This output of the public project in period t is given as

$$g_t = \gamma(e_{kt} + a_k), \tag{3.1}$$

  where e_{kt} represents the effort exerted by the policy-maker k in period t, a_k represents his ability and $\gamma > 0$ is a parameter. The ability a_k is a random variable distributed uniformly on $[-A, A]$, $A > 0$. After the office-holder has exerted e_{kt}, he will know how able he is. This will remain private information. Voters will only observe g_t. All citizens derive the same utility from the public project according to the instantaneous utility function $U^P(g_t) = g_t$.
- Ideological (or Redistribution) Policy: I
  In each period, the policy-maker decides on an ideological policy I that affects voters differently. The choice set for I is represented by a one-dimensional policy space [0, 1]. We assume that voters are ordered according to their ideal points regarding I. Voter i has preferences over I according to the instantaneous utility function

$$U_i^I(i_{kt}) = -(i_{kt} - i)^2, \tag{3.2}$$

  where i_{kt} is the policy chosen by policy-maker k on I and i is the ideal point of voter i.

3.3.3 Utilities

In this section we describe the utilities of voters and candidates. Voters and politicians are equally patient. Their common discount factor is denoted by β with $0 < \beta \leq 1$.

The expected utility of voter i evaluated at the beginning of $t = 1$ is given by the discounted sum of the benefits from the public project and from the ideological policy. We distinguish two cases.

(i) If the same politician k is in office in both periods, lifetime utility is given by

$$V_i = g_1 + U_i^I\left(i_{k1}\right) + \beta\left[g_2 + U_i^I\left(i_{k2}\right)\right]. \tag{3.3}$$

(ii) If politician k is in office in period $t = 1$ and politician k' ($k' \neq k$) holds office in period $t = 2$, lifetime utility is given by

$$V_i = g_1 + U_i^I\left(i_{k1}\right) + \beta\left[g_2 + U_i^I\left(i_{k'2}\right)\right]. \tag{3.4}$$

The candidates derive utility from two sources.

- Office-holding
  A policy-maker derives private benefits $b > 0$ from holding office, including monetary and non-monetary benefits such as power and enhanced career prospects. He incurs costs of $C(e_{kt}) = ce_{kt}^2$, where $c > 0$, from exerting effort.
- Benefits from policies
  We assume that candidate L is a left-wing candidate, i.e. his most preferred point, denoted by μ_L with regard to policy I, satisfies $\mu_L < \frac{1}{2}$. Similarly, candidate R is a right-wing candidate with ideal point $\mu_R > \frac{1}{2}$. To simplify the exposition, we assume that $\frac{1}{2} - \mu_L = \mu_R - \frac{1}{2}$. Hence the candidates' ideal points are symmetrically distributed around the median's ideal point of $\frac{1}{2}$. Moreover, the candidates derive the same benefits from public projects as voters.

To describe the overall utility of politicians, we have to distinguish four cases. For example, politician R's lifetime utility, denoted by V_R, can be computed as follows:

(i) If R is in office over both periods:

$$V_R = b - (i_{R1} - \mu_R)^2 - ce_{R1}^2 + g_1 + \beta[b - (i_{R2} - \mu_R)^2 - ce_{R2}^2 + g_2].$$

(ii) If R is in office in $t = 1$ only:

$$V_R = b - (i_{R1} - \mu_R)^2 - ce_{R1}^2 + g_1 + \beta[-(i_{L2} - \mu_R)^2 + g_2].$$

(iii) If R is in office in $t = 2$ only:

$$V_R = -(i_{L1} - \mu_R)^2 + g_1 + \beta[b - (i_{R2} - \mu_R)^2 - ce_{R2}^2 + g_2].$$

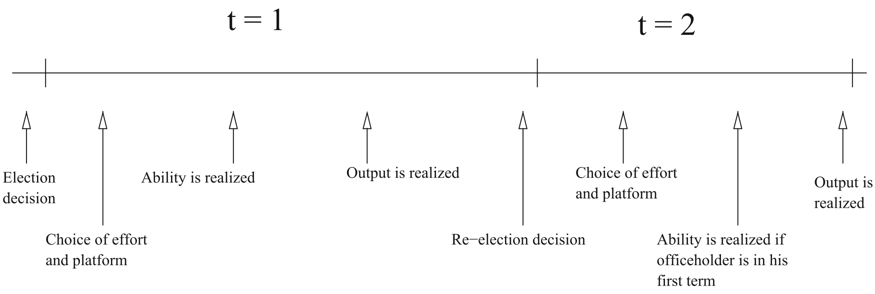

Fig. 3.1 Time-line for the overall game

(iv) If R is never in office:

$$V_R = -(i_{L1} - \mu_R)^2 + g_1 + \beta[-(i_{L2} - \mu_R)^2 + g_2].$$

3.3.4 Parameter Assumptions

We assume that b is sufficiently large, so that candidates will prefer to be in office under any of the circumstances we consider. To simplify the exposition, we assume $\beta = 1$. The extension to $\beta < 1$ is straightforward and the qualitative effects remain the same for β sufficiently close to one.

3.3.5 The Overall Game

We summarize the overall game in Fig. 3.1.

3.3.6 Assumptions and Equilibrium Concept

We assume that politicians cannot commit themselves to a policy platform during campaigns. Voters observe the policy maker's choice with regard to policy I. Moreover, we assume that voters observe only output g_1 and not its composition between effort and ability.[2] Output g_1 is not contractible so it cannot be used to generate rewards for politicians beyond elections. Moreover, citizens are assumed to vote sincerely, i.e. they vote for the candidate from whom they expect a higher utility.[3] We are looking for perfect Bayesian Nash equilibria for the game under these assumptions.

[2]This assumption follows Alesina and Tabellini (2007).

[3]Obviously, with a continuum of voters, the individual voter has no influence on the outcome of an election. The optimality of sincere voting can be justified for a model variant with a large but finite number of voters or when the act of voting generates benefits.

3.4 Elections Alone

We first examine the standard case where elections are held before the first and second term start. The candidate with the higher share of votes will be elected. If both candidates obtain the same share of votes, the probability of each candidate to win in the first period is 0.5. In the second period, we consider the tie-breaking rule that prescribes that in this case, the incumbent will be elected.

3.4.1 *The Second Period*

As candidates cannot commit to policy platforms, a policy-maker will choose his most preferred platform in the second period. The expected amount of the public project depends on whether the policy-maker is in his first term and does not know his ability, or whether he is in his second term and has observed his ability in period 1. For the analysis of the second period, we momentarily assume that the ability of the office-holder in the first term will be perfectly inferred by all agents at the end of the first term. This will be proven in Subsect. 3.4.2. In the Appendix, we show:

Proposition 3.1 *Suppose that candidate k is elected at date $t = 2$. Then*

(i) he will choose $i_{k2} = \mu_k$ for policy I;
(ii) irrespective of whether k is in his first or second term, he will choose $e^*_{k2} = \frac{\gamma}{2c}$;
(iii) the expected utility of a policy maker at the beginning of period 2 is given by

$$(\alpha)\ \textit{first-term policy maker:}\quad V^*_{k2} = b + \frac{\gamma^2}{4c},$$
$$(\beta)\ \textit{second-term policy maker:}\ V^*_{k2} = b + \frac{\gamma^2}{4c} + \gamma a_k.$$

(iv) The expected utility of the politician $k' \neq k$ who has lost the second election is given by

$$(\alpha)\ V^D_{k'2} = \gamma\left(\frac{\gamma}{2c}\right) + \gamma a_k - (\mu_R - \mu_L)^2, \textit{ if } k \textit{ has been in office in the first period}$$
$$(\beta)\ V^D_{k'2} = \gamma\left(\frac{\gamma}{2c}\right) - (\mu_R - \mu_L)^2, \textit{ if } k' \textit{ has been in office in the first period.}$$

3.4.2 *The First Period*

We now look at equilibria in the first period. As the candidates' ideal points are distributed symmetrically around the median voter's ideal point, the probability of each candidate winning is one half. Once in office, the candidate has to choose e_{k1} and i_{k1}. Without loss of generality, we assume that candidate R has been elected. We first make two simple observations that will hold in every equilibrium with pure strategies.

Fact 3.1 *Suppose that candidate R is elected at date $t = 1$. Then*

(i) he will choose $i_{R1} = \mu_R$;
(ii) voters will perfectly infer the ability of the policy-maker at the end of period 1.

The first part of this fact is obvious, as voters know that policy-makers will choose their bliss points in the last period. So politician R will not gain more votes in the second election by choosing a different platform than μ_R in period 1. The second part follows from the informational structure of the game. As candidates will observe their ability after they have exerted effort, in any pure strategy equilibrium, exactly one level of effort will be chosen and expected by the voters. Any deviation of g_1 from the expected effort multiplied by γ will be interpreted correctly as variation in ability.[4]

Now we derive the optimal choice of effort by the office-holder in the first period. For this purpose, a few preliminary steps are necessary. Let $\hat{e}_1$ denote the public's expectations about the incumbent's effort level in the first period. Moreover, let $p(e_{R1}, \hat{e}_1)$ denote the probability with which office-holder R will be reelected and $\widetilde{a}_R(e_{R1}, \hat{e}_1)$ denote candidate R's expected level of ability conditional on the fact that he is reelected. In the Appendix, we show that the following holds:

Fact 3.2 *We have*

$$p(e_{R1}, \hat{e}_1) = \frac{1}{2}\left(1 + \frac{(e_{R1} - \hat{e}_1)}{A}\right), \tag{3.5}$$

$$\widetilde{a}_R(e_{R1}, \hat{e}_1) = \frac{A + \hat{e}_1 - e_{R1}}{2}. \tag{3.6}$$

Note that the probability of R being reelected, which is given by $p(e_{R1}, \hat{e}_1)$, increases in e_{R1}. In terms of given expectations about his effort $\hat{e}_1$, the office-holder can improve the public's estimate of his ability by exerting more effort. A more favorable evaluation of his ability increases the incentives of voters to vote for him. Similarly, we can explain why the expected level of R's ability contingent on the fact of being reelected decreases with e_{R1}. Increases in e_{R1} imply that R will be reelected even if he displays lower levels of ability. As a consequence, $\widetilde{a}_R(e_{R1}, \hat{e}_1)$ is decreasing.

Now the incumbent's optimization problem can be stated in the following way:

$$\begin{aligned}\max_{e_{R1} \geq 0} \Bigg\{ b + \gamma e_{R1} - c e_{R1}^2 + p(e_{R1}, \hat{e}_1)\left(b + \gamma\left(\frac{\gamma}{2c} + \widetilde{a}_R(e_{R1}, \hat{e}_1)\right) - \frac{\gamma^2}{4c}\right) \\ + \left(1 - p(e_{R1}, \hat{e}_1)\right)\left(\frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2\right)\Bigg\}.\end{aligned} \tag{3.7}$$

Here, we have used the facts that candidate R is reelected with probability $p(e_{R1}, \hat{e}_1)$ and dismissed with probability $1 - p(e_{R1}, \hat{e}_1)$.

---

[4]Formally, $a_R = \frac{g_1 - \gamma \hat{e}_1}{\gamma}$, where $\hat{e}_1$ is the effort level expected by the electorate.

We are now in a position to calculate the effort level chosen by candidate R in the first period. In the Appendix we prove the following proposition:

Proposition 3.2

(i) The policy-maker R chooses $e^*_{R1} = \frac{1}{2c}\left\{\gamma + \frac{1}{2A}[b - \frac{\gamma^2}{4c} + (\mu_R - \mu_L)^2]\right\}$.

(ii) The probability of R being reelected is given by

$$p(e^*_{R1}, e^*_{R1}) = \frac{1}{2}. \tag{3.8}$$

(iii) The average ability level of a reelected candidate is

$$\widetilde{a}_R(e^*_{R1}, e^*_{R1}) = \frac{A}{2}. \tag{3.9}$$

The equilibrium effort e^*_{R1} depends on the parameters in an intuitive way. The larger the utility loss of the incumbent if he is deselected, i.e. the larger $(\mu_R - \mu_L)^2$ and b, the higher the effort the politician is willing to invest. The higher A, the lower the marginal gain in reelection chances when R marginally increases effort. Accordingly, greater uncertainty regarding quality, i.e. a large value of A, will depress effort. The impact of γ is more subtle. On the one hand, higher γ increases the marginal value of higher effort today and the value of office tomorrow, which both motivate R to invest more effort. On the other hand, higher γ increases the utility in period 2 when the opponent is in office and increases the losses if the incumbent is reelected with lower ability than average. These two effects reduce the effort choice of R.

3.5 Vote-share Contracts

3.5.1 Vote-shares as Political Contracts

In this section, we allow both candidates to offer Vote-share Contracts by stipulating a vote-share threshold s_k with $\frac{1}{2} \leq s_k \leq 1$. Throughout the section, we assume that $\frac{2\mu_R - 1}{2A\gamma} < \frac{1}{2}$, which ensures interior solutions.[5] The interpretation is as follows: If politician k takes office in $t = 1$, he must win a share of votes at least equal to s_k at the next election date if he wants to retain office. Otherwise, the challenger will take office. Hence, the incumbent faces a self-imposed vote threshold in the election at the end of period 1.

The vote-share threshold is a particular type of Political Contract. Generally, a Political Contract consists of verifiable election promises and the associated rewards

[5] Corner solutions are an important variant of our model. If $\frac{2\mu_R - 1}{2A\gamma} > \frac{1}{2}$, the incumbent may have an incentive to renounce exerting high effort, since reelection chances are too low or zero when vote-share thresholds are high.

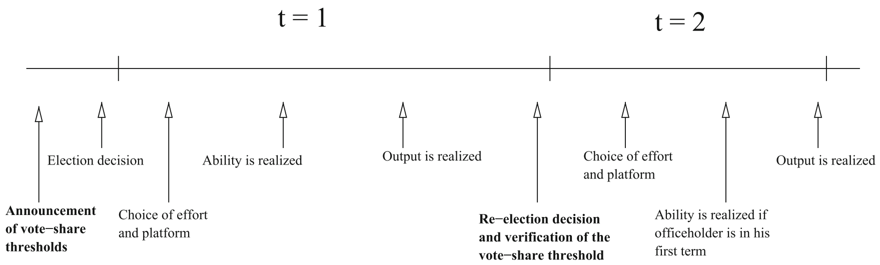

Fig. 3.2 Time-line for the extended game

or sanctions, depending on whether promises are kept or not. These type of contracts describe what a politician is willing to offer to society. Political contracts have to be approved by an independent body.

The timing of the extended game is summarized in Fig. 3.2.

3.5.2 The Second and First Period

For the first step of the analysis, we assume that a candidate k, say R, has been elected after having offered a vote-share threshold $s_R \geq \frac{1}{2}$.

In the second period, the choice regarding P and I by R (if he remains in office), or by L (if he enters office) will remain the same as in Proposition 3.1. However, the election probabilities of R and L will change in period 2, which will be examined next.

In the Appendix, we show that the equations in Fact 3.2 have to be modified in the following way:

Fact 3.3 *We have*

$$p(e_{R1}, \hat{e}_1) = \frac{1}{2}\left(1 + \frac{1}{A}\left(e_{R1} - \hat{e}_1 - \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1)\right)\right), \tag{3.10}$$

$$\widetilde{a}_R(e_{R1}, \hat{e}_1) = \frac{A + \hat{e}_1 + \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1) - e_{R1}}{2}. \tag{3.11}$$

It is straightforward to see that these equations correspond to Eqs. (3.5) and (3.6) for $s_R = \frac{1}{2}$.

The optimal choice of e_{R1} is the solution to the optimization problem (3.7), together with Eqs. (3.10) and (3.11). Equilibrium values with vote-shares are labelled by V. In the Appendix, we show:

Proposition 3.3

(i) $e_{R1}^{*V} = \frac{1}{2c}\left\{\gamma + \frac{1}{2A}\left[b - \frac{\gamma 2}{4c} + (2\mu_R - 1)(2s_R - 1) + (\mu_R - \mu_L)^2\right]\right\}.$

(ii) The probability of R being reelected is given by

$$p^V(e_{R1}^{*V}, e_{R1}^{*V}) = \frac{1}{2} - \frac{(2\mu_R - 1)(2s_R - 1)}{2A\gamma}. \tag{3.12}$$

(iii) The average ability level of a reelected candidate corresponds to

$$\widetilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V}) = \frac{A + \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1)}{2}. \tag{3.13}$$

We observe that the equilibrium effort level and the average ability level of a reelected candidate, given by $\widetilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V})$, are increasing in s_R. This shows that vote-share thresholds for incumbents have two effects. First, a higher vote-share threshold stimulates effort, as it raises the marginal gain in reelection chances which can be reaped with higher effort. Second, a higher vote-share threshold raises the lowest-possible ability of the incumbents that are reelected, as only those incumbents will be able to garner enough votes for the purpose. This is socially desirable as long as incumbents with above-average ability are reelected. If the threshold is too high, even incumbents with above-average qualities will be deselected, which is socially undesirable. A socially optimal vote-share threshold for incumbents balances these effects, and the socially optimal vote-share threshold for incumbents is typically larger than one half, which we will show next.

3.5.3 Competition for Vote-share Contracts and Welfare

Finally, we consider the initial stage when both candidates compete for office with Vote-share Contracts. We call a vote-share threshold "ex ante optima" if it maximizes expected aggregated utility.[6] For that purpose, we define the optimal vote-share from the perspective of the median voter. This vote-share is denoted by s^* and is the solution of the following problem:[7]

$$\max_{\frac{1}{2} \le s_R \le 1} \left\{ \gamma e_{R1}^{*V} + \left(p^V(e_{R1}^{*V}, e_{R1}^{*V}) \right) \gamma \tilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V}) \right\}. \tag{3.14}$$

For the solution s^* we obtain the following fact:

Fact 3.4

$$s^* = \min\left\{ \frac{1}{2} + \frac{\gamma^2}{4c(2\mu_R - 1)}; 1 \right\}. \tag{3.15}$$

[6] Precisely, an optimal vote-share threshold maximizes aggregate utility when voters can impose vote-share thresholds and use elections to select a candidate.

[7] Note that the expected ability of a new candidate is zero.

The fact is proven in the Appendix.[8] We are now ready to state our main theorem.

Theorem 3.1

(i) In the first campaign, both candidates R and L offer s^*. Each candidate wins the election with probability 0.5.
(ii) $s^* > \frac{1}{2}$.
(iii) s^* is the ex ante optimal vote-share.

Proof of Theorem 3.1

We first observe that the reelection probability of an incumbent offering s^* is larger than zero, as $p^V(e_{R1}^{*V}, e_{R1}^{*V}) > 0$ according to our assumption $\frac{2\mu_R - 1}{2A\gamma} < \frac{1}{2}$. Hence, according to our general assumption that b is sufficiently large, the incumbent has no incentive to exert lower effort, thereby losing his chances of getting reelected. If a candidate deviates from s^* (higher or lower vote-shares), he will not be elected, as the median voter is better off with the candidate offering s^*. Hence, deviation is not profitable. Uniqueness of the equilibrium choice s^* follows from the same considerations. If a candidate chooses a share $s_k \neq s^*$, the other candidate k' can win the election with certainty by choosing a vote-share threshold marginally closer to s^*. The second point is obvious. For the third point, we observe that any other vote-share threshold lowers the expected utility derived from public projects, as citizens are homogeneous with respect to public project provision. Due to the symmetry of ideal points of candidates and voters, aggregate utility from the ideological project does not depend on whether the left- or right-wing candidate is elected. This proves the Theorem. □

From Theorem 3.1, it follows that Vote-share Contracts lead to higher welfare than standard elections. Vote-share Contracts induce higher efforts and raise the ability of reelected incumbents.

A final remark is in order: The utility of the politicians in office is negligible in our model, as we have a continuum of voters. Thus, their utility does not affect welfare considerations. In a finite version of our model, the utility of the politician and the cost of exerting effort will affect the welfare optimizing vote-share threshold. As a result, the welfare-optimal vote-share in a finite version of our model tends to be slightly lower.

3.6 Extensions and Ramifications

We have illustrated the working of Vote-share Contracts in a simple model. Numerous extensions can and should be pursued to address the robustness and validity of the argument for using Vote-threshold Contracts in a broader context.

[8] The vote-share threshold s^* is larger than $\frac{1}{2}$, which is the vote-share threshold ensuring that the incumbent will be reelected if and only if his ability is equal to or greater than zero. The median voter trades off higher effort versus lower reelection probability of incumbents with high ability.

3.6.1 Incumbency Advantages

In particular, one could incorporate various sources of incumbency advantages into our model. For this purpose, it is useful to start with a brief overview of possible sources of incumbency advantages.

At least three explanations have been advanced for the existence of incumbency advantages. First, the incumbent may be perceived as a safer bet than his challengers as developed in the original papers by Bernhardt and Ingberman (1985) and Anderson and Glomm (1992). For example, the incumbent may have gained a communication advantage over his challengers. Second, incumbents may have, on average, higher qualities than challengers. The reason is two-fold: candidates who have won in the past are of higher quality[9] and challengers may be deterred from running against them (Cox and Katz 1996, Jacobson and Kernell 1983, Stone et al. 2004, and Gordon et al. 2007). Third, the incumbent may be able to increase his reelection prospects by the provision of constituency service (Cain et al. 1987) or (socially) costly actions like government expenditures or war (Alesina and Cuckierman 1990, Hess and Orphanides 1995, 2001, Rogoff and Sibert 1988, and Cukierman and Tommasi 1998).[10]

We next explore three different types of incumbency advantages and how they impact on the results in Sect. 3.5.

Communication Advantage

Suppose that candidates can commit to a specific platform regarding ideological policy during campaigns. The final position a candidate will adopt when he is in office differs, however, by some random disturbance. Suppose a candidate can achieve a communication advantage when he is in office, e.g. uncertainty (variance) about implemented policies is usually lower for incumbents than it is for challengers, as discussed above. Such an incumbent will move towards his own preferred position in the next election. Vote-share Contracts can draw the platform choice of the incumbent towards the center, and by using the approach set out in Gersbach (1992), one can show that it is welfare-improving from a utilitarian perspective.[11]

Learning by Doing

Another fruitful extension is learning by doing. Suppose the politician in office experiences learning effects during the first term in office. Then, his marginal effort

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[9]See Ashworth (2005), Banks and Sundaram (1998), Londregan and Romer (1993), Samuelson (1984), Zaller (1988), and Diermeier et al. (2005).

[10]Other explanations of incumbency advantage are based on the incumbents' voting behavior and face-recognition (Ansolabehere et al. 2000 and Prior 2006). Finally, challengers may have less access to campaign funds (Gerber 1998). Whether these explanations can themselves be explained by a quality-based incumbency advantage is addressed in Ashworth and Bueno de Mesquita (2008). Given the existence of large incumbency advantages, Buchler (2007) challenges the assumption that competitive elections are a priori socially desirable.

[11]The situation is more complicated, but qualitatively the same, if two new candidates with different communication skills compete for office on the basis of Vote-share Contracts.

costs may decline for the second term. The incumbent will thus have an election probability higher than one-half. The source of the incumbency advantage is socially desirable ex ante. As competing candidates will choose welfare-optimal vote-share thresholds, the positive welfare effect of vote-share thresholds continues to hold.[12]

Output-Shift Policy

We next outline an extension that allows politicians to take costly social actions to improve their reelection prospects. Specifically, suppose we allow that the incumbent can decide whether or not to shift the realization of a specific part of the output from one period to the next. If he is still in office in the next period, he can realize a part of the shifted output. However, a new office-holder cannot reap the benefits of the effort invested by the preceding policy-maker. This makes deselection costly and induces voters to reelect an incumbent even if his ability is lower than the expected ability of the challenger (for details, see Gersbach 2007a, b).

Output shifts are possible on policies that require policy-specific efforts by the policy-maker, and enable him to determine the time at which the output is realized. Examples are international treaties, foreign policy or new regulatory frameworks for specific industries such as the health care system. Such policies require policy-specific human capital that is lost at least partially when a new government comes into office. Moreover, the timing of the realization of the benefits from such policies lies in the hands of the policy-maker. The option to shift output across time is a simple device generating an incumbency advantage that is, however, socially detrimental.

As shown in Gersbach (2007a, b), in such circumstances, Vote-share Contracts have additional benefits beyond those identified in this chapter, as they cause the deselection of incumbents with below average ability and reduce socially costly output shifts.

3.6.2 Ramifications and Applications

Alternative Election Procedures

Two alternative election procedures involving Vote-share Contracts can be considered. First, an election procedure could be a separate election between a new right-wing candidate and candidate L if the incumbent R does not win at least the self-imposed share of votes s_R. Such a procedure ensures that politicians are only elected if they receive at least 50% of the votes. Second, instead of the candidates, society may impose a term-dependent vote share or reelection threshold. Both variants of the model yield the same (latter version) or qualitatively similar results (former version). The result is obvious for the latter version. The public will set the threshold s^*, as any other threshold will lower the utility of all voters, given the election of one of the candidates. Details on the former version are available upon request.

[12] Details are available upon request.

Repeated Competition with Vote-share Contracts

A useful extension of the model is to consider a larger time horizon or a version of the model with an infinite horizon, where candidates for public office compete in each term on the basis of Vote-share Contracts. In such a framework, the election hurdle will typically increase with the number of terms an incumbent stays in office. We conjecture that Vote-share Contracts are also welfare-improving in this type of dynamic versions of our model.

Constraining Government Debt Accumulation

Increasing vote-share thresholds can also be used to constrain government debt accumulation. Suppose the government wants to issue debt beyond normal rules. A standard rule is for example to constrain public debt financing by a government net investment. Possible exceptions are recessions or natural disasters. We suggest using the following correction mechanism when governments have issued debt beyond normal rules: The government can roll over the exceptional debt from year to year, but for this it needs the support of the parliament. The required vote-share threshold is increasing over time, which makes rolling over debt more and more difficult. Such a rule allows the legislature to determine the timing of fiscal consolidation and also ensures that exceptional debt will eventually be repaid if the limit of the vote-share threshold schedule is set close to the unanimity rule. Moreover, if the same vote-share threshold needs to be applied to situations when the government wants to issue new exceptional debt although past exceptional debt has not yet been repaid, accumulation of exceptional debt is excluded.

3.7 Conclusion

We have introduced Vote-share Contracts, which would improve the functioning of a liberal democracy. Of course, institutional changes may trigger feedback and consequences that are unintended and unknown yet, both when the change is proposed and when it actually happens. Nevertheless, Vote-share Contracts are a new institution that liberal democracies would be well-advised to explore.

Appendix

Proof of Proposition 3.1

The first point is obvious. Suppose next that in $t = 2$, the politician is in his first term. Accordingly, he does not know his ability yet. His problem is given by

$$\max_{e_{k2}}\{\mathbb{E}[\gamma(e_{k2} + a_k)] - ce_{k2}^2\}.$$

The solution is given by $e_{k2} = \frac{\gamma}{2c}$. Suppose that the politician is in his second term and has observed his ability in the first period. His problem is given by

$$\max_{e_{k2}}\{\gamma(e_{k2} + a_k) - ce_{k2}^2\},$$

which yields the same solution. The expected utility for the first-term office-holder from the public project is given by $\gamma\left(\frac{\gamma}{2c}\right) - c\left(\frac{\gamma}{2c}\right)^2 = \frac{\gamma^2}{4c}$. For an office-holder in his second term, the corresponding utility is

$$\gamma\left(\frac{\gamma}{2c} + a_k\right) - c\left(\frac{\gamma}{2c}\right)^2 = \frac{\gamma^2}{4c} + \gamma a_k.$$

With these expressions, the formulas in the proposition follow at once. □

Proof of Fact 3.2

In the following we consider the reelection decision of the median voter $i = \frac{1}{2}$. It is optimal for the median voter to reelect R if electing the challenger yields less expected utility in the second period. Formally, this condition can be stated as

$$\gamma\left(e_2^* + (a_R + e_{R1} - \hat{e}_1)\right) \geq \gamma e_2^*, \tag{3.16}$$
$$\gamma\left(a_R + e_{R1} - \hat{e}_1\right) \geq 0, \tag{3.17}$$
$$a_R \geq -e_{R1} + \hat{e}_1, \tag{3.18}$$

where we have applied the observation that upon observing g_1, the median voter expects the ability level of R to be $\frac{g_1}{\gamma} - \hat{e}_1 = a_R + e_{R1} - \hat{e}_1$, with $e_2^* = e_{R2}^* = e_{L2}^* = \frac{\gamma}{2c}$. The above condition states that R is reelected if his ability level is equal to or higher than the critical level $-e_{R1} + \hat{e}_1$.[13] Applying the fact that a_R is uniformly distributed on $[-A, A]$, we conclude that the probability of a_R being higher than $-e_{R1} + \hat{e}_1$ amounts to $p(e_{R1}, \hat{e}_1) = \frac{A+(e_{R1}-\hat{e}_1)}{2A}$.

It remains to derive the expression for $\widetilde{a}_R(e_{R1}, \hat{e}_1)$ stated in the text. Recall that this variable denotes the ability level of R, conditional on the fact that he is reelected. We have already shown that R is reelected if and only if $a_R \geq -e_{R1} + \hat{e}_1$. Thus, the arithmetical average of $-e_{R1} + \hat{e}_1$ and A yields the desired expression, i.e. $\widetilde{a}_R(e_{R1}, \hat{e}_1) = \frac{A+\hat{e}_1-e_{R1}}{2}$. □

[13]For simplicity, we use the tie-breaking rule that the incumbent is reelected if he receives exactly half of the votes.

Proof of Proposition 3.2

Fact 3.2 and the maximization problem (3.7) yield the following first-order condition:

$$\gamma - 2ce_{R1} + \frac{1}{2A}\left(b + \frac{\gamma^2}{4c} + \frac{\gamma\left(A + \hat{e}_1 - e_{R1}\right)}{2}\right)$$
$$-\frac{\gamma}{2}\left(\frac{e_{R1} - \hat{e}_1}{2A} + \frac{1}{2}\right) - \frac{1}{2A}\left(\frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2\right) = 0.$$

In equilibrium, $\hat{e}_1 = e_{R1}$ will hold, so the equilibrium effort e^*_{R1} is given by

$$2ce^*_{R1} = \gamma\left(1 + \frac{1}{4} - \frac{1}{4}\right) + \frac{b}{2A} - \frac{\gamma^2}{8Ac} + \frac{1}{2A}(\mu_R - \mu_L)^2$$

or

$$e^*_{R1} = \frac{1}{2c}\left\{\gamma + \frac{1}{2A}[b - \frac{\gamma^2}{4c} + (\mu_R - \mu_L)^2]\right\}.$$

The assertions now follow by substitution. □

Proof of Fact 3.3

The derivation of (3.10) and (3.11) is very similar to the derivation of (3.5) and (3.6). However, with $s_R > \frac{1}{2}$, candidate R is reelected only if voter $i = 1 - s_R$ prefers to vote for R, which implies that all voters with $i > 1 - s_R$ also prefer R to L.[14] This leads to the following condition:

$$\gamma\left(e_2^{*V} + (a_R + e_{R1} - \hat{e}_1)\right) - (\mu_R - (1 - s_R))^2 \geq \gamma e_2^{*V} - (\mu_L - (1 - s_R))^2. \tag{3.19}$$

Using $\mu_L = 1 - \mu_R$, this can be rewritten as

$$a_R \geq -e_{R1} + \hat{e}_1 + \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1). \tag{3.20}$$

The right-hand side of this inequality gives the minimum ability that R must have in order to be reelected. The minimum ability is increasing in s_R.

With this condition it is straightforward to show that (3.5) and (3.6) generalize to (3.10) and (3.11). □

Proof of Proposition 3.3

The problem of the incumbent is the same as in Proposition 3.2, except that we have to use Eqs. (3.10) and (3.11) rather than (3.5) and (3.6). Then, the first-order condition of the maximization problem (3.7) is given by

[14] We use the tie-breaking rule that the incumbent is reelected if he receives exactly s_R votes.

$$\gamma - 2ce_{R1} + \frac{1}{2A}\left(b + \frac{\gamma^2}{4c} + \frac{\gamma\left(A + \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1) + \hat{e}_1 - e_{R1}\right)}{2}\right)$$
$$-\frac{\gamma}{2}\left(\frac{A - \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1) + e_{R1} - \hat{e}_1}{2A}\right)$$
$$-\frac{1}{2A}\left(\frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2\right) = 0.$$

In equilibrium, $\hat{e}_1 = e_{R1}$ must hold, so the equilibrium effort e_{R1}^{*V} is given as

$$e_{R1}^{*V} = \frac{1}{2c}\left\{\gamma + \frac{1}{2A}\left[b - \frac{\gamma^2}{4c} + (2\mu_R - 1)(2s_R - 1) + (\mu_R - \mu_L)^2\right]\right\}.$$

Points (ii) and (iii) follow by inserting this equilibrium effort level into expressions (3.10) and (3.11). □

Proof of Fact 3.4

Together with Eqs. (3.12) and (3.13) the maximization problem (3.14) yields the following first-order condition:

$$\frac{(2\mu_R - 1)\gamma}{2Ac} - \frac{(2\mu_R - 1)}{A\gamma}\left(\frac{A\gamma + (2\mu_R - 1)(2s_R - 1)}{2}\right)$$
$$+(2\mu_R - 1)\left(\frac{1}{2} - \frac{(2\mu_R - 1)(2s_R - 1)}{2A\gamma}\right) = 0.$$

Solving for s_R yields $s^* = \frac{1}{2} + \frac{\gamma^2}{4c(2\mu_R - 1)}$. □

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Chapter 4
Vote Thresholds With Signaling of Competence

4.1 Background

In this chapter, we will again study why an incumbent might *offer* to accept reelection only if he has reached a certain (higher) vote percentage and why this might be socially beneficial. Yet, why would a politician encumber himself with such an impediment? To do this, he must find it advantageous. Once in office, an office-holder has no reason to waive his incumbency advantage: it will only make reelection more difficult.[1] But at the time of his *candidacy*, he might find that the commitment to a higher reelection vote percentage helps his (first) election. If he promises to accept reelection only if he is reelected with a higher, pre-defined vote percentage, he is perceived as willing to put so much effort into his first term in office that the voters will reward him with more votes than his incumbency advantage alone would yield. Such a campaign promise enhances the candidate's credibility, all the more if it is ascertained by a Political Contract. There is another issue: How can the candidate be sure that he will do well? His intended policy can prove more difficult to implement than he thought, be it for institutional or practical reasons. Circumstances may change, rendering a task more costly, more time-consuming or more dependent on parameters that cannot be influenced. An office-holder might also lose some support in parliament, or his projects might lose priority. Finally, an essential factor for success is this politician's *ability*, and more specifically, how well a candidate can signal his own ability at the time he offers to reach a higher vote percentage for reelection.

In the preceding chapter, we dealt with the simplest case: The candidate does *not know anything about his own ability* as an office-holder. As we have seen, this makes it more difficult to set the vote percentage he promises to reach at reelection. As he is unsure about the quality of his future performance, the candidate will set the reelection vote percentage a bit higher than the one for election, but not too high—as otherwise, he might not reach it. Another candidate might offer a better vote percentage—and win the election.

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[1]For a discussion of several important aspects of the incumbency advantage that are not discussed in this chapter, see the references at the end of this chapter.

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The situation is quite different if the candidate does have some knowledge about his ability. At first sight, one should think this renders it easier to set a vote percentage for reelection. It allows the candidate to be less prudent if he is able: He can offer to reach a higher reelection vote percentage than a candidate who does not know how able he is—as well as a higher vote percentage than a less able candidate, of course. Although the reelection vote percentage is easier to set, the consequences of this knowledge are more complex, as less able candidates might want to mimic more able candidates when they set their vote-threshold[2]: This is the issue we analyze in this chapter.[3]

4.2 Introduction

We illustrate the workings of vote-share thresholds in a model in which a polity selects an office-holder from a pool of candidates which differ in privately observed abilities. Office-holders undertake public projects whose output depends on their effort and ability. Elected candidates of low ability can try to mimic high-ability office-holders by exerting higher efforts in order to secure reelection. As a consequence, all office-holders or at least the vast majority may be reelected, as their expected ability is no worse than the expected ability of a challenger. Moreover, the efforts needed for low-ability office-holders to mimic high-ability ones may be low.

Imposing higher vote-thresholds for incumbents essentially eliminates the worst possible equilibria and thus improves the average ability of reelected incumbents on balance, and tends to improve efforts exerted by office-holders. This is socially desirable. As an illustration, consider the polar case where all policy-makers pool and produce the same output of public projects. Low-ability policy-makers exert more effort than high-ability ones to produce the same output, but the average level of effort tends to be low compared to equilibria in which low-ability policy-makers and high-ability ones choose different output levels and only the latter group is reelected. In such an equilibrium, in which all policy-makers pool, they will merely obtain 50% of the votes and get reelected. From the perspective of the voters, the expected abilities of the incumbent and of a challenger are the same. Imposing a higher vote threshold than 50% for incumbents essentially eliminates such bad equilibria. In order to reach higher vote shares, higher-ability office-holders must necessarily distinguish themselves from the low-ability ones. As a consequence, only those equilibria survive which, on balance, yield higher welfare.

Technically, we characterize in this chapter the set of equilibria with standard elections and those where higher vote thresholds for incumbents are imposed. We also

[2]In this chapter, we use the term "Vote-threshold Contract" rather than "Vote-share Contract" as in the preceding chapter. This choice is due to the fact that we leave the contractual implementation aside and now focus on the level of the vote-threshold. However, both terms are synonyms.

[3]This chapter is an updated version of the working paper, CEPR Discussion Paper No. 7320. A short version of the working paper was published in the Economics Bulletin, 2010, vol. 30(1), pages 774–785.

identify the circumstances under which vote thresholds higher than 50% eliminate the worst equilibria and improve welfare on balance. We also establish the existence of a socially optimal vote threshold.

As discussed in the preceding chapter, which is based on Gersbach (2007), this chapter is closely related to the large body of literature dealing with incumbency advantages. It is organized as follows: In Sect. 4.3 we introduce the model. Section 4.4 discusses the benchmark case where there are only standard elections. In Sect. 4.5 we investigate how higher thresholds for incumbents affect effort and ability of office-holders and overall welfare. In Sect. 4.6 we discuss various extensions, applications, and generalizations of the model. Section 4.7 concludes.

4.3 The Model

4.3.1 Agents

We consider elections in a two-period model. At the beginning of each of two periods, $t = 1$ and $t = 2$, voters must elect a politician. The same two candidates compete for office on both election dates. Candidates are denoted by k or $k' \in \{R, L\}$. Candidate R (L) is the right-wing (left-wing) candidate. The ability of a candidate is a random variable a_k distributed uniformly on $[-A, A]$, where $A > 0$. The realization of a_k, drawn at the beginning of period 1, is private information to candidate k. There is a continuum of voters. Individual voters are indexed by $i \in [0, 1]$.

4.3.2 Policies

There are two types of policy problems the policy-maker faces.

- Public Project: P
  In each period the office-holder can undertake a public project. The result is determined by the effort invested by the policy-maker and by his ability. The output of this public project in period t is given by [4]

$$g_t = \gamma(e_{kt} + a_k), \tag{4.1}$$

  where e_{kt} represents the effort exerted by the policy-maker k in period t, a_k represents his ability and $\gamma > 0$ is a parameter. Voters will observe g_t. The citizens derive utility from the public project in accordance with the instantaneous utility function $U^P(g_t) = g_t$.

[4]See Alesina and Tabellini (2007) for this formulation.

- Ideological (or Redistribution) Policy: I
  In each period the policy-maker decides on an ideological policy I that affects each voter differently. The choice set for I is represented by the one-dimensional policy space [0, 1]. We assume that voters are ordered according to their ideal points regarding I. Voter i has preferences about I according to the instantaneous utility function
$$U_i^I(i_{kt}) = -(i_{kt} - i)^2, \tag{4.2}$$
  where i_{kt} is the platform chosen by the policy-maker and i is the ideal point of voter i.[5]

Some remarks are in order here. The only advantage we assume the incumbent may have when he stands for reelection is that he may be able to signal his ability to voters by choosing a particular output g. In the extension we consider further advantages from being in office.

4.3.3 Utilities

In this section we describe the utilities of voters and candidates. The discount factor of voters and politicians is denoted by β with $0 < \beta \leq 1$.

The expected utility of voter i evaluated at the beginning of $t = 1$ is given by the discounted sum of the benefits from the public project and the ideological policy. Given that k is in office in both periods, the lifetime utility of voter i is given by

$$V_i = g_1 + U_i^I(i_{k1}) + \beta\left(g_2 + U_i^I(i_{k2})\right). \tag{4.3}$$

The candidates themselves derive utility from two sources:

- *Office-holding*
  A policy-maker derives private benefits b (with $b > 0$) from holding office, including monetary and non-monetary benefits such as power and enhanced career prospects. He incurs costs from the exertion of effort, amounting to $C(e_{kt}) = ce_{kt}^2$, with $c > 0$.
- *Benefits from policies*
  We assume that candidate L is a left-wing candidate, i.e. his most preferred point, denoted by μ_L with regard to policy I, satisfies $\mu_L < \frac{1}{2}$. Similarly, candidate R is a right-wing candidate with an ideal point $\mu_R > \frac{1}{2}$. To simplify the exposition we assume that $\frac{1}{2} - \mu_L = \mu_R - \frac{1}{2}$. Hence the candidates' ideal points are symmetrically distributed around the median's ideal point $\frac{1}{2}$. Moreover, the candidates derive the same benefits from public projects as voters, given by U^p.

[5]The quadratic utility function (4.2) solely eases the presentation. The results remain quantitatively unchanged for any concave or convex utility representation as long as there exists a trade off between a higher level of g and a more distant platform I.

To describe the overall utility of politicians we have to distinguish four cases. The lifetime utility of for example politician R, denoted by V_R, can be computed as follows:

(i) If R is in office over both periods,

$$V_R = b - (i_{R1} - \mu_R)^2 - ce_{R1}^2 + g_1 + \beta \left(b - (i_{R2} - \mu_R)^2 - ce_{R2}^2 + g_2 \right).$$

(ii) If R is in office in $t = 1$ only,

$$V_R = b - (i_{R1} - \mu_R)^2 - ce_{R1}^2 + g_1 + \beta \left(-(i_{L2} - \mu_R)^2 + g_2 \right).$$

(iii) If R is in office in $t = 2$ only,

$$V_R = -(i_{L1} - \mu_R)^2 + g_1 + \beta \left(b - (i_{R2} - \mu_R)^2 - ce_{R2}^2 + g_2 \right).$$

(iv) If R never is in office,

$$V_R = -(i_{L1} - \mu_R)^2 + g_1 + \beta \left(-(i_{L2} - \mu_R)^2 + g_2 \right).$$

The lifetime utility of politician L can be determined analogously.

4.3.4 Parameter Assumptions

We assume that b is not too small. More specifically, we require that

$$b > \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2.$$

As we will see, this assumption ensures that candidates with the lowest ability still aspire for reelection and that effort levels are always positive. Moreover, we assume $\frac{\gamma}{2c} > A$, which, as we show later, ensures that output will be non-negative when an incumbent with the lowest ability chooses effort. To simplify the exposition we assume $\beta = 1$.[6]

4.3.5 The Overall Game

We summarize the overall game in Fig. 4.1.

[6]The extension to $\beta < 1$ is straightforward, with utility levels in the second period scaled down by a factor β.

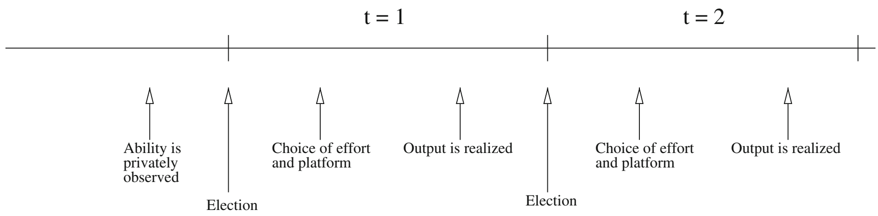

Fig. 4.1 Time-line for standard elections

4.3.6 Assumptions and Equilibrium Concept

We assume that politicians cannot commit to a policy platform. Voters observe the policy-maker's choice with regard to policy I. Moreover, we assume that voters observe output g_1 only and not its composition between effort and ability. The remaining parameters and assumptions of the model are common knowledge. Output g_1 is not contractable, so it cannot be used to generate rewards for politicians beyond elections. Moreover, citizens are assumed to vote sincerely, i.e. they vote for the candidate from whom they expect higher utility.[7] To break ties we assume that voters reelect the incumbent if they are indifferent between him and the competitor. We are looking for perfect Bayesian Nash equilibria for the described game.

4.4 Elections Alone

We first examine the standard case where elections are held. As a tie-breaking rule we assume that the probability of either candidate winning in the first period is 0.5 if they both have the same share of votes. In the second period the incumbent will be elected if he has 50% of the votes.

4.4.1 The Second Period

As candidates cannot commit to policy platforms, a policy-maker will choose his most preferred platform in the second period. In the Appendix we prove:

Proposition 4.1 *Suppose candidate k is elected at the beginning of period $t = 2$. Then*

[7] With a continuum of voters the individual voter has no influence on the outcome of an election. The optimality of sincere voting can be justified for a model variant with a large but finite number of voters or when the act of voting generates benefits.

(i) *he will choose* $i_{k2} = \mu_k$ *for policy I;*
(ii) *irrespective of whether k is in his first or second term, he will choose* $e_{k2} = \frac{\gamma}{2c}$*;*
(iii) *the utility the policy maker realizes in period* 2 *is given by*

$$V_{k2} = b + \frac{\gamma^2}{4c} + \gamma a_k;$$

(iv) *the utility of the politician* $k' \neq k$ *who has lost the second election is given by*

$$V_{k'2} = \gamma\left(\frac{\gamma}{2c}\right) - (\mu_R - \mu_L)^2;$$

We note that even the policy-maker with the worst ability $a = -A$ would like to remain in office if $b > \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2$, which we did indeed assume.

4.4.2 The First Period

We now look at the equilibria in the first period. As the candidates' ideal points are distributed symmetrically around the median voter's ideal point, the probability of either candidate winning is one half. Once in office, the candidate has to choose e_{k1} and i_{k1}. Without loss of generality we assume that candidate R has been elected. We first make a simple observation that will hold in every equilibrium with pure strategies.

Fact 4.1 *Suppose candidate R is elected at the beginning of period* $t = 1$. *Then he will choose* $i_{R1} = \mu_R$.

This fact follows from the observation that policy-makers will choose their bliss points in the last period. The reason is that politician R will not increase his reelection chances by choosing a different platform than μ_R in period 1, so deviating from his ideal policy μ_R will not be profitable.

We next derive the equilibrium effort choices made by the office-holder in the first period.

Semi-separating Equilibria

We first look at equilibria that divide the type of candidates into two groups. We call such equilibria semi-separating. For this purpose a few preliminary steps are necessary. We first construct a separation of the type of candidates into two groups as follows: The first group with ability equal to or higher than some critical threshold a^{cut}, $a^{\text{cut}} \in (-A, A)$, expects to be reelected with probability 1. A second group with ability smaller than a^{cut} expects to be deselected with probability 1, and will exert little effort. An office-holder with $a = a^{\text{cut}}$ is indifferent between being part of the first or the second group.

As a preparation for the formulation of semi-separating equilibria, we examine the conditions for the indifference of an office-holder with $a = a^{\text{cut}}$ between rejection and reelection. Without loss of generality we assume that k is a right-wing politician R. If he does not expect to be reelected, his utility is given by

$$V_R^{\text{rejection}} = b + \gamma(e_{\text{R1}} + a^{\text{cut}}) - ce_{\text{R1}}^2 + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2. \quad (4.4)$$

Given this expectation, the optimal choice of e_{R1} is given as $e_{\text{R1}} = \frac{\gamma}{2c}$, which yields

$$V_R^{\text{rejection}} = b + \frac{3\gamma^2}{4c} + \gamma a^{\text{cut}} - (\mu_R - \mu_L)^2. \quad (4.5)$$

If he expects to be reelected, his utility is

$$V_R^{\text{reelection}} = b + \gamma(e_{\text{R1}} + a^{\text{cut}}) - ce_{\text{R1}}^2 + b + \frac{\gamma^2}{4c} + \gamma a^{\text{cut}}. \quad (4.6)$$

The office-holder is indifferent between rejection and reelection if $V_R^{\text{rejection}} = V_R^{\text{reelection}}$, which yields

$$ce_{\text{R1}}^2 - \gamma e_{\text{R1}} + \frac{\gamma^2}{2c} - b - \gamma a^{\text{cut}} - (\mu_R - \mu_L)^2 = 0. \quad (4.7)$$

The solutions of this quadratic equation are given by

$$e_{\text{R1}} = \frac{\gamma \pm \sqrt{4c\left(b + \gamma a^{\text{cut}} + (\mu_R - \mu_L)^2\right) - \gamma^2}}{2c}. \quad (4.8)$$

The effort choice of an office-holder who will be rejected equals $\frac{\gamma}{2c}$, as the same reasoning as in Proposition 4.1, point (ii) applies. An incumbent who will be reelected will not choose a lower effort level than an incumbent who will be rejected. Hence the only viable solution is [8]

$$e_{a^{\text{cut}}} = \frac{1}{2c}\left(\gamma + \sqrt{4c\left(b + \gamma a^{\text{cut}} + (\mu_R - \mu_L)^2\right) - \gamma^2}\right). \quad (4.9)$$

After these preparations we can now characterize the set of semi-separating equilibria. For that purpose we use $E_g[a]$ to denote the beliefs of voters regarding the expected ability of an office-holder if he produces output g.

[8]We note that the assumption $b > \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2$ in Sect. 4.3.4 ensures that $e_{a^{\text{cut}}}$ is well-defined and positive.

Proposition 4.2 *There exists a continuum of semi-separating equilibria parameterized by $a^{cut} \in (-A, +A)$. An equilibrium associated with a^{cut} is characterized as follows:*

(i) Policy-makers with $a < a^{cut}$ choose

$$e_{R1} = \frac{\gamma}{2c}. \tag{4.10}$$

Voters perfectly infer their ability and deselect those policy-makers.

(ii) Policy-makers with $a \geq a^{cut}$ choose

$$e_{R1} = e_{a^{cut}} + a^{cut} - a = \frac{1}{2c}\left(\gamma + \sqrt{4c\left(b + \gamma a^{cut} + (\mu_R - \mu_L)^2\right) - \gamma^2}\right) + a^{cut} - a. \tag{4.11}$$

They generate the same output, given by

$$g^{sep}(a^{cut}) = \gamma(e_{a^{cut}} + a^{cut}), \tag{4.12}$$

and are reelected.

(iii) The beliefs of the voters are characterized by

α.) $E_{\gamma(e_{a^{cut}}+a^{cut})}[a] = \frac{A+a^{cut}}{2}$,
$E_{\gamma(\frac{\gamma}{2c}+a)}[a] = a, \ \forall a \in \left[-A, a^{cut}\right)$;

β.) $E_g[a]$ arbitrary if $g > \left(\gamma(e_{a^{cut}} + a^{cut})\right)$,
$E_g[a] < 0$ if $g < (\gamma(e_{a^{cut}} + a^{cut}))$ and $g \notin \left[\gamma(\frac{\gamma}{2c} - A), \gamma(\frac{\gamma}{2c} + a^{cut})\right]$.

The proof of Proposition 4.2 is given in the Appendix. The beliefs in (iii)α.) are on the equilibrium path, while (iii)β.) are conditions for out-of-equilibrium beliefs. On the equilibrium path, an incumbent reveals his ability if he chooses the effort level $e_{\mathrm{R1}} = \frac{\gamma}{2c}$. Voters believe that the average ability is $\frac{A+a^{\mathrm{cut}}}{2}$, if the incumbent chooses e_{R1} according to Eq. (4.11). Proposition 4.2 reveals that the selection power of the reelection mechanism may be severely limited. There are equilibria for which almost all incumbents are reelected, that occur when a^{cut} is low. In such cases the output is also low. We next consider pooling equilibria.

Pooling Equilibria

In a pooling equilibrium all office-holders choose the same output levels. Such equilibria are characterized in the following proposition.

Proposition 4.3 *There exists a continuum of pooling equilibria characterized by output levels*

$$g^p \in \left[g^p_{low}, g^p_{high}\right]$$

where

$$g^p_{low} = \frac{\gamma^2}{2c} + \gamma A \tag{4.13}$$

and [9]

$$g^p_{high} = \frac{\gamma^2}{2c} - \gamma A + \frac{\gamma}{2c}\sqrt{4c\left(b - \gamma A + (\mu_R - \mu_L)^2\right) - \gamma^2}. \tag{4.14}$$

At an equilibrium characterized by g^p, the following statements hold:

(i) Office-holders choose

$$e_{R1} = \frac{g^p}{\gamma} - a \tag{4.15}$$

and produce the same output g^p.

(ii) All office-holders are reelected.

(iii) Voters' beliefs are given by

α.) $E_{g^p}[a] = 0$,
β.) $E_g[a]$ arbitrary for $g > g^p$
$E_g[a] < 0$ for $g < g^p$.

The proof of Proposition 4.3 is given in the Appendix. Again, conditions in (iii)β.) restrict out-of-equilibrium beliefs. We note that in the pooling equilibrium associated with g^p_{low}, we have $e_{\text{R1}} = \frac{\gamma^2}{2c}$ for the effort undertaken by the office-holder with the highest ability. This effort level would also be chosen by a politician when he has no chance to get reelected (see Proposition 4.2).

Pooling equilibria can only exist if $g^p_{\text{high}} \geq g^p_{\text{low}}$. Otherwise the interval $[g^p_{\text{low}}, g^p_{\text{high}}]$ is empty, and no pooling equilibria exist. Comparing the expressions for g^p_{high} and g^p_{low} we observe that pooling equilibria exist if the utility from holding office or the disutility in the case where the ideological platform of the opposing candidate is adopted, is relatively high.

We would like to stress that one particular pooling equilibrium, characterized by g^p, requires that all voters associate the level of g^p with the equilibrium value, while all other values of g are interpreted as a deviation from this equilibrium value.[10]

We further note that pooling equilibria deter voters completely from gathering information regarding the ability of candidates. As a consequence, all incumbents are reelected. This represents an extreme case where the election mechanism has no power to select able candidates for public office.

Other Equilibria

Finally we discuss whether further equilibria exist. For that purpose we introduce the following plausible refinement:

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[9]The assumptions on b in Sect. 4.3.4 ensure that g^p_{high} is well-defined, as the expression under the root is positive.

[10]This requires coordination among voters by some commonly-known criterion or norm. For instance, voters may coordinate on a norm that the performance of policy-makers is weak and thus believe that the equilibrium associated with g^p_{low} is played.

Definition 4.1 An equilibrium satisfies output-ability monotonicity (henceforth OAM) if voters believe that an office-holder who produces a higher output than another one has equal or higher ability. Formally, the belief function $E_g[a]$ is non-decreasing in g in an equilibrium.

The OAM refinement makes sense because higher ability for given effort levels translates into higher output. We then obtain:

Proposition 4.4 *There are no other equilibria in pure strategies satisfying OAM than the semi-separating and pooling equilibria described in Propositions 4.2 and 4.3.*

The proof of Proposition 4.4 is given in the Appendix.

Proposition 4.4 demonstrates that no other equilibrium exists if we impose OAM. In the following we can reduce the set of equilibria by applying a plausible refinement. We impose the widely-applied Intuitive Criterion (see e.g. Cho and Kreps 1987) regarding the set of pooling and semi-separating equilibria identified in the last sections.[11]

The pooling equilibria satisfy the Intuitive Criterion. For the semi-separating equilibria we obtain the following Proposition:

Proposition 4.5 *Semi-separating equilibria fulfill the Intuitive Criterion if and only if $a^{cut} \leq 0$.*

Hence the Intuitive Criterion rules out equilibria with high levels of a^{cut}. The intuitive reasoning is as follows. Suppose a^{cut} is high. Then, only a small share of policy-makers with the highest ability are reelected. Policy-makers of high ability, but below a^{cut}, are deselected.

If a policy-maker with ability close to a^{cut}, but $a < a^{\text{cut}}$ still, increases both his effort and the output in order to separate himself from lower-ability ones, then the out-of-equilibrium beliefs will cause voters to believe that he has below-average ability. As only high-ability policy-makers are willing to increase output by a sufficiently high amount, these out-of-equilibrium beliefs are implausible.

For the remainder we concentrate on equilibria that fulfill the Intuitive Criterion, i.e. on pooling equilibria and semi-separating equilibria with $a^{\text{cut}} \leq 0$.

4.5 Vote-share Thresholds

In this section we assume that the public sets a reelection threshold for incumbents m with $\frac{1}{2} \leq m \leq 1$. The interpretation is as follows: If politician k takes office in

[11] The criterion says that if the information set following a message is off the equilibrium path and if this message is equilibrium-dominated for a certain type, then the receiver's belief should give this type zero probability.

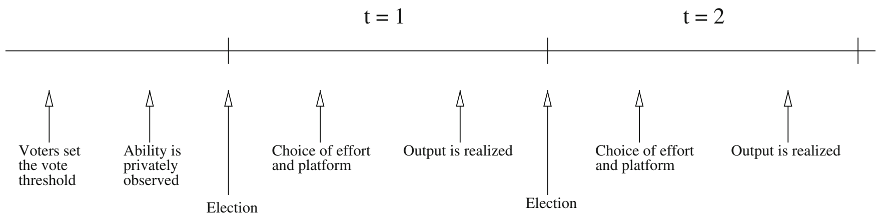

Fig. 4.2 Time-line for elections and vote-share thresholds

$t = 1$, he must win a share of votes at least equal to m at the next election if he wants to retain office. Otherwise the challenger will take office.[12]

The timing of the extended game is summarized in Fig. 4.2.

For the following analysis we assume that a candidate k, say R, has been elected and that the vote-share threshold has been set at $m \geq \frac{1}{2}$.

In the second period the choice regarding P and I by R (if he remains in office) or by L (if he enters office) will remain the same as in Proposition 4.1. Hence we can concentrate on the first period.

4.5.1 The First Period

For the first period we assume without loss of generality that candidate R has been elected. We obtain

Proposition 4.6 *Suppose $m > \frac{1}{2}$. Then,*

(i) the pooling equilibria do not exist, and
(ii) semi-separating equilibria parameterized by a^{cut} exist if and only if $a^{cut} \geq a^{crit}(m)$, where the critical quality level $a^{crit}(m)$ is given by

$$a^{crit}(m) := -A + \frac{2}{\gamma}(2\mu_R - 1)(2m - 1). \tag{4.16}$$

The proof of Proposition 4.6 is given in the Appendix. We note that $a^{\text{crit}}(m)$ is larger than $-A$ and monotonically increasing in m. Result (ii) of Proposition 4.6, can be extended to $m = \frac{1}{2}$. We obtain $a^{\text{crit}} = -A$, and thus all semi-separating equilibria exist.

Proposition 4.6 shows that higher thresholds for incumbents destroy pooling equilibria and eliminate semi-separating equilibria where the average ability of reelected incumbents is low. The reason is that an incumbent can only gain a vote share that exceeds 50% marginally if his perceived average ability exceeds 0 marginally.

[12] Another practical solution is to allow for a runoff between two new candidates. Such a procedure ensures that all candidates elected to public office gain at least 50% of the votes.

Proposition 4.6 reveals how political polarization measured by $\mu_R - \frac{1}{2}$ impacts on the set of semi-separating equilibria that are eliminated by higher vote thresholds. When polarization is high $a^{\text{crit}}(m)$ is higher for a given vote threshold than when polarization is low. If $\frac{2(2\mu_R-1)}{\gamma A} < 1$, even the most extreme vote threshold $m = 1$ would not eliminate all semi-separating equilibria with $a^{\text{cut}} \leq 0$.

In the next section we discuss the welfare implications of these results.

4.5.2 Welfare Properties

To prepare the ground for welfare implications we first calculate the welfare associated with a particular equilibrium. As aggregate utility from the ideological policy is independent of the type of politician, we use W to denote the expected welfare that voters derive from public project P in a particular equilibrium. Specifically, $W^{\text{pool}}(g^p)$ denotes the expected welfare in a pooling equilibrium associated with output $g^p \in [g^p_{\text{low}}, g^p_{\text{high}}]$. $W^{\text{sep}}(a^{\text{cut}})$ is the expected welfare in a semi-separating equilibrium with cut-off ability a^{cut}.

Using Propositions 4.1, 4.2, and 4.3, we derive that

$$\begin{aligned}
W^{\text{pool}}(g^p) &= \frac{1}{2A}\int_{-A}^{+A} \gamma\left(\frac{g^p}{\gamma} - a + a\right) da + \frac{1}{2A}\int_{-A}^{+A} \gamma\left(\frac{\gamma}{2c} + a\right) da \\
&= \frac{1}{2A}\left[g^p a + \frac{\gamma^2}{2c}a + \frac{1}{2}\gamma a^2\right]_{-A}^{+A} \\
&= \frac{1}{2A}\left(g^p A + \frac{\gamma^2}{2c}A + \frac{1}{2}\gamma A^2 + g^p A + \frac{\gamma^2}{2c}A - \frac{1}{2}\gamma A^2\right) \\
&= \frac{1}{2A}\left(2g^p A + \frac{\gamma^2}{c}A\right) \\
&= \frac{\gamma^2}{2c} + g^p,
\end{aligned} \tag{4.17}$$

and

$$\begin{aligned}
W^{\text{sep}}(a^{\text{cut}}) &= \frac{1}{2A}\left(\int_{-A}^{a^{\text{cut}}}\left(\frac{\gamma^2}{2c} + \gamma a + \frac{\gamma^2}{2c}\right) da + \int_{a^{\text{cut}}}^{+A}\left(\gamma e_{a^{\text{cut}}} + \gamma a^{\text{cut}} + \frac{\gamma^2}{2c} + \gamma a\right) da\right) \\
&= \frac{1}{2A}\left(\left[\frac{\gamma^2}{c}a + \frac{1}{2}\gamma a^2\right]_{-A}^{a^{\text{cut}}} + \left[\gamma e_{a^{\text{cut}}} a + \gamma a^{\text{cut}} a + \frac{\gamma^2}{2c}a + \frac{1}{2}\gamma a^2\right]_{a^{\text{cut}}}^{+A}\right) \\
&= \frac{1}{2A}\Big(\frac{\gamma^2}{c}a^{\text{cut}} + \frac{1}{2}\gamma (a^{\text{cut}})^2 + \frac{\gamma^2}{c}A - \frac{1}{2}\gamma A^2 + \gamma e_{a^{\text{cut}}} A + \gamma a^{\text{cut}} A \\
&\quad + \frac{\gamma^2}{2c}A + \frac{1}{2}\gamma A^2 - \gamma e_{a^{\text{cut}}} a^{\text{cut}} - \gamma (a^{\text{cut}})^2 - \frac{\gamma^2}{2c}a^{\text{cut}} - \frac{1}{2}\gamma (a^{\text{cut}})^2\Big)
\end{aligned}$$

$$= \frac{1}{2A}\left(\frac{\gamma^2 a^{\text{cut}}}{2c} - \gamma(a^{\text{cut}})^2 + \frac{3\gamma^2 A}{2c} + \gamma e_{a^{\text{cut}}} A + \gamma a^{\text{cut}} A - \gamma e_{a^{\text{cut}}} a^{\text{cut}}\right)$$
$$= \frac{\gamma}{4cA}\left(\gamma(3A + a^{\text{cut}}) + 2c(e_{a^{\text{cut}}} + a^{\text{cut}})(A - a^{\text{cut}})\right). \tag{4.18}$$

It is useful to study the properties of $W^{\text{sep}}(a^{\text{cut}})$ in more detail

Lemma 4.1 *It holds that:*

(i) $\frac{\partial W^{sep}}{\partial a^{cut}} > 0$ *if* $\dfrac{\gamma}{2c} + A > e_{a^{cut}} + 2a^{cut}$*, and*
(ii) W^{sep} *is concave*

The proof is given in the Appendix. Lemma 4.1 exhibits how a^{cut} impacts welfare. On the one hand, office-holders with $a \geq a^{\text{cut}}$ exert more effort when a^{cut} is higher. This fosters welfare. On the other hand, the share of types of office-holders who choose low effort increases if a^{cut} rises. This is detrimental for welfare. For a^{cut} close to $-A$ the first effect dominates if $\dfrac{\gamma}{2c} + A > e_{a^{\text{cut}}} + 2a^{\text{cut}}$ while the second effect dominates for higher values of a^{cut}. We obtain:

Proposition 4.7 *(i) It holds that:*

$$\lim_{a^{cut} \to -A} W^{sep}(a^{cut}) = W^{pool}(g^p_{high}) = \frac{\gamma^2}{c} - A\gamma + \frac{\gamma}{2c}\sqrt{4c\left(b - \gamma A + (\mu_R - \mu_L)^2\right) - \gamma^2}$$

(ii) There exists a^{cut*} *that maximizes* $W^{sep}(a^{cut})$.

The proof of Proposition 4.7 is given in the Appendix.

In the next Proposition we state a sufficient condition for $a^{\text{cut}} = -A$ to constitute the worst semi-separating equilibrium.

Proposition 4.8 *There exists a fixed number* $\delta > 0$ *such that* $W^{sep}(-A) < W^{sep}(a^{cut})$ *for all* $a^{cut} \in (-A, 0]$ *if*

$$b \leq \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2 + \delta.$$

The proof of Proposition 4.8 is given in the Appendix. Note that the condition given in Proposition 4.8 is compatible with the assumption $b > \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2$ made in Sect. 4.3.4. Proposition 4.8 reveals that the semi-separating equilibrium with $a^{\text{cut}} = -A$ is the worst one in the class of semi-separating equilibria. This holds if b is not too large. Otherwise the welfare losses from office-holders with low ability who do not aspire for reelection in semi-separating equilibria, outweight the gains from higher effort choices of office-holders with $a > a^{\text{cut}}$. A further result will help to draw welfare comparisons:

Corollary 4.1 *(i) The effort of a policy-maker with* $a \geq a^{cut}$ *in a semi-separating equilibrium is strictly higher than the effort in any pooling equilibrium.*

(ii)

$$\lim_{a^{cut}\to -A} g^{sep}(a^{cut}) = \lim_{a^{cut}\to -A} (e_{a^{cut}} + a^{cut}) = g^{p}_{high}$$

The corollary follows from comparisons (4.11) in Proposition 4.2 and (4.14) in Proposition 4.3. The corollary states that output generated by policy-makers with $a \geq a^{\text{cut}}$ in semi-separating equilibria, is always higher than in pooling equilibria. As a^{cut} decreases to $-A$, the semi-separating equilibrium essentially becomes a pooling equilibrium with output g^{p}_{high}.

4.5.3 Welfare Impact of Higher Vote Thresholds

To describe the welfare impact of higher vote thresholds, we distinguish two cases. First we assume that the semi-separating equilibrium with $a^{\text{cut}} = -A$ is the worst among the semi-separating equilibria. A sufficient condition for this case is given in Proposition 4.8. Here, Proposition 4.7 reveals that there are two types of equilibria associated with low welfare: Pooling equilibria and semi-separating equilibria with a^{cut} close to $-A$.
Combining Proposition 4.6 and Proposition 4.7 yields our main result:

Theorem 4.1 *Suppose that* $b \leq \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2 + \delta$ *for* $\delta > 0$ *obtained in Proposition 4.8. Then:*

(i) Higher vote thresholds than $\frac{1}{2}$ *eliminate the worst equilibria (pooling equilibria).*
(ii) Vote thresholds $m = \frac{1}{2} + \Delta$ *for* $\Delta > 0$ *arbitrarily small, eliminate semi-separating equilibria with* a^{cut} *close to* $-A$*, while semi-separating equilibria with higher values of* a^{cut} *continue to exist.*

Theorem 4.1 is the rationale for advocating higher vote thresholds than $\frac{1}{2}$. To determine a socially optimal vote threshold, one has to make an assumption regarding the likelihood with which an equilibrium will be established. As an example, we perform an exercise assuming that each equilibrium has the same chance of being realized. Then we obtain

Proposition 4.9 *Suppose that each equilibrium has the same chance of being attained. Then, there exists a welfare optimal vote threshold* m^**, with* $m^* > \frac{1}{2}$.

The proof is given in the Appendix.

Two remarks are in order here. First, it is important to stress that vote thresholds slightly higher than 50% eliminate a small set of low-welfare semi-separating equilibria but destroy all pooling equilibria. Both effects are desirable from a welfare point of view.

Second, the utility of politicians in office is negligible in the aggregate in our model, as we have a continuum of voters. Here their utility does not affect welfare considerations. In a finite version of our model the utility of the politician and the cost of exerting effort will affect the welfare-optimizing vote-share threshold. As a result the welfare-optimal vote-share in a finite version of our model tends to be slightly lower.

We next consider the case when the semi-separating equilibrium with $a^{\text{cut}} = -A$ is not the worst equilibrium of this type. This occurs when b is very large. In this case slightly higher vote thresholds than $m = \frac{1}{2}$ still eliminate all pooling equilibria, but they do not eliminate the worst semi-separating equilibria.

Still, we observe from (4.17) and (4.18) that the pooling equilibrium with the lowest output is the worst in terms of welfare among all equilibria, if b is large. Hence higher vote thresholds than $\frac{1}{2}$ eliminate the worst equilibria but the overall effects on welfare are more involved. Depending on assumptions about the likelihood of equilibria, it can now happen that $m = \frac{1}{2}$ is the optimal vote threshold.[13]

4.5.4 Competition for Vote Thresholds

We can allow candidates to compete with vote thresholds. That is, both candidates offer vote thresholds $m_k \geq \frac{1}{2}$ during campaigns to which they are committed if they are elected. Since the policy on which preferences of voters differ is one-dimensional and preferences are single-peaked, a candidate will obtain a majority of votes if he offers a vote-threshold that is more attractive to the median voter than the one offered by the competitor. Since both candidates like to be elected, they will offer the vote threshold that maximizes the utility of the median voter, which in our model, is equivalent to welfare maximization. We note that there appears to be no risk for welfare, if candidates are allowed to compete with vote thresholds. If $m = \frac{1}{2}$ is the desired level by the median voter, candidates will offer $m_k = \frac{1}{2}$. Otherwise, they will offer higher vote thresholds. In any case, allowing candidates to compete with vote thresholds has no effect or strictly improves welfare.

4.6 Extensions, Applications and Generalizations

We have illustrated how higher vote thresholds for incumbents can improve the selection power of elections. Numerous extensions can and should be pursued to test the robustness and validity of the argument.

4.6.1 Further Incumbency Advantages

It is useful to allow for further sources of incumbency advantage discussed in the literature. Suppose a candidate can gain advantages in office that make him more

[13] Details are available upon request.

attractive than a challenger, even if his ability is the same. Such advantages may be name recognition or lower uncertainty (variance) about ideological positions for incumbents than for challengers. Such advantages generated by holding office further weaken the selection power of elections, as office-holders with expected ability slightly below average will be reelected. In such circumstances higher vote thresholds for incumbents are even more advantageous, as setting them sufficiently high ensures that only office-holders with above-average ability will still be reelected.

4.6.2 Learning by Doing

Another fruitful extension is *learning by doing of office-holders*. Suppose the politician in office experiences learning effects during the first term in office. Then his marginal effort costs may decline for the second term. In such circumstances it is socially desirable to reelect incumbents with expected ability slightly below average. As a consequence, higher vote thresholds for incumbents than for the election of a newcomer are still welfare-improving, but they have to be set at a lower level than in the model variant without learning-by-doing effects.

4.6.3 Alternative Election Procedures

Two alternative election procedures involving vote-share thresholds for incumbents can be considered. First, if the incumbent fails to reach the threshold, a separate election between a new right-wing candidate and candidate L will take place. Such a procedure ensures that politicians are only elected if they receive at least 50% of the votes. Second, instead of the public, candidates may propose vote thresholds for themselves which become effective if they take office. Both variants of the model yield the same (latter version) or qualitatively similar results (former version). The result is obvious for the latter version. Competing newcomers will tend to offer the socially optimal vote threshold, as the voters in the center will prefer those candidates that are closest to the socially optimal threshold.

4.6.4 Repeated Competition With Vote Thresholds

A useful extension of the model is to consider a larger time horizon or a version of the model with an infinite horizon. In a finitely repeated version of our model there is no a priori welfare reason why vote thresholds for incumbents should increase further at the end of their second or third term. However, if the incumbency advantages discussed in Sect. 4.6.1, increase with the number of terms in office, this could be a justification for vote thresholds increasing in the number of terms an incumbent has been in office.

4.6.5 Generalizations of the Model

Finally, it is useful to study a general version of the present model that does not rely on specific functional forms. A generalization of the model can be stated as follows:

- Ability is distributed on $[-A, +A]$ according to some density function $f(A)$. The expected ability is normalized to zero.
- The amount of the public project in period t is given by $g_t = h(e_{kt}, a_k)$ with
  - $\lim_{e_{kt}\to\infty} h(e_{kt}, a_k) = \infty$ for all $a_k \in [-A, +A]$,
  - $\frac{\partial h(\cdot,\cdot)}{\partial e_{kt}} > 0$, $\frac{\partial^2 h(\cdot,\cdot)}{\partial e_{kt}^2} \leq 0$, $\frac{\partial h(\cdot,\cdot)}{\partial a_k} > 0$, $\frac{\partial^2 h(\cdot,\cdot)}{\partial a_k^2} \leq 0$.
- $U_i^I(i_{kt}) = -k(|i_{kt} - i|)$ with $k(0) = 0$, $k'(\cdot) > 0$ and $k''(\cdot) > 0$.
- The function $C(e_{kt})$ satisfies $C(0) = 0$, $C'(0) = 0$, $C'(e_{kt}) > 0$ for $e_{kt} > 0$, $\lim_{e_{kt}\to\infty} C'(e_{kt}) = \infty$ and $C''(\cdot) > 0$.

The assumptions on $h(e_{kt}, a_k)$ ensure that it is always possible for agents with low ability to mimic the output of high-ability office-holders, by exerting a sufficiently high level of effort. The same analysis as in this chapter can also be performed in this more general setting. While no explicit solutions can be derived for the general case and the set of equilibria may vary, the qualitative considerations remain unchanged.[14]

4.7 Conclusion

The main insight of this chapter is that higher vote thresholds increase the selection power of elections, which is socially desirable.[15] As it is easy to implement in practice, it will be useful to experiment with this new institution.

[14] Conditions for the existence of the equilibria and for socially optimal thresholds above $\frac{1}{2}$ are available on request.

[15] That higher bars for incumbents yield welfare gains has recently been shown in other circumstances as well. Even if office-holders gain experience, higher bars for incumbents continue to be socially desirable (see Gersbach and Müller 2016).

Appendix

Proof of Proposition 4.1

The first point is obvious as the office-holder is in his last period. The optimization problem of the office-holder regarding his effort choice is given by

$$\max_{e_{k2}}\{\gamma(e_{k2}+a_k)-ce_{k2}^2\},$$

which yields $e_{k2}^* = \frac{\gamma}{2c}$. Therefore, the utility of a policy-maker with ability a_k, at the beginning of period 2, is given by

$$b+\gamma\left(\frac{\gamma}{2c}+a_k\right)-c\left(\frac{\gamma}{2c}\right)^2 = b+\frac{\gamma^2}{4c}+\gamma a_k.$$

This proves (iii). The last point then follows directly from (i) and (ii). □

Proof of Proposition 4.2

Suppose a^{cut} is given. We prove that the strategies and beliefs as given in the proposition, form a semi-separating equilibrium.

Step 1
Office-holders with $a < a^{\text{cut}}$ could mimic the output generated by incumbents with $a \geq a^{\text{cut}}$ in order to get reelected. Mimicking requires an office-holder with ability $a < a^{\text{cut}}$, to put in effort

$$e_{\text{R1}} = e_{a^{\text{cut}}} + a^{\text{cut}} - a, \tag{4.19}$$

which would yield a utility

$$V_{\text{R1}}^{\text{dev}} = b+\gamma(e_{a^{\text{cut}}}+a^{\text{cut}}-a+a)-ce_{\text{R1}}^2+b+\frac{\gamma^2}{4c}+\gamma a. \tag{4.20}$$

This will be smaller than its equilibrium utility

$$V_{\text{R1}} = b+\frac{3\gamma^2}{4c}+\gamma a-(\mu_R-\mu_L)^2, \tag{4.21}$$

if and only if the following condition holds:

$$b+\frac{3\gamma^2}{4c}+\gamma a-(\mu_R-\mu_L)^2 > b+\gamma(e_{a^{\text{cut}}}+a^{\text{cut}}-a+a)-ce_{\text{R1}}^2+b+\frac{\gamma^2}{4c}+\gamma a. \tag{4.22}$$

Rearranging terms yields

$$ce_{\text{R1}}^2-\gamma e_{a^{\text{cut}}}+\frac{\gamma^2}{2c}-b-\gamma a^{\text{cut}}-(\mu_R-\mu_L)^2 > 0. \tag{4.23}$$

As $a < a^{\text{cut}}$, we have $e_{a^{\text{cut}}} < e_{\text{R1}}$. For $e_{\text{R1}} = e_{a^{\text{cut}}}$ the left-hand side of (4.23) is zero as it is the same condition as in (4.7). As $e_{\text{R1}} > e_{a^{\text{cut}}}$, Condition (4.23) holds and thus mimicking by an office-holder with $a < a^{\text{cut}}$ is not profitable.

Step 2
Candidates with $a \geq a^{\text{cut}}$ could choose to lower their effort in order to increase utility, although thereby they risk deselection. Suppose that an office-holder with $a = a^{\text{cut}}$ considers to lower effort. The equilibrium utility is given by

$$V_{\text{R1}} = b + \gamma(e_{a^{\text{cut}}} + a^{\text{cut}} - a + a) - ce_{\text{R1}}^2 + b + \frac{\gamma^2}{4c} + \gamma a. \tag{4.24}$$

Deviating by $e_{\text{R1}} = \frac{\gamma}{2c}$,[16] yields

$$V_{\text{R1}}^{\text{dev}} = b + \frac{3\gamma^2}{4c} + \gamma a - (\mu_R - \mu_L)^2. \tag{4.25}$$

Deviation is not profitable if

$$ce_{\text{R1}}^2 - \gamma e_{a^{\text{cut}}} + \frac{\gamma^2}{2c} - b - \gamma a^{\text{cut}} - (\mu_R - \mu_L)^2 \leq 0. \tag{4.26}$$

As $a \geq a^{\text{cut}}$, we have $e_{a^{\text{cut}}} \geq e_{\text{R1}}$. Again, the left-hand side of (4.26) is zero for $e_{\text{R1}} = e_{a^{\text{cut}}}$. Hence the deviation is not profitable.

Step 3
Voters' equilibrium beliefs about utility and voting decisions are given as follows:

- If output is $\gamma(e_{a^{\text{cut}}} + a^{\text{cut}})$, expected ability is given by

$$E_a(\gamma(e_{a^{\text{cut}}} + a^{\text{cut}})) = \frac{A + a^{\text{cut}}}{2} > 0,$$

  and office-holders producing this output are reelected.
- If output is $\gamma(\frac{\gamma}{2c} + a)$ with $-A \leq a < a^{\text{cut}} \leq 0$, voters will believe that the candidate has ability a and he will be deselected because his ability is below-average.
- If output is below $\gamma(e_{a^{\text{cut}}} + a^{\text{cut}})$ and out of the equilibrium, voters will believe that candidates' ability is below zero.
- If output is above $\gamma(e_{a^{\text{cut}}} + a^{\text{cut}})$, then the belief of voters is arbitrary.

Steps 1–3 prove the properties of a semi-seperating equilibrium associated with a particular value of a^{cut}. □

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[16] This is the most attractive deviation.

Proof of Proposition 4.3

We first observe that properties (i)–(iii) constitute a potential pooling equilibrium. We next derive the conditions for such pooling equilibria to be attainable. If a politician with ability a plays the equilibrium strategy, his utility is given by

$$V_{\mathrm{R1}}^{\mathrm{pool}} = b + \gamma\left(\frac{g^p}{\gamma} - a + a\right) - c\left(\frac{g^p}{\gamma} - a\right)^2 + b + \frac{\gamma^2}{4c} + \gamma a. \tag{4.27}$$

If he deviates to a slightly higher effort $e_{\mathrm{R1}} = \frac{g^P}{\gamma} - a + \epsilon$, his utility would amount to

$$V_{\mathrm{R1}}^{\mathrm{hdev}} := b + \gamma\left(\frac{g^p}{\gamma} - a + \epsilon + a\right) - c\left(\frac{g^p}{\gamma} - a + \epsilon\right)^2 + b + \frac{\gamma^2}{4c} + \gamma a, \tag{4.28}$$

where ϵ is small and positive. Such a deviation is not attractive if $V_{\mathrm{R1}}^{\mathrm{pool}} \geq V_{\mathrm{R1}}^{\mathrm{hdev}}$, which yields

$$g^p \geq \frac{\gamma^2}{2c} + a\gamma - \frac{\gamma}{2}\epsilon. \tag{4.29}$$

Condition (4.29) has to hold for all $\epsilon > 0$ and for all $a \in [-A, +A]$. For type $a = A$ not to deviate,

$$g_{\mathrm{low}}^p = \frac{\gamma^2}{2c} + A\gamma. \tag{4.30}$$

Deviation to a lower effort than in the pooling equilibrium will result in deselection and would yield

$$V_{\mathrm{R1}}^{\mathrm{ldev}} = b + \frac{\gamma^2}{4c} + \gamma a + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2. \tag{4.31}$$

There will be no downward deviation if $V_{\mathrm{R1}}^{\mathrm{pool}} \geq V_{\mathrm{R1}}^{\mathrm{ldev}}$, which yields

$$\begin{aligned} g^p \in [\, & \tfrac{\gamma^2}{2c} + a\gamma - \tfrac{\gamma}{2c}\sqrt{4c\left(b + \gamma a + (\mu_R - \mu_L)^2\right) - \gamma^2}, \\ & \tfrac{\gamma^2}{2c} + a\gamma + \tfrac{\gamma}{2c}\sqrt{4c\left(b + \gamma a + (\mu_R - \mu_L)^2\right) - \gamma^2}\ ]. \end{aligned} \tag{4.32}$$

The condition has to hold for all $a \in [-A, +A]$. The worst type $a = -A$ will not want to lower his effort if

$$g^p \leq \frac{\gamma^2}{2c} - A\gamma + \frac{\gamma}{2c}\sqrt{4c\left(b - \gamma A + (\mu_R - \mu_L)^2\right) - \gamma^2}, \tag{4.33}$$

which gives us g_{high}^p.

Moreover, the comparison of $V_{\mathrm{R1}}^{\mathrm{pool}}$ and $V_{\mathrm{R1}}^{\mathrm{ldev}}$ provides another condition that has to be fulfilled for no upward deviation to occur:

$$g^p \geq \frac{\gamma^2}{2c} + A\gamma - \frac{\gamma}{2c}\sqrt{4c\left(b + \gamma A + (\mu_R - \mu_L)^2\right) - \gamma^2}. \tag{4.34}$$

As this condition is less strict than Condition (4.29), g^p_{low} is given by Eq. (4.30). Hence, for $g^p \in \left[g^p_{\text{low}}, g^p_{\text{high}}\right]$, the properties (i)–(iii) constitute a pooling equilibrium associated with output level g^p. □

Proof of Proposition 4.4

The strategy of the proof is as follows. We first show no other semi-separating equilibria than those identified in Proposition 4.2 with two groups exists. Then, we show that no equilibrium exists in which all office-holders separate themselves. Finally, we show that no semi-separating equilibrium with two or more groups of pooling office-holders exists.

Case 1

Suppose that there exists another type of semi-separating equilibrium with two groups, in which one group, say group 1, chooses output g^1 and those policy-makers are reelected. The policy-makers in the other group, say group 2, choose effort level $e = \frac{\gamma}{2c}$ and are deselected. Output g^1 has to be larger than the maximal output produced by policy-makers in the second group, as otherwise policy-makers in the second group could mimic policy-makers in the first group at no cost.

Next we observe that minimal ability of agents in the first group is equal to maximal ability in the second group. Indeed, suppose that a_1 belongs to group 1 and a_2 belongs to group 2 with $a_2 > a_1$, then we will obtain a contradiction by the following argument: As a_1 is in group 1, his utility from producing g^1 is equal to or higher than that associated with choosing $e = \frac{\gamma}{2c}$, i.e.

$$\begin{aligned} U_{a_1}\left(g^1\right) &= b + \gamma\left(\frac{g^1}{\gamma} - a_1 + a_1\right) - c\left(\frac{g^1}{\gamma} - a_1\right)^2 + b + \frac{\gamma^2}{4c} + \gamma a_1, \\ &\geq \\ U_{a_1}\left(\gamma\left(\frac{\gamma}{2c} + a_1\right)\right) &= b + \frac{\gamma^2}{2c} + \gamma a_1 - \frac{\gamma^2}{4c} + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2. \end{aligned}$$

The opposite must hold for a_2:

$$\begin{aligned} U_{a_2}\left(g^1\right) &= b + \gamma\left(\frac{g^1}{\gamma} - a_2 + a_2\right) - c\left(\frac{g^1}{\gamma} - a_2\right)^2 + b + \frac{\gamma^2}{4c} + \gamma a_2, \\ &\leq \\ U_{a_2}\left(\gamma\left(\frac{\gamma}{2c} + a_2\right)\right) &= b + \frac{\gamma^2}{2c} + \gamma a_2 - \frac{\gamma^2}{4c} + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2. \end{aligned}$$

Together, this yields $\left(\frac{g^1}{\gamma} - a_1\right)^2 \leq \left(\frac{g^1}{\gamma} - a_2\right)^2$. From the first paragraph we can draw upon $g^1 > \gamma\left(\frac{\gamma}{2c} + a_2\right)$, so that $a_2 < \frac{g^1}{\gamma}$. Moreover, as no policy-maker will choose a lower effort than $\frac{\gamma}{2c}$, we have $g^1 > \gamma\left(\frac{\gamma}{2c} + a_1\right)$ and thus $a_1 < \frac{g^1}{\gamma}$.

Hence $\left(\frac{g^1}{\gamma} - a_1\right)^2 \leq \left(\frac{g^1}{\gamma} - a_2\right)^2$ implies $a_2 < a_1$, which contradicts the assumption $a_2 > a_1$. We may conclude that there exists no semi-seperating equilibrium of the kind described at the beginning of Case 1.

Case 2
Suppose that policy-makers completely separate themselves by choosing effort $e = \frac{\gamma}{2c}$ and all policy-makers with $a \geq 0$ are reelected. Such a constellation cannot be an equilibrium, as a policy-maker with $a = -\epsilon$ and $\epsilon > 0$ being small can increase his effort marginally and mimic a policy-maker with $a = 0$, thereby securing reelection. As the value of office satisfies $b > \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2$ the utility of the policy-maker with $a = -\epsilon$ increases and thus, this deviation is profitable. Thus, no fully separable equilibrium exists.

Case 3
Suppose that there exists a semi-separating equilibrium with three different groups, from which two groups pool, say group 1 and group 2. Pooling necessarily requires that office-holders are reelected, as otherwise every policy-maker will choose the same effort, but produce a different output. Suppose that output in group 1 is g^1 and in group 2 it is g^2, where $g^1 > g^2$. Then policy-makers in group 1 can lower their effort and choose g^2, thus still securing reelection. Such a deviation is not profitable if $g^2 < \gamma\left(\frac{\gamma}{2c} + a\right)$ for all policy-makers with ability a in group 1, as $e_1 = \frac{\gamma}{2c}$ is the minimal effort any policy-maker will choose. Hence the ability of all policy-makers in group 2 is equal to or smaller than the lowest ability level in group 1.

According to OAM, any policy-maker in group 1 would therefore be reelected if he chose $e_1 = \frac{\gamma}{2c}$ and produced output $\gamma\left(\frac{\gamma}{2c} + a\right)$, which is larger than g^2. We thus arrive at a contradiction, as we have assumed that such policy-makers will not pool. Therefore, no semi-seperating equilibrium as described at the beginning of Case 3, exists.

Case 4
The argument in Case 3 immediately generalizes to semi-separating equilibria with a finite number of groups and office-holders who pool in such groups. Thus, such equilibria cannot exist either. □

Proof of Proposition 4.5

Consider a semi-separating equilibrium with $a^{\text{cut}} > 0$. Suppose that an office-holder with $a = 0$ expects to be reelected with certainty if he deviates. The effort he would be willing to exert so that he obtains the same utility as in equilibrium is given by

$$e^{\text{dev}}_{\text{R1},a=0} = \frac{1}{2c}\left(\gamma + \sqrt{4c\left(b + \gamma a^{\text{cut}} + (\mu_R - \mu_L)^2\right) - \gamma^2}\right) =: e^{\text{dev}}_{\text{R1},a=0}(a^{\text{cut}}).$$

Suppose now that any policy-maker with $a \in [0, a^{\text{cut}})$ chooses $e^{\text{dev}}_{\text{R1},a=0}(a^{\text{cut}}) - a$. Such a policy-maker will benefit from the deviation if voters believe that he actually has an ability of $a \geq 0$ and thus reelect him. Policy-makers with $a < 0$, however, are worse off by choosing $e^{\text{dev}}_{\text{R1},a=0}(a^{\text{cut}}) - a$ as they need to exert more effort than $e^{\text{dev}}_{\text{R1},a=0}(a^{\text{cut}})$, even if voters believe that they are of high ability and reelect them.

Hence equilibria with $a^{\text{cut}} > 0$ do not satisfy the Intuitive Criterion. Equilibria with $a^{\text{cut}} < 0$ do satisfy this criterion, as no office-holder with above-average ability would want to deviate. □

Proof of Proposition 4.6

(i) We have to show that pooling equilibria in which all policy-makers are reelected with certainty do not exist for $m > \frac{1}{2}$. Suppose that such a pooling equilibrium exists. The expected ability of an office-holder in such a pooling equilibrium is 0. The median voter is indifferent between reelecting the office-holder and electing a new candidate. This would imply that no office-holder can obtain a share of votes equal to m, as they get 50% and m was assumed to be strictly higher than $\frac{1}{2}$.

We note that no pooling equilibrium exists in which policy-makers expect that they will not be reelected. Office-holders would choose the same effort level, but the outputs would be different, and voters could perfectly infer their ability. This is a contradiction.

(ii) We now look at semi-separating equilibria. With $m > \frac{1}{2}$, candidate R is reelected only if voter $i = 1-m$ prefers to vote for him, which implies that all voters with $i > 1 - m$ will also prefer R to L. Voter $i = 1 - m$ prefers R over L if

$$\gamma\left(\frac{\gamma}{2c} + \frac{A + a^{\text{cut}}}{2}\right) - (\mu_R - (1-m))^2 \geq \gamma\left(\frac{\gamma}{2c}\right) - (\mu_L - (1-m))^2. \quad (4.35)$$

Using $\mu_L = 1 - \mu_R$, we obtain

$$a^{\text{cut}} \geq \frac{2}{\gamma}(2\mu_R - 1)(2m-1) - A =: a^{\text{crit}}. \quad (4.36)$$

□

Proof of Lemma 4.1

(i) Taking the derivative of W^{sep} with respect to a^{cut} yields

$$\frac{\partial W^{\text{sep}}}{\partial a^{\text{cut}}} = \frac{\gamma^2 - 4\gamma a^{\text{cut}} c + 2\gamma A c - 2\gamma e_{a^{\text{cut}}} c}{4cA}.$$

As the parameters γ, c and A are assumed to be positive, we can see that $\frac{\partial W^{\text{sep}}}{\partial a^{\text{cut}}} > 0$ if

$$\frac{\gamma}{2c} + A > e_{a^{\text{cut}}} + 2a^{\text{cut}}.$$

(ii) As $\frac{\partial^2 W^{\text{sep}}}{\partial (a^{\text{cut}})^2} = -\frac{\gamma}{A}$, we conclude that $W^{\text{sep}}(a^{\text{cut}})$ is concave in a^{cut}. □

Proof of Proposition 4.7

(i) We obtain the result of part (i) of the statement by inserting g^p_{high} for g^p in Eq. (4.17) and by inserting $-A$ for a^{cut} in Eq. (4.18).

$$\begin{aligned}\lim_{a^{\text{cut}}\to -A} W^{sep}(a^{\text{cut}}) &= \frac{\gamma}{4cA}\left(\gamma(3A-A)+2c(e_{a^{\text{cut}}=-A}-A)(A+A)\right)\\ &= \frac{\gamma^2}{c} - A\gamma + \frac{\gamma}{2c}\sqrt{4c\left(b-\gamma A+(\mu_R-\mu_L)^2\right)-\gamma^2} = W^{\text{pool}}(g^p_{\text{high}}).\end{aligned}$$

(ii) The existence of an optimal value a^{cut*} is guaranteed, as $W^{\text{sep}}(a^{\text{cut}})$ is continuous and is maximized on the compact set $[-A, 0]$. The necessary condition for an interior solution is $\frac{\partial W^{\text{sep}}(a^{\text{cut}})}{\partial a^{\text{cut}}} > 0$, which holds, since

$$\frac{\partial W^{\text{sep}}(a^{\text{cut}})}{\partial a^{\text{cut}}} = \frac{\gamma^2 - 4\gamma a^{\text{cut}} c + 2\gamma A c - 2\gamma e_{a^{\text{cut}}} c}{4cA} > 0.$$

□

Proof of Proposition 4.8

We show that under the condition stated in Proposition 4.8, welfare in a semi-separating equilibrium with $a^{\text{cut}} = a$ is higher than with $a^{\text{cut}} = -A$ for some $a \in (-A, 0]$. We have

$$\begin{aligned}W^{\text{sep}}(a^{\text{cut}}=a) &= \frac{\gamma^2}{c} + \frac{\gamma a(A-a)}{2A} + \frac{\gamma(A-a)}{4Ac}\sqrt{4c\left(b+(\mu_R-\mu_L)^2+\gamma a\right)-\gamma^2},\\ W^{\text{sep}}(a^{\text{cut}}=-A) &= \frac{\gamma^2}{c} + \frac{\gamma}{2c}\sqrt{4c\left(b+(\mu_R-\mu_L)^2-A\gamma\right)-\gamma^2} - \gamma A.\end{aligned}$$

Hence the difference $\Delta = W^{\text{sep}}(a^{\text{cut}} = a) - W^{\text{sep}}(a^{\text{cut}} = -A)$ is given by

$$\begin{aligned}\Delta = {}& \gamma(A + \frac{a}{2} - \frac{a^2}{2A})\\ & - \frac{\gamma a}{4Ac}\sqrt{4c\left(b+(\mu_R-\mu_L)^2+\gamma a\right)-\gamma^2}\\ & + \frac{\gamma}{2c}\left(\frac{1}{2}\sqrt{4c\left(b+(\mu_R-\mu_L)^2+\gamma a\right)-\gamma^2} - \sqrt{4c\left(b+(\mu_R-\mu_L)^2-A\gamma\right)-\gamma^2}\right).\end{aligned}$$

The first summand is positive, as $-A < a \le 0$. The second summand is positive, as $a \le 0$. The third term is zero for $a = -A$ and $b = \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2$. As $\frac{\partial \Delta}{\partial b}$ is bounded, there exists $\delta > 0$ such that Δ remains positive for

$$b < \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2 + \delta.$$

This ensures that $a^{\text{cut}} = -A$ is the worst semi-separating equilibrium. □

Proof of Proposition 4.9

For $m > \frac{1}{2}$ expected welfare $W(a^{\text{crit}}(m))$ is given by:

$$W(a^{\text{crit}}(m)) = \int_{a^{\text{crit}}(m)}^{0} \frac{1}{-a^{\text{crit}}(m)} W^{\text{sep}}(a^{\text{cut}}) da^{\text{cut}}$$
$$= \int_{a^{\text{crit}}(m)}^{0} \frac{1}{-a^{\text{crit}}(m)} \Bigg(\frac{\gamma^2}{c} + \frac{\gamma a^{\text{cut}}(A - a^{\text{cut}})}{2A}$$
$$+ \frac{\gamma(A - a^{\text{cut}})}{4Ac} \sqrt{4c\left(b + (\mu_R - \mu_L)^2 + \gamma a^{\text{cut}}\right) - \gamma^2} \Bigg) da^{\text{cut}}.$$

If $m = \frac{1}{2}$, expected welfare is given by

$$\int_{a^{\text{crit}}}^{0} \frac{1}{-a^{\text{crit}} + g^h - g^l} W^{\text{sep}}(a^{\text{cut}}) da^{\text{cut}} + \int_{g^l}^{g^h} \frac{1}{-a^{\text{crit}} + g^h - g^l} W^{\text{pool}}(g^P) dg^P.$$

We next observe that there exists an arbitrarily small $\epsilon > 0$ and $\hat{m} \in \left(\frac{1}{2} + \epsilon, 1\right)$ such that expected welfare is higher for $\hat{m}$ than for any $m \in \left[\frac{1}{2}, \frac{1}{2} + \epsilon\right]$. The reason is that a slightly higher m than $m = \frac{1}{2}$ eliminates the pooling equilibrium and the worst semi-separating equilibria. Accordingly, welfare is higher for $m \geq \frac{1}{2}$, but arbitrarily close to $m = \frac{1}{2}$, which proves the observation. So, we can find the optimal m by maximizing $W(a^{\text{crit}}(m))$ on $\left[\frac{1}{2} + \epsilon, 1\right]$. $W(a^{\text{crit}}(m))$ is continuous on the compact interval $\left[\frac{1}{2} + \epsilon, 1\right]$, which guarantees the existence of an optimal m^* with $m^* > \frac{1}{2}$. □

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Chapter 5
Information Markets, Elections and Threshold Contracts

5.1 Background

Our research on the use of information markets to foster welfare dates back to 2005. In 2006, we published a first paper on the use of information markets as a tool to assess the office-holders' performance. We further expanded this theme and were able to show that information markets are useful when Political Contracts on a certain performance are difficult—if not impossible—to achieve.

We started from the observation that the relative brevity of an office-term is an obstacle to the implementation of socially beneficial long-term projects. For the voters, it would be desirable to have a proof of an office-holder's total achievements before the next elections, a proof that is very difficult to produce if the duration of a project exceeds the office-term. As they do not foster reelection, or may even render an office-holder unpopular when he introduces them, socially beneficial *long-term projects* might be neglected—if they are addressed at all.

Had an office-holder built a bridge during his term, it would be visible proof of his performance. With long-term projects, things are more complex: The reform of a country's health care system or the tightening of equity capital regulations, for instance, are bound to span many years, and tangible results might still be scarce by reelection day.[1] Often, such projects are unpopular when freshly started, as they might meet opposition from powerful interest groups.

Thus, office-holders need incentives to implement such long-term projects. If there is no possibility to divide such a project into shorter sub-projects whose completion can be credited to the office-holder's "performance account" by the end of each term, there should be a possibility to evaluate the project as a whole before it is completed.

[1]Other examples are the slowing down of climate change or the reduction of long-term unemployment. The bibliography provides literature on these issues and lists key articles on democratic decision-making that are relevant for this chapter.

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To do this, we suggest to use information markets to predict the expected output of a project, the overall performance of an office-holder, or the viability and benefits of a policy package. Such an information market collects and processes all kinds of available data to yield an evaluation in the form of a “price” attributed to a project, the office-holder or a policy package.

The competition between candidates might induce them to offer a minimal price they have to achieve on this information market to obtain the right to run for reelection. An office-holder who has started a long-term project will be able to campaign for reelection with the valuation of the information market, i.e. the price of his project or the prediction of his reelection chances. If this price is high, he will be perceived as having worked well—and be worth reelecting.

As such a price can also be set for an office-holder’s entire performance, the suggestion developed in the two preceding chapters can now be expanded: Instead of promising to reach a certain *vote percentage* in reelection, a candidate could commit to do so well that he will reach a certain *price* on the information market by the end of his term. If he fails to reach this price, he will waive candidacy for a second term.

If a price on an information market is closely connected to the right to be a candidate, it might fall prey to manipulation. After a thorough analysis of the interdependencies of the office-holders’ actions and information markets, we will thus examine what types of manipulation such a system might incur, and analyze ways to prevent them.[2]

5.2 Introduction

As discussed above, voters may lack information about the achievements of an office-holder at the election date, and we examine a triple mechanism involving political information markets, threshold incentive contracts, and democratic elections to solve this fundamental problem.

At the end of the first term, a political information market is held. Here investors can bet on whether the incumbent will be reelected at the end of the second term and hence whether he has undertaken socially beneficial long-term policies of which the effects will become visible in the second term. As it is uncertain whether the politician will be reelected for the first time at the end of period 1, this is a conditional information market. It aggregates the information on whether the incumbent has undertaken socially desirable long-term projects or whether the incumbent has merely pandered to current public opinion. A high price on the political information market indicates high probability that the incumbent will be elected a second time.

[2]This chapter is an updated version of the CESifo Working Paper No. 3327 with title “Information Markets, Elections and Contracts”. I am most grateful to Markus Müller for his permission to include this paper in my book.

The second element of the triple mechanism involves reelection threshold contracts that competing politicians can offer before they start on their first term. The reelection threshold contract stipulates a critical price threshold the information market must reach or exceed for the incumbent to have the right to stand for first reelection. The critical price thresholds are offered competitively by politicians campaigning for their first term in office.

The third element of our mechanism are democratic elections that take place at three dates. In a first election, an office-holder with a particular reelection threshold contract is elected by citizens. If the office-holder fulfills his contract, he can stand for reelection at the second date. If he succeeds, he can try to get reelected a second time.

The main idea is as follows: Political information markets, price thresholds on these markets, and democratic elections increase the motivation of politicians to undertake long-term beneficial policies that may be unpopular at the time they are introduced. We develop this insight in the framework of a simple political agency model. We show that a carefully designed combination of political information markets and threshold contracts can—on balance—improve welfare. In Sect. 5.6 we explore the robustness of the triple mechanism and we address several potential pitfalls such as attempts to manipulate information markets.

Our model is most closely related to the suggestion to combine contracts and democratic elections introduced by Gersbach (2003), extended by Gersbach and Liessem (2008), and surveyed in Gersbach (2012). These papers show how the dual mechanism—contracts offered competitively during campaigns and elections—can improve political outcomes. All these papers rely on verifiable data by which contracts can be conditioned. As a contrast, we also analyze the case where the results from current policy can only be observed in a future period and may never be verifiable. We suggest a novel triple mechanism where a political information market produces verifiable information in the form of prices at a time when policy results are not observable.

A comprehensive presentation of the the latest development of our ideas on incentive contracts can be found in Gersbach (2012).

Political information markets have attracted a lot of attention recently. The basic idea behind information markets is the accumulation of scattered information in order to predict uncertain future events. Political information markets have turned out to be quite successful in predicting election results (see e.g. Berg et al. 1996 or Berlemann and Schmidt 2001) and are already established in practice. Information markets have been suggested to improve public policy decisions (see e.g. the recent surveys and discussions by Wolfers and Zitzewitz (2004), or Hanson (2013), who suggests to use information markets to select policies that are expected to raise GDP.) A comprehensive summary on this relatively new topic can be found, for example, in Hahn and Tetlock (2004). Musto and Yilmaz (2003) present the first theoretical model that analyzes markets with contingent securities, and identifies critical characteristics of an efficient prediction market, which we will take up in Sect. 5.6.

We suggest a new type of information market. While standard markets predict the result of the next election, we use a market that predicts the result of the next but one election in order to obtain an approximation of the long-term effects of current policies. The idea is that the incumbent will only be reelected in the next but one election if the voters are satisfied with the long-term project results they learn about over time. The information our prediction market aggregates could, in principle, also be provided by other sources, in particular by a free press. The information market has the advantage that it generates a verifiable signal in the form of a price on which reelection threshold contracts can be conditioned. This is not the case for information provision by the media, even if such provision were unbiased.

This chapter is broadly related to political agency and accountability theory. While this literature developed by Barro (1973) and Ferejohn (1986) has established the advantages and drawbacks of democratic elections in making office-holders accountable, we present a new institutional framework to address the accountability of politicians. We would like to point out that our analysis is a theoretical exercise on how such a new institutional framework would function and how it might improve electoral processes on balance.

The chapter is organized as follows: In the next section we introduce the model. The results for elections only are analyzed in Sect. 5.3. In Sect. 5.4 we examine the triple mechanism involving political information markets, threshold incentive contracts, and democratic elections. In Sect. 5.5 we look at some extensions to our basic model. Section 5.6 concludes. Appendix A contains the proofs and Appendices B and C describe the political information market in more detail. In Appendix D we provide a numerical example.

5.3 The Basic Model

Our basic model draws on Maskin and Tirole (2004). There are three periods, denoted by $t = 1, 2, 3$.

5.3.1 *The Election Framework*

There is a continuum of identical voters of measure 1. We assume that there are two politicians denoted by $i = 1, 2$. They compete for office before the first period starts. The elected politician has to take some kind of action during the first period. He can choose between action $a_1 = 1$ and action $a_1 = 0$. All voters have the same preference ranking for the two possible actions,[3] but they do not know their preferences when they decide about the office-holder for the first term. There are two possible states of the world $s_1 = 1$ and $s_1 = 0$, which are drawn randomly. State $s_1 = 1$ will occur with probability z, and state $s_1 = 0$ will occur with probability $1 - z$. We assume that $\frac{1}{2} < z < 1$. The state of the world determines which action is optimal for the

[3]For the relevance of this assumption and for an outline of how to accommodate heterogeneous preferences of voters, see Maskin and Tirole (2004).

voters. If state $s_1 = 1$ is drawn, then the optimal action for the voters will be $a_1 = 1$. The optimal action for the voters will be $a_1 = 0$ in state $s_1 = 0$. If $a_1 = s_1$, voters get a payoff of 1, otherwise they get a payoff of 0. Voters are risk-neutral and want to maximize their expected utility. As $z > \frac{1}{2}$, we will refer to $a_1 = 1$ as the popular action and to $a_1 = 0$ as the unpopular action.

There are two types of politicians, either congruent or dissonant. Both politicians know their own type and the type of their opponent.[4] However, voters cannot observe the politicians' types. A politician is congruent with a probability of $\frac{1}{2}$. In this case he has the same preferences as the voters. A politician is dissonant with a probability of $\frac{1}{2}$, i.e. if $a_1 = 1$ is optimal for the voters, then $a_1 = 0$ is optimal for the dissonant politician and vice versa. The two political candidates may differ as to congruence or dissonance. In all other respects they are identical.

5.3.2 The Information Structure

At the beginning of the game, voters and politicians have a priori probabilities of z that state $s_1 = 1$ will occur and of $1 - z$ that state $s_1 = 0$ will occur. In the first period, the elected politician can learn precisely which state of the world has occurred, thus knowing with certainty which action is best for the voters and which action is best for himself.

We assume that voters are able to observe the action of the incumbent immediately and that the action is verifiable.[5] We also assume that, while it is impossible to verify which state of the world has occurred, the voters will be able to observe it. However, it is not clear when the voters will make this observation. We assume that before their first reelection decision, voters will observe with probability μ which state of the world has been realized, while the probability that they will observe the state in period 2, i.e. after their first reelection decision, is $1 - \mu$. Further, we assume that $0 \leq \mu \leq \frac{1}{2}$ to analyze a situation where the possibility that the performance of a project is not observable in the short term is a serious problem.[6] Note that regardless of whether there is early observability or not, the project result will never be verifiable in court. Hence, political contracts conditioned directly on performance cannot be used here. We have to find other solutions to this non-verifiability problem.

We assume that the value of μ does not depend on the realized state of the world. This means that early observability is as likely in state $s_1 = 1$ as in state $s_1 = 0$. The incumbent has to undertake the action in the first period before he knows whether the voters will be able to observe the realized state in period 1.

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[4]The assumption that politicians have knowledge about each other's type may appear to be plausible because of their daily interaction. However, a candidate cannot use his knowledge about the type of his opponent in his election campaign, since he is not able to credibly communicate this information.

[5]Verification means that it can be proved in a court of law.

[6]The assumption that $\mu \leq \frac{1}{2}$ is not crucial for our qualitative results. It is only of importance for our quantitative welfare analysis in Appendix D.

Some remarks about our informational assumptions are in order here. We model a situation where politicians obtain information earlier than voters. At the time the policy is undertaken, the incumbent can precisely identify the correct state of the world, while voters are still completely ignorant. Voters will observe the state of the world at a later point in time. If voters only observe the realized state in period 2, they do not know whether the incumbent has undertaken the socially optimal action at the time of their first reelection decision.

5.3.3 Reelection Schemes

Voters are able to observe the realized state in period 1 with a probability of μ. In this case, they know whether the politician has undertaken the socially optimal action, and we assume that they will reelect the incumbent if $a_1 = s_1$, while they will deselect him if $a_1 \neq s_1$.[7] If voters are not able to observe the state of the world in period 1, which happens with a probability of $1 - \mu$, they do not know whether the incumbent has acted congruently. Voters will reelect the politician if $a_1 = 1$, while they will deselect him if $a_1 = 0$, as $a_1 = 1$ is the action that is more likely to be correct.[8] We use $r_1 \in [0, 1]$ to denote reelection probability for the incumbent after his first period in office. When politicians undertake their actions, their beliefs regarding reelection are given as

$$r_1 = \begin{cases} \mu + (1-\mu) = 1 & \text{if } a_1 = 1, s_1 = 1 \\ 0 & \text{if } a_1 = 0, s_1 = 1 \\ 1 - \mu & \text{if } a_1 = 1, s_1 = 0 \\ \mu & \text{if } a_1 = 0, s_1 = 0 \end{cases} \tag{5.1}$$

We assume that reelection probability at the end of period 2 depends only on the outcomes realized in period 2 from the policy action undertaken in period 1. Further policy actions during the second term are assumed to be irrelevant for reelection chances at the end of period 2. This assumption greatly simplifies our analysis and can be justified in several ways. First, if the politician undertakes only long-term policies in the second period, then there may be no new information available at the end of the second period, when the second reelection decision takes place. Second, the policy actions during his second term in office may be much less relevant than the

[7] Note that voters are indifferent between reelection schemes, as the politician will undertake no further action during his second or third term in office. The retrospective voting scheme used in this chapter is an optimal response of voters in our simple model and hence an equilibrium outcome. Retrospective voting is a particular resolution of the indifference of voters creating the highest possible disciplining device. The voting behavior can be further justified as a unique equilibrium outcome when we allow for an arbitrarily small amount of reciprocity. This justification has been developed by Hahn (2009). Of course, retrospective voting is a polar case and thus highlights the trade-offs the politician faces.

[8] Again, retrospective voting is a best response of voters.

first-period choices, so the performance of his policy depends only on his first-period action. Later we will extend our model to cover the case where the incumbent has to undertake further actions and discuss how this influences our result.

We use r_2 to denote the reelection probability for the incumbent at the end of period 2, and we assume that voters will reelect the incumbent if and only if he has acted congruently. This means that both types of politician are deselected with certainty after the second period at the latest if they behaved dissonantly in the first period, while both types of politicians are reelected with certainty at the end of the second period if they behaved congruently in the first period.[9] Thus, the beliefs of the politicians regarding reelection at the end of period 2 are given by

$$r_2 = \begin{cases} 1 & \text{if } a_1 = 1 \text{ and } s_1 = 1 \text{ or if } a_1 = 0 \text{ and } s_1 = 0, \\ 0 & \text{if } a_1 = 1 \text{ and } s_1 = 0 \text{ or if } a_1 = 0 \text{ and } s_1 = 1. \end{cases} \tag{5.2}$$

5.3.4 Preferences of Politicians

The elected politician has personal benefits R ($R > 0$) from being in office. Furthermore, he obtains a private benefit or personal satisfaction G ($G > 0$) if he undertakes the action that is optimal for himself. This benefit G accrues to the politician in the period in which he performs the action.[10] We assume that the candidate receives no utility from the realization of his preferred action if another politician undertakes the action.[11] We use δ with $0 < \delta \leq 1$ to denote the discount factor of the politician. The utility of the politician in office is denoted by U^P and given by

$$U^P = R + r_1[\delta R + r_2\delta^2 R] + \begin{cases} G & \text{if a congruent politician acts congruently,} \\ G & \text{if a dissonant politician acts dissonantly,} \\ 0 & \text{if a congruent politician acts dissonantly,} \\ 0 & \text{if a dissonant politician acts congruently,} \end{cases} \tag{5.3}$$

where r_1 is given by Eq. (5.1) and r_2 is given by Eq. (5.2). Some examples will illustrate Eq. (5.3). An elected politician who is congruent has utility $R + (1 - \mu)\delta R$

[9] Note that it is possible that a politician who behaved congruently in his first term may be ousted from office by the voters when they make their first reelection decision.

[10] It may be useful to think that the action is irreversible, e.g. investment in public infrastructure, such that it cannot be overturned by a future office-holder.

[11] We might also assume that the politician receives the same utility as an ordinary voter if his opponent performs the action. However, this assumption may be less plausible in the case of a dissonant politician. At all events, the results of our analysis are not affected as long as the value of G is sufficiently large in comparison to the utility of ordinary voters.

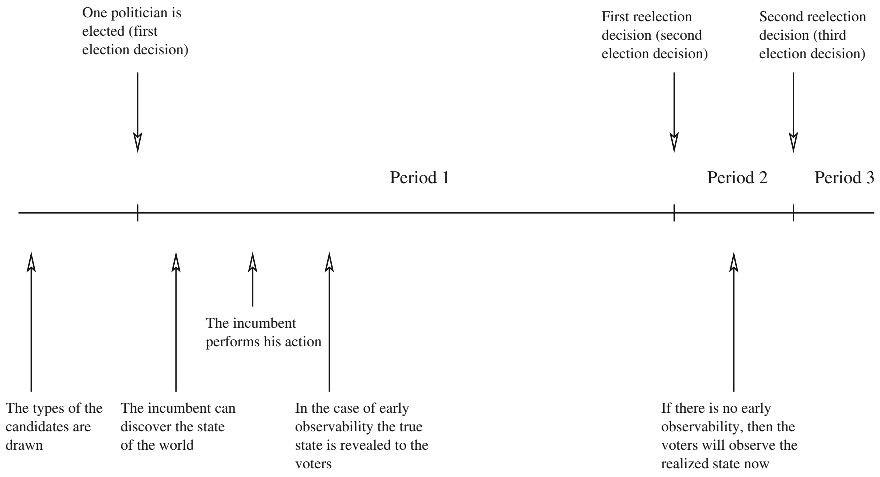

Fig. 5.1 Time-line of the basic game

if he chooses $a_1 = 1$ in state $s_1 = 0$, while his utility is $R + G + \mu[\delta R + \delta^2 R]$ if he chooses $a_1 = 0$ in state $s_1 = 0$. A politician of the dissonant type has utility $R + G + (1 - \mu)\delta R$ if he chooses $a_1 = 1$ in state $s_1 = 0$, while his utility is $R + [\delta R + \delta^2 R]$ if he chooses $a_1 = 1$ in state $s_1 = 1$.

We now need to examine the circumstances under which the elected politician will act congruently. Obviously, it is always optimal for the voters if the incumbent behaves congruently.[12] We will use the following tie-breaking rule: If the elected politician is indifferent as to the two actions, he will undertake the action that is optimal for the voters.

5.3.5 Summary and Welfare Criterion

The timing of the whole game in its basic version is summarized in Fig. 5.1.

The welfare criterion we adopt is the expected utility of voters at the time when the first election starts. Maximization of voters' utility is equivalent to the maximization of the likelihood that the correct action is undertaken.[13]

[12]Note that, in contrast to Maskin and Tirole (2004), there is no "selection effect" in our model, as the politician only acts during his first term in office. Thus there is no welfare-enhancing effect when the voters discover that the incumbent is of the dissonant type and accordingly select a new one.

[13]As we have a continuum of voters, we neglect the utility of the politician in aggregate welfare.

5.4 Elections Only

In this section we consider the behavior of both types of politicians in the scenario without threshold contracts and information markets. Here elections are the only instrument used to discipline the incumbent.

5.4.1 Behavior of Dissonant Politicians

We first look at case $s_1 = 1$, where the popular action is optimal from the voters' point of view but the politician would prefer the unpopular action. Using the beliefs and utilities from (5.1)–(5.3), the dissonant politician will only undertake the socially optimal action if

$$\begin{aligned} R + \delta R + \delta^2 R &\geq R + G \\ \Leftrightarrow \delta R(1+\delta) &\geq G. \end{aligned} \tag{5.4}$$

Condition (5.4) will be violated if the personal gain from choosing the individually optimal action is sufficiently larger than the gains from holding office.

We next examine $s_1 = 0$. Here voters prefer the unpopular action while the politician prefers the popular action. The dissonant politician will only undertake the socially optimal action if

$$\begin{aligned} & R + \mu(\delta R + \delta^2 R) && \geq R + G + (1-\mu)\delta R \\ \Leftrightarrow \quad & \delta R(2\mu + \delta\mu - 1) && \geq G. \end{aligned} \tag{5.5}$$

This condition can only be fulfilled for certain values of δ and μ, as (5.5) cannot be satisfied if $(2\mu + \delta\mu - 1)$ is not positive. Note that $(2\mu + \delta\mu - 1)$ is monotonically increasing in δ. For $\delta = 1$ the condition $(2\mu + \delta\mu - 1) > 0$ is equivalent to $\mu > \frac{1}{3}$. This means that, even in the case of $\delta = 1$ (which is the value of δ that makes the condition most likely to be fulfilled), it is only possible to fulfill Eq. (5.5) for $\frac{1}{3} < \mu < \frac{1}{2}$. Hence, there are large parameter ranges where a dissonant politician cannot be motivated to perform the socially optimal action if the unpopular state has occurred. In particular, this will not be possible if the probability of early observation by voters is small, as reflected in a low value of μ. Furthermore, it is obvious that Condition (5.4) is easier to fulfill than Condition (5.5).

Finally, we obtain the following intuitive results: If the parameters are such that Condition (5.4) is fulfilled while Condition (5.5) is not fulfilled, then there will be a distortion in favor of the popular action $a_1 = 1$. If neither Condition (5.4) nor Condition (5.5) are fulfilled, then there will be a distortion in favor of the unpopular action $a_1 = 0$.[14] It is useful to summarize these key observations in the following proposition.

Proposition 5.1
Dissonant politicians will not choose the socially optimal action if

(i) $s_1 = 1$ *and* $\delta R(1+\delta) < G$*, or*
(ii) $s_1 = 0$ *and* $\delta R(2\mu + \delta\mu - 1) < G$.

Three particularly interesting special cases of Proposition 5.1 are summarized in the following corollary:

Corollary 5.1
Suppose $\delta = 1$. *A dissonant politician will not choose the socially optimal action if*

(A) $s_1 = 1$ *has occurred and* $G > 2R$*, or*
(B) $s_1 = 0$ *has occurred and* $G > \frac{1}{2}R$*, or*
(C) $s_1 = 0$ *has occurred and* $\mu < \frac{1}{3}$.

Note that $\delta = 1$ is most favorable for the public. If a dissonant incumbent cannot be motivated to act congruently in case $\delta = 1$, then it will never be possible.

5.4.2 Behavior of Congruent Politicians

By (5.1)–(5.3), a congruent politician will undertake the socially optimal action in state $s_1 = 1$ if

$$R + G + \delta R + \delta^2 R \geq R. \tag{5.6}$$

This condition is always fulfilled, which means that, in this state of the world, congruent politicians always undertake the socially optimal action, as both voters and the politician prefer the popular action.

We now look at case $s_1 = 0$, meaning that voters and the politician prefer the unpopular action. The congruent politician will only undertake the optimal action for the voters if

$$\begin{aligned} & R + G + \mu(\delta R + \delta^2 R) && \geq R + (1-\mu)\delta R \\ \Leftrightarrow \quad & G + \delta R(2\mu + \delta\mu - 1) && \geq 0. \end{aligned} \tag{5.7}$$

In contrast to the case of $s_1 = 1$, it may now be the case that even a congruent politician will not undertake the socially optimal policy, although he too would prefer this policy. The socially optimal action is unpopular, but the politician would

[14] Note that $z > \frac{1}{2}$, so—under the assumption that neither (5.4) nor (5.5) are fulfilled—the probability that the incumbent will undertake $a_1 = 0$ in a situation where he should perform $a_1 = 1$ is higher than the probability for undertaking $a_1 = 1$ instead of the socially optimal action $a_1 = 0$.

still like to be reelected. This condition resembles Eq. (5.5), but now G is on the left-hand side because a congruent politician receives personal benefits G by acting congruently, while a dissonant politician receives G by acting dissonantly. Hence, if Condition (5.5) is fulfilled, then Condition (5.7) will also hold: if it is possible to motivate a dissonant politician to undertake the socially optimal action, then it is always possible to motivate a congruent politician to undertake the socially optimal action. Clearly, the reverse is not true. Furthermore, we have a distortion in favor of the popular action, given that it is possible for $a_1 = 1$ to be chosen too often, while the incumbent may not always carry out the unpopular action $a_1 = 0$ when he should. We summarize the results in the following proposition:

Proposition 5.2 *A politician of the congruent type will not undertake the socially optimal action if* $s_1 = 0$ *and* $G + \delta R(2\mu + \delta\mu - 1) < 0$.

5.5 The Triple Mechanism

We now introduce reelection threshold contracts and analyze their effect on the behavior of politicians and on social welfare. We assume that there exists a political information market that yields a price predicting the reelection chances of the incumbent in the second reelection decision. Investors receive private signals about which state of the world has occurred, and information is aggregated in the information market.

In Appendix B we provide a detailed microfoundation of how prices are formed in this information market, and how the information market enters and affects the political process. The basic result is that the equilibrium price p^* in the information market will be higher if the incumbent undertakes the socially optimal action, as choosing the optimal action ensures his success in the second reelection decision. In Appendix B we prove the following result:

Proposition 5.3 (short version)
If the signals of investors are sufficiently informative, then the equilibrium price on the information market is larger than one-half if the incumbent undertakes the action that is socially optimal, while it is smaller than one-half if the incumbent chooses the socially undesirable action.

The detailed version of Proposition 5.3 and its proof can be found in Appendix B.

5.5.1 Reelection Thresholds

Before the first period starts, politician i can offer conditional reelection threshold contracts $C_i(p_i^1, p_i^0)$ with $0 \le p_i^1 \le 1$ and $0 \le p_i^0 \le 1$, which means that the incumbent will only be allowed to stand for reelection after the first period if the price p^* on a political information market fulfills the condition

$$p^* \geq \begin{cases} p_i^1 & \text{if } a_1 = 1, \\ p_i^0 & \text{if } a_1 = 0, \end{cases}$$

where p_i^1 is the threshold price if the incumbent undertakes $a_1 = 1$ and p_i^0 is the threshold price if he chooses $a_1 = 0$. As the action of the politician is observable and verifiable, politicians can condition the threshold prices on the action and therefore p_i^1 and p_i^0 may differ. Note that offering a contract with $p_i^1 = p_i^0 = 0$ is equivalent to offering no contract at all.

5.5.2 Reelection Schemes

Reelection schemes are given by Eq. (5.1) for the first reelection and by Eq. (5.2) for the second reelection.[15] Recall from Eq. (5.1) that the scheme for the first reelection is such that a politician will always be deselected if he acts dissonantly in state $s_1 = 1$. Thus, threshold contracts will have no effect in state $s_1 = 1$, as in this state the reelection scheme from Eq. (5.1) effectively deters the politician from acting dissonantly. Adding threshold contracts prohibiting a politician who has behaved dissonantly from running for reelection will not change the results, as the politician would not be reelected anyway. By contrast, threshold contracts will have a positive effect in state $s_1 = 0$. As a consequence, only the threshold price p_i^1 will impact on the behavior of the politician, as dissonant behavior in state $s_1 = 0$ means choosing $a_1 = 1$ and thus p_i^1 applies.

5.5.3 Summary

The timing of the whole game including threshold contracts and political information markets is summarized in Fig. 5.2.

5.5.4 Robust Election Scheme

We assume that both politicians have to decide simultaneously about offering conditional threshold contracts. Moreover, we assume that voters use the following election

[15] If information markets are allowed and actually used, they might be taken into account by voters when making reelection decisions. Such feedback effects will be discussed in our extensions.

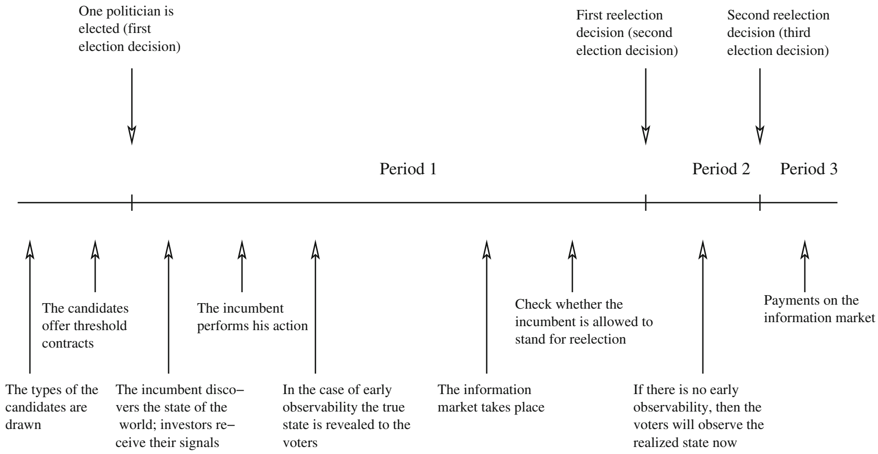

Fig. 5.2 Time-line of the game including threshold contracts and political information markets

scheme, where $e_1(p_1^1, p_1^0, p_2^1, p_2^0)$ denotes the probability of candidate 1 being elected at the first election decision:

$$e_1(p_1^1, p_1^0, p_2^1, p_2^0) = \begin{cases} 1 & \text{if } p_1^k \geq \frac{1}{2}\ \forall k \in \{0,1\} \text{ and } \exists l \in \{0,1\} : p_2^l < \frac{1}{2}, \\ 1 & \text{if } \exists k \in \{0,1\} : p_1^k \geq \frac{1}{2} \text{ and } p_2^l < \frac{1}{2}\ \forall l \in \{0,1\}, \\ 0 & \text{if } p_1^k < \frac{1}{2}\ \forall k \in \{0,1\} \text{ and } \exists l \in \{0,1\} : p_2^l \geq \frac{1}{2}, \\ 0 & \text{if } \exists k \in \{0,1\} : p_1^k < \frac{1}{2} \text{ and } p_2^l \geq \frac{1}{2}\ \forall l \in \{0,1\}, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{5.8}$$

We call this voting scheme *robust election scheme* (*RES*). The idea behind it is the following: Voters will elect a politician if and only if the threshold offers indicate that the politician will choose the socially optimal action, i.e. if $p \geq \frac{1}{2}$.[16] The precise values of p do not matter. Under *RES* a politician is elected with certainty if he offers prices for both actions that are equal to or above $\frac{1}{2}$ if the other politician does not do the same. If both candidates offer threshold values that are qualitatively similar with regard to the comparison to $\frac{1}{2}$, then both candidates will be elected with a probability of one-half. Later we will show that the assumptions of the voters in equilibrium are correct regarding the behavior of politicians. Accordingly, we call this an optimal voting scheme.

[16] Recall that the equilibrium price on the information market will be larger than one-half if and only if the incumbent undertakes the action that is socially optimal.

5.5.5 Equilibrium Notion

We are looking for perfect Bayesian equilibria of the game depicted in Fig. 5.2 among politicians and investors. Voters are not highly sophisticated players. They vote according to *RES*, as described above, and to the reelection schemes given in Eqs. (5.1) and (5.2). Henceforth a Bayesian equilibrium will simply be called “equilibrium”. The entire game is solved by assuming *RES* and the property that a price equal to or above $\frac{1}{2}$ indicates that the politician has chosen the optimal action, while a price below $\frac{1}{2}$ indicates the opposite. The optimality of *RES* will be shown later. The property of the prices in the information market is established in Appendix B. There we show that sophisticated investors use their private signals and their updated beliefs from the signalling subgame when politicians choose their action to trade on the information market. The equilibrium price indicates whether the office-holder has chosen the socially desirable action.

5.5.6 Equilibria

In this subsection we examine equilibria that involve robust election schemes. It is important to note that, when threshold contracts are offered, politicians do not know which state of the world will occur. We use the following plausible refinement:

Minimal Price Offer (MPO)
If a candidate is indifferent between two sets of prices for p_i^1 and p_i^0 given the contract choice of the other politician and *RES*, then he will choose the contract with the minimal values for p_i^1 and p_i^0 in the corresponding sets.

A formal description of MPO is as follows: Suppose a politician is indifferent between $C_i(p_i^1, p_i^0)$ and $\tilde{C}_i(\tilde{p}_i^1, \tilde{p}_i^0)$. Then he will choose $C_i(p_i^1, p_i^0)$ if $p_i^k \leq \tilde{p}_i^k \ \forall k \in \{0, 1\}$ and $\exists l \in \{0, 1\} : p_i^l < \tilde{p}_i^l$, but $\tilde{C}_i(\tilde{p}_i^1, \tilde{p}_i^0)$ if $p_i^k \geq \tilde{p}_i^k \ \forall k \in \{0, 1\}$ and $\exists l \in \{0, 1\} : p_i^l > \tilde{p}_i^l$.

The refinement can be justified by the fact that the likelihood of fulfilling a given threshold is non-increasing in the value of the prices.[17] Through the observation that the utility of an elected politician weakly decreases in his price offers we obtain the following lemma:

Lemma 5.1
Under MPO and RES, equilibrium price offers satisfy $p_i^k \leq \frac{1}{2} \ \forall k \in \{0, 1\}, i = 1, 2.$

[17] MPO can be justified by arbitrarily small errors of investors. Suppose there is a possibility of such errors by investors. Then, the probability of fulfilling a given threshold is strictly decreasing in prices. Without MPO, other prices than $\frac{1}{2}$ in threshold contracts can emerge in equilibrium in Proposition 5.4. The implications for the behavior of office-holders, however, are the same.

Next, as a consequence of Lemma 5.1 and *RES*, we can restrict ourselves to four cases for a particular politician i:

(i) $p_i^1 = \frac{1}{2}, p_i^0 = \frac{1}{2}$
(ii) $p_i^1 = \frac{1}{2}, p_i^0 < \frac{1}{2}$
(iii) $p_i^1 < \frac{1}{2}, p_i^0 = \frac{1}{2}$
(iv) $p_i^1 < \frac{1}{2}, p_i^0 < \frac{1}{2}$

As only the threshold price p_i^1 will impact on the behavior of the incumbent, we thus obtain the following Lemma:

Lemma 5.2

(A) Cases (i) and (ii) induce the same behavior by an elected politician.
(B) Cases (iii) and (iv) induce the same behavior by an elected politician.

In Appendix A we show:

Proposition 5.4
Both politicians will offer threshold contracts $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ under election scheme RES, irrespective of their own type and irrespective of the type of their opponent.

Given this result of Proposition 5.4, we next show that the voting behavior of the *RES* is indeed optimal:

Proposition 5.5
The robust election scheme is optimal for voters.

The proof is given in Appendix A. The strength of *RES* is that voters do not need to have specific information regarding the parameters of projects or the signals of investors in the information market. They simply judge whether politicians are willing to compete against a fair coin. The next theorem is our main result.

Theorem 5.1
The conditions under which politicians in state $s_1 = 0$ behave congruently with threshold contracts are less strict, and dissonant behavior is less attractive, than without threshold contracts. This holds for both types of politicians. In particular, with the triple mechanism we obtain:

(i) A dissonant politician behaves congruently in state $s_1 = 1$ if and only if $\delta R(1 + \delta) \geq G$.
(ii) A dissonant politician behaves congruently in state $s_1 = 0$ if and only if $\delta R\mu(1 + \delta) \geq G$.
(iii) A congruent politician always behaves congruently in both states.

The proof is given in Appendix A. The intuition is as follows: Given equilibrium threshold contracts $C_i(p_i^1 = \frac{1}{2})$, politicians have no chance of being reelected in state $s_1 = 0$ if they behave dissonantly, i.e. if they undertake $a_1 = 1$. If they behave congruently, their reelection chances are given by probability μ. If threshold contracts are absent, a politician who behaves dissonantly still has a chance to get reelected, while congruent behavior does not yield higher reelection probabilities than μ. Hence, threshold contracts make dissonant behavior in state $s_1 = 0$ less attractive than congruent behavior.

In Appendix D we provide a brief example of the welfare gains that can be achieved with the triple mechanism. The example illustrates, among other things, that threshold contracts have the largest effect on welfare when R is larger than G and when the probability of the unpopular state $s_1 = 0$ is rather high, i.e. if z lies close to $\frac{1}{2}$.

5.6 Extensions, Robustness and Pitfalls

In the following we discuss several extensions of the model, thereby exploring the robustness and potential pitfalls of the triple mechanism.

5.6.1 *Monotonic Election Scheme and Overpromising*

As we showed in Proposition 5.5, the robust election scheme used in the last section is optimal for voters. However, it is not clear whether the scheme is unique. In this section we consider further candidates for election schemes. We start with a simple and intuitive scheme that we call the *monotonic election scheme* (*MES*):

$$e_1(p_1^1, p_1^0, p_2^1, p_2^0) = \begin{cases} 1 & \text{if } p_1^k \geq p_2^k \ \forall k \in \{0, 1\} \text{ and } \exists k \in \{0, 1\} : p_1^k > p_2^k, \\ 0 & \text{if } p_1^k \leq p_2^k \ \forall k \in \{0, 1\} \text{ and } \exists k \in \{0, 1\} : p_1^k < p_2^k, \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

The *MES* is intuitive in the sense that voters simply elect the candidate who offers tighter constraints on his reelection thresholds. The problem is, however, that *overpromising* may occur under *MES*. We call a threshold contract with prices p^1, p^0 overpromising if at the date of the offer the politician already knows that at least one of the thresholds can never be reached. Such overpromising may occur if it is more profitable for a politician to be elected with certainty in the first election and to be certainly not reelected in the next election, in comparison to being elected with probability $\frac{1}{2}$ in the first election and having a positive reelection probability. In Appendix A we show:

Proposition 5.6
Under the monotonic election scheme, overpromising may occur.

Overpromising invites extreme short-termism, where both types of politicians simply behave in accordance with their first-period preferences and maximize their first period utility. In the case of overpromising, dissonant politicians will always behave dissonantly, while a congruent politician will behave congruently.[18] Hence, the monotonic election scheme is not optimal and thus is not an equilibrium response for voters.

5.6.2 Sophisticated Election Scheme

We next examine a voting scheme which we call the *sophisticated election scheme* (*SES*):

$$e_1(p_1^1, p_1^0, p_2^1, p_2^0) = \begin{cases} 1 & \text{if } p_1^1 \geq z, p_1^0 \geq 1-z, \text{ and } p_2^1 < z \text{ or } p_2^0 < 1-z, \\ 1 & \text{if } p_2^1 < z, p_2^0 < 1-z, \text{ and } p_1^1 \geq z \text{ or } p_1^0 \geq 1-z, \\ 0 & \text{if } p_1^1 < z, p_1^0 < 1-z, \text{ and } p_2^1 \geq z \text{ or } p_2^0 \geq 1-z, \\ 0 & \text{if } p_2^1 \geq z, p_2^0 \geq 1-z, \text{ and } p_1^1 < z \text{ or } p_1^0 < 1-z, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{5.9}$$

This voting scheme is similar to *RES*, but the critical values are not $\frac{1}{2}$ but z for action $a_1 = 1$ and $1 - z$ for action $a_1 = 0$. This reflects the fact that the a priori probability for $s_1 = 1$ (where $a_1 = 1$ is socially optimal) is z and for $s_1 = 0$ (where $a_1 = 0$ is socially optimal) is $1 - z$. Under *SES*, voters demand that the prices on the information market at least reach the a priori probabilities. In this voting scheme voters use the following information:

Proposition 5.7 (short version)
If the signals of investors in the information market are sufficiently informative, then the following holds:

(i) If the incumbent undertakes $a_1 = 1$, then the price on the information market will be larger than z if $s_1 = 1$ and smaller than z if $s_1 = 0$.
(ii) If the incumbent chooses $a_1 = 0$, then the price on the information market will exceed $1 - z$ if $s_1 = 0$, while the price will be below $1 - z$ if $s_1 = 1$.

[18] It is obvious that overpromising is socially detrimental in the case of dissonant politicians. If the incumbent is congruent, there will be no immediate negative effect on social welfare. However, as a congruent incumbent who overpromises will be replaced by a new politician who can either be congruent or dissonant, overpromising by congruent politicians would have negative effects on social welfare in an extended version of the model, where the incumbent would undertake further action in periods 2 or 3.

The detailed version of Proposition 5.7 and its proof can be found in Appendix B. Furthermore, one can show that under *SES* both politicians will offer $C_i(p_i^1 = z, p_i^0 = 1 - z)$ and that *SES* is also optimal for voters. The proof follows the same lines as the proof of Propositions 5.4 and 5.5 and is therefore omitted here. Note that, with a sophisticated election scheme, voters anticipate that the market price will be higher under congruent behavior of the politician in state $s_1 = 1$ than under congruent behavior of the politician in state $s_1 = 0$. While both *RES* and *SES* are optimal for voters, we will show in Corollary 5.2 of Appendix B that the conditions for Proposition 5.7 to hold are weaker than the conditions required for Proposition 5.3 to hold.

5.6.3 Market-Based Voting

In our basic model we have assumed that the price on the information market has no influence on reelection probability. Now we assume the other polar case, where voters only use the price on the information market as a basis for their first reelection decision. In this case, threshold contracts are without effect (either positive or negative). The existence of a political information market that predicts the reelection chances after the next term is sufficient to generate all efficiency gains when voters solely use information markets as their forward-looking reelection scheme.[19] The reason is that the price on the information market is the best predictor regarding the quality of the decisions of the politician. Purely market-based voting is a polar case. It is likely that actual voting will be between both polar cases (no market-based voting and purely market-based voting). Then, reelection threshold contracts will continue to have beneficial welfare effects.

5.6.4 Repeated Action

Another potential extension is to examine repeated actions by the politician. Suppose that the incumbent stays in office as long as he gets reelected, that he undertakes an action a_t in each period t in office,[20] and that the candidates are allowed to offer threshold contracts before each election. In our working paper Gersbach and Müller (2006), we have shown that the results of the two-period case still hold when actions are repeated. In particular, threshold contracts are always socially advantageous compared to elections alone, since the probability of a politician behaving congruently is higher when threshold contracts are used.

[19]The same would occur if there existed other means that perfectly aggregate the information of investors.

[20]We assume that the politician will undertake no action in the last two periods, which corresponds to our assumption in the basic model where the politician does not take any action in the second and third periods.

5.6.5 More Candidates

Our analysis can be extended to more than two candidates running for office in the first election. If n denotes the number of candidates, the election probability of a candidate would decrease accordingly to $\frac{1}{n}$. Equilibrium threshold contract offers and behavior of office-holders, as well as the price behavior in the information market, would remain the same.

5.6.6 Manipulations

A serious concern is manipulation. The incumbent might try to push the equilibrium price above the price in his threshold contract through trading in such markets.[21] The most obvious way to prevent manipulation is to prohibit trading by politicians and to punish the use of stooges. However, such prohibitions may not be sufficient to prevent manipulations. A robust possibility to ensure that the incumbent is not interested in manipulating the market is to use an average price calculated over a longer time-span. One can use the entire time span between the action of the incumbent and the end of the first term to operate the information market. An incumbent who wants to raise the average market price above his threshold price via manipulation would be forced to manipulate the price in the information market every day over several years, which would become very costly over time.

5.7 Conclusion

In this chapter we have suggested a triple mechanism for improving the functioning of democracies when information is not observable or not verifiable. The results seem to be quite robust for various extensions as well. Moreover, the idea of the triple mechanism could be extended to multi-task settings, where the politician decides on many issues in his first term. As the threshold contract depends on the average long-term performance of the politician, the standard problem may aggravate distortions in favor of tasks with better observability.

Political information markets are an instrument for solving the problem of short-term unobservability coupled with long-term non-verifiability. Hence threshold contracts combined with information markets can be used successfully when projects have long-term effects and information on project results is not available in the short

[21] Rhode et al. (2004) discuss several historical manipulations episodes and provide important insights how these can be engineered.

term. Of course, any suggestion of a new institution such as the one made in this chapter has to be subjected to further scrutiny.[22]

Appendix A: Proofs

Proof of Proposition 5.4

Suppose that voters use the robust election scheme *RES*. Both candidates decide simultaneously about their threshold contracts. We show that for $i = 1, 2$, the unique equilibrium of the politician's contract choice is $C_i(p_i^1 = p_i^0 = \frac{1}{2})$.

Equilibrium Property: Given that candidate $g \in \{1, 2\}$ offers $C_g(p_g^1 = p_g^0 = \frac{1}{2})$, politician $h \neq g$, $h \in \{1, 2\}$ will not offer $p_h^k < \frac{1}{2}$ for any $k \in \{0, 1\}$, since he would have no chance of winning the election. Furthermore, he has no incentive to offer $p_h^k > \frac{1}{2}$ for any $k \in \{0, 1\}$, since this does not increase his chances of winning the election. Thus, given that candidate g offers $C_g(p_g^1 = p_g^0 = \frac{1}{2})$, a best response for candidate h is to offer $C_h(p_h^1 = p_h^0 = \frac{1}{2})$, independently of his type. Hence, offering $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ $\forall i \in \{1, 2\}$ is an equilibrium. In the next steps we show that it is unique.

Uniqueness: We know from Lemma 5.1 that $p_i^k \leq \frac{1}{2}$ $\forall k \in \{0, 1\}$, $i = 1, 2$, so we only have to examine whether there may exist other equilibria with threshold offers below $\frac{1}{2}$. Suppose that candidate g offers a contract with $p_g^k \leq \frac{1}{2} \forall k \in \{0, 1\}$ and $p_g^k < \frac{1}{2}$ for at least one $k \in \{0, 1\}$. We distinguish three cases:

Case 1: First, consider a constellation with candidates g and h offering contracts $C_g(p_g^1 < \frac{1}{2}, p_g^0 < \frac{1}{2})$ and $C_h(p_h^1 < \frac{1}{2}, p_h^0 < \frac{1}{2})$. Then candidate h has an incentive to deviate by offering $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$. Indeed, under *RES*, his election chances are strictly higher when he offers $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$ and offering $p_h^0 = \frac{1}{2}$ does not reduce the reelection chances of h, whether he behaves congruently or dissonantly.[23]

Case 2: Consider next a constellation with candidates g and h offering contracts $C_g(p_g^1 = \frac{1}{2}, p_g^0 < \frac{1}{2})$ and $C_h(p_h^1 = \frac{1}{2}, p_h^0 < \frac{1}{2})$. Then candidate h can profitably deviate by offering $C_h(p_h^1 = \frac{1}{2}, p_h^0 = \frac{1}{2})$ as this induces the same behavior (Lemma 5.2) and gives the same reelection chances, while increasing the election chances from $\frac{1}{2}$ to 1.

Case 3: We are left with the optimal response of politician $h \neq g$ if candidate g offers $C_g(p_g^1 < \frac{1}{2}, p_g^0 = \frac{1}{2})$. There are two possibilities for optimal responses: $C_h(p_h^1 = p_h^0 = \frac{1}{2})$ and $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$. We show now that both types of politicians will prefer to offer $C_h(p_h^1 = p_h^0 = \frac{1}{2})$ rather than $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$ in response

[22] One might, for example, wonder how the triple mechanism could be introduced. The best way to try to implement the triple mechanism is political competition. If one candidate proposes the idea, then competing candidates are forced to offer the same in order to avoid losing votes, as the triple mechanism is welfare-improving.

[23] Recall that only threshold p_i^1 can affect the reelection chances of the incumbent.

to a contract $C_g(p_g^1 < \frac{1}{2}, p_g^0 = \frac{1}{2})$. Suppose that a candidate, say candidate 2, offers $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$.

Case 3a: We consider candidate 1 and assume first that he is of the congruent type. If a congruent politician offers a contract with $p_1^1 = p_1^0 = \frac{1}{2}$ and gets elected, then he will always behave congruently.[24] If a congruent politician offers a contract with $p_1^1 < \frac{1}{2}$ and $p_1^0 = \frac{1}{2}$ and gets elected,[25] then his behavior in state $s_1 = 0$ will depend on whether $R + G + \mu[\delta R + \delta^2 R]$ is larger or smaller than $(R + (1 - \mu)\delta R)$. Candidate 1 is better off by choosing $p_1^1 = p_1^0 = \frac{1}{2}$ if

$$\begin{gathered} z[R + G + \delta R + \delta^2 R] + (1 - z)\{R + G + \mu[\delta R + \delta^2 R]\} \\ \geq \\ \frac{1}{2} z[R + G + \delta R + \delta^2 R] + \frac{1}{2}(1 - z) \max\{R + G + \mu[\delta R + \delta^2 R]; R + (1 - \mu)\delta R\}. \end{gathered} \tag{5.10}$$

To analyze this inequality, we consider the two possible cases, starting with $R + G + \mu[\delta R + \delta^2 R] \geq (R + (1 - \mu)\delta R)$. In this case, inequality (5.10) simplifies to $1 \geq \frac{1}{2}$ and thus holds. Next we look at $R + G + \mu[\delta R + \delta^2 R] < (R + (1 - \mu)\delta R)$. Then inequality (5.10) can be simplified to

$$\begin{aligned} \frac{1}{2} z[R + G {+} \delta R + \delta^2 R] + (1 - z)\{R + G + \mu[\delta R + \delta^2 R]\} \\ \geq \frac{1}{2}(1 - z)[R + (1 - \mu)\delta R]. \end{aligned}$$

This condition is always fulfilled because $\frac{1}{2} z[R + \delta R] > \frac{1}{2}(1 - z)[R + (1 - \mu)\delta R]$ and the other terms on the left hand side of the condition are positive. Thus, a congruent politician 1 will offer a contract with $p_1^1 = p_1^0 = \frac{1}{2}$ in response to $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$.

Case 3b: Next, we analyze the behavior of politician 1 if he is dissonant and candidate 2 offers $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$. In contrast to our considerations for congruent politicians above, it is no longer clear this time whether politician 1 will behave congruently or dissonantly. Nevertheless, it still holds that he will offer a contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$. To substantiate this claim we distinguish four cases:

(i) Suppose candidate 1 is elected and behaves in a dissonant manner regardless of the threshold contract he has offered.[26] Then we obtain

$$EU^1\left(p_1^1 = p_1^0 = \frac{1}{2}\right) = z(R + G) + (1 - z)(R + G) = R + G \tag{5.11}$$

---

[24] This is obvious in state $s_1 = 1$. In state $s_1 = 0$, the politician has utility $R + G + \mu[\delta R + \delta^2 R]$ when he behaves congruently and utility R when be behaves dissonantly. Hence, the politician will always behave congruently. Closer reasoning will be given in Theorem 5.1.

[25] Note that in this case the election probability is only $\frac{1}{2}$.

[26] Intuitively, this will occur if the value of G is sufficiently large.

and

$$EU^1\left(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2}\right) = \frac{1}{2}\{z[R+G] + (1-z)[R+G+(1-\mu)\delta R]\}$$
$$= \frac{1}{2}[R+G+(1-z)(1-\mu)\delta R] < R + \frac{G}{2}, \quad (5.12)$$

where EU^1 denotes the expected utility of politician 1 depending on the contract he has offered. Hence, expected utility will be larger if he offers $p_1^1 = p_1^0 = \frac{1}{2}$.

(ii) Suppose candidate 1 is elected and behaves in a congruent manner, regardless of the threshold contract he has offered.[27] For such circumstances we obtain

$$EU^1\left(p_1^1 = p_1^0 = \frac{1}{2}\right) = z[R+\delta R+\delta^2 R] + (1-z)[R+\mu(\delta R+\delta^2 R)]$$
$$= R + [z+(1-z)\mu](1+\delta)\delta R \quad (5.13)$$

and

$$EU^1\left(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2}\right) = \frac{1}{2}\{z[R+\delta R+\delta^2 R] + (1-z)[R+\mu(\delta R+\delta^2 R)]\}$$
$$= \frac{1}{2}\{R+[z+(1-z)\mu](1+\delta)\delta R\}. \quad (5.14)$$

As the expression (5.13) is larger than the expression in (5.14), candidate 1 is better off by offering $p_1^1 = p_1^0 = \frac{1}{2}$.

(iii) Suppose candidate 1 is elected and behaves dissonantly with a contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ and congruently with a contract $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$. According to Eqs. (5.12) and (5.14) acting congruently after having offered $p_1^1 < \frac{1}{2}$ is only optimal if $G < [z+(1-z)\mu](1+\delta)\delta R$. However, for $G<[z+(1-z)\mu](1+\delta)\delta R$ the politician would act congruently after having offered $p_1^1 = \frac{1}{2}$ according to Eqs. (5.11) and (5.13). This is a contradiction and hence, case (iii) cannot occur.

(iv) Suppose candidate 1 is elected and behaves congruently with the contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ while behaving dissonantly with $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$. The utility of acting dissonantly with contract $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$ is smaller than the utility of acting dissonantly with contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$. As we have assumed that the candidate behaves congruently under $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ and thus achieves higher or equal utility than by acting dissonantly, the utility of acting dissonantly with $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$ is smaller than the utility of behaving congruently with contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$. Hence, we can conclude that if politician 1 is of the dissonant type, he will always offer a contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ given that candidate 2 offers a contract $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$.

[27] This will occur if the value of G is sufficiently small.

To sum up, $C_i(p_i^1 = p_i^0 = \frac{1}{2})\ \forall i \in \{1, 2\}$ is the unique equilibrium under the election scheme *RES*. □

Proof of Proposition 5.5

In Proposition 5.4 we have shown that both politicians will offer $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ if they believe that voters will use *RES*. Now we show that *RES* is optimal for voters.

The proof of Proposition 5.3 in Appendix B shows that the equilibrium price on the information market will be larger than $\frac{1}{2}$ if the incumbent chooses the socially optimal action, while it will be smaller than $\frac{1}{2}$ if the incumbent chooses the socially undesirable action. So *RES* is optimal, as it induces the socially optimal action. Specifically, under *RES* a politician (say $i = 2$) who offers a contract with a price smaller than $\frac{1}{2}$ will never generate a higher utility than a politician who offers thresholds p_1^1 and p_1^0 equal to $\frac{1}{2}$. Thus in this case electing politician 1 can never be worse than electing politician 2. Finally, we note that under *RES* a politician (say $i = 2$) who offers a contract with a threshold strictly larger than $\frac{1}{2}$ will never generate a higher utility than a politician who offers thresholds p_1^1 and p_1^0 equal to $\frac{1}{2}$. In this case, electing politician 1 can never be worse than electing politician 2. This completes the proof. □

Proof of Theorem 5.1

We start with dissonant politicians and look first at the case $s_1 = 1$ where the popular action is optimal from the voters' point of view. The politician, however, would prefer the unpopular action. In the scenario with threshold contracts, the dissonant politician will undertake the socially optimal action if and only if

$$\begin{aligned} & R + \delta R + \delta^2 R \geq R + G \\ \Leftrightarrow \quad & \delta R(1 + \delta) \geq G. \end{aligned} \tag{5.15}$$

Comparison with the condition when threshold contracts are absent shows that Condition (5.15) is identical to Condition (5.4). The reason is that threshold contracts have no impact in state $s_1 = 1$.

We next consider the case $s_1 = 0$. In this state, voters prefer the unpopular action, while the dissonant politician prefers the popular action. The dissonant politician will only undertake the socially optimal action if

$$\begin{aligned} & R + \mu(\delta R + \delta^2 R) \geq R + G \\ \Leftrightarrow \quad & \delta R\mu(1 + \delta) \geq G. \end{aligned} \tag{5.16}$$

Comparison with the condition in the scenario without threshold incentive contracts shows that Condition (5.5) is tighter than Condition (5.16), i.e. the set of parameter values fulfilling (5.16) is larger than the corresponding set for Condition (5.5). For instance, Eq. (5.16) is always fulfilled if R is sufficiently high, which is not true in general under Condition (5.5).

Next consider congruent politicians. In case $s_1 = 1$, a congruent politician will undertake the socially optimal action if

$$R + G + \delta R + \delta^2 R \geq R. \tag{5.17}$$

This condition is always fulfilled. In case $s_1 = 0$, a congruent politician will undertake the optimal action if

$$R + G + \mu(\delta R + \delta^2 R) \geq R. \tag{5.18}$$

Again, this condition is always fulfilled. Hence, in both states of the world, the politician will always pursue the policy optimal for the voters if he has offered a threshold contract with $p_i^1 = p_i^0 = \frac{1}{2}$. As showed in Eq. (5.7), this is not necessarily true for congruent politicians in the scenario without threshold contracts. □

Proof of Proposition 5.6

Suppose that relative to R, G is sufficiently large to ensure that congruent politicians will always act congruently and dissonant politicians will always act dissonantly, irrespective of the threshold contracts they have offered. In Appendix B we show that, for G sufficiently large relative to R, the equilibrium price will be smaller than 1, even if politicians act in a socially optimal way. Thus, if both candidates offered contracts $p_i^1 = p_i^0 = 1$, neither of them would ever be able to fulfill their contract. This is an example of overpromising.

Suppose next that both candidates are of the congruent type. Then no candidate will deviate from the Nash equilibrium $p_i^1 = p_i^0 = 1$, as a deviating candidate would never be elected.

Now we show that the Nash equilibrium $p_i^1 = p_i^0 = 1$ is unique for certain parameters. Suppose both candidates offer threshold contracts with $p_1^1 = p_2^1 < 1$ and $p_1^0 = p_2^0 < 1$. Politicians face the trade-off between offering the largest thresholds that can be reached by acting congruently and deviating from this offer to higher values, thereby increasing election chances to 1. Deviation to higher threshold values is profitable if

$$\begin{aligned} \frac{1}{2}\{z[R + G + \delta R + \delta^2 R] + (1 - z)[R + G + \mu(\delta R + \delta^2 R)]\} &< (R + G) \\ \Leftrightarrow R\{[z + \mu(1 - z)](\delta + \delta^2) - 1\} &< G. \end{aligned} \tag{5.19}$$

We see that this condition will always be fulfilled if G is sufficiently large relative to R.[28] □

[28] There exist other constellations where overpromising occurs. Details are available on request.

Appendix B: Political Information Market

In this appendix we describe the functioning of the political information market in detail. First, we describe the assets and the investors. As investors receive information from two sources—the private signals and actions of politicians—we have to examine how both sources of information jointly determine the beliefs of the investors, step by step. Finally, we determine the equilibrium price in the market.

B.1 Assets

We assume that a political information market is organized during the first period after politicians have chosen their actions.

There are two assets, D and E, in which investors can trade. If the office-holder is reelected after the second period, the owners of asset D receive one monetary unit for a single unit of D. If the politician stands for reelection but is not reelected after the second period, the owners of asset E receive one monetary unit for a single unit of E. If the politician is not able to run for second reelection, e.g. if he was already deselected at the first reelection or if he does not want to stand for reelection, then all transactions that have occurred will be neutralized. This means that each investor will be paid back the money he has invested.[29]

The information market works as follows: A bank or an issuer offers an equal amount of assets D and E. On the secondary market, traders can buy assets D or E.[30] Trading in the secondary market results in price p for one unit of asset D. As buying one unit of D and one unit of E pays one monetary unit with certainty, the price of asset E must be $1-p$, otherwise either traders or the issuer could make riskless profits. An equilibrium on the information market is a price p^* such that traders demand an equal amount of assets D and E.[31]

It is useful to look more closely at the event tree associated with the assets. If, for example, an investor buys one unit of asset D at price p, then the event tree and the payoffs for the information market are given as in Fig. 5.3.

In this chapter we specifically design information markets to allow for the design of reelection threshold contracts. If threshold contracts are offered, then the event tree and the payoffs for the information market have to be modified in the following way as depicted in Fig. 5.4.

Finally, note that with probability μ there is complete information in period 1. Then the price in the information market will be either 1 or 0, depending on whether the politician undertook the socially optimal action or not.

[29] See Berg et al. (2003) for alternative ways to implement conditional prediction markets in practice.

[30] We could allow for short-selling, but this is immaterial to our analysis.

[31] This is equivalent to an information market with only asset D where traders can buy or sell D and an equilibrium is obtained when supply equals demand.

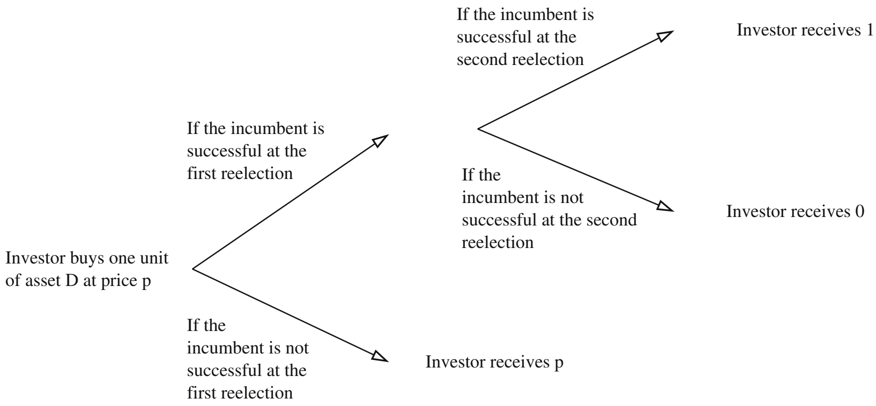

Fig. 5.3 Event tree and payoffs for the information market

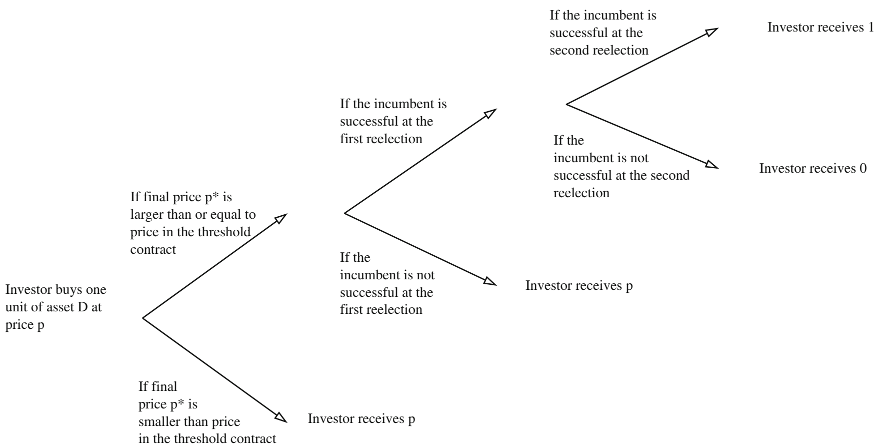

Fig. 5.4 Event tree and payoffs for the information market including threshold contracts

B.2 Investors

There are N potential investors.[32] Investors are a subgroup of voters. We assume that there are many investors in the market. However, compared to the total number of voters investors constitute a minority and can not influence the voting outcome.

We assume that investors have log-utility with

$$U_j(Y_j + W_j) = \ln(Y_j + W_j), \tag{5.20}$$

[32]It is sensible for only individuals to be allowed to trade in such information markets and for the trading volume per person to be limited so as to avoid large-scale manipulation attempts.

where W_j is the investor's wealth and Y_j is gain or loss in the information market.[33] Each investor j obtains a signal $\sigma_j \in \{0; 1\}$ about the state of the world at the point in time when the politician in office discovers the state of the world.[34] The probability that investor j receives a correct signal, i.e. that $\sigma_j = s_1$, is given by $h_j \in (\frac{1}{2}, 1)$, where each investor j knows his personal signal quality h_j. Our assumption $h_j > \frac{1}{2}$ implies that the signals are not completely uninformative.[35] We assume that h_j does not depend on the state that has occurred.[36]

We first calculate the investors' posterior probability estimations of the state after they have received their signals. We obtain:

$$Prob(s_1 = 1|\sigma_j = 1) = \frac{zh_j}{zh_j + (1-z)(1-h_j)}, \tag{5.21}$$

$$Prob(s_1 = 1|\sigma_j = 0) = \frac{z(1-h_j)}{(1-z)h_j + (1-h_j)z}, \tag{5.22}$$

$$Prob(s_1 = 0|\sigma_j = 1) = \frac{(1-z)(1-h_j)}{zh_j + (1-z)(1-h_j)}, \tag{5.23}$$

$$Prob(s_1 = 0|\sigma_j = 0) = \frac{(1-z)h_j}{(1-z)h_j + (1-h_j)z}. \tag{5.24}$$

B.3 Information from the Politician's Choice

Investors may receive additional information about the state by observing the action of the incumbent. Recall that a politician of the congruent type will always behave congruently in equilibrium when threshold contracts are used. The behavior of a dissonant incumbent depends on the parameters R, G, δ, and μ, which are common knowledge among investors. Three cases can occur: First, the value of G may be sufficiently low relative to R. Then dissonant politicians will behave congruently. Second, the value of G is at an intermediate level, and dissonant politicians will behave congruently in the popular state $s_1 = 1$, while they will behave dissonantly in the unpopular state $s_1 = 0$. Third, the value of G may be rather high relative to

[33] Note that we neglect utility from the action of the politician in the utility function of investors, as policy outcomes have no influence on the trading behavior of investors.

[34] There are several justifications why investors may be better informed than voters. It is fair to assume that investors spend time collecting information concerning the state of the world and thus have more knowledge than ordinary voters.

[35] We could also allow for poor signal qualities, i.e. $h_j \in (0, \frac{1}{2})$. As investor j knows h_j, a value of h_j near to 0 is as informative as a value near to 1. The lowest information gain is received by a signal which is correct with a probability of $\frac{1}{2}$. Nevertheless, we restrict the signal quality to $h_j \in (\frac{1}{2}, 1)$ in order to avoid additional case differentiations.

[36] The extension to state contingent values of h_j does not change the qualitative results of our model.

Table 5.1 Actions of politicians

Case	Condition	a_1^c if $s_1 = 1$	a_1^c if $s_1 = 0$	a_1^d if $s_1 = 1$	a_1^d if $s_1 = 0$
1	$G \leq \mu\delta R(1+\delta)$	1	0	1	0
2	$\mu\delta R(1+\delta)$ $< G$ $\leq \delta R(1+\delta)$	1	0	1	1
3	$\delta R(1+\delta) < G$	1	0	0	1

Table 5.2 Conditional probabilities

Case	Condition	$Prob_c(s_1 = 1\|a_1 = 1)$	$Prob_c(s_1 = 0\|a_1 = 0)$
1	$G \leq \mu\delta R(1+\delta)$	1	1
2	$\mu\delta R(1+\delta) < G \leq \delta R(1+\delta)$	$\dfrac{2z}{z+1}$	1
3	$\delta R(1+\delta) < G$	z	$1-z$

the benefits from holding office. Then dissonant politicians will behave dissonantly in both states of the world. We summarize the three cases in Table 5.1, where a_1^c denotes the action of a congruent politician and a_1^d denotes the action of a dissonant politician.

In the following we use $c \in \{1, 2, 3\}$ to denote the cases. In the next step we will calculate the conditional probabilities $Prob_c(s_1 = 1|a_1 = 1)$ and $Prob_c(s_1 = 0|a_1 = 0)$ for an individual investor without private signals updating his beliefs in the signalling game with politicians choosing their action. For example, we obtain $Prob_2(s_1 = 1|a_1 = 1)$ as $\frac{z}{z+\frac{1}{2}(1-z)} = \frac{2z}{z+1}$ for $c = 2$. We summarize the conditional probabilities in Table 5.2.

B.4 Private Signals and Information from Politicians

Finally, we calculate the conditional probabilities $Prob_c(s_1|\sigma_j, a_1)$ for $c \in \{1, 2, 3\}$ when voters have received their private signals σ_j and draw inferences from the signalling games among politicians.

Case $c = 1$

Suppose $c = 1$. Then investors will learn the state with certainty by observing the action of the incumbent and can disregard their signals σ_j. We obtain

$$Prob_1(s_1 = 1|\sigma_j = 1, a_1 = 1) = Prob_1(s_1 = 1|\sigma_j = 0, a_1 = 1) = 1,$$

$$Prob_1(s_1 = 1|\sigma_j = 1, a_1 = 0) = Prob_1(s_1 = 1|\sigma_j = 0, a_1 = 0) = 0,$$

$$Prob_1(s_1 = 0|\sigma_j = 1, a_1 = 0) = Prob_1(s_1 = 0|\sigma_j = 0, a_1 = 0) = 1,$$

$$Prob_1(s_1 = 0|\sigma_j = 1, a_1 = 1) = Prob_1(s_1 = 0|\sigma_j = 0, a_1 = 1) = 0.$$

Case $c = 2$

In case 2, investors know with certainty that the true state of the world is $s_1 = 0$ when they observe $a_1 = 0$, i.e.

$$Prob_2(s_1 = 1|\sigma_j = 1, a_1 = 0) = Prob_2(s_1 = 1|\sigma_j = 0, a_1 = 0) = 0,$$

$$Prob_2(s_1 = 0|\sigma_j = 1, a_1 = 0) = Prob_2(s_1 = 0|\sigma_j = 0, a_1 = 0) = 1.$$

If investors observe $a_1 = 1$, then the signalling game reveals that the probability of $s_1 = 1$ after observing $a_1 = 1$ is equal to $\frac{2z}{z+1}$. Using the additional information from signal σ_j, investor j forms the following a posteriori belief[37]:

$$Prob_2(s_1 = 1|\sigma_j = 1, a_1 = 1) = \frac{\frac{2z}{z+1}h_j}{\frac{2z}{z+1}h_j + \frac{1-z}{z+1}(1-h_j)} = \frac{2zh_j}{2zh_j + (1-z)(1-h_j)}.$$

In a similar way we obtain

$$Prob_2(s_1 = 1|\sigma_j = 0, a_1 = 1) = \frac{2z(1-h_j)}{2z(1-h_j) + (1-z)h_j},$$

$$Prob_2(s_1 = 0|\sigma_j = 1, a_1 = 1) = \frac{(1-z)(1-h_j)}{2zh_j + (1-z)(1-h_j)},$$

$$Prob_2(s_1 = 0|\sigma_j = 0, a_1 = 1) = \frac{(1-z)h_j}{2z(1-h_j) + (1-z)h_j}.$$

Case $c = 3$

In case 3, the investors do not gain any information from the politician's action, as there is complete pooling. A congruent politicians behaves congruently, while all dissonant politicians behave dissonantly, and the probability for both types of politician equals $\frac{1}{2}$. Hence,

$$Prob_3(s_1 = 1|\sigma_j = 1, a_1 = 1) = Prob_3(s_1 = 1|\sigma_j = 1, a_1 = 0) = \frac{zh_j}{zh_j + (1-z)(1-h_j)},$$

[37] Alternatively, one could calculate the a posteriori belief of investor j in the following way:

$$Prob_2(s_1 = 1|\sigma_j = 1, a_1 = 1) = \frac{2Prob(s_1 = 1|\sigma_j = 1)}{Prob(s_1 = 1|\sigma_j = 1) + 1} = \frac{2zh_j}{2zh_j + (1-z)(1-h_j)}.$$

Both methods lead to the same result.

$$Prob3(s_1 = 1|\sigma_j = 0, a_1 = 1) = Prob3(s_1 = 1|\sigma_j = 0, a_1 = 0) = \frac{z(1-h_j)}{(1-z)h_j + z(1-h_j)},$$

$$Prob3(s_1 = 0|\sigma_j = 1, a_1 = 1) = Prob3(s_1 = 0|\sigma_j = 1, a_1 = 0) = \frac{(1-z)(1-h_j)}{zh_j + (1-z)(1-h_j)},$$

$$Prob3(s_1 = 0|\sigma_j = 0, a_1 = 1) = Prob3(s_1 = 0|\sigma_j = 0, a_1 = 0) = \frac{(1-z)h_j}{(1-z)h_j + z(1-h_j)}.$$

B.5 Price Formation Process

For ease of exposition, we assume that all investors are homogeneous concerning the quality of their signals σ_j, i.e. we assume that $h_j = h \, \forall j \in \{1, ...N\}$.[38] Thus, investors only differ as to whether they receive signal $\sigma_j = 1$ or $\sigma_j = 0$. When the number of investors is sufficiently large a fraction h of the investors will receive the correct signal, i.e. they receive $\sigma_j = 1$ if $s_1 = 1$ or $\sigma_j = 0$ if $s_1 = 0$, respectively.[39] A fraction $1 - h$ will receive a misleading signal, i.e. they receive $\sigma_j = 1$ if $s_1 = 0$ or $\sigma_j = 0$ if $s_1 = 1$.

From Corollary 5.3 in Appendix C we know that the price in the information market will be a weighted average of the prices that would arise in the two subgroups of investors. This means that the price will be h times the price that would arise in a market where all investors receive a correct signal plus (1-h) times the price in a market where investors only receive incorrect signals. Again, we go through all three cases.

Case $c = 1$

We start with case $c = 1$. In this scenario, the action of the incumbent will perfectly reveal the state of the world. Thus, we obtain

$$p^{*1}_{1,1} = p^{*1}_{0,0} = 1 \tag{5.25}$$

and

$$p^{*1}_{1,0} = p^{*1}_{0,1} = 0, \tag{5.26}$$

[38] Further, we assume that investors are homogeneous concerning their wealth and their subjective confidence in their own signals. In Appendix C we will derive some general results for heterogeneous investors. Using the notation of Appendix C we assume $W_j = W \, \forall j$ and $b_j = b \, \forall j$ in this section. At the cost of additional notational complexity, the results can be extended to heterogeneous investors by using the formulas derived in Appendix C.

[39] For a finite number of investors the variance of the fraction of investors receiving the correct signal is not zero. However, for a sufficiently large number of investors the variance becomes arbitrarily small. For instance, for $N = 10000$ the probability that the share of investors with a correct signal is in [0.89, 0.91] for $h = 0.9$ is larger than 99.9%.

where $p^{*c}_{a_1,s_1}$ denotes the equilibrium price in case c given action a_1 and state s_1. The equilibrium price will equal 1 if the incumbent chooses the socially optimal action, while the price will be 0 if the politician chooses the non-optimal action.
Case $c = 2$
If $c = 2$, we obtain

$$p^{*2}_{0,0} = 1 \tag{5.27}$$

and

$$p^{*2}_{0,1} = 0, \tag{5.28}$$

which reflects the fact that the equilibrium price will be equal to zero or one upon observing $a_1 = 0$, as this action reveals the true state of the world with certainty. If the incumbent undertakes $a_1 = 1$ in case $c = 2$, then we obtain

$$p^{*2}_{1,1} = 1 - \frac{(1-z^2)h(1-h)}{[2zh + (1-z)(1-h)][2z(1-h) + (1-z)h]} \tag{5.29}$$

and

$$p^{*2}_{1,0} = \frac{2z(1+z)h(1-h)}{[2zh + (1-z)(1-h)][2z(1-h) + (1-z)h]}. \tag{5.30}$$

Case $c = 3$
If $c = 3$, we obtain

$$p^{*3}_{1,1} = 1 - \frac{(1-z)h(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]}, \tag{5.31}$$

$$p^{*3}_{1,0} = \frac{zh(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]}, \tag{5.32}$$

$$p^{*3}_{0,0} = 1 - \frac{zh(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]}, \tag{5.33}$$

$$p^{*3}_{0,1} = \frac{(1-z)h(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]}. \tag{5.34}$$

We observe that $p^{*3}_{1,1} = 1 - p^{*3}_{0,1}$ and $p^{*3}_{0,0} = 1 - p^{*3}_{1,0}$. The next proposition is the main result of Appendix B and the extended version of Proposition 5.5 in the text.

Proposition 5.3 (detailed version)
Suppose that $h > \hat{h}(z)$ *with*

$$\hat{h}(z) = \frac{1 + \sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2} < 1. \tag{5.35}$$

Then the equilibrium price in the information market fulfills the following conditions:

$$p^{*c}_{1,1} > \frac{1}{2} \; \forall c,$$

$$p^{*c}_{0,0} > \frac{1}{2} \; \forall c,$$

$$p^{*c}_{1,0} < \frac{1}{2} \; \forall c$$

and

$$p^{*c}_{0,1} < \frac{1}{2} \; \forall c.$$

Proposition 5.3 shows that for $h > \hat{h}(z)$, the equilibrium price will be larger than one-half in all circumstances, if the incumbent behaves congruently, while the equilibrium price will be smaller than one-half if the politician behaves dissonantly. Note that for $z \in (\frac{1}{2}, 1)$, $\hat{h}(z)$ is increasing in z and that $\hat{h}(z) \in (\frac{1}{2} + \sqrt{\frac{3}{44}}, 1)$. The intuition that $\hat{h}(z)$ must be larger than $\frac{1}{2}$ runs as follows: In the unpopular state $s_1 = 0$ in case $c = 2$, where $Prob(s_1 = 0|a_1 = 1)$ is rather low, the signal must be sufficiently informative in order to detect dissonant behavior of a politician. A formal derivation and explanation for Condition (5.35) is given in the following proof of Proposition 5.3.

Proof of Proposition 5.3 (detailed version)

We will prove the statement in three steps:

Step 1: First, it is obvious that $p^{*1}_{1,1} > \frac{1}{2}, p^{*1}_{0,0} > \frac{1}{2}, p^{*1}_{1,0} < \frac{1}{2}, p^{*1}_{0,1} < \frac{1}{2}, p^{*2}_{0,0} > \frac{1}{2}$ and $p^{*2}_{0,1} < \frac{1}{2}$ for any values of $h \in (\frac{1}{2}, 1)$.

Step 2: The condition $p^{*2}_{1,1} > \frac{1}{2}$ is equivalent to

$$\frac{(1-z^2)h(1-h)}{[2zh + (1-z)(1-h)][2z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{5.36}$$

Some manipulations yield the condition

$$2z(1-z) + h(1-h)(11z^2 - 6z - 1) > 0. \tag{5.37}$$

We note that $h(1-h) < \frac{1}{4} \, \forall h \in (\frac{1}{2}, 1)$ and that $2z(1-z) > -\frac{1}{4}(11z^2 - 6z - 1) \, \forall z \in (\frac{1}{2}, 1)$. Thus, Condition (5.37) is always fulfilled.

Next we examine $p^{*2}_{1,0} < \frac{1}{2}$, which is equivalent to

$$\frac{2z(1+z)h(1-h)}{[2zh + (1-z)(1-h)][2z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{5.38}$$

Rearranging terms yields

$$h(1-h) < \frac{2z(1-z)}{-5z^2 + 10z - 1}. \tag{5.39}$$

Solving for h leads to

$$h > \frac{1 + \sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2}. \tag{5.40}$$

Step 3: The next condition $p^{*3}_{1,1} > \frac{1}{2}$ is equivalent to

$$\frac{(1-z)h(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{5.41}$$

This condition can be transformed to

$$z(1-z) + h(1-h)(4z^2 - 2z - 1) > 0. \tag{5.42}$$

We note that $h(1-h) < \frac{1}{4}\ \forall h \in (\frac{1}{2}, 1)$ and that $z(1-z) > -\frac{1}{4}(4z^2 - 2z - 1)\ \forall z \in (\frac{1}{2}, 1)$. Thus, Condition (5.42) is always fulfilled.

The condition $p^{*3}_{1,0} < \frac{1}{2}$ is equivalent to

$$\frac{zh(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{5.43}$$

After some manipulations, we obtain

$$h(1-h) < \frac{z(1-z)}{-4z^2 + 6z - 1}, \tag{5.44}$$

which then yields

$$h > \frac{1 + \sqrt{\frac{2z-1}{-4z^2+6z-1}}}{2}. \tag{5.45}$$

Condition (5.45) is a weaker condition than Condition (5.40) as the following inequality holds for all $z \in (\frac{1}{2}, 1)$:

$$\sqrt{\frac{2z-1}{-4z^2+6z-1}} < \sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}. \tag{5.46}$$

Hence, if $h > \frac{1+\sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2}$, then Condition (5.45) will always be fulfilled.

Next we investigate $p^{*3}_{0,0} > \frac{1}{2}$, which leads to

$$\frac{zh(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]} < \frac{1}{2}. \tag{5.47}$$

This condition is the same as (5.43) and thus always fulfilled for $h > \frac{1+\sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2}$.

Finally, we consider $p^{*3}_{0,1} < \frac{1}{2}$, which yields

$$\frac{(1-z)h(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]} < \frac{1}{2}. \tag{5.48}$$

This is identical to Condition (5.41), which always holds as shown above. □

B.6 Sophisticated Election Scheme

In this subsection we prove Proposition 5.7 by proving the following detailed version of Proposition 5.7:

Proposition 5.7 (detailed version)
Suppose that $h > \hat{\hat{h}}$ with

$$\hat{\hat{h}} = \frac{1+\sqrt{\frac{z+1}{9z+1}}}{2} < \frac{1}{2} + \sqrt{\frac{3}{44}} \approx 0.761. \tag{5.49}$$

Then the equilibrium price in the information market fulfills the following conditions:

$$p^{*c}_{1,1} > z \ \forall c,$$

$$p^{*c}_{0,0} > 1-z \ \forall c,$$

$$p^{*c}_{1,0} < z \ \forall c$$

and

$$p^{*c}_{0,1} < 1-z \ \forall c.$$

Proof of Proposition 5.7

The proof follows the same line as the proof of Proposition 5.3.

Step 1: It is obvious that $p^{*1}_{1,1} > z$, $p^{*1}_{0,0} > 1-z$, $p^{*1}_{1,0} < z$, $p^{*1}_{0,1} < 1-z$, $p^{*2}_{0,0} > 1-z$ and $p^{*2}_{0,1} < 1-z$.

Step 2: We explore the condition $p^{*2}_{1,1} > z$, which is equivalent to

$$\frac{(1-z^2)h(1-h)}{[2zh+(1-z)(1-h)][2z(1-h)+(1-z)h]} < 1-z. \tag{5.50}$$

This can be rewritten into

$$2(1-z)^2 + h(1-h)(-9z^2 + 16z - 7) > 0. \tag{5.51}$$

Using $h(1-h) < \frac{1}{4}\,\forall h \in (\frac{1}{2}, 1)$ and $2(1-z)^2 > \frac{1}{4}(9z^2 - 16z + 7)\,\forall z \in (\frac{1}{2}, 1)$ shows that Condition (5.51) is fulfilled for all $z \in \{\frac{1}{2}, 1\}$.

Next we examine $p^{*2}_{1,0} < z$, which yields

$$\frac{2(1+z)h(1-h)}{[2zh + (1-z)(1-h)][2z(1-h) + (1-z)h]} < 1. \tag{5.52}$$

Rearranging terms leads to

$$h(1-h) < \frac{2z(1-z)}{-9z^2 + 8z + 1} = \frac{2z}{9z+1}, \tag{5.53}$$

which implies

$$h > \frac{1 + \sqrt{\frac{z+1}{9z+1}}}{2}. \tag{5.54}$$

Step 3: Finally, conditions $p^{*3}_{1,1} > z, p^{*3}_{1,0} < z, p^{*3}_{0,0} > 1-z$ and $p^{*3}_{0,1} < 1-z$ are equivalent to

$$\frac{h(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]} < 1, \tag{5.55}$$

which in turn holds if and only if

$$h(1-h) < \frac{1}{4}. \tag{5.56}$$

As $h \geq \frac{1}{2}$ Condition (5.56) is always fulfilled and the assertion is proven. □

By comparing $\hat{h}$ and $\hat{\hat{h}}$ we obtain the following corollary:

Corollary 5.2

For all z with $\frac{1}{2} < z < 1$, $\hat{\hat{h}} < \hat{h}$.

Hence, for all values $z \in (\frac{1}{2}, 1)$ Condition (5.49) is easier to fulfill than Condition (5.35).[40] As a consequence, *SES*, which uses the results from Proposition 5.7, is applicable for signals with lower information content than *RES*. Note that Corollary 5.2 follows directly from comparing $\hat{h}$ and $\hat{\hat{h}}$. The claim $\hat{\hat{h}} < \hat{h}$ can be transformed to $2z^2 + z > 1$, which proves the corollary.

[40]For $z = \frac{1}{2}$ Eq. (5.35) would be identical to Condition (5.49).

Appendix C: General Price Formation Process

In this appendix we determine a general formula for an information market with heterogeneous agents. Suppose, without loss of generality, that politician 1 has been elected after offering a contract $C_1(p_1^1, p_1^0)$, that the politician undertakes $a_1 = 1$, and hence that p_1^1 applies.

For a price $p < p_1^1$, no investor will have a strict incentive to buy assets, as he will be paid back p. Thus, suppose $p \geq p_1^1$. An investor j with signal σ_j has to weigh up the state of his information and the information the market price will reveal.[41] One way of modeling the information aggregation process is as follows:

$$Prob_j(RE|p) = b_j\, Prob_j(RE) + (1 - b_j)\, p, \tag{5.57}$$

where $Prob_j(RE|p)$ is the probability assessment of investor j that the incumbent will be reelected, taking into account the information inferred from the market price. The term $Prob_j(RE)$ is given as the individual reelection probability estimation of an investor and depends on his signal σ_j, the signal quality h_j, the action a_1, and the case c. If, e.g., $c = 3$, $a_1 = 1$, and $\sigma_j = 1$, then $Prob_j(RE) = \frac{zh_j}{zh_j+(1-z)(1-h_j)}$, where we assume that z and h_j are known to investor j. The weight b_j (with $0 < b_j \leq 1$) describes self-assessed confidence, i.e. the subjective confidence of an investor in his estimation $Prob_j(RE)$ relative to the market belief expressed by the price p.[42] The information aggregation formula (5.57) is flexible. It captures the case $b_j = 1$ when investors rely only on their own signal, which would occur if they can only submit a quantity (and not an entire demand/supply schedule depending on the price) to the market. For small values of b_j, investors rely mainly on the information aggregated by the market.[43]

Given price p and signal $Prob_j(RE)$, an investor j aims to maximize his expected utility, which can be written as the following optimization problem:

$$\max_{d_j} EU_j = Prob_j(RE|p)\, ln(W_j + d_j(1-p)) + (1 - Prob_j(RE|p))\, ln(W_j - d_j p), \tag{5.58}$$

where d_j denotes his demand. If d_j is positive, investor j will want to buy d_j units of asset D. If d_j is negative, investor j will want to buy d_j units of asset E. The solution of the investor's optimization problem is

[41] Note that investors learn nothing from the threshold contract offers of the candidates because in equilibrium both types of politicians will offer the same contract, as we will show later.

[42] For a statistical foundation, see Morris (1983) and Rosenblueth and Ordaz (1992). Wolfers and Zitzewitz (2006) have independently suggested a similar procedure.

[43] Note that it can never be rational to set $b_j = 0 \quad \forall j$ as the price would contain no information contradicting the assumption of investors to rely only on the information inferred from the market price. This is the information paradox addressed by Grossman and Stiglitz (1980).

$$d_j^* = W_j \, \frac{b_j \, Prob_j(RE) + (1 - b_j)p - p}{p(1-p)}$$
$$\Leftrightarrow \quad d_j^* = W_j \, \frac{b_j \, Prob_j(RE) - p \, b_j}{p(1-p)}. \tag{5.59}$$

We thus obtain:

Proposition 5.8
There is a unique equilibrium in the information market given by

$$p^* = \sum_{j=1}^{N} Prob_j(RE) \, \frac{W_j \, b_j}{\sum_{k=1}^{N} W_k \, b_k} \, . \tag{5.60}$$

Proof of Proposition 5.8

Equilibrium in the information market requires that condition $\sum\limits_{j=1}^{N} d_j^* = 0$ be fulfilled, which implies $\sum\limits_{j=1}^{N} W_j \, b_j \, Prob_j(RE) \; - p \sum\limits_{j=1}^{N} W_j \, b_j \; = 0$. The assertion follows from this equation. □

The market price is a wealth- and confidence-weighted average belief on the part of investors. We note that the market price is equal to the simple average belief of investors if traders are homogeneous with respect to wealth and confidence in their own belief. If confidence levels are homogeneous, the market price is a wealth-weighted average belief on the part of traders. We summarize both cases in the following corollary:

Corollary 5.3

(i) Suppose $W_j = W \;\; \forall j$ *and* $b_j = b \;\; \forall j$. *Then* $p^* = \frac{1}{N} \sum\limits_{j=1}^{N} Prob_j(RE)$.

(ii) Suppose $b_j = b \;\; \forall j$. *Then* $p^* = \sum\limits_{j=1}^{N} Prob_j(RE) \frac{W_j}{\sum_{k=1}^{N} W_k}$.

Appendix D: Welfare Gains

Here we provide an example of the welfare gains that can be achieved with the triple mechanism. Suppose that, at a time when this institution is introduced, it is only known that δ is equal to 1 and that μ is uniformly distributed in $[0, \frac{1}{2}]$. Since only the proportion of R and G is important for our analysis, we write $G = \alpha R$ with $0 \leq \alpha < \infty$. In the following, we calculate the values of μ that enable congruent behavior by the incumbent. We use *eo* to denote the case with elections only and *tm*

Table 5.3 Conditions—behaviour of politicians

	Congruent politician		Dissonant politician	
	$s_1 = 1$	$s_1 = 0$	$s_1 = 1$	$s_1 = 0$
Elections only	$\alpha \geq -2$	$\mu \geq \frac{1-\alpha}{3}$	$\alpha \leq 2$	$\mu \geq \frac{1+\alpha}{3}$
Triple mechanism	$\alpha \geq -2$	$\mu \geq -\frac{\alpha}{2}$	$\alpha \leq 2$	$\mu \geq \frac{\alpha}{2}$

to denote the scenario with the triple mechanism. From Condition (5.6) we conclude that, in the case of elections alone, a congruent politician will only behave congruently in state $s_1 = 1$ if

$$\alpha R + 3R \geq R.$$

This condition is equivalent to $\alpha \geq -2$. In the same way we obtain the other conditions summarized in Table 5.3.

Note that congruent politicians will always behave congruently in the scenario with the triple mechanism, as conditions $\alpha \geq -2$ and $\mu \geq -\frac{\alpha}{2}$ are always fulfilled. Furthermore, if $\alpha \geq 1$ congruent politicians will always behave congruently in the scenario with elections only. Finally, it is apparent that a dissonant politician will never act congruently for $\alpha > 2$, which clearly derives from Corollary 5.1 and Theorem 5.1. In the next stage, we calculate expected utilities, starting with the triple mechanism scenario:

$$EU^{tm} = \frac{1}{2} + \frac{1}{2}z \begin{cases} \int_0^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 2 \\ 0 & \text{if } \alpha > 2 \end{cases} + \frac{1}{2}(1-z) \begin{cases} \int_{\frac{\alpha}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 1, \\ 0 & \text{if } \alpha > 1. \end{cases}$$

The reasoning for the above expression is as follows: A politician is of the congruent type with probability $\frac{1}{2}$. He always behaves congruently and thus generates a voter utility of 1. The probability that a politician is of the dissonant type and that state $s_1 = 1$ occurs is given by $\frac{1}{2}z$. In this case, the politician generates a utility of 1 for all feasible values of μ, as long as α is not larger than 2. Finally, the probability that a politician is of the dissonant type and that state $s_1 = 0$ occurs is given by $\frac{1}{2}(1 - z)$. In this case, the politician generates a utility of 1 for all values of μ with $\mu \geq \frac{\alpha}{2}$, as long as α is not larger than 1.[44] The calculation in the scenario with elections alone is similar and yields

[44]Note that we have assumed that μ is uniformly distributed in $[0, \frac{1}{2}]$.

$$EU^{eo} = \frac{1}{2}z + \frac{1}{2}(1-z)\begin{cases} \int_{\frac{1-\alpha}{3}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 1 \\ \int_{0}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 1 \end{cases}$$

$$+\frac{1}{2}z\begin{cases} \int_{0}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 2 \\ 0 & \text{if } \alpha > 2 \end{cases} + \frac{1}{2}(1-z)\begin{cases} \int_{\frac{1+\alpha}{3}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq \frac{1}{2} \\ 0 & \text{if } \alpha > \frac{1}{2}. \end{cases}$$

These expressions can be simplified to

$$EU^{tm} = \begin{cases} \frac{1}{2} + \frac{1}{2}[1-\alpha(1-z)] & \text{if } \alpha \leq 1, \\ \frac{1}{2} + \frac{1}{2}z & \text{if } 1 < \alpha \leq 2, \\ \frac{1}{2} & \text{if } \alpha > 2 \end{cases} \tag{5.61}$$

and

$$EU^{eo} = \begin{cases} z + \dfrac{1}{3}(1-z) & \text{if } \alpha \leq \frac{1}{2}, \\ z + (1-z)\dfrac{(1+2\alpha)}{6} & \text{if } \frac{1}{2} < \alpha \leq 1, \\ \frac{1}{2} + \frac{1}{2}z & \text{if } 1 < \alpha \leq 2, \\ \frac{1}{2} & \text{if } \alpha > 2. \end{cases} \tag{5.62}$$

We illustrate the relationships by calculating the utilities for four different values of α. We choose one value of α that is smaller than 1, one value larger than 1, and α equal to 1. These values correspond to the cases where, for the politician, utility G is lower/higher than or equal to utility R. Furthermore, we add the special case $\alpha = 0$, where the politician has no private benefits G. The expected utilities in these four cases are summarized in the following Table 5.4:

Table 5.4 Expected utilities

	$\alpha = 3$	$\alpha = 1$	$\alpha = 0.1$	$\alpha = 0$
EU^{eo}	$\frac{1}{2}$	$\frac{1+z}{2}$	$\frac{1+2z}{3}$	$\frac{1+2z}{3}$
EU^{tm}	$\frac{1}{2}$	$\frac{1+z}{2}$	$\frac{19+z}{20}$	1
$EU^{tm} - EU^{eo}$	0	0	$\frac{37(1-z)}{60}$	$\frac{2(1-z)}{3}$
$\Delta_{EU} = \frac{EU^{tm} - EU^{eo}}{EU^{eo}}$	0	0	$\frac{37(1-z)}{20+40z}$	$\frac{2(1-z)}{1+2z}$

Note that in all cases we have $EU^{tm} \geq EU^{eo}$. Further, we see that EU^{tm} is strictly larger than EU^{eo} if $z < 1$ and $\alpha < 1$. The difference between EU^{tm} and EU^{eo} depends on z for $0 < \alpha < 1$. The last row in the table shows the relative welfare gains (Δ_{EU}). Δ_{EU} is maximal for $\alpha = 0$. The example illustrates the following insights:

(i) Threshold contracts have the highest effect in the case $\alpha = 0$, i.e. if the politicians are only motivated by benefits R acquired from holding office and not from choosing his personally preferred action. Note that threshold contracts may reduce the reelection chances of the incumbent. Thus, threshold contracts will be more effective if politicians are mainly interested in getting reelected, which is expressed in a low value of α.

(ii) If α is at least equal to 1, i.e. if politicians are at least as motivated by G as by R, then there is no effect from threshold contracts. This is due to the fact that in state $s_1 = 0$ congruent politicians always behave congruently, while dissonant candidates always behave dissonantly. The conditions for congruent behavior in state $s_1 = 1$ are the same in the scenarios with or without threshold contracts. If α is at least equal to 2, then congruent politicians will always behave congruently, while dissonant candidates will always behave dissonantly. Thus, the expected utility is equal to $\frac{1}{2}$.

(iii) Finally, for a given value of α we discover that Δ_{EU} is (weakly) decreasing in z. Thus, the higher the probability of the unpopular state $s_1 = 0$, the larger is the effect of threshold contracts.

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Chapter 6
Limits of Contractual Democracy – Competition for Wages and Office

6.1 Background

Our research on wage competition dates back to 2000: We started from the observation that office-holders' wages do not depend on their performance. As the improvement of the office-holders' efficiency and the selection of competent office-holders are main goals of our research, we wanted to try to use salaries as an incentive for good performance. We developed a model in which an office-holder who performs badly incurs a salary reduction, as developed in Chap. 2 in the context of long-term projects. As a special feature of our model, this reduction was not meant to be *imposed* on the office-holders, but to be *offered* by the candidates during their campaign.

As this reward/retribution system is based on the concept of a *flexible salary*, we tried to widen the application range of this idea, and found that wage flexibility might be put to good use not only *after* performance, but also *before* elections—as a selection tool. We had assessed that such flexibility might yield better results and could be less difficult to implement if the candidates offer it *themselves*. We wanted to see what happens if candidates are allowed to determine their future wages themselves too, as part of the service-package they *offer* in exchange for election.

Our goal was to compare a situation in which an office-holder's wages are set by the public to a setting where competing candidates for office are allowed make a contractual "wage-offer", i.e. to determine the salary they demand in exchange for their services. At first sight, this might seem beneficial with regard to costs, as each candidate should try to be a better bargain than the other. Yet, a very able candidate might be able to advertise his high ability, ask for higher wages—and still be elected, despite the costs of hiring him. On the flip side, the public might also choose a candidate asking for a lower salary—and hire a less able office-holder to save money.

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Again, as we have seen in Chaps. 3 and 4 in a different setting, a candidate's ability and the extent of his knowledge about this ability play an essential part in electoral competition.[1]

6.2 Introduction

In a simple model, we examine two institutional frameworks in which pay rates of politicians are determined. In the first, politicians face a given remuneration schedule, which is determined by law before they run for office. In the second framework, politicians have a major impact on their own remuneration. For instance, most parliaments design the laws that stipulate compensation for their members.[2]

We compare both these institutional settings determining the remuneration of politicians. We consider a citizen-as-candidate model, where an elected politician undertakes policy projects for a society. Candidates may differ in competence, and wages for politicians are financed by taxes. Our main insights are as follows: First, as a rule, the competence of elected candidates is equal or higher when the public determines wages optimally as opposed to remuneration being self-designed by candidates. Second, in the case of competitive wage offers by candidates, social welfare is usually lower than in the case of predetermined remuneration. The intuition for this result is as follows: Since taxation is distortionary, higher wages impose economic costs on the electorate. On the other hand, higher wages may prompt the more competent politicians (as well as less competent ones) to run for office, which generates economic benefits as voters can elect the more competent candidate for office. The two wage schemes-publicly determined wages and competitive wages—solve this trade-off differently. Competition bids up wages beyond the level required for an efficient selection of politicians. The more competent candidate is the residual claimant as he can ask for wages that make voters indifferent between both candidates. The more competent candidate—who knows that he produces a larger surplus—proposes a wage that allows him to capture all the extra surplus he generates. If wages are set by the electorate, then the wage must be just high enough to induce the better candidate to run for office, thus ensuring that the extra surplus that candidate generates goes to the voters. In this case voters are the residual claimants. Since wages are financed by distortionary taxes, welfare is higher with predetermined wages.

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[1] This chapter is an updated version of the article "Competition of Politicians for Wages and Office" published in *Social Choice and Welfare 32(4)*, in 2009.

[2] How self-designed remuneration packages influence politicians' decisions on whether to run has been demonstrated in a wealth of research. For instance, Hall and van Houwelling (1995) analyze the impact of a 1990 law that significantly increased pensions for US-congressmen who retired after 1992. They find that a significant number of congressmen who otherwise would have retired in 1990 decided to re-run for office in order to receive this financial windfall. Groseclose and Krehbiel (1994), Diermeier et al. (2005), and Besley (2004) also identify the importance of financial considerations for politicians when they run for office.

The current analysis draws on four strands in the literature. First, there exist a number of recent papers that discuss how the value of office affects the quality of politicians and their incentives to pursue socially efficient policies. Besley (2004) examines how paying politicians can solve the agency problems of incumbents who are subject to a two-period term limit. Caselli and Morelli (2004) examine how the quality of elected politicians is affected by the value of office when candidates know in advance whether they can convince the electorate of their abilities. Messner and Polborn (2003) develop a new type of citizen-candidate model by assuming that the abilities of candidates are observable to voters, whereas their opportunity costs are private information. Poutvaara and Takalo (2007) develop a tractable citizen-candidate model that allows for unobserved ability differences, informative campaigning, and political parties. These recent advances in modeling representative democracies illustrate that increasing the value of office does not necessarily increase the average quality of candidates. None of the preceding papers, however, focuses on the comparison between remuneration set by the public and self-designed wages as attempted in this chapter.

Second, incentive elements in politics, other than elections, have been examined in a series of papers, starting with Gersbach (2003) and surveyed in Gersbach (2012) as well as in the first chapter of this book. With incentive contracts, the value of holding office in the second term is made dependent on the realization of macroeconomic variables. This increases the incentive for politicians to undertake socially desirable policies with long-term consequences in the first term. Politicians are allowed to offer their own long-term wage contracts during campaigns. In this chapter, by contrast, we consider the competition of politicians for wages and office in a single term in the context of a citizen-as-candidate set-up. While the above literature shows that competition with long-term wage contracts between politicians is welfare-improving, the results in this chapter show that this may not hold when politicians only make wage offers for a single term. Indeed, we show that politicians should not be allowed to offer their own remuneration schemes for the next term.

Third, candidates holding office will provide a public good, so we may face the standard free-riding and underprovision problem when public goods are privately supplied by a set of actors. This problem is discussed e.g. in Palfrey and Rosenthal (1984), Bergstrom et al. (1986), Güth and Hellwig (1986), and recently Hellwig (2003). In our model, the interaction of the entry decisions of two actors is the only factor determining the level of a public good. Hence, for this simple public-good problem the public can overcome the underprovision problem by setting wages or by allowing politicians to offer wage schemes.

Fourth, we use a simplified version of the citizen-as-candidate model, as developed by Osborne and Slivinski (1996) and Besley and Coate (1997). In such settings, citizens who consider running for office must take into account the private costs incurred by running for office, benefits from policies they would like to undertake, and benefits from policies other potential candidates are likely to implement.

This chapter is organized as follows: In the next section we introduce the model. We then examine fixed wages set by the public. In Sect. 6.5 we identify equilibria in cases where politicians can propose their remuneration themselves. Section 6.6

contains the welfare comparison. In Sect. 6.7 we discuss the importance of our assumptions and several possible extensions of our model. Section 6.8 concludes.

6.3 The Model

6.3.1 The Set-Up

We consider a society with N voters who have to elect a politician undertaking policy projects for all members of the group. There are two potential candidates, $i = 1, 2$, for this job. The remaining $N - 2$ individuals cannot be candidates and only act as voters.[3] Candidates differ in their competence: with his policies candidate i ($i = 1, 2$) can generate a net benefit $b_i > 0$ for every member of the society. We label candidates in such a way that $b_1 > b_2$. The value of b_i can be associated with the competency of candidates.

For each candidate i, there is an individual cost c_i incurred by serving in office. This cost includes effort, opportunity costs and any individual gains from being in office. If the latter source of utility is dominating, we have $c_i < 0$. The parameters c_i are assumed to be perfectly observable by the voters. The elected politician receives a wage that is financed by distortionary taxation, which is levied on all other members of the society.[4] Let $\lambda \geq 0$ denote the shadow cost of public funds. That is, taxation uses $(1 + \lambda)$ of tax payers' resources in order to levy 1 unit of resources for paying wages to candidates in office. The utility of candidate i if he is elected and earns the wage W is

$$b_i + W - c_i, \tag{6.1}$$

while the utility of any other member of the society is

$$b_i - \frac{W(1 + \lambda)}{N - 1}. \tag{6.2}$$

If no potential candidate is willing to run, then a default policy will be implemented yielding a benefit of $b_0 = 0$ for every voter. If only one candidate runs for office, then he will automatically assume power.

[3]We assume that N is greater than 4, i.e. there are more voters not seeking office than there are candidates.

[4]In principle, our model allows for negative wages when candidates are highly interested in power and bid for office. In such cases, shadow costs of public funds should be set at zero.

6.3.2 Assumptions and Economic Problem

We compare two institutional systems of determining wages for elected politicians: remunerations are either set by the public or are offered competitively by the candidates during campaigning. We make two types of assumptions.

The first assumption defines the economic problem. We assume complete information about benefits and costs that candidates produce when in office. In addition however, we assume that the competency of candidates, i.e. the values of b_i, are observable by the public but not verifiable in a court. Hence, the public cannot make individualized wage offers. The rationale for this assumption is discussed extensively in the incomplete-contract literature (see Hart 1995 or Watson 2007). For instance, abilities of candidates may become known to other agents, but it is impossible to prove in a court that one individual has greater competence than another for undertaking future tasks in a public office.

This assumption introduces the following trade-off: When candidates themselves offer wages, they can offer different remunerations reflecting their interests. However, candidates do not care about social welfare as such. The public, by contrast, is concerned about social welfare but cannot offer different wages to the candidates and thus cannot replicate the outcome itself under competitive wage offers. If the public were offering different wages, the candidate with a lower wage could go to court claiming that he has the same level of competency and would win because of the verification problem discussed above.[5]

The second set of assumptions is made for tractability. In particular, we assume zero cost for running as candidate, status-quo utility of zero when no candidate is running for office, observable utility from holding office expressed by c_i, two candidates, and linear dead weight costs λ. In Sect. 6.7 on robustness, we discuss the importance of these assumptions for our results.

6.3.3 The Institutions

Here we outline the timing for both scenarios. In the first scenario, we discuss how voters would determine the wages for politicians. The timing in the first scenario is as follows:

Stage 1: Voters decide on the level of the politician's wage denoted by W.
Stage 2: The candidates decide simultaneously whether to run for office or not.
Stage 3: The voters elect one of the two candidates.

It is obvious in this first scenario that, if both candidates run for office, it is always optimal for the voters to elect candidate 1, because $b_1 > b_2$ and the wages for both

[5]Note that public law in modern democracies prohibits different wage settings for public office without verifiable evidence.

candidates are identical. Note that we assume complete information. That is, voters observe the parameters $\{b_1, b_2, c_1, c_2\}$ before they set their wages.

In the second scenario, candidates themselves can offer wages, denoted by W_1 and W_2, which become effective if a candidate runs and is elected. Therefore, in the second scenario, the first two stages are replaced by:

Stage 1′: Candidates offer W_1 and W_2.

Note that it is always possible for a candidate to propose a salary so large that he will never get elected. Therefore we do not explicitly model a stage where candidates decide whether to run or not in the second scenario. Throughout the chapter, we use the weak dominance concept in the following way: In every possible voting game in the first or second scenario, voters are assumed to employ only strategies that are weakly undominated in that subgame. If we include tie-breaking rules when voters are indifferent, this refinement produces unique voting outcomes for every subgame. Given the equilibrium voting behavior, we look at running equilibria (first scenario) or wage offer equilibria (second scenario) of candidates, where we again eliminate weakly dominated strategies, if they exist.

6.4 Fixed Wages

We first consider fixed wages set by the public and obtain our first result.

Proposition 6.1 *There exists an equilibrium for stages* 2 *and* 3 *that depends on the wage level in the following way:*

(i) *If* $W \geq c_2 - b_2$ *and*

$$W \geq \frac{N-1}{N+\lambda}\big(c_1 - (b_1 - b_2)\big), \tag{6.3}$$

then both candidates run for office and candidate 1 *is elected.*

(ii) *If* $W \geq c_2 - b_2$ *and*

$$W < \frac{N-1}{N+\lambda}\big(c_1 - (b_1 - b_2)\big), \tag{6.4}$$

then candidate 2 *runs for office and is elected.*

(iii) *If* $W < c_2 - b_2$ *and* $W \geq c_1 - b_1$*, then candidate* 2 *does not run for office. Candidate* 1 *runs for office and is elected.*

(iv) *If* $W < c_2 - b_2$ *and* $W < c_1 - b_1$*, no candidate runs for office.*

The proof of Proposition 6.1 is given in the Appendix. Proposition 6.1 indicates the considerations the public has to weigh up in determining optimal wages. A higher wage may prompt the more competent candidate to run for office. Higher wages, however, will also attract the bad candidate. Nevertheless, as the more competent candidate will be elected if he runs for office, the public can always ensure that the more competent candidate will take office by specifying an appropriate wage. As higher wages imply more deadweight costs, the public has to trade off competency of office-holders against the deadweight costs of financing the remuneration of politicians. We will later determine the optimal wage levels the public should set for the political race.

6.5 Competition for Wage Contracts

In this section we explore what happens if candidates can offer to perform political duties for a certain wage. After the candidates have proposed their remuneration scheme, the voters elect the candidate whom they believe will create the highest utility for them. Thus, the timing is as follows:

Stage 1′: Each candidate proposes a remuneration scheme W_i.
Stage 2: The voters observe W_1 and W_2 and elect one of the two candidates.

We first observe from Eqs. 6.1 and 6.2, that candidate 1 is elected if [6]

$$b_1 - \frac{W_1}{N-1}(1+\lambda) \geq b_2 - \frac{W_2}{N-1}(1+\lambda), \tag{6.5}$$

$$b_1 - \frac{W_1}{N-1}(1+\lambda) \geq 0.$$

Note that the latter constraint guarantees that voters are better off with the first politician than without any politician and sticking to the status quo, which would produce zero utility. In Propositions 6.2 and 6.3, we will state suitable and mild conditions to ensure that this assumption is not binding. First, we look at conditions for which equilibria exist in which candidate 1 is elected.

Proposition 6.2 *Suppose* $(1+\lambda)(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2)$. *Then, candidate* 1 *is elected in every equilibrium and wage offers satisfy*

$$W_1 = (b_1 - b_2)\frac{N-1}{1+\lambda} + W_2.$$

Wages are indeterminate. In particular, there exists an equilibrium in which candidate 1 *is elected with minimal wages* W_1^{min} *and* W_2^{min} *given by*

[6] For convenience, we use a tie-breaking rule in favor of candidate 1 if voters are indifferent between candidates. Otherwise we would need to work with ε considerations.

$$W_1^{min} = \frac{N-1}{N+\lambda} c_1,$$
$$W_2^{min} = \frac{N-1}{N+\lambda} c_1 - (b_1 - b_2)\frac{N-1}{1+\lambda}.$$

There also exists an equilibrium in which candidate 1 *is elected with maximal wages* W_1^{max} *and* W_2^{max} *given by* [7]

$$W_1^{max} = (b_1 - b_2)\frac{N-1}{1+\lambda} + \frac{N-1}{N+\lambda} c_2.$$
$$W_2^{max} = \frac{N-1}{N+\lambda} c_2.$$

The proof is given in the Appendix. An important consequence of Proposition 6.2 is that wages are indeterminate, i.e., that there are infinitely many combinations of pairs (W_1, W_2) that can constitute an equilibrium.

The reason for the multiplicity of equilibria can be summarized as follows: Within the range $[W_1^{\min}, W_1^{\max}]$, candidate 2 is either better off when candidate 1 is elected, or he has no chance of winning the election if he proposes a high wage of W_2. Which candidate is elected depends solely on the wage difference $W_1 - W_2$. Hence there is no anchor for wage W_2, which causes the indeterminacy.

Candidate 2 and all voters will strictly prefer the equilibrium associated with $[W_1^{\min}, W_2^{\min}]$ over all other equilibrium wage combinations. Candidate 1, however, benefits most if $[W_1^{\max}, W_2^{\max}]$ is realized. Hence simple refinement criteria, such as the Pareto principle, cannot reduce the multiplicity of equilibria. In the next step, we look at equilibria in which candidate 2 wins the election. Proposition 6.2 indicates that candidate 1 can ask for higher wages than candidate 2. The wage difference is naturally closely related to the additional benefits $b_1 - b_2$ that candidate 1 will generate for voters.

For $\lambda = 0$, the condition in Proposition 6.2 $(1+\lambda)(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2)$ requires that the differential benefits the highly competent politician generates for the society are larger than the difference of the costs of the politician to provide the public good. Hence, the condition $(1+\lambda)(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2)$ from Proposition 6.2 appears to be the more plausible than the opposite. For completeness, we supplement our discussion by characterizing the equilibria in the opposite case.[8]

Proposition 6.3 *Suppose* $(1+\lambda)(c_1 - c_2) > (N+\lambda)(b_1 - b_2)$. *Then, in any equilibrium candidate* 2 *is elected and wage offers satisfy*

$$W_1 = (b_1 - b_2)\frac{N-1}{1+\lambda} + W_2$$

[7]The net utility from electing candidate must be positive. Hence, $b_1 - \frac{W_1^{max}}{N-1}(1+\lambda) \geq 0$, which is equivalent to the condition $b_2 - \frac{1+\lambda}{N+\lambda} c_2 \geq 0$. This is a mild condition which is assumed to hold.

[8]In this case tie-breaks are resolved in favor of candidate 2 in order to simplify the exposition.

and wages are indeterminate. In particular, there exists an equilibrium in which candidate 2 is elected with minimal wages W_2^{min} and W_1^{min} given by

$$W_1^{min} = (b_1 - b_2)\,\frac{N-1}{1+\lambda} + \frac{N-1}{N+\lambda}\,c_2,$$
$$W_2^{min} = \frac{N-1}{N+\lambda}\,c_2.$$

There also exists an equilibrium in which candidate 2 is elected with maximal wages W_2^{max} and W_1^{max} given by[9]

$$W_1^{max} = \frac{N-1}{N+\lambda}\,c_1,$$
$$W_2^{max} = \frac{N-1}{N+\lambda}\,c_1 - (b_1 - b_2)\,\frac{N-1}{1+\lambda}.$$

The proof of Proposition 6.3 follows the lines of the proof of Proposition 6.2 and is therefore omitted. Again, there exists a continuum of pairs (W_1, W_2) that can constitute an equilibrium.

6.6 Welfare Comparisons

6.6.1 The General Case

In this section we discuss welfare comparisons. We assume that the public determines the wage in the first scenario in order to maximize welfare in terms of the utilitarian welfare function. Two views on welfare are present in the literature. Either utilities of ordinary voters alone are counted, or utilities of all individuals, i.e. including the candidates. We choose the latter approach for two reasons: First, it is difficult to justify excluding individuals from welfare considerations (see e.g. Besley and Coate 1997). Second, our results tend to be reinforced if we exclude candidates from welfare considerations, since wage competition yields higher remunerations than fixed wages when the same candidate is elected.

While in principle the equilibria of Propositions 6.2 and 6.3 allow for negative wage proposals, we restrict our welfare analysis to the plausible case of non-negative wages.

Following the logic of Sect. 6.4 in the case of a fixed wage, candidate 2 will run for office for any wage $W \geq c_2 - b_2$, because $b_2 + W - c_2 \geq 0$. Candidate 1 will also enter the political competition if

[9] For voters to be better off by electing candidate 2 than with the status quo, the condition $b_2 - \frac{W_2^{max}}{N-1}(1+\lambda) > 0$ must hold, which in terms of exogenous parameters is $b_1 - \frac{1+\lambda}{N+\lambda}c_1 \geq 0$. This mild condition is assumed to hold.

$$W \geq \tilde{W} := \frac{N-1}{N+\lambda}\big(c_1 - (b_1 - b_2)\big).$$

From the discussion on fixed wages, we can draw the following observations regarding welfare: If $\tilde{W} \leq 0$, then the welfare-maximizing wage under a fixed remuneration scheme W^{opt} is zero. Indeed, candidate 1 runs for office for any non-negative wage W and is elected with certainty.[10] Therefore the public sets $W^{opt} = 0$, because otherwise they would have to incur the wage costs. In this case, welfare, denoted by U^{fix}, is given by

$$U^{fix} = Nb_1 - c_1 - \lambda W^{opt} = Nb_1 - c_1.$$

If on the other hand $\tilde{W} > 0$ and $\tilde{W} > c_2 - b_2$, there exist two potentially optimal wage offers, taking into account that the public always prefers that at least one candidate is running for office (also see Footnote 9). The first of these wage levels is $W^{opt} = c_2 - b_2$, as this wage offer optimizes the total welfare in the case where candidate 1 does not run for office, whereas candidate 2 does run and is elected. Overall welfare would be given by

$$U^{fix} = Nb_2 - c_2 - \lambda W^{opt} = Nb_2 - c_2 - \lambda(c_2 - b_2).$$

The second potentially optimal wage level is $W^{opt} = \tilde{W}$. In this case, candidate 1 would run for office and would be elected with certainty. Overall welfare would be given by

$$U^{fix} = Nb_1 - c_1 - \lambda\tilde{W}.$$

Therefore, if additionally $Nb_1 - c_1 - \lambda\tilde{W} \geq Nb_2 - c_2 - \lambda(c_2 - b_2)$, the optimal remuneration for politicians would be $W^{opt} = \tilde{W}$, for $\tilde{W} > 0$ and $\tilde{W} > c_2 - b_2$.

If however $\tilde{W} > 0$, but $\tilde{W} < c_2 - b_2$, the welfare maximizing wage under a fixed remuneration scheme is $W^{opt} = \tilde{W}$: Candidate 1 runs for office for $\tilde{W}$ and is elected with certainty. In this case, overall welfare is given by

$$U^{fix} = Nb_1 - c_1 - \lambda\tilde{W}.$$

Now we turn to compensation schemes offered competitively by the politicians. According to Sect. 6.5, if $(1+\lambda)\,(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2)$, candidate 1 offers the wage

$$W_1 = (b_1 - b_2)\frac{N-1}{1+\lambda} + W_2$$

and is elected. Overall welfare, denoted by U^{var}, is given in this case by

$$U^{var} = Nb_1 - c_1 - \lambda W_1.$$

[10] This follows from the assumption that $\tilde{W} \leq 0$: That implies that for any non-negative wage offer we have $W \geq 0 \geq \tilde{W}$ and $c_1 - b_1 \leq 0$, so that we are in either case (i) or (iii) of Proposition 6.1.

Recall that the minimal and maximal wages are given by Proposition 6.2.

For $(1+\lambda)(c_1-c_2) > (N+\lambda)(b_1-b_2)$, candidate 2 is elected with a wage W_2 which must satisfy the equilibrium boundaries. Overall welfare is simply:

$$U^{var} = Nb_2 - c_2 - \lambda W_2,$$

$$W_2 = W_1 - (b_1 - b_2)\frac{N-1}{1+\lambda}.$$

Recall that the minimal and maximal wages in this case are given by Proposition 6.3.

The preceding observations lead to the following result:

Proposition 6.4 *(i) Suppose $\lambda > 0$. For sufficiently large N, welfare is always higher under fixed wages than under competitive wages. In both scenarios the more competent candidate is always elected.*

(ii) Suppose $\lambda = 0$. For sufficiently large N, fixed and competitive wages yield the same welfare.

The proof of Proposition 6.4 is given in the Appendix. Proposition 6.4 indicates that fixed wages outperform self-designed remuneration packages as long as the size of the society is not too small.

The comparisons in the proof illustrate that, under competitive wage offers by candidates, realized wage costs become higher than they would under fixed and predetermined remunerations for politicians. The main intuition for the result is as follows: Both wage schemes provide a solution for the following trade-off. Higher wages prompt the more competent politician (as well as the less competent one) to run for office. This enables voters to elect a competent office-holder, which increases welfare. Higher wages imply higher tax distortions, which lowers welfare. Consider now the competitive wage regime. The more able candidate proposes a wage that allows him to capture all the extra surplus which he generates. This means that wages end up being too high. Given the cost of raising public funds, the welfare-optimal wage must be just high enough to induce the better candidate to run for office so that the extra surplus generated by that candidate goes to the voters.

It is a little surprising that wage competition leads to excessive wages if we think of Bertrand competition. However, candidates compete with "differentiated products" and do not fully take into account the tax distortions they thereby create for society.

6.6.2 A Special Case

There are two reasons why it is instructive to consider the case $c_2 = 0$. This case enables us to provide a simple illustration of the role of candidate competency and the impact of the shadow costs of funds on the relative welfare comparison between fixed and flexible wages.[11] As we do not make an assumption regarding N, we can state an analogous result to Proposition 6.4.

Proposition 6.5 *Suppose* $\lambda > 0$. *Then*

(i) welfare is always higher under fixed wages than under competitive wages, and
(ii) candidate 1 is elected equally or more often under fixed wages than under competitive wages.

The proof of Proposition 6.5 is given in the Appendix.

Proposition 6.6 *For* $\lambda = 0$, *candidate 1 is elected under fixed wages and competitive wages equally often as candidate 2. Both scenarios yield the same welfare.*

The proof of Proposition 6.6 is given in the Appendix. Propositions 6.5 and 6.6 provide further insight into the role of tax distortions. The more competent candidate can capture all the surplus under competitive wages, which creates tax distortions and lowers welfare compared to fixed wages. Such tax distortions may, however, help candidate 2 to get elected under competitive wages, while candidate 1 is elected under fixed wages. This further lowers welfare in a competitive wage setting. The higher competency with fixed wages can only occur if N is not large, as otherwise Proposition 6.4 applies. If there are no tax distortions, i.e. $\lambda = 0$, neither of the two potentially welfare-reducing effects are present, and, as shown in Proposition 6.6, both scenarios yield the same welfare.

6.7 Robustness and Extensions

Our results show that wage-setting competition does not have welfare-enhancing effects, as is usually the case with Bertrand competition.

Of course, our model builds on several assumptions, the importance of which we will discuss in this section. First, we have restricted the number of candidates to two. In principle, wage competition might become fiercer the more potential candidates there are. However, as long as individual costs of serving in office vary much less than the net benefits that candidates can generate, wage competition will depend on the two best candidates, i.e. the two candidates for whom b_i is highest. As a consequence, our welfare results will still hold in such a setting.[12]

Second, suppose that citizens incur some small cost of acting as candidates in the electoral competition. Our equilibria need to be slightly altered as candidates will only run for office if they have a chance of being elected. While this has no effect in the case of publicly determined wages, the remuneration the more able candidate can obtain with competitive wages increases. This further lowers social welfare when

[11] The case $c_1 = 0$ yields qualitatively the same results, but it is more cumbersome to present.

[12] Details are available on request.

wages are offered competitively. Hence, the welfare comparison remains qualitatively the same in this case.

Third, we could assume that the status quo is causing an infinitely negative utility. This would exclude the fourth case in Proposition 6.1 and would simplify its proof. The welfare results remain the same.

Fourth, we have assumed linear deadweight losses. This can be justified as a first-order approximation to tax distortions when a member of the society has to make a non-negligible contribution to paying elected public officials. However, since our main arguments in comparing fixed wages to competitive wages (notably in Sect. 6.6) only rely on the existence of positive deadweight costs, our results tend to be robust to non-linear deadweight costs.

Fifth, one could imagine a world where there are only imperfect signals about the competence of candidates, and where these signals are observed only after a candidate has decided to run for office. In this context, pooling equilibria under fixed wages might occur where some low-competence and low-opportunity-cost candidates mimic the other types, thus possibly causing a bad-selection problem as identified by Caselli and Morelli (2004) and Poutvaara and Takalo (2007). But politicians can also destroy pooling equilibria by operating with not directly informative campaigns (see Gersbach 2004). When separating equilibria in terms of competency are the only ones that exist, our main insight can be applied again, and wage competition will also tend to be excessive in this case.

Sixth, an interesting variant of our model[13] is to assume that, as before, there are two candidates with allocations (b_1, c_1) and (b_2, c_2), but where c_i is an increasing function of b_i. Candidate 1 may mimic the somewhat less competent candidate, i.e. he can undertake b_1 at cost c_1 or b_2 at cost c_2. If $b_1 - c_1 > b_2 - c_2$, our result can be applied in this framework, as candidate 1 has no incentive to imitate candidate 2. If the cost function is strictly concave, we have $b_1 - c_1 < b_2 - c_2$, thus such a framework creates two kinds of economic problems:

(i) Even if the public has complete information, candidate 1 may simply implement b_2. This commitment problem will seriously inhibit the functioning of both wage-setting schemes as candidate 1 will never implement b_1. As both candidates will effectively play type 2, it is straightforward to show that both wage schemes will lead to the same wage and the same welfare.

(ii) Suppose there is incomplete information for the public regarding the type of politician and also suppose $b_1 - c_1 < b_2 - c_2$, such that candidate 1 has an interest in claiming that he is type 2 if he gets elected. Then we will have pooling equilibria under competitive wages where both candidates will offer wages according to type 2 as candidate 1 cannot credibly signal his type. Again, both wage schemes will yield identical results.

The situation will be different if there is punishment (e.g. reciprocal behavior of voters, career concerns, reputation losses) when a candidate announces a wage, claims to be of type 1, and imitates type 2 when elected. How such punishment schemes

[13] I am grateful to a reviewer for this suggestion.

can be integrated into our model and how it will affect the balance between publicly determined wages and competitive wage offers will be an important avenue for future research.

6.8 Conclusion

Our results can be interpreted in several ways. The drawback of competitively offered wages can be understood as an argument against the general application of the dual mechanism — incentive contracts and elections — in politics, as examined in a series of papers, starting with Gersbach (2003) and surveyed in Gersbach (2012) as well as in the first chapter of this book. Allowing candidates to design the conditions of their term may cause excessive wage costs or cause less competent politicians to be elected.

In a broader perspective, the most important drawback of competitively offered remuneration packages might be less competency in politics. In addition, allowing politicians to compete with self-designed compensation packages might involve further adverse consequences. Wealthy candidates running for office may be able to forgo remuneration from the public completely. Accordingly, other less wealthy candidates may not be able to compete on equal terms in political campaigns. As we intend to examine in subsequent research, this might undermine a core principle of democracies which says that the pool of candidates for political positions should not be constrained a priori. Hence, allowing for competitively offered wages in each term does not appear to be a priority in broadening the scope of democracies.

Appendix

Proof of Proposition 6.1

Note that if candidate 1 decides to run for office, he will be elected independently of whether candidate 2 decides to run for office or not. Therefore, candidate 2 should run for office if and only if his utility from serving as a politician is greater than zero, which is his utility from the default outcome when no candidate runs for office. Thus "run for office" is weakly dominant for candidate 2 if $b_2 + W - c_2 \geq 0$. If $b_2 + W - c_2 < 0$, "do not run" is weakly dominant. Hence, if and only if $W \geq c_2 - b_2$, candidate 2 will run for office. If $W \geq c_2 - b_2$, then candidate 1 will also run for office if

$$b_1 + W - c_1 \geq b_2 - \frac{W}{N-1}(1+\lambda),$$

i.e. if his utility from holding office is higher than the utility obtained when candidate 2 is in office, based on the utilities in Eqs. (6.1) and (6.2). This condition can be transformed into

$$W \geq \frac{N-1}{N+\lambda}\big(c_1 - (b_1 - b_2)\big).$$

If $W < c_2 - b_2$, candidate 1 will run for office if $b_1 + W - c_1 \geq 0$, i.e. if $W \geq c_1 - b_1$. $\square$

Proof of Proposition 6.2

First note that in order for candidate 1 to be elected, W_1 must satisfy

$$W_1 \leq (b_1 - b_2)\frac{N-1}{1+\lambda} + W_2,$$

because otherwise the public is better off electing candidate 2. This follows from Eq. (6.5). Therefore, when candidate 1 wants to be elected, he offers the wage

$$W_1 = (b_1 - b_2)\frac{N-1}{1+\lambda} + W_2. \tag{6.6}$$

A downward deviation can be excluded, because in that case candidate 1 could raise his utility by offering a higher wage, and he would still be elected. Deviation to a higher wage leads to the election of candidate 2.

Candidate 1 will not deviate to a higher wage than in (6.6) and will not leave the office to candidate 2 if

$$b_1 + W_1 - c_1 \geq b_2 - \frac{W_2}{N-1}(1+\lambda).$$

Inserting the equilibrium value of W_1 from Eq. (6.6) as a function of W_2, this condition becomes

$$b_1 + (b_1 - b_2)\frac{N-1}{1+\lambda} + W_2 - c_1 \geq b_2 - \frac{W_2}{N-1}(1+\lambda),$$

which can be transformed into

$$(b_1 - b_2)\left(1 + \frac{N-1}{1+\lambda}\right) + W_2\left(1 + \frac{1+\lambda}{N-1}\right) \geq c_1,$$

which then yields

$$W_2 \geq \frac{N-1}{N+\lambda}c_1 - (b_1 - b_2)\frac{N-1}{1+\lambda} =: W_2^{min}. \tag{6.7}$$

Thus, candidate 1 will want to run for office if condition (6.7) is fulfilled, i.e. if the proposed remuneration W_2 exceeds a certain threshold. By substitution, the corresponding threshold for W_1 is then given by $W_1^{min} = \frac{N-1}{N+\lambda}c_1$.

We next examine the optimal choice of W_2 by candidate 2. A possible deviation from the proposed equilibrium in the proposition for candidate 2 would be to offer a wage $W_2' = W_2 - \epsilon$ for some small $\epsilon > 0$, which would lead to his election. Candidate 2 will not choose this option if

$$b_1 - \frac{W_1}{N-1}(1+\lambda) \geq b_2 + W_2' - c_2,$$

i.e. if his utility from being a citizen under candidate 1 is higher than his utility from holding office himself. By inserting the equilibrium value of W_1, as given by (6.6), we obtain the condition

$$b_1 - \frac{W_2}{N-1}(1+\lambda) - (b_1 - b_2) \geq b_2 + W_2 - \epsilon - c_2,$$

which can be transformed into

$$W_2 \leq \frac{N-1}{N+\lambda}(c_2 + \epsilon). \tag{6.8}$$

Therefore, if wage W_2 is small enough, candidate 2 would prefer to be a citizen under candidate 1 rather than running for office for a lower wage.

Concluding, there only exist equilibrium values for wage offers W_2 that satisfy both conditions (6.8) and (6.7) if

$$W_2^{max} := \frac{N-1}{N+\lambda} c_2 \geq \frac{N-1}{N+\lambda} c_1 - (b_1 - b_2)\frac{N-1}{1+\lambda}$$

and hence we obtain the assumption of the proposition given by

$$(1+\lambda)(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2).$$

From condition (6.6), we also obtain threshold wage

$$W_1^{max} := (b_1 - b_2)\frac{N-1}{1+\lambda} + \frac{N-1}{N+\lambda} c_2.$$

□

Proof of Proposition 6.4

We first prove statement (i). In principle, six different cases can occur.

Case 1: $\tilde{W} \leq 0$, $(1+\lambda)(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2)$.
Case 2: $\tilde{W} \leq 0$, $(1+\lambda)(c_1 - c_2) > (N+\lambda)(b_1 - b_2)$.
Case 3: $\tilde{W} > 0$, $\tilde{W} > c_2 - b_2$ and $(1+\lambda)(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2)$.
Case 4: $\tilde{W} > 0$, $\tilde{W} < c_2 - b_2$ and $(1+\lambda)(c_1 - c_2) \leq (N+\lambda)(b_1 - b_2)$.
Case 5: $\tilde{W} > 0$, $\tilde{W} > c_2 - b_2$ and $(1+\lambda)(c_1 - c_2) > (N+\lambda)(b_1 - b_2)$.
Case 6: $\tilde{W} > 0$, $\tilde{W} < c_2 - b_2$ and $(1+\lambda)(c_1 - c_2) > (N+\lambda)(b_1 - b_2)$.

If N is sufficiently large, we obtain $(1+\lambda)(c_1 - c_2) < (N+\lambda)(b_1 - b_2)$. This implies that we can drop the cases 2, 5, and 6. Now we examine the three remaining cases.

Case 1: As candidate 1 is elected under competitive wages by Proposition 6.2, welfare is given by

$$U^{var} = Nb_1 - c_1 - \lambda W_1$$

As discussed in Sect. 6.6.1, under a fixed wage, the wage is set at zero, candidate 1 runs for office and is elected. We obtain

$$U^{fix} = Nb_1 - c_1.$$

Thus welfare under the fixed wage scenario is no smaller than under competitive wages. Note that in both scenarios candidate 1 is elected.

Case 3: To derive our results in case 3, we proceed in four steps.

Step 1: Due to Proposition 6.2, candidate 1 is again elected under competitive wages. From the same proposition and the assumption of non-negative wages, we obtain

$$W_2^{min} = \max\left\{0, \frac{N-1}{N+\lambda}c_1 - (b_1 - b_2)\frac{N-1}{1+\lambda}\right\},$$

which yields

$$W_1^{min} = \max\left\{\frac{N-1}{1+\lambda}(b_1 - b_2), \frac{N-1}{N+\lambda}c_1\right\}.$$

For sufficiently large N we obtain

$$W_1^{min} = \frac{N-1}{1+\lambda}(b_1 - b_2).$$

Therefore maximal welfare under competition for wages is given by

$$U_{max}^{var} = Nb_1 - c_1 - \lambda\frac{N-1}{1+\lambda}(b_1 - b_2).$$

Step 2: Under a fixed wage, welfare depends on which candidate is elected. Following the logic in Sect. 6.6.1 and given the assumptions of case 3 and the non-negativity of wages, we have

$$U^{fix} = \max\left\{Nb_2 - c_2 - \lambda \max\{0, c_2 - b_2\}, Nb_1 - c_1 - \lambda\tilde{W}\right\}.$$

We now show that for sufficiently large N, the public will always set the wage at $\tilde{W}$, so that candidate 1 runs for office and is elected. As

$$Nb_2 - c_2 - \lambda \max\{0, c_2 - b_2\} \leq Nb_2 - c_2,$$

it suffices to show that

$$Nb_2 - c_2 < Nb_1 - c_1 - \lambda\tilde{W}.$$

Step 3: To prove the assertion, we insert $\tilde{W}$ and obtain

$$Nb_2 - c_2 < Nb_1 - c_1 - \lambda\frac{N-1}{N+\lambda}(c_1 - (b_1 - b_2)).$$

This inequality can be transformed into

$$c_1(1 + \lambda\frac{N-1}{N+\lambda}) - c_2 < (N + \lambda\frac{N-1}{N+\lambda})(b_1 - b_2),$$

which holds for sufficiently large N (note that $\frac{N-1}{N+\lambda} \to 1$ for $N \to \infty$).

Step 4: We can state now that welfare under fixed wages is given by

$$U^{fix} = Nb_1 - c_1 - \lambda\tilde{W}.$$

Welfare is higher under a fixed wage scenario if

$$Nb_1 - c_1 - \lambda\tilde{W} > Nb_1 - c_1 - \lambda\frac{N-1}{1+\lambda}(b_1 - b_2).$$

Inserting $\tilde{W}$ yields

$$(1+\lambda)(c_1 - (b_1 - b_2)) < (N+\lambda)(b_1 - b_2),$$

which holds for sufficiently large N. Again, candidate 1 is elected in both scenarios.

Case 4: Case 4 is analogue to case 3.
Under competitive wages, candidate 1 is elected and the maximal welfare is given by

$$U^{var}_{max} = Nb_1 - c_1 - \lambda\frac{N-1}{1+\lambda}(b_1 - b_2),$$

according to the same considerations as in case 3.
Under fixed wages, welfare is given by

$$U^{fix} = Nb_1 - c_1 - \lambda\tilde{W}.$$

Thus, as in case 3, welfare is higher under a fixed wage if N is sufficiently large, and candidate 1 is elected in both scenarios.

Statement (ii) of the proposition follows immediately from the above considerations. If we insert $\lambda = 0$, welfare is given under both wage-setting regimes by $Nb_1 - c_1$, as candidate 1 is always elected. □

Proof of Proposition 6.5

In principle, three different cases can occur:

Case 1: $\tilde{W} \leq 0$.
Case 2: $\tilde{W} > 0$ and $(1 + \lambda)\, c_1 \leq (N + \lambda)(b_1 - b_2)$.
Case 3: $\tilde{W} > 0$ and $(1 + \lambda)\, c_1 > (N + \lambda)(b_1 - b_2)$.

We prove the statement by showing the assertions for each case.

Case 1: Suppose $\tilde{W} \leq 0$. This implies $c_1 < b_1 - b_2$, which can be easily verified by checking the definition of $\tilde{W}$. Then $(1 + \lambda)c_1 \leq (N + \lambda)(b_1 - b_2)$ holds. Therefore candidate 1 is elected under competition for wages with $W_2 = 0$, since $W_2^{max} = 0$. Accordingly, W_1 is given by $\frac{N-1}{1+\lambda}(b_1 - b_2)$ and welfare is given by

$$U^{var} = Nb_1 - c_1 - \lambda \frac{N-1}{1+\lambda}\,(b_1 - b_2).$$

Since $\tilde{W} < 0$, the wage is set at zero in case of a fixed wage, and candidate 1 runs for office and is elected. We obtain

$$U^{fix} = Nb_1 - c_1,$$

and thus welfare is higher under the fixed wage scenario. In both scenarios candidate 1 is elected.

Case 2: Suppose $\tilde{W} > 0$ and $(1 + \lambda)\, c_1 \leq (N + \lambda)(b_1 - b_2)$. Then candidate 1 is elected under competition for wages. Since $W_2^{max} = 0$, in this case welfare is given by

$$U^{var} = Nb_1 - c_1 - \lambda \frac{N-1}{1+\lambda}\,(b_1 - b_2). \tag{6.9}$$

Under a fixed wage, the public sets the wage at $\tilde{W}$ so that candidate 1 runs for office and is elected if $Nb_1 - c_1 - \lambda\tilde{W} \geq Nb_2$.
As

$$\tilde{W} = \frac{N-1}{N+\lambda}\,\big(c_1 - (b_1 - b_2)\big),$$

this inequality can be transformed into

$$Nb_1 - c_1 - \lambda \frac{N-1}{N+\lambda} \left(c_1 - (b_1 - b_2)\right) \geq Nb_2.$$

This implies

$$(b_1 - b_2) \geq c_1 \frac{1+\lambda}{N + \lambda(2 - \frac{1}{N})},$$

which always holds for $(1+\lambda)\, c_1 \leq (N+\lambda)(b_1 - b_2)$ because

$$(b_1 - b_2) \geq c_1 \frac{1+\lambda}{N+\lambda} \geq c_1 \frac{1+\lambda}{N + \lambda(2 - \frac{1}{N})}.$$

This implies that under a fixed wage scenario, candidate 1 will run and will be elected with certainty. We have welfare

$$U^{fix} = Nb_1 - c_1 - \lambda \tilde{W}. \tag{6.10}$$

Comparing (6.9) and (6.10), welfare is higher under a fixed wage scenario if

$$\tilde{W} < \frac{N-1}{1+\lambda} (b_1 - b_2).$$

We insert $\tilde{W}$ and rearrange the terms, and obtain

$$(1+\lambda)(c_1 - (b_1 - b_2)) < (N+\lambda)(b_1 - b_2).$$

According to the assumptions in case 2, this inequality holds. Again, in both scenarios candidate 1 is elected.

Case 3: Suppose $\tilde{W} > 0$ and $(1+\lambda)\, c_1 > (N+\lambda)(b_1 - b_2)$. In this case, candidate 2 is elected under competitive wages. The welfare under competition for wages is given by

$$U^{var} = Nb_2 - \lambda W_2.$$

Under the fixed wage framework, welfare is

$$U^{fix} = \max \left\{ Nb_1 - c_1 - \lambda \tilde{W}, Nb_2 \right\}.$$

Hence welfare with wages set by the public is higher than, or equal to, what it would be under competitive wages.

While it is unambiguously clear that welfare is higher under fixed wages, it is not clear which wage the public will set in this scenario. The wage is set at $\tilde{W}$ such that candidate 1 runs for office and is elected if and only if

$$Nb_1 - c_1 - \lambda \tilde{W} \geq Nb_2,$$

which can be transformed into

$$(b_1 - b_2) \geq c_1 \frac{1 + \lambda}{N + \lambda(2 - \frac{1}{N})}.$$

According to the assumption made in case 3, the upper inequality can either hold or not. This implies that candidate 1 may be elected under fixed wages, while under competition for wages candidate 2 will be elected for sure.

All in all, welfare is always higher under fixed wages, while candidate 1 is elected equally or more often under fixed wages than under competitive wages. □

Proof of Proposition 6.6

We explore the same cases as in Proposition 6.5, but now with $\lambda = 0$.

Case 1: Suppose $\tilde{W} \leq 0$. This implies $c_1 < b_1 - b_2$, hence $c_1 < N(b_1 - b_2)$ holds as well. By Proposition 6.2 and using $\lambda = 0$ and $c_2 = 0$, we conclude that candidate 1 is elected under competition for wages. Welfare is given by

$$U^{var} = Nb_1 - c_1.$$

Under a fixed wage, the wage is set at zero, candidate 1 runs for office and is elected. We obtain

$$U^{fix} = Nb_1 - c_1.$$

In both scenarios, candidate 1 is elected, and welfare is given by $Nb_1 - c_1$ both under fixed wages and under competition for wages.

Case 2: Suppose $\tilde{W} > 0$ and $c_1 \leq N(b_1 - b_2)$. This implies that candidate 1 is elected under competition for wages. Welfare is given by

$$U^{var} = Nb_1 - c_1.$$

Under fixed wages, the public sets a wage no smaller than $\tilde{W}$, so that candidate 1 runs for office and is elected if and only if $Nb_1 - c_1 \geq Nb_2$. But this inequality holds by the assumptions made in case 2. Therefore welfare is given by

$$U^{fix} = Nb_1 - c_1.$$

As in case 1, candidate 1 is elected in both scenarios, and welfare is given by $Nb_1 - c_1$.

Case 3: Suppose $\tilde{W} > 0$ and $c_1 > N(b_1 - b_2)$. Under competition for wages, candidate 2 is elected and welfare is given by

$$U^{var} = Nb_2.$$

The public sets a wage strictly smaller than $\tilde{W}$ so that only candidate 2 will run for office and be elected if and only if $Nb_2 > Nb_1 - c_1$. But this inequality must hold in case 3 by assumption, thus welfare is given by

$$U^{fix} = Nb_2.$$

Therefore, fixed wages and competitive wages yield the same welfare, and in both scenarios candidate 2 is elected. □

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Part II
Rules for Decision-Making and Agenda-Setting

Chapter 7
Introduction to Part II

7.1 Motivation

Reflection on the role and governance of a democratically-organized state can start at various levels. At the most fundamental level, founding principles such as the monopoly of coercion of the state, the power to levy taxes, the validation of property rights and of contracts between citizens, or equal voting and agenda-setting rights, the basic right to be a candidate for office, and the separation of the legislative, judicial and executive powers are the basis of governmental authority. At the next level, we find the definition of roles and of governmental activities such as the appointment of office-holders and the procedures for provision of services and public goods. We will focus on this second level, taking the founding governmental principles as given, but may allow democratically-founded modifications of voting rights over the course of a decision-taking process.

Throughout this second part of this book, we will continue to adopt a normative perspective. Besides complying with the basic principles of democracy, collective decision-making must try to achieve as high a level of social welfare as possible. Ideally, it might also contribute to social cohesion by protecting minorities from exploitation. There are at least three different approaches to the functioning of a democratic state: (i) constitutional economics, (ii) mechanism design, and (iii) election and voting. Let us briefly examine these three areas. They will be integrated into our research on "Democratic Mechanisms".

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7.2 Constitutional Economics

In their classic work on the foundations of constitutional economics, Buchanan and Tullock (1962) have outlined how a society might—or should—choose the rules that will govern its political processes. For instance, this society weighs the costs and benefits of rules such as the majority and supermajority rules.[1]

Today, constitutional economics encompass the design and analysis of the rules governing proposal-making, the treatment of citizens[2] and collective decisions, as well as agenda-setting rules (see Gersbach 2009a).

We will adopt a constitutional perspective when we explore a new area: Democratic Mechanisms.

7.3 Mechanism Design

Mechanism design is a well-defined core area of microeconomic theory. It deals with the optimal choice of rules for games. Traditionally, its key issues are how optimal outcomes can be achieved through a set of rules when agents have private, i.e. hidden, information. A particular branch of mechanism design deals with public goods provision, and thus with one of the main activities of any government.

Democracy is not the main concern of standard mechanism theory, which tries to find payment schemes that generate incentives for individuals to reveal their preferences, such that an efficient level of public good can be provided. With their schemes, Clarke (1971) and Groves (1973) gave a first solution to this problem. Such a mechanism specifies two terms which, combined, determine an agent's payment. A first part depends on the agent's message and aligns this agent's interests with social welfare. A second part is independent of the agent's message and can be chosen to satisfy individual rationality constraints. In general, however, such mechanisms do not achieve budget balance, and D'Aspremont and Gérard-Varet (1979) and Arrow (1979) developed the expected externality mechanism that achieves budget balance. However, this mechanism violates interim individual rationality.

These two examples are representative for a fundamental problem in mechanism design: There are deep conflicts between the requirements of incentive compatibility, efficiency and voluntary participation in settings with private goods (see Myerson and Satterthwaite 1983). These conflicts are accentuated in the context of public-good provision and persist when the society is large.[3]

[1] Explicit contractual incompleteness for the design of optimal majority rules and the role of vested interests ex-post have been developed by Aghion and Bolton (2003).

[2] An example of treatment rules is the requirement that citizens with the same income have to pay the same income tax.

[3] See Jackson (2001), Mailath and Postlewaite (1990), Güth and Hellwig (1986), and Hellwig (2003). In contrast, there exist mechanisms that are interim individually rational, approximately

We will abandon voluntary participation in our approach and assume that the government can tax people. However, we will impose other properties the mechanisms of democratic societies will have to fulfill. This will be our starting point to introduce Democratic Mechanisms.

7.4 Voting Rules

The invention, the analysis and real-world applications of election and voting rules have been the research focus of social choice, public choice and political economy scholars. Thus, a rich body of theoretical and empirical knowledge about such rules has been accumulated over time.

In recent years, a variety of new voting rules was introduced. This has opened up new opportunities for democratic societies or for committees to organize themselves and to reach decisions. Storable votes, Minority Voting, qualitative voting, and the flexible majority rule, for instance, have been implemented and evaluated thoroughly.[4]

In our new research area of Democratic Mechanisms, we will embed existing or new voting rules into a set of constitutional rules, and invent some new collective decision rules that enhance the potential of democratic constitutions to foster their citizens' well-being.

7.5 Democratic Mechanisms

To bring constitutional economics, mechanism design and the design of voting rules and electoral rules together, we proceed in three steps. First, we introduce Democratic Mechanisms—or equivalently, democratic constitutions—as originally introduced in Gersbach (2009a). Suppose that a society faces the task to decide what level of a certain public good should be provided by the government. The individual utilities of the public good are the citizens' private information. Then, a Democratic Mechanism is a set of rules that specifies

(i) the costs and benefits for the proposal-makers, and
(ii) the restrictions on proposals that can be made, and
(iii) how the society decides on the proposal.

(Footnote 3 continued)
efficient, and budget-balanced in large societies in private-goods settings. For an excellent and unified treatment of the theory of mechanism design, see Börgers (2015).

[4]For storable votes, see Casella (2005), for Minority Voting, see Chap. 9 of this book, and Fahrenberger and Gersbach (2010, 2012). For qualitative voting, see Hortala-Vallve (2012) and for the flexible majority rule Gersbach (2004a, b, 2005, 2009b).

These rules must satisfy the liberal-democracy constraint, which consists of the following sub-constraints:

(i) Every agent has the same chance of making a proposal[5];
(ii) Every individual is allowed to abstain from proposal-making;
(iii) Decision rules have to satisfy the anonymity principle;
(iv) Every individual has the right to vote; and
(v) Only yes/no-votes or abstention are allowed at voting stages.

Democratic Mechanisms differ from procedures of the standard mechanism design framework with regard to the absence of a mechanism designer, the need to respect the liberal-democracy constraint, and the power of a government enforcing participation within its own jurisdiction.

The absence of a mechanism designer makes it necessary to add a pre-stage, i.e. the stage necessary to put the set of rules of a Democratic Mechanism into place before it can be used. This pre-stage is called the "Constitutional Stage". It has the same purpose as in constitutional economics: Behind a veil of ignorance, in which individuals cannot know how future generations will be affected by future collective decisions, a society decides which Democratic Mechanism it wants to use when deciding on the provision of a specific public good.

The most simple game that incorporates the constitutional period and the use of Democratic Mechanisms works as follows:

- *Stage 1.* In the constitutional period, the society decides unanimously about the rules of a Democratic Mechanism.
- *Stage 2.* Citizens observe their own utility from a certain public good and decide simultaneously whether to apply for agenda-setting or not.
- *Stage 3.* Among all citizens that apply, one citizen is determined by fair randomization to set the agenda. The agenda-setter proposes a public good level/financing package.
- *Stage 4.* Given the proposal, the citizens decide simultaneously whether to accept it or not.

Summarizing, a Democratic Mechanism is a set of rules that specifies

(i) whether there is special treatment for the agenda-setter (*Agenda-setter Rules*), and
(ii) the set of restrictions on the proposals (*Agenda Rules*)—a proposal consisting of a level of public good and a financing package—, and
(iii) how the society decides about a proposal (*Decision Rules*).

The set of all conceivable Democratic Mechanisms is large, even in this most simple of games. The reason is that there are many possible ways to define such rules, as outlined in Gersbach (2009b, 2011).

[5]This right can be delegated to representatives in parliament.

We have not explored all conceivable rules yet. In more complex Democratic Mechanism games, one could include the opportunity for amendments, for instance, or the formation of initiative groups. These are the issues we take up.

This second part of *Redesigning Democracy* comprises our latest research in this area. It first focuses on Democratic Mechanisms as a whole and then examines the invention of particular rules for decision-making or proposal-making.

7.6 Overview

7.6.1 Divisible Public Goods

Democratic Mechanisms were introduced in Gersbach (2009b). We showed that there exist mechanisms that yield first-best allocations for indivisible public goods—essentially for a decision between the status quo and a specific indivisible public good.

More specifically, we introduced Democratic Mechanisms as a set of rules that must obey liberal democracy's fundamental principles of equal voting and agenda rights. We showed that an appropriate combination of three rules may yield efficient provision of public projects: First, we need *flexible and double majority rules*, where the size of the majority depends on the proposal,[6] and taxed and non-taxed individuals need to support the proposal; Second, we require *flexible agenda costs*, where the agenda-setter has to pay a certain amount of money if his proposal does not generate enough supporting votes; Third, there should be a *a ban on subsidies*, as universal equal treatment with regard to taxation is undesirable in this context. Finally, we showed how simple constitutions involving fixed supermajority rules yield socially desirable outcomes if the agenda-setter is benevolent.[7]

In Chap. 8, we will explore whether suitable Democratic Mechanisms exist when public goods are *divisible* and the society can choose either from an entire range—or even a continuum—of public goods. New issues will arise, as both under- and over-provision of public goods have to be avoided. With a suitable version of the game outlined above, we will search for first-best Democratic Mechanisms for situations with and without aggregate shocks. It will turn out that such mechanisms can be found, and that democracy can yield an efficient provision of public goods in much more general settings.

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[6]Over the course of the last decade, several variants and aspects of flexible majority rules have been developed, which started with Erlenmaier and Gersbach (2001) and are surveyed in Gersbach (2017).

[7]The limitations of such Democratic Mechanisms with regard to the dimension of uncertainty—and with regard to uncertainty about the size of the utility losses—is dealt with in Gersbach (2011).

7.6.2 *Minority Voting and Public Project Provision*

Minority Voting is a simple method to avoid the repeated exploitation of minorities. Our research on this subject goes back to 2005 and started from the observation that the narrower the majority of a voting outcome, the larger the minority—and with it, the number of voting losers. For efficiency, and fairness reasons, we need some kind of compensation for the voting losers, all the more if they are nearly as numerous as the voting winners. Minority Voting is a concept that partially compensates the loss in the first voting round through exclusive voting rights in the next voting round. This alternative to majority voting can be used in any voting process with several rounds, and in two-round voting procedures, in particular.

Minority Voting for voting procedures with several projects was developed in Fahrenberger and Gersbach (2010, 2012). In this book, we will explore an alternative version in which a single public project provision is split in two parts, the first being the decision whether the project should be provided or not, and the second being the decision how this project should be *financed* if the first decision is in favor of the project. The central idea of this variant of Minority Voting is that citizens who oppose a project and are in the minority in the first decision have a greater say in the second, when the polity decides who should bear the costs of providing the public project.[8]

7.6.3 *Initiative-Group Constitutions and the Democratic Provision of Public Projects*

Our research on initiative groups aims at efficient public-good provision by focusing on the beneficiaries of a project instead of trying to compensate voting losers in the next voting round(s), as in Minority Voting. In particular, the concept of an initiative group requires that a public project can only be provided if a sufficient number of project beneficiaries is willing to bear higher taxes for its implementation.

Thus, instead of compensating the losers, winning may come at a cost, and this cost can be pre-determined to a certain extent. We examine the optimal design and the functioning of initiative groups. We introduce a two-stage process to allocate a public project: In the first stage, an initiative group can be formed to support a particular project. If this group reaches a certain, pre-defined size, it is allowed to make a financing proposal for the project. This proposal may contain higher taxes for the group members, but definitely not less than the rest of the polity. We examine and discuss whether this scheme yields another way to induce efficient public good provision.

[8] A first version of this chapter has appeared as Gersbach (2009c).

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Chapter 8
Democratic Provision of Divisible Public Goods

8.1 Background

When choosing among a wide range of democratic decision-making methods, the most important task is to find those who yield the socially desirable levels of public goods.

This chapter explores democratic mechanisms—or synonymously democratic constitutions. We analyze the pre-voting phase during which, under a veil of ignorance, the collectivity chooses the rules that will govern decision-making on public good provision.

Ideally, public goods should be neither under- nor overprovided, and the structure underlying any provision of public goods should provide for situations with and without knowledge about the underlying distribution of costs and benefits. The problem is accentuated when aggregate shocks to costs and benefits can occur. How democratic constitutions could still induce optimal public goods provision for each realization of such shocks is unknown.

A democratic constitution is a set of rules that must satisfy the liberal-democracy constraints: Every agent has the same chance of making a proposal; every individual has the right to vote; only yes/no votes are allowed at the voting stages; every individual is allowed to abstain from proposal-making. A democratic constitution that avoids under- or over-provision of public goods is called a "first-best" constitution.

We show that optimal public goods provision can be achieved through democratic constitutions involving *tax-sensitive majority rules*, where the size of the majority required to approve them depends on the aggregate tax revenues.[1]

[1]This chapter is an updated version of the CESifo Working Paper 2939, 2010, under the same title.

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8.2 Introduction

We consider public-good provision and public-good financing in a large-economy version of Hellwig (2005), embedded in a four-stage game. In the constitutional period, the society decides unanimously about the constitutional principles governing legislative decision-making. There is uncertainty regarding to who will benefit how much from the public good. Moreover, there may be aggregate uncertainty regarding the benefits and costs of the public good. At the start of the legislative period, citizens observe the realization of aggregate benefits and costs as well as their own utility and decide simultaneously whether to apply for agenda-setting or not. Among all citizens that apply, one citizen is determined by fair randomization to set the agenda. The agenda-setter proposes a project-financing package. Citizens decide simultaneously whether to accept the proposal or not. This game is a direct translation of the four sub-constraints that constitute the liberal-democracy constraint. We explore the potential of a democratic constitution restricted to this kind of game. A democratic constitution is a set of rules specifying (i) how the agenda-setter is treated, (ii) which types of proposal are allowed, and (iii) how the society decides on a proposal.

The first insight of this chapter is that the combination of the four following rules yields efficient provision of public goods when aggregate shocks are absent: (1) a supermajority rule under which the adoption of a particular level of a public good requires a prespecified vote-share; (2) a tax rule that levies the same tax rate on all individuals except the proposal-maker, who is exempted from taxation; (3) subsidies are forbidden; (4) the agenda-setter has to pay a fixed amount for agenda-setting.

The second insight is that in the case of aggregate shocks to benefits and costs, the replacement of supermajority rules by tax-sensitive majority rules can preserve the efficiency of democratic constitutions. With a tax-sensitive majority rule, the majority required to put a proposal through increases, as the aggregate tax revenues in a proposal get higher. As long as it holds that a higher amount of taxes in one state of the world is associated with a higher share of beneficiaries in comparison with the status quo, then appropriately designed tax-sensitive majority rules—in conjunction with the other constitutional rules—lead to first-best allocation.

The third insight of this chapter is that it is always possible to find a democratic constitution that implements a Pareto-improvement over the status quo. There are also circumstances for which we can find first-best constitutions that fully compensate voting losers, so that the socially optimal level of the public good can be implemented as a Pareto-improvement.

In this novel constitution, two rules deserve particular attention. By making the required majority threshold a strictly monotonically increasing function of aggregate tax revenues, an agenda-setter cannot induce the adoption of an amount of the public good that is higher than socially desirable. The reason is that the required majority varies with the socially optimal amount of public goods, measured by the aggregate tax revenues in different states of the world. In addition, the required majority is set equal to the share of individuals who strictly benefit from the proposal. By exempting the proposal-maker from taxation, while requiring equal tax treatment for all other individuals, the proposal-maker is forced to propose the highest possible level of the public good that would be adopted. This rule avoids under-provision of public goods.

This chapter is a study in constructive constitutional economics, as outlined in the classic contribution by Buchanan and Tullock (1962). Under a veil of ignorance, individuals decide which rules should govern legislative decision-making. In a long tradition dating back to Rousseau (1762), Buchanan and Tullock (1962) have examined the costs and benefits of majority rules chosen by a society operating under a veil of ignorance.

Aghion and Bolton (2003) have introduced contractual incompleteness for the design of optimal majority rules. They show how the simple or qualified majority rule can help to overcome ex-post vested interests. Gersbach (2009) has introduced the liberal-democracy constraint and explored democratic mechanisms for indivisible public goods. He has shown how increasingly sophisticated treatment and agenda rules, in conjunction with flexible or double majority rules, can yield first-best allocations for binary decisions. Gersbach (2011) explores the limits of this approach.

We also use the liberal-democracy constraint to define the set of admissible mechanisms in this chapter. In contrast to Gersbach (2009, 2011), however, we consider a model in which a society chooses among a continuum of possible public-good levels and we allow that benefits and costs may be affected by aggregate shocks during the legislative period. We introduce two novel rules which help to construct first-best allocations in such circumstances: aggregate-tax-sensitive majority rules and exemption of the proposal-maker from taxation. These rules, together with the other rules discussed in the introduction, avoid under- and over-provision of public good provision in circumstances with many different possible levels of public goods. Moreover, they induce that democratic public good provision adjusts optimally to fluctuations in costs and benefits of public goods.

The twin problem of societies—the risk of tyranny by the majority and the risk of legislation-blocking by the minority, as outlined in Aghion and Bolton (2003)—has been further examined in Aghion et al. (2004), who derive optimal supermajority governing rules that balance both of these dangers. Harstad (2005) develops a theory of majority rules based on the incentives of members of a club to invest in order to benefit from anticipated projects. Optimal majority rules balance two opposing forces. Large required majorities provide little incentive to invest because of hold-up problems, while the members of small majorities invest too much to become members of a majority coalition. We use aggregate-tax-sensitive majority rules to balance the power of majorities and minorities, in order to avoid under- or overprovision of public-good provision.

As a workhorse, we will use the large-economy model of Hellwig (2005). Our analysis is, however, more closely related to Hellwig (2003) who has examined public-good provision with many participants. In Sect. 5 we will discuss in detail how our results relate to Hellwig (2003).

The chapter is organized as follows: In the next section we introduce the model and the constitutional rules we want to use. In Sect. 8.4, we study first-best constitutions when aggregate uncertainty is absent. In Sect. 8.5, we examine first-best constitutions when benefits or costs of public goods are subject to aggregate shocks. In Sect. 8.6, we explore the possibility of subsidizing voting losers to achieve voluntary participation. Section 8.7 concludes.

8.3 Model and Constitutional Rules

8.3.1 Model

We consider a social-choice problem in public-good provision and financing in the large-economy model of Hellwig (2005). Time is indexed by $\tau = 0, 1$. The first period $\tau = 0$ is the constitutional period, when a society of risk-neutral members decides how public-good provision and financing should be governed in the legislative period $\tau = 1$. The society consists of a continuum of voters of measure 1, represented by [0, 1].

In the legislative period $\tau = 1$, each citizen is endowed with y units of a private consumption good. The society has an aggregate production capacity of Y, which can be used to provide an amount C of aggregate consumption of a private commodity and a public good of level Q. The resource constraint amounts to

$$C + K(Q) = Y.$$

The cost function $K(\cdot)$ is assumed to be strictly increasing, strictly convex and continuously differentiable, with $K(0) = 0$, $K'(0) = 0$ and $\lim_{Q\to\infty} K'(Q) = \infty$. Citizens are assumed to be risk-neutral.

A citizen derives utility zQ from the level of public good Q. The parameter z is the citizen's private information. From the perspective of the other citizens, or of the system as a whole, z is the realization of a random variable that takes values in [0, 1] and has a probability distribution $F(\cdot)$, with mean $\bar{z}$ and density $f(\cdot)$. By applying a suitable version of the law of large numbers, $F(\cdot)$ can be interpreted as the distribution of z in the population, and $\bar{z}$ is its mean. A citizen will be associated with his preference parameter z. As a shortcut, such a citizen is called citizen z.

The public good is financed by taxes, and citizens may be subsidized. We use $t(z)$ and $s(z)$ respectively to denote the tax payment and subsidy of a citizen with preference parameter z. Given a level Q of public-good provision, the utility of citizen z in the legislative period is given by

$$U(z) = y + zQ - t(z) + s(z).$$

Throughout the chapter, we assume that $s(\cdot)$ and $t(\cdot)$ are integrable functions. Furthermore, we assume that y is sufficiently large for the individuals to be able to pay the taxes proposed under any of the constitutions we will discuss. Finally, the budget constraint on the society in the legislative period is given by

$$\int_0^1 t(z)\, f(z)\, dz = K(Q) + \int_0^1 s(z)\, f(z)\, dz. \tag{8.1}$$

The aggregate tax revenue is denoted by T.[2]

8.3.2 Socially Optimal Solutions

We next characterize socially optimal solutions, where we use utilitarian welfare function to measure social welfare. Thus, as citizens are risk-neutral, the optimal level of the public good, from an ex ante point of view, is the solution of the following problem:

$$\max_Q \left\{ \int_0^1 zQ\, f(z)\, dz - K(Q) \right\},$$

subject to the budget constraint, given in (8.1). Our assumptions on the cost function imply that for this maximization problem, there exists a unique solution Q^*, which is determined by the equation

$$K'(Q^*) = \bar{z}. \tag{8.2}$$

Hence, at the socially optimal level of the public good, the marginal cost of provision equals the expected marginal benefit. The first-best allocation does not determine the financing scheme and hence neither the taxes nor the subsidy functions. The sole constraint is the budget constraint. We also note that all individuals with $z \geq \bar{z}$ benefit from public-good provision if the costs are shared equally, i.e. $zQ^* > K(Q^*)$. To prove this fact, we observe that

$$K'(Q) > \frac{K(Q)}{Q},$$

since $K(0) = 0$ and $K''(Q) > 0$. Hence, $\bar{z} = K'(Q^*) > \frac{K(Q^*)}{Q^*}$ indeed, so individuals with $z \geq \bar{z}$ will strictly benefit from the provision of Q^*.

8.3.3 Democratic Provision

We use the *liberal-democracy constraint*, which requires that the legislative process operates under the following sub-constraints:

- Every agent has the same chance to make a proposal.
- Every individual has the right to vote.

[2]Since the continuum of voters has measure 1, the terms in Eq. (8.1) represent the expected per capita tax burden and the expected per capita subsidy payments, respectively.

- Only yes/no messages are allowed at the voting stages.
- Every individual is allowed to abstain from voting or applying for proposal-making.

Several remarks are in order. First, Gersbach (2009), drawing on the philosophical foundations of democracy, provides an extensive justification of this constraint. Second, every citizen has the right to refrain from applying for agenda-setting. Once a citizen has applied and is selected, however, he may have to pay a cost, and thus becoming an agenda setter may be costly. The precise formalization of the liberal-democracy constraint is embodied in the game in the next subsection.

8.3.4 The Game

We consider the standard game that represents the sequence of constitutional and legislative periods:

Stage 0: In the constitutional period, the polity decides by the unanimity rule about the constitutional rules that govern the legislative processes.

Stage 1: At the start of the legislative period, citizens observe their preference parameter z. Citizens decide simultaneously whether to apply for agenda-setting: $\psi(z) = 1$ when they apply and or $\psi(z) = 0$) when they don't.

Stage 2: Among all citizens that apply, one citizen is determined randomly to set the agenda. The preference parameter of the agenda-setter is denoted by $z_a \in [0, 1]$. The agenda-setter proposes a project/financing package $\big(Q, t(\cdot), s(\cdot)\big)$. Denote this choice by P_{z_a}.

Stage 3: Given P_{z_a}, citizens decide simultaneously whether to accept the proposal: $\delta_z(P_{z_a}) = 1$ when they accept it and $\delta_z(P_{z_a}) = 0$ when they don't. The polity decides about the proposal according to some majority rule specified in the constitution.

The game fulfills the conditions constituting the liberal-democracy constraint. If nobody applies for agenda-setting, the status quo will prevail, which is characterized by $Q = 0, t(\cdot) \equiv s(\cdot) = 0$. Hence the utility of each citizen is y in this case.

We use $\mathcal{P} = \{P_z\}_{z\in[0,1]:\psi(z)=1}$ to denote the set of possible proposals. The set of strategies can be summarized by

$$\Big\{\big(\psi(z), P_z, \delta_z(\cdot)\big)\Big\}_{z\in[0,1]}.$$

In deriving an equilibrium, we face the problem that as we have a continuum of voters an individual vote has no influence on the outcome. To describe the application and voting outcome in our model, we use the weak dominance criterion that mimics the optimal voting and application behavior of a society with a large but finite number of agents (see Gersbach 2005). In our model, voting is a simple binary decision, so individuals have nothing to gain from strategic voting. Hence the above criterion implies that agents vote sincerely, i.e. agents will vote for their most-preferred alternative.

It is obvious that sincere voting on a proposal selects a unique voting equilibrium. Hence we can use the weak dominance criterion for the decision on whether to apply for agenda-setting (stage 1). This concept is applied in the following way: We first look at the set of agents who can strictly improve their utility by making a proposal, compared to the status quo. In all of our constitutions, this set will be non-empty, and those agents will apply for agenda-setting. Moreover, in all of our constitutions an agenda-setter can never fare better if somebody other than himself makes a proposal. As a consequence, all individuals will apply for agenda-setting.

To simplify the exposition, we assume as a tie-breaking rule that, if an agent z is indifferent between applying for agenda-setting and renouncing such an application, he will apply for agenda-setting, as we also assume that a citizen who is indifferent between voting yes or no will choose the former. In what follows we always assume sincere voting and the above behavior regarding agenda-setting.

We are now ready to characterize the expected level of the public good that a particular constitution can deliver. We say that a constitution $\mathcal{C}$ implements level Q if all possible perfect Bayesian equilibria under constitution $\mathcal{C}$ that satisfy the above refinements and tie-breaking rules, yield Q.

We call a constitution *first-best* if it implements the level Q^*, given by Eq. (8.2). To prove that the constitutions we propose are first-best, we show that

- all individuals apply for agenda-setting,
- each agenda-setter makes a proposal involving Q^*,
- this proposal will be adopted.

We finally note that in the constitutional period (stage 0), the society decides about the constitution by the unanimity rule. It is obvious that if a set of constitutional rules yields a first-best allocation, it will be approved unanimously in stage 0, since individuals are identical at this point and risk-neutral.

8.3.5 Constitutional Principles

The rules of the constitution have to specify

1. whether there is to be special treatment for the agenda-setter (**agenda-setter rules**);
2. restrictions on the agendas, i.e. definition of all constitutional agendas (**agenda rules**). An agenda consists of a project proposal and a financing package;
3. how the society decides on a proposal (**decision rules**).

We consider the following rules that will enable us to construct first-best constitutions.

Agenda-setter rules

- *Costs of agenda-setting [* ***CA*** *(b)]*
  The agenda-setter pays a fixed amount $b \geq 0$.

Agenda rules

- *Equal taxation of citizens except the agenda-setter [$\boldsymbol{ETT}^{-z_a}$]*
  All citizens except the agenda-setter have to pay the same taxes.
- *No subsidies [**NS**]*
  The agenda-setter is not allowed to propose any subsidies.
- *Budget constraint [**BC**]*
  The financing package must satisfy the budget constraint.

Decision rules

- *m-majority rule [$\boldsymbol{M}(m)$]*
  If a proposal to change the status quo receives at least a majority of m percent of the citizens ($0 \leq m \leq 1$), the proposal will be adopted.
- *Tax-sensitive majority rule [$\boldsymbol{FM}(m(T))$]*
  Under a tax-sensitive majority rule, the required majority to support a proposal depends on aggregate taxes $T = \int_0^1 t(z)dz$.

A priori we allow m to be smaller than $\frac{1}{2}$. In Sect. 8.4 we will discuss whether it is sensible to restrict m to $m \geq \frac{1}{2}$. Note that the tax-sensitive majority rule may depend on information generated by the proposal. By contrast, the rules [CA(b)] and [M(m)] do not depend on the proposal but may depend on other parameters. We will call a proposal P_{z_a} constitutional if the triple $\left(z_a, P_{z_a}, \{\delta_z^*(P_{z_a})\}_{z\in[0,1]}\right)$ does not violate the constitutional rules. The set $\{\delta_z^*(P_{z_a})\}_{z\in[0,1]}$ denotes the collection of equilibrium voting strategies of all individuals if P_{z_a} is proposed.

Throughout the chapter we assume that if a proposal violates the budget constraint, the status quo will prevail. If taxes exceed project costs and subsidies, we assume that excess revenues will be paid back uniformly to citizens, with the exception of the agenda-setter, as lump-sum transfer. As agenda-setters will never have an incentive to make a proposal with an unbalanced budget, we neglect this possibility in the following.

8.4 First-Best Constitutions

8.4.1 The Main Theorem

In this section we explore the structure of first-best constitutions. For the remainder of the chapter we use $AV(Q) = \frac{K(Q)}{Q}$ to denote the average cost function.

We start with our first main result. We consider the following constitution

$$\mathcal{C}_1 := \left\{[CA(\hat{b})], [NS], [ETT^{-z_a}], [M(\hat{m})]\right\},$$

where

$$\hat{b} = K(Q^*),$$
$$\hat{m} = 1 - F\left(\frac{K(Q^*)}{Q^*}\right) = 1 - F\big(AV(Q^*)\big).$$

Proposition 8.1 *Constitution $\mathcal{C}_1$ is first-best.*

The proof of Proposition 8.1 is given in the Appendix. The intuition for the result is as follows. First notice that the cost $\hat{b}$ paid by the agenda setter is independent of the proposal. As the agenda setter is exempted from taxation, he is therefore interested in the maximal level of public goods that is adopted by the polity.

Consider next an individual z with $zQ^* > K(Q^*)$. As observed in Sect. 8.3.2, such individuals exist. For these individuals the proposal $(Q = Q^*, t^{-z_a} = K(Q^*))$ is optimal, since this is the maximal level of the public good supported by at least $\hat{m}$ voters. An agenda-setter can avoid bearing taxes, but he has to pay $\hat{b} = K(Q^*)$. Hence individuals with $zQ^* > K(Q^*)$ are strictly better off with respect to the status quo if they can be the proposal-makers, and will therefore apply for agenda-setting.

Consider an individual with $zQ^* \leq K(Q^*)$. If he can set the agenda, he can at most obtain $zQ^* - \hat{b}$. If he does not set the agenda, he will obtain $zQ^* - K(Q^*) = zQ^* - \hat{b}$. According to our tie-breaking rules, those individuals will apply for proposal-making and will propose $(Q = Q^*, t^{-z_a} = K(Q^*))$. Accordingly, all individuals will apply for agenda-setting.

We stress that constitution $\mathcal{C}_1$ with any value of $\hat{b}$ in $[0, K(Q^*)]$ yields first-best allocations. A similar observation holds for all constitutions developed in this chapter. In order to ensure that all agenda-setters will also contribute to the provision of public goods, we set $\hat{b}$ at the highest level that guarantees first-best allocations.[3]

A remark regarding the size of $\hat{m}$ is in order. A priori $\hat{m}$ can be smaller than $\frac{1}{2}$. As long as undertaking the public project is irreversible, $m < \frac{1}{2}$ is feasible. If, however, the static process in this chapter is repeated and the public good can be undone at little cost, $\hat{m} < \frac{1}{2}$ would invite cycling. In such cases it is sensible to restrict $\hat{m}$ to $\hat{m} \geq \frac{1}{2}$.[4]

[3]This approach neutralizes the benefits from agenda-setting, and thus indirectly imposes equal burden sharing in this section. In a setting with a finite number of citizens, a particular level of agenda-setting costs may be important to balance the budget.

[4]To ensure that such constitutions still yield first-best allocations, the tax rule has to be modified. A fraction of individuals does not pay taxes, while the remaining group of individuals shares the tax burden equally.

8.4.2 An Example

Throughout the chapter we will use a simple example to illustrate the results. We assume

$$K(Q) = aQ^2,$$

for some constant a, and that

$$f(z) = \begin{cases} 1 & \text{if } z \in [0, 1], \\ 0 & \text{otherwise.} \end{cases}$$

Then, $\bar{z} = \frac{1}{2}$, so the first-best solution is given by

$$Q^* = \frac{1}{4a}.$$

Hence the set of parameters for the constitutional rules is given by

$$\hat{b} = \frac{1}{16a},$$
$$\hat{m} = 1 - \frac{aQ^{*2}}{Q^*} = \frac{3}{4}.$$

We note that a majority rule with a 75 %-vote threshold is needed to pass a proposal. An agenda-setter will propose ($Q^* = \frac{1}{4a}, t^{-z_a} = \frac{1}{16a}$). For voter $\bar{z} = \frac{1}{2}$, benefits from public-good provision are $\bar{z}Q^* - K(Q^*) = \frac{1}{8a} - \frac{1}{16a} = \frac{1}{16a}$.

8.5 First-Best Constitution and Aggregate Uncertainty

In this section we explore what are the first-best constitutions when there is aggregate uncertainty regarding costs, benefits, or jointly regarded costs and benefits.

Different states of nature, e.g. different costs, imply different levels of socially desirable public good provision. Accordingly, the aggregate tax revenue may differ. In this case, we will use a tax-sensitive majority rule *[**FM**$(m(T))$]* under which the required majority depends on aggregate tax revenue. It will turn out that such rules can be constructed in a way that the majority requirement optimally adjusts to aggregate events.

To obtain first-best constitutions including this rule, we need the following monotonicity condition (MC):

(MC): Suppose that socially optimal tax revenues associated with two different states of nature, say state 1 and state 2 satisfy $T^1 > T^2$ then the shares of individuals benefiting from the provision of the socially optimal levels of public goods satisfy $m^1 \geq m^2$ in both states.

The (MC) states that a higher amount of taxes in one state of the world is associated with an equal or higher share of beneficiaries relative to the status quo. Note that in case of equal aggregate tax revenues, no conditions are imposed.

8.5.1 Aggregate Uncertainty Regarding Benefits

In this section we consider the case of aggregate uncertainty regarding the benefits of the public good. In particular, we assume that a citizen derives utility $V = d^X zQ$ from the level of the public good. The random variable z is the same as before, while d^X denotes an aggregate shock, with $X \in \{l, h\}$, such that $d^l < d^h$. In particular, X equals h with probability $p \in [0, 1]$ and l with probability $1 - p$.

We assume that the aggregate shock is realized after the constitution has been put in place, but before the legislative process starts. The socially optimal allocation is now characterized by two levels of public goods (Q^{*h}, Q^{*l}) given by $d^l\bar{z} = K'(Q^{*l})$ or $d^h\bar{z} = K'(Q^{*h})$, depending on whether d^h or d^l has been realized. We note that $Q^{*h} > Q^{*l}$ and hence $T^h = K(Q^{*h}) > T^l = K(Q^{*l})$. Moreover, we introduce

$$\hat{m}^X = 1 - F\left(\frac{AV(Q*^X)}{d^X}\right) \text{ for } X = h, l.$$

We note that $\hat{m}^h$ and $\hat{m}^l$ are equal to the shares of individuals who benefit from the proposal $(Q^{*h}, K(Q^{*h}))$ and $(Q^{*l}, K(Q^{*l}))$, where the states h or l occur respectively. Therefore, the monotonicity condition (MC) requires that

$$\hat{m}^h \geq \hat{m}^l. \tag{8.3}$$

This condition states that a higher level of socially optimal tax revenue T^h in the event of d^h is associated with a higher share of citizens supporting the proposal $(Q = Q^{*h}, t^{-z_a} = K(Q^{*h}))$ than the share of individuals supporting $(Q = Q^{*l}, t^{-z_a} = K(Q^{*l}))$ if d^l occurs. Hence (MC) is equivalent to

$$\frac{d^l}{d^h} \leq \frac{AV(Q^{*l})}{AV(Q^{*h})}.$$

As

$$\frac{d^l}{d^h} = \frac{K'(Q^{*l})}{K'(Q^{*h})},$$

(MC) is equivalent to

$$\frac{K'(Q^{*l})}{AV(Q^{*l})} \leq \frac{K'(Q^{*h})}{AV(Q^{*h})},$$

and is thus ensured if $\frac{d(\ln K(Q))}{d(\ln Q)}$ is weakly monotonically increasing in Q.

We consider the following constitution:

$$\mathcal{C}_2 := \left\{[CA(\hat{b})], [NS], [ETT^{-z_a}], [FM(m^*(T))]\right\},$$

where

$$\hat{b} = K(Q^{*l}),$$
$$m^*(T) = \begin{cases} \hat{m}^h & \text{if} \quad T > T^l, \\ \hat{m}^l & \text{otherwise.} \end{cases}$$

The following proposition shows that $\mathcal{C}_2$ can yield first-best allocations independently of whether d^h or d^l occurs.

Proposition 8.2 *Suppose (MC) holds. Then constitution $\mathcal{C}_2$ is first-best.*

The proof of Proposition 8.2 is given in the Appendix. The intuition for the result is as follows: Suppose that d^l is realized, then, any agenda-setter will propose $(Q^{*l}, t^{-z_a} = K(Q^{*l}))$, which will be supported by a share of $\hat{m}^l$ voters. Since $m^*(T^l) = \hat{m}^l$, the proposal is adopted. A higher amount of the public good will gain less than $\hat{m}$ voters and thus will not be adopted, as $\hat{m}^h \geq \hat{m}^l$. Suppose that d^h is realized. The proposal $(Q^{*h}, t^{-z_a} = K(Q^{*h}))$ will be made by an agenda-setter, as it gains just $\hat{m}^h$ voters.

We observe that the scheme $m^*(T)$ has a built-in flexibility, as the majority threshold depends on the aggregate tax outlays. In equilibrium, these tax revenues will vary according to whether d^h or d^l occurs. This feedback from aggregate tax revenues to the majority threshold means that the first-best allocations can be produced in both states.

We note that we can generalize the result to more complicated discrete or continuous distributions of d as long as (MC) holds. If (MC) holds, the ensuing function $m^*(T)$ is monotonically increasing and thus we can apply the construction from the proof of Proposition 8.2.

We illustrate Proposition 8.2 with the same example we introduced in the last section. We assume, in addition, that $d^l = \frac{1}{2}$ and $d^h = \frac{3}{2}$. The socially optimal levels of the public good are $Q^{*l} = \frac{1}{8a}$, $Q^{*h} = \frac{3}{8a}$. Hence $T^l = \frac{1}{64a}$, $T^h = \frac{9}{64a}$.

$$\hat{m}^h = 1 - F\left(\frac{\frac{9}{64a}}{\frac{3}{8a} \cdot \frac{3}{2}}\right) = 1 - F\left(\frac{1}{4}\right) = \frac{3}{4},$$

$$\hat{m}^l = 1 - F\left(\frac{\frac{1}{64a}}{\frac{1}{8a} \cdot \frac{1}{2}}\right) = 1 - F\left(\frac{1}{4}\right) = \frac{3}{4},$$

Therefore (MC) holds, and constitution $\mathcal{C}_2$ yields a socially efficient outcome.

8.5.2 Aggregate Uncertainty Regarding Costs

We next consider the other case: aggregate uncertainty regarding the costs of public goods. In particular, we assume that the cost of providing the public good is equal to $\omega^V K(Q)$. Here, ω^V represents the aggregate shock, with $V \in \{l, h\}$. V equals h with probability $\tilde{p}$ and l with probability $1 - \tilde{p}$, and $\omega^h > \omega^l$. Again, the aggregate shock is realized at the beginning of the legislative process.

The socially optimal allocation is characterized by two levels of public goods, Q^{*h}, Q^{*l}, defined by $\bar{z} = \omega^h K'(Q^{*h})$, $\bar{z} = \omega^l K'(Q^{*l})$. We note that $Q^{*h} < Q^{*l}$.

As in the case of aggregated uncertainty regarding benefits, for $V \in \{l, h\}$ we define

$$T^V = \omega^V K(Q^{*V}), \quad \hat{m}^V = 1 - F\left(\frac{\omega^V K(Q^{*V})}{Q^{*V}}\right).$$

The monotonicity condition (MC) is fulfilled if and only if *neither*

$$T^h > T^l \text{ and } m^l > m^h$$

nor

$$T^l > T^h \text{ and } m^h > m^l,$$

so the requirement is that the tax levels and the corresponding fractions of supporting individuals are "co-monotonic." We note that (MC) is always fulfilled for cost functions $K(Q) = aQ^n$ with $n \in \mathbb{N}$, $n > 1$, since in this case we have

$$Q^{*V} = \left(\frac{\bar{z}}{\omega^V an}\right)^{1/(n-1)}.$$

Therefore it follows that

$$m^V = 1 - F\left(\frac{\omega K(Q^{*V})}{Q^{*V}}\right) = 1 - F(\bar{z}/n)$$

is independent of ω^V.

We consider the following constitution:

$$\mathcal{C}_3 := \left\{[CA(\hat{b})], [NS], [ETT^{-z_a}], [FM(m^*(T))]\right\},$$

where

$$\hat{b} = \min\{T^l, T^h\}$$

and

$$m^*(T) = \begin{cases} \max\{\hat{m}^h, \hat{m}^l\} & \text{if } T > \min\{T^h, T^l\}, \\ \min\{\hat{m}^h, \hat{m}^l\} & \text{otherwise.} \end{cases}$$

We obtain the following result:

Proposition 8.3 *Suppose (MC) holds. Then constitution $\mathcal{C}_3$ is first-best.*

The proof of Proposition 8.3 is given in the Appendix. The intuition is similar to the case where there is aggregate uncertainty regarding benefits. The majority rule is a step function, and in equilibrium the required majority threshold becomes contingent on the aggregate state. This yields socially efficient allocations.

For our parametrized example we choose $\omega^h = \frac{3}{2}$ and $\omega^l = \frac{1}{2}$. The socially optimal amounts of public goods are $Q^{*h} = \frac{1}{6a}$ and $Q^{*l} = \frac{1}{2a}$, therefore $T^h = \frac{1}{2a}$ and $T^l = \frac{1}{8a}$, and

$$\hat{m}^h = 1 - F\left(\frac{\frac{1}{36a}}{\frac{1}{6a} \cdot \frac{2}{3}}\right) = 1 - F\left(\frac{1}{4}\right) = \frac{3}{4},$$

$$\hat{m}^l = 1 - F\left(\frac{\frac{1}{4a}}{\frac{1}{2a} \cdot \frac{2}{1}}\right) = 1 - F\left(\frac{1}{4}\right) = \frac{3}{4}.$$

We now give a second example, this time with $T^h = T^l$, so that the monotonicity condition is trivially satisfied. Suppose that $K(Q) = e^{2Q}$ and $F(z) = z$ for all $z \in [0, 1]$, i.e. z is uniformly distributed on $[0, 1]$. As before, $\omega^l = \frac{1}{2}$ and $\omega^h = \frac{3}{2}$. Then, using $\overline{z} = \frac{1}{2}$, the first-best solutions are given by

$$Q^{*h} = \frac{1}{2}\log\left(\frac{1}{4\omega^h}\right) \text{ and } Q^{*l} = \frac{1}{2}\log\left(\frac{1}{4\omega^l}\right).$$

Hence $Q^{*l} > Q^{*h}$. Aggregate tax revenue is given by

$$T^h = \omega^h e^{\frac{2}{2}\log(\frac{1}{4\omega^h})} = \frac{1}{4} = T^l = \omega^l e^{\frac{2}{2}\log(\frac{1}{4\omega^l})}.$$

In this case, the corresponding majorities differ:

$$\hat{m}^h = 1 - F\left(\frac{T^h}{Q^{*h}}\right) = 1 - \frac{\overline{T}}{Q^{*h}} < 1 - \frac{\overline{T}}{Q^{*l}} = \hat{m}^l, \quad \text{where } \overline{T} := T^h = T^l,$$

as $Q^{*l} > Q^{*h}$. According to Proposition 8.3, constitution $\mathcal{C}_3$ is first-best.

8.5.3 *Joint Aggregate Uncertainty*

In this section we consider the most demanding case, joint aggregate uncertainty regarding benefits and costs.

In particular, we assume that the citizens' utility is $d^X zQ$ with $X \in \{h, l\}$ and $d^h > d^l$, while the aggregate cost of providing the amount Q is $\omega^V K(Q)$ with $V \in \{h, l\}$ and $\omega^h > \omega^l$. The four possible states of the world are thus

$$(d^h, \omega^h), (d^h, \omega^l), (d^l, \omega^h), (d^l, \omega^l).$$

The socially optimal allocation is characterized by $d^X \bar{z} = \omega^V K'(Q^{*XV})$, with $X, V \in \{l, h\}$. We obtain:

$$Q^{*hl} > \max\left\{Q^{*hh}, Q^{*ll}\right\} > \min\left\{Q^{*hh}, Q^{*ll}\right\} > Q^{*lh}.$$

Moreover, $Q^{*hh} > Q^{*ll}$ if and only if $\frac{d^h}{\omega^h} > \frac{d^l}{\omega^l}$. We define the corresponding levels of aggregate taxes as

$$T^{XV} = \omega^V K(Q^{*XV}), \text{ for } X, V \in \{l, h\}.$$

The first index X of T^{XV} denotes the level of benefits, the second index V denotes the level of costs. It is not possible in general to determine how the different realizations of T^{XV} relate to each other. For example, it is obvious that $T^{hl} = \omega^l K(Q^{*hl}) > \omega^l K(Q^{*ll}) = T^{ll}$, as $Q^{*hl} > Q^{*ll}$, but without further assumptions we cannot say whether $T^{hl} = \omega^l K(Q^{*hl}) > \omega^h K(Q^{*hh}) = T^{hh}$ holds, although we know that $Q^{*hl} > Q^{*hh}$ is true as well.

We define

$$\hat{m}^{XV} = 1 - F\left(\frac{\omega^V K(Q^{*XV})}{d^X Q^{*XV}}\right).$$

The monotonicity condition implies that no state $(X, V) \in \{(h, h), (h, l), (l, h), (l, l)\}$ exist such that $T^X > T^V$ and $m^V > m^X$. Thus the monotonicity condition implies that the four states of the world can be named A, B, C, D such that

$$T^A \geq T^B \geq T^C \geq T^D \text{ and } m^A \geq m^B \geq m^C \geq m^D.$$

We consider the following constitution:

$$\mathcal{C}_4 := \left\{[CA(\hat{b})], [NS], [ETT^{-z_a}], [FM(m^*(T))]\right\},$$

where $\hat{b} = T^D$ and

$$m^*(T) = \begin{cases} m^D & \text{if } T \leq T^D, \\ m^C & \text{if } T^D < T \leq T^C, \\ m^B & \text{if } T^C < T \leq T^B, \\ m^A & \text{if } T^B < T. \end{cases}$$

The monotonicity condition ensures that with this constitution a fraction of *at least* m^X of supporting voters is needed to bring through a proposal that involves a tax level *strictly higher* than T^X. We point out that this statement holds true, even if one of the cases in the definition of m^* collapses to the empty set. Assume, for example, the parameters are such that $T^C = T^B$. Then, to bring through a proposal involving a tax level strictly higher that T^B, one would need a fraction of at least $m^A \geq m^B$ supporting voters.

This characteristic of the constitution is the key to the following Proposition:

Proposition 8.4 *Suppose (MC) holds. Then constitution* $\mathcal{C}_4$ *is first-best.*

The proof of Proposition 8.4 is given in the Appendix.

Example 1 For the example introduced in Sect. 8.4.2, i.e. $K(Q) = aQ^2$ and $F(x) = x$, we may choose $\omega^l = \frac{1}{2}$, $\omega^h = \frac{3}{2}$, $d^h = \frac{3}{2}$ and $d^l = \frac{1}{2}$. The socially optimal levels of the public good are $Q^{*hh} = \frac{1}{4a} = Q^{*ll}$, $Q^{*hl} = \frac{3}{4a}$ and $Q^{*lh} = \frac{1}{12a}$. It follows that $T^{hh} = \frac{3}{32a}$, $T^{ll} = \frac{1}{32a}$, $T^{hl} = \frac{9}{32a}$ and $T^{lh} = \frac{3}{288a}$. We obtain the following ordering:

$$T^{hl} > T^{hh} > T^{ll} > T^{lh}.$$

The required majorities are again given by

$$\hat{m}^{hl} = \hat{m}^{hh} = \hat{m}^{ll} = \hat{m}^{lh} = \frac{3}{4}.$$

Hence constitution $\mathcal{C}_4$ is first-best.

Example 2 As a second example, take the same set-up as in Example 1, but we use $K(Q) = e^{\frac{1}{8}Q} - 1$ instead of $K(Q) = aQ^2$. As $K'(Q) = \frac{1}{8}e^{\frac{1}{8}Q}$ we obtain that $Q^{*XV} = 8\log(\frac{4d^X}{\omega^V})$. The socially optimal levels of public good are given by

$$\begin{aligned} Q^{*hh} &= Q^{*ll} = 8\log(4) \approx 11.1, \\ Q^{*hl} &= 8\log(12) \approx 19.9 \text{ and} \\ Q^{*lh} &= 8\log(\frac{4}{3}) \approx 2.3 \end{aligned}$$

i.e. we have $Q^{*hl} > Q^{*hh} = Q^{*ll} > Q^{*lh}$. The general form of the corresponding tax level is $T^{XV} = \omega^V K(Q^{*XV}) = 4d^X - \omega^V$. Hence we obtain

$$\begin{aligned} T^{hh} &= \frac{9}{2}, \\ T^{ll} &= \frac{3}{2}, \\ T^{hl} &= \frac{11}{2} \text{ and} \\ T^{lh} &= \frac{1}{2}, \end{aligned}$$

so that $T^{hl} > T^{hh} > T^{ll} > T^{lh}$. Finally, the associated $m^{XV} = 1 - \frac{T^{XV}}{d^X Q^{*XV}}$ are given by

$$m^{hh} = m^{ll} \approx 0.73,$$
$$m^{hl} \approx 0.82 \text{ and}$$
$$m^{lh} \approx 0.57.$$

The ordering is given by $m^{hl} > m^{hh} = m^{ll} > m^{lh}$. Again constitution $\mathcal{C}_4$ implements a first-best allocation.

8.6 Ex Post Constraints

Common to all our constitutions is the property that individuals with low valuations of the public good are worse off with democratic provision of public goods than with the status quo. In principle, as democratic constitutions are chosen by unanimity under a veil of ignorance, ex post constraints do not need to be honored, as the constitution legitimizes the government's power to tax people.

In this section, we nevertheless explore whether ex post constraints could be honored. The reason is threefold. First, individuals may leave ex post the jurisdiction if they suffer too much. Second, as in Hellwig (2005), citizens may opt for social welfare functions with inequality aversion. Third, honoring ex post constraints allows us to relate the constitutions to the standard mechanism literature.

In particular, we explore in this section the scope of subsidizing voting losers without sacrificing the efficiency properties of the constitution. In particular, we assume that all individuals who voted against the proposal will receive a subsidy $s^L > 0$, which we will denote by the rule $SL(s^L)$. In a continuum model, such a rule would destroy the efficiency properties of our constitutions, as knowing that they have no impact on the voting outcome, individuals would oppose any proposal in order to receive s^L. In a finite version of our economy this is less clear, as individuals may be pivotal. To explore the potential of our constitutions under such circumstances, we keep the tractable structure of our continuum framework, but we assume the following:

No-switching assumption: All individuals in a subset $\Omega \subseteq [0, 1]$ will support a proposal P if the following conditions are met:

(i) All individuals $i \in \Omega$ are better off with P than with the status quo, or they are indifferent. All $i \in [0; 1] \setminus \Omega$ are worse off with the proposal than without.
(ii) If P is seconded by all agents in Ω, except for a subset of measure 0, then the proposal is adopted. Otherwise, it is rejected.

The no-switching assumption mimics being pivotal in a finite population. It is the best possible assumption for constitutions to work in the continuum version of our model.[5] Our main result in this section shows that, with this assumption, participation may be eased and general voluntary participation and first-best allocation may be compatible. We use the model variant with no aggregate risk.

Proposition 8.5 *(i) There exists an* $s^L > 0$ *and an* $\hat{m}$ *such that the constitution*

$$\tilde{C}_1 = \left\{[CA(\hat{b})], [SL(s^L)], [ETT^{-z_a}], [M(\hat{m})]\right\}$$

with

$$\hat{b} = 0,$$

yields a positive level of Q. *The resulting allocation is a Pareto-improvement over the status quo.*

(ii) There exist constellations (e.g. $f(z) = 1$, $K(Q) = aQ^2$*), an* $s^L > 0$ *and* $\hat{m}$*, such that* $\tilde{C}_1$ *yields* Q^* *and the allocation is a Pareto-improvement over the status quo.*

(iii) There are constellations for which there exist no $s^L > 0$ *and* $\hat{m}$*, such that* $\tilde{C}_1$ *yields* Q^* *and no individual is worse off compared to the status quo.*

The proof is given in the Appendix. Proposition 8.5 shows that, under the favorable no-switching assumption, democratic constitutions can engineer a Pareto-improvement. When z is uniformly distributed it is even possible for democratic constitutions to simultaneously yield first-best levels of Q and voluntary participation.

When we allow voluntary participation, we address the same question as Hellwig (2003), albeit with an infinite number of agents. It is thus important to relate the above proposition to his results.

We first observe that the impossibility result of Hellwig (Proposition 3.10), according to which incentive-compatibility obviates the implementation of first-best outcomes, does not apply in our framework, as Hellwig's argument is based on the agents' uncertainty about the amount of the public good that will be provided. This uncertainty, in turn, results from the agents' uncertainty about the actual distribution of types in the society. The uncertainty vanishes if—as is the case in our model—the number of agents tends to infinity. In the limit, the implementation of first-best outcomes may be possible, as is shown by the example given above in Proposition 8.5, part (ii).

Second, Proposition 6.1 of Hellwig's paper states that when the number of agents becomes large, the quantity of the public good provided under a second-best, incentive-compatible mechanism follows approximately a (truncated) normal distribution around the first-best quantity. The variance is proportional to n when the utility is linear and costs are quadratic. For our model—in which the set of agents is normalized to the unit interval—to be obtained as a limit of a model with finitely many

[5] To ensure no-switching in a continuum model, one would need open ballots and coordination of voting behavior, such that switching by one individual $i \in \Omega$ would simultaneously trigger deviation by a subset in Ω of positive measure.

agents, quantities must be rescaled with $1/n$, which gives a variance proportional to $1/n$. In the limit, the variance vanishes, and the mechanism implements the first-best outcome. Our example is thus also in line with Hellwig's result in Proposition 6.1.

8.7 Conclusion

Our analysis has shed light on the potential of liberal democracies for achieving first-best allocations. Numerous issues deserve further attention in this research program. Most importantly, it will be useful to investigate optimal constitutions for circumstances where the monotonicity condition is violated.

Appendix

Proof of Proposition 8.1

Step 1: We first consider individuals for which $zQ^* > K(Q^*)$. Such individuals always exist, as set out in Sect. 8.3.2. Suppose that such an individual applies for agenda-setting and is recognized. We claim that he will propose $(Q = Q^*, t^{-z_a} = K(Q^*))$. For notational convenience, as all proposals will involve $s(\cdot) \equiv 0$, the subsidy function is neglected in the following.

This proposal will be accepted, as a share of $\hat{m} = 1 - F(AV(Q^*))$ voters are better off than with the status quo and will thus support the proposal. The utility of the agenda-setter is $zQ^* - \hat{b} = zQ^* - K(Q^*) > 0$. A proposal $(Q < Q^*, t^{-z_a} = K(Q))$ would also be accepted but generates smaller utility $zQ - \hat{b} = zQ - K(Q^*)$ for the agenda-setter. A proposal $(Q > Q^*, t^{-z_a} = K(Q))$ would not be adopted as the share of supporting voters is smaller than $\hat{m}$. Hence individuals with $zQ^* > K(Q^*)$ strictly benefit from setting the agenda, relative to the status quo, and will thus apply for agenda-setting and make a proposal $(Q = Q^*, t^{-z_a} = K(Q^*))$ if recognized.
Step 2: Consider an individual with $zQ^* \leq K(Q^*)$. Suppose that he applies for agenda-setting and is chosen to make a proposal. As he has to pay $\hat{b} = K(Q^*)$ and can renounce taxing himself, the same considerations as in Step 1 imply that the best proposal is $(Q = Q^*, t^{-z_a} = K(Q^*))$. According to our tie-breaking rule, all individuals with $zQ^* \leq K(Q^*)$ apply for agenda-setting, as they are indifferent between applying and not applying. $\square$

Proof of Proposition 8.2

Step 1: Suppose that d^l has been realized. As the costs for an agenda-setter are fixed, a citizen that applies for agenda-setting and who is recognized as such proposes the maximum level of public goods that will be supported by the electorate. The candidate proposal $(Q^{*l}, t^{-z_a} = K(Q^{*l}))$ is supported by a share of $\hat{m}^l$ voters, and the proposal will be accepted, since $T \leq T^l$ and the vote threshold $\hat{m}^l$ applies.

Any proposal $(Q, t^{-z_a} = K(Q))$ with $Q > Q^{*l}$ would be rejected, as fewer than $\hat{m}^l$ voters will support the proposal and $\hat{m}^h \geq \hat{m}^l$ supporting votes would be required. Hence the candidate proposal is optimal.

Following the same logic as in Proposition 8.1, all citizens will apply for agenda-setting.

Step 2: Suppose that d^h has been realized. Any agenda-setter will propose $(Q^{*h}, t^{-z_a} = K(Q^{*h}))$ in this case.

The proposal would be supported by $\hat{m}^h$ voters. As $T > T^l$, the required threshold is also $\hat{m}^h$, so the proposal is adopted. A higher level of public goods would be supported by a fraction of voters less than $\hat{m}^h$.

By the same logic as in Proposition 8.1, all citizens will apply for agenda-setting. □

Proof of Proposition 8.3

We have seen that the monotonicity condition (MC) is fulfilled if and only if *neither*

$$T^h > T^l \text{ and } m^l > m^h$$

nor

$$T^l > T^h \text{ and } m^h > m^l.$$

Therefore, the two possible states of the world, h and l, can be renamed A and B, i.e. $V \in \{h, l\} = \{A, B\}$, such that both $T^A \geq T^B$ and $m^A \geq m^B$. Now

$$m^*(T) = \begin{cases} \hat{m}^A & \text{for } T > T^B, \\ \hat{m}^B & \text{for } T \leq T^B. \end{cases}$$

Step 1: Suppose that ω^B has been realized and that an individual is recognized as the agenda-setter. By the same line of reasoning as in Proposition 8.2, he proposes $(Q^{*B}, t^{-z_a} = \omega^B K(Q^{*B}))$, which is supported by a share of $\hat{m}^B$ voters and accepted, since $T \leq T^B$ and the vote threshold $\hat{m}^B$ applies. Any proposal $(Q, t^{-z_a} = \omega^B K(Q))$ with $Q > Q^{*B}$ would be rejected, as strictly fewer than $\hat{m}^B$ voters would support the proposal but $\hat{m}^A \geq \hat{m}^B$ supporting voters would be required. Following the same logic as in Proposition 8.1, all citizens will apply for agenda-setting.

Step 2: Suppose that ω^A has been realized. Again, using the same logic, an agenda-setter will propose $(Q^{*A}, t^{-z_a} = \omega^A K(Q^{*A}))$, which will be adopted as it is supported by $\hat{m}^A$ voters. A strictly higher level of the public good would not be adopted, as it would be supported by a strictly smaller measure of voters than $\hat{m}^A$, while the required threshold would be $\hat{m}^A$ since the corresponding tax-level would be strictly higher than $T^A \geq T^B$. Again, all citizens apply for agenda-setting. □

Proof of Proposition 8.4

Suppose that state $W = (X, V) \in \{A, B, C, D\}$ has been realized, associated with the event (d^X, ω^V), with $X, V \in \{l, h\}$. Suppose that an individual is recognized as the agenda-setter. We claim that he will propose $(Q = Q^W, t^{-z_a} = \omega^V K(Q^W) = T^W)$.

The candidate proposal is supported by a share of $\hat{m}^{XV}$ voters. The proposal will be accepted, since $T \le T^W$ and a vote threshold of at most $\hat{m}^W$ applies. Any proposal $(Q, t^{-z_a} = \omega^j K(Q))$ with $Q > Q^W$ would be rejected, since strictly fewer than $\hat{m}^W$ voters would support the proposal, but, as we have already pointed out above, just before Proposition 8.4, at least $\hat{m}^W$ supporting voters would be required. Every agenda-setter proposes the maximum level of public goods that will be supported by the electorate. Any proposal $(Q, t^{-z_a} = \omega^j K(Q))$ with $Q < Q^W$ will not maximize the utility of the agenda-setter, hence he will not make such a proposal. The candidate's proposal is therefore optimal.

Following the same logic as in Proposition 8.1, all citizens will apply for agenda-setting. □

Proof of Proposition 8.5

Step 1: We construct a Pareto-improvement. First of all, notice that making a proposal with $Q > 0$ that is a Pareto-improvement over the status quo, requires that

$$s^L = t.$$

Indeed, as there exist individuals with z so small that $zQ \in [0, t)$ with positive probability, they would be harmed if $s^L < t$. Thus, individuals rejecting the proposal need to be compensated for their taxes.
Step 2: Suppose that a level $Q > 0$ is proposed. Recall that the budget constraint is

$$t = K(Q) + t \cdot F\left(\frac{t}{Q}\right), \tag{8.4}$$

which implicitly determines the tax rate t. All citizens with $zQ < t$ will reject the proposal, so a share of $F(t/Q)$ individuals have to be subsidized by $s^L = t$. Costs to provide the public good and subsidies have to be covered by taxes.
Step 3: We prove point (i). Let $\kappa \in (0, 1)$ such that $F(\kappa) = \frac{1}{2}$. By our assumptions on the cost function, we know that there exists a $\tilde{Q} > 0$, such that $K''(\tilde{Q})\tilde{Q} < \frac{1}{2}\kappa$, and that this holds for any $Q < \tilde{Q}$. Taking the second order Taylor expansion of the cost function $K(\cdot)$ around 0, yields

$$K(Q) = K'(Q)Q + \frac{1}{2}K''(Q)Q^2 + \mathrm{O}(Q^3),$$

so it follows that we can find a sufficiently small $\hat{Q}$, such that for $t = \kappa\hat{Q}$, we have

$$\begin{aligned} K(\hat{Q}) + t \cdot F\left(\frac{t}{\hat{Q}}\right) &= K'(\hat{Q})\hat{Q} + \frac{1}{2}K''(\hat{Q})\hat{Q}^2 + \mathrm{O}(\hat{Q}^3) + \frac{1}{2}\kappa\hat{Q} \\ &< \kappa\hat{Q} = t. \end{aligned}$$

For $t = 0$ on the other hand, we have that

$$K(Q) + t \cdot F\left(\frac{t}{Q}\right) = K(Q) > 0 = t.$$

Hence, by the Intermediate Value Theorem, there exists $\hat{t} \in (0, \kappa \hat{Q})$ such that

$$\hat{t} = K(\hat{Q}) + \hat{t} F\left(\frac{\hat{t}}{\hat{Q}}\right),$$

with $F(\hat{t}/\hat{Q}) < \frac{1}{2}$.

Therefore, if we set

$$\hat{m} = 1 - F\left(\frac{\hat{t}}{\hat{Q}}\right) > \frac{1}{2}$$

and invoke the no-switching assumption, constitution $\tilde{\mathcal{C}}_1$ will implement $\hat{Q}$. Indeed, only individuals with $z = \frac{\hat{t}}{\hat{Q}}$ are indifferent, and by continuity of the distribution $F(\cdot)$ this occurs with probability 0.

Implementing $\hat{Q}$ is a Pareto-improvement, as a fraction $\hat{m}$ of individuals is strictly better off with the public good and paying $\hat{t}$, and a fraction $1 - \hat{m}$ is also better off with the public good, as they receive subsidies $s^L = \hat{t}$.

Step 4: We prove the existence of the constellations in point (ii) and (iii) by giving two explicit examples. First, note that the budget constraint can be rewritten as

$$\big(1 - F(t/Q^*)\big)t = K(Q^*).$$

Example A: Let us assume that z is uniformly distributed and $K(Q) = \frac{1}{2}Q^2$. Then

$$Q^* = \frac{1}{2}\,.$$

The budget constraint is

$$(1 - 2t)t = \frac{1}{8}\,.$$

This equation has a unique solution at $t^* = \frac{1}{4}$. We set $\hat{m} = \frac{1}{2}$. At a tax rate $t^* = \frac{1}{4}$ all citizens with $z > \frac{1}{2}$ have a utility $zQ^* - t^* > 0$. All individuals with $z < \frac{1}{2}$ would be worse off without subsidization. With subsidies their utility is zQ^*. Example A proves (ii).

Example B: Again let $K(Q) = \frac{1}{2}Q^2$, but now let $f(z) = 2 - 2z$, which yields $F(z) = 2z - z^2$. The mean value is given by $\bar{z} = \int_0^1 \big(2z - 2z^2\big)dz = \frac{1}{3}$. Hence, $Q^* = \frac{1}{3}$. The budget constraint becomes

$$\big(1 - 6t + 9t^2\big)t = \frac{1}{18}\,,$$

which has no solution in the interval $[0; Q^*]$. This example proves (iii). □

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Chapter 9
Minority Voting and Public Project Provision

9.1 Background

The main goal of Minority Voting is to compensate the tyranny of the majority through the "counter-tyranny" of a protected minority. Ideally, voting rules should award a just share of decision power across decisions to every voter. After a first voting round, the members of the majority are rewarded with a decision that corresponds to their wishes, contrary to the members of the minority, who incur a loss. With Minority Voting, this loss in the first round is compensated by the exclusive right to vote on a second issue.

Unless the second decision is unanimous, compensating the members of the minority with an exclusive voting right cannot ensure that all members will win in the second round: Some members of the minority will be in the minority again. Yet, even such "double losers" will realize that overall, they received more voting rights than the first majority. This extra-power of decision could partly make up for their losing, even if they lose repeatedly. Theoretically, Minority Voting might even be applied to further voting rounds, thus reducing the number of losers continually.

Once established, the concept of Minority Voting sets the basis for many new ways to structure collective decision-making. All of them aim at some kind of balance between the majority and the minority from a first voting round. Yet, the restriction of voting rights in a second vote might not yield the expected balance, as the two voting rounds might not have the same weight. The voters might care very much for the first decision, while being indifferent about the second. This might be the case if the two votes are not connected at all or if the first has far-reaching implications while the second is of reduced importance, so that sole decision power on that issue has no consolatory power.[1]

[1] Another way to account for the differing importance of certain decisions is to modify Minority Voting itself (see Gersbach and Wickramage 2015).

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Thus, it might be useful to apply Minority Voting to *connected* decisions, allowing the minority to *maintain* some power of decision on an issue despite their losing in the first voting. Instead of splitting the voters into winners and losers, Minority Voting would reintegrate the losers into the decision process, and yield a *second majority* out of the first minority, which would join the first majority to support the voting outcomes.

This would lead to a different way to structure voting proposals. Instead of trying to connect decisions of equal importance for the voters, the simplest way to achieve connected voting rounds is to split any decision into two sub-decisions, and then apply Minority Voting.

As an example, let us examine a decision about a public project: With Minority Voting, the first decision should be whether to implement the project or not, and would be taken by all voters by simple majority decision. If the proposal is rejected, there is no further voting round, while if it is accepted, the voters who voted against it and lost are compensated through an exclusive voting right in a second vote. This second voting determines the financing scheme for the project, and the decision is taken according to the unanimity rule or the simple majority rule.

In this chapter, we compare this variant of Minority Voting to majority voting with regard to welfare, determine the chances and drawbacks of our scheme, and assess a strategy for further research.[2]

9.2 Introduction

We compare Minority Voting to simple majority voting with regard to allocating and financing public goods. We first focus on the case where the unanimity rule is applied in the second round, under minority voting. In Sect. 9.8 we discuss the alternative setting in which the simple majority rule is applied in the second round.

The following properties characterize equilibria under Minority Voting: When the public project is proposed in the first round, only those individuals will support the proposal who value the project highly, i.e. more than the maximum tax payment that may occur in the second round. If the project is supported in the first round, the supporting majority is minimal. Every supporting individual must be pivotal, since those individuals lose their voting right for the second round.

If the project is rejected in the first round, the collective choice process ends. If the project is adopted, an equilibrium financing scheme will involve subsidies for project losers in order to gain the support of all voting losers from the first round. All voting winners from the first round pay the highest admissible tax rate to finance the project and the subsidies. The agenda setter will also tax all other beneficiaries of the project in order to generate subsidies for himself.

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[2]This chapter is an updated version of an article with the same title, which appeared in *Economics: The Open-Access, Open-Assessment E-Journal* in Gersbach (2009).

The attractive feature of the Minority Voting scheme is that individuals who benefit largely from a project pay more taxes, while individuals who benefit little, or are disadvantaged by it, will be protected from high tax payments. Moreover, Minority Voting with the unanimity rule in the second round ensures that only Pareto improvements occur and that three standard inefficiencies in democratic decision-making are avoided: Inefficient projects are neither proposed nor adopted; inefficient redistribution schemes are neither proposed nor adopted; when proposed, efficient projects are not rejected.

The drawback of Minority Voting is that efficient projects may not be proposed in the first round. Accordingly, we compare Minority Voting with the standard simple-majority-rule framework, both coupled with the same tax-protection rule, and compare the relative social welfare of the schemes. In this chapter, we provide a first pass of relative welfare comparisons between Minority Voting and simple majority voting. On balance, the Minority Voting outperforms the simple majority voting in all circumstances except in the following constellation: A socially desirable project is adopted under the simple majority rule and redistribution costs do not outweigh the social gains while the project is not provided under Minority Voting.

We would also like to stress that the scheme analyzed in this chapter may be weakly inferior in terms of aggregate utility to other possible schemes as we will explore in the concluding section. The current chapter, therefore, is only a first pass to explore the virtues and drawbacks of Minority Voting in the context of public project provision. Numerous further analyses and extensions of our model should and can be performed as we will discuss at the end of the chapter.

This chapter is part of the recent literature on linking voting across problems. Casella (2005) introduced storable votes mechanisms, where a committee makes binary decisions repeatedly over time and where agents may store votes over time.[3] Experimental evidence has supported the efficiency gains of storable votes (Casella et al. 2006). Jackson and Sonnenschein (2007) show that, when problems repeat themselves many times, full efficiency can be reached at the limit, and that this insight essentially applies to any collective-decision problem. In Fahrenberger and Gersbach (2010), Minority Voting is developed for repeated project decisions where projects have a durable impact.[4] Linkages of voting across problems can also occur through vote trading, which goes back at least to Buchanan and Tullock (1962) and Coleman (1966) and has been developed, among others, by Brams and Riker (1973), Ferejohn (1974), Philipson and Snyder (1996) or Piketty (1994).

We propose to split project and financing decisions and to introduce Minority Voting in such a way that, at the outset, all individuals have the same right to influence outcomes and minorities are protected (e.g. Guinier 1994 or Issacharoff et al. 2002). Our proposal is aimed at resolving the “tyranny of the majority” problem by giving

[3] Storable voting is closely related to cumulative voting, as individuals can cast more than one vote for one alternative under such schemes (see e.g. Sawyer and MacRae 1962, Brams 1975, Cox 1990, Guinier 1994 or Gerber et al. 1998).

[4] In Fahrenberger and Gersbach (2012), Minority Voting is developed for situations in which citizens have a desire for harmony.

an emerging minority the exclusive right to decide about the financing scheme for a public project that a society has previously approved.

This chapter is organized as follows: In the next section, we introduce the model and the constitutional principles. In Sect. 9.4, we characterize the equilibria under simple majority voting, while Minority Voting is discussed in Sect. 9.5. In Sect. 9.6 we present the relative welfare comparison. In Sect. 9.7 we discuss an example, and Sect. 9.8 deals with possible extensions and alternatives of the model. Section 9.9 concludes. All proofs can be found in the appendix.

9.3 Model and Constitutional Principles

9.3.1 Model

We consider a standard social-choice problem of public project provision and financing. Time is indexed by $\tau = 0, 1$. The first period $\tau = 0$ is the constitutional period. In the constitutional period, a society Ω of N ($N > 3$, N odd) risk-neutral members decides how public project provision and financing should be governed in the legislative period. Citizens are indexed by $j \in \Omega = \{1, \dots, N\}$.

In the legislative period, $\tau = 1$, each citizen is endowed with e units of a private consumption good. The community can adopt a public project with per capita costs $k > 0$. We use V_j to denote the benefit of agent j from the provision of the public project. At $\tau = 0$, the benefit V_j is unknown and can hence be interpreted as a random variable.

We assume that V_j is uniformly distributed on $[\underline{V}, \overline{V}]$ with $\underline{V}, \overline{V} \in \mathbb{R}$ and $\underline{V} < \overline{V}$. In the legislative period we index members of the society according to their realized benefit levels, i.e. individual j is associated with the benefit $V_j \in [\underline{V}, \overline{V}]$ with $V_1 \leq V_2 \leq V_3 \leq \dots \leq V_N$. The vector $(V_1, \dots, V_N)$ is denoted by V.

Public projects must be financed by taxes. We assume that taxation is distortionary. Let $\lambda > 0$ denote the shadow cost of public funds. Accordingly, taxation uses $(1+\lambda)$ of taxpayer resources in order to levy 1 unit of resources for public projects and for transfers to citizens. Hence the overall per capita costs of the public project amount to $(1 + \lambda)k$. We assume that $0 < \lambda < 1$. Plausible values for tax distortions are considerably smaller than 100%.

We use t_j and s_j to denote citizen j's tax payment or subsidy, respectively. We introduce two separate variables (taxes and subsidies) rather than a single variable for the "net" contribution, because it makes the exposition more transparent and reduces the formal complexity.[5] Taxes are associated with distortions and there will be a tax protection rule, while subsidies are unlimited. Hence, it is useful to distinguish taxes and subsidies by different symbols.

[5]Formally, it would be possible to define a net contribution $n_i = t_i - s_i$. By using the $\max\{n_i, 0\}$ and $\min\{n_i, 0\}$ operators one could then distinguish between taxes and subsidies.

We define the variable g as indicating whether the public project is proposed ($g = 1$) or not ($g = 0$). The utility of citizen j, denoted by U_j, in the legislative period is given by[6]

$$U_j = e + gV_j - t_j + s_j. \tag{9.1}$$

Finally, the budget constraint of the society in the legislative period is given by

$$\sum_{j\in\Omega} t_j \; = (1+\lambda)\Big[gNk + \sum_{j\in\Omega} s_j \;\Big]. \tag{9.2}$$

We assume throughout the chapter that e is sufficiently large for agents to be able to pay taxes in all circumstances that may occur. We summarize the set of parameters that, together with random variable V, define the characteristics of the public project as (k, λ, N).

9.3.2 Socially Efficient Solutions

The fact that citizens are risk-neutral implies that, from an ex ante point of view or from an utilitarian perspective, it is socially efficient to provide the public project if and only if

$$\hat{V} := \frac{1}{N}\sum_{j\in\Omega} V_j \geq k(1+\lambda),$$

and taxes are raised solely to finance the public project. Any redistribution activities are detrimental from an ex ante point of view. A socially efficient tax scheme, for instance, is one where a socially desirable public project is financed by project winners and no subsidies are paid. In order to implement such a solution, a complete social contract would be necessary. We summarize our observations as follows:

Ex Ante First-best Allocation

Any allocation that provides the public project if and only if $\hat{V} \geq k(1+\lambda)$, *and that raises taxes only to finance the public project, is ex ante socially efficient.*

We follow the literature on incomplete social contracting (see Aghion and Bolton 2003 and Gersbach 2005) and assume that society allocates public projects by democratic procedures. Given socially efficient allocations, it is important at this stage to identify the sources of inefficiencies that may arise in legislative decision-making: There are four types of inefficiencies:

(1) inefficient projects are proposed and adopted
(2) pure redistribution proposals are made and adopted

[6] All tax and subsidy functions t_j and s_j respectively are assumed to be integrable. We only discuss mechanisms where this condition is trivially fulfilled.

(3) efficient projects are proposed and rejected
(4) efficient projects are not proposed

The latter two inefficiencies mean that delay in undertaking efficient public projects is costly. In this chapter we assume that not adopting projects results in the status quo. In the following we examine two ways of designing the democratic process for the provision of a public project, (1) the simple majority voting scheme and (2) the Minority Voting scheme.

9.3.3 Simple Majority Voting

In the constitutional period the society decides about the rules governing the legislative processes. The first democratic procedure is a standard simple majority voting scheme called SM.

Stage 1: At the start of the legislative period, the benefits of all citizens become common knowledge. Citizens decide simultaneously whether to apply for agenda setting ($\psi_j = 1$) or not ($\psi_j = 0$).

Stage 2: Among all citizens who apply, one citizen a is determined by fair randomization to set the agenda. The agenda setter proposes a project/financing package $\left(g, t_j, s_j\right)_{j\in\Omega}$. This choice is denoted by A_a.

Stage 3: Given A_a, citizens decide simultaneously whether to accept ($\delta_j(A_a) = 1$) or not ($\delta_j(A_a) = 0$). The proposal is accepted if a majority of members adopt it.

Note that if nobody applies for agenda setting, the status quo will prevail. Moreover, individuals know when they cast their votes in stage 3 who will be taxed and who will receive subsidies if the proposal is accepted. Obviously, the status quo also prevails if a proposal to change it does not receive enough yes–votes, as required by the majority voting rule.

An equilibrium for stages 1 to 3 can be described as a set of strategies

$$\left(\psi, A, \delta\right),$$

where $\psi = (\psi_j)_{j\in\Omega}$, $A = (A_a)_{a\in\Omega}$, $\delta = (\delta_j)_{j\in\Omega}$ and where $\delta_j = \delta_j(A_a)$ depends on the proposed agenda A_a.

To describe the application and voting outcome in our model, we use weak dominance criteria. Elimination of weakly dominated strategies is a standard assumption for eliminating the multiplicity of equilibria based on the trembling-hand perfection of Nash equilibria.

As individuals cannot gain anything from strategic voting, since voting in our model is a simple binary decision, this procedure implies that agents participate and vote according to their preferences, i.e. they vote for their most preferred alternative. The elimination of weakly dominated strategies with respect to voting, henceforth (EWSV), is thus captured by the following rule:

- **(EWSV)** *Suppose an agenda setter a has been drawn randomly. Then, given his proposed agenda* A_a*, the voting strategies are* $\delta_j^*(A_a) = 1$ *if the net utility* $u_j = gV_j + s_j - t_j$ *from* A_a *is nonnegative and* $\delta_j^*(A_a) = 0$ *otherwise.*

It is obvious that (EWSV) implies unique voting equilibria, so we can also use the weak dominance criterion for the decision on whether to apply for agenda setting (stage 1), henceforth (EWSA):

- **(EWSA)** *Agents eliminate weakly dominated strategies in stage 1.*

Since the requirement (EWSV) ensures that the voting outcome is unique, we can use $U_j(A_a)$ to define the utility level that an agent j will achieve if agent a has proposed agenda A_a and voting has taken place. Moreover, let the set of all possible agendas be denoted by $\mathcal{A}$. In order to simplify the exposition, we assume that the following three tie-breaking rules are applied:

- If an agent j cannot strictly improve his utility by agenda setting, he will not apply for agenda setting.
- If an agenda setter knows with certainty that any agenda with $g = 1$ will be rejected, he will propose an agenda with $g = 0$.
- If an agenda setter is indifferent between an agenda that leads to $g = 1$ and another that yields $g = 0$, he will propose the former.

Note that $U_j(A_a)$ is based on the optimal voting strategies of all agents. For instance, $U_j(A_a) = e$ if A_a is rejected. In what follows we will assume throughout—without referring to the fact explicitly—that (EWSV), (EWSA), and the tie-breaking rules are all applied.

9.3.4 Minority Voting

In this section we introduce an alternative democratic decision process called *Minority Voting* (MV).

Stage 1: At the start of the legislative period, citizens observe their own benefit V_j and the utilities of all other individuals. Citizens decide simultaneously whether to apply for agenda setting ($\psi_j = 1$) or not ($\psi_j = 0$).

Stage 2: Among all citizens who apply, one citizen a_1 is determined by fair randomization to set the agenda. The agenda setter decides whether undertaking the public project should be considered or whether a pure redistribution proposal should be considered. Denote this choice by $g_{a_1}^{MV} \in \{1, 0\}$. If nobody applies for agenda setting, the status quo prevails.

Stage 3: Citizens decide whether to accept $\big(\delta_j(g_{a_1}^{MV}) = 1\big)$ or not $\big(\delta_j(g_{a_1}^{MV}) = 0\big)$. The proposal is accepted if a majority of members adopt it. We use $\mathcal{M} = \{j \mid \delta_j(g_{a_1}^{MV}) = 0\}$ to denote the set of individuals who voted against the proposal.

Stage 4: If $g_{a_1}^{MV}$ has been adopted, i.e. if $|\mathcal{M}| < \frac{N+1}{2}$, all agents of the minority can apply to propose a financing scheme. Among those, a citizen a_2 is determined by fair randomization and proposes a package $(t_j, s_j)_{j\in\Omega}$. Denote this choice by T_{a_2}. If nobody applies for agenda setting, the status quo prevails.

Stage 5: Given T_{a_2}, citizens who belong to $\mathcal{M}$ decide simultaneously whether to accept the financing scheme $T_{a_2}\big(\delta_j(T_{a_2}) = 1\big)$ or not $\big(\delta_j(T_{a_2}) = 0\big)$. T_{a_2} is accepted if, and only if, all individuals in $\mathcal{M}$ vote $\delta_j(T_{a_2}) = 1$, i.e. the unanimity rule applies. If T_{a_2} is accepted, the plan $(g_{a_1}^{MV} = 1, T_{a_2})$ is implemented. Otherwise the status quo $(g_{a_1}^{MV} = 0,\ t_j = s_j = 0\ \ \forall\ j)$ prevails.

A number of remarks are in order here. First, there are several alternatives for resolving a situation where $g_{a_1}^{MV} = 1$ is accepted and T_{a_2} is rejected. For instance, one could allow for further rounds of financing proposals or one could design a default financing scheme to be applied together with $g_{a_1}^{MV} = 1$.[7]

Second, as all individuals would like to keep their voting right in stage 3, no majority can be formed for a proposal $g_{a_1}^{MV} = 0$ as supporting agents are worse off than when the status quo prevails. Therefore pure redistribution proposals will never be adopted under MV. The situation is different when $g_{a_1}^{MV} = 1$ has been proposed. Without support, the public project will not be provided. This may create incentives for individuals who benefit highly from a public project to support a proposal $g_{a_1}^{MV} = 1$.

Third, as with simple majority, to derive equilibria we use weak dominance to characterize subgame perfect equilibria. Moreover, we use the same tie-breaking rules that apply in simple majority voting for agenda setting with regard to public project provision (Stage 2). In Stage 4, we assume that all individuals apply for agenda setting and make a financing proposal as long as they are not worse off (relative to the status quo) if their proposals are adopted in Stage 5. Again, these tie-breaking rules merely simplify the exposition.

9.3.5 Tax Protection Rule

In the following sections we prepare the ground for the comparison of the two systems SM and MV by characterizing the equilibrium of the games. We do not impose any further rules on proposal-making, but we do assume an upper limit on taxes, denoted by $\hat{t}$. That is, a proposal that involves $t_j > \hat{t}$ for some individual j is unconstitutional, and the status quo prevails. Such tax protection rules are ubiquitous in modern democracies (Rangel 2005).[8] Note that the tax protection rule does not preclude an agenda setter voluntarily contributing more than $\hat{t}$ to the financing of the public project. Moreover, it could happen that an individual j is burdened by

[7]The implications of such extensions are left for future research.

[8]In 1983, for instance, the German Constitutional Court declared excessive tax burdens that would fundamentally impair wealth to be unconstitutional (Reding and Müller 1999).

a tax exceeding $\hat{t}$, but receives large subsidies and hence the net contribution is substantially smaller than $\hat{t}$. As we will see, in all equilibria with the simple majority rule or with Minority Voting, an individual will be either taxed or subsidized (or none) and hence it never occurs that t_j and s_j are both non-zero.

9.4 Equilibria Under Simple Majority Voting

We first characterize the equilibria under SM. For this purpose we use Ω_{-j} to denote the set $\Omega\backslash\{j\}$, i.e. the society with exception of individual j. Under simple majority voting everybody stands to gain from agenda setting as this will always enable the agenda setter to propose a pure redistribution proposal that benefits him. Hence we will have $\psi_j = 1$ in any equilibrium. We use I to denote an arbitrary subset of the society with $|I| = \frac{N-1}{2}$. In Stage 2 an agenda setter a solves the following problem:

$$\max_{(g,t_j,s_j)_{j\in\Omega}} \{U_a = e + gV_a + s_a - t_a\},$$

$$\text{s.t.} \quad \sum_{j=1}^{N} t_j = (1+\lambda)\Big[gNk + \sum_{j=1}^{N} s_j\Big],$$

and

$$\exists I \subset \Omega_{-a}, \text{ with } |I| = \frac{N-1}{2},$$

$$\text{s.t.} \quad U_j - e = gV_j + s_j - t_j \geq 0, \, j \in I.$$

We obtain:

Lemma 9.1
Suppose that the simple majority rule is applied. An equilibrium proposal $g = 0$ is associated with the redistribution scheme

$$t_j := \begin{cases} \hat{t} & \text{if } j \notin I_{+a} := I \cup \{a\}, \\ 0 & \text{if } j \in I_{+a}, \end{cases}$$

and

$$s_j := \begin{cases} 0 & \text{if } j \in \Omega_{-a}, \\ \frac{N-1}{2(1+\lambda)}\,\hat{t} & \text{if } j = a. \end{cases}$$

The lemma is obvious as all individuals in I_{+a} support the proposal and I_{+a} is the smallest majority the agenda setter can form. The minority of size $|I|$ is taxed by the highest possible rate allowed by the tax protection rule, $\hat{t}$. All individuals in the winning majority except the agenda setter do neither pay taxes nor receive subsidies. The agenda setter therefore extracts the highest amount of subsidies.

We next investigate the case $g = 1$. For this purpose we introduce the set

$$LW := \{j \in \Omega \mid V_j \geq \hat{t}\}.$$

Individuals belonging to LW are called large project winners. We also introduce the set

$$LW_{-a} := \begin{cases} LW \backslash \{a\} & \text{if } a \in LW \\ LW & \text{otherwise.} \end{cases}$$

We obtain:

Lemma 9.2
Under the simple majority rule, an equilibrium proposal $g = 1$ is associated with

- $s_j = 0$ *and* $t_j = \hat{t}$, *if* $j \in LW_{-a} \cup \left(\Omega \backslash I_{+a}\right)$;
- $s_j = 0$, *and* $t_j = V_j$, *if* $j \in I \backslash LW_{-a}$ *and* $V_j \geq 0$;
- $s_j = -V_j$ *and* $t_j = 0$, *if* $j \in I$ *and* $V_j < 0$;
- $s_a = \max\{0, \bar{s}_a\}$ *and* $t_a = \max\{0, -(1+\lambda)\bar{s}_a\}$,

where

$$\bar{s}_a = \frac{1}{1+\lambda}\left(\sum_{j \in LW_{-a} \cup (\Omega \backslash I_{+a})} \hat{t} + \sum_{j \in I \backslash LW_{-a},\ V_j \geq 0} V_j - (1+\lambda)\Big[Nk - \sum_{j \in I,\ V_j < 0} V_j\Big]\right),$$

and

$$I = \begin{cases} \{\frac{N+3}{2}, \ldots, N\} & \text{if } a \leq \frac{N+1}{2}, \\ \{\frac{N+1}{2}, \ldots, N\} \backslash \{a\} & \text{if } a > \frac{N+1}{2}. \end{cases}$$

The proof can be found in the appendix.[9] Lemma 9.2 indicates that the choice of $g = 1$ is associated with both large-project winners and the minority paying the highest amount of taxes up to the level allowed by the tax protection rule, $\hat{t}$. Citizens who do not belong to the set of large project winners, but to the majority necessary to adopt the proposal, are taxed according to their benefits, or they are subsidized. Such a proposal maximizes the subsidies for the agenda setter.

The crucial question is whether $g = 1$ will be chosen in equilibrium, which is equivalent to the question whether the following condition (G) holds:

$$(G): \quad V_a + (1+\lambda)^{1-\mathrm{sg}(\bar{s}_a)}\bar{s}_a(g=1) \geq s_a(g=0),$$

where

$$\mathrm{sg}(\bar{s}_a) = \begin{cases} 1, & \bar{s}_a > 0, \\ 0, & \bar{s}_a \leq 0. \end{cases}$$

[9]Note that the tax payment of the agenda setter may be higher than $\hat{t}$ if he voluntarily decides to contribute more in order to secure the financing of the project.

Condition (G) compares the gains from choosing $g = 1$ (V_a and the maximal subsidies) and $g = 0$ (maximal subsidies). By using $|LW_{-a} \cup \Omega \setminus I_{+a}| - |I| = |LW_{-a} \cap I|$, and substituting $\bar{s}_a$, condition (G) can be detailed for both cases $\mathrm{sg}(\bar{s}_a) = 1$ and $\mathrm{sg}(\bar{s}_a) = 0$ respectively:

$$(G^+): \quad (1+\lambda)V_a + \sum_{j \in LW_{-a} \cap I} \hat{t} + \sum_{j \in I \setminus LW_{-a}, V_j \geq 0} V_j \geq (1+\lambda)(Nk - \sum_{j \in I, V_j < 0} V_j),$$

$$(G^-): V_a + \sum_{j \in LW_{-a} \cap I} \hat{t} + |I| \tfrac{\lambda}{1+\lambda} \hat{t} + \sum_{j \in I \setminus LW_{-a}, V_j \geq 0} V_j \geq (1+\lambda)(Nk - \sum_{j \in I, V_j < 0} V_j).$$

In other words, if and only if the agenda setter can generate tax revenues from project winners (under $g = 1$) that are sufficiently high to finance the project and to compensate project losers, he will propose $g = 1$.

In Appendix A we provide a general characterization of the equilibria under simple majority voting.

9.5 Equilibria With Minority Voting

9.5.1 Financing

We next consider MV. To prepare the equilibria, it is instructive to consider voting in Stage 3 first, assuming that financing will occur with certainty in Stages 4 and 5 if $g_{a_1}^{MV} = 1$ has been adopted. We obtain:

Proposition 9.1
Suppose Minority Voting is applied.

(i) Suppose individual a_1 has been chosen to set the agenda. If $|LW| \geq \frac{N+1}{2}$, the agenda setter proposes $g_{a_1}^{MV} = 1$. Exactly $\frac{N+1}{2}$ large project winners will accept the proposal.
(ii) If $|LW| < \frac{N+1}{2}$, nobody applies for agenda setting and the status quo prevails.

The proof can be found in the appendix. Recall that a proposal $g_{a_1}^{MV} = 0$ will never be supported under MV. An immediate consequence is the following:

Corollary 9.1
The voting equilibria in case (i) are indeterminate with respect to which of the set of large project winners will accept the proposal if $|LW| > \frac{N+1}{2}$.

In principle, all individuals with $V_j \geq \hat{t}$ prefer the project to be accepted, but they would like to reject the proposal $g_{a_1}^{MV} = 1$ in order to keep their voting rights. We use the following plausible refinement of voting equilibria:

Maximal Magnanimity

Suppose $g_{a_1}^{MV} = 1$ and $|LW| \geq \frac{N+1}{2}$, then all individuals with $j \geq \frac{N+1}{2}$ cast the vote $\delta_j(g_{a_1}^{MV} = 1) = 1$, while all individuals with $j < \frac{N+1}{2}$ vote $\delta_j(g_{a_1}^{MV} = 1) = 0$.

Under Maximal Magnanimity, those individuals who benefit most exclude themselves from the financing decision in order to enable that the project may be undertaken if the financing proposal is adopted in the fifth stage. Those individuals who benefit less and are not needed to form a majority reject the proposal. Their taxes will never exceed their benefits from the project. It is in this sense that such equilibria fulfill Maximal Magnanimity. For future references, we note that the set of voters $\mathcal{M}$ who voted against a project proposed is equal to $\{1, \ldots, \frac{N-1}{2}\}$ if $g_{a_1}^{MV}$ has been adopted.

We next consider the financing decision under MV. For this purpose, define

$$LW^{>} := \{j \mid V_j > \hat{t}\}$$

and suppose that $g_{a_1}^{MV} = 1$ has been adopted. An agenda setter a_2 has to gain unanimous support among the members of $\mathcal{M}$. Moreover, an individual applies for agenda setting if he can increase his utility. Hence, if $a_2 \in LW^{>}$ the project can be financed if[10]

$$(F^-): \quad V_{a_2} + |LW_{-a_2}| \cdot \hat{t} + \sum_{j \in \Omega_{-a_2} \setminus LW} \max\{V_j, 0\} \geq (1+\lambda)\big[Nk - \sum_{j \in \Omega_{-a_2}} \min\{V_j, 0\}\big].$$

It is not necessary for the agenda setter a_2 to be part of $LW^{>}$ for the project to be financed if

$$(F^+): \quad |LW| \cdot \hat{t} + \sum_{j \in \Omega \setminus LW} \max\{V_j, 0\} \geq (1+\lambda)[Nk - \sum_{j \in \Omega} \min\{V_j, 0\}]$$

holds. In this way, given a certain realization $(V_j)_{j \in \Omega}$, all projects (characterized by per capita cost k) that satisfy

$$(F) = \begin{cases} (F^-), & \text{if } a_2 \in LW^{>} \\ (F^+), & \text{otherwise} \end{cases}$$

can be provided. The condition (F) states that tax revenues from both large and small project winners are weakly larger than aggregate project costs and subsidy payments to project losers. The left side represents the maximal tax revenues that can be generated in the political process. The right side represents the minimal aggregate expenditure needed to implement a project. The next lemma determines which agents will apply for agenda setting in Stage 1.

[10] An agenda setter $a_2 \in LW^{>}$ may pay higher taxes than $\hat{t}$ in order to ensure the financing of the public project.

Lemma 9.3
Suppose that Minority Voting is applied.

(i) *If* $|LW| > \frac{N+1}{2}$ *and* (F^+) *holds with strict inequality, then all individuals will apply for agenda setting and would propose* $g_{a_1}^{MV} = 1$.
(ii) *If* $|LW| = \frac{N+1}{2}$ *and* (F^+) *holds with strict inequality, all individuals except those with* $V_j = \hat{t}$ *will apply for agenda setting and would propose* $g_{a_1}^{MV} = 1$.
(iii) *If* $|LW| \geq \frac{N+1}{2}$ *and* (F^+) *holds with equality, all individuals in* $LW^> := \{j \mid V_j > \hat{t}\}$ *will apply for agenda setting and would propose* $g_{a_1}^{MV} = 1$.
(iv) *If* $|LW| > \frac{N+1}{2}$ *and* (F^-) *holds with strict inequality for all* $a_2 \in LW^> \cap \mathcal{M}$ *but* (F^+) *is not satisfied, then all individuals in* $LW^>$ *will apply for agenda setting and would propose* $g_{a_1}^{MV} = 1$.
(v) *If* $|LW| > \frac{N+1}{2}$, *and* (F^-) *holds with equality for at least one* $a_2 \in LW^> \cap \mathcal{M}$, *then all individuals in*

$$LW^> \setminus \left\{ j \in \mathcal{M} \;\middle|\; (F^-) \text{ does not hold or holds with equality if } j = a_2 \right\}$$

will apply for agenda setting and would propose $g_{a_1}^{MV} = 1$.
(vi) *In all other cases nobody will apply for agenda setting.*

The proof of Lemma 9.3 follows directly from the fact that the project can only be financed if (F) holds and from the tie-breaking rule that agents will not apply for agenda setting if they cannot strictly improve their utility.

9.5.2 Overall Equilibria

After these preliminary considerations, we can characterize the equilibria of the five-stage game. For convenience, let $\mathcal{F} = \{j \in \mathcal{M} \mid (F) \text{ holds if } a_2 = j\}$.

Proposition 9.2
Suppose that the Minority Voting rule is applied.

(i) *If* $|LW| < \frac{N+1}{2}$ *or* $\mathcal{F} = \emptyset$, *then* $\psi_j = 0 \; \forall j \in \Omega$ *and the status quo prevails with* $E[U_j] = e$ *for all individuals.*
(ii) *If* $|LW| \geq \frac{N+1}{2}$ *and* $\mathcal{F} \neq \emptyset$, *we obtain the following subgame perfect equilibrium:*

Stage 1: *The individuals apply for agenda-setting as described in items (i)–(v) of Lemma 9.3.*

Stage 2: $g_{a_1}^{MV} = 1$.

Stage 3: $\delta_j(g_{a_1}^{MV} = 1) = \begin{cases} 1, & j \geq \frac{N+1}{2}, \\ 0, & j < \frac{N+1}{2}. \end{cases}$

Stage 4: *All individuals $j \in \mathcal{F}$ apply to propose a financing package and the randomly chosen agenda setter a_2 proposes*

$$T^*_{a_2} = \begin{cases} t_j = \hat{t} \;\; \textit{if} \;\; j \in LW_{-a_2}; \\ t_j = V_j \;\; \textit{if} \;\; j \in \Omega_{-a_2} \backslash LW \; \textit{and} \; V_j > 0; \\ s_j = -V_j \;\; \textit{if} \;\; j \in \Omega_{-a_2} \; \textit{and} \; V_j < 0; \\ t_{a_2} = \max\{0, -(1+\lambda)\bar{s}_{a_2}\}; \\ s_{a_2} = \max\{0, \bar{s}_{a_2}\}, \end{cases}$$

where

$$\bar{s}_{a_2} := (1+\lambda)^{-1} \sum_{j \in \Omega_{-a_2}} t_j - Nk - \sum_{j \in \Omega_{-a_2}} s_j.$$

Stage 5: $\delta_m(T^*_{a_2}) = 1$, *for all* $m \in \mathcal{M}$.

In such an equilibrium, the expected payoffs are

$$E[U_j] = \begin{cases} e + V_j - \hat{t} \;\; \textit{if}\; j \in LW \setminus \mathcal{F}; \\ e + (1 - \frac{1}{|\mathcal{F}|})(V_j - \hat{t}) + \frac{1}{|\mathcal{F}|}(V_j + (1+\lambda)^{1-sg(\bar{s}_{a2})}\bar{s}_{a2}), \;\; \textit{if}\; j \in LW \cap \mathcal{F}, ; \\ e + \frac{1}{|\mathcal{F}|}(V_j + (1+\lambda)^{1-sg(\bar{s}_{a2})}\bar{s}_{a2}), \;\; \textit{if}\; j \in \mathcal{F} \setminus LW; \\ e, \;\; \textit{if}\; j \notin LW \cup \mathcal{F}. \end{cases}$$

The proof can be found in the appendix.

9.6 Welfare Comparisons

9.6.1 Welfare Criteria

In this section we examine which voting scheme for the legislative period the society prefers to choose in the constitutional period. For a comparison of the two voting regimes at the constitutional period, three kinds of uncertainty have to be considered: The vector $(V_j)_{j \in \Omega}$ of project benefits; who the agenda setters, a or a_1, a_2 respectively will be; and what type j, the agent himself will be. An agent's ex ante expected utility in the simple majority voting scheme when all three types of uncertainty are present is denoted by $E_0[U^{SM}]$. It can be written as

$$E_0\big[U^{SM}\big] = \int_{\mathcal{V}} \Big(h(V) \sum_{m \in \Omega} P(a=m) \cdot E\big[U_j^{SM} \big| V, a\big]\Big)\, dV, \tag{9.3}$$

where $\mathcal{V} = [\underline{V}, \overline{V}]^N$ is the N-dimensional cube, $h(V)$ is the density function on $\mathcal{V}$, $P(a = m)$ represents the probability that individual m will be the agenda setter, and $E\big[U_j^{SM}\big|V, a\big]$ denotes the expected utility of an agent given (V, a), without knowing which j he will be.

With regard to Minority Voting, we have to distinguish the cases in which there is an agenda setter a_2 and those where the project will not be financed. We note that whether the project will be provided under MV depends solely on the conditions in Lemma 9.3 and not on who is the agenda setter a_1. It is therefore convenient to introduce an imaginary agenda setter $a_2 = 0$ if the project will not be provided. More precisely, we introduce the following definition:

Definition 9.1

$$a_2 = \begin{cases} \text{randomly chosen from } \mathcal{F}, & \text{if } \ |LW| \geq \frac{N+1}{2} \wedge \mathcal{F} \neq \emptyset, \\ 0, & \text{if } \ |LW| < \frac{N+1}{2} \vee \mathcal{F} = \emptyset. \end{cases}$$

The probability that $a_2 = m$, where $m \in \mathcal{F} \cup 0$, is

$$P(a_2 = m) = \begin{cases} \frac{1}{|\mathcal{F}|}, & \text{if } \ m \in \mathcal{F} \wedge |LW| \geq \frac{N+1}{2}, \\ 0, & \text{if } \ m = 0 \wedge \mathcal{F} \neq \emptyset \wedge |LW| \geq \frac{N+1}{2}, \\ 1, & \text{if } \ m = 0 \wedge \mathcal{F} = \emptyset, \end{cases}$$

$$E\big[U_j^{MV}\big|V, 0\big] = e.$$

With this definition we can write the ex ante expected utility in the Minority Voting scheme in a similar way as for majority voting:

$$E_0\big[U^{MV}\big] = \int_{\mathcal{V}} \Big(h(V) \sum_{m \in \mathcal{F} \cup 0} P(a_2 = m) \cdot E\big[U_j^{MV}\big|V, a_2\big]\Big)\, dV. \tag{9.4}$$

First, it would be interesting to identify the constellations (V, a, a_2) in which an agent would prefer the Minority Voting scheme from an ex ante perspective, that is, if he does not know his type j. The overall comparison from an ex ante perspective then depends on how the different situations are weighted in the aggregation process. More precisely, it depends on how large the difference is in expected utilities conditional on (V, a, a_2) and what the probability weights are. In this section we take the first step. As all individuals have the same probability of being some type j, we can define social welfare as

$$W^{SM/MV} = \sum_{j \in \Omega} U_j^{SM/MV},$$

which can be interpreted as the sum of ex ante expected utilities given (V, a, a_2), though the agents do not know what j they will be. More precisely,

$$E\big[U_j^{MV}\big|V, a_2\big] = \frac{W^{MV}}{N}, \ \forall j \in \Omega.$$

Similar definitions can be made for SM.

9.6.2 Comparisons

For the following comparisons, it is useful to note:

Fact 9.1
Under a Minority Voting rule, it depends only on the benefit vector V, if the project is proposed and accepted.[11] *This is different under simple majority voting, where it depends on the benefit level V_a of the agenda setter whether the project will be proposed or not.*

Consequently, the realization (V, a) directly determines the pair (g^{SM}, g^{MV}). It will transpire that most statements only require knowledge of (g^{SM}, g^{MV}).

Proposition 9.3
Suppose $|LW| < \frac{N+1}{2}$ or $\mathcal{F} = \emptyset$. Suppose that (G) does not hold. Then

$$E\big[U_j^{MV}\big|V, 0\big] > E\big[U_j^{SM}\big|V, a\big].$$

The proof can be found in the appendix.

The preceding proposition rests on the fact that the MV rule protects a society against inefficient redistribution proposals that will occur under SM if no project is proposed.

Proposition 9.4
If the project is not proposed, i.e. $g = 0$, the welfare loss due to redistribution is strictly higher under SM than under MV. If the project is provided, welfare costs of redistribution activities are weakly higher with SM than with MV.

The proof can be found in the appendix.

For the intuition of Proposition 9.4, we note that $|LW| \geq \frac{N+1}{2}$ must hold if $g^{MV} = g^{SM} = 1$. As $|LW| \geq \frac{N+1}{2}$, the agenda setter in SM does not have to care about the voting behavior of all individuals $\Omega_{-a} \setminus LW$ and consequently proposes the highest tax for them. This is different for the agenda setter a_2 in MV, as he needs the unanimous support of the votes of the minority. In this way, total tax payments, and hence welfare losses from redistribution must be weakly higher with SM than with MV.

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[11] The benefit vector V determines the set of agenda setters and whether the financing condition holds.

Further we observe:

Lemma 9.4
In MV only socially desirable projects will be proposed and adopted.

The proof can be found in the appendix.

We are now in a position to formulate the following result.

Proposition 9.5
From an ex ante social welfare perspective, simple majority voting is strictly preferable to Minority Voting if and only if $(g^{SM}, g^{MV}) = (1, 0)$ *and*

$$\sum_{\Omega} V_j > (1+\lambda)Nk + \lambda \sum_{\Omega} s_j^{SM}(g^{SM} = 1).$$

The proof can be found in the appendix.

The previous propositions and lemmata have shown that, under the proposed Minority Voting scheme, the first three possible inefficiencies of legislative decision making listed in Sect. 9.3.2 are avoided. For instance, Lemma 9.4 ensures that no inefficient projects are proposed and adopted. Proposition 9.3 shows that MV protects against pure redistribution proposals. However, Minority Voting suffers from the last inefficiency: In certain situations efficient projects are not proposed. In such cases, a simple majority rule may be preferable from an ex ante welfare perspective. Using Lemma 9.3 the necessary condition $(g^{SM}, g^{MV}) = (1, 0)$ for SM to be strictly preferable to MV is given by

$$\left[|LW| < \frac{N+1}{2} \vee \neg(F)\right] \wedge (G).$$

Consider the case where $|LW| \geq \frac{N+1}{2}$. Then a project would be provided in SM but not in MV if condition (G) holds and the financing condition (F) is violated ($\mathcal{F} = \emptyset$). In order to further characterize this case, denote by $\bar{a}_2$ the individual with the highest valuation of the project in the minority. That is, $\bar{a}_2$ is the agent in $\mathcal{M}$ for whom $V_{\bar{a}_2} \geq V_j, \ \forall j \in \mathcal{M}$. We note that if (F) is violated when $\bar{a}_2$ is the agenda setter, it must be violated for all $j \in \mathcal{M} \setminus \bar{a}_2$. Now we can formulate the following lemma:

Lemma 9.5
Suppose $|LW| \geq \frac{N+1}{2}$, *then* $(g^{SM}, g^{MV}) = (1, 0)$ *if either*

(i) $\frac{N-1}{1+\lambda}\hat{t} \leq Nk$ *and*

$$V_{\bar{a}_2} + \sum_{LW_{-\bar{a}_2}} \hat{t} + \sum_{\substack{\Omega_{-\bar{a}_2} \setminus LW \\ V_j > 0}} V_j + (1+\lambda) \sum_{\substack{\Omega_{-\bar{a}_2} \setminus LW \\ V_j < 0}} V_j$$

$$< (1+\lambda)Nk \le V_a + \tfrac{N-1}{2}\hat{t} + |I|\tfrac{\lambda}{1+\lambda}\hat{t}, \quad \textit{for } \bar{a}_2 \in LW^{>},$$

$$\sum_{LW} \hat{t} + \sum_{\substack{\Omega \setminus LW \\ V_j > 0}} V_j + (1+\lambda) \sum_{\substack{\Omega \setminus LW \\ V_j < 0}} V_j$$

$$< (1+\lambda)Nk \le V_a + \tfrac{N-1}{2}\hat{t} + |I|\tfrac{\lambda}{1+\lambda}\hat{t}, \quad \textit{for } \bar{a}_2 \notin LW^{>},$$

or

(ii) $\frac{N-1}{1+\lambda}\hat{t} > Nk$ *and*

$$V_{\bar{a}_2} + \sum_{LW_{-\bar{a}_2}} \hat{t} + \sum_{\substack{\Omega_{-\bar{a}_2} \setminus LW \\ V_j > 0}} V_j + (1+\lambda) \sum_{\substack{\Omega_{-\bar{a}_2} \setminus LW \\ V_j < 0}} V_j$$

$$< (1+\lambda)Nk \le (1+\lambda)V_a + \tfrac{N-1}{2}\hat{t}, \quad \textit{for } \bar{a}_2 \in LW^{>}$$

$$\sum_{LW} \hat{t} + \sum_{\substack{\Omega \setminus LW \\ V_j > 0}} V_j + (1+\lambda) \sum_{\substack{\Omega \setminus LW \\ V_j < 0}} V_j$$

$$< (1+\lambda)Nk \le (1+\lambda)V_a + \tfrac{N-1}{2}\hat{t}, \quad \textit{for } \bar{a}_2 \notin LW^{>}.$$

The proof can be found in the appendix.

According to Proposition 9.5, it is socially desirable for a project that would not be proposed under MV to be provided under SM if

$$\sum_{\Omega} V_j > (1+\lambda)Nk + \lambda \sum_{\Omega} s_j^{SM}(g^{SM} = 1).$$

With $|LW| \ge \frac{N+1}{2}$ and Proposition 9.9, this condition transforms into

$$\sum_{\Omega} V_j > Nk + \frac{\lambda}{1+\lambda}(N-1)\hat{t}.$$

Hence we obtain:

Corollary 9.2
If $|LW| \geq \frac{N+1}{2}$*, the situations in which simple majority voting is superior to Minority Voting from an ex ante social welfare point of view are characterized by*

$$\sum_{\Omega} V_j > Nk + \frac{\lambda}{1+\lambda}(N-1)\hat{t}$$

and if the case in Proposition 9.5 occurs.

9.6.3 Ramification

The previous welfare comparison discussed which voting scheme will result in higher expected utilities conditional on the realizations (V, a, a_2) when the individuals do not know their type. Additionally, considering uncertainty about who will be the agenda setter in SM, we can formulate the following proposition with respect to expected utilities conditional on V:

Proposition 9.6
If and only if $(|LW| < \frac{N+1}{2} \vee \mathcal{F} = \emptyset)$ *and*

$$\underbrace{\frac{1}{N}\sum_{a\in\mathcal{G}} E\left[U_j^{SM}\middle|V, a\right] + (1-p(G))E\left[U_j^{SM}\middle|V, a \notin \mathcal{G}\right]}_{E\left[U_j^{SM}\middle|V\right]} - e > 0, \tag{9.5}$$

simple majority voting yields strictly higher levels of expected utility than Minority Voting.

The proof can be found in the appendix.

Alongside a comparison of the voting regimes with respect to ex ante expected utility, one could ask whether the outcomes under the different voting schemes would be Pareto improvements to the status quo $(U_j = e,\ \forall j \in \Omega)$.

Proposition 9.7
Project provision under Minority Voting is always a Pareto improvement over the status quo. The simple majority voting scheme will result in a Pareto improvement if and only if $V_j \geq \hat{t},\ \forall j \neq a$ *and* V_a *satisfies (G).*

The proof can be found in the appendix.

9.7 Example

In this section we present a simple example with a homogeneous society.

Suppose that $V_j = \tilde{V} \in [\underline{V}, \overline{V}],\ \forall j \in \Omega$.

Proposition 9.8
If

(i) $\tilde{V} \geq \hat{t} \quad \wedge \quad \left[\left(\tilde{V} \geq (1+\lambda)Nk - \frac{N-1}{2}\hat{t} - |I|\frac{\lambda}{1+\lambda}\hat{t} \quad \wedge \quad (N-1)\hat{t} \leq (1+\lambda)Nk \right) \vee \left(\tilde{V} \geq Nk - \frac{N-1}{2(1+\lambda)}\hat{t} \quad \wedge \quad (N-1)\hat{t} > (1+\lambda)Nk \right) \right]$, *simple majority voting and the Minority Voting scheme yield equal levels of welfare;*

(ii) $\tilde{V} \geq \hat{t} \quad \wedge \quad \neg\left[\left(\tilde{V} \geq (1+\lambda)Nk - \frac{N-1}{2}\hat{t} - |I|\frac{\lambda}{1+\lambda}\hat{t} \quad \wedge \quad (N-1)\hat{t} \leq (1+\lambda)Nk \right) \vee \left(\tilde{V} \geq Nk - \frac{N-1}{2(1+\lambda)}\hat{t} \quad \wedge \quad (N-1)\hat{t} > (1+\lambda)Nk \right) \right]$, *Minority Voting is strictly better than simple majority voting;*

(iii) $\max\left\{ \frac{(1+\lambda)Nk}{1+\lambda+\frac{N-1}{2}}, \frac{Nk+\frac{\lambda}{1+\lambda}\frac{N-1}{2}\hat{t}}{N-\frac{\lambda}{1+\lambda}\frac{N-1}{2}} \right\} =: V^c < \tilde{V} < \hat{t}$, *simple majority voting is strictly preferable from a social perspective;*

(iv) $\tilde{V} \leq V^c$, *the Minority Voting scheme is superior to simple majority voting.*

The proof can be found in the appendix.

The example illustrates the advantages and drawbacks of Minority Voting. It also illustrates the importance of the tax protection expressed by the upper limit $\hat{t}$. If $\hat{t} > \tilde{V}$, a socially desirable project may not be proposed under MV.

9.8 Extensions and Alternative Voting Rules

There is a number of fruitful extensions and variations of Minority Voting which may bring the scheme closer to practical applications.

9.8.1 Extensions

There are immediate extensions of the basic model. First, we can reach a further level of design by varying the maximal tax level $\hat{t}$ in order to maximize social welfare. One might even consider a pre-voting step in which $\hat{t}$ is determined. Second, we have focussed on unanimous decisions in the financing round under MV. It is important to stress that this scheme is still weakly inferior in terms of aggregate utility compared to a scheme where every individual has the chance to make a proposal and the unanimity rule applies if it were possible to forbid pure redistribution proposals. Hence, it is important to consider other schemes. For instance, one could compare the results in this chapter with the outcome when the simple majority rule is used for the financing round under MV. The latter scheme makes financing much easier and thus increases the chances that the project is accepted, but it may lead to the adoption of socially inefficient projects.

Third, we could allow agents to differ with respect to endowments (incomes) and taxes to increase with income. In order to preserve the incentives of citizens to support the project in the first round, tax protection has to be income-dependent, i.e. when a citizen loses his voting right by favoring the project, the maximal tax burden has to increase with income. When the unanimity rule is used in the financing round, such a scheme may hamper the scope of the Minority Voting scheme, as individuals with high income may never support the project (or financing) proposal, as their net gain is negative. However, if we use the simple majority rule in the financing round, the advantages of the scheme can be preserved, as high-income individuals cannot block the adoption of financing proposals anymore.

Fourth, we have assumed that voters observe their own utility, as well as everybody else's. This allows large project winners to coordinate their voting decision in the first round. If individuals only observe their own utility, coordination is much less likely, and we need to examine mixed voting strategies of large project winners. This might tend to decrease the chances for a project to be approved in the first round and decrease the attractiveness of Minority Voting. However, the alternative rules discussed in the next subsection could alleviate this problem.

Fifth, Minority Voting eliminates the adoption of pure redistribution proposals that are inefficient from an ex ante perspective because taxation is socially wasteful and citizens are risk-neutral. Existing welfare states with redistribution schemes, however, are often justified by risk aversion of individuals who may not be able to insure themselves against risk in private markets. This problem may be handled as follows: MV is applied to project decisions, while there is a separate collective decision on a general redistribution scheme, using standard majority rules.

9.8.2 Alternative Voting Rules

Some of the potential problems of MV discussed in the last subsection may be alleviated by an "Initiative Group Scheme". The scheme works as follows: In a first round, individuals can decide whether to join an initiative group by paying a fee. If and only if the initiative group is formed, i.e. if and only if it passes a predetermined size-threshold, the electorate decides about the financing scheme. By raising or lowering the size-threshold for initiative groups, as well as the fees, the formation of initiative groups may be fostered, even if individuals cannot coordinate. The initiative group scheme might work for larger electorates if the fee levels are set below the maximal tax rate.[12]

[12] We currently examine whether this conjecture holds. Examples are available upon request.

9.9 Conclusion

In this chapter we propose a two-round collective decision process called Minority Voting which can avoid a variety of inefficiencies in democratic decision-making. Minority voting and variations of it may inform designers of democratic rules how to improve provision of public projects.

Appendix A: Characterization of Simple Majority Voting

A.1 Description of Equilibria

We first state a simple observation that facilitates the characterization of the equilibria.

Lemma 9.6
In the simple majority voting scheme, an agenda setter who is not one of the large project winners ($a \notin LW$) will never make a proposal that involves a tax payment for himself in order to finance the public project.

The proof can be found in the Appendix B.

With these preliminary observations we obtain

Proposition 9.9
Suppose that all individuals have applied for agenda setting. Then simple majority voting is characterized by the following equilibria:

(i) If $|LW_{-a}| \geq \frac{N-1}{2}$ and (G) holds for a proposal maker a, he offers

$$A_a^* = \begin{cases} g = 1, \\ s_j = 0, \ \textit{if } j \in \Omega_{-a}, \\ t_j = \hat{t}, \ \textit{if } j \in \Omega_{-a}, \\ t_a = \max\{0, -(1+\lambda)\bar{s}_a\}, \\ s_a = \max\{0, \bar{s}_a\}, \\ \bar{s}_a = \frac{(N-1)\hat{t}}{(1+\lambda)} - Nk. \end{cases}$$

Voting strategies are

$$\delta_j^* = \begin{cases} 1 & \textit{if } j \in LW, \\ 1 & \textit{if } j = a, \\ 0 & \textit{otherwise.} \end{cases}$$

(ii) If $|LW_{-a}| < \frac{N-1}{2}$ and (G) holds for a proposal maker a, he offers

$$A_a^* = \begin{cases} g = 1, \\ t_j = \hat{t}, \quad \textit{if} \;\; j \in LW_{-a} \cup \Omega \backslash I_{+a}, \\ t_j = V_j, \quad \textit{if} \;\; j \in I \backslash LW \;\textit{with}\; V_j \geq 0, \\ t_j = 0, \quad \textit{if} \;\; j \in I \backslash LW \;\textit{with}\; V_j < 0, \\ s_j = -V_j, \quad \textit{if} \;\; j \in I \quad \textit{with}\; V_j < 0, \\ t_a = \max\{0, -(1+\lambda)\bar{s}_a, \} \\ s_a = \max\{0, \bar{s}_a\}, \end{cases}$$

where

$$\bar{s}_a = (1+\lambda)^{-1} \Big\{ \Big(\frac{N-1}{2} + |LW_{-a}| \Big) \hat{t} + \sum_{j \in I \backslash LW, V_j \geq 0} V_j \Big\} - Nk + \sum_{j \in I, V_j < 0} V_j.$$

Voting strategies are

$$\delta_j^* = \begin{cases} 1, \quad \textit{if} \;\; j \geq \frac{N+3}{2}, \\ 1, \quad \textit{if} \;\; j = \frac{N+1}{2} \;\textit{and}\; a \geq \frac{N+1}{2}, \\ 1, \quad \textit{if} \;\; j = a, \\ 0, \quad \textit{if} \;\; j = \frac{N+1}{2} \;\textit{and}\; a < \frac{N+1}{2}, \\ 0, \quad \textit{otherwise.} \end{cases}$$

(iii) If (G) does not hold for a proposal maker, he offers

$$A_a^* = \begin{cases} g = 0, \\ t_j = \hat{t}, \quad \textit{for an arbitrary subset}\; I \subset \Omega_{-a} \;\textit{with}\; |I| = \frac{N-1}{2}, \\ t_j = 0, \quad \textit{if} \;\; \in \Omega \backslash I, \\ s_j = 0, \quad \textit{if} \;\; j \neq a, \\ s_a = \frac{N-1}{2(1+\lambda)} \; \hat{t}. \end{cases}$$

Voting strategies are

$$\delta_j^* = \begin{cases} 1, \quad \textit{if} \;\; j \in \Omega \backslash J, \\ 1, \quad \textit{if} \;\; j = a, \\ 0, \quad \textit{if} \;\; j \in J. \end{cases}$$

The proof of Proposition 9.9 is straightforward. The expressions for s_a are obtained from the budget constraint (9.2). Proposition 9.9 immediately implies that a proposal maker can always strictly improve his utility relative to the status quo. Hence we obtain:

Corollary 9.3
Under the simple majority rule, every individual applies for agenda setting.

As condition (G) may hold for some proposal makers but not for others, we provide a general characterization of the equilibria in the next subsection.

A.2 Expected Utilities

For later use, we derive the expected utility for the following scenario. The vector $(V_j)_{j\in\Omega}$ of project benefits is known, but the agenda setter has not been chosen. To derive the expected utility, we introduce the set $\mathcal{G}$:

$$\mathcal{G} := \left\{ j \,\middle|\, (G) \text{ holds for } a = j \right\}.$$

Thus, $\mathcal{G}$ is the set of individuals who propose $g = 1$ if they can set the agenda. We define

$$p(G) := \frac{|\mathcal{G}|}{N},$$
$$\tilde{p}(G) := \max\{p(G) - \frac{1}{N}, 0\}.$$

The expression $p(G)$ denotes the share of individuals who will propose $g = 1$ in equilibrium. Hence, $p(G)$ is the probability that the public project will be proposed before the agenda setter is chosen. As every individual j knows whether $j \in \mathcal{G}$ or $j \notin \mathcal{G}$ we obtain the following proposition.

Proposition 9.10
The expected utilities are given by:

(i) *Let* $|LW| \geq \frac{N-1}{2}$.

(α) *If* $j \in \mathcal{G}$,

$$E[U_j] = e + \tilde{p}(G)(V_j - \hat{t}) + \frac{1}{N}\big(V_j + (1+\lambda)^{1-sg(\bar{s}_a)}\bar{s}_a\big) - \big(1 - p(G)\big)\frac{N-1}{2N}\hat{t}.$$

(β) *If* $j \notin \mathcal{G}$,

$$E[U_j] = e + p(G)(V_j - \hat{t}) + \frac{1}{N}\big(s_a(g=0)\big) - \left(1 - p(G) - \frac{1}{N}\right)\hat{t}\,\frac{N-1}{2N}.$$

(ii) *Let* $|LW| < \frac{N-1}{2}$.

(α) *If* $j \in \mathcal{G}$,

$$E[U_j] = \begin{cases} e + \tilde{p}(G)(V_j - \hat{t}) + \frac{1}{N}\big(V_j + (1+\lambda)^{1-sg(\bar{s}_a)}\bar{s}_a\big) - \big(1 - p(G)\big)\frac{N-1}{2N}\,\hat{t}, \\ \qquad \textit{if } j \in LW \cup \{j | j < \frac{N+1}{2}\}; \\ e + \frac{1}{N}\big(V_j + s_a(g=1)\big) - \big(1 - p(G)\big)\frac{N-1}{2N}\,\hat{t}, \\ \qquad \textit{if } j \notin LW \textit{ and } j \geq \frac{N+3}{2}; \\ e + \tilde{p}(G)\frac{N-1}{2N}(V_j - \hat{t}) + \frac{1}{N}\big(V_j + s_a(g=1)\big) - (1 - p(G))\frac{N-1}{2N}\hat{t}, \\ \qquad j \notin LW,\, j = \frac{N+1}{2}. \end{cases}$$

(β) *If* $j \notin \mathcal{G}$,

$$E[U_j] = \begin{cases} e + p(G)(V_j - \hat{t}) + \frac{1}{N}s_a(g=0) - \big(1 - p(G) - \frac{1}{N}\big)\frac{N-1}{2N}\hat{t}, \\ \qquad \textit{if } j \in LW \cup \{j | j < \frac{N+1}{2}\} \\ e + \frac{1}{N}s_a(g=0) - \big(1 - p(G) - \frac{1}{N}\big)\frac{N-1}{2N}\hat{t}, \\ \qquad \textit{if } j \notin LW \textit{and } j \geq \frac{N+3}{2} \\ e + p(G)\frac{N-1}{2N}(V_j - \hat{t}) + \frac{1}{N}s_a(g=0) - \big(1 - p(G) - \frac{1}{N}\big)\frac{N-1}{2N}\hat{t}, \\ \qquad \textit{if } j \notin LW,\, j = \frac{N+1}{2}. \end{cases}$$

Proposition 9.10 follows directly from Proposition 9.9.

Appendix B: Proofs

Proof of Lemma 9.2
For this lemma the following observation is important: For the agenda setter it is optimal in the case of $g = 1$ to select the majority supporting his proposal by choosing set I as

$$I = \begin{cases} \{\frac{N+3}{2} \cdots, N\} & \text{if } a \leq \frac{N+1}{2} \\ \{\frac{N+1}{2}, \cdots, N\}\backslash\{a\} & \text{if } a > \frac{N+1}{2}. \end{cases}$$

Set I comprises the people with the highest values of V_j. Individuals in I can be charged with higher taxes or need fewer subsidies while still supporting $g = 1$ than the other individuals. As he can impose $t_j = \hat{t}$ on the individuals in $\Omega \setminus I_{+a}$, he will obtain maximal tax revenues (or minimal subsidies) by choosing I. □

Proof of Lemma 9.6
Suppose that $a \notin LW$. The agenda setter will propose $g = 1$ if (G) is satisfied. As $s_a(g=0) = \frac{N-1}{2(1+\lambda)}\hat{t}$, (G) can be written as

$$V_a + (1-\lambda)^{1-\mathrm{sg}(\bar{s}_a)}\bar{s}_a(g=1) \geq \frac{N-1}{2(1+\lambda)}\hat{t}. \tag{9.6}$$

Now suppose that a proposal that involves tax for the agenda setter himself, i.e. $\bar{s}_a < 0$. Then, by the condition above, the project will only be proposed if

$$V_a > \frac{N-1}{2(1+\lambda)}\hat{t} > \hat{t}, \tag{9.7}$$

since $0 < \lambda < 1$ and $N \geq 5$. This contradicts $a \notin LW$ however, thus the assertion follows. □

Proof of Proposition 9.1
Suppose an individual a_1 proposes $g_{a_1}^{MV} = 1$. By the rules of MV, an individual who supports $g_{a_1}^{MV} = 1$ faces two possibilities. Either he is in a minority and $g_{a_1}^{MV} = 0$ prevails, or he is in the majority. As he will lose his voting rights, he will be taxed by $\hat{t}$ in the subsequent financing round. Hence voting $\delta_j\left(g_{a_1}^{MV} = 1\right) = 0$ weakly dominates $\delta_j\left(g_{a_1}^{MV} = 1\right) = 1$ for all individuals with $V_j < \hat{t}$. By our tie-breaking rule, result (ii) follows.

If $|LW| \geq \frac{N+1}{2}$ and if $\frac{N+1}{2}$ large project winners accept the proposal, the best response for other large project winners is to vote $\delta_j\left(g_{a_1}^{MV} = 1\right) = 0$ as they then have a chance of becoming agenda setter in the financing round. In turn, given the voting behavior of all other individuals, it is the best response for large project winners in the tight majority supporting $g_{a_1}^{MV} = 1$, as otherwise the status quo would prevail. □

Proof of Proposition 9.2
The proof follows from a backward induction argument. In Stage 4 the agenda setter solves the following problem:

$$\max_{(t_j, s_j)_{j\in\Omega}} U_{a_2} = e + V_{a_2} + s_{a_2} - t_{a_2},$$

$$\begin{aligned}
\text{s.t.} \qquad & U_m - e = V_m + s_m - t_m \geq 0, && \forall m \in \mathcal{M} \\
& \sum_{j\in\Omega} t_j = (1+\lambda)(Nk + \sum_{j\in\Omega} s_j), && \forall t_j \leq \hat{t}, \forall j,
\end{aligned}$$

which yields the solution in the proposition. Note also that Maximal Magnanimity applies in Stage 3. □

Proof of Proposition 9.3
Since the project is not proposed under MV and SM, by using Lemma 9.1, we obtain:

$$\begin{aligned}
E\left[U_j^{MV}\middle|V, 0\right] - E\left[U_j^{SM}\middle|V, a\right] &= e - \left(e + \frac{1}{N}s_a(g=0) - \frac{N-1}{2N}\hat{t}\right) \\
&= \frac{N-1}{2N}\,\hat{t} - \frac{1}{N}s_a(g=0) \\
&= \hat{t}\,\frac{N-1}{2N}\left(\frac{\lambda}{1+\lambda}\right) > 0.
\end{aligned}$$

□

Proof of Proposition 9.4
The first part of the proposition is obvious, as if the project is not proposed, there will be redistribution in SM but not in MV. Hence $W^{MV} = eN$ and $W^{SM} = eN + \frac{N-1}{2(1+\lambda)}\hat{t} - \frac{N-1}{2}\hat{t} < eN$.

As for the second part, suppose the project is to be provided under both voting schemes, that is, $g^{MV} = g^{SM} = 1$. Redistribution activities cause a welfare loss of

$$\lambda \sum_{j\in\Omega} s_j.$$

Accordingly, the proposition claims that

$$\sum_{j\in\Omega} s_j^{SM} \geq \sum_{j\in\Omega} s_j^{MV}.$$

Using the budget constraint of Eq. (9.2), the above condition can be written as

$$\sum_{j\in\Omega} t_j^{SM} \geq \sum_{j\in\Omega} t_j^{MV}.$$

This holds true, as in MV the tax payments according to Proposition 9.2 are

$$\sum_{j\in\Omega} t_j^{MV} = \sum_{LW_{-a_2}} \hat{t} + \sum_{\substack{\Omega_{-a_2}\setminus LW \\ V_j>0}} V_j,$$

whereas in SM, according to Lemma 9.2, they amount to

$$\begin{aligned}\sum_{j\in\Omega} t_j^{SM} &= \sum_{LW_{-a}\cup\Omega\setminus I_{+a}} \hat{t} + \sum_{I\setminus LW_{-a}, V_j\geq 0} V_j \\ &\geq \sum_{LW_{-a}} \hat{t} + \sum_{\Omega_{-a}\setminus LW, V_j\geq 0} V_j.\end{aligned}$$

Note that either $I \subseteq LW_{-a}$ or $LW_{-a} \subseteq I$. In the former case we have $I \setminus LW_{-a} = \emptyset$ and $\Omega_{-a}\setminus LW \subseteq \Omega\setminus I_{+a}$, whereas in the latter case, $\Omega\setminus I_{+a} \cup I \setminus LW_{-a} = \Omega_{-a}\setminus LW$. Combining this with the fact that $V_j < \hat{t}$ for $j \notin LW$, the last inequality follows. □

Proof of Lemma 9.4
As the agenda setter a_2 is not able to make any member of society worse off as compared to the status quo, the total taxes collected must be weakly smaller than the sum of the benefits derived from the public project by those individuals who benefit from its provision. Hence we have

$$\sum_{\Omega} \max\{V_j, 0\} \geq \sum_{\Omega} t_j = (1+\lambda)\Big[Nk + \sum_{\Omega} s_j\Big]. \tag{9.8}$$

The sum $\sum_{\Omega} s_j$ can be split into

$$\sum_{\Omega} s_j = -\sum_{\Omega} \min\{V_j, 0\} + \sum_{\Omega} s_j^{pr}.$$

The first term on the right-hand side reflects compensatory payments to the project losers in $\mathcal{M}$, while the second term represents purely redistributional subsidies (hence the superscript "pr"), which in equilibrium can only be positive if individual j is the agenda setter.[13] Consequently, using $\sum\limits_{\Omega} V_j = \sum\limits_{\Omega} \max\{V_j, 0\} + \sum\limits_{\Omega} \min\{V_j, 0\}$, inequality (9.8) can be rewritten as

$$\sum_{\Omega} V_j \geq (1+\lambda)\Big[Nk + \sum_{\Omega} s_j^{pr}\Big] - \lambda \sum_{\Omega} \min\{V_j, 0\}.$$

As $(1+\lambda)\sum_{\Omega} s_j^{pr} - \lambda \sum_{\Omega} \min\{V_j, 0\} \geq 0$, we obtain

$$\sum_{\Omega} V_j \geq (1+\lambda)Nk.$$

If the above condition held with equality, then an agenda setter could not realize positive subsidies. In this case, nobody would apply for agenda-setting.

Consequently, if the project is proposed and adopted, the inequality must be strict, implying that the project is socially desirable. $\square$

Proof of Proposition 9.5

As from an ex ante point of view, each individual is equally likely to assume any of the values V_j, total welfare can be measured as the sum of utilities. Since all members of the society are risk-neutral, this translates into

$$W = \sum_{\Omega}(e + gV_j) - (1+\lambda)gNk - \lambda \sum_{\Omega} s_j,$$

where we have used the budget constraint in Eq. (9.2).

From Proposition 9.4 we know that redistribution losses are weakly higher under SM than under MV if $g^{SM} = g^{MV} = 1$, and strictly higher if $g^{SM} = g^{MV} = 0$. Consequently, in these cases social welfare is weakly or strictly higher in MV than in SM, respectively. This must also be the case if $(g^{SM}, g^{MV}) = (0, 1)$, because from Lemma 9.4 we know that, when the project is adopted in MV,

$$W^{MV}(g^{MV} = 1) = \sum_{\Omega} e + \underbrace{\sum_{\Omega} V_j - (1+\lambda)Nk - \lambda \sum_{\Omega} s_j^{MV}(g^{MV} = 1)}_{\geq 0},$$

[13] Note that in the Minority Voting case $s_{a_2}^{pr} = \bar{s}_{a_2}$ if $V_{a_2} > 0$ and $s_{a_2}^{pr} = \bar{s}_{a_2} + V_{a_2}$ if $V_{a_2} < 0$. The same rule applies for simple majority voting.

whereas in SM without project provision,

$$W^{SM}(g^{SM}=0)=\sum_{\Omega} e \underbrace{-\lambda \sum_{\Omega} s_j^{SM}(g_a^{SM}=0)}_{<0}.$$

Consequently, the only possibility for SM to be strictly socially preferable is when $(g^{SM}, g^{MV})=(1,0)$. A simple welfare comparison then reveals that

$$W^{SM}(g^{SM}=1)=\sum_{\Omega}(e+V_j)-(1+\lambda)Nk-\lambda\sum_{\Omega} s_j^{SM}(g^{SM}=1)$$

$$>\sum_{\Omega} e = W^{MV}(g^{MV}=0)$$

if and only if

$$\sum_{\Omega} V_j > (1+\lambda)Nk+\lambda\sum_{\Omega} s_j^{SM}(g^{SM}=1).$$

□

Proof of Lemma 9.5
With $|LW| \geq \frac{N+1}{2}$, $\bar{s}_a \gtreqless 0$ is equivalent to $\frac{N-1}{1+\lambda}\hat{t} \gtreqless Nk$. Further, (G) can be rewritten as

$$(G)=\begin{cases}(G^-) & \text{if } \; V_a+\frac{N-1}{2}\hat{t}+|I|\frac{\lambda}{1+\lambda}\hat{t} \geq (1+\lambda)Nk \\ (G^+) & \text{if } \; (1+\lambda)V_a+\frac{N-1}{2}\hat{t} \geq (1+\lambda)Nk.\end{cases}$$

Consider the first of the above cases characterized by $\bar{s}_a \leq 0$. $(g^{SM}, g^{MV})=(1,0)$ then requires $(G^-)\wedge\neg(F^-)$. As $\neg(F^-)$ can be written as

$$V_{\bar{a}_2}+|LW_{-\bar{a}_2}|\cdot\hat{t}+\sum_{j\in\Omega_{-\bar{a}_2}\setminus LW}\max\{V_j,0\}<(1+\lambda)\Big[Nk-\sum_{j\in\Omega_{-\bar{a}_2}}\min\{V_j,0\}\Big],$$

both (G^-) and $\neg(F^-)$ hold if

$$V_{\bar{a}_2}+\sum_{LW_{-\bar{a}_2}}\hat{t}+\sum_{\substack{\Omega_{-\bar{a}_2}\setminus LW\\ V_j>0}}V_j+(1+\lambda)\sum_{\substack{\Omega_{-\bar{a}_2}\setminus LW\\ V_j<0}}V_j<(1+\lambda)Nk\leq V_a+\frac{N-1}{2}\hat{t}+|I|\frac{\lambda}{1+\lambda}\hat{t}.$$

The other conditions of the lemma are derived analogously. □

Proof of Proposition 9.6
From the discussion in the previous section we know that the simple majority voting scheme will only yield strictly higher expected utility compared to Minority Voting if $(g^{SM}, g^{MV}) = (1, 0)$. According to Fact 9.1, V directly determines g^{MV}. However, given V, there may be uncertainty about g^{SM}, as not every agenda setter under SM would propose the project. Hence SM would be strictly preferable to MV if the weighted expected utilities when it is socially desirable to provide the project, are large enough to compensate for the situations in which adhering to the status quo would yield higher welfare. Note that if the project is proposed, the expected utility depends on who will be the agenda setter. The reason is that different agenda setters can charge different amounts of taxes from the majority, which involves different levels of redistributional shadow costs. □

Proof of Proposition 9.7
As under MV the minority must agree to the project by the unanimity rule and the majority will only approve project provision if they are members of the set LW, no individual will be worse off compared to the status quo. If no agent is strictly better off by providing the public project, no one will apply for agenda setting in the first stage. Hence public project provision must involve a Pareto improvement to the status quo.

Under SM at least the members of the minority will be taxed by $\hat{t}$, as they are not necessary for proposal approval. Hence only a benefit from the project that is at least $\hat{t}$ will prevent an individual in the minority from being worse off when the project is provided compared to the status quo. V_a satisfying (G) implies that the project will be proposed and that at least the agenda setter will strictly gain in utility.[14] It is easy to see that in all other cases SM will not lead to a Pareto improvement. More precisely, if the project is not proposed, pure redistribution will leave the minority with utility lower than e. Further, if the project is proposed but there is an individual $j \neq a$ with $V_j < \hat{t}$, this person will be a member of the minority (as we know from Lemma 9.2) and hence will face taxes $\hat{t}$. □

Proof of Proposition 9.8

Case (i)

Let $\tilde{V} > \hat{t}$. This implies that $|LW| \geq \frac{N+1}{2}$. As $a_2 \in LW^{>}$, the public project will be proposed and adopted under MV if

$$(F^{-}) \quad \tilde{V} + (N-1)\hat{t} \geq (1+\lambda)Nk.$$

With respect to SM, project provision implies

[14]The reason is that (G) implies that his utility gain is at least as high as the one he could achieve by pure redistribution.

$$(G^-) \qquad \tilde{V} + \frac{N-1}{2}\hat{t} + |I|\frac{\lambda}{1+\lambda}\hat{t} \geq (1+\lambda)Nk, \qquad \text{if } (N-1)\hat{t} \leq (1+\lambda)Nk$$

$$(G^+) \qquad (1+\lambda)\tilde{V} + \frac{N-1}{2}\hat{t} \geq (1+\lambda)Nk, \qquad \text{if } (N-1)\hat{t} > (1+\lambda)Nk.$$

Suppose that (G^-) holds. Then (F^-) also holds. Hence the project will be provided under both regimes SM and MV. As $|LW| = N$, both agenda setter a and a_2 will propose $\hat{t}$ for every individual except himself. They will close the budget gap with a tax payment of their own. Both voting schemes yield equivalent tax revenues and no subsidies and thus result in equal levels of welfare.

The reasoning for (G^+) is similar. (G^+) also implies (F^-). In this case however, the agenda setters a and a_2 receive subsidies that are the same under both voting schemes.

In the case of $\tilde{V} = \hat{t}$, the proof has to be adapted in the following way: As $a_2 \notin LW^>$, the public project will be proposed and adopted if

$$N\hat{t} > (1+\lambda)Nk$$

holds. We denote this condition by $(F^+)^>$. The assumptions involved in case (i) imply that $(F^+)^>$ holds, therefore the same reasoning applies as before.

Case (ii)
In the case of (ii), we have either $\neg(G) \wedge (F)$ or $\neg(G) \wedge \neg(F)$. Although $\neg(G) \wedge (F)$ might imply that there are higher shadow costs of public funds under MV, the sum of utilities derived from public project provision must overcompensate them, as no individual can be worse off in this voting scheme (see also the proof of Proposition 9.7). Further, we know from Lemma 9.4 that only socially desirable projects will be provided under MV. In this way, MV is superior to SM. The same holds true if $\neg(G) \wedge \neg(F)$, as verified in Proposition 9.4.

Case (iii)
Now consider situation (iii), where

$$\max\left\{\frac{(1+\lambda)Nk}{1+\lambda+\frac{N-1}{2}}, \frac{Nk+\frac{\lambda}{1+\lambda}\frac{N-1}{2}\hat{t}}{N-\frac{\lambda}{1+\lambda}\frac{N-1}{2}}\right\} =: V^c < \tilde{V} < \hat{t}.$$

The project will not be provided under MV, as $|LW| < \frac{N+1}{2}$. The project will be proposed under SM if (G^+) holds, which can be transformed to[15]

$$\tilde{V} \geq \frac{(1+\lambda)Nk}{1+\lambda+\frac{N-1}{2}}.$$

[15]Note that according to Lemma 9.6 if $V < \hat{t}$ the agenda setter under SM would not propose $g = 1$ if he had to accept a tax for himself. Hence the project will be provided if (G^+) holds.

According to the condition in Proposition 9.5, it would be socially desirable to do so if

$$N\tilde{V} > Nk + \frac{\lambda}{1+\lambda}\left(\frac{N-1}{2}\tilde{V} + \frac{N-1}{2}\hat{t}\right). \tag{9.9}$$

This inequality holds if the utilities derived from the project satisfy

$$\tilde{V} > \frac{Nk + \frac{\lambda}{1+\lambda}\frac{N-1}{2}\hat{t}}{N - \frac{\lambda}{1+\lambda}\frac{N-1}{2}}.$$

Hence, if both (G^+) and (9.9) hold, a socially desirable project is provided under SM that would not be provided under MV. So in this case SM is strictly preferable to MV.

Case (iv)

Finally, for $\tilde{V} \leq V^c$, the project is not provided under either voting scheme or is only proposed under SM. However, provision under SM is not desirable from a social welfare perspective, as the redistribution losses are higher than the sum of additional utilities derived from the public project. Consequently, MV is superior to SM. □

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Chapter 10
Initiative-Group Constitutions

10.1 Background

Our research on initiative groups is best described as companion research to our work on Minority Voting: In both areas, we try to balance the gains and losses incurred in democratic decision-taking, from different vantage points. As Minority Voting compensates *voting losers* by granting them extra-rights *after this voting*, our suggestion on initiative groups takes effect *before the voting* and affects the (potential) *voting winners* by putting a price on winning. Thus, our initiative group concept addresses the core issue of Minority Voting at another voting stage, and does not reward the losers, but encumbers the winners.

A democracy should produce socially desirable outcomes, and it will be particularly viable if it appeals to an elementary sense of fairness. As we saw in the chapter on Minority Voting, it is desirable to counterbalance the tyranny of the majority to some extent, in particular if this majority is a narrow one, or if some groups of voters find themselves in the minority repeatedly.

This challenge might be addressed from another point of view, taxation. If taxation is non-discriminatory, every citizen incurs the same cost for the provision of public goods. Yet, the benefits derived from a given public good are not the same for each individual. Thus, a proposal on the provision of a public good might be voted down by those voters for whom the benefits of this public good might be lower than the corresponding taxes. This might however be inefficient, as a minority may benefit a lot, while the majority only loses little, this being a typical case of a tyranny of the majority.

As non-discriminatory taxation might impede the socially desirable provision of public goods, taxation can be restructured to allow such provision, if not to foster it. The most simple solution to this problem would be to tax the beneficiaries of a certain public good more than non-beneficiaries. Yet, the assessment of these benefits seems too complex a task, and more often than not, individual benefits are private information.

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It seems easier to assess the *expected* benefits, i.e. how much every citizen is interested in the provision of a certain public good, and to do this *before* the voting. The best method would be to assess whether a citizen is willing to incur a certain amount of extra-taxes as a trade-off for the public good.

To ascertain this willingness, one might imagine "tax promises" of the type: 'If public good A is provided, I am willing to pay 10% more taxes'. But such a type of commitment would require a complex procedure, with a "commitment round" before the voting and a "tax-invoicing round" after it, which would probably be more costly than the taxes collected. Another problem might be that too few citizens commit to higher taxes, while other potential beneficiaries offer no higher taxation and gamble, hoping for a decision in their favor without their having to pay for it.

To prevent this type of free-riding and to avoid a complicated administrative procedure, one might envision a different solution – an initiative group advocating the provision of a certain public good. This initiative group should follow two rules. First, the initiative group should reach a certain pre-defined number of members to initiate voting on a public good. If the initiative group fails to reach this size, the *status quo* prevails. Second, if the pre-defined number is reached, the initiative group makes a financing proposal containing the possibility to tax the members of the initiative group at a higher rate than the rest of the electorate. This can make initiative group membership costly. Finally, the society as a whole decides on the financing proposal, by a simple majority rule or a supermajority rule. If it is adopted, the project and the financing scheme are implemented.

In this chapter, we examine constitutions with initiative groups, and assess their potential as a building block of democracy.

10.2 Introduction

We consider a standard two-stage problem of public project provision and examine how initiative groups as a vehicle to select proposal-makers and proposals perform with respect to social welfare. In the first stage, the individuals decide whether to join an initiative group. If the size of the initiative group reaches a critical level, the process moves to the second stage. Otherwise, it ends and the status quo prevails. In the second stage, one member of the initiative group is chosen randomly and has the right to make a financing proposal for the public project. If this proposal is accepted by a simple or super-majority of the society, the project is undertaken and financed according to the proposed scheme. Otherwise, the status quo prevails. This two-stage democratic institution is called an *initiative-group constitution*.

As suggested above, initiative-group constitutions involve two essential assumptions. First, members of the initiative group can be charged a higher tax than the rest of the society. This represents an endogenously determined cost or fee of joining the initiative group. Second, taxation is required to be non-discriminating within the initiative group on the one hand, and within the remaining society on the other.

Our analysis yields three main insights.

First, if benefits from public projects vary in the electorate but are positive for all citizens, appropriately chosen initiative-group constitutions yield efficient allocations. This result is a benchmark. It shows that initiative-group constitutions can replicate what would be obtained in a Coasian setting when agents bargain frictionless over the surplus they can generate by undertaking a public project. Democratic rules such as the formation of the initiative group, non-discriminant with regard to taxation and voting, yield social efficiency. If costs of public projects are stochastic, minimal sized interest groups and the unanimity vote lead to efficient outcomes. The initiative-group constitutions deter the formation of interest groups precisely in the cases when projects are socially undesirable.

Second, if there are strict losers from public projects, efficiency can still be obtained but the size of the initiative group has to be raised. The reason is that equal taxation within the group of citizens not joining the interest group, makes it impossible to reach efficiency if project winners do not contribute more which can only be achieved by making the size of the initiative group larger. For the same reason, it may be socially desirable to reduce the set of agents that must be subsidized, as citizen are protected against discriminatory taxation.

Third, the initiative-group constitution outperforms majority voting constitutions in which a proposal-maker is chosen at random, makes a proposal and the polity decides according to an optimally chosen super-majority vote. The initiative-group constitution allows for a two-tiered tax scheme where the members of the initiative group potentially bear a higher financial burden. This allows project winners to self-select into the initiative group and, if needed, to lower the tax burden for the rest of the electorate.

Practical forms of democratic decision-making through initiative groups do already exist. There are two different forms of initiative groups. In direct democracies such as Switzerland and California (see, for example, Feld and Kirchgässner 2000), large initiative groups are required to obtain the right for a public vote on a public project. In some parliaments of representative democracies on the other hand, members have the right to force discussion and decision if they can manage to gather a critical number of signatures supporting their proposal. An example is the German Bundestag.[1]

Comparing such practical forms of initiative groups with our scheme reveals one important difference. Typically, real world initiative groups are not associated with higher financial contributions by the members of the initiative group. We allow additional flexibility in forming the initiative groups, as a proposal-maker can impose higher taxes on members of the initiative group than on the rest of the society. Thus, joining an initiative group may be costly because the tax burden may be larger. In general, there are non-monetary costs of signing an initiative such as the time used to understand and sign the process. In case of large-scale initiative groups, the cost of organizing the signature process may be larger. Such costs are neglected in our

[1] See for instance http://www.bundestag.de/service/glossar/I/initiativrecht.html, retrieved 29/08/2013.

analysis but could be integrated and added to the tax burden of members of the initiative group.[2]

Much of the older and the more recent research on alternative democratic procedures has focused on voting procedures and how they can be chosen optimally. Buchanan and Tullock (1962) examined the costs and benefits of majority rules chosen by a society that operates under a veil of ignorance. Aghion and Bolton (2003) introduced contractual incompleteness for the design of optimal majority rules. They show how the simple or qualified majority rule can help to overcome ex-post vested interests. The twin-problem of societies—the risk of tyranny by the majority and the risk of blocking by the minority—was further examined by Aghion et al. (2004), who derive optimal supermajority rules to balance these risks. Harstad (2005) developed a theory of majority rules where agents can invest in order to benefit more from future projects. Optimal majority rules balance two opposing forces: the incentives to invest may be too small if the majorities required are large, but they may be excessively high if only a small majority is required. Gersbach (2009) allowed the majority rules to depend on the proposal, and showed how such flexible rules can yield efficient allocations if redistribution is costly and taxation can be discriminatory.

An essential assumption in our analysis is that taxation is non-discriminatory, in the sense that all citizens are allowed to opt for the lower tax burden. There is a long tradition in law and economics which provides justification for this assumption.[3] By varying the size of the initiative group and the size of the majority for the second round, this scheme allows to achieve efficient allocations in cases when project winners are a minority of the electorate. Moreover, the initiative-group constitution is flexible and retains largely its efficient properties when the size of project winners or costs of projects are stochastic. Finally, the first stage in our democratic process to provide public projects is a participation problem. Single-stage participation games when a single unit of a public good can be produced have been throughly examined in the literature (Palfrey and Rosenthal 1984, Dixit and Olson 2000, and Shinohara 2009). In this chapter we combine a participation problem—joining or not an initiative group—with proposal-making and voting in a second stage to describe an entire political process.

This chapter is structured as follows: In Sect. 10.3 we introduce a basic version of the model, in which only two types of agents exist: project winners, who strictly benefit from the project, and (weak) project losers, who are indifferent with respect to the project as long as they are exempted from taxation. In Sect. 10.4 we analyze the equilibria of this game. Our main result will be that a unanimity rule, combined with no constitutional constraints on the minimum size of the initiative group, yields an efficient outcome. In Sect. 10.5 we extend the model by a third group of agents, whom we call strict project losers. We illustrate that there are circumstances under

[2] We suggest that the two-tier tax scheme is desirable when initiative groups are formed from public project provision.

[3] Explanations from an equity and/or efficiency perspective have been reviewed and provided, e.g., in Gersbach et al. (2013).

which constraints on the minimum size of the initiative group can improve welfare. In Sect. 10.6 we discuss our results. Section 10.7 concludes.

10.3 Model and Constitutional Principles

In this section we describe the model. We start with a basic version with only two types of agents. We define initiative-group constitutions and characterize the strategy space of the game. For later use as a benchmark case, we introduce majority-voting constitutions.

10.3.1 Model

We consider a standard social-choice problem of public project provision and financing by a society of $N \geq 2$ risk-neutral members, who are indexed by $1, \ldots, N$. Each agent is endowed with the same amount of a private consumption good. The amount is sufficiently large to pay the taxes under any policy considered in the model. The society can adopt a public project with per capita cost $K > 0$. Let V_j denote the utility of agent j from the provision of the public project. V_j can take two values (expressed in terms of the consumption good), $V_j = 1$ and $V_j = 0$. To simplify the language, we call the individuals with $V_j = 1$ project winners and those with $V_j = 0$ (weak) project losers. By $\mathbf{V} = (V_1, \ldots, V_N)$ we denote the vector of types. For any vector $\mathbf{z} = (z_1, \ldots, z_m)$, where m is an arbitrary natural number, we write

$$\mathcal{N}(\mathbf{z}) := \#\{j \mid z_j > 0\}.$$

With this notation, $\mathcal{N}(\mathbf{V})$ is the number of project winners in the society; it is equal to the aggregate benefit from the public project.

The pair $(K, \mathbf{V})$ is drawn from some probability distribution on the set $\mathfrak{X} := (0, 1) \times \{0, 1\}^N$ of possible states of the world. As we shall see in Sect. 10.3.3, a project with per-capita cost above 1 will never be proposed under the rules that we discuss. If $K = 1$, the project might be proposed, but nobody will be *strictly* in favor of it. We therefore neglect such projects and restrict the potential values of K to $(0, 1)$. The public project must be financed by taxes, which we assume to be non-distortionary. By t_j we denote agent j's tax payment; a negative value of t_j means a subsidy. Let $\mathbf{t} := (t_1, \ldots, t_N)$ be the vector of taxes. We define the variable D as indicating whether the public project is realized ($D = 1$) or not ($D = 0$). With the status quo utility normalized to zero, the utility of agent j is

$$D \cdot (V_j - t_j) .$$

The budget constraint of the society is thus given by

$$\sum_j t_j = D \cdot KN. \tag{10.1}$$

10.3.2 Initiative-Group Constitutions

We consider a two-stage democratic process, called an *initiative-group constitution* and denoted by $\mathcal{C}^{\mathrm{IG}}(a, b)$. It is characterized by a pair (a, b) of thresholds, with $a \in \{0, \ldots, N\}$ and $b \in \{1, \ldots, N\}$. In the process, an initiative group is first formed. If the group size reaches the threshold b, a proposal is made, specifying whether the project should be realized and how taxes and subsidies should be set. If the proposal is approved by a (sub- or super-) majority of all individuals in the society, determined by the parameter a, the taxes are collected, subsidies are paid, and the project is undertaken. The overall democratic game is as follows:

Initial situation: Nature draws the vector $(K, \mathbf{V})$, where $\mathbf{V} = (V_1, \ldots, V_N)$ is the vector denoting the agents' types. Each agent j observes the realization $(\kappa, \mathbf{v})$ of $(K, \mathbf{V})$.

Stage 1 (Group formation): Each project winner decides whether he wants to join the initiative group or not. In this baseline set-up, we assume that project losers do not join the initiative group.[4] After the initiative group has been formed, everybody can observe who participates in the initiative group and who does not. The initiative group can be described by a vector $\mathbf{g} = (g_1, \ldots, g_N) \in \{0; 1\}^N$, with $g_j = 1$ if agent j joined the initiative group, and $g_j = 0$ if he did not. If $\mathcal{N}(\mathbf{g}) < b$, the game ends and the status quo prevails.

Stage 2 (Voting): If on the other hand $\mathcal{N}(\mathbf{g}) \geq b$, the initiative group has been formed successfully. In this case, an individual out of the group is chosen by fair randomization; he makes a proposal specifying whether the public project should be realized and how taxes and subsidies should be set. Given the proposal, all agents decide simultaneously in a voting whether they want to support the proposal or not. The proposal is accepted if it is supported by at least a agents. If the proposal involving the realization of the project is accepted, the project is implemented and financed according to the proposal. Otherwise, the project is not realized.[5]

Since we model the decision process basically as a one-shot game, without a "second chance" for the realization of the project as soon as it has been rejected once, the proposal-maker is in a powerful position, which allows him to achieve acceptance of plans that result in the exploitation of other agents. In addition, a majority may be tempted to shift the tax burden to a minority of agents that cannot alter the decision. In order to balance the power of the proposal-maker and to establish some kind of minority protection, we demand the proposal to meet the following requirements:

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[4] In Sect. 10.6, we discuss whether and how our results would be affected if every agent could join the initiative group.

[5] Either a proposal containing the project is rejected or the status quo is proposed and accepted.

1. Differences in taxes (or subsidies) may only be based on membership or non-membership in the initiative group. We call this condition the *equal taxation within groups* constraint (ETG).
2. Non-members of the initiative group must not be taxed higher than members. We call this condition *higher taxation of initiators* constraint (HTI).

Formally, a proposal is characterized by a tuple (d, t, t'), specifying if the project should be realized ($d = 1$) or not ($d = 0$), as well as a tax level t for the members of the initiative group and a tax level t' for the non-members. A valid proposal must satisfy both the social budget constraint (10.1), which due to (ETG) simplifies to

$$\mathcal{N}(\mathbf{g}) \cdot t + \big(N - \mathcal{N}(\mathbf{g})\big) \cdot t' = d \cdot KN, \tag{10.2}$$

and the inequality

$$t \geq t', \tag{10.3}$$

which reflects (HTI). A situation in which the proposal-maker does not come up with a valid proposal (i.e. with an invalid proposal or no proposal at all) shall be treated as if he had proposed to keep the status quo, i.e. $d = 0$, $t = 0$ and $t' = 0$. In our model, the voting in Stage 2 is a simple binary decision. The elimination of weakly dominated strategies amounts to sincere voting, i.e. the agents vote for their most preferred alternative. Further, to simplify the presentation of the steps in the analysis, we use the following tie-breaking rule[6]:

(T1): If voting for the proposal and voting against it lead to the same expected utility, the agent will vote for the proposal.

Sincere voting and the tie-breaking rule obviously select a unique voting outcome. In the following, we will always assume that sincere voting and the tie-breaking rule (T1) apply.

10.3.3 Proposals and Strategies

The social budget constraint, as well as the conditions (ETG) and (HTI) imposed above, have immediate consequences for the proposals that will actually be made:

1. Since the proposal-maker can secure a utility of at least 0 for himself by proposing not to realize the project, and since a pure redistribution proposal, i.e. a proposal $(0, t, t')$ with $t \neq t'$, would always imply a strictly negative utility for the proposal-maker due to (HTI), pure redistribution proposals will not be made. In other words, if $d = 0$ is proposed, the proposal will be $(0, 0, 0)$. We denote this proposal $\mathcal{P}_0$. Due to sincere voting and the tie-breaking rule (T1), proposal $\mathcal{P}_0$ will be accepted unanimously in the voting stage.

[6] As said, the tie-breaking rule simplifies the presentation. It could also be integrated as a property of equilibrium strategy profiles.

2. Since non-members cannot be taxed higher than members, any proposal involving project realization must impose a tax of *at least* K on the members and a tax of *at most* K on non-members. As a consequence, if $K > 1$, the realization of the project would be connected with a strictly negative utility for the proposal-maker; hence, the project would never be realized if $K > 1$. If $K = 1$, the project will only be realized if c agents are project winners with $c = \max\{a, b\}$. Then, all agents will be indifferent between realization and non-realization of the project. This justifies our assumption that per-capita cost K is drawn from the interval (0, 1).
3. As lower taxes for non-members imply higher taxes for members of the initiative group, proposing a tax level strictly below K for non-members is only rational if this has influence on voting behavior. Due to sincere voting and the tie-breaking rule (T1), a valid proposal $(1, t, t')$ involving realization of the project will be accepted

   (i) by the *project winners within* the initiative group if and only if $t \leq 1$;
   (ii) by the *project winners outside* the initiative group, in any case;
   (iii) by the *project losers* if and only if $t' \leq 0$.

   Since the proposal-maker will not want to set an unnecessarily high tax level for himself, the only potentially rational proposals involving project realization are $(1, K, K)$ and $\big(1, KN/\mathcal{N}(\mathbf{g}), 0\big)$. With the first of these two proposals, which we denote by $\mathcal{P}_1$, the project is financed by the society as a whole. With the second proposal, denoted by $\mathcal{P}_2$, the cost is borne by the members of the initiative group only.
4. The above considerations show that subsidies will never be proposed.

Using these considerations, we can now formally characterize the agents' strategies. Denote by

$$\mathfrak{X}_j := \big\{(v_1, \ldots, v_N) \in \mathfrak{X} \;\big|\; v_j = 1\big\}$$

the set of all states of the world in which agent j is a project winner and where κ and v_j are realizations of K and V_j. A strategy of agent j can be described by a pair $s = (s^{[1]}, s^{[2]})$. Its first component $s^{[1]}$ is a (measurable) function

$$s^{[1]} \colon \mathfrak{X}_j \to \{0, 1\},$$

with $s^{[1]}(\kappa, \mathbf{v})$ indicating whether the agent is going to join the initiative group after having observed $(\kappa, \mathbf{v})$. The second component $s^{[2]}$ describes proposal-making. With

$$\mathfrak{Y}_j := \Big\{(\kappa, \mathbf{v}, \mathbf{g}) \;\Big|\; (\kappa, \mathbf{v}) \in \mathfrak{X}_j, \mathbf{g} = (g_1, \ldots, g_N) \in \{0, 1\}^N, \\ g_j = 1,\; g_i = 0 \;\forall i : v_i \neq 1\Big\},$$

being the set of possible states of the world in which agent j is a project winner, together with the initiative groups involving participation of agent j, we take $s^{[2]}$ to be a (measurable) function

$$s^{[2]} \colon \mathfrak{Y}_j \to \{\mathcal{P}_0\,, \mathcal{P}_1\,, \mathcal{P}_2\}$$

which satisfies the condition

$$s^{[2]}(\kappa, \mathbf{v}, \mathbf{g}) \in \{\mathcal{P}_0\,, \mathcal{P}_1\} \text{ for } \mathcal{N}(\mathbf{g}) = N. \tag{10.4}$$

Thus, $s^{[2]}(\kappa, \mathbf{v}, \mathbf{g})$ is the proposal the agent is going to make if the state of the world is $(\kappa, \mathbf{v})$ and the initiative group is given by $\mathbf{g}$. Condition (10.4) is purely notational if all agents have joined the initiative group, the proposals $\mathcal{P}_1$ and $\mathcal{P}_2$ are equivalent, and we assume that $\mathcal{P}_1$ is proposed instead of $\mathcal{P}_2$ in this case.

Suppose some agent j has been chosen as proposal-maker and that he faces the situation $(\kappa, \mathbf{v}, \mathbf{g})$ with $\mathcal{N}(\mathbf{g}) \geq b$. Then, if he proposes $\mathcal{P} \in \{\mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_2\}$, the utility of any project winner i is given by[7]

$$u_i{}^{[2]}(\mathcal{P}; j; \kappa, \mathbf{v}, \mathbf{g}) = \begin{cases} 0 & \text{if } \mathcal{P} = \mathcal{P}_0, \\ \mathbb{1}_{\{\mathcal{N}(\mathbf{v}) \geq a\}} \cdot (1 - \kappa) & \text{if } \mathcal{P} = \mathcal{P}_1, \\ 1 - \mathbb{1}_{\{g_i = 1\}} \cdot \dfrac{\kappa N}{\mathcal{N}(\mathbf{g})} & \text{if } \mathcal{P} = \mathcal{P}_2, \mathcal{N}(\mathbf{g}) \geq \kappa N, \\ \mathbb{1}_{\{N - \mathcal{N}(\mathbf{g}) \geq a\}} \cdot \left(1 - \mathbb{1}_{\{g_i = 1\}} \cdot \dfrac{\kappa N}{\mathcal{N}(\mathbf{g})}\right) & \text{if } \mathcal{P} = \mathcal{P}_2, \mathcal{N}(\mathbf{g}) < \kappa N. \end{cases} \tag{10.5}$$

This can be seen as follows: We start by noting that the identity of the proposal-maker i does not directly impact on the utility of agent i. However, it is useful to keep the proposal-maker as an argument in the utility function to avoid confusion in the subsequent analyses. Since $\mathcal{P}_0$ involves neither the realization of the project nor transfers, it results in a utility of 0. If $\mathcal{P}_1$ is accepted, the project is realized and everybody pays κ. Hence $\mathcal{P}_1$ results in a strictly positive utility of $1 - \kappa$ for each project winner and in a strictly negative utility of $-\kappa$ for each project loser. Proposal $\mathcal{P}_1$ therefore passes the voting stage if and only if the number of project winners is at least a. This consideration justifies the second expression in the above equation. If proposal $\mathcal{P}_2$ is accepted, the project is realized, but non-members of the group are not taxed. This implies a utility of $1 - \kappa N / \mathcal{N}(\mathbf{g})$ for the project winners within the initiative group, a utility of 1 for the project winners outside the group, and a utility of 0 for the project losers. If $\mathcal{N}(\mathbf{g}) \geq \kappa N$, no agent's utility is strictly negative, and the proposal is accepted unanimously in the voting stage. If $\mathcal{N}(\mathbf{g}) < \kappa N$, the project winners within the initiative group will oppose the proposal, and the proposal will be accepted only if the number of agents outside the group is at least a. Of course,

[7] Recall the definition of the indicator function $\mathbb{1}$ for an arbitrary logical statement A:

$$\mathbb{1}_{\{A\}} = \begin{cases} 1 & \text{if } A \text{ true} \\ 0 & \text{if } A \text{ false}. \end{cases}$$

.

since the proposal-maker is a member of the initiative group himself, we shall never observe this case as long as the proposal-maker behaves rationally.

The expected utility of a project winner j at the beginning of Stage 1 is

$$u_j(\mathbf{s}; \kappa, \mathbf{v}, \mathbf{g}) = \begin{cases} 0 & \text{if } \mathcal{N}(\mathbf{g}) < b, \\ \dfrac{1}{\mathcal{N}(\mathbf{g})} \displaystyle\sum_{i=1, g_i=1}^{N} u_j{}^{[2]}\Big(s_i{}^{[2]}(\kappa, \mathbf{v}, \mathbf{g});\ i;\ \kappa, \mathbf{v}, \mathbf{g}\Big) & \text{if } \mathcal{N}(\mathbf{g}) \geq b, \end{cases}$$

with $\mathbf{g} = (g_1, \ldots, g_N)$ where $g_i = s_i{}^{[1]}(\kappa, \mathbf{v})$.

10.3.4 Majority Voting

In order to assess the performance of the initiative group constitutions, we will use majority voting as a benchmark case. Under majority voting, no discrimination in taxes is possible, so the society has to decide between realizing the project and sharing the taxes equally (proposal $\mathcal{P}_1$) and not realizing the project (proposal $\mathcal{P}_0$). We will not only consider simple (50 %) majority voting, but sub- and supermajority rules as well.

For any $a \in \{0, \ldots, N\}$, we define the constitution $\mathcal{C}^{\mathrm{MV}}(a)$ as follows: The agents decide between $\mathcal{P}_1$ and $\mathcal{P}_0$ in a simultaneous voting. Proposal $\mathcal{P}_1$ is accepted and realized if it is supported by at least a agents. Otherwise, it is rejected, and the project is not realized. We again impose the sincere-voting assumption as well as the tie-breaking rule (T1). This implies that the project winners will vote in favor of $\mathcal{P}_1$, while the project losers will vote against it. Hence, under $\mathcal{C}^{\mathrm{MV}}(a)$, proposal $\mathcal{P}_1$ will be accepted if and only if the number of project winners in the society reaches the required threshold, i.e. if and only if $\mathcal{N}(\mathbf{V}) \geq a$.

10.4 Equilibrium

In this section we introduce the equilibrium concept and define efficiency. We determine under which conditions the project is realized. Finally, we characterize efficient constitutions, first for the case of deterministic cost, then for the case where the cost is stochastic.

10.4.1 Equilibrium Concept, Efficiency

We use the concept of coalition-proof subgame-perfect equilibrium. We define such an equilibrium as follows:

Definition 10.1 In the game defined by the constitution $\mathcal{C}^{\mathrm{IG}}(a, b)$, a *coalition-proof subgame-perfect equilibrium* is a strategy profile $\mathbf{s} = (s_1, \ldots, s_N)$ for which the following requirements are fulfilled:

(i) The proposal-maker behaves rationally, i.e. for any $j \in \{1, \ldots, N\}$, any $(\kappa, \mathbf{v}, \mathbf{g}) \in \mathfrak{Y}_j$, and all proposals $\mathcal{P} \in \{\mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_2\}$, one has

$$u_j{}^{[2]}(\mathcal{P};\ j;\ \kappa, \mathbf{v}, \mathbf{g}) \le u_j{}^{[2]}\big(s_j{}^{[2]}(\kappa, \mathbf{v}, \mathbf{g});\ j;\ \kappa, \mathbf{v}, \mathbf{g}\big);$$

(ii) Any coalition of agents who want to deviate from $\mathbf{s}$ is unstable. Formally: For each state of the world $(\kappa, \mathbf{v}) \in \mathfrak{X}$, each set of agents $C' \subseteq \{1, \ldots, N\}$, $C' \neq \emptyset$, and each strategy profile $\mathbf{s}' = (s'_1, \ldots, s'_N)$ with $s_i = s'_i$ for all $i \notin C'$ and $u_i(\mathbf{s}'; \kappa, \mathbf{v}) > u_i(\mathbf{s}; \kappa, \mathbf{v})$ for all $i \in C'$, there exist a subset $C'' \subseteq C'$, $C'' \neq \emptyset$, and a strategy profile $\mathbf{s}''$ such that $s''_i = s'_i$ for all $i \notin C''$, and $u_i(\mathbf{s}''; \kappa, \mathbf{v}) > u_i(\mathbf{s}'; \kappa, \mathbf{v})$ for all $i \in C''$.

Under majority voting, as defined in Sect. 10.3.4, the agents' behavior is fully determined by the tie-breaking rule (T1) and the sincere voting assumption. Formally, each agent's strategy space consists of exactly one strategy. We treat the resulting single strategy profile as being trivially a coalition-proof subgame-perfect equilibrium.

The notion of an efficient mechanism transforms to the notion of an efficient constitution:

Definition 10.2 A constitution is called *efficient* if it satisfies the following requirement: A coalition-proof subgame-perfect equilibrium exists, and in any coalition-proof subgame-perfect equilibrium the project is realized[8] if the events

$$\{\mathcal{N}(\mathbf{V}) > K\,N,\ \mathit{but\ the\ project\ is\ not\ realized}\}$$

and

$$\{\mathcal{N}(\mathbf{V}) < K\,N\,,\ \mathit{but\ the\ project\ is\ realized}\}$$

both occur with probability zero.

10.4.2 Proposal-Making

We analyze the second stage of the game. The next proposition describes proposal-making in equilibrium.

Proposition 10.1 *Suppose the constitution is $\mathcal{C}^{\mathrm{IG}}(a, b)$, $a \in \{0, \ldots, N\}$, $b \in \{1, \ldots, N\}$, and there is complete information. Consider an agent j the proposal-maker, who faces a situation described by $(\kappa, \mathbf{v}, \mathbf{g}) \in \mathfrak{Y}_j$ with $\mathcal{N}(\mathbf{g}) \ge b$.*

---

[8] A project is realized if it is proposed and approved.

(i) If $\mathcal{N}(\mathbf{v}) \geq a$, proposing $\mathcal{P}_1$ is a strictly dominant strategy. The proposal will be accepted in the voting stage.
(ii) If $\mathcal{N}(\mathbf{v}) < a$ and $\mathcal{N}(\mathbf{g}) > \kappa N$, proposing $\mathcal{P}_2$ is a strictly dominant strategy. The proposal will be accepted in the voting stage.
(iii) If $\mathcal{N}(\mathbf{v}) < a$ and $\mathcal{N}(\mathbf{g}) = \kappa N$, the proposal-maker is indifferent between $\mathcal{P}_0$, $\mathcal{P}_1$ and $\mathcal{P}_2$. The proposals $\mathcal{P}_0$ and $\mathcal{P}_2$ will be accepted in the voting stage, $\mathcal{P}_1$ will not.
(iv) If $\mathcal{N}(\mathbf{v}) < a$ and $\mathcal{N}(\mathbf{g}) < \kappa N$, the proposal-maker is indifferent between $\mathcal{P}_0$ and $\mathcal{P}_1$. He considers $\mathcal{P}_2$ as strictly inferior if it is going to be accepted in the voting stage; otherwise, it is equivalent to $\mathcal{P}_0$ and $\mathcal{P}_1$. The project will not be realized.

The proof of Proposition 10.1 is given in Appendix.

The different cases in Proposition 10.1 are intuitive and show how for a given vector of utilities, the size of the initiative group, together with the constitution, shapes the proposal-making and the outcomes. With the help of Proposition 10.1, we can now give necessary and sufficient conditions for the realization of the project in equilibrium.

Proposition 10.2 *Consider any constitution $\mathcal{C}^{\mathrm{IG}}(a, b)$ with $a \in \{0, \ldots, N\}$, $b \in \{1, \ldots, N\}$. Then,*

(a) a coalition-proof subgame-perfect equilibrium exists, and
(b) in every coalition-proof subgame-perfect equilibrium and for every state of the world $(\kappa, \mathbf{v})$, the following statements hold:

(i) If

$$\mathcal{N}(\mathbf{v}) < \max\Big\{b, \min\big\{a, \lfloor \kappa N \rfloor\big\}\Big\}, \tag{10.6}$$

the project is not realized.

(ii) If

$$\mathcal{N}(\mathbf{v}) \geq \max\Big\{b, \min\big\{a, \lfloor \kappa N \rfloor + 1\big\}\Big\}, \tag{10.7}$$

the project is realized. More precisely, $\mathcal{P}_1$ is made and accepted if

$$\mathcal{N}(\mathbf{v}) \geq \max\{a, b\}, \tag{10.8}$$

and $\mathcal{P}_2$ is made and accepted if

$$\max\big\{b, \lfloor \kappa N \rfloor + 1\big\} \leq \mathcal{N}(\mathbf{v}) < a. \tag{10.9}$$

The proof of Proposition 10.2 is given in Appendix.

Proposition 10.2 displays how for given costs and vector of utilities, the constitution shapes the realization of the project. This is expressed in the critical Condition (10.7). A higher value of the threshold b for the size of the initiative group or the majority rule threshold a, tends to make it more difficult to realize the project.

Moreover, there are two ways to achieve acceptance of the project, depending on the relationship between a and b and the project parameters.

We note that the strategies defined in the proof of part (a) are not symmetric. The strategies ensure that the initiative group has enough members to realize the project whenever this is possible. With our strategy profile, those project winners join the group who have the lowest indices j. Of course, this specification is arbitrary. Since with $\mathcal{P}_2$, project winners outside the initiative group are better off than group members, there is a coordination problem. In our analysis, we have assumed implicitly that this coordination problem can be solved, e.g., by using the labeling of agents. Other coordination devices are conceivable and will be discussed in Sect. 10.6.

10.4.3 Deterministic Cost

With the results of Sect. 10.4.2, we can now characterize efficient constitutions. We first analyze the case of deterministic cost, i.e. we assume $P(K = \kappa) = 1$ for some $\kappa \in (0; 1)$. We will deal with stochastic cost in Sect. 10.4.4.

Proposition 10.3 *If $P(K = \kappa) = 1$ for some $\kappa \in (0, 1)$, the following statements hold:*

(i) For all $a \in \{0, \dots, N\}$, $b \in \{1, \dots, N\}$ with

$$b \leq \lfloor \kappa N \rfloor + 1 \quad \textit{and} \quad \max\{a, b\} > \lceil \kappa N \rceil - 1, \tag{10.10}$$

the constitution $\mathcal{C}^{\mathrm{IG}}\big(a, b\big)$ is efficient.

(ii) If in addition $P\big(\mathcal{N}(\mathbf{V}) = m\big) > 0$ for all $m \in \{0, \dots, N\}$, Condition (10.10) is necessary for $\mathcal{C}^{\mathrm{IG}}(a, b)$ to be efficient.

The proof of Proposition 10.3 is given in Appendix.

In the following proposition, we consider majority voting:

Proposition 10.4 *If $P(K = \kappa) = 1$ for some $\kappa \in (0, 1)$, the following statements hold:*

(i) For a with

$$\kappa N \leq a \leq \kappa N + 1, \tag{10.11}$$

the constitution $\mathcal{C}^{\mathrm{MV}}\big(a\big)$ is efficient.

(ii) If also $P\big(\mathcal{N}(\mathbf{V}) = m\big) > 0$ for all $m \in \{0, \dots, N\}$, Condition (10.11) is necessary for $\mathcal{C}^{\mathrm{MV}}\big(a\big)$ to be efficient.

The proof of Proposition 10.4 is given in Appendix.

We observe that with deterministic costs, initiative groups constitutions, as well as majority-voting constitutions, yield social efficiency if (a, b) in the former and (a) in the latter are chosen appropriately. The reason is that with deterministic costs, there exists a deterministic threshold for the number of winners, above which the public project is efficient. These deterministic thresholds can be used to construct supermajority rules that induce adoption of the project if and only if it is efficient.

With stochastic costs, this will change, and only initiative-group constitutions will continue to yield social efficiency.

10.4.4 Stochastic Cost

We now assume that both the agents' types V_j and the cost K of the project are stochastic.

Proposition 10.5 *(i) The initiative-group constitution* $\mathcal{C}^{\mathrm{IG}}(a, b)$ *is efficient if*

$$a = N \quad \textit{and} \quad b = 1. \tag{10.12}$$

(ii) Suppose K *and* $\mathcal{N}(\mathbf{V})$ *are stochastically independent. In addition, suppose that* $P\big(0 < KN < 1\big) > 0$ *and* $P\big(N - 1 < KN < N\big) > 0$*, as well as* $P\big(\mathcal{N}(\mathbf{V}) = m\big) > 0$ *for all* $m \in \{0, \ldots, N\}$*. Then (10.12) is necessary for* $\mathcal{C}^{\mathrm{IG}}(a, b)$ *to be efficient. Further, except for the knife-edge cases* $a = KN$ *and* $a = KN + 1$*, there is no parameter* a *such that the rule* $\mathcal{C}^{\mathrm{MV}}(a)$ *is efficient.*

The proof of Proposition 10.5 is given in Appendix.

The proposition states that an initiative-group constitution without a minimum requirement on the size of the initiative group, but with a unanimity requirement in the voting stage, yields an efficient outcome. This is due to the fact that under an initiative-group constitution, the project losers can be compensated by the project winners indirectly. If the project is socially desirable, the number of project winners joining the initiative group in equilibrium is large enough to make compensation possible. Since losers cannot be compensated under majority voting, majority-voting constitutions are not efficient, as the majority that would have to be fixed in the constitution, depends on the cost of the project.

To sum up, initiative-group constitutions can replicate what would be obtained in a Coasian setting in which agents bargain frictionlessly over the surplus they can generate by undertaking a public project. This property is much more challenging to achieve—but still possible—when there are strict losers. This will be addressed next.

10.5 Strict Losers

In this section, we modify the model by introducing strict project losers. As in Sect. 10.4, we first analyze proposal-making. For the case in which the number of strict losers is deterministic, we give necessary and sufficient conditions for project realization in equilibrium. We shall see that the efficient constitution may now involve non-trivial requirements for the minimum size of the initiative group and that unanimity in the voting stage needs no longer be optimal. The case in which the number of strict losers is stochastic as well, is illustrated by a numerical example, showing that initiative-group constitutions can be strictly welfare-superior than any majority-voting constitution.

10.5.1 Modification of the Model

We consider an extension of our model, where two different types of project losers exist: "weak" project losers, whose utility from the project is 0, and "strict" project losers, whose utility is $-w$, with $w > 0$. Formally, an agent's type is now given by either $V_j = 1$, $V_j = 0$ or $V_j = -w$, which means that the states of the world are from the set

$$\mathfrak{X} := (0; 1) \times \{-w, 0, 1\}^N.$$

The rules of initiative-group constitution are maintained. In particular, only project winners can become members of the initiative group, and differences in taxation may only be due to membership or non-membership in the initiative group.

The key difference to the model in the previous section is that now, proposal $\mathcal{P}_2$ will not necessarily be accepted by all agents outside the initiative group, as the proposal does not involve any compensation for the utility loss of the strict project losers. Hence, in order to achieve the necessary majority, the proposal-maker may be willing to propose such compensation. Since discrimination among non-members of the initiative group is forbidden, this means that all non-members must be subsidized. Consequently, a rational proposal-maker will make one of the following four proposals:

$$\begin{aligned}
\mathcal{P}_0 &: \quad (0,\ 0,\ 0), \\
\mathcal{P}_1 &: \quad (1,\ K,\ K), \\
\mathcal{P}_2 &: \quad \big(1,\ KN/\mathcal{N}(\mathbf{g}),\ 0\big), \quad \text{or} \\
\mathcal{P}_3 &: \quad \Big(1,\ (K+w)N/\mathcal{N}(\mathbf{g}) - w,\ -w\Big).
\end{aligned}$$

If $\mathcal{N}(\mathbf{g}) = N$, the proposals $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ are equivalent, and we again assume that $\mathcal{P}_1$ is proposed instead of $\mathcal{P}_2$ or $\mathcal{P}_3$ in this case. The definitions of $\mathfrak{X}_j$ and $\mathfrak{Y}_j$ carry over. In essence, the definition of a strategy $s = (s^{[1]}, s^{[2]})$ does so as well, but with proposal-making now being described by a function

$$s^{[2]} : \mathfrak{Y}_j \to \{\mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\},$$

which again is required to satisfy Condition (10.4). For $\mathbf{v} = (v_1, \ldots, v_N)$ we use the notation

$$\mathcal{N}_{\geq 0}(\mathbf{v}) := \#\{i \mid v_i \geq 0\}$$

to denote the number of agents who are not a strict loser.

In order to describe the utility of a project winner, we have to modify Eq. (10.5) as follows. Let i be a project winner facing the situation $(\kappa, \mathbf{v}, \mathbf{g})$ with $\mathcal{N}(\mathbf{g}) \geq b$. His utility from Proposal $\mathcal{P}_0$ is

$$u_i^{[2]}(\mathcal{P}_0; j; \kappa, \mathbf{v}, \mathbf{g}) = 0.$$

His utility from $\mathcal{P}_1$ is

$$u_i^{[2]}(\mathcal{P}_1; j; \kappa, \mathbf{v}, \mathbf{g}) = \mathbb{1}\big\{\mathcal{N}(\mathbf{v}) \geq a\big\} \cdot (1 - \kappa).$$

If $\mathcal{N}_{\geq 0}(\mathbf{v}) \geq a$ and $\mathcal{N}(\mathbf{g}) \geq \kappa N$ or if $\mathcal{N}_{\geq 0}(\mathbf{v}) - \mathcal{N}(\mathbf{g}) \geq a$, his utility from $\mathcal{P}_2$ is

$$u_i^{[2]}(\mathcal{P}_2; j; \kappa, \mathbf{v}, \mathbf{g}) = 1 - \mathbb{1}\big\{g_i = 1\big\} \cdot \frac{\kappa N}{\mathcal{N}(\mathbf{g})}.$$

Otherwise, $\mathcal{P}_2$ is not accepted in the voting stage and hence,

$$u_i^{[2]}(\mathcal{P}_2; j; \kappa, \mathbf{v}, \mathbf{g}) = 0.$$

If $\mathcal{N}(\mathbf{g}) \geq (\kappa + w)N$ or $N - \mathcal{N}(\mathbf{g}) \geq a$, proposal $\mathcal{P}_3$ gives him a utility of

$$u_i^{[2]}(\mathcal{P}_3; j; \kappa, \mathbf{v}, \mathbf{g}) = 1 + w - \mathbb{1}\big\{g_i = 1\big\} \cdot \frac{(\kappa + w)N}{\mathcal{N}(\mathbf{g})}.$$

Otherwise, $\mathcal{P}_3$ is not accepted in the voting stage and hence,

$$u_i^{[2]}(\mathcal{P}_3; j; \kappa, \mathbf{v}, \mathbf{g}) = 0.$$

10.5.2 Results

The following proposition describes proposal-making.

Proposition 10.6 *Suppose the constitution is $\mathcal{C}^{\mathrm{IG}}(a, b)$ and there is complete information. Consider an agent j becoming the proposal-maker, who faces a situation described by $(\kappa, \mathbf{v}, \mathbf{g}) \in \mathfrak{Y}_j$ with $\mathcal{N}(\mathbf{g}) \geq b$.*

(i) If $\mathcal{N}(\mathbf{v}) \geq a$, proposing $\mathcal{P}_1$ is a strictly dominant strategy. The proposal will be accepted in the voting stage.

(ii) *If* $\mathcal{N}(\mathbf{v}) < a$, $\mathcal{N}_{\geq 0}(\mathbf{v}) \geq a$, *and* $\mathcal{N}(\mathbf{g}) > \kappa N$, *proposing* $\mathcal{P}_2$ *is a strictly dominant strategy. The proposal will be accepted in the voting stage.*

(iii) *If* $\mathcal{N}(\mathbf{v}) < a$, $\mathcal{N}_{\geq 0}(\mathbf{v}) \geq a$, *and* $\mathcal{N}(\mathbf{g}) = \kappa N$, *the proposal-maker is indifferent between* $\mathcal{P}_0$, $\mathcal{P}_1$ *and* $\mathcal{P}_2$. *The proposals* $\mathcal{P}_0$ *and* $\mathcal{P}_2$ *will be accepted in the voting stage,* $\mathcal{P}_1$ *will not. Proposal* $\mathcal{P}_3$ *is strictly inferior.*

(iv) *If* $\mathcal{N}_{\geq 0}(\mathbf{v}) < a$ *and* $\mathcal{N}(\mathbf{g}) > (\kappa + w)/(1 + w) \cdot N$, *proposing* $\mathcal{P}_3$ *is a strictly dominant strategy. The proposal will be accepted in the voting stage.*

(v) *If* $\mathcal{N}_{\geq 0}(\mathbf{v}) < a$ *and* $\mathcal{N}(\mathbf{g}) = (\kappa + w)/(1 + w) \cdot N$, *the proposal-maker is indifferent between all four proposals. The proposals* $\mathcal{P}_0$ *and* $\mathcal{P}_3$ *will be accepted in the voting stage,* $\mathcal{P}_1$ *and* $\mathcal{P}_2$ *will not.*

(vi) *If*

$$\mathcal{N}(\mathbf{v}) < a \quad \textit{and} \quad \mathcal{N}(\mathbf{g}) < \kappa N$$

or

$$\mathcal{N}_{\geq 0}(\mathbf{v}) < a \quad \textit{and} \quad \mathcal{N}(\mathbf{g}) < \frac{\kappa + w}{1 + w} \cdot N,$$

the proposal-maker is indifferent between $\mathcal{P}_0$ *(which will be accepted in the voting stage),* $\mathcal{P}_1$ *(which will not be accepted) and any proposal* $\mathcal{P} \in \{\mathcal{P}_2, \mathcal{P}_3\}$ *which will not be accepted in the voting stage. Each proposal in* $\{\mathcal{P}_2, \mathcal{P}_3\}$ *that will be accepted in the voting stage is strictly inferior. The project will not be realized.*

The proof of Proposition 10.6 is given in Appendix.

The proof follows the logic outlined in the proof of Proposition 10.1, but the additional proposal $\mathcal{P}_3$ has to be taken into account.

The next proposition is an analogue to Proposition 10.2. It gives necessary and sufficient conditions for project realization in equilibrium.

Proposition 10.7 *Consider a constitution* $\mathcal{C}^{\mathrm{IG}}(a, b)$ *for some* $a \in \{0, \ldots, N\}$, $b \in \{1, \ldots, N\}$. *Then,*

(a) *a coalition-proof subgame-perfect equilibrium exists, and*

(b) *in every coalition-proof subgame-perfect equilibrium and for every state of the world* $(\kappa, \mathbf{v})$, *the following statements hold:*

(i) *If*

$$\mathcal{N}(\mathbf{v}) < b$$

or

$$\mathcal{N}(\mathbf{v}) < a \quad \textit{and} \quad \mathcal{N}(\mathbf{v}) < \kappa N$$

or

$$\mathcal{N}_{\geq 0}(\mathbf{v}) < a \quad \textit{and} \quad \mathcal{N}(\mathbf{v}) < \frac{\kappa + w}{1 + w} \cdot N,$$

the project is not realized.

(ii) Proposal $\mathcal{P}_1$ is made and accepted if

$$\mathcal{N}(\mathbf{v}) \geq \max\{a, b\},$$

$\mathcal{P}_2$ is made and accepted if

$$\max\big\{b, \lfloor \kappa N \rfloor + 1\big\} \leq \mathcal{N}(\mathbf{v}) < a \leq \mathcal{N}_{\geq 0}(\mathbf{v}),$$

and $\mathcal{P}_3$ is made and accepted if

$$\max\left\{b, \left\lfloor \frac{\kappa + w}{1 + w} \cdot N \right\rfloor + 1\right\} \leq \mathcal{N}(\mathbf{v}) \leq \mathcal{N}_{\geq 0}(\mathbf{v}) < a.$$

The proof of Proposition 10.7 is given in Appendix.

By giving an explicit example, we will now show that if strict losers exist, setting $b > 1$ can be welfare-improving. In particular, it follows that Proposition 10.5 does not necessarily hold. For this purpose, we note that the project is strictly socially desirable if total utility strictly exceeds total cost, i.e. if

$$1 \cdot \mathcal{N}(\mathbf{V}) - w \cdot \big(N - \mathcal{N}_{\geq 0}(\mathbf{V})\big) > \kappa N.$$

If total utility is strictly below total cost, the project is socially disadvantageous.

We assume that K is constant, i.e. that $P(K = \kappa) = 1$ for some $\kappa \in (0; 1)$. Further, we assume that the number of strict losers in the society is constant, i.e. that $P\big(\big(N - \mathcal{N}_{\geq 0}(\mathbf{V})\big) = L\big) = 1$ for some $L \in \{0, 1, \ldots, N\}$. Then, the project is strictly socially desirable if and only if

$$\mathcal{N}(\mathbf{V}) > \kappa N + wL,$$

and it is strictly disadvantageous if the reverse inequality (with "$<$" instead of "$>$") holds.

Proposition 10.8 *Suppose the above assumptions hold.*

(i) The constitution $\mathcal{C}^{\mathrm{IG}}(a, b)$ with

$$a = N - L \quad \textit{and} \quad b = \lfloor \kappa N + wL \rfloor + 1, \tag{10.13}$$

is efficient.

(ii) Suppose that $\mathcal{N}(\mathbf{V})$ takes each of the values $0, \ldots, N - L$ with positive probability. Furthermore, suppose that

$$\kappa N + wL + 1 < N - L - w^{-1} \tag{10.14}$$

and

$$wL > 1. \tag{10.15}$$

Then, a necessary condition for the constitution $\mathcal{C}^{\mathrm{IG}}(a, b)$ to be efficient is:

$$a \leq N - L \quad \textit{and} \quad \kappa N + wL \leq b \leq \kappa N + wL + 1. \tag{10.16}$$

(iii) Suppose that $\mathcal{N}(\mathbf{V})$ takes each of the values $0, \ldots, N - L$ with positive probability and that $\kappa N + wL < N - L$. Then the constitution $\mathcal{C}^{\mathrm{MV}}(a)$ is efficient if and only if

$$\kappa N + wL \leq a \leq \kappa N + wL + 1.$$

The proof of Proposition 10.8 is given in Appendix.

It is easy to see that there exist parameter values, for which (10.14) and (10.15) are fulfilled. Take for instance, $N = 10$, $\kappa = 0.1$, $L = 1$, and $w = 1.0$. Then Condition (10.16) reads

$$a \leq 9 \quad \text{and} \quad b = 2.$$

We thus have shown that if strict losers exist, a non-trivial size of the initiative group can be necessary to achieve efficiency.[9]

What is the intuition behind these results? As *equal taxation within groups* is required, it is not possible to balance merely the higher losses of the strict losers by higher subsidies. To achieve consent of the strict losers, all non-members of the group have to be subsidized with an amount of $\kappa + w$. Since all non-winners are outside the initiative group, at least L individuals have to be subsidized. By Assumption (10.14) this is not possible if the society comprises exactly $\lceil \kappa N + wL \rceil + 1$ project winners—a case in which the project is desirable. Hence, if efficiency is to be achieved, consent of the strict losers cannot be required in the voting stage. In particular, contrary to Proposition 10.5, unanimity in the voting stage does not yield an efficient outcome. If however, strict losers' consent is not needed in the voting stage, it is sufficient for the initiative group to subsidize all non-members with an amount of κ, thus receiving consent of the non-strict losers. Assumption (10.15) ensures that the strict losers' utility loss is so high that, from a utilitarian point of view, the society must comprise strictly more than $\lfloor \kappa N + wL \rfloor + 1$ project winners to outweigh this utility loss, in order to make project realization socially desirable. This can and must be guaranteed by an appropriate requirement on the minimal size of the initiative group (i.e. by some $b > 1$).

Since the cost of the project and the strict losers' collective loss are deterministic, and since under majority voting, no subsidies for weak losers are possible, an efficient outcome can be reached by $\mathcal{C}^{\mathrm{MV}}(a)$ as well, with the parameter a suitably chosen. In the subsequent section we give a numerical example demonstrating that initiative-group constitutions can be strictly superior if the number of strict losers (and thus the collective loss) is stochastic.

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[9] With these parameter values, the same goal can be reached by the constitution $\mathcal{C}^{\mathrm{MV}}(a)$ with $a = 2$.

10.5.3 A Numerical Example

To illustrate that initiative-group constitutions can be welfare-superior to any majority-voting constitution, we give a numerical example.

We consider a society of $N = 10$ agents. The agents' types are independent. Each agent is a project winner with a probability of $p_1 = 0.3$, a strict loser with a probability of $p_2 = 0.2$, and a weak loser with a probability of $p_3 = 1 - p_1 - p_2 = 0.5$. Hence the numbers of winners, weak losers, and strict losers are jointly trinomially distributed. Each winner's utility from the project is 1, each weak loser's utility is 0, and each strict loser's utility is $-w$ with $w = 1.0$. The per-capita cost of the project is $\kappa = 0.2$.

If the project were realized if and only if the project yields a non-negative total welfare, the expected welfare would be given by

$$\sum_{\substack{m_1, m_2, m_3 \\ m_1 + m_2 + m_3 = N}} \binom{N}{m_1, m_2, m_3} p_1^{m_1} p_2^{m_2} p_3^{m_3} \left[1 \cdot m_1 - \kappa N - w m_2\right]^+, \tag{10.17}$$

with

$$[x]^+ := \begin{cases} x & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$

Expression (10.17) is the maximal expected welfare that could be reached if a completely informed, benevolent dictator decided on the realization of the project after observing the agents' types. For the above parameter values, this maximal expected welfare amounts to 0.458, as a numerical evaluation of Expression (10.17) shows. The corresponding probability of project realization, which is given by

$$\sum_{\substack{m_1, m_2, m_3 \\ m_1 + m_2 + m_3 = N}} \binom{N}{m_1, m_2, m_3} p_1^{m_1} p_2^{m_2} p_3^{m_3} \cdot \mathbb{1}\left\{1 \cdot m_1 - \kappa N - w m_2 \geq 0\right\},$$

amounts to 0.251.

Table 10.1 states the expected welfares and the corresponding probabilities of project realization if majority voting $\mathcal{C}^{\mathrm{MV}}(a)$ is used to decide on project realization. As we can observe, setting $a = 4$ results in an expected welfare of 0.370, which is the optimum among all majority-voting constitutions. Hence if a majority voting is to be used, the project should be realized if at least four individuals vote in favour of the project. Since under majority voting, an agent is in favour of the project if and only if he is a project winner, this is equivalent to the society's comprising at least four project winners. The probability of project realization is 0.350. Compared to the optimum of 0.251 calculated above, the project is realized too frequently.

The numerical results for the initiative-group constitutions $\mathcal{C}^{\mathrm{IG}}(a, b)$ are listed in Tables 10.2 and 10.3, under the assumption that the project is realized whenever the conditions from Part (b)(i) of Proposition 10.7 are not fulfilled. For $a = 8$ and $b = 4$ expected welfare is maximal, amounting to 0.430, which is larger than what can

Table 10.1 Constitutions $\mathcal{C}^{\mathrm{MV}}(a)$ in the example from Sect. 10.5.3.

a	EW	PR	PNR
1	−0.863	0.972	0.028
2	−0.430	0.851	0.149
3	0.103	0.617	0.383
4	0.370	0.350	0.650
5	0.313	0.150	0.850
6	0.151	0.047	0.953
7	0.046	0.011	0.989
8	0.009	0.002	0.998
9	0.001	0.000	1.000
10	0.000	0.000	1.000

The figures in the second column represent the expected welfare (EW) under the constitution $\mathcal{C}^{\mathrm{MV}}(a)$; the figures in the third and fourth column are the corresponding probabilities that the project is realized (PR) or not realized (PNR). Choosing $a = 4$ yields the highest expected welfare

be achieved by the optimal majority-voting constitution $\mathcal{C}^{\mathrm{MV}}(4)$. The probability of project realization is 0.287, which is lower than under $\mathcal{C}^{\mathrm{MV}}(4)$. How can this welfare gain be explained? As the figures indicate, $\mathcal{P}_3$ is (almost) never implemented under $\mathcal{C}^{\mathrm{IG}}(8, 4)$, which means that strict losers are never fully compensated for their losses. Hence, roughly speaking, the project is realized if and only if the society comprises at most $10 - 8 = 2$ strict losers and at least 4 project winners. Thereby, unlike the majority-voting constitution $\mathcal{C}^{\mathrm{MV}}(4)$, the initiative-group constitution $\mathcal{C}^{\mathrm{IG}}(8, 4)$ prevents project realization not only in cases in which the society comprises a reasonable number of strict project winners, but a high number of strict losers as well. Note, further, that under $\mathcal{C}^{\mathrm{IG}}(8, 4)$, proposal $\mathcal{P}_1$ is implemented very rarely (the probability amounts to 0.002). Proposal $\mathcal{P}_2$ (its probability being 0.285) accounts for nearly all cases of project realization. Since $\mathcal{P}_2$, unlike majority voting, involves subsidies for weak losers, the utility differences among the agents will be weaker under $\mathcal{C}^{\mathrm{IG}}(8, 4)$.

10.6 Discussion and Directions of Future Research

Our analysis rests on some simplifying assumptions which we address in more detail in this section. Regarding the game theoretic treatment, we have assumed sincere voting and the tie-breaking rule (T1). Both are not essential as there is only one voting round at the end of the game, backward induction dictates that any subgame perfect equilibrium will involve sincere voting. The tie-breaking rule simplifies the exposition.

Essential, however, is the assumption of perfect coordination among project winners who will join the initiative group. There are several ways how such coordination

Table 10.2 Constitutions $\mathcal{C}^{\mathrm{IG}}(a, b)$ in the example from Sect. 10.5.3.

b	a	EW	PR	PNR	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$
1	1	−0.863	0.972	0.028	0.972	0.000	0.000
1	2	−0.430	0.851	0.149	0.851	0.000	0.000
1	3	0.103	0.617	0.383	0.617	0.000	0.000
1	4	0.103	0.617	0.383	0.350	0.267	0.000
1	5	0.108	0.616	0.384	0.150	0.466	0.000
1	6	0.135	0.609	0.391	0.047	0.562	0.000
1	7	0.225	0.573	0.427	0.011	0.563	0.000
1	8	0.370	0.468	0.532	0.002	0.466	0.000
1	9	0.407	0.279	0.721	0.000	0.277	0.002
1	10	0.220	0.091	0.909	0.000	0.085	0.006
2	≤2	−0.430	0.851	0.149	0.851	0.000	0.000
2	3	0.103	0.617	0.383	0.617	0.000	0.000
2	4	0.103	0.617	0.383	0.350	0.267	0.000
2	5	0.108	0.616	0.384	0.150	0.466	0.000
2	6	0.135	0.609	0.391	0.047	0.562	0.000
2	7	0.225	0.573	0.427	0.011	0.563	0.000
2	8	0.370	0.468	0.532	0.002	0.466	0.000
2	9	0.407	0.279	0.721	0.000	0.277	0.002
2	10	0.220	0.091	0.909	0.000	0.085	0.006
3	≤3	0.103	0.617	0.383	0.617	0.000	0.000
3	4	0.103	0.617	0.383	0.350	0.267	0.000
3	5	0.108	0.616	0.384	0.150	0.466	0.000
3	6	0.135	0.609	0.391	0.047	0.562	0.000
3	7	0.225	0.573	0.427	0.011	0.563	0.000
3	8	0.370	0.468	0.532	0.002	0.466	0.000
3	9	0.407	0.279	0.721	0.000	0.277	0.002
3	10	0.220	0.091	0.909	0.000	0.085	0.006
4	≤4	0.370	0.350	0.650	0.350	0.000	0.000
4	5	0.370	0.350	0.650	0.150	0.200	0.000
4	6	0.376	0.348	0.652	0.047	0.301	0.000
4	7	0.399	0.336	0.664	0.011	0.325	0.000
4	8	0.430	0.287	0.713	0.002	0.285	0.000
4	9	0.381	0.183	0.817	0.000	0.181	0.002
4	10	0.195	0.066	0.934	0.000	0.059	0.006

The table is continued by Table 10.3

Table 10.3 Continuation of Table 10.2

b	a	EW	PR	PNR	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$
5	≤ 5	0.313	0.150	0.850	0.150	0.000	0.000
5	6	0.313	0.150	0.850	0.047	0.103	0.000
5	7	0.316	0.147	0.853	0.011	0.137	0.000
5	8	0.313	0.133	0.867	0.002	0.131	0.000
5	9	0.264	0.093	0.907	0.000	0.091	0.002
5	10	0.142	0.039	0.961	0.000	0.033	0.006
6	≤ 6	0.151	0.047	0.953	0.047	0.000	0.000
6	7	0.151	0.047	0.953	0.011	0.037	0.000
6	8	0.149	0.045	0.955	0.002	0.043	0.000
6	9	0.130	0.035	0.965	0.000	0.033	0.002
6	10	0.084	0.020	0.980	0.000	0.014	0.006
7	≤ 7	0.046	0.011	0.989	0.011	0.000	0.000
7	8	0.046	0.011	0.989	0.002	0.009	0.000
7	9	0.046	0.011	0.989	0.000	0.009	0.002
7	10	0.046	0.011	0.989	0.000	0.004	0.006
8	≤ 8	0.009	0.002	0.998	0.002	0.000	0.000
8	9	0.009	0.002	0.998	0.000	0.001	0.000
8	10	0.009	0.002	0.998	0.000	0.001	0.001
9	≤ 9	0.001	0.000	1.000	0.000	0.000	0.000
9	10	0.001	0.000	1.000	0.000	0.000	0.000
10	≤ 10	0.000	0.000	1.000	0.000	0.000	0.000

The figures in third column represent the expected welfare (EW) under the constitution $\mathcal{C}^{\mathrm{IG}}(a,b)$. The figures in the fourth column are the corresponding probabilities that the project is realized (PR). The fourth column gives the probability that the project is not realized (NPR). The fifth, sixth and seventh column contain the probabilities that proposal $\mathcal{P}_1$, $\mathcal{P}_2$, or $\mathcal{P}_3$ is made and accepted. The highest expected welfare achieved for $a = 8$ and $b = 4$

can be achieved such as conventions or communication (see, e.g., Fahrenberger and Gersbach 2010). Finally, when strict project losers are present, they may have a strategic incentive to join the initiative group in order to deter the adoption of the public project as they may not be subsidized if a project winner makes a proposal. As long as the size of the interest group is at least two members and proposal-making in the initiative group is governed by majority decisions, all of our equilibria remain equilibria when project losers may join the initiative group. Joining would have no impact on the proposal made, but would expose strict project losers to equal or higher taxation than the rest of the electorate. This makes joining the initiative group unattractive.

However, allowing all agents to join the interest group can lead to a host of additional interesting issues that can be pursued in future research. As mentioned before, project losers could attempt to masquerade as project winners to join the initiative group and to propose $\mathcal{P}_0$, the null project, in order to block the project. Then, we may ask whether equilibria with initiative groups consisting of project losers and winners exist when individual utilities are private information. Moreover, how initiative groups might ferret out such rogue agents might be an interesting issue. One can imagine a pre-commitment to differential taxation, for instance, where group members pay more than the rest, or membership costs in general, as well as collective decision-making procedures within the initiative group, to decide on the proposal to be made by the group.

The present inquiry raises a number of further issues that can – and should – be pursued in future research. First, the current model and results could be linked to the dominant strategy mechanism design literature. Vickrey–Clarke–Groves Mechanisms for instance, induce agents to report their type truthfully. In the present model, eliciting information in not the problem, as types are known, but initiative-group constitutions aim at agents volunteering the information that they are project winners, joining the initiative group, and paying higher taxes. Thus, there is a similarity to a standard mechanism design problem, and one might try more general approaches when agents sort themselves into three or more groups (such as beneficiaries, weak project losers, and strict project losers, for instance), each facing different taxation. A suitable constitution may then dictate that the project will be executed if and only if the sizes of the groups according to this endogenous sorting indicate that the execution would be efficient.

10.7 Conclusion

This chapter is a first exploration of the virtues and potential drawbacks of initiative-group constitutions in the context of public project provision. Initiative group constitutions combine the selection of proposal-makers, protection of citizen from excessive taxation and majority voting in a way that promises to be a fruitful institutional design for other collective decisions.

Appendix

Proof of Proposition 10.1

The statements are verified by inspecting Eq. (10.5). We go through them one-by-one.

(i) Consider the case $\mathcal{N}(\mathbf{v}) \geq a$. Proposal $\mathcal{P}_1$ implies a utility of

$$u_j{}^{[2]}\big(\mathcal{P}_1; j; \kappa, \mathbf{v}, \mathbf{g}\big) = 1 - \kappa > 0$$

for the proposal-maker. The proposal will be accepted in Stage 2. It obviously dominates $\mathcal{P}_0$. If $\mathcal{N}(\mathbf{g}) = N$, Proposal $\mathcal{P}_2$ is not made by assumption. If $\mathcal{N}(\mathbf{g}) < N$, the utility from $\mathcal{P}_2$ is at most $1 - \kappa N/\mathcal{N}(\mathbf{g})$, which is less than the utility from $\mathcal{P}_1$. Hence, proposing $\mathcal{P}_1$ is a strictly dominant strategy.

(ii) Let $\mathcal{N}(\mathbf{v}) < a$ and $\mathcal{N}(\mathbf{g}) > \kappa N$. $\mathcal{P}_2$ implies a utility of

$$u_j{}^{[2]}\big(\mathcal{P}_2;\, j;\, \kappa, \mathbf{v}, \mathbf{g}\big) = 1 - \kappa N / \mathcal{N}(\mathbf{g}) > 0$$

for the proposal-maker. $\mathcal{P}_2$ will be accepted unanimously in the voting stage. By Eq. (10.5), the proposals $\mathcal{P}_0$ and $\mathcal{P}_1$ lead to a utility of 0. Hence, proposing $\mathcal{P}_2$ is a strictly dominant strategy.

(iii) Let $\mathcal{N}(\mathbf{v}) < a$ and $\mathcal{N}(\mathbf{g}) = \kappa N$. Proposal $\mathcal{P}_0$ gives every agent a utility of zero. Proposal $\mathcal{P}_2$ implies a utility of $1 - \kappa N / \mathcal{N}(\mathbf{g}) = 0$ for members of the initiative group and a non-negative utility for non-members, since non-members are not taxed. These two proposals will be accepted unanimously in the voting stage. Proposal $\mathcal{P}_1$ will not pass the voting stage, being opposed by all project losers, as acceptance would mean a negative utility of $-\kappa$ for project losers. Hence, each of the three proposals leads to a utility of zero for the proposal-maker.

(iv) Let $\mathcal{N}(\mathbf{v}) < a$ and $\mathcal{N}(\mathbf{g}) < \kappa N$. By the same reasoning as in (ii) and (iii), $\mathcal{P}_1$ will not pass the voting stage. Hence, the proposal-maker is indifferent between $\mathcal{P}_0$ and $\mathcal{P}_1$. The acceptance of proposal $\mathcal{P}_2$ would mean a utility of $1 - \kappa N / \mathcal{N}(\mathbf{g}) < 0$ for the proposal-maker, such that $\mathcal{P}_2$ is strictly dominated by $\mathcal{P}_0$ and $\mathcal{P}_1$ if it is going to pass the voting stage; otherwise, the proposal-maker is indifferent between all three proposals. □

Proof of Proposition 10.2

We order agents according to their benefits and use the labels of agents to construct the equilibrium. (a) For $j \in \{1, \ldots, N\}$, define the strategies $s_j^* = \big(s_j^{*[1]}, s_j^{*[2]}\big)$ by

$$s_j^{*[1]} \colon \mathfrak{X}_j \to \{0, 1\},$$
$$s_j^{*[1]}(\kappa; v_1, \ldots, v_N) = \begin{cases} 1 & \text{if } \#\{i \le j \mid v_i = 1\} < \max\big\{b, \lceil \kappa N \rceil\big\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$s_j^{*[2]} \colon \mathfrak{Y}_j \to \{\mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_2\},$$
$$s_j^{*[2]}(\kappa, \mathbf{v}, \mathbf{g}) = \begin{cases} \mathcal{P}_2 & \text{if } \mathcal{N}(\mathbf{v}) < a \text{ and } \mathcal{N}(\mathbf{g}) \ge \kappa N, \\ \mathcal{P}_1 & \text{otherwise.} \end{cases}$$

We show that the strategy profile $\mathbf{s}^* = (s_1^*, \ldots, s_N^*)$ constitutes a coalition-proof subgame-perfect equilibrium. From Proposition 10.1 we observe that the strategy profile implies rational behavior in proposal-making; hence Requirement (i) of Definition 10.1 is fulfilled, and it remains to show that for any state of the world $(\kappa, \mathbf{v}) \in \mathfrak{X}$, no coalition wants to deviate, which then implies Requirement (ii) of Definition 10.1. We note that the labeling of agents is used as a coordination device in the construction of the equilibrium.

We start with the case $\mathcal{N}(\mathbf{v}) < b$. In this case, the initiative group cannot be formed successfully; hence the project will not be realized, thus no coalition can gain anything by deviation.

Now, consider the case $\mathcal{N}(\mathbf{v}) \geq \max\{a, b\}$. If the strategy profile $\mathbf{s}^*$ is played,

$$\min\Big\{\mathcal{N}(\mathbf{v}),\ \max\big\{b, \lceil \kappa N \rceil\big\}\Big\}$$

project winners will join the initiative group. Since $\mathcal{N}(\mathbf{v}) \geq b$, the size of the initiative group will reach the required threshold. The initiative group will thus be successfully formed and proposal $\mathcal{P}_1$ will be made. Each project winner will get a utility of $1 - \kappa$. Now suppose a group of agents thinks about deviating. As, by Part (i) of Proposition 10.1, proposal $\mathcal{P}_1$ will be made whenever the initiative group has been formed successfully, a project winner who participates in a coalition of deviating agents will get the same utility of $1 - \kappa$ if the project is realized, he will get a utility of 0 if the project is not realized, but he will never get a utility strictly higher than $1 - \kappa$. Thus, no project winner has an incentive to participate in a coalition of deviating agents.

Next, consider the case $b \leq \kappa N \leq \mathcal{N}(\mathbf{v}) < a$. With the strategy profile $\mathbf{s}$ being played, exactly $\max\big\{b, \lceil \kappa N \rceil\big\}$ project winners will join the initiative group, which will hence reach the required threshold b. Proposal $\mathcal{P}_2$ will be made and be unanimously accepted. Hence, each project winner within the initiative group will get a utility of

$$1 - \frac{\kappa N}{\mathcal{N}(\mathbf{g})} \geq 1 - \frac{\kappa N}{\lceil \kappa N \rceil} \geq 0,$$

whereas each project winner outside the group will get a utility of 1. Consider again a coalition of agents who think about deviating. A project winner will not participate in a deviating coalition if deviation prevents project realization. In addition, by Part (ii) of Proposition 10.1, deviation will not lead to the implementation of $\mathcal{P}_1$. Thus, a coalition might only want to deviate if after deviation $\mathcal{P}_2$ is still implemented. Project winners outside the initiative group will not want to deviate by joining the group, as project winners outside the group are (at least weakly) better off than a member of an initiative group of any size. It remains to be shown that group members will not want to deviate either. Recall that the size of the initiative group is $\mathcal{N}(\mathbf{g}) = \max\big\{b, \lceil \kappa N \rceil\big\}$. Consequently, (a) $\mathcal{N}(\mathbf{g}) = b$ or (b) $b < \kappa N$ and $\mathcal{N}(\mathbf{g}) = \lceil \kappa N \rceil$. Consider first case (a). If group members deviate by leaving the group, the group size falls below b, which implies that the group is not successfully formed, the project is not realized, and the project winners' utility is zero. In case (b), if group members leave the group, the size of the initiative group falls below κN. By Part (iv) of Proposition 10.1, this means that the project will not be realized. Again, deviation is not strictly profitable.

Finally, we consider the case $b \leq \mathcal{N}(\mathbf{v}) < \kappa N$. With strategy profile $\mathbf{s}^*$ being played, all project winners will join the initiative group. Proposal $\mathcal{P}_1$ will be made, but will be rejected in the voting stage. The parameter a is irrelevant for this observation. Consider now any deviation of a coalition of agents. Since the number of project winners in the society is strictly lower than κN, the initiative group will always consist of less than κN members, so that by Part (iv) of Proposition 10.1, the deviation will not lead to project realization and hence not alter the utility of the members of the coalition.

Our arguments show that the strategy profile $\mathbf{s}^*$ indeed is a coalition-proof (in fact, even a strong) subgame-perfect equilibrium.

(b) Consider a subgame-perfect coalition-proof equilibrium and some state of the world $(\kappa, \mathbf{v})$. We first prove Part (i). Suppose Condition (10.6) is fulfilled. If $\mathcal{N}(\mathbf{v}) < b$, the size of the initiative group will miss the required threshold, since only project winners are allowed to join. The project will not be realized in that case. Part (iv) of Proposition 10.1 tells us that if $b \leq \mathcal{N}(\mathbf{v}) < \min\{\kappa N, a\}$, the project will not be realized, either. This shows Part (i).

It remains to demonstrate Part (ii). If Condition (10.7) is fulfilled, at least one of the Inequalities (10.8) or (10.9) holds. Assume that Inequality (10.8) is fulfilled. In this case, $\mathcal{P}_1$ will be accepted whenever it is made. Suppose proposal $\mathcal{P}_1$ is not made (either because too few agents join the initiative group or because another proposal is made). Then, the resulting utility for the project winners is strictly less than $1 - \kappa$, such that a coalition of all project winners can deviate profitably by following the strategies s_j^* in the state $(\kappa, \mathbf{v})$. As we have seen above, no sub-coalition has the incentive to deviate further. This contradicts the assumption of a subgame-perfect coalition-proof equilibrium, so $\mathcal{P}_1$ must be made.

Now assume that Inequality (10.9) is fulfilled. Again, proposal $\mathcal{P}_2$ is accepted whenever it is made. Proposal $\mathcal{P}_1$ will not be accepted; consequently, if $\mathcal{P}_2$ is not made, the project is not realized at all. Suppose now that $\mathcal{P}_2$ is not made. Then project winners can deviate profitably by following the strategies s_j^* in the state $(\kappa, \mathbf{v})$, and no sub-coalition has the incentive to deviate further. Thus, we again obtain a contradiction and $\mathcal{P}_2$ must be made when Inequality (10.9) is fulfilled. $\square$

Proof of Proposition 10.3

(i) Consider a constitution $\mathcal{C}^{IG}(a, b)$ with $a \geq 0, b > 0$ satisfying (10.10). By Part (a) of Proposition 10.2, a coalition-proof subgame-perfect equilibrium exists. It remains to be shown that *any* coalition-proof subgame-perfect equilibrium yields an efficient outcome.

Suppose $\mathcal{N}(\mathbf{v}) < \kappa N$. Then

$$\mathcal{N}(\mathbf{v}) \leq \lceil \kappa N \rceil - 1 < \min\{\kappa N, \max\{a, b\}\} \leq \max\{b, \min\{a, \kappa N\}\},$$

which is (10.6). By Part (b)(i) of Proposition 10.2, the project is not realized.

If $\mathcal{N}(\mathbf{v}) > \kappa N$, then by Condition (10.10), $\mathcal{N}(\mathbf{v}) \geq \lfloor \kappa N \rfloor + 1 \geq b$. Thus Condition (10.7) is fulfilled, and Part (b) of Proposition 10.2 tells us that the project is realized.

(ii) Suppose first $b > \lfloor \kappa N \rfloor + 1$. Then there is a positive probability that $\kappa N < \mathcal{N}(\mathbf{V}) < b$, in which case the initiative group cannot be formed, and the project is not realized, although $\mathcal{N}(\mathbf{v}) > \kappa N$.

Suppose $\max\{a, b\} \leq \lceil \kappa N \rceil - 1$. Then the condition

$$\max\{a, b\} \leq \mathcal{N}(\mathbf{V}) < \kappa N \tag{10.18}$$

holds with positive probability. If (10.18) holds, project winners can secure themselves a strictly positive utility by joining the initiative group and proposing $\mathcal{P}_1$, in which case the project is realized, although $\mathcal{N}(\mathbf{v}) < \kappa N$. □

Proof of Proposition 10.4

(i) Consider $\mathcal{C}^{\mathrm{MV}}\big(a\big)$ with a satisfying Condition (10.11). As we have seen in Sect. 10.3.4, an agent will vote in favor of the project if and only if he is a project winner, and thus the project is realized if and only if $\mathcal{N}(\mathbf{V}) \geq a$. If $\mathcal{N}(\mathbf{V}) > \kappa N$, then $\mathcal{N}(\mathbf{V}) \geq \lfloor \kappa N \rfloor + 1 \geq a$, since $\mathcal{N}(\mathbf{V})$ and a take only integer values; hence the project is realized. If $\mathcal{N}(\mathbf{V}) < \kappa N$, then $\mathcal{N}(\mathbf{V}) < a$, such that the project is not realized in this case.

(ii) Suppose $a > \kappa N + 1$. The project will not be realized if $\mathcal{N}(\mathbf{V}) = \lfloor \kappa N \rfloor + 1$, which implies $\mathcal{N}(\mathbf{V}) > \kappa N$ and happens with positive probability. Hence the constitution is not efficient. Now suppose $a < \kappa N$. The project will be realized if $\mathcal{N}(\mathbf{V}) = \lceil \kappa N \rceil - 1$, which implies $\mathcal{N}(\mathbf{V}) < \kappa N$; this happens with positive probability. Again, the constitution is not efficient. □

Proof of Proposition 10.5

(i) The proof is analogous to that of Part (i) of Proposition 10.3. Suppose Condition (10.12) is fulfilled. From Part (a) of Proposition 10.2 we know that a coalition-proof subgame-perfect equilibrium exists. Part (b)(i) of that proposition states that the project is not realized if

$$\mathcal{N}(\mathbf{V}) < \max\Big\{b, \min\big\{\lfloor KN \rfloor, a\big\}\Big\},$$

which, due to (10.12), is equivalent to $\mathcal{N}(\mathbf{V}) < KN$. Part (b)(ii) of Proposition 10.2 states that the project is realized if

$$\mathcal{N}(\mathbf{V}) \geq \max\Big\{b, \min\big\{\lfloor KN \rfloor + 1, a\big\}\Big\},$$

which, by similar arguments, is equivalent to $\mathcal{N}(\mathbf{V}) > KN$.

(ii) Suppose Condition (10.12) is not fulfilled. Then $b > 1$ or $\max\{a, b\} < N$. Since, by assumption, $P\big(0 < KN < 1\big) > 0$ and $P\big(N - 1 < KN < N\big) > 0$, there is a positive probability that

$$\lfloor KN \rfloor + 1 < b \quad \text{or} \quad \lceil KN \rceil - 1 \geq \max\{a, b\}.$$

Continuing as in the proof of Part (ii) of Proposition 10.3, we can conclude that the constitution $\mathcal{C}^{\mathrm{IG}}(a, b)$ is not efficient. To prove that $\mathcal{C}^{\mathrm{MV}}(a)$ is not efficient, one can argue as for Part (ii) of Proposition 10.4. □

Proof of Proposition 10.6

The proof extends the proof of Proposition 10.1. The arguments are similar, but we must take $\mathcal{P}_3$ into account now.

(i) Suppose $\mathcal{N}(\mathbf{v}) \geq a$. If $\mathcal{N}(\mathbf{g}) = N$, the proposal-maker has to choose between $\mathcal{P}_0$ and $\mathcal{P}_1$. He will strictly prefer $\mathcal{P}_1$. If $\mathcal{N}(\mathbf{g}) < N$, all proposals will be accepted in the voting stage. The proposal-maker will propose $\mathcal{P}_1$, as this proposal yields the strictly highest outcome for him.

(ii) As $\mathcal{N}(\mathbf{v}) < a$, proposal $\mathcal{P}_1$ will not be accepted in the voting stage. Hence if $\mathcal{P}_1$ is proposed, it will yield a utility of 0. As $\mathcal{N}_{\geq 0}(\mathbf{v}) \geq a$ and $\mathcal{N}(\mathbf{g}) > \kappa N$, both $\mathcal{P}_2$ and $\mathcal{P}_3$ will be accepted in the voting stage, with $\mathcal{P}_2$ giving the proposal-maker a strictly positive outcome, which is strictly higher than the outcome of $\mathcal{P}_3$, hence proposing $\mathcal{P}_2$ is a strictly dominant strategy. We note that P_3 will be accepted only if $\mathcal{N}(\mathbf{g}) > \kappa N + (N - \mathcal{N}(\mathbf{g}))w$. This fact does not change the conclusion of point (ii).

(iv) Under the given conditions, $\mathcal{P}_3$ will be accepted in the voting stage, but $\mathcal{P}_1$ and $\mathcal{P}_2$ will not. The condition $\mathcal{N}(\mathbf{g}) > (\kappa + w)/(1 + w) \cdot N$ is equivalent to $(\kappa + w)N/\mathcal{N}(\mathbf{g}) - w < 1$, which guarantees that $\mathcal{P}_3$ yields a strictly positive outcome for the proposal-maker. Hence, proposing $\mathcal{P}_3$ is a strictly dominant strategy.

Parts (iii) and (v) are clear.

(vi) If $\mathcal{N}(\mathbf{g}) < \kappa N$, then both $\mathcal{P}_2$ and $\mathcal{P}_3$ yield a strictly negative outcome for the proposal-maker if they pass the voting stage. If $\mathcal{N}(\mathbf{g}) < (\kappa + w)/(1 + w) \cdot N$, $\mathcal{P}_3$ will be accepted, and $\mathcal{P}_2$ will not pass the voting stage. The assertion follows. □

Proof of Proposition 10.7

Part (a): We define strategies $s_j^* = \left(s_j^{*[1]}, s_j^{*[2]}\right)$. For $(\kappa; \mathbf{v}) \in \mathfrak{X}_j$ with $\mathbf{v} = (v_1, \ldots, v_N)$, let

$$s_j^{*[1]} := 1$$

if

$$\mathcal{N}_{\geq 0}(\mathbf{v}) \geq a \quad \text{and} \quad \#\{i \leq j \mid v_i = 1\} < \max\left\{b, \kappa N\right\}$$

or if

$$\mathcal{N}_{\geq 0}(\mathbf{v}) < a \quad \text{and} \quad \#\{i \leq j \mid v_i = 1\} < \max\left\{b, (\kappa + w)/(1 + w) \cdot N\right\},$$

and let

$$s_j^{*[1]} := 0$$

otherwise. For $(\kappa, \mathbf{v}, \mathbf{g}) \in \mathfrak{Y}_j$ let

$$s_j^{*[2]}(\kappa, \mathbf{v}, \mathbf{g}) := \begin{cases} \mathcal{P}_3 & \text{if } \mathcal{N}_{\geq 0}(\mathbf{v}) < a \text{ and } \mathcal{N}(\mathbf{g}) \geq (\kappa + w)/(1 + w) \cdot N, \\ \mathcal{P}_2 & \text{if } \mathcal{N}(\mathbf{v}) < a \leq \mathcal{N}_{\geq 0}(\mathbf{v}) \text{ and } \mathcal{N}(\mathbf{g}) \geq \kappa N, \\ \mathcal{P}_1 & \text{otherwise.} \end{cases}$$

Then the strategy profile $\mathbf{s}^* = (s_1^*, \dots, s_N^*)$ is a coalition-proof subgame-perfect equilibrium. To verify this, we argue as in the proof of Proposition 10.2, skipping the details here. If $\mathcal{N}(\mathbf{v}) < b$, the initiative group cannot be formed, yielding a utility of 0 for every agent. In the following, we will assume that $\mathcal{N}(\mathbf{v}) \geq b$.

First, suppose that $\mathcal{N}_{\geq 0}(\mathbf{v}) \geq a$. Then, exactly

$$\min\Big\{\mathcal{N}(\mathbf{v}),\ \max\big\{b,\ \lceil \kappa N \rceil\big\}\Big\}$$

project winners join the initiative group. If $\mathcal{N}(\mathbf{v}) \geq a$, Proposal $\mathcal{P}_1$ will be made and accepted. If $b \leq \kappa N \leq \mathcal{N}(\mathbf{v}) < a$, the group will consist of exactly $\max\big\{b, \lceil \kappa N \rceil\big\}$ members, and Proposal $\mathcal{P}_2$ will be made and accepted. If $b \leq \mathcal{N}(\mathbf{v}) < \kappa N$, all project winners will join the initiative group. Proposal $\mathcal{P}_1$ will be made, but rejected in the voting state. One checks that in each of these cases, no coalition has an incentive to deviate.

Second, suppose $\mathcal{N}_{\geq 0}(\mathbf{v}) < a$. Then, exactly

$$\min\Big\{\mathcal{N}(\mathbf{v}),\ \max\big\{b,\ \lceil (\kappa + w)/(1 + w) \cdot N \rceil\big\}\Big\}$$

project winners join the group. If $b \leq (\kappa + w)/(1 + w) \cdot N \leq \mathcal{N}(\mathbf{v}) < a$, the group will consist of exactly $\max\big\{b, \lceil (\kappa + w)/(1 + w) \cdot N \rceil\big\}$ members, and Proposal $\mathcal{P}_3$ will be made and accepted. If $b \leq \mathcal{N}(\mathbf{v}) < (\kappa + w)/(1 + w) \cdot N$, all project winners will join the initiative group. Again, proposal $\mathcal{P}_1$ will be made, but rejected in the voting state. In each of these cases, no coalition has an incentive to deviate.

Part (b)(i): If $\mathcal{N}(\mathbf{v}) < b$, the initiative group cannot be formed successfully. By Part (vi) of Proposition 10.6, if one of the remaining two conditions is satisfied, any proposal involving project realization yields a strictly negative utility for the proposal-maker.

Part (b)(ii) is proved in a similar way as the corresponding part of Proposition 10.2.
□

Proof of Proposition 10.8

Part (i): Consider a constitution $\mathcal{C}^{\mathrm{IG}}(a, b)$ satisfying (10.13). By Part (a) of Proposition 10.7, a coalition-proof subgame-perfect equilibrium exists. It remains to be shown that every coalition-proof subgame-perfect equilibrium yields an efficient outcome. If the project is disadvantageous, i.e. if $\mathcal{N}(\mathbf{V}) < \kappa N + wL$, then $\mathcal{N}(\mathbf{V}) < b$, so the initiative group cannot be successfully formed, and the project is not realized. If the project is socially desirable, i.e. if $\mathcal{N}(\mathbf{V}) > \kappa N + wL$, then $\mathcal{N}(\mathbf{V}) \geq b$. In addition, $\mathcal{N}_{\geq 0}(\mathbf{V}) = a$. Hence, by Part (b)(ii) of Proposition 10.7, the project is realized.

Part (ii): Consider a constitution $\mathcal{C}^{\mathrm{IG}}(a, b)$. Suppose, first, $a > N - L$. Then $\mathcal{N}_{\geq 0}(\mathbf{V}) < a$. Assumption (10.14) yields the inequality

$$\kappa N + wL + 1 + w\big(\kappa N + wL + 1\big) < \kappa N + wN,$$

from which it follows that

$$\lfloor \kappa N + wL \rfloor + 1 \leq \kappa N + wL + 1 < \frac{\kappa + w}{1 + w} \cdot N.$$

Hence, by Part (b)(i) of Proposition 10.7, the project is not realized if $\mathcal{N}(\mathbf{V}) = \lfloor \kappa N + wL \rfloor + 1$. This however, happens with positive probability.

Now suppose $b > \lfloor \kappa N + wL \rfloor + 1$. Consider the event $\mathcal{N}(\mathbf{V}) = \lfloor \kappa N + wL \rfloor + 1$, which happens with positive probability. In this case, the initiative group cannot be formed, which implies that the project is not realized, although it is socially desirable.

Finally, suppose $b < \kappa N + wL$. This implies $\kappa N + wL > 1$. The event $\mathcal{N}(\mathbf{V}) = \lceil \kappa N + wL \rceil - 1$ occurs with positive probability. Since, then, $\mathcal{N}(\mathbf{V}) < \kappa N + wL$, the project is socially disadvantageous. Assumption (10.15) yields the inequality

$$\kappa N + wL > \kappa N + 1,$$

which, since $\mathcal{N}(\mathbf{V}) = \lceil \kappa N + wL \rceil - 1$, implies

$$\mathcal{N}(\mathbf{V}) \geq \lfloor \kappa N \rfloor + 1.$$

Hence, by Part (b)(ii) of Proposition 10.7, the project is realized, although it is socially disadvantageous.

Part (iii): The proof of this statement is analogous to the proof of Proposition 10.4. □

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Chapter 11
Perspectives

We are always working on new ideas. Generally, we start by describing them in a short paper to assess their potential drawbacks. If possible, such texts are first published as a newspaper article or as a column. In a second step, an in-depth analysis and the development of the model follow. At the moment, we are addressing three issues: (i) History-bound Reelections, (ii) Assessment Voting, and (iii) Co-voting Democracy.

(i) History-bound Reelections. We have been working on procedures that allow to mitigate the so-called "incumbency advantage" for a long time. The issue is important, as incumbents are often reelected (too) easily, so that they might lack incentives to give the best-possible performance after elections or may indulge too strongly in their own preferences. We now suggest to introduce a new reelection rule requiring incumbents to achieve at least the vote-share of their best past electoral performance for the same office. Such a procedure would encourage the office-holders to invest more effort when in office in order to earn the necessary votes and would better correspond to the median voter's preferences. As a variant, we also suggest to take the best previous vote-share as a basis for the vote-share needed for reelection, but to reduce it by some margin.[1]

(ii) Assessment Voting. We suggest a new voting procedure consisting of two voting rounds. In the first round, a randomly-selected group of citizens votes on a given issue, and the results of this voting round are published. This allows the group or person who made an original proposal through an initiative to withdraw it or amend it. If the proposal is not taken back, all citizens who have not voted yet cast their vote in a second voting round, the results of both rounds are added, and the so-taken decision is implemented.

Such a voting procedure has several advantages: As it comprises only a subset of the citizens, the first round costs much less than a vote by all citizens. This first round

[1] A first assessment on this problem was published in 2016 as CEPR Discussion Paper No. DP11103.

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serves as an assessment of the citizens' wishes and allows those who have the right to put the issue to a vote to stop the procedure if they see that their proposal might not be accepted. They might also be given an opportunity to adapt the proposal after the first round, which restarts the two-round voting procedure. As a complementary measure, the parliament could make a suitable counterproposal, again restarting the elective procedure.[2]

(iii) Co-voting Democracy. A relatively new area of research on our workbench is the issue how to generate the best-possible support for the governments of representative democracies as soon as an important decision is at hand. That such support can be crucial was demonstrated impressively by the recent Brexit referendum. We are exploring the potential of a special voting process for such decisions, which consists in handing back part of governmental decision power to the voters. Such a procedure would foster the voters' support for the decision, and help them accept its (possibly) negative consequences. To include the voters in an important decision without reverting to direct democracy, one could imagine a randomly-chosen representative subset of all voters, so-called "Vote-holders", who are given a one-time voting right on the decision at hand. Although the majority of the voters could not vote, this majority would still feel it has a say in the decision – through the Vote-holders. The Parliament would vote on the same issue, and the two decisions would be weighted according to a pre-defined key. The outcome of the weighting would yield the final decision.

Based on this outline, we are currently exploring procedural issues such as sequential voting rounds versus parallel voting rounds, anonymity for the Vote-holders and secure electronic voting supported by encryption technologies, together with ETH-researchers who are specialized in information security.[3]

As you can see, there is still a lot to do—and we are still at work!

[2]We published a first description of this idea in the *Neue Zürcher Zeitung*. It also appeared in 2015 as CER-ETH Working Paper 15/214. We are currently developing the corresponding model.

[3]A first description and assessment of Co-voting appeared in 2016 as CER-ETH Working Paper 16/256.