Chapter 13
The Written Report of the Results of an Experiment

13.1 Introduction

The results of an experiment may be rendered useless if they are not presented in written form, so that they may be used by other researchers or the person who performed the measurements at a later time, if needed. Regarding the way in which an experimental procedure and its results may be presented, there is no unique recipe which could be followed at all times. The form of the report depends on the purpose for which it is written. In order of increasing 'importance', the following forms may be mentioned:

1. The presentation of an experiment performed in an educational laboratory. The aim here is usually for the student to give the instructor the opportunity to appreciate the experimental work performed and obtain the highest possible grade.
2. The presentation of the results of an experiment to a small circle of people, such as, for example, the members of the group to which a researcher belongs. Based on the report, the research group may possibly make important decisions regarding the future of their work.
3. The writing of a scientific article, either for presentation at a conference or for publication in a scientific journal.
4. The presentation of experimental results in the framework of a diploma thesis or of postgraduate studies.
5. The exposition of experimental results in a monograph, intended for publication so that it becomes available to a wide public.

The way in which the report is written differs, depending on the purpose for which it is intended. There is a rich bibliography concerning the writing of scientific reports, articles and theses [1] as well as the presentation of the results to audiences [2]. Publishers have their own specific rules which must be followed in the writing of books [3] and scientific papers [4]. The rules and the advice for the best

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presentation of a subject apply, of course, not only for the presentation of experimental results but for the results of purely theoretical work as well.

In this chapter, we will discuss the simplest problem of the writing of a report for the presentation of the experimental results and of the conclusions drawn from them, in the case of experimental work performed in the framework of an educational laboratory.

13.2 The Preparation of the Writing of the Report While Performing the Experiment

It must be stated from the start that, even in an educational laboratory in which the results of the experiment are usually known beforehand, a strict experimental procedure must be followed. If the purpose of the exercise is for the student to be trained in the proper experimental practice, he or she must behave as if performing an important experiment for the first time.

The writing of the report on the results of an experiment begins with the entrance of the experimenter in the laboratory (or even earlier). The recording of all the information which are relevant to the experiment as well as the actions taken in the laboratory are of fundamental importance and a prerequisite for the writing of a useful report.

The experimenter must record all the information acquired as well as the details of his actions during the experiment. It is recommended that for this purpose a *notebook* is used rather than loose sheets of paper as the latter may be easily lost or not be kept in the right order to describe what was performed in the right time sequence.

What is worth recording is largely a matter of judgment by the experimenter. Obviously the numerical readings of instruments must be written down, but the experimenter must decide which other information is necessary for the interpretation of the results (for example, in some experiments knowing the temperature in the laboratory might be needed but not in others). Since the possibilities are countless, the experimenter must have as complete an understanding of the phenomenon to be studied as possible. As a complete understanding is of course impossible (otherwise there would be no need for the experiment to be performed!) there is obviously a need for very good *preparation* for the experiment with a thorough study of the relevant bibliography before starting the experimental work.

Recording the information in the laboratory notebook must be made having in mind that the analysis of the observations will possibly take place some considerable time later. Facts that the experimenter knows well at the time of the experiment may not be accurately recorded in his memory. This means that the information must be recorded (in legible form!) with clarity and with as much detail as would be needed for the notes to be understood by another person. Drawings, makeshift and by hand, are extremely useful and indispensable in the description of apparatus or experimental arrangements.

A general but by no means complete enumeration of the useful information that must be recorded during the experiment, could be the following:

1. The title (if there is one) of the experiment to be performed.
2. The names of the instructor and of the co-workers for the particular experiment.
3. The date and time. The time should be recorded whenever a new procedure is started.
4. The aim of the experiment.
5. Environmental conditions which might prove useful.
6. The relevant bibliography used, as well as any additional documents that might be used in the analysis of the results.
7. The procedure of preparation of the experiment.
8. Drawings and a description of the experimental setup used.
9. The results of the measurements. Wherever necessary, the numerical values should be recorded in well designed tables, with clear descriptions of the magnitudes recorded (magnitude, units, common multiplying factor if there is one etc.).
10. The problems met and the questions that arose during the experiment and the actions taken for their resolution. Anything that is considered relevant for the analysis of the results.
11. Ideas the experimenter may have had during the experiment, which, although in a crude form at the time, might be adequately developed later and help in the better understanding of the results.
12. Suggestions for further work that might be done or for improvement of the procedures followed that might help the same or another experimenter in a repetition of the experiment at a later time. Ideas for similar research that might be done.

It is obvious that this list is simply indicative and may differ from experiment to experiment.

One good piece of advice is that if the need arises for the rejection of some results, these should be crossed out in such a way that they remain legible. A thin straight line drawn over them, preferably in a different color, is all that is needed for the purpose. Very frequently, results which were considered useless at some stage of the experiment turn out to be correct. The same is true for a whole experimental procedure which had been followed before being replaced by another which was considered at the time to be a better one.

The writing of the report describing an experiment and analyzing its results commences in the laboratory. During the experiment, the experimenter must compute intermediate results and draw, roughly, graphs, in order to check the development of the experiment and take the necessary measures for the optimum continuation of it, at every stage. The accuracy of the calculations does not have to be high and the graphs may be drawn by hand whenever that is satisfactory. Their value, however, is enormous.

The experimental setup must not be dismantled before the experimenter is satisfied that there is no need for additional measurements to be taken. In scientific research, this happens after the results have been analyzed completely. Very often, during the analysis of the results, it is found that additional measurements have to be made or that some needed information has not been recorded. In an educational laboratory this is usually impossible to do. For this reason, before leaving the laboratory, the experimenter should be satisfied that the experiment has been concluded.

13.3 The Written Report of an Experiment

The written report of an experiment in the framework of an educational laboratory must contain the following parts:

1. *Title.* The title must be brief and comprehensive.
2. *Names.* The names of the people who conducted the experiment should be given, as well as that of the supervisor, if any.
3. *Dates and time.* The date and time on which the experiment was conducted.
4. *Summary.* In about one paragraph, a short description should be given of the work done, the scientific reasons for which this was done and the method followed. The main results obtained may also be mentioned.
5. *Theory.* A *short* summary of the theoretical background of the experiment should be given, without this being an extensive presentation of theory and unnecessary equations. The method which is to be followed for the extraction of the information needed from the experimental results should also be described.
6. *Apparatus used.* A detailed list should be given of the equipment available and used in the experiment. All relevant technical specifications should be mentioned.
7. *Experimental results.* The crude results of the measurements should be presented. Whenever possible, the results should be given in tables, in the order in which these were performed, before being subjected to any arithmetical treatment. Both the experimental arrangement and the procedures followed in taking the results must be described with as much detail as needed. In this part, drawings showing the experimental setup are particularly useful.
8. *Analysis of the results.* The analysis of the results should be presented with clarity, without it being necessary to show all the intermediate results of the arithmetic calculations. On the other hand, results should not appear out of nowhere! In this part, extensive use of tables and graphs should be made.

9. *Conclusions.* The conclusions that follow from the analysis of the measurements should be presented and justified. A short general assessment of the success or not of the whole exercise should also be given.
10. *Bibliography.* A list of the bibliography used should be given (books, journals, theses, manuals etc.). There are many ways of presenting references and bibliography. For the purposes of an educational laboratory, the following method is adequate:

    1. Books:
       H.D. Young and R.A. Freedman, *University Physics with Modern Physics* (Pearson—Addison Wesley, 11th edition, 2004). Chaps. 6, 9.
       If a single reference is made to a book, the relevant chapters or pages are given in the bibliography and this is referred to in the text as (Young and Freeman 2004). Otherwise, multiple references are made in the text: (Young and Freeman 2004, Chap. 6) and elsewhere (Young and Freeman 2004, pp 345–9).
    2. Journal articles:
       C. Christodoulides (2017), 'The construction of a perpetual motion machine of the fourth kind', *Journal of Non-Reproducible Results*, **1**, 1–665.
       The name of the journal is given in full or in the accepted abbreviated form. The title of the article is optional but can be of great help if given. The volume number of the journal is given in bold. The number of the article's first page is then given. The end page might also be useful.
       A. Genius (2003), 'The creation of a mini black hole in the educational laboratory'. *J. Black Hole Res.*, **13**, (7), 123–4.
       Here, the issue number of the journal (7) is also given.
    3. Reference books—tables:
       G.W.C. Kaye and T.H. Laby, *Tables of Physical and Chemical Constants* (Longmans, 14th ed., 1973). Page 165, Table 15.6.
       W.M. Haynes (ed.), *CRC Handbook of Chemistry and Physics* (CRC Press, 95th ed., 2014). Page 983.
       The reference is given in a manner similar to the way a book is referred to (1). The table or graph used should be mentioned with enough detail so that a reader may easily find the information which has been used.
    4. Web pages:
       The NIST reference on Constants, Units and Uncertainty. http://physics.nist.gov/cuu/Constants/
       National Physical Laboratory, *Tables of Physical and Chemical Constants*. http://www.kayelaby.npl.co.uk/

Some general rules for the write-up of the report are the following:

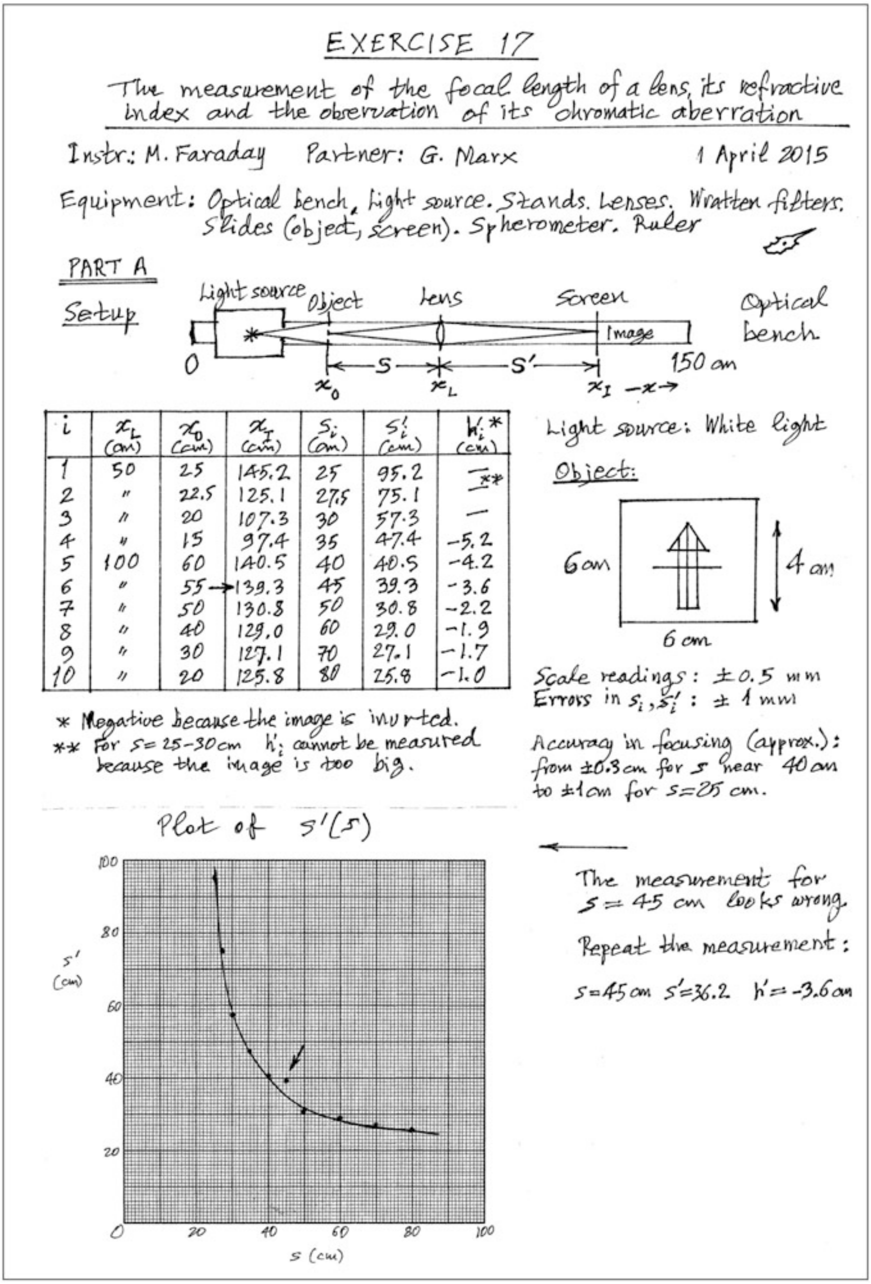
(a) The tense used is that appropriate to what is described. The experiment was performed in the past, so the past tense must be used in describing it. Phrases are used such as: '*The experimental arrangement used is shown in Figure 1*' or '*The resistance was varied from 0 to 1000 Ω, in steps of 100 Ω*' etc.
(b) The use of impersonal expressions is preferred. For example, instead of the statement '*We decided to investigate further the range of current values between 20 and 30 mA*', it is preferable to write: '*It was judged necessary for the range of current values between 20 and 30 mA to be investigated in greater detail*'.
(c) When referring to decisions that are taken by us or to conclusions we reached during the analysis of the results, the use of first person present tense is allowed and, in some cases, necessary. For example, we may write: '*We conclude that the two results for the electronic charge agree with each other, within experimental error*'. This can also be written as: '*It is concluded that the two results for the electronic charge agree with each other, within experimental error*'.
(d) Bibliographic references should be given in the text with adequate detail. For example, a reference may be given as (Kittel, p. 467). If the same author is mentioned more than once for the same year in the bibliography, the distinction between the two or more books or articles can be achieved by using letters (a, b, …) after the year of publication (e.g. Genius 2003a, b). The details of the book or article (publisher, place and time of publication, edition number, volume) are given in the bibliography table. It must be borne in mind that there are many different ways of giving bibliographic references.

13.4 An Example of a Written Report of an Experiment

An example of a written report of an experiment conducted in the framework of an educational laboratory is given below. The method used is by no means unique but it may be used as a rough guide for similar reports.

Although software is available for evaluating mean values, standard deviations and least-squares parameters, in this exercise these magnitudes will be evaluated 'by hand', in order to understand the principles of the methods.

Figure 13.1 shows the notes taken in the laboratory during an experiment.



EXERCISE 17

The measurement of the focal length of a lens, its refractive index and the observation of its chromatic aberration

Instr.: M. Faraday Partner: G. Marx 1 April 2015

Equipment: Optical bench, light source. Stands. Lenses. Wratten filters. Slides (object, screen). Spherometer. Ruler

PART A

Setup

i	x_L (cm)	x_0 (cm)	x_I (cm)	s_i (cm)	s'_i (cm)	h'_i* (cm)
1	50	25	145.2	25	95.2	— **
2	"	22.5	125.1	27.5	75.1	
3	"	20	107.3	30	57.3	—
4	"	15	97.4	35	47.4	−5.2
5	100	60	140.5	40	40.5	−4.2
6	"	55 →	139.3	45	39.3	−3.6
7	"	50	130.8	50	30.8	−2.2
8	"	40	129.0	60	29.0	−1.9
9	"	30	127.1	70	27.1	−1.7
10	"	20	125.8	80	25.8	−1.0

* Negative because the image is inverted.
** For $s = 25$–30 cm h'_i cannot be measured because the image is too big.

Light source: White light

Object:

Scale readings: ± 0.5 mm
Errors in s_i, s'_i: ± 1 mm

Accuracy in focusing (approx.): from ±0.3 cm for s near 40 cm to ±1 cm for $s = 25$ cm.

Plot of $s'(s)$

The measurement for $s = 45$ cm looks wrong.

Repeat the measurement:

$s = 45$ cm $s' = 36.2$ $h' = -3.6$ cm

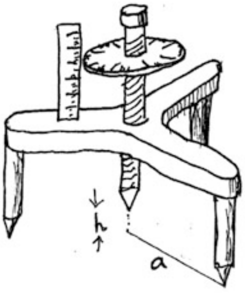
Fig. 13.1 The notes taken in the laboratory during an experiment

PART B

Sherometer

Distance of central pointed end from 3 others: $a = 3$ cm (given by manufacturer)

Accuracy of h readings: $\pm$ 0.0005 cm



Radii of lens using the spherometer (both positive)

For R_1

i	h_{1i} (cm)
1	0.1875
2	0.1820
3	0.1845
4	0.1860
5	0.1825

For R_2

i	h_{2i} (cm)
1	0.1865
2	0.1820
3	0.1835
4	0.1850
5	0.1825

PART C

1. The image is formed using red or blue light:

Filters used: red: Wratten 24 blue: Wratten 38 A

For $s = 40$ cm
Distance of image with red light $s'_R = 40.5$ cm
" " " " blue " $s'_B = 38.5$ cm (± 1 cm)

2. Chromatic aberration of lens:

Form image of the filament of the light source. Spectral analysis is visible near edges of image.

3. Chromatic aberration of telscope:

Lens has diameter $d = 12$ cm and $f \approx 25$ cm.

Holding the lens with one hand and a jeweller's eyepice near the eye, we have a tesescope (astronomical). Magnification ~×8.

Chromatic aberration is clearly visible.
Blue and red images slightly different.
Spectral analysis visible near the edges of the image

KX

Fig. 13.1 (continued)

Exercise 17

The Measurement of the Focal Length of a Lens, its Refractive Index and the Observation of its Chromatic Aberration
A. Fresher

Instructor: M. Faraday Partner: G. Marx

Date the experiment was performed: 1 April, 2017

Aim of the Experiment

In this experiment the focal length of a converging lens was measured. From the knowledge of the lens's focal length and the measurement of the radii of its spherical surfaces, the refractive index of the glass the lens is made of was evaluated. Finally, the phenomenon of chromatic aberration in the lens was observed.

Bibliography

C. Christodoulides and G. Christodoulides, *Analysis and Presentation of Experimental Results* (Springer, 2017). To be referred to as APER.
E. Hecht, *Optics* (Addison-Wesley, 3rd ed., 1998).
W.M. Haynes (ed.), *CRC Handbook of Chemistry and Physics* (CRC Press, 95th ed., 2014).
H.D. Young and R.A. Freedman, *University Physics with Modern Physics* (Pearson–Addison Wesley, 11th edition, 2004). To be referred to as Y&F.

Theory

A converging lens may be used for the formation of the real image of an object. If s and s' are the distances from the center of the lens of the object and of the image, respectively, the relation between them is (Y&F, Sect. 34.4)

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{1}$$

where f is a length characteristic of the lens, its focal length (Fig. 1a). By measuring the values of s' for a series of values of s, the focal length of the lens may be determined.

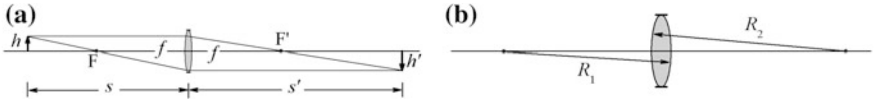


Figure 1 **a** Formation of the real image of an object by a converging lens. **b** The radii of the lens's spherical surfaces

The size of the image is related to the size of the object through the expression

$$\frac{h'}{h} = -\frac{s'}{s}, \tag{2}$$

where the negative sign indicates the fact that the image is inverted relative to the object.

If R_1 and R_2 are the radii of the lens's two spherical surfaces (Fig. 1b), then its focal length is given by the lens maker's equation (Y&F, Sect. 34.4)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \tag{3}$$

where n is the index of refraction of the lens's glass. With the sign convention used, R_2 must be taken as negative. The equation is valid for thin lenses, otherwise a correction is needed.

Equipment Available

The following equipments was supplied (Fig. 2):

An optical bench of 150 cm length.
A source of white light.
A set of Wratten filters.
Stands which can be placed on the optical bench and can hold various optical elements.
Lenses: The lens to be studied, a hand held magnifier and a jeweler's eyepiece.
A slide with the shape of an arrow drawn in transparent lines on it. To be used as the object.
A screen on which the image may be formed.
A ruler.

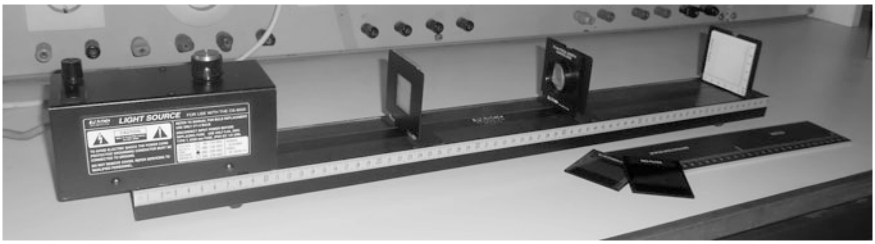


Figure 2 The experimental setup

Experimental Procedure

The experiment was performed in three parts:

Part A. In this part of the experiment the focal length of a converging lens was determined by measuring the distances of the images from the lens for various distances of an object.
Part B. The index of refraction of the glass of the lens was determined by measuring the radii of its surfaces and using the value of its focal length found in Part A.
Part C. Phenomena were observed which are associated with the chromatic aberration of the lens.

Presentation of the Experimental Procedure and Analysis of the Results

Part A. The Determination of the Focal Length of a Converging Lens by Measuring the Distances of the Images from the Lens for Various Distances of the Object

The experimental arrangement used in this part of the exercise is shown in Figs. 2 and 3. The experiment was conducted in a darkened part of the laboratory. On an optical bench a source of white light was placed and stands that could be moved along the bench. The position of each element on the bench could be read on a scale along it, with an accuracy of 1 mm. The length of the optical bench was 150 cm.

A converging lens was placed on a stand approximately at the middle of the bench, at a distance of x_L from the end of the bench on which the source was situated. As object, an opaque slide provided was used, which was transparent along thin lines on it forming an arrow of height $h = 4.0 \pm 0.1$ cm (Fig. 4). The object was placed at x_O. The light from the source illuminated the slide from behind

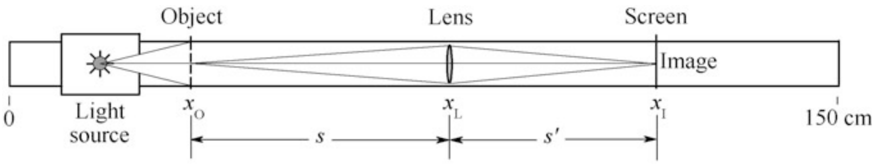


Figure 3 The optical bench used with the light source, the object, the lens and the screen in position

Figure 4 The slide used as object. The thin lines were transparent

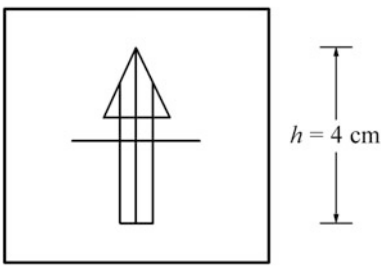


Table 1 The measurements for the first part of the exercise

i	x_L (cm)	x_O (cm)	x_I (cm)	s_i (cm)	s'_i(cm)	h'_i(cm)
1	50	25	145.2	25	95.2	–
2	50	22.5	125.1	27.5	75.1	–
3	50	20	107.3	30	57.3	–
4	50	15	97.4	35	47.4	−5.2
5	100	60	140.5	40	40.5	−4.2
6	100	55	139.3	45	39.3	−3.6
7	100	50	130.8	50	30.8	−2.2
8	100	40	129.0	60	29.0	−1.9
9	100	30	127.1	70	27.1	−1.7
10	100	20	125.8	80	25.8	−1.0

so that light rays would emerge from it through the transparent lines. The image of the arrow was formed on a screen placed on the other side of the lens, at x_I. Keeping the lens in a fixed position, the object was placed at various distances s from the center of the lens. At each position the screen was moved along the bench until a clear well focused image of the object was formed on it. Theory suggests that s and s' are related to each other by the expression of Eq. (2). Since the optical bench was only 150 cm long, in order to use values of s and s' in as large a range as possible, the first 4 measurements were performed with the lens at $x_L = 50\,\text{cm}$ and then the lens was moved to $x_L = 100$ cm. The magnitudes read off the scale were x_L, x_O and x_I. The distance of the object from the lens, $s = x_L - x_O$, was chosen to be a round number of centimeters. The distance of the image from the lens, $s' = x_I - x_L$, was determined as described above.

The values of x_L, x_O and x_I, together with the calculated values of s and s' are shown in Table 1. The height of every image formed, h'_i, was also measured and is given in the table. The values given are negative, to state the fact that the image was inverted relative to the object. It was not possible to measure the size of the image for the distances of $s = 25\,\text{cm}, s = 27.5\,\text{cm}$ and $s = 30\,\text{cm}$, as in these cases the image was too big, bigger than the slide serving as screen.

A plot of s' as a function of s drawn in the laboratory (Fig. 5) suggested that point #6 might be the result of a mistake in reading the scale as its deviation from the curve between the points seems rather large. The measurement for $s = 45\,\text{cm}$ was repeated and it was found that $s' = 36.2\,\text{cm}$. The results, with the new value of measurement #6 are given in columns 2 and 3 of Table 3.

The possibility that result #6 is spurious will be tested by evaluating the focal length f_i of the lens substituting each pair of values of s and s' in Eq. (1). This is done with the aid of Table 2.

From these 10 values it is found (APER, Sect. 4.2) that:

The mean value of the focal distance: $\overline{f} = \frac{198.56}{10} = 19.86\,\text{cm}$.

The graph drawn in Fig. 5 is $s'(s)$ from Eq. (1) with $f = 19.9\,\text{cm}$.

The standard deviation of the distribution of f values: $\sigma_f = \sqrt{\frac{2.4328}{10}} = 0.49\,\text{cm}$.

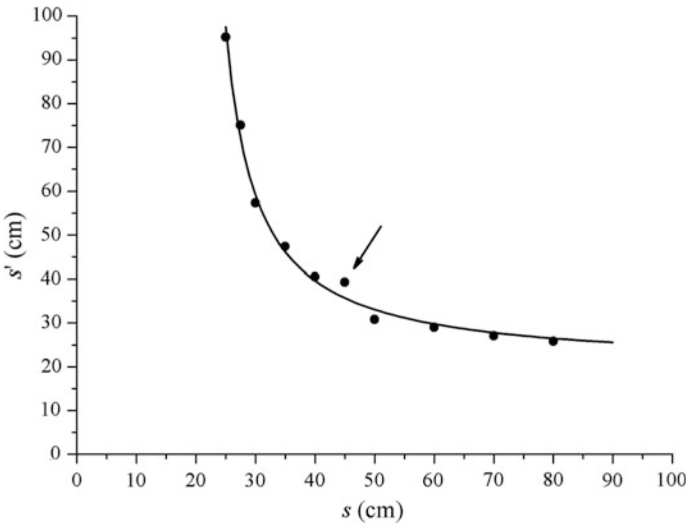


Figure 5 Plot of s' as a function of s. Initial measurements. The measurement for s = 45 cm is suspected to deviate too much from the best curve through the points. The line drawn is $s'(s)$ from Eq. (1) with f = 19.9 cm

Table 2 The initial measurements for the first part of the exercise

i	s_i (cm)	s'_i (cm)	f_i (cm)	$f_i - \bar{f}$ (cm)	$(f_i - \bar{f})^2$ (cm^2)
1	25	95.2	19.80	−0.06	0.0036
2	27.5	75.1	20.13	0.27	0.0729
3	30	57.3	19.69	−0.17	0.0289
4	35	47.4	20.13	0.27	0.0729
5	40	40.5	20.12	0.26	0.0676
6	45	39.3	20.98	1.12	1.2544
7	50	30.8	19.06	−0.80	0.6400
8	60	29.0	19.60	−0.26	0.0676
9	70	27.1	19.54	−0.32	0.1024
10	80	25.8	19.51	−0.35	0.1225
Sums			198.56		2.4328

It is seen that measurement #6 differs from the mean by 1.12 cm or by 2.3 σ_f.

Chauvenet's criterion (APER, Chap. 10) states that, for 10 measurements, a measurement differing from the mean by more than 1.96 σ_f should be rejected. This is the case here, so we reject the initial measurement for $s = 45$ cm is rejected.

Using the new measurement for $s = 45$ cm, we plot $s'(s)$ in Fig. 6.

Table 3 The measurements for the first part of the exercise, with the repeated measurement

i	s_i (cm)	s'_i (cm)	h'_i (cm)	f_i (cm)	$f_i - \bar{f}$ (cm)	$(f_i - \bar{f})^2$ (cm^2)
1	25	95.2	–	19.80	0.04	0.0016
2	27.5	75.1	–	20.13	0.37	0.1369
3	30	57.3	–	19.69	−0.07	0.0049
4	35	47.4	−5.2	20.13	0.37	0.1369
5	40	40.5	−4.2	20.12	0.36	0.1296
6	45	36.2	−3.6	20.06	0.30	0.0900
7	50	30.8	−2.2	19.06	−0.70	0.4900
8	60	29.0	−1.9	19.60	−0.16	0.0256
9	70	27.1	−1.7	19.54	−0.22	0.0484
10	80	25.8	−1.0	19.51	−0.25	0.0625
Sums				197.64		1.1264

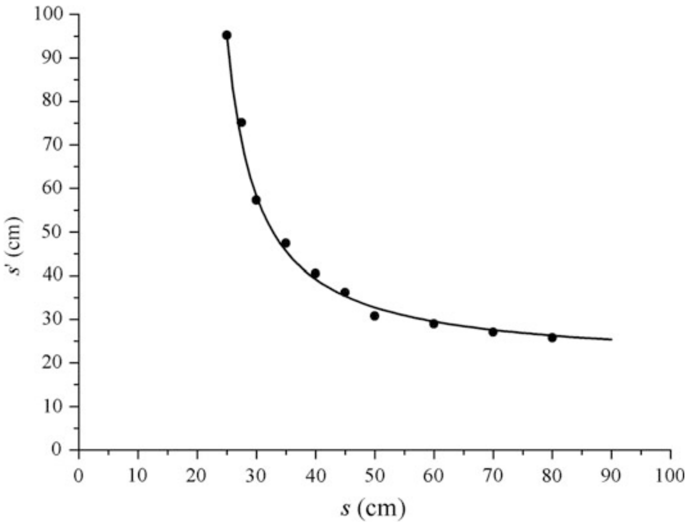


Figure 6 Plot of s' as a function of s. With the measurement for $s = 45$ cm repeated. The line drawn is $s'(s)$ from Eq. (1) with $f = 19.8$ cm

Determination of the focal length of the lens using the equation $1/f = 1/s + 1/s'$ *and the pairs of values of* s *and* s'.

The focal length of the lens may be evaluated by substituting in Eq. (1) each pair of the values of s and s'. The values of f_i thus found are given in the 5th column of Table 3. From these 10 values of f the mean may be found as well as the standard deviation of the distribution of the values and the standard deviation of the mean.

Mean value of the focal distance: $\bar{f} = \frac{197.64}{10} = 19.76\,\text{cm}$.

The graph drawn in Fig. 6 is $s'(s)$ from Eq. (1) with $f = 19.8\,\text{cm}$.

Table 4 The data used for testing the validity of equation $-h'/h = s'/s$

i	s_i (cm)	s'_i (cm)	$-h'_i$ (cm)	s'_i/s_i	$-h'_i/h$
1	25.0 ± 0.2	95.2 ± 4.8	–	3.81 ± 0.19	–
2	27.5 ± 0.2	75.1 ± 3.8	–	2.73 ± 0.14	–
3	30.0 ± 0.2	57.3 ± 2.9	–	1.91 ± 0.10	–
4	35.0 ± 0.2	47.4 ± 2.4	5.2 ± 0.3	1.35 ± 0.07	1.30 ± 0.10
5	40.0 ± 0.2	40.5 ± 2.0	4.2 ± 0.2	1.01 ± 0.05	1.05 ± 0.07
6	45.0 ± 0.2	36.2 ± 1.8	3.6 ± 0.2	0.80 ± 0.04	0.90 ± 0.07
7	50.0 ± 0.2	30.8 ± 1.5	2.2 ± 0.2	0.62 ± 0.03	0.55 ± 0.06
8	60.0 ± 0.2	29.0 ± 1.5	1.9 ± 0.2	0.48 ± 0.02	0.48 ± 0.06
9	70.0 ± 0.2	27.1 ± 1.4	1.7 ± 0.2	0.39 ± 0.02	0.43 ± 0.05
10	80.0 ± 0.2	25.8 ± 1.3	1.0 ± 0.1	0.32 ± 0.02	0.25 ± 0.03

The standard deviation of the values of f is: $\sigma_f = \sqrt{\frac{1.1264}{10}} = 0.34\,\text{cm}$.

Having rejected one of the initial measurements, the next candidate for rejection is the measurement for $s = 50\,\text{cm}$. It differs from the new mean by 0.7 cm or 2 σ_f. We consider this measurement to be acceptable.

From Table 4 it is found that the standard deviation of the mean $\overline{f}$ is: $\sigma_{\overline{f}} = \sqrt{\frac{1.1264}{10\times 9}} = 0.11\,\text{cm}$.

The final result for the focal length of the lens used is, therefore,

$$\boxed{f = 19.8 \pm 0.1\,\text{cm}}$$

Test of the validity of the equation $-h'/h = s'/s$ for the size of the image

The values of the image size, h' are used in order to check the validity of Eq. (2). In this experiment, the object size was $h = 4.0 \pm 0.1\,\text{cm}$. Due to the uncertainties in determining the positions of the lens and the image, the error in s was of the order of 2 mm. The errors in s' were estimated to be of the order of 5%. The best estimate for the errors in h' is (1 mm + 5%). The first term results from the procedure of measuring the size of the image with a ruler and the second from the limited definition in the image. In Table 4 we give the values of s, s', h', s'/s and h'/h, with their errors.

In Fig. 7, $-h'/h$ is plotted as a function of s'/s. The errors in both variables are also marked. Drawing a straight line passing through the origin and between the points of the graph, it is seen that the linear relation is verified within the accuracy of the measurement, in agreement with Eq. (2).

Determination of the focal length of the lens using the method of least squares on the linearized graph of the equation $1/f = 1/s + 1/s'$.

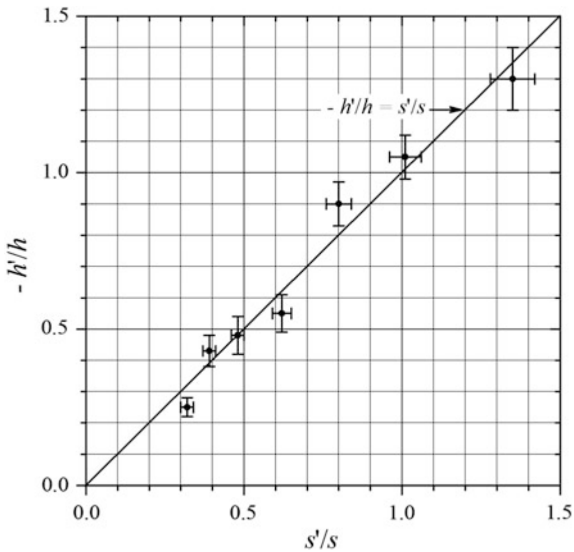


Figure 7 The ratio of the image height to the object height as a function of the ratio s'/s of their respective distances from the lens

In Table 5 are recorded: the values of s and s', the values of $x = 1/s$ and $y = 1/s'$, and of the products needed for the application of the method of least squares.

In terms of x and y, Eq. (1) takes the form $y = 1/f - x$. The graph of $y = 1/s'$ as a function of $x = 1/s$ is plotted in Fig. 8. As expected, the points lie near a straight line which has a slope of about -1. This line was determined using the method of least squares.

Assuming that the best fit to the experimental points is given by the straight line $y = a + \lambda x$, the coefficients are found from (APER, Sect. 11.3)

$$\alpha = \frac{[y][x^2] - [x][xy]}{N[x^2] - [x]^2} = 0.0512\,\text{cm}^{-1} \qquad \lambda = \frac{N[xy] - [x][y]}{N[x^2] - [x]^2} = -1.031$$

If $d_i \equiv y_i - \alpha - \lambda x_i$, the errors in α and λ are

$$\delta\alpha = \sigma_\alpha = \sqrt{\frac{1}{N-2} \cdot \frac{[x^2][d^2]}{N\,[x^2] - [x]^2}} = 0.0012\,\text{cm}^{-1} \quad \text{and}$$

$$\delta\lambda = \sigma_\lambda = \delta\alpha\sqrt{\frac{N}{[x^2]}} = 0.048$$

so, finally, $\alpha = 0.0512 \pm 0.0012\ \text{cm}^{-1}$ and $\lambda = -1.031 \pm 0.048$.

Table 5 The quantities needed in the application of the method of least squares

i	s_i (cm)	s'_i (cm)	$x_i = 1/s_i$ (cm^{-1})	$y_i = 1/s'_i$ (cm^{-1})	x_i^2 (10^{-4} cm^{-2})	$x_i y_i$ (10^{-4} cm^{-2})	d_i (cm^{-1})	d_i^2 (10^{-6} cm^{-2})
1	25	95.2	0.0400	0.0105	16.00	4.20	0.00054	0.292
2	27.5	75.1	0.0364	0.0133	13.22	4.84	−0.00037	0.137
3	30	57.3	0.0333	0.0175	11.11	5.82	0.00063	0.397
4	35	47.4	0.0286	0.0211	8.16	6.03	−0.00061	0.372
5	40	40.5	0.0250	0.0247	6.25	6.17	−0.00073	0.533
6	45	36.2	0.0222	0.0276	4.94	6.14	−0.00298	8.880
7	50	30.8	0.0200	0.0325	4.00	6.49	0.00192	3.686
8	60	29.0	0.0167	0.0345	2.78	5.75	0.00048	0.230
9	70	27.1	0.0143	0.0369	2.04	5.27	0.00044	0.194
10	80	25.8	0.0125	0.0388	1.56	4.84	0.00049	0.240
		Sums	0.2490 = [x]	0.2555 = [y]	70.07×10^{-4} = [x^2]	55.56×10^{-4} = [xy]		14.961×10^{-6} = [d^2]

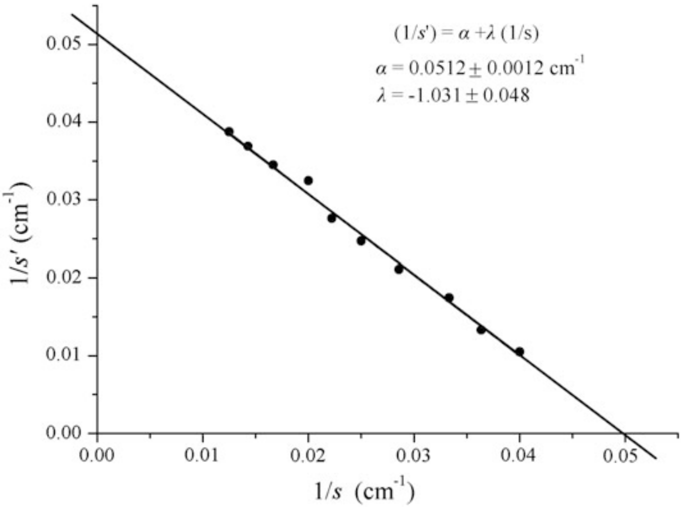


Figure 8 Plot of $1/s'$ as a function of $1/s$

The focal length of the lens is $f = 1/\alpha$ and its error $\delta f = \delta\alpha/\alpha^2$. Substituting,

$$\boxed{f = 19.5 \pm 0.5\,\text{cm}}$$

Part B. The Index of Refraction of the Glass of the Lens from its Radii and its Focal Length

The index of refraction of the glass the lens is made of may be determined from a knowledge of the lens's radii and its focal length.

Measurement of the radii of the lens using a spherometer

A spherometer is shown on Fig. 9. The pointed ends of the three legs of the spherometer, forming an equilateral triangle, are placed on the surface of the lens whose curvature is to be measured. A central leg is then lowered until it makes contact with the surface. The height h of the point of the central leg above the plane of the other three is indicated on the spherometer's scale. One complete turn of the dial shifts the central leg by 0.5 mm. The dial is graduated into subdivisions, making it possible to measure h with greater precision. With the spherometer used, the accuracy with which this is measured is ± 0.0005 cm

If the distance of the three fixed legs of the spherometer from each other is a and the radius of the sphere being measured is R, then, with reference to Fig. 9, it is

$$(\text{OC}) = h + \sqrt{(\text{OA})^2 - a^2} \quad \text{or} \quad R = h + \sqrt{R^2 - a^2}.$$

Squaring, $\cancel{R^2} - a^2 = \cancel{R^2} - 2Rh + h^2$, from which it follows that

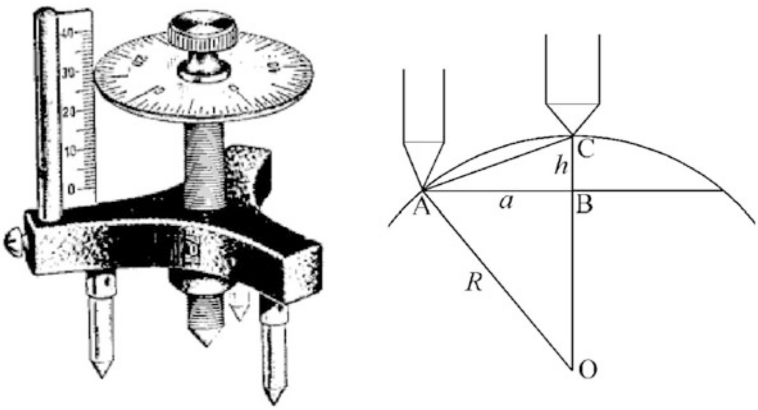


Figure 9 A spherometer and its geometry

$$R = \frac{a^2}{2h} + \frac{h}{2}. \tag{4}$$

The results of the measurements performed on the two surfaces of the lens are given in the second columns of the two following tables.

Measurements for R_1			
i	h_{1i} (cm)	$h_{1i} - \overline{h_1}$ (cm)	$\left(h_{1i} - \overline{h_1}\right)^2$ (10^{-6} cm^2)
1	0.1875	0.0030	9.00
2	0.1820	−0.0025	6.25
3	0.1845	0	0
4	0.1860	0.0015	2.25
5	0.1825	−0.0020	4.00
Sums	0.9225	0	21.50

Measurements for R_2			
i	h_{2i} (cm)	$h_{2i} - \overline{h}_2$ (cm)	$\left(h_{2i} - \overline{h}_2\right)^2$ (10^{-6} cm^2)
1	0.1865	0.0026	6.76
2	0.1820	−0.0019	3.61
3	0.1835	−0.0004	0.16
4	0.1850	0.0011	1.21
5	0.1825	−0.0014	1.96
Sums	0.9195	0	13.70

The first table gives: $\overline{h}_1 = 0.9225/5 = 0.1845$ cm and $\sigma_{\overline{h}_1} = \sqrt{21.50 \times 10^{-6}/5 \times 4} = 0.0010$ cm.

The second table gives: $\overline{h}_2 = 0.9195/5 = 0.1839$ cm and $\sigma_{\overline{h}_2} = \sqrt{13.70 \times 10^{-6}/5 \times 4} = 0.0008$ cm.

The final results are $h_1 = 0.1845 \pm 0.0010$ cm and $h_2 = 0.1839 \pm 0.0008$ cm. The errors are 0.54% for h_1 and 0.44% for h_2.

The radii corresponding to these values of h are found, given that, according to the manufacturers of the spherometer, $a = 3.000$ cm with an error of the order of 0.05%. The values of R_1 and R_2 are found using Eq. (4): $R_1 = 24.48$ cm, $R_2 = 24.56$ cm.

The errors in R_1 and R_2 are given by (APER, Sect. 6.2.3)

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial h}\right)^2 (\delta h)^2 + \left(\frac{\partial R}{\partial a}\right)^2 (\delta a)^2} = \sqrt{\left(\frac{1}{2} - \frac{a^2}{2h^2}\right)^2 (\delta h)^2 + \left(\frac{a}{h}\right)^2 (\delta a)^2}.$$

By substitution it is found that $\delta R_1 = 0.13$ cm and $\delta R_2 = 0.11$ cm. The final results for the radii of the lens are

$$R_1 = 24.48 \pm 0.13\,\text{cm} \quad \text{and} \quad R_2 = 24.56 \pm 0.11\,\text{cm}.$$

Determination of the index of refraction of the glass of the lens

The value of the refractive index of the glass of the lens may be found using the value of $f = 19.8 \pm 0.1$ cm for the focal length of the lens found in Part A and $R_1 = 24.48 \pm 0.13$ cm and $R_2 = 24.56 \pm 0.11$ cm in the lens maker's equation (Y&F, Sect. 34.4)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \quad \text{from which it follows that} \quad n = 1 + \frac{R_1 R_2}{f(R_2 - R_1)}$$

(with R_2 negative, here). Substituting, it is found that $n = 1.6150$.

The error in n is found from the last equation and the general formula

$$\delta n = \sqrt{\left(\frac{\partial n}{\partial f}\delta f\right)^2 + \left(\frac{\partial n}{\partial R_1}\delta R_1\right)^2 + \left(\frac{\partial n}{\partial R_2}\delta R_2\right)^2}$$

to be

$$\delta n = (n-1)\sqrt{\left(\frac{\delta f}{f}\right)^2 + \frac{1}{(R_2 - R_1)^2}\left[\left(\frac{R_2}{R_1}\delta R_1\right)^2 + \left(\frac{R_1}{R_2}\delta R_2\right)^2\right]}.$$

Substituting, $\delta n = 0.6150\sqrt{0.00505^2 + 0.00266^2 + 0.00224^2} = 0.0038$.

Since it is

$$\delta n = 0.6150 \times 0.00505 \times \sqrt{1 + 0.4742}$$
$$= 0.6150 \times 0.00505 \times \sqrt{1.4742} \times \left\{ \frac{1}{\sqrt{1.4742}} + \frac{\sqrt{1.4742} - 1}{\sqrt{1.4742}} \right\}$$
$$= 0.0038 \times (0.82 + 0.18)$$

it is seen that 82% of the error in n is due to the error in f and 18% to the errors in R_1 and R_2.

Finally,

$$\boxed{n = 1.615 \pm 0.004}$$

is the index of refraction of the glass the lens is made of.

Part C. The Observation of the Chromatic Aberration of a Lens

The image of the object was focused using blue light and then red light, isolated by suitable filters (Fig. 10).

From tables (Hecht 1998, Fig. 3.37) it is found that the index of refraction for light flint glass is $n_B = 1.65$ for blue light and $n_R = 1.62$ for red light, approximately. For a lens with two convex surfaces with radii equal to $R = 24.5$ cm, the focal lengths for the two colors would be $f_B = 18.8$ cm and $f_R = 19.8$ cm. For an object at a distance of $s = 40$ cm from the lens, the images formed by the lens at the two colors would appear at distances $s'_B = 35.5$ cm and $s'_R = 39.2$ cm. The lens used, with $f_W = 19.8 \pm 0.1$ cm measured using white light, would give for $s = 40$ cm an image distance of $s'_W = 39.2$ cm.

Two filters were used in order to obtain blue or red light: A Wratten 38A filter in order to obtain blue light and Wratten 24 filter for red. From tables (*Handbook of Chemistry and Physics,* 2014), it is found that the dominant wavelength transmitted by each of the two filters are: for filter 38A $\lambda_B = 479$ nm and for filter 24 $\lambda_R = 611$ nm. With the object at $s = 40$ cm, the image was formed at a distance of $s'_B = 38.5 \pm 1.0$ cm for blue light and $s'_R = 40.5 \pm 1.0$ cm for red. The focusing was rather difficult and the uncertainties in the image distances might be greater than ± 1 cm. Nevertheless, the phenomenon of the dispersion of light and the chromatic aberration of the lens used were observed, at least qualitatively.

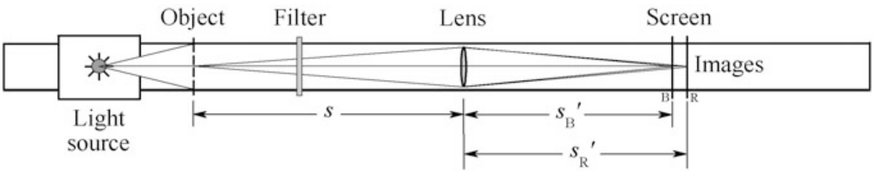


Figure 10 Forming the image of an object using *blue* (B) or *red* (R) light

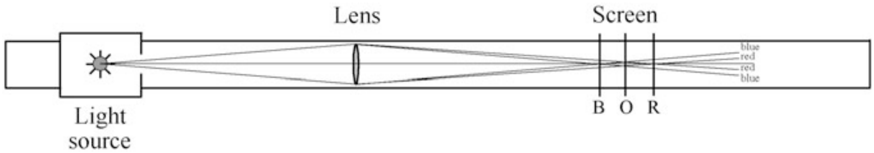


Figure 11 The observation of chromatic aberration by forming the image of the filament of the light source

Observation of chromatic aberration by forming the image of the filament of the light source. With the lens in a given position, an image of the light source's filament was formed on the screen (position O in Fig. 11). The image at position O had the best focusing that could be achieved. Moving the screen towards the light source (point B) the focusing changes, with blue being in focus but not red. The separation of the colors is clearly visible. Moving the screen away from the light source focuses red at the expense of blue. The distribution of colors is now different.

Observation of the chromatic aberration of a telescope. A hand-held stamp collector's lens and a jeweler's magnifier were held so that they formed an astronomical telescope, as shown in Fig. 12. The objective lens had a diameter of 12 cm and a focal length of about 25 cm. While the eyepiece was held in position using one hand, the objective lens, held in the other hand, was moved forward or backwards until focusing was achieved. The telescope had a magnification of 8, approximately. Chromatic aberration was clearly visible as the blue and red colors were separated at the edges of the objects being observed.

Figure 12 A makeshift astronomical telescope

Conclusions

The focal length of the converging lens studied was determined by two methods: By using the relation $1/f = 1/s + 1/s'$ to evaluate f directly and by applying the method of least squares to this relation, suitably linearized. The results were $f = 19.8 \pm 0.1$ cm and $f = 19.5 \pm 0.5$ cm, respectively. The first method is seen to be more accurate.

The refractive index of the glass the lens is made of was determined using the focal length found and the values of the lens's radii which were measured. The result was $n = 1.615 \pm 0.004$. The error in n is due largely to the error in f. This means that the best way to improve the accuracy in n is to lower the error in f.

The dispersion of light, although difficult to observe by focusing the images of an object illuminated by light of different colors, was observed qualitatively. This effect is important in the construction of telescopes, as verified experimentally.

Focusing the image was found to be easier for distances of s and s' near the value of $2f$. For this reason, it is expected that, if more measurements were performed in this region of values, a more accurate value of the lens's focal length f would have been obtained. Better results would be obtained if the determination of the position of the lens (its center) could be determined with more accuracy. The use of monochromatic light would also give a more accurate value of f for particular wavelengths.

Problem

13.1 **[E.O.P.R.]** Perform the analysis of the data in the tables and draw the figures of the report above.

References

1. L. Finkelstein Jr., *Pocket Book of Technical Writing for Engineers and Scientists* (McGraw-Hill, New York, 2000)
2. P. Kenney, *A Handbook of Public Speaking for Scientists and Engineers* (Adam Hilger, Bristol, 1982)
3. Springer, *Manuscript Guidelines*. Prentice-Hall, *Author's Guide*. Both frequently revised
4. American Institute of Physics, *AIP Style Manual*, 4th edn (1990)