Chapter 5
The Presentation of Numerical Results

We will now discuss the way in which numerical results should be presented. First, however, we will say a few things about significant figures and the rounding of numbers. In this chapter, the system of units known as S.I. is also presented, as well as the basic rules that must be followed in its use for the presentation of experimental results.

5.1 Significant Figures and Rounding of Numbers

Significant figures of a number are all its digits, except any consecutive leading zeros if the number is written in decimal form.

The position of the decimal point or a multiplying factor of a power of ten do not affect the number of significant figures. Some examples are given in Table 5.1. It should be noted that a zero at the end of the number counts as a significant figure. So, for example, the number 2.70 has three significant figures, while 2.7 has only two. The number of significant figures of integers may be designated if they are written in the form: (decimal) $\times$ (power of 10). For example, the number 12 300, which has 5 significant figures (s.f.), may be written with 4 s.f. as 1.230×10^4, with 3 s.f. as 1.23×10^4, with 2 s.f. as 1.2×10^4 and with 1 s.f. as 1×10^4. A slight problem arises with two-digit integers ending in 0. For example, it does not look natural for 40 to be written as 4×10. In these cases, the number may be written with two digits (40) and be stated that only one figure is significant, in the sense that the zero may not necessarily be zero but it could be 1, 2, 3 or 4.

In order to decrease the number of significant figures of a decimal number we use *rounding*. Rounding is performed as follows:

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Table 5.1 Examples of numbers with various numbers of significant figures

1 significant figure	2 significant figures	3 significant figures	4 significant figures
3	31	305	3050
0.3×10^2	0.31×10^2	3.05×10^2	3.050×10^3
0.2	0.23	0.234	0.2336
0.06	0.058	0.0582	0.05815
0.002	0.0016	0.00156	0.001558
7×10^{-4}	7.3×10^{-4}	7.30×10^{-4}	7.300×10^{-4}
6×10^6	6.1×10^6	6.06×10^6	6.063×10^6

If the least significant figure (the one on the right) is 0, 1, 2, 3 or 4, it is simply omitted. If it is 5, 6, 7, 8 or 9, the digit is omitted and the next digit on the left is increased by unity.

Examples: $1.42 \approx 1.4 \approx 1$, $3.6285 \approx 3.629 \approx 3.63 \approx 3.6 \approx 4$.

To round an integer, we first convert it into the form (decimal) × (power of 10) and then we round the decimal.

Example: $3506 = 3.506 \times 10^3 \approx 3.51 \times 10^3 \approx 3.5 \times 10^3 \approx 4 \times 10^3$.

If we round but keep the zeros, e.g. $3506 \approx 3510 \approx 3500 \approx 4000$, we give the wrong impression that the number of significant figures remains equal to 4.

5.2 The Presentation of a Numerical Result of a Series of Measurements

If we have performed N measurements x_i $(i = 1, 2, \ldots, N)$ of the magnitude x, the results of which have a mean $\bar{x}$ and a standard deviation of the mean $\sigma_{\bar{x}}$, the result of the measurements is usually presented in the form:

$$x = \bar{x} \pm \sigma_{\bar{x}} \text{ units.} \tag{5.1}$$

Examples:
The length of the rod was measured to be equal to $l = 18.25 \pm 0.13$ cm.
The electrical resistance is equal to $R = 103.6 \pm 1.2\ \Omega$.
During the measurement, $\Delta N = 169 \pm 13$ *fissions of nuclei were counted.*

It should be noted that the units given refer to both the mean value and its standard deviation. Parentheses which would stress this fact are not necessary.

By this we mean that the result of the first example should not necessarily be written as $l = (\ 18.25 \pm 0.13)$ cm, without, of course, this being forbidden.[1]

When a multiplying factor of a power of ten is present in the numerical result, this power must be the same for both $\bar{x}$ and $\sigma_{\bar{x}}$. The way of presenting the result in this case is:

$$D = (1.49 \pm 0.15) \times 10^6\,\text{km}$$

The parentheses state that the multiplying factor applies to both quantities.

An alternative way of presenting the error in the mean, especially if a large number of significant figures is used, is

$$c = 2.997\ 924\ 59(12) \times 10^8\ \text{m/s}.$$

By this notation, it is meant that the two digits in parentheses state the standard deviation of the mean and appear at the same position as the last two digits of the mean value, i.e. it is:

$$c = 2.997\ 924\ 59(12) \times 10^8\ \text{m/s} \equiv (2.997\ 924\ 59 \pm 0.000\ 000\ 12) \times 10^8\ \text{m/s}$$

In the examples given, we applied the basic rule:

The numerical values of the mean and of the standard deviation of the mean are given with the same number of decimal digits (or, with the same accuracy).

Therefore, it would be wrong to write

$$32.263 \pm 0.14 \quad \text{instead of the correct} \quad 32.26 \pm 0.14,$$

$$\text{or} \quad 4.63 \pm 0.1348 \quad \text{instead of the correct} \quad 4.63 \pm 0.13.$$

It is of course understood that value and error are given in the same units.

It is recommended that the use of commas as delimiters which separate the digits in triads in multi-digit numbers be avoided. Instead, a small empty space should be used. For example, instead of $c = (2.997{,}924{,}59 \pm 0.000{,}000{,}12) \times 10^8\text{m/s}$ we write $c = (2.997\,924\,59 \pm 0.000\,000\,12) \times 10^8\text{m/s}$ and instead of 32,467.63 we write 32 467.63.

This is done in order to avoid the confusion between dots and the commas used as decimal points in non-English speaking countries. The space is not necessary if the integral part of the number consists of four digits. For example, we write 4632, without a space.

[1]In fact, according to the suggestions of ISO 31-0: 1992 (E), the form $l = (\ 18.25 \pm 0.13)$ cm is the correct one. This, however, has not known any significant acceptance in scientific work, possibly because the proliferation of parentheses is judged to be unnecessary.

So far we have said nothing about the accuracy with which $\bar{x}$ and $\sigma_{\bar{x}}$ should be presented. We will examine this question in the next section. For the moment we state that:

For the presentation of numerical results, we first decide on the accuracy with which the standard deviation of the mean will be given (i.e. the error in the mean), $\sigma_{\bar{x}}$*, and then give the mean* $\bar{x}$ *with the same accuracy (i.e. with the same number of decimal figures). Any rounding of the numbers which is necessary must be taken into account.*

5.3 The Number of Significant Figures Used in the Presentation of Numerical Results

The question of the number of significant figures with which numerical results should be presented is not as simple as it appears. Initially, we present the problem with a numerical example:

If our measurements gave the numerical results

$$\bar{x} = 876.12345, \quad \sigma_{\bar{x}} = 1.2345 \tag{5.2}$$

in suitable units, with how many decimal figures should each one of the results be finally stated?

Given that in the example the standard deviation of the mean is approximately equal to unity, the last integral digit as well as the decimal part of the mean are uncertain. Obviously, there is no sense in giving the mean with so many decimals. The same is true for the standard deviation. We remind that, according to what was said in Sect. 4.3, the standard deviation of the standard deviation is equal to

$$\sigma(\sigma_{\bar{x}}) = \frac{\sigma_{\bar{x}}}{\sqrt{2(N-1)}}, \tag{5.3}$$

where N is the number of the measurements used in the evaluation of $\bar{x}$ and $\sigma_{\bar{x}}$. If, for example, for the results (5.2) it is $N = 10$, the standard deviation of the standard deviation is

$$\sigma(\sigma_{\bar{x}}) = \frac{\sigma_{\bar{x}}}{\sqrt{2(N-1)}} = 0.236 \times \sigma_{\bar{x}} = 0.236 \times 1.2345 = 0.291 \tag{5.4}$$

Thus, the uncertainty in the value of $\sigma_{\bar{x}}$ is approximately equal to 24%, and the decimal digits are uncertain.

We will present and discuss the various views on the subject before we decide which one we adopt in this book:

(A) *The standard deviation of the mean is always given with one significant figure and the mean with the same accuracy.*

Thus, the numerical results (5.2) are presented as $x = 876 \pm 1$. Other examples are:

$$56 \pm 2 \quad 0.73 \pm 0.05 \quad 0.0069 \pm 0.0007 \quad (4.2 \pm 0.3) \times 10^{-5} \quad (8.0 \pm 0.1) \times 10^{6}.$$

The justification is the following: Given that, especially in an experiment in an educational laboratory, the number of measurements is rarely larger than 10 and often is about 5, the error in the standard deviation is equal to about 25% or even larger. The digits beyond the first are uncertain and are omitted (in our numerical example, the digits 0.2345 represent about 20% of $\sigma_{\bar{x}}$).

(B) *The standard deviation of the mean is given with one significant figure, unless its first two digits lie between 10 and 25, in which case it is given with two significant figures. The mean is given with the same accuracy.*

Thus, the numerical results (5.2) are presented as: $x = 876.1 \pm 1.2$.

Other examples are:

$$(4.2 \pm 0.3) \times 10^{-5} \quad (8.1 \pm 0.5) \times 10^{6} \quad 56 \pm 3 \quad 0.006\,97 \pm 0.000\,07,$$

$$(4.20 \pm 0.15) \times 10^{-5} \quad (8.05 \pm 0.23) \times 10^{6} \quad 56.4 \pm 1.2 \quad 0.006\,97 \pm 0.000\,17.$$

The justification is the following: If, for example, the standard deviation of the mean has 1 as its initial digit, a decimal part equal to 0.4 represents about 30% of the whole value and is larger than 25%, which is the percent error in the standard deviation for N between 5 and 10. For this reason, the second significant figure is not omitted. A similar argument holds for all cases where the first two digits lie between 10 and 25.

The general criterion to be used is whether, given the number of measurements used in extracting the numerical results, the standard deviation of the standard deviation of the mean justifies the presentation of the standard deviation with two significant figures or not.

(C) *The standard deviation of the mean is always given with two significant figures and the mean with the same accuracy.*

In an educational laboratory, the number of measurements used in the evaluation of the mean may not be large enough to justify the presentation of the standard deviation with two significant figures. In scientific research, however, things are different for two main reasons:

1. The number of measurements may be large enough
2. The result may be final for the researcher who performed the measurements but it must be considered as initial or intermediate result if it is to be used in calculations by other researchers.

With this last possibility in mind, the result must be given with enough accuracy so that no errors are introduced in the calculations of other magnitudes based on these results due to its rounding that has been performed. This practice of retaining one more significant figure than justified in intermediate results in order to evaluate the final result, should always be followed.

When, therefore, it is anticipated that the result of an experiment may be used for further calculations, the standard deviation of the mean should be given with two significant figures and the mean should be given with the same accuracy. Based on these facts, the well known mathematician Harold Jeffreys has commented: '*If one wants to render a series of good measurements useless, the best way in which one may achieve this is to give the value of the error with one significant figure and not mention the number of measurements*'.

We accept, therefore, that, in results of serious scientific research,

1. The standard deviation of the mean should be given with two significant figures,
2. The mean should be given with the same accuracy and
3. The number of measurements should be given.

Under these conditions, a numerical result must be presented in the form

$$x = \bar{x} \pm \sigma_{\bar{x}} \text{ units } (N \text{ measurements}) \tag{5.5}$$

as, for example, $x = 6.327 \pm 0.017\ \mu\text{m}$ (12 measurements).

Alternatively, instead of the number of measurements, the fractional standard deviation of the standard deviation $\sigma(\sigma_{\bar{x}})/\sigma_{\bar{x}}$ [see Eq. (4.43)] should be stated, based on the number of measurements performed. The numerical result is then presented in the form

$$x = \bar{x} \pm \sigma_{\bar{x}}(1 \pm \sigma(\sigma_{\bar{x}})/\sigma_{\bar{x}}) \text{ units} \tag{5.6}$$

as, for example, $x = 6.327 \pm 0.017(1 \pm 0.2)\ \mu\text{m}$.

Despite all this, the number of measurements is rarely given in scientific works, due mainly to the fact that, usually, the results are not obtained by repeating the same measurement many times under the same conditions but in other ways. Nevertheless, the standard deviation of the mean is given with two significant figures. Thus, one sees in tables the values of the fundamental physical constants to be given in the concise form [1]:

Newtonian constant of gravitation:	$G = 6.673\,84(80) \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$
Planck constant:	$h = 6.626\,069\,59(27) \times 10^{-34}\,\mathrm{J\,s}$
Electron charge:	$e = -1.602\,176\,565(35) \times 10^{-19}\,\mathrm{C}$
Rydberg constant:	$R_\infty = 10\,973\,731.568\,539(55)\,\mathrm{m^{-1}}$

The values given here are those accepted since 2010. As new results for the constants become available, the values are modified to take into account both the old and the new values of the constants. For this purpose, it is necessary to have the standard deviations of the two values with adequate accuracy. Hence the justification for the two significant figures. By the way, included in the values given above are, on one hand, the constant known with the largest uncertainty, G, for which it is $\delta G/G = 1.2 \times 10^{-4}$ (δG is the error or the standard deviation of G) and, on the other hand, Rydberg's constant, R_∞, one of the constants known with the highest accuracy, $\delta R_\infty/R_\infty = 5.0 \times 10^{-12}$.

Conclusion

Having discussed the three possibilities for the number of significant figures in the presentation of $\sigma_{\bar{x}}$ and $\bar{x}$, in this book we will adopt the following:

1. The standard deviation of the mean, $\sigma_{\bar{x}}$, will be given with one significant figure, unless its two first digits lie between 10 and 25, included, in which cases it will be given with two significant figures.
2. The mean, $\bar{x}$, will be given with the same accuracy as $\sigma_{\bar{x}}$.
3. In cases when it is known that the number of measurements is large enough and in cases of reliable values of physical constants, we will give $\sigma_{\bar{x}}$ with two significant figures and the mean, $\bar{x}$, with the same accuracy.
4. In all cases, the intermediate results will be evaluated and presented with one more significant figure than justified in the final presentation of $\sigma_{\bar{x}}$.

5.4 The International System of Units (S.I.) and the Rules of Its Use

The International System of Units (S.I., *Système International d' Unités*) was adopted in 1960 by the 11th General Conference on Weights and Measures (CGPM, *Conférence Générale des Poids et Mesures*), under the auspices of The International Bureau of Weights and Measures (BIPM, *Bureau International des Poids et Mesures*). See http://physics.nist.gov/cuu/Units/index.html

It is based on seven *base units,* which are considered to be independent of each other as regards their dimensions. All the other units used are *derived units*, which

Table 5.2 The base units of S.I.

Quantity	Unit	
	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

In the use of the unit mol, the kind of the entities being measured must be mentioned (atoms, molecules, ions, electrons, other particles or groups of such particles)

are formed by combinations of multiplications and divisions of powers of the base units, as these are dictated by the physical laws connecting the relative physical magnitudes. The base units are given in Table 5.2 while some derived units are given in Table 5.3. The names of the units are given in the tables. There are differences in the spelling of the names of the units in different languages (e.g. kilogramme and ampère in French, instead of the English kilogram and ampere), but the symbols of the units are the same in all languages.

Multiples and sub-multiples of the S.I. units are expressed with the use of prefixes. These represent powers of 10 in steps of the factor 10^3, with some smaller steps (of the factor) used near unity. These prefixes are given in Table 5.4. The prefixes *deci*, *deca* and *hecto* are not widely used.

We mention below the main rules that must be obeyed in the use of S.I. units.

1. The symbols for physical quantities are written in italics, while those of units in upright characters (e.g. $m = 1.3$ kg, $f = 1.034$ MHz, $V = 1.2$ V). An empty space is always left between the numerical value and the symbols of the units.
2. The symbols of units named after scientists, have their first letter in upper case (e.g. Pa, Bq, W, A, Hz, Wb, F, H, S). The names of the units, however, are written in lower case characters (e.g. joule, volt, siemens). The spelling and syntax of the names of units may vary from language to language. Only the use of the symbols is mandatory. For example, although the use of the symbol V is mandatory for the unit of volt, the use of the name *volt* is not.
3. The symbols of the units do not change in the plural. For example, we write 3 kg and not 3 kgs. However, the plurals of the names of the units are formed freely in each language, according to its rules. For example, in English we may write 5 kilograms or 3 volts. It should be noted that the word siemens is in the singular, despite the s at the end.

Table 5.3 The main derived units of S.I.

Quantity	Unit			
	Name	Symbol	Equivalent	
			In terms of other S.I. units	In terms of base S.I. units
Angle	radian	rad	m/m = 1	m/m = 1
Solid angle	steradian	sr	$m^2/m^2 = 1$	$m^2/m^2 = 1$
Velocity			m/s	m/s
Acceleration			m/s^2	m/s^2
Angular velocity			rad/s	rad/s
Angular acceleration			rad/s^2	rad/s^2
Frequency	hertz	Hz	s^{-1}	s^{-1}
Force, weight	newton	N	$kg \cdot m/s^2$	$kg \cdot m/s^2$
Pressure, stress	pascal	Pa	N/m^2	N/m^2
Energy, work, heat	joule	J	$N \cdot m$	$kg \cdot m^2/s^2$
Momentum, impulse			$N \cdot s$	$kg \cdot m/s$
Power, radiant flux	watt	W	J/s	$kg \cdot m^2/s^3$
Electric charge	coulomb	C	$A \cdot s$	$A \cdot s$
Electric potential, emf	volt	V	J/C, W/A	$kg \cdot m^2/(s^3 \cdot A)$
Electrical resistance	ohm	Ω	V/A	$kg \cdot m^2/(s^3 \cdot A^2)$
Electrical conductance	siemens	S	A/V, Ω^{-1}	$s^3 \cdot A^2/(kg \cdot m^2)$
Magnetic flux	weber	Wb	$V \cdot s$	$kg \cdot m^2/(s^2 \cdot A)$
Induction	henry	H	Wb/A	$kg \cdot m^2/(s^2 \cdot A^2)$
Capacity	farad	F	C/V	$s^4 \cdot A^2/(kg \cdot m^2)$
Strength of electric field			V/m, N/C	$kg \cdot m/(s^3 \cdot A)$
Magnetic flux density	tesla	T	Wb/m^2, $N/(A \cdot m)$	$kg/(s^2 \cdot A)$
Electric displacement			C/m^2	$A \cdot s/m^2$
Degree Celsius	degree Celsius	°C	K	K
Luminous flux	lumen	lm	$cd \cdot sr$	
Illuminance	lux	lx	lm/m^2	
Radioactivity (activity)	becquerel	Bq	s^{-1}	s^{-1}
Absorbed dose	gray	Gy	J/kg	m^2/s^2
Biologically equivalent dose	sievert	Sv	J/kg	m^2/s^2
Catalytic activity	katal	kat	mol/s	mol/s

4. The word 'degree' and the symbol ° are used for the Celsius scale but not for temperatures on the thermodynamic scale (Kelvin). Thus, we write 36 °C or 36 degrees Celsius, but 287 K (or, less often, 287 kelvin). For a temperature interval it is 1 °C = 1 K. The two scales are related through the exact equation $T(\mathrm{K}) \equiv t(^\circ\mathrm{C}) + 273.15$.

Table 5.4 Prefixes used with S.I. units

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{-1}	deci	d	10^{1}	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-24}	yocto	y	10^{24}	yotta	Y

5. The prefixes for factors equal to or greater than 10^6 are written in capitals.[2] The rest are written in lower case letters. There is no empty space between the prefix and the symbol for the unit. Compound prefixes, such as mμ, μμ etc, should be avoided. An exponent on the symbol of the unit applies for the prefix as well: For example, $\text{cm}^3 = (\text{cm})^3 = (10^{-2} \times \text{m})^3 = 10^{-6}\ \text{m}^3$. When a multiple or sub-multiple of a unit is written in full, so should the prefix, in lower case letters: For example, megahertz, but not Mhertz or Megahertz.
6. The kilogram is, for historical reasons, the only base unit of the S.I. which is written with a prefix. Multiples and sub-multiples of kg are formed with the prefix to the symbol applied to g and not to kg, as, for example, μg, mg etc.
7. In multiplying units, a raised multiplication dot is placed between the two symbols. Alternatively, a small empty space is left between the symbols, as in N · m, N · s, A · s and cd · sr or N m, N s, A s and cd sr.
8. The division of units is denoted by forward slashes (e.g. m/s) or using negative exponents (e.g. $\text{m}\,\text{s}^{-1}$). The multiple uses of the forward slash (m/s/s) is not allowed. When more than one units appear in the denominator, they should be enclosed in parentheses, as, for example, N/(A · m). In these cases, it is recommended that negative exponents are used, $\text{NA}^{-1}\text{m}^{-1}$. Units with prefixes can appear both in the numerator and the denominator as, for example, $\text{m}\Omega/\text{m}$, K/ms. Special care is needed to distinguish between the use of m as a symbol for the *meter* and its use as the prefix 10^{-3}. There is a difference between m N and mN. In cases like this the use of a multiplication dot helps in avoiding confusion (m · N).

[2]It should be stressed that the prefix kilo- must be written as a lower case k. Symbols such as Kg and Km are wrong (they might be misunderstood to mean kelvin gram or kelvin meter, respectively).

Table 5.5 Units which do not belong to the S.I. but are used in practice, mainly for historical reasons

Quantity	Unit		
	Name	Symbol	Definition
Time	minute	min	1 min = 60 s
	hour	h	1 h = 60 min = 3600 s
	day	d	1 d = 24 h = 86 400 s
	year (Julian)	a (also y or yr)	365.25 d
Angle	degree	°	$1° = (\pi/180)$ rad
	minute	′	$1' = (1/60)° = (\pi/10\,800)$ rad
	second	″	$1'' = (1/60)' = (\pi/648\,000)$ rad
Volume	liter	L	$1\ \text{L} = 1\ \text{dm}^3 = 10^{-3}\ \text{m}^3$
Mass	tonne, metric ton	t	1 t = 1000 kg
Ratio	neper	Np	(*)
	bel	B	(**)

(*) Two signals differ by N neper (Np), if it is $N = \ln\left|\frac{I}{I_0}\right|$ or $I = I_0\,e^N$

(**) It is 1 B = 10 dB (decibel). When referring to power, P, two signals differ by N db if it is $N = 10\ log\left|\frac{P}{P_0}\right|$ or $P = P_0 10^{N/10}$. When referring to field quantities, V, two signals differ by N db if it is $N = 20\ log\left|\frac{V}{V_0}\right|$ or $V = V_0 10^{N/20}$.

Table 5.6 Units which are acceptable for use in S.I and whose values are determined experimentally

Quantity	Unit		
	Name	Symbol	Value
Energy	electron volt	eV	$1.602\ 176\ 565(35) \times 10^{-19}$ J
Mass	unified atomic mass unit	u, Da*	$1.660\ 538\ 921(73) \times 10^{-27}$ kg

*Da = dalton

Also used, mainly for historical reasons, are a group of units which, strictly speaking, do not belong to the S.I. These units are shown in Table 5.5.

Given in Table 5.6 are some units which are accepted for use in the S.I. and which are determined experimentally.

Other units used in parallel to the S.I. units are:

astronomical unit	1 ua (also au or AU) = originally defined as the length of the semi-major axis of the Earth's orbit, is now defined as 149 597 870 700 m (exactly)
light year	1 ly = distance travelled by light in one Julian year (365.25 d) $= 9.460\,730\,472\,580\,800 \times 10^{15}$ m (exactly)

parsec	1 pc = distance at which one astronomical unit subtends an angle of one arc second = $3.085\,677\,6 \times 10^{16}$ m = 3.2616 ly = 206 264.81 ua. (pc = **par**allax **sec**ond. A star having a parallax of 1″ is at a distance of 1 pc. For a parallax of 0.5″ the distance is 2 pc etc.)
nautical mile	=1852 m
knot	=1 nautical mile per hour
hectar	$1\ \text{ha} = 100\ \text{ar} = 10^4\ \text{m}^2$
bar	$1\ \text{bar} = 1000\ \text{mbar} = 10^5\ \text{N} / \text{m}^2 = 10^5\text{Pa}$
angstrom	$1\text{Å} \equiv 10^{-8}\ \text{cm} = 10^{-10}\ \text{m} = 0.1\ \text{nm}$
barn[3]:	$1\ \text{barn} = 1\ \text{b} \equiv 10^{-24}\ \text{cm}^2 = 10^{-28}\text{m}^{z}$

5.5 Recommendations on the Notation Used for Mathematical Constants, Algebraic Parameters, Variables, Indices, Mathematical Functions, Operators, Physical Units, Elementary Particles and Isotopes

The ISO recommendation for the way of writing constants, variables, operators, units, elementary particles and isotopes are as follows:

Magnitude	Examples
Mathematical constants	
should be written in upright letters	$\mathrm{\pi}$, e, i, $\mathrm{\gamma}$, $\mathrm{\phi}$
Algebraic parameters, variables	
should be written in italics	a, b, x, y
Indices	
numerical indices should be written in upright letters	a_0, x_3
indices taking numerical values should be written in italics	a_i, x_l
indices which are variables or functions should be written in italics	a_z, $y_{x=\bar{x}}$
non-numerical indices should be written in upright letters	f_{UV}, $x_{\mathrm{exper.}}$, $\iint_{\mathrm{ellipse}}$

(continued)

[3]Used as a unit of cross-section (area). Legend has it that in the early days of measuring the cross-sections of reactions of neutrons with nuclei, a value so large was once measured that somebody remarked 'this is as big as a barn!'. The unit for cross-section was thus christened. We take this opportunity to suggest that the barn be renamed the Rutherford (R or rd). The unit of radioactivity named the Rutherford (one million decays per second) is obsolete, as it has been replaced by the megabecquerel.

(continued)

Magnitude	Examples
Mathematical functions	
should be written in upright letters	$\sin x$, $\cos x$, e^x, $\exp(x)$, $\mathrm{erf}(x)$
special functions	$\Gamma(x)$, $\mathrm{J}_l(x)$, $\mathrm{P}_l(x)$
functions in the general sense should be written in italics	$f(x)$, $g(x)$, $V(t)$
Operators	
should be written in upright letters	δ, $\frac{\mathrm{d}}{\mathrm{d}}$, $\frac{\partial}{\partial}$, Δ, ∇, $\sum$ $\left(\text{frequent exceptions: } \delta \text{ and } \frac{d}{d}\right)$
Physical units and their prefixes	
should be written in upright letters	s, μs, m, km, V, eV, TeV, Ω
Elementary particles	
should be denoted by symbols in upright letters	α, β, γ, p, n, ν_e, π^0, K, τ
Isotopes	
should be denoted by upright letters	H, Au, ${}^{15}_{8}\mathrm{O}$, ${}^{12}_{6}\mathrm{C}^{2+}$, $\mathrm{H_2O}$

Problems

5.1 The following numbers are given:

$$0.0,017,624 \quad 8.14369 \quad 267980 \quad 1.27386 \times 10^5 \quad 8.13227 \times 10^{-8}.$$

Enter, in a table, all the numbers with 1, 2, 3 and 4 significant figures.

5.2 The following numbers are given:

$$4.7386 \times 10^4 \quad 5.13227 \times 10^{-8} \quad 0.00029764 \quad 3.14159 \quad 937980.$$

Enter, in a table, all the numbers with 1, 2, 3 and 4 significant figures.

5.3 Write the following results giving the standard deviation of the mean with 2 significant figures and the mean with the same accuracy:

$$523.5782 \pm 5.367 \quad 0.0078321 \pm 0.0000632 \quad 4.7301 \times 10^6 \pm 942105.$$

5.4 Write the following results correctly:

$$263.582 \pm 0.2467 \quad 0.003321 \pm 0.002572 \quad 4.6308 \times 10^3 \pm 1210.$$

5.5 Given that the results

$$823.6 \pm 5.4 \quad 0.007832 \pm 0.000043 \quad (4.73 \pm 0.44) \times 10^6$$

were obtained by performing 5, 10 and 25 measurements, respectively, express each one of them in the form $x = \overline{x} \pm \sigma_{\overline{x}}(1 \pm \sigma(\sigma_{\overline{x}})/\sigma_{\overline{x}})$.

5.6 Given the results

$$23.62 \pm 0.24 \times (1 \pm 0.35) \qquad 0.003321 \pm 0.0015 \times (1 \pm 0.24)$$
$$(4.6 \pm 1.2 \times (1 \pm 0.16)) \times 10^3,$$

find in each case the approximate number of measurements, N, used in the determination of the result.

Reference

1. *The NIST Reference on Constants, Units and Uncertainty* at http://physics.nist.gov/cuu/Constants/. For a comprehensive list of values, see Appendix 4