

Chapter 9

Channel Capacity of Cognitive Radio in a Fading Environment with CSI and Interference Power Constraints

9.1 Introduction

In general, channel capacity is used as a basic performance measurement tool for the analysis and design of new and more efficient techniques to improve the spectral efficiency of wireless communication systems. An adaptive power transmission scheme that achieves the Shannon capacity under the fading environment was discussed in [1], and average transmit power constraints along with the availability of channel state information (CSI) at the cognitive transmitter were initially considered in [2]. The power optimization problem with peak and average transmit power constraints was investigated [3]. In spectrum sharing systems, CSI can be used at the cognitive/secondary transmitter to adaptively adjust the transmission resources as discussed in [4, 5]. In [5], knowledge of the secondary link CSI and information at the secondary transmitter (ST) (CR transmitter) about the channel between the secondary transmitter and the primary receiver (PR) was used to obtain the optimal power transmission policy of the secondary user (SU) under constraints on the peak and average received-power at the primary receiver. Ghasem and Sousa [6] demonstrated that the secondary user may take advantage in the fading environment between the primary and secondary user by opportunistically transmitting with high power when the signal received by the licensed receiver is deeply faded.

One of the most efficient ways to determine the spectrum occupancy is to sense the activity of primary users operating in the secondary user's range of communication [7]. Practically, it is difficult for a secondary user to have direct access to the CSI pertaining to the primary user link. Recent work on spectrum sharing systems has concentrated on sensing the primary transmitter's activity, and is based on local processing at the secondary user side [8]. In this context, the sensing ability is provided by a sensing detector mounted on the secondary user's equipment, which scans the spectrum for specific times [9]. The activity statistics of the primary user's signal in the shared spectrum is computed and, based on the sensing information [10], the cognitive user is capable of determining the local presence of the

primary transmitter in a specific spectrum band. For instance, the received signals at the energy-based detector [11, 12] were used to detect the presence of unknown primary transmitters. However, by using this sensing information obtained from the spectrum sensor and considering that the secondary transmitter does not have information about the state of its corresponding channel, the power adaptation strategy that maximizes the channel capacity of the secondary user's link is investigated in [13]. Rezki and Alouini in [14] considered the limited/imperfect CSI at the secondary transmitter and computed the Ergodic channel capacity. Further, in [15] the power allocation for erroneous estimated channel gain between the secondary user and primary base station was performed through a geometric programming problem which was solved by Lagrange dual decomposition. However, only the underlay spectrum sharing model was considered in [15]. Parsaeefard and Sharafat in [16] considered the cognitive nodes as relay nodes and illustrated the power and channel allocation strategy to the cognitive users in the Rayleigh fading environment. In [17], the rate loss constraint (RLC) is considered instead of conventional interference power constraints in order to protect the primary user, and the channel capacity of a cognitive user that utilizes primary users' OFDM (orthogonal frequency division multiplexing) subcarriers, is maximized by RLC and cognitive user transmit power constraints. In [14–18] the authors computed the channel capacity of the cognitive user without considering the channel sensing information available at the secondary transmitter.

In this chapter, we focus on a cognitive radio wireless communication system with maximum achievable Ergodic channel capacity, considering a single cognitive user. In a collaborative communication framework, either extra relay terminals assist the communication between some dedicated sources and their corresponding destinations, and/or they allow the users in a network to help each other to achieve higher communication system capacity than the single point-to-point communication between source and destination [19, 20]. In this chapter we have considered point-to-point communication between the cognitive users without any kind of cooperation/collaboration among them. Therefore, if more than one cognitive users are competing to access the primary user's spectrum hole, then due to probable inter-cognitive users' interference, the maximum achievable channel capacity is upper bounded by only the single cognitive user's case. The proposed spectrum sharing system has a pair of primary transmitter (PT) and PR as well as a pair of ST and secondary receiver (SR), as shown in Fig. 9.1. Further, the small-scale fading effects over the transmit power of the secondary transmitter in the proposed system has been explored. However, in [21] this type of system model is considered without fading in the link channel between the ST and the PR. Therefore, the Ergodic channel capacity for the Nakagami- m fading channel in the secondary and primary links is the basic motivation of this chapter. The power of the secondary transmitter is controlled based on the:

- (i) Sensing information about the primary user's activity, and
- (ii) CSI of the secondary and primary link.

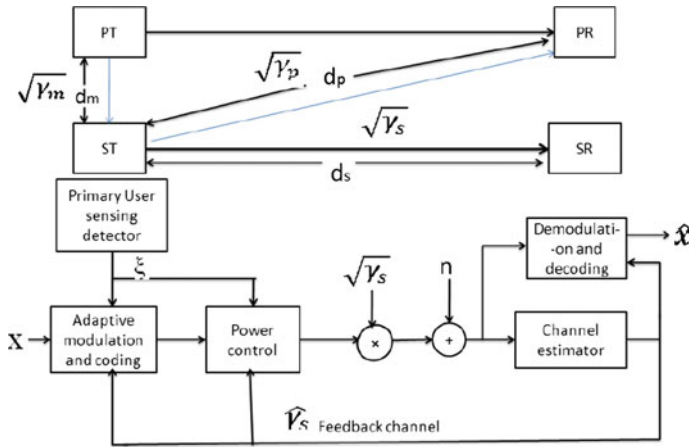


Fig. 9.1 The proposed spectrum sharing system model

Moreover, the constraint on average interference at the primary radio receiver is considered for the channel capacity. Since the cognitive user is able to adapt any modulation strategy, it can change its modulation strategy according to the fading environment, and hence both policies in the rate and power are established [22], which is referred as the variable rate and power transmission scheme. In this context, we have also considered the variable rate and power M -QAM transmission strategy in the cognitive radio communication system where the rate and power of the ST is adaptively controlled based on the availability of the secondary user’s link CSI and sensing information about the primary user’s activity. Therefore, in this chapter we have numerically computed the channel capacity in the fading environment under the average interference power constraint with two adaptation policies for spectrum sharing. The channel capacity is maximized for these two policies by considering the Lagrange optimization problem for average interference power constraint. The small-scale fading effect over the transmit power of the secondary transmitter is also presented.

The remainder of this chapter is organized as follows. Section 9.2 concerns the spectrum sharing system model. Section 9.3 discusses the power and rate adaptation policy, and in Sect. 9.4 Ergodic channel capacity of the adaptation policies under Nakagami- m fading is computed. In Sect. 9.5, the numerical simulation results of the proposed spectrum sharing model are presented, and finally, Sect. 9.6 summarizes the work.

9.2 Spectrum Sharing System

9.2.1 System Model

This proposed spectrum sharing system consists of a PT and PR pair as well as an ST and SR pair, as shown in Fig. 9.1. In this scenario, the secondary user is allowed to use the spectrum band assigned to the primary user as long as the interference power imposed by secondary transmitter on the primary receiver is less than a predefined threshold value, which is the interference temperature limit. We consider the primary user link that is the channel between the PT and PR to be a stationary block-fading channel. According to the definition of block-fading, the channel gain remains constant over some block length T and after that time, the channel gain changes to a new independent value based on its distribution [21].

The average transmit power of the PT is assumed to be P_t , its average ON/active time is α , and its average OFF/inactive time is $\bar{\alpha} = 1 - \alpha$ [13]. In addition, we have assumed a discrete-time flat-fading channel with perfect CSI at the receiver and transmitter of the secondary user. As shown in Fig. 9.1, the secondary/cognitive receiver generates and estimates the channel power gain ($\hat{\gamma}_s$) between the secondary transmitter and secondary receiver (SR). We have assumed that the channel power gain is fed back to the secondary transmitter error-free and without delay. Further, the channel gain between the transmitter and receiver of the secondary user, ST and PR as well as between the PT and ST, are given by $\sqrt{\gamma_s}$, $\sqrt{\gamma_p}$, and $\sqrt{\gamma_m}$, respectively. The channel power gains γ_s , γ_p , and γ_m are independent of each other. We have obtained the cognitive radio communication system's Ergodic channel capacity by considering the distribution of γ_s and γ_p as the Nakagami- m distribution. d_m , d_s and d_p are the distances between ST to PR, ST to SR, and ST to PR, respectively. Moreover, the channel between the PT and SR is considered an additive white Gaussian noise (AWGN) channel, denoted as n , and can be modeled as a zero-mean Gaussian random variable with variance N_0B , where N_0 and B denote the noise power spectral density and the signal bandwidth, respectively. x is the data transmitted from ST and \hat{x} is the estimated transmitted data at SR as shown in Fig. 9.1.

9.2.2 Spectrum Sensing Module

As is clear from Fig. 9.1, the secondary transmitter is equipped with a spectrum sensing detector whose function is to sense the frequency band of the primary user for the secondary user's transmission. Based on the received signals, the detector computes a single sensing metric denoted by ξ , [12]. The sensing metric is the total primary signal power in the number of independent signal samples [13]. We consider that the statistics of ξ conditioned on the primary user being active or idle are known prior to the ST. Using the energy detection method for sensing

information on the primary user being active or idle, the sensing parameter ξ is modeled according to Chi-square probability distribution functions (pdfs) with ν degrees of freedom as discussed in [11], where ν is related to the number of samples used in the sensing period, N . We define the pdf of ξ , given that the PT is active or idle, by $f_1(\xi)$ and $f_0(\xi)$, respectively, that is, $f_1(\xi)$ and $f_0(\xi)$ are conditional probabilities. According to [23, pp. 941], for a large number of ν (for example ≥ 30), one can approximate the Chi-square distribution with a Gaussian pdf. Since the number of observation samples can be large enough for the approximation to be valid, we choose $f_1(\xi) \sim \mathcal{N}(\mu_1, \delta_1^2)$ and $f_0(\xi) \sim \mathcal{N}(\mu_0, \delta_0^2)$, where (μ_1, δ_1^2) and (μ_0, δ_0^2) are given by [8]. The probability distribution of ξ depends on [13]:

$$\left. \begin{aligned}
 \mu_1 &= N \left(\frac{P_t}{d_m^2} + 1 \right) \\
 \delta_1^2 &= 2N \left(\frac{P_t}{d_m^2} + 1 \right)^2, \text{ and} \\
 \mu_0 &= N \\
 \delta_0^2 &= 2N
 \end{aligned} \right\} \begin{array}{l} \text{when PT is active} \\ \\ \text{when PT is idle} \end{array} \quad (9.1a)$$

and the probability distributions of ξ are given as [8]:

$$\left. \begin{aligned}
 f_0(\xi) &= \frac{1}{\sqrt{2\pi\delta_0^2}} \exp\left(\frac{-(\xi-\mu_0)^2}{2\delta_0^2}\right) \\
 f_1(\xi) &= \frac{1}{\sqrt{2\pi\delta_1^2}} \exp\left(\frac{-(\xi-\mu_1)^2}{2\delta_1^2}\right)
 \end{aligned} \right\} \quad (9.1b)$$

In this chapter, we have used the energy detector for spectrum sensing due to its easy implementation and low computational complexity, as discussed in [11]. The other sensing detectors can also be used for spectrum sensing since the authors' main motive is to compute the sensing metric ξ , which represents the total signal power observed or the correlation between the observed signal and a known signal pattern [13]. However, the main difference lies in the number of samples required for the same performance in different detectors, and that depends on the required signal-to-noise ratio [11]. In addition, the cognitive radio user transmission should be limited so that it does not cause harmful interference to the primary user. Therefore, a limit or constraint is set at PR called the average interference power constraint or simply interference constraint. When PU is active, ST cannot transmit at a power which exceeds the average interference power constraint at the primary receiver, which is given as [21]:

$$E_{\gamma_s, \xi, \gamma_p} [P(\gamma_s, \gamma_p, \xi) \gamma_p | PUs \text{ ON}] \leq Q_{\text{int}}; \forall \gamma_s, \gamma_p, \xi \quad (9.2)$$

where the transmit power of SU is $P(\gamma_s, \gamma_p, \xi)$ and expectation over the joint pdf of random variables γ_s, γ_p and ξ is denoted by $E_{\gamma_s, \xi, \gamma_p}[\cdot]$. Q_{int} is the interference limit set at PR, that is, the maximum interference power that it can tolerate without degrading its own performance. The constraint defined in Eq. (9.2) is used to compute the Ergodic channel capacity. However, the average interference power constraint is considered only because we have assumed that the licensed user performance is measured by the average signal-to-noise ratio (SNR) and not by instantaneous SNR. Moreover, the Ergodic channel capacity under the average received power constraint is, in general, higher than that of the peak received power constraint due to the more restrictive nature of the peak power, as opposed to the average interference power constraint.

9.3 Rate and Power Adaptation Policy for M -QAM

The data rate and power adaptation is a potential transmission strategy which adjusts the transmit power and data rate of a cognitive radio system to improve the spectrum efficiency for utilizing the shared spectrum [21, 24–26]. Data rate adaptation is a spectrally efficient technique, and its adaptation can be achieved either through variation of the symbol time duration [27] or by varying the constellation size [28]. However, the former method is spectrally inefficient and requires variable-bandwidth system design as discussed in [29]. The variable data rate adaptation policy using varying constellation size is fixed bandwidth with a spectrally efficient method [29]. The Ergodic channel capacity under adaptation policy of the variable data rate and power transmission strategy in M -QAM signal constellation is considered with the knowledge of CSI and spectrum sensing information at the secondary transmitter side, which satisfies the predefined bit-error-rate (BER) requirements and adheres to the constraints on the average interference power at the primary user. In this case, the cognitive radio adapts the transmit power according to:

- (i) the primary and secondary channel power gain γ_p and γ_s , respectively,
- (ii) the primary user's activity states ξ , subjected to the average interference, and
- (iii) the instantaneous bit-error-rate constraint $P_b(\gamma_s, \xi) = P_b$.

The P_b bound for each value of γ_s and ξ is given as [21]:

$$P_b(\gamma_s, \xi) \leq 0.2 \exp\left(\frac{-1.5}{M-1} \times \frac{P(\gamma_s, \gamma_p, \xi) \gamma_s}{N_0 B}\right) \quad (9.3)$$

where M is the constellation size or the number of symbols in the particular modulation format. $P(\gamma_s, \gamma_p, \xi)$ is the transmit power of ST. To satisfy the conditions

as discussed in Eq. (9.3), we can adjust the values of M and $P(\gamma_s, \gamma_p, \xi)$. However, the instantaneous bit error rate constraint given by Eq. (9.3) holds for $M \geq 4$ [21]. We can also express Eq. (9.3) by the following mathematical expression:

$$P_b(\gamma_s, \xi) \leq 0.2 \exp\left(\frac{-1.5}{M-1} SNR_{ss}\right) \quad (9.3a)$$

where SNR_{ss} is the signal-to-noise power ratio of the ST to SR. For both the adaptive data rate and adaptive power transmission policy, Eq. (9.3) should be satisfied for the following constraint on average interference power:

$$\frac{P(\gamma_s, \gamma_p, \xi) \gamma_p}{N_0 B} \leq Q_{\text{int}} \quad (9.3b)$$

or

$$SNR_{\text{sp}} \leq Q_{\text{int}}$$

where SNR_{sp} is the signal-to-noise power ratio of secondary transmitter to primary receiver. After some mathematical manipulation of Eq. (9.3), we obtain the following maximum constellation size for a given $P_b(\gamma_s, \xi)$:

$$M(\gamma_s, \xi) = 1 + K \left(\frac{P(\gamma_s, \gamma_p, \xi) \gamma_s}{N_0 B} \right) \quad (9.3c)$$

Moreover, we can achieve the constellation size that is the value of M in M -QAM modulation format for an arbitrary chosen bit-error-rate, the average interference power and the ratio of $\frac{\gamma_s}{\gamma_p}$, and is given by the following expression:

$$M = 1 + K \left(\frac{\gamma_s}{\gamma_p} \right) Q_{\text{int}}$$

and,

$$M = 2^n = 2^{\log_2 \left(1 + K \left(\frac{\gamma_s}{\gamma_p} \right) Q_{\text{int}} \right)} \quad (9.4)$$

where

$$K = \frac{-1.5}{\ln(5P_b)} < 1 \quad (9.5)$$

and n is the number of bits per symbol. However, for $M < 4$ which is assumed for BPSK, the error rate is given in [29]. Therefore, the Ergodic channel capacity under average interference power constraint and given P_b is:

$$\frac{C_{er}}{B} = \max_{P(\gamma_s, \gamma_p, \xi)} \iint \log_2 \left(1 + \frac{K\gamma_s P(\gamma_s, \gamma_p, \xi)}{N_0 B} \right) f_s(\gamma_s) f_p(\gamma_p) (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) d\gamma_s d\gamma_p \quad (9.6)$$

With the constraint:

$$\iint \gamma_p P(\gamma_s, \gamma_p, \xi) f_s(\gamma_s) f_p(\gamma_p) f_1(\xi) d\gamma_s d\gamma_p \leq Q_{int} \quad (9.7)$$

The transmitter power $P(\gamma_s, \gamma_p, \xi)$ of the cognitive transmitter is the joint function of secondary channel gain, primary channel gain and sensing metric. Asghari and Aissa [21] provided a mathematical expression for the channel capacity of the secondary user's link for power adaptation policies under the interference and peak power constraint with the sensing pdf's. However, the primary user's link channel power gain γ_p , which is presented in Eq. (9.6), was not considered in [30]. Now, we have to maximize the Ergodic capacity of the system as given by Eq. (9.6) by simultaneously satisfying the constraint given in Eq. (9.7). Therefore, to yield the optimal power allocation $P(\gamma_s, \gamma_p, \xi)$, we form the Lagrangian multiplier, λ [31] and construct the following Lagrangian function:

$$\begin{aligned} L(P(\gamma_s, \gamma_p, \xi), \lambda) &= \iint \log_2 \left(1 + \frac{K\gamma_s P(\gamma_s, \gamma_p, \xi)}{N_0 B} \right) f_s(\gamma_s) f_p(\gamma_p) (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) d\gamma_s d\gamma_p \\ &\quad - \lambda \left(\iint \gamma_p P(\gamma_s, \gamma_p, \xi) f_s(\gamma_s) f_p(\gamma_p) f_1(\xi) d\gamma_s d\gamma_p - Q_{int} \right) \end{aligned} \quad (9.8)$$

$L(P(\gamma_s, \gamma_p, \xi), \lambda)$ is the concave function of $P(\gamma_s, \gamma_p, \xi)$, and the interference constraint defined in Eq. (9.7) is convex, therefore the first order condition that is the derivative of $L(P(\gamma_s, \gamma_p, \xi), \lambda)$ with respect to $P(\gamma_s, \gamma_p, \xi)$ is a sufficient KKT condition for the optimality [32] and the sufficient condition allows us to obtain a solution. Now, the optimization problem being convex (i.e. this problem is a maximization problem with a concave cost function and a convex set of constraints), there is a unique solution. Hence, the solution given by the sufficient condition is the only solution and is given by:

$$\begin{aligned} \frac{\partial L(P, \lambda)}{\partial P} &= \frac{1}{1 + \frac{K\gamma_s P(\gamma_s, \gamma_p, \xi)}{N_0 B}} \frac{K\gamma_s}{N_0 B} (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) f_s(\gamma_s) f_p(\gamma_p) \\ &\quad - \lambda \gamma_p f_1(\xi) f_s(\gamma_s) f_p(\gamma_p) = 0 \end{aligned}$$

or

$$\begin{aligned} \frac{\partial L(P, \lambda)}{\partial P} &= \frac{K\gamma_s}{N_0B + K\gamma_s P(\gamma_s, \gamma_p, \xi)} (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) \\ - \lambda \gamma_p f_1(\xi) &= 0 \end{aligned} \quad (9.9)$$

and

$$P(\gamma_s, \gamma_p, \xi) = \frac{\gamma_\mu(\xi)}{\lambda \gamma_p} - \frac{N_0B}{\gamma_s K} \quad (9.10a)$$

If we assume $P(\gamma_s, \gamma_p, \xi) = 0$ for some values of γ_s, γ_p , and ξ , which take place in the condition defined below and after putting $P(\gamma_s, \gamma_p, \xi) = 0$ in Eq. (9.10a), we get:

$$\frac{\gamma_p}{\gamma_s} > \frac{\gamma_\mu(\xi)K}{\lambda N_0B} \quad (9.10b)$$

Therefore, from Eqs. (9.10a) and (9.10b), the power $P(\gamma_s, \gamma_p, \xi)$ is adapted to maximize the Ergodic channel capacity as defined in Eq. (9.6), which is given as:

$$P(\gamma_s, \gamma_p, \xi) = \begin{cases} \frac{\gamma_\mu(\xi)}{\lambda \gamma_p} - \frac{N_0B}{\gamma_s K}, & \frac{\gamma_p}{\gamma_s} \leq \frac{\gamma_\mu(\xi)K}{\lambda N_0B} \\ 0, & \frac{\gamma_p}{\gamma_s} > \frac{\gamma_\mu(\xi)K}{\lambda N_0B} \end{cases} \quad (9.10c)$$

where

$$\gamma_\mu(\xi) = \alpha + \bar{\alpha} \frac{f_0(\xi)}{f_1(\xi)}. \quad (9.11)$$

The optimal power allocation obtained by Eq. (9.10a) represents the greater transmission power, which can be used when γ_s increases and γ_p decreases and the average interference constraint at the primary receiver is satisfied. This is due to the primary user's fading channel advantage which enhances the cognitive user's capacity. The sensing decision is considered in Eq. (9.11), where we observe that when the conditional probability that the PU is idle ($f_0(\xi)$) gets higher than that of being active ($f_1(\xi)$), then the value of $\gamma_\mu(\xi)$ has an ascending behavior and $\gamma_\mu(\xi) > 1$, otherwise, $\gamma_\mu(\xi) < 1$. Therefore, as the conditional probability distribution of the primary user being idle gets higher than being active, $\gamma_\mu(\xi)$ increases and, consequently, we can increase the secondary user's transmission power without causing harmful interference to the PR. Note that when $\gamma_\mu(\xi) = 1$, the ST has no information about the primary user's activity. Accordingly, it considers that

the primary user is always active ($\frac{f_0(\xi)}{f_1(\xi)} = 1$) and continuously transmits with the same power level with which it is already transmitting. For $\gamma_\mu(\xi)$, the values of $f_0(\xi)$ and $f_1(\xi)$ should be taken at that value of ξ which is computed by the sensing detector for a given detection and false alarm probabilities. A higher value of ξ as compared to threshold that is the energy computed in a particular time interval over a spectrum, indicates the presence of PU signal, and vice versa [13]. However, if we modify the probability of false alarm, the value of ξ is also modified. By substituting Eq. (9.10a) in Eq. (9.7), we get:

$$\iint_0^{K\gamma_\mu(\xi)} \left(\frac{\gamma_\mu(\xi)}{\lambda_0} - \frac{N_0 B \gamma_p}{\gamma_s K} \right) f_s(\gamma_s) f_p(\gamma_p) f_1(\xi) d\gamma_s d\gamma_p = Q_{\text{int}}$$

where λ_0 is determined in such a way that the average interference power constraint in Eq. (9.7) is equal to Q_{int}

$$\iint_0^{K\gamma_\mu(\xi)} \left(\frac{\gamma_\mu(\xi)}{\lambda_0 N_0 B} - \frac{\gamma_p}{\gamma_s K} \right) f_s(\gamma_s) f_p(\gamma_p) f_1(\xi) d\gamma_s d\gamma_p = \frac{Q_{\text{int}}}{N_0 B} = \Phi$$

or

$$\iint_0^{K\gamma_\mu(\xi)\gamma_0} \left(\gamma_\mu(\xi)\gamma_0 - \frac{\gamma_p}{\gamma_s K} \right) f_s(\gamma_s) f_p(\gamma_p) f_1(\xi) d\gamma_s d\gamma_p = \Phi \quad (9.12)$$

where $\gamma_0 = \frac{1}{\lambda_0 N_0 B}$, and $\Phi = \frac{Q_{\text{int}}}{N_0 B}$ is the average SNR [4]. By substituting Eq. (9.10a) in Eq. (9.6), gives the following Ergodic channel capacity expression:

$$\frac{C_{\text{er}}}{B} = \int_{\gamma_p}^{\infty} \frac{N_0 B \lambda_0}{K \gamma_\mu(\xi)} = \frac{1}{K \gamma_0 \gamma_\mu(\xi)} \log_2 \left(1 + \frac{K \gamma_s}{N_0 B} \left[\frac{\gamma_\mu(\xi)}{\lambda_0 \gamma_p} - \frac{N_0 B}{\gamma_s K} \right] \right) f_s(\gamma_s) f_p(\gamma_p) (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) d\gamma_s d\gamma_p$$

or

$$\frac{C_{\text{er}}}{B} = \int_{\gamma_p}^{\infty} \frac{N_0 B \lambda_0}{K \gamma_\mu(\xi)} \geq \frac{1}{K \gamma_0 \gamma_\mu(\xi)} \log_2 \left(\frac{K \gamma_s \gamma_\mu(\xi)}{N_0 B \lambda_0 \gamma_p} \right) f_s(\gamma_s) f_p(\gamma_p) (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) d\gamma_s d\gamma_p$$

or

$$\frac{C_{\text{er}}}{B} = \int_{\gamma_p}^{\infty} \frac{N_0 B \lambda_0}{K \gamma_\mu(\xi)} \geq \frac{1}{K \gamma_0 \gamma_\mu(\xi)} \log_2 \left(\frac{K \gamma_s \gamma_\mu(\xi) \gamma_0}{\gamma_p} \right) f_s(\gamma_s) f_p(\gamma_p) (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) d\gamma_s d\gamma_p \quad (9.13)$$

or

$$\frac{C_{er}}{B} = \frac{E_{\gamma_s, \gamma_p, \xi}}{\gamma_s} \geq \frac{N_0 B \lambda_0}{K \gamma_\mu(\xi)} \left[\log_2 \left(\frac{K \gamma_u(\xi) \gamma_s}{\lambda_0 N_0 B \gamma_p} \right) \right] \quad (9.14)$$

where C_{er} denotes the Ergodic capacity and $E[\cdot]$ denotes the expectation operator. Equation (9.14) is similar to that presented in [21, Eq. (30)] except the term γ_p , which is due to the consideration of the primary channel gain in the cognitive user's system capacity. However, when only the power adaptation policy is considered instead of power and rate adaptation policy, then the additional constraint of Eq. (9.5) is not needed, and the Ergodic channel capacity of adaptive power transmission policy is given by the following mathematical expression, substituting $K = 1$ in Eq. (9.14):

$$\frac{C_{er}}{B} = \frac{E_{\gamma_s, \gamma_p, \xi}}{\gamma_s} \geq \frac{N_0 B \lambda_0}{\gamma_\mu(\xi)} \left[\log_2 \left(\frac{\gamma_u(\xi) \gamma_s}{\lambda_0 N_0 B \gamma_p} \right) \right] \quad (9.15)$$

Comparing the Ergodic capacity of power adaptation policy as given by Eq. (9.15) and rate and power adaptation policy for M -QAM modulation format in Eq. (9.14), Eq. (9.14) reveals that there is an effective power loss of K for adaptive M -QAM compared to that of Eq. (9.15). However, for the adaptive power transmission policy, the probability of error is significantly greater and is fixed, at 0.0446, in comparison to that of the adaptive rate and power transmission policy, where the probability of bit error can vary according to the quality-of-service requirement.

9.4 Effect of Channel Conditions

In this section, we explore the fading channel effect on the cognitive radio communication system performance and numerically compute the Ergodic channel capacity in different fading environments.

- Nakagami- m fading

The Nakagami- m distribution often provides the best fit to the urban [33] and indoor [34] multipath propagation and gives AWGN, Rayleigh and Rician fading channel models by adjusting the fading parameter m , which is the ratio of line-of-sight (LOS) signal power to the multipath signal power. The channel fading model based on Nakagami distribution, both γ_s and γ_p , would be distributed according to the following Gamma distribution [6]:

$$f(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m)} e^{-m\gamma}$$

where m and γ are shape parameter and channel power gain, respectively. Therefore, the pdf $f_s(\gamma_s)f_p(\gamma_p)$ is given as:

$$f_s(\gamma_s)f_p(\gamma_p) = \binom{m_0}{m_1} \frac{z^{m_1-1}}{\beta(m_0, m_1) \left(x + \frac{m_0}{m_1}\right)^{m_0+m_1}} \quad (9.16)$$

where m_0 and m_1 are m parameters [6] for γ_p and γ_s , respectively. $\frac{\gamma_p}{\gamma_s} = z$, and z is a random variable. $\beta(\cdot)$ is the beta function. When $m_0 = m_1 = m$, the Eq. (9.16) becomes:

$$f_s(\gamma_s)f_p(\gamma_p) = \frac{z^{m-1}}{\beta(m, m)(z+1)^{2m}} \quad (9.17)$$

By substituting Eq. (9.17) in (9.12), we yield the following value of secondary transmit power, which satisfies the average interference constraint for the Nakagami- m fading channel:

$$\iint_0^{K\gamma_\mu(\xi)\gamma_0} \left(\gamma_\mu(\xi)\gamma_0 - \frac{\gamma_p}{\gamma_s K} \right) \frac{z^{m-1}}{\beta(m, m)(z+1)^{2m}} f_1(\xi) d\gamma_s d\gamma_p = \frac{Q_{\text{int}}}{N_0 B} \quad (9.18)$$

and the Ergodic channel capacity from Eq. (9.13), for the Nakagami- m fading environment is given by:

$$\frac{C_{\text{er}}}{B} = \int_{\gamma_p}^{\infty} \frac{N_0 B \lambda_0}{K \gamma_\mu(\xi)} = \frac{1}{K \gamma_0 \gamma_\mu(\xi)} \log_2 \left(\frac{K \gamma_s \gamma_\mu(\xi) \gamma_0}{\gamma_p} \right) \frac{z^{m-1}}{\beta(m, m)(z+1)^{2m}} (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) d\gamma_s d\gamma_p \quad (9.19)$$

9.4.1 Rayleigh Fading

The Nakagami- m distribution with fading parameter equal to 1 represents the Rayleigh fading channel, and the pdf $f_s(\gamma_s)f_p(\gamma_p)$ will have log-logistic distribution [6]. By substituting $m = 1$ in Eq. (9.18), we get:

$$\int_0^{K\gamma_\mu(\xi)\gamma_0} \left(\gamma_\mu(\xi)\gamma_0 - \frac{z}{K} \right) \frac{1}{(1+z)^2} f_1(\xi) dz = \frac{Q_{\text{int}}}{N_0 B}$$

or

$$f_1(\xi) \left(-\frac{1}{K} \log_2(1 + K\gamma_\mu(\xi)\gamma_0) + \gamma_\mu(\xi)\gamma_0 \right) = \frac{Q_{\text{int}}}{N_0 B} = \Phi \quad (9.20)$$

Therefore the capacity of the cognitive radio communication system in the Rayleigh fading environment is achieved by putting $m = 1$ in Eq. (9.19):

$$\frac{C_{\text{er}}}{B} = \int_{\frac{1}{\gamma_0 \gamma_\mu(\xi)}}^{\infty} \log_2(K\gamma_0\gamma_\mu(\xi)z) \frac{1}{(1+z)^2} (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) dz \quad (9.21)$$

or $\frac{C_{\text{er}}}{B} = (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) \log_2(1 + K\gamma_\mu(\xi)\gamma_0(\Phi))$

where $\gamma_0(\Phi)$ is from the Eq. (9.20) for a given Φ . Equation (9.21) gives the Ergodic channel capacity of adaptive rate and power transmission policy under the Rayleigh fading environment. Further, the capacity of adaptive power transmission policy under the Rayleigh fading environment is as given below:

$$\frac{C_{\text{er}}}{B} = (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) \log_2(1 + \gamma_\mu(\xi)\gamma_0(\alpha)) \quad (9.22)$$

9.4.2 Rician Fading

The Nakagami- m distribution with the fading parameter greater than or equal to 2 represents the Rician fading channel. Now, by substituting $m = 2$ in Eq. (9.18), we get the following expression for the Rician fading channel:

$$\int_0^{K\gamma_\mu(\xi)\gamma_0} \left(\gamma_\mu(\xi)\gamma_0 - \frac{z}{K} \right) \frac{6z}{(1+z)^4} f_1(\xi) dz = \frac{Q_{\text{int}}}{N_0 B}$$

or

$$f_1(\xi) \left(\frac{3K\gamma_0\gamma_\mu(\xi) + 2}{6K(1 + K\gamma_0\gamma_\mu(\xi))^2} + \frac{\gamma_0\gamma_\mu(\xi)}{6} - \frac{2}{6K} \right) = \frac{Q_{\text{int}}}{N_0 B} = \Phi \quad (9.23)$$

Therefore, for the spectrum sharing system operating under the predefined power constraints and a target BER value P_b , the Rician fading channel capacity expression of the secondary user's link, based on the adaptive rate and power M-QAM transmission policy, is obtained by putting $m = 2$ in Eq. (9.19):

$$\frac{C_{er}}{B} = \int_{\frac{1}{K\gamma_0(\Phi)}}^{\infty} \log_2(K\gamma_0(\Phi)\gamma_\mu(\xi)z) \frac{6z}{(1+z)^4} (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) dz \quad (9.24)$$

where $\gamma_0(\Phi)$ is from Eq. (9.23) for a given Φ . Furthermore, the Ergodic channel capacity of adaptive power transmission policy in the Rician fading environment is given by the following expression:

$$\frac{C_{er}}{B} = \int_{\frac{1}{\gamma_0\gamma_\mu(\xi)}}^{\infty} \log_2(\gamma_0\gamma_\mu(\xi)z) \frac{6z}{(1+z)^4} (\alpha f_1(\xi) + \bar{\alpha} f_0(\xi)) dz \quad (9.25)$$

Similarly, we can compute the channel capacity for different fading parameter values, however it leads to cumbersome mathematical expressions.

9.5 Simulation Results

In this section, we numerically simulate the proposed spectrum sharing system model that operates under the constraints on the average received-interference power in the Nakagami- m fading environment for adaptation strategies such as variable power and variable rate and power, as presented in the preceding Sects. 9.3 and 9.4.

The position of terminals as shown in Fig. 9.1 is assumed in such a way that $d_s = d_p = 1$ (unit) and $d_m = 3$ (unit). The channel gains $(\gamma_s)^{1/2}$ and $(\gamma_p)^{1/2}$ are distributed according to the Nakagami- m fading pdf. Furthermore, we assumed $N_0B = 1$ and the sensing detector computes the sensing-information metric for $N = 30$ observation samples. We suppose that the primary user remains active at 50% of the time ($\alpha = 0.5$) and have set the PU's transmit power $P_t = 1$. Figure 9.2a illustrates the distribution of conditional probabilities $f_0(\xi)$ and $f_1(\xi)$ corresponding to the different values of energy detected by sensing detector in the particular number of samples. Moreover, these distributions are used for the computation of $\gamma_\mu(\xi)$ for different detected energy values in a particular interval as shown in Fig. 9.2b. Three regions have been recognized for the parameter $\gamma_\mu(\xi)$, namely, $\gamma_\mu(\xi) > 1$, $\gamma_\mu(\xi) = 1$ and $\gamma_\mu(\xi) < 1$. In Fig. 9.2b, when $\gamma_\mu(\xi) > 1$ represent that the probability of the PU to be idle is higher than that of being active otherwise, $\gamma_\mu(\xi) < 1$. The power and rate are adapted according to the channel gains and the sensing information. Moreover, the higher power levels are used by secondary users when the probability of the primary user being inactive is significantly more (higher values of $\gamma_\mu(\xi)$) in comparison to the case for which $\gamma_\mu(\xi)$ is less. We have considered the bit-error-probability 10^{-2} , 10^{-4} and 10^{-6} for the adaptive rate and power transmission policy for these two cases: ($\gamma_\mu(\xi) > 1$ and $\gamma_\mu(\xi) < 1$).

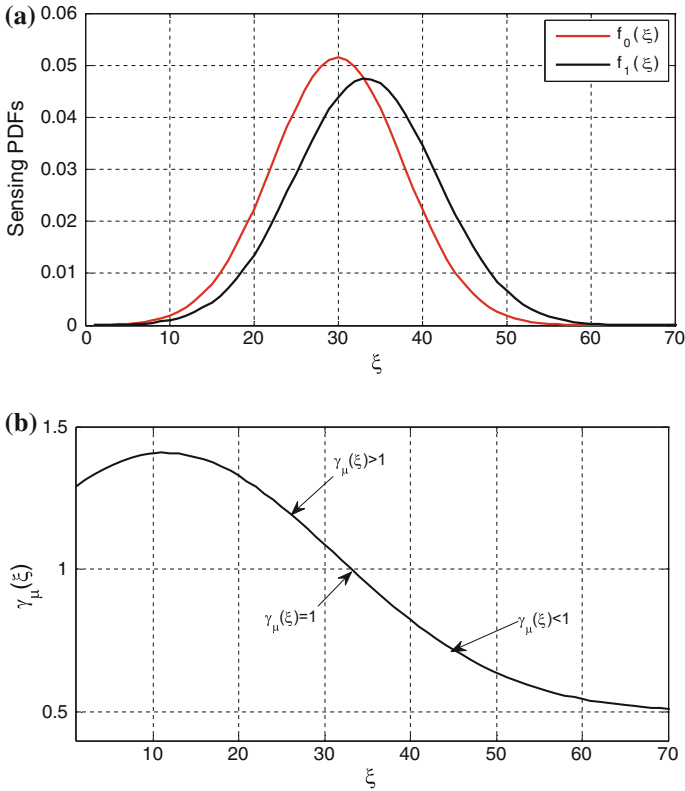


Fig. 9.2 The soft sensing information **a** Spectrum sensing probability density functions given that the primary user is idle $f_0(\xi)$ and active $f_1(\xi)$ [21], and **b** $\gamma_\mu(\xi)$ variation for $N = 30$, $P_t = 1$, $\alpha = 0.5$ and $d_m = 3$ [21]

For the Rayleigh fading environment or Nakagami- m distribution with $m = 1$, Fig. 9.3a, b shows the variation of the Lagrangian parameter λ and Ergodic channel capacity with Q_{int} for the adaptive power and adaptive rate and power transmission policy, while considering the sensing information metric available at the cognitive user. The simulation results in Fig. 9.3 are presented for the value of parameter $\gamma_\mu(\xi) < 1$. Moreover, Fig. 9.3a shows the optimum value of the Lagrangian parameter for the given Q_{int} and $\gamma_\mu(\xi)$, which satisfy (9.20) and provide the adaptation in transmit power needed for the Rayleigh fading channel. It is clear from Fig. 9.3b that as the interference tolerance (Q_{int}) at the primary receiver increases, the capacity of the secondary user increases due to the increase in transmit power of the secondary user. The Ergodic capacity of adaptive rate and power transmission policy is less in comparison to that of the adaptive power transmission policy, since there is an additional constraint on target BER in the former policy. In addition, as the required BER decreases, the Ergodic capacity of

the system is less, as depicted from Fig. 9.3b. For example, the capacity for P_b of 10^{-6} is less than that for $P_b = 10^{-2}$ due to the stricter constraint on the required error rate.

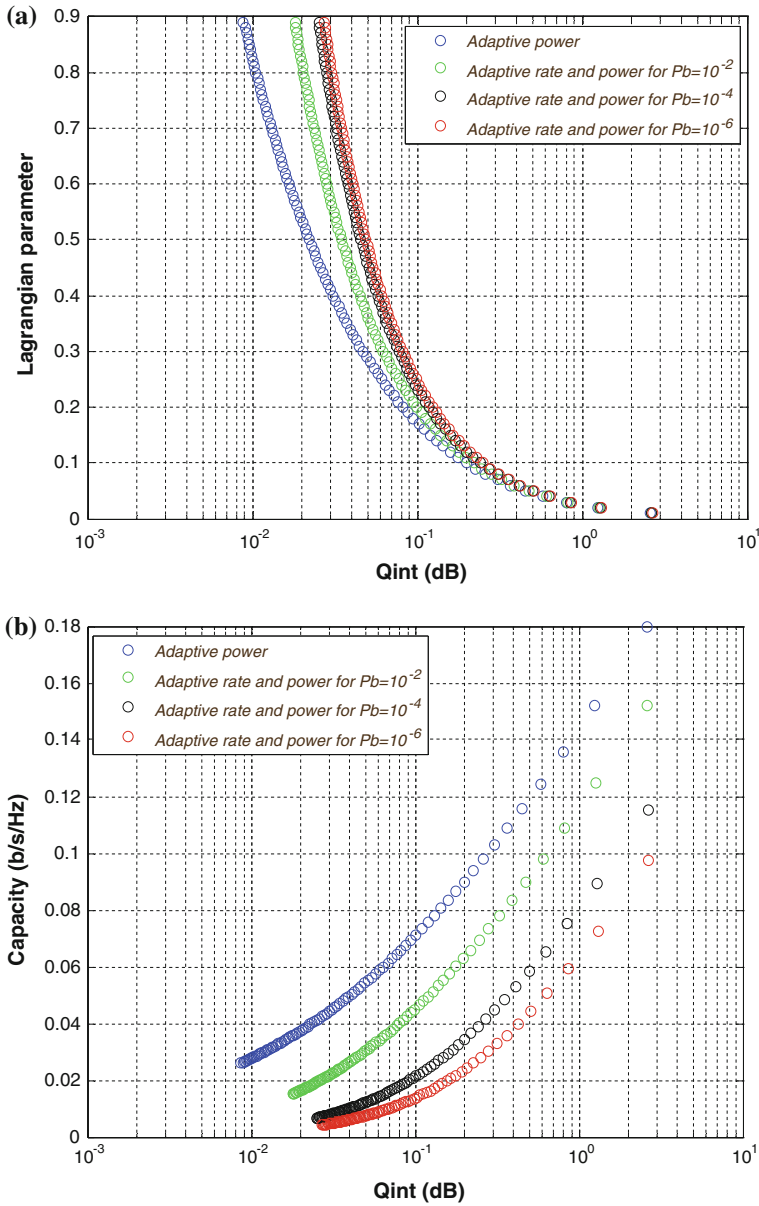


Fig. 9.3 The response of primary receiver interference power constraint for the adaptive power and adaptive rate and power transmission policies in the Rayleigh fading channel for M -QAM modulation and $\gamma_\mu(\xi) = 0.8$ over **a** the Lagrangian parameter, and **b** Ergodic channel capacity

In Fig. 9.4a, b, we have considered the value of the parameter $\gamma_\mu(\xi) > 1$, which shows that the probability of the primary user being active is greater than that of it being inactive so it leads to an increase in the transmit power; consequently the

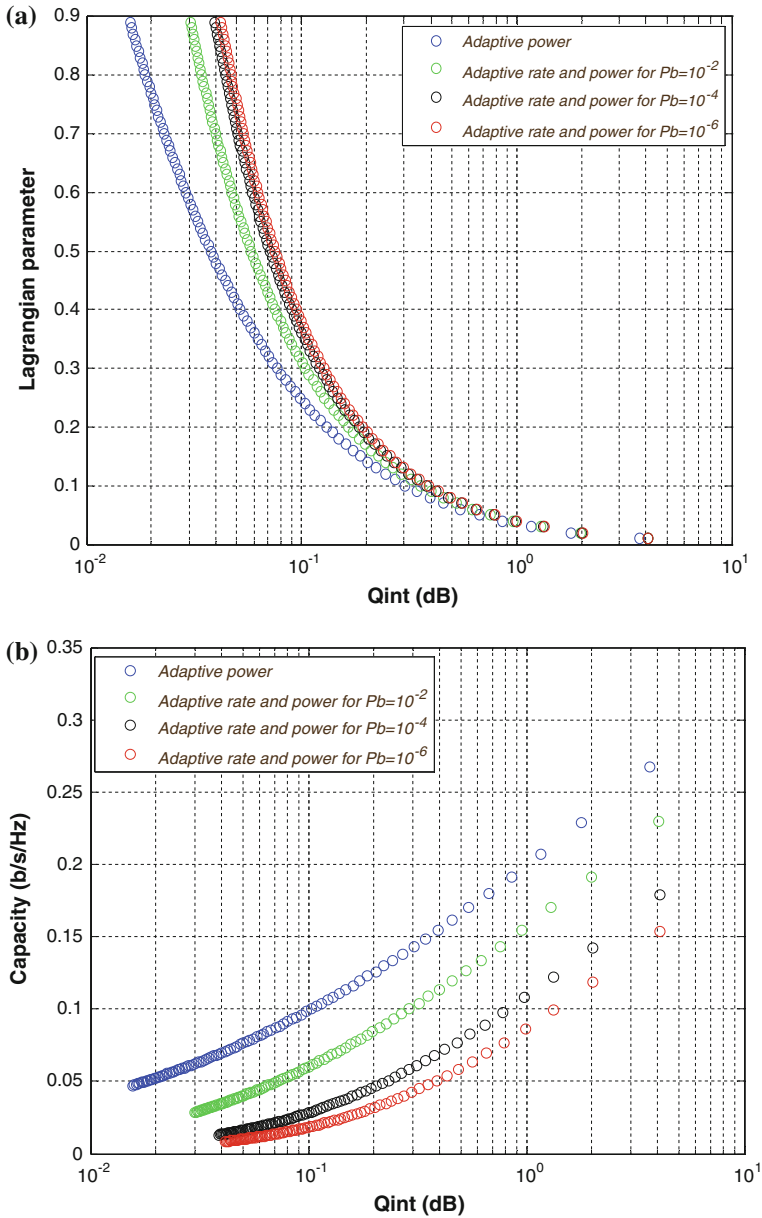


Fig. 9.4 The response of primary receiver interference power constraint for the adaptive power and adaptive rate and power transmission policies in the Rayleigh fading channel for M -QAM modulation and $\gamma_\mu(\xi) = 1.2$ over **a** the Lagrangian parameter, and **b** Ergodic channel capacity

result is an increase in capacity of the secondary user in comparison to the capacity that is shown in Fig. 9.3b, where $\gamma_\mu(\xi) < 1$. Further, without considering the sensing information available at the secondary user, the capacity variations with Q_{int} presented in Fig. 9.5 have been validated with Fig. 3 of [6], which is the case when only the average interference power constraint is considered. The effect of average interference power constraint Q_{int} on the capacity and Lagrangian parameter λ in the Nakagami- m fading environment with $m = 2$, that is, for the Rician fading channel for the adaptive power and adaptive rate and power transmission, is shown in Fig. 9.6a, b for the case when $\gamma_\mu(\xi) < 1$. Moreover, for the adaptive power and adaptive rate and power transmission policy, the comparison of the capacity for three cases of BER that is 10^{-2} , 10^{-4} and 10^{-6} is presented in Fig. 9.6b.

Figure 9.7a, b present the Lagrangian parameter and capacity in the Rician fading environment (Nakagami- m distribution with $m = 2$) for $\gamma_\mu(\xi) > 1$. The comparison of Fig. 9.6b with 9.7b reveals that the significant enhancement in the capacity is due to the higher power adaptation of the secondary transmitter. Moreover, the capacity comparison between Rayleigh and Rician fading environments demonstrates that the capacity of the cognitive radio network for the latter case is less than that of the former for a given Q_{int} . The reason lies in the fact that

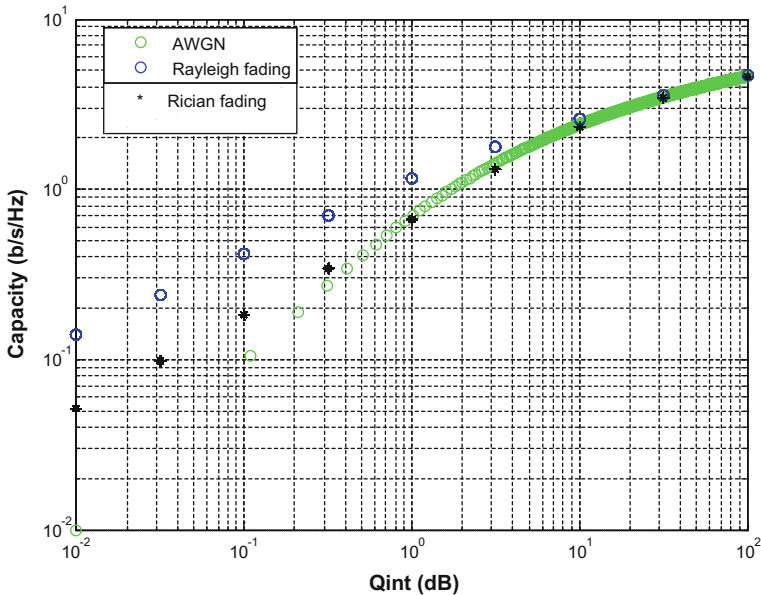


Fig. 9.5 The capacity under the average interference-power constraint as reported in [6]

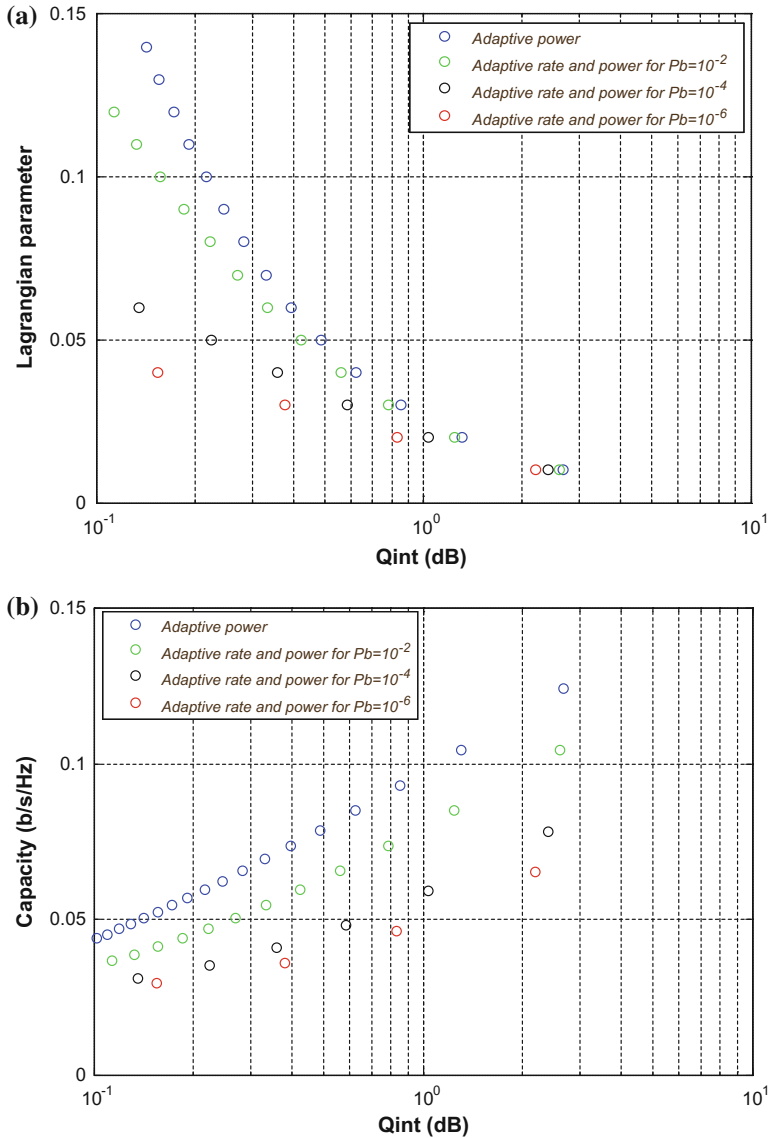


Fig. 9.6 The response of primary receiver interference power constraint for the adaptive power and adaptive rate and power transmission policies in the Rician fading channel for M -QAM modulation and $\gamma_{\mu}(\xi) = 0.8$ over **a** the Lagrangian parameter, and **b** Ergodic channel capacity

severe primary channel Rayleigh fading gives an advantage to the secondary transmitter to increase its transmission power while keeping the interference power constraint constant in comparison to the Rician fading channel with $m = 2$, which is less severe due to the presence of a line-of-sight (LOS) component. Moreover,

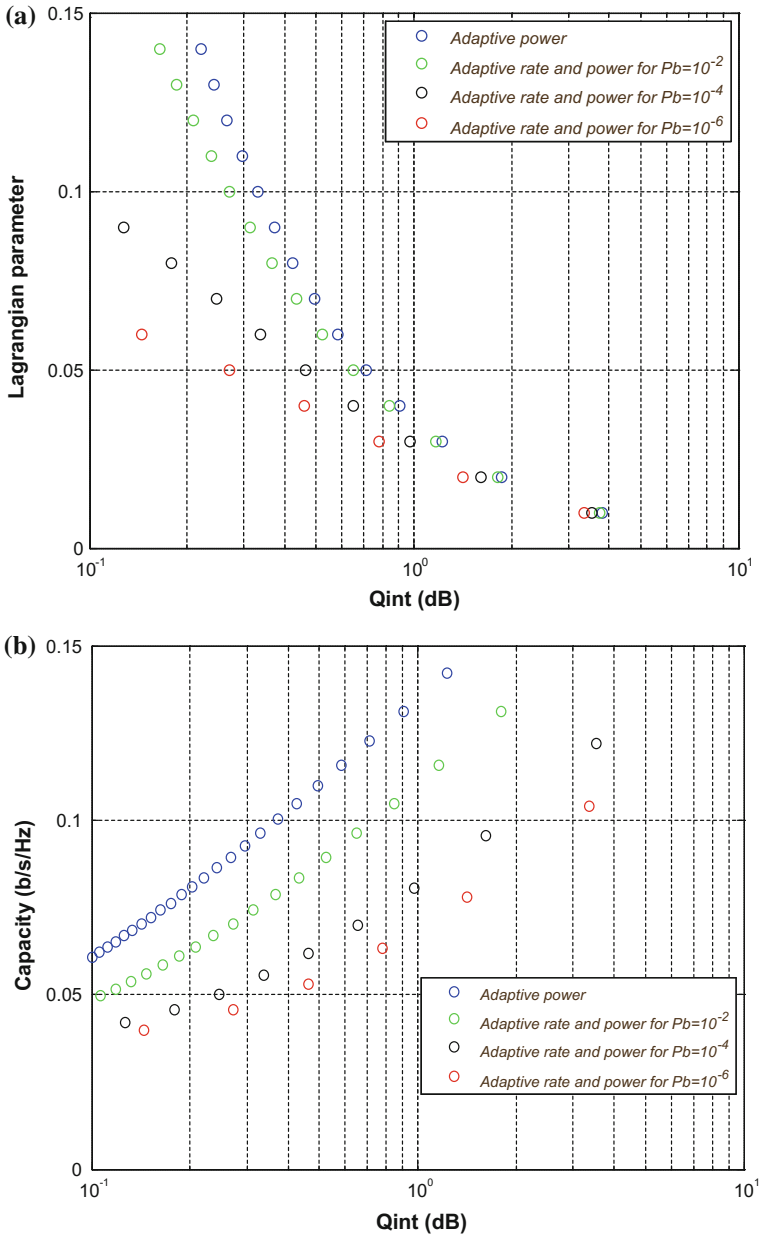


Fig. 9.7 The response of primary receiver interference power constraint for the adaptive power and adaptive rate and power transmission policies in the Rician fading channel for M -QAM modulation and $\gamma_\mu(\xi) = 1.2$ over **a** the Lagrangian parameter, and **b** Ergodic channel capacity

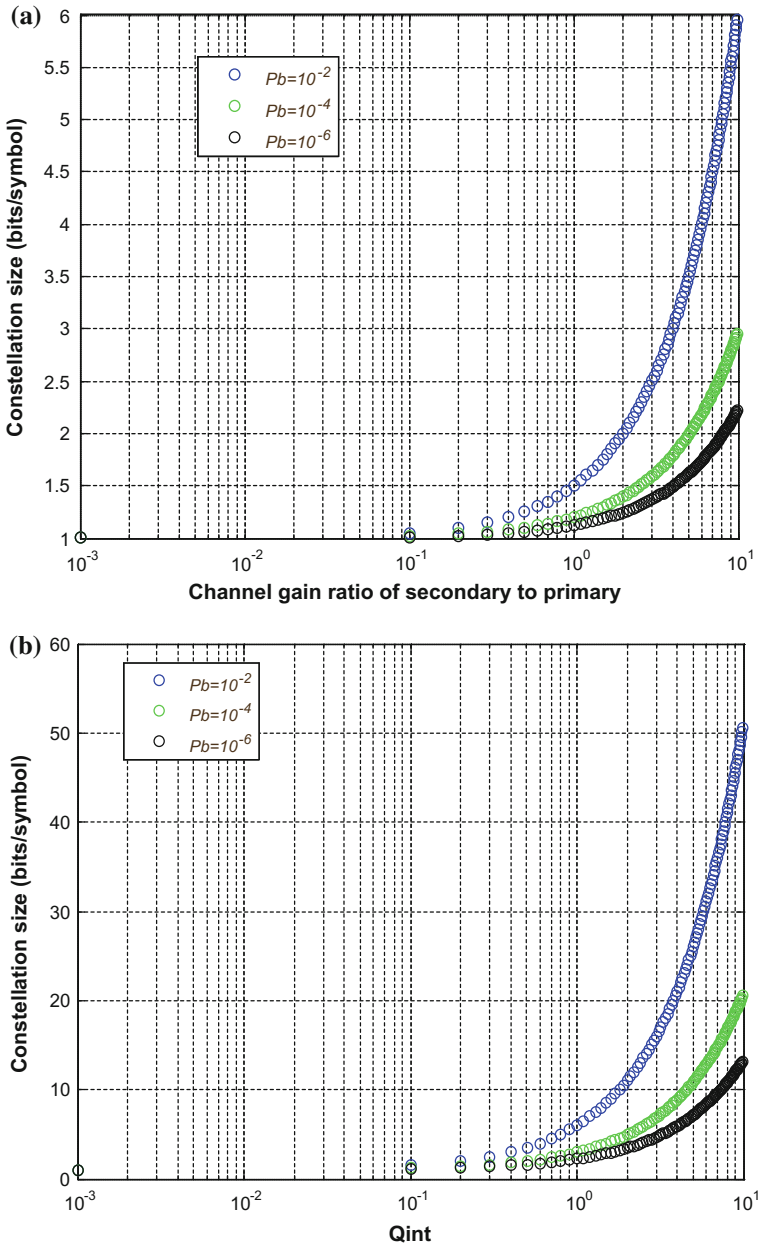


Fig. 9.8 The constellation size adaptation **a** according to the signal-to-noise power ratios of secondary-to-primary user for $Q_{int} = 0\text{dB}$, and **b** with the interference power constraint, for the given signal-to-noise power ratios (10 dB) of secondary-to-primary user for adaptive power and rate transmission policy with $P_b = 10^{-2}$, 10^{-4} , and 10^{-6}

Fig. 9.8a, b shows the adaptation in the constellation size according to the channel gain ratio of the secondary-to-primary user and average interference power for different BER, respectively. It is also clear from Fig. 9.8a, b that the number of bits per symbol or the constellation size of the modulation technique increases as the channel gain ratio of the ST to PR increases, or the average interference power limit at PR increases for the chosen BER. Thus significantly better channel conditions of the secondary link lead to the adaptation of higher modulation format.

9.6 Summary

In this chapter, we have considered a spectrum sharing concept for the cognitive radio system where the secondary user's transmit power and rate can be adjusted based on the sensing information of the primary user and secondary user, as well as secondary-to-primary user's fading environment. In addition, the spectrum sharing system operates under the average interference power constraints of the PR. In this context, we have demonstrated the Ergodic capacity of the cognitive radio communication system with power and rate adaptation policy in different fading environments for a chosen BER. Since the Nakagami- m distribution is fit for both the Rayleigh and Rician fading distributions by varying the fading parameter, the Ergodic capacity for both these distributions were presented. The numerically simulated results for the Ergodic capacity were presented for both the adaptive power and adaptive rate and power transmission policies, which revealed that the adaptive power transmission has more capacity than that of the adaptive rate and power transmission policy at the cost of BER. Moreover, we have demonstrated that knowledge of the sensing parameter provides an opportunity to control the secondary user's transmission parameters, such as rate and power, according to different primary users activity levels observed by the sensing detector. However, the secondary transmitter can adapt different modulation by varying the value of M in M -QAM according to the channel conditions, BER and interference constraints. Further, it was illustrated that the capacity, in the case of Rician fading environment, is lower than that of Rayleigh fading because a LOS component present in the ST to PR has provided a more prominent effect on the capacity of the secondary user in comparison to that present in the ST-to-SR link.

References

1. E. Biglieri, J. Proakis, S. Shamai, Fading channels: Information theoretic and communications aspects. *IEEE Trans. Inf. Theory* **44**(6), 2619–2692 (1998)
2. A.-J. Goldsmith, P. Varaiya, Capacity of fading channels with channel side information. *IEEE Trans. Inf. Theory* **43**(6), 1986–1992 (1997)

3. M.-A. Khojastepour, B. Aazhang, The capacity of average and peak power constrained fading channels with channel side information, *Proceedings of IEEE Wireless Communication and Networking Conference (WCNC'04)* (Atlanta, CA, USA, Mar. 2004), pp. 77–82
4. A. Ghasemi, E.S. Sousa, Capacity of fading channels under spectrum-sharing constraints, *Proceedings of IEEE International Conference on Communication (ICC'06)* (Istanbul, Turkey, June 2006), pp. 4373–4378
5. L. Musavian, S. Aissa, Ergodic and outage capacities of spectrum sharing systems in fading channels, *Proceedings of IEEE Global Telecommunication Conference (GLOBECOM'07)* (Washington, DC, USA, Nov. 2007), pp. 3327–3331
6. A. Ghasemand, E.-S. Sousa, Fundamental limits of spectrum-sharing in fading environments. *IEEE Trans. Wirel. Commun.* **6**(2), 649–658 (2007)
7. B. Wang, K.-J.-R. Liu, Advances in cognitive radio networks: a survey. *IEEE J. Sel. Top. Sign. Proces.* **5**(1), 5–23 (2011)
8. V. Asghari, S. Aissa, Resource sharing in cognitive radio systems: outage capacity and power allocation under soft sensing. *Proceedings of IEEE Global Telecommunication Conference (GLOBECOM'08)* (New Orleans, LA, USA, Nov. 2008), pp. 1–5
9. I.-F. Akyildiz, W.-Y. Lee, M.-C. Vuran, S. Mohanty, NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey. *Comput. Netw.* **50**(13), 2127–2159 (2006)
10. S. Pandit, G. Singh, Throughput maximization with reduced data loss rate in cognitive radio network. *Telecommun. Syst.* **57**(2), 209–215 (2014)
11. T. Yucek, H. Arslan, A survey of spectrum sensing algorithms for cognitive radio applications. *IEEE Commun. Surv. Tutorials* **11**(1), 116–130 (2009)
12. H. Urkowitz, Energy detection of unknown deterministic signals. *Proc. IEEE* **55**(4), 523–531 (1967)
13. S. Srinivasa, S.A. Jafar, Soft sensing and optimal power control for cognitive radio. *IEEE Trans. Wirel. Commun.* **9**(12), 3638–3649 (2010)
14. Z. Rezk, M.-S. Alouini, Ergodic capacity of cognitive radio under imperfect channel state information. *IEEE Trans. Veh. Technol.* **61**(5), 2108–2119 (2012)
15. S. Pandit, Medium access control protocol for spectrum sharing in cognitive radio communication system. *Ph D Thesis, Jaypee University of Information Technology, Wanknaghat, Solan-173234, India*
16. S. Parsaeefard, A.-R. Sharafat, Robust distributed power control in cognitive radio networks. *IEEE Trans. Mob. Comput.* **12**(4), 609–620 (2013)
17. G. Zhao, C. Yang, G.-Y. Li, D. Li, A.-C.-K. Soong, Power and channel allocation for cooperative relay in cognitive radio networks, *IEEE J. Sel. Top. Sign. Process.* **5**(1), 151–159 (2011)
18. X. Kang, H.-K. Garg, Y.-C. Liang, R. Zhiang, Optimum power allocation for OFDM-based cognitive radio with new primary transmission protection criteria. *IEEE Trans. Wirel. Commun.* **9**(6), 2066–2075 (2010)
19. Q. Li, K.-H. Li, K.-C. Teh, Diversity-multiplexing tradeoff of wireless communication systems with user cooperation. *IEEE Trans. Inf. Theory* **57**(9), 5794–5819 (2011)
20. A. Sendonaris, E. Erkip, B. Aazhang, User cooperation diversity—part I: system description. *IEEE Trans. Commun.* **51**(11), 1927–1938 (2003)
21. V. Asghari, S. Aissa, Adaptive rate and power transmission in spectrum-sharing systems. *IEEE Trans. Wirel. Commun.* **9**(10), 3272–3280 (2010)
22. A.-J. Goldsmith, S.-G. Chua, Variable-rate variable-power MQAM for fading channels. *IEEE Trans. Commun.* **45**(10), 1218–1230 (1997)
23. M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions: with Formulas, Graphs, and Mathematical Tables*, 9th edn. (Dover, New York, 1972)
24. S.-S. Tzeng, C.-W. Huang, Effective throughput maximization for in-band sensing and transmission in cognitive radio networks. *Wireless Netw.* **17**(4), 1025–1029 (2011)
25. S. Haykin, Cognitive radio: brain-empowered wireless communications. *IEEE J. Sel. Areas Commun.* **23**(2), 201–220 (2005)

26. P. Potier, C. Sorrells, Y. Wang, L. Qian, H. Li, Spectrum inpainting: a new framework for spectrum status determination in large cognitive radio networks. *Wireless Netw.* **20**(3), 423–439 (2014)
27. J.-K. Cavers, Variable rate transmission for Rayleigh fading channels. *IEEE Trans. Commun.* **20**(1), 15–22 (1972)
28. W.-T. Webb, R. Steele, Variable rate QAM for mobile radio. *IEEE Trans. Commun.* **43**(7), 2223–2230 (1995)
29. M.-S. Alouini, A.-J. Goldsmith, Adaptive modulation over Nakagami fading channels. *Wireless Pers. Commun.* **13**(1–2), 119–143 (2000)
30. X. Liu, S. Shankar, Sensing-based opportunistic channel access. *Mob. Netw. Appl.* **11**(4), 577–591 (2006)
31. S. Boyd, L. Vandenberghe, *Convex Optimization* (Cambridge University Press, New York, 2004)
32. D.P. Bertsekas, *Nonlinear Programming* (Athena Scientific, Belmont, MA, 1995)
33. H. Suzuki, A statistical model for urban multipath propagation. *IEEE Trans. Commun.* **25**(7), 673–680 (1977)
34. A.-U. Sheikh, M. Abdi, M. Handforth, Indoor mobile radio channel at 946 MHz: measurements and modeling. *Proceedings of IEEE Vehicular Technology Conference (VTC '93)* (Secaucus, NJ, May 1993), pp. 73–76