Chapter 9
Adaptive Fault Detection for Uncertain Time-Delay Systems

9.1 Introduction

Time delay phenomenon often exists in the practical applications because of information transmission. It has been proven that such time delay will causes the performance degradation of the controlled systems, even instability. Hence, the research of such class of time delayed systems has become a hot issue on [1–6]. Design of observer including fault detection observer is an important and challenging problem. The main difficulty lies in handling the time delay [7]. For example, consider a simple system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu \\ y(t) = Cx(t) \end{cases} \tag{9.1}$$

where $x,\ y$ and u denote state, output and control input, respectively; $A,\ A_d,\ B$ and C are known real matrices; d is a constant. In most of the existing results such as [8], its observer often is designed as:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-d) + Bu + L(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{9.2}$$

where L is observer gain matrix. Notice that, the first equation in (9.2) contains time delay term $\hat{x}(t-d)$. Obviously, if d is unknown, then observer (9.2) is not reasonable and does not work in practical applications. Hence, how to avoid the above shortcoming and design a proper observer for dynamical systems becomes important and practically useful, which is the first motivation of our work.

On the other hand, faults/failures inevitable occur in the system parts such as actuators and sensors, which will lead to the decreasing of the system performance. In order to compensate for these faults/failures, various fault-tolerant control (FTC) methods are proposed [9–68]. Among these FTC methods, active FTC methods is more common and important useful [43–53]. Fault detect (FD) is the first and

© Springer International Publishing AG 2017

Q. Shen et al., *Fault Diagnosis and Fault-Tolerant Control Based on Adaptive Control Approach*, Studies in Systems, Decision and Control 91,
DOI 10.1007/978-3-319-52530-3_9

important step in active FTC method [9]. In general, the so-called FD observer is designed to detect the faults occurred in the system. Recently, the FD problem of time delay systems has drawn wide attentions. For time delay systems, however, most of the FD observers proposed in literature are similar to (9.2), which also have the same shortcoming, i.e., the FD observers contain the unknown time delay terms. In [8], an asymptotic value of the norm of state estimation error vector is taken as a fault indicator. However, the asymptotic value cannot be accessed in practical applications. Therefore, how to design an efficient FD mechanism is another motivation of this work.

Uncertainty/nonlinearity is common in the controlled systems. In general, as [8], the uncertainty is assumed to be known and to satisfy the so-called Lipschitz condition. Indeed, under the condition, control design and system stability analysis are simplified largely. It should be pointed out that, however, this condition could not be always satisfied in practical applications. Hence, how to efficiently detect the fault occurred in nonlinear systems where the uncertainties do not satisfy Lipschitz condition is particularly valuable and helpful, which also motivate us for this work.

In this chapter, based on the above-mentioned works, the FD problem of time delay systems is considered, where neural networks (NNs) [59, 69, 70] are used to approximate the unknown smooth functions. Compared with the existing results, the contributions of our work are as follows:

(1) First, a novel adaptive neural networks-based fault detection observer is constructed for a class of uncertain time delay systems. In the observer design, by using a suitable adaptation mechanism, the real value of time delay can be estimated online, which means that the conditions (the time delay should be known) and shortcoming (the fault detection observer contains the unknown time delay) are removed.

(2) Next, different from [8] where the uncertainty was assumed to satisfy the Lipschitz condition, the condition is relaxed in our work, and it is just required that the norm of the uncertainty is less than the sum of unknown functions. Thus, the algorithm proposed in this chapter can be used in the widespread practical applications.

(3) Furthermore, a novel fault detection mechanism is proposed, which is more efficient for FD under practical conditions.

The rest of this chapter is organized as follows. Section 9.2 gives the problem formulation and the preliminaries of neural networks are presented. In Sect. 9.3, a novel adaptive NNs-based fault detection observer is proposed. In Sect. 9.4, simulations are presented. Finally, Sect. 9.5 draws the conclusions.

9.2 Problem Statement and Description of NNs

In this section, we will first formulate the fault detection problem. Then, the mathematical description of NNs is introduced.

9.2.1 Problem Statement

Consider the time-delayed system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + g(x(t), x(t-d); t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\bar{d}, 0] \end{cases} \tag{9.3}$$

where $x(t) \in R^n$ is state, $u(t) \in R^m$ is input and $y(t)$ denote output; A, A_d, B and C are known real matrices with appropriate dimensions; $d \in R$ is unknown and satisfies $0 < d \leq \bar{d}$, $\bar{d}$ is a known real constant;

$$g(\cdot) = [g_1(\cdot), g_2(\cdot); \cdots, g_n(\cdot)]^T \in R^n,$$

$g_i(\cdot) = g_i(x(t), x(t-d); t) \in R$, $i = 1, 2, \ldots, n$ are the uncertainties, which denote model uncertainty, external disturbance, time-varying parameter variation, and system nonlinearity; $\varphi(t)$ is an arbitrarily known continuous bounded function.

Throughout this chapter, (A, C) is assumed to be observable and only system output y is measurable.

In this chapter, the faulty system can be described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + \\ \qquad g(x(t), x(t-d); t) + f(x(t), u(t); t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \ t \in [-\bar{d}, 0] \end{cases} \tag{9.4}$$

where $f(\cdot) \in R^n$ denotes the unknown faults occurred in actuators or the other system components.

The aim of this chapter in this chapter is to design a suitable adaptive observer and more efficient fault detection mechanism for system (9.3) to detect the occurred faults.

For notational convenience, let us define the following notations: $g_i = g_i(\cdot)$ and $g = [g_1, g_2, \cdots, g_n]^T$. In addition, $\star(t)$ will be abbreviated as $\star$.

Assumption 9.1 There exist two unknown smooth functions $g_{i1}(x(t)) \geq 0 \in R$, $g_{i2}(x(t-d)) \geq 0 \in R$ and an unknown real constant $g_{i3} \geq 0$ satisfying

$$|g_i| \leq g_{i1}(x(t)) + g_{i2}(x(t-d)) + g_{i3}.$$

Assumption 9.2 Time delay d is bounded, namely, there exist two known real constants $\bar{d} > 0 \in R$ and $\underline{d} > 0 \in R$ such $\underline{d} < d \leq \bar{d}$.

Remark 9.1 In [8], the nonlinear function g_i was assumed to be known satisfying the Lipschitz condition. However, this condition could be not always satisfied in practical applications. In such case, the results in [8] would not work. In this chapter, the condition is replaced by Assumption 9.1. What's important, it is not necessary that g_{i1}, g_{i2} and g_{i3} are known, which relaxes largely the condition in [8]. Thus, the proposed method in this chapter can be used in the widespread practical applications.

9.2.2 Mathematical Description of NNs

NNs have been widely used in controlling of nonlinear systems due to their capabilities of nonlinear function approximation [69]. In this chapter, RBF NNs

$$h(Z,\theta) = \theta^T \xi(Z)$$

will be used to approximate a smooth function $h(Z)$, where the weight vector θ, the basis function vector $\xi(Z)$ are defined as follows:

$$\theta = (\theta_1, \theta_2, \ldots, \theta_N)^T,$$

$$\xi(Z) = (\xi_1(Z), \xi_2(Z), \ldots, \xi_N(Z))^T,$$

$$\theta_i(Z) = \exp(-(\sum_{j=1}^{p} (z_j - a_{ij})^2)/(\mu_i)^2),$$

$\mu_i > 0$ denotes the width of the receptive field, and a_{ij} denotes the center of the Gaussian function, z_j denotes the jth element of Z, p denotes the dimension of Z, N is the number of the NNs nodes.

In this chapter, for $i = 1, \ldots, n$, $g_{i1}(x(t))$ and $g_{i2}(x(t-d))$ are approximated by NNs as:

$$\hat{g}_{i1}(\hat{x}(t), \hat{\theta}_{i1}) = \hat{\theta}_{i1}^T \xi_{i1}(\hat{x}(t))$$

$$\hat{g}_{i2}(\hat{x}(t-\hat{d}), \hat{\theta}_{i2}) = \hat{\theta}_{i2}^T \xi_{i2}(\hat{x}(t-\hat{d}))$$

Optimal parameter vectors θ_{gi1}^* and θ_{gi2}^* are defined as

$$\theta_{i1}^* = \arg\min_{\theta_{i1}\in\Omega_{i1}} [\sup_{x\in U, \hat{x}\in\hat{U}} |g_{i1}(x(t)) - \hat{\theta}_{i1}^T \xi_{i1}(\hat{x}(t))|]$$

$$\theta_{i2}^* = \arg\min_{\theta_{i2}\in\Omega_{i2}} [\sup_{x\in U, \hat{x}\in\hat{U}} |g_{i2}(x(t-d)) - \hat{\theta}_{i2}^T \xi_{i2}(\hat{x}(t-\hat{d}))|]$$

where Ω_{i1}, Ω_{i2}, U and $\hat{U}$ are compact regions for $\hat{\theta}_{i1}$, $\hat{\theta}_{i2}$, x and $\hat{x}$, $\hat{d}$, $\hat{\theta}_{i1}$ and $\hat{\theta}_{i2}$ are the estimates of d, θ_{i1}^* and θ_{i2}^*, respectively.

The NNs minimum approximation errors are defined as

$$\varepsilon_{i1} = g_{i1}(x(t)) - \theta_{i1}^{*T}\xi_{i1}(\hat{x}(t)),$$

$$\varepsilon_{i2} = g_{i2}(x(t-d)) - \theta_{i2}^{*T}\xi_{i2}(\hat{x}(t-\hat{d})).$$

Now, the following assumptions are made throughout this chapter.

Assumption 9.3 $|\varepsilon_{i1}| \leq \varepsilon_{i1}^{*}$ and $|\varepsilon_{i2}| \leq \varepsilon_{i2}^{*}$, where $\varepsilon_{i1}^{*} > 0 \in R$ and $\varepsilon_{i2}^{*} > 0 \in R$ are unknown constants.

9.3 Fault Detection Observer Design

For (9.3), the FD observer is designed as:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-\hat{d}) + A_d\Delta_1 + Bu(t) + \\ \qquad L(\hat{y}(t) - y(t)) + sgn(e_y^T F)\hat{g} + \Delta_2 \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{x}(t) = 0, \ t \in [-\bar{d}, 0] \end{cases} \tag{9.5}$$

where $\hat{x}(t) \in R^n$ is observer state, $u(t) \in R^m$ is observer control input, and $\hat{y}(t)$ is observer output;

$$sgn(e_y^T F) = diag\{sgn(e_y^T F_1), \ldots, sgn(e_y^T F_n)\}$$

$$\hat{g} = \hat{g}_1 + \hat{g}_2 + \hat{g}_3$$

$$\hat{g}_1 = [\hat{g}_{11}, \ldots, \hat{g}_{n1}]^T$$

$$\hat{g}_2 = [\hat{g}_{12}, \ldots, \hat{g}_{n2}]^T$$

$$\hat{g}_3 = [\hat{g}_{13}, \ldots, \hat{g}_{n3}]^T$$

$\hat{g}_{i1}$ $(= \hat{g}_{i1}(x(t)))$, $\hat{g}_{i2}$ $(= \hat{g}_{i2}(x(t-\hat{d})))$ and $\hat{g}_{i3}$ are the estimates of unknown smooth functions g_{i1}, g_{i2} and unknown constant g_{i3}, respectively; g_{i1}, g_{i2} and g_{i3} are defined in Assumption 9.1, F_i $(i = 1, 2, \ldots, n)$ is the ith column of matrix F, which satisfies the following condition

$$(F^T C)^T = P \tag{9.6}$$

real matrix $P = P^T > 0$ will be defined later, $e_y = y - \hat{y}$, $\hat{d}$ is an estimate of d, Δ_1 and Δ_2 are robust terms to be defined later.

Denote

$$e_x(t) = x(t) - \hat{x}(t), \quad e_d = x(t-d) - \hat{x}(t-\hat{d})$$

Then, from (9.3) and (9.5), the observer error dynamics can be described as follows:

$$\begin{aligned}\dot{e}_x(t) &= (A - LC)e_x(t) + A_d x(t-d) - A_d\hat{x}(t-\hat{d}) + \\ &\quad \tilde{g} - A_d\Delta_1 - \Delta_2 \\ &= (A - LC)e_x(t) + A_d x(t-d) - A_d\hat{x}(t-\hat{d}) - \\ &\quad A_d\hat{x}(t-d) + A_d\hat{x}(t-d) + \tilde{g} - A_d\Delta_1 - \Delta_2 \\ &= (A - LC)e_x(t) + A_d e_x(t-d) + A_d\hat{x}(t-d) - \\ &\quad A_d\hat{x}(t-\hat{d}) + \tilde{g} - A_d\Delta_1 - \Delta_2\end{aligned} \tag{9.7}$$

where $\tilde{g} = [\tilde{g}_1, \ldots, \tilde{g}_n]^T$ and

$$\tilde{g}_i = g_i - sgn(e_y^T F_i)(\hat{g}_{i1} + \hat{g}_{i2} + \hat{g}_{i3}),$$

Note that, $A_d\hat{x}(t-d)$ is added to and subtracted from the right side of (9.7).

Remark 3: Many researchers study the observer design of time-delayed systems in literature. For example, consider

$$\begin{cases}\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\bar{d}, 0]\end{cases}$$

where x, y and u denote the system state, output and input, $d > 0 \in R$ denotes the time delay. In general, as doing in [8], the FD observer was given as:

$$\begin{cases}\dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-d) + Bu(t) + L(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{x}(t) = 0, \; t \in [-\bar{d}, 0]\end{cases}$$

then we obtain the error dynamics

$$\dot{e}_x(t) = (A - LC)e_x(t) + A_d e_x(t-d)$$

However, as Jiang pointed out in [7], the shortcoming of the aforementioned observer is that d must be known. If not, the observer does not work in the practical applications. Hence, for avoiding the shortcoming, a novel fault detection observer (9.5) is designed in this chapter.

Define the following smooth function

$$V_{De_x} = e_x^T(t)Pe_x(t) \tag{9.8}$$

where $P = P^T > 0$ is defined as in (9.6).

Differentiating V_{De_x} with respect to time t, we have

$$\begin{aligned}\dot{V}_{De_x} =& e_x^T(t)(P(A-LC)+(A-LC)^T P)e_x(t)+ \\ & 2e_x^T(t)PA_d e_x(t-d)+2e_x^T(t)PA_d(\hat{x}(t-d)- \\ & \hat{x}(t-\hat{d}))+2e_x^T(t)P\tilde{g}-2e_x^T(t)P(A_d\Delta_1+\Delta_2)\end{aligned} \tag{9.9}$$

From Young's inequality, we have

$$\begin{aligned}& 2e_x^T(t)PA_d e_x(t-d) \\ & \leq e_x^T(t)PA_d S^{-1}A_d^T Pe_x(t)+e_x^T(t-d)Se_x(t-d)\end{aligned} \tag{9.10}$$

where real matrix $S=S^T>0$.

From Assumption 9.1, it follows

$$\begin{aligned}& 2e_x^T(t)P\tilde{g} \\ & =\sum\nolimits_{i=1}^n 2e_x^T(t)P_i\tilde{g}_i \\ & =\sum\nolimits_{i=1}^n 2e_x^T(t)P_i(g_i-sgn(e_y^T F_i)\hat{g}_i) \\ & \leq \sum\nolimits_{i=1}^n(|2e_x^T(t)P_i||g_i|-sgn(e_y^T F_i)2e_x^T(t)P_i\hat{g}_i)\end{aligned}$$

where P_i is the ith column of matrix P.

From (9.6), we know, $P=(F^T C)^T$ and $P=P^T>0$. Further, we have

$$e_x^T(t)P_i=e_y^T(t)F_i$$

Hence, we have

$$\begin{aligned}& 2e_x^T(t)P\tilde{g} \\ & \leq \sum\nolimits_{i=1}^n |2e_y^T(t)F_i||g_i|-\sum\nolimits_{i=1}^n |2e_y^T(t)F_i|\hat{g}_i \\ & \leq \sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(g_{i1}+g_{i2}+g_{i3})- \\ & \quad \sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(\hat{g}_{i1}+\hat{g}_{i2}+\hat{g}_{i3}) \\ & =\sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(\tilde{g}_{i1}+\tilde{g}_{i2}+\tilde{g}_{i3}) \\ & =\sum\nolimits_{i=1}^n |2e_y^T(t)F_i|[\theta_{i1}^{*T}\xi_{i1}(\hat{x}(t))+\varepsilon_{i1}(\hat{x}(t))- \\ & \quad \hat{\theta}_{i1}\xi_{i1}(\hat{x}(t))+\theta_{i2}^{*T}\xi_{i2}(\hat{x}(t-\hat{d}))+ \\ & \quad \varepsilon_{i2}(\hat{x}(t-\hat{d}))-\hat{\theta}_{i2}\xi_{i2}(\hat{x}(t-\hat{d}))+\tilde{g}_{i3}] \\ & \leq \sum\nolimits_{i=1}^n |2e_y^T(t)P_i|(\tilde{\theta}_{i1}^T\xi_{i1}+\tilde{\theta}_{i2}^T\xi_{i2}+\tilde{g}_{i3})+ \\ & \quad \sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(\varepsilon_{i1}^*+\varepsilon_{i2}^*)\end{aligned} \tag{9.11}$$

where $\tilde{\theta}_{i1} = \theta_{i1}^* - \hat{\theta}_{i1}, \tilde{\theta}_{i2} = \theta_{i2}^* - \hat{\theta}_{i2}$, ξ_{i1} and ξ_{i2} are the abbreviations of $\xi_i(\hat{x}(t))$ and $\xi_{i2}(\hat{x}(t-\hat{d}))$, respectively.

Substituting (9.10) and (9.11) into (9.9), it yields

$$\begin{aligned}\dot{V}_{De_x} \leq & e_x^T(t)(P(A-LC)+(A-LC)^T P)e_x(t)- \\ & 2e_x^T(t)P(A_d\Delta_1+\Delta_2)+ \\ & e_x^T(t)PA_dS^{-1}A_d^T Pe_x(t)+ \\ & e_x^T(t-d)Se_x(t-d)+ \\ & 2e_x^T(t)PA_d(\hat{x}(t-d)-\hat{x}(t-\hat{d}))+ \\ & \sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(\tilde{\theta}_{i1}^T\xi_{i1}+\tilde{\theta}_{i2}^T\xi_{i2}+\tilde{g}_{i3})+ \\ & \sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(\varepsilon_{i1}^*+\varepsilon_{i2}^*)\end{aligned} \tag{9.12}$$

Define the following smooth function

$$\begin{aligned}V_{D1} = & V_{De_x} + \int_{t-d}^{t} e_x^T(s)Se_x(s)ds+ \\ & \sum\nolimits_{i=1}^n [\frac{1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1}+\frac{1}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2}]+ \\ & \sum\nolimits_{i=1}^n [\frac{1}{2\eta_3}\tilde{g}_{i3}^2+\frac{1}{2\eta_4}\tilde{\varepsilon}_i^2]\end{aligned} \tag{9.13}$$

where $\tilde{\varepsilon}_i = \varepsilon_i^* - \hat{\varepsilon}_i, \varepsilon_i^* = \varepsilon_{i1}^* + \varepsilon_{i2}^*, \hat{\varepsilon}_i$ is the estimate of $\varepsilon_i^*, \eta_l > 0 \in R, l = 1, 2, 3, 4$ are adaptive rates, I is an identity matrix.

Differentiating V_{D1} with respect to time t, it yields

$$\begin{aligned}\dot{V}_{D1} = & \dot{V}_{De_x} + e_x^T(t)Se_x(t)- \\ & e_x^T(t-d)(S+2I)e_x(t-d)- \\ & \sum\nolimits_{i=1}^n [\frac{1}{\eta_1}\tilde{\theta}_{i1}^T\dot{\hat{\theta}}_{i1}+\frac{1}{\eta_2}\tilde{\theta}_{i2}^T\dot{\hat{\theta}}_{i2}]- \\ & \sum\nolimits_{i=1}^n [\frac{1}{\eta_3}\tilde{g}_{i3}\dot{\hat{g}}_{i3}+\frac{1}{\eta_4}\tilde{\varepsilon}_i\dot{\hat{\varepsilon}}_i]\end{aligned} \tag{9.14}$$

Substituting (9.12) into (9.14), it yields

$$\begin{aligned}\dot{V}_{D1} \leq\ & e_x^T(t)\Xi_1 e_x(t) - 2e_x^T(t)P(A_d\Delta_1 + \Delta_2)+ \\ & 2e_x^T(t)PA_d(\hat{x}(t-d) - \hat{x}(t-\hat{d}))+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})]+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})]+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})]+ \\ & \sum\nolimits_{i=1}^{n}[|2e_y^T(t)F_i|\varepsilon_i^* - \frac{1}{\eta_4}\tilde{\varepsilon}_i\dot{\hat{\varepsilon}}_i]\end{aligned} \tag{9.15}$$

where

$$\Xi_1 = (P(A - LC) + (A - LC)^T P + PA_d S^{-1} A_d^T P + S) \tag{9.16}$$

Now, Δ_1 and Δ_2 are designed as follows:

$$\begin{aligned}\Delta_1 &= sgn(e_y^T(t)F_{Ad})(|\hat{x}(t-\hat{d})| + |\hat{x}_m|) \\ \Delta_2 &= sgn(e_y^T(t)F)\hat{\varepsilon}\end{aligned} \tag{9.17}$$

where

$$sgn(e_y^T F_{Ad}) = diag\{sgn(e_y^T F_{Ad1}), \ldots, sgn(e_y^T F_{Adn})\},$$

$$sgn(e_y^T(t)F) = diag\{sgn(e_y^T F_1), \ldots, sgn(e_y^T F_n)\},$$

F_{Adi} and F_i, $i = 1, \ldots, n$, denote the ith column of matrix F_{Ad} and F, respectively,

$$|\hat{x}(t-\hat{d})| = [|\hat{x}_1(t-\hat{d})|, \cdots, |\hat{x}_n(t-\hat{d})|]^T,$$

$$|\hat{x}_m| = [\hat{x}_{m1}, \ldots, \hat{x}_{mn}]^T,$$

$$\hat{x}_{mi} = \max\nolimits_{0\leq\tau\leq\bar{d}}\{|\hat{x}_i(t-\tau)|\}, i = 1, \ldots, n,$$

matrix F satisfies (9.6), while F_{Ad} satisfies the following condition

$$PA_d = (F_{Ad}^T C)^T, \tag{9.18}$$

and

$$\hat{\varepsilon} = [\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_n]^T$$

From (9.6), (9.17) and (9.18), we have

$$-2e_x^T(t)PA_d\Delta_1 + 2e_x^T(t)PA_d(\hat{x}(t-d) - \hat{x}(t-\hat{d})) \leq 0 \tag{9.19}$$

$$2e_x^T(t)P\Delta_2 = \sum\nolimits_{i=1}^{n} |2e_y^T(t)F_i|\hat{\varepsilon}_i \tag{9.20}$$

Substituting (9.19) and (9.20) into (9.15) and considering (9.6) and (9.18), one has

$$\begin{aligned}\dot{V}_{D1} \leq & e_x^T(t)\Xi_1 e_x(t) + \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})] + \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})] + \\ & \sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})] + \\ & \sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_y^T(t)F_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i)\end{aligned} \tag{9.21}$$

In order to derive the adaptive law of $\hat{d}$, $\tilde{d}e_y^T(t)e_y(t)$ is added to and subtracted from the the right hand of (9.21), then, we have

$$\begin{aligned}\dot{V}_{D1} \leq & e_x^T(t)\Xi_1 e_x(t) + \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})] + \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})] + \\ & \sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})] + \\ & \sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_y^T(t)F_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i) + \\ & \tilde{d}e_y^T(t)e_y(t) - \tilde{d}e_y^T(t)e_y(t)\end{aligned} \tag{9.22}$$

where $\tilde{d} = d - \hat{d}$.

Since $e_y = Ce_x$, we have

$$\tilde{d}e_y^T(t)e_y(t) = \tilde{d}e_x^T(t)C^TCe_x(t)$$

And since

$$\tilde{d} = d - \hat{d} \;\text{ and }\; e_x^T(t)C^TCe_x(t) \geq 0,$$

we have

$$\begin{aligned}\tilde{d}e_y^T(t)e_y(t) &= \tilde{d}e_x^T(t)C^TCe_x(t) \\ &= (d - \hat{d})e_x^T(t)C^TCe_x(t) \\ &= de_x^T(t)C^TCe_x(t) - \hat{d}e_x^T(t)C^TCe_x(t) \\ &\leq \bar{d}e_x^T(t)C^TCe_x(t)\end{aligned}$$

where the properties: $0 \le d \le \bar{d}$ (Assumption 9.2) and $0 \le \hat{d}$, are used. Note that, $0 \le \underline{d} \le \hat{d} \le \bar{d}$ is ensured by adaptive law (9.31). Further,

$$\begin{aligned}\dot{V}_{D1} \le e_x^T(t)(\Xi_1 + \bar{d}C^T C)e_x(t) + \\ &\sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})] + \\ &\sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})] + \\ &\sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})] + \\ &\sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_x^T(t)P_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i) - \\ &\tilde{d}e_y^T(t)e_y(t)\end{aligned} \tag{9.23}$$

If $Q > 0 \in R^{n\times n}$, $L \in R^{n\times n}$ and $P = P^T > 0 \in R^{n\times n}$ are chosen to satisfy the following inequality,

$$\begin{aligned}P(A - LC) + (A - LC)^T P + \\ PA_d S^{-1} A_d^T P + S + \bar{d}C^T C \le -Q\end{aligned} \tag{9.24}$$

then (9.23) can be developed as follows:

$$\begin{aligned}\dot{V}_{D1} \le - e_x^T(t)Qe_x(t) + \\ &\sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})] + \\ &\sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})] + \\ &\sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})] + \\ &\sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_y^T(t)F_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i) - \\ &\tilde{d}e_y^T(t)e_y(t)\end{aligned} \tag{9.25}$$

Define the following Lyapunov function

$$V_D = V_{D1} + \frac{1}{2\eta_5}\tilde{d}^2$$

where $\eta_5 > 0$ is a design parameter.

Differentiating V_D with respect to time t and considering (9.25), it yields

$$\begin{aligned}\dot{V}_D \leq & - e_x^T(t)Qe_x(t) + \\ & \sum_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})] + \\ & \sum_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})] + \\ & \sum_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})] + \\ & \sum_{i=1}^{n} \tilde{\varepsilon}_i(|2e_y^T(t)F_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i) - \\ & \tilde{d}(e_y^T(t)e_y(t) + \frac{1}{\eta_5}\dot{\hat{d}})\end{aligned} \tag{9.26}$$

Define the following adaptive laws

$$\dot{\hat{\theta}}_{i1} = \eta_1|2e_y^T(t)F_i|\xi_{i1} - \sigma_1\hat{\hat{\theta}}_{i1} \tag{9.27}$$

$$\dot{\hat{\theta}}_{i2} = \eta_2|2e_y^T(t)F_i|\xi_{i2} - \sigma_2\hat{\hat{\theta}}_{i2} \tag{9.28}$$

$$\dot{\hat{g}}_{i3} = \eta_3|2e_y^T(t)F_i| - \sigma_3\hat{\hat{g}}_{i3} \tag{9.29}$$

$$\dot{\hat{\varepsilon}}_i = \eta_4|2e_y^T(t)F_i| - \sigma_4\hat{\hat{\varepsilon}}_i \tag{9.30}$$

$$\dot{\hat{d}} = \begin{cases} \kappa, \text{ if } \underline{d} \leq \hat{d} \leq \bar{d} \text{ or} \\ \quad (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa \leq 0 \\ 0, \text{ if } (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa > 0 \end{cases}, \ \underline{d} < \hat{d}(0) < \bar{d} \tag{9.31}$$

where $i = 1, \ldots, n$, $\sigma_l > 0, l = 1, \ldots, 5$ are design parameters, $\kappa = -\eta_5 e_y^T(t) e_y(t) - \sigma_5\hat{d}$.

Note that, under the initial condition that $\underline{d} < \hat{d}(0) < \bar{d}$, the adaptive law (9.31) can guarantees that

$$\underline{d} \leq \hat{d}(t) \leq \bar{d}, \quad \text{for } t \geq 0$$

In fact, it is easily derived by lyapunov stability theory. Let us define the following Lyapunov function

$$V_d = \frac{1}{2}\hat{d}^2$$

Differentiating V_d with respect to time t, we have

$$\dot{V}_d = \hat{d}\dot{\hat{d}}$$

The following analysis will be derived in two cases.

Case 1: the first condition of (9.31) holds

Since

$$\underline{d} \le \hat{d} \le \bar{d} \text{ or } (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa \le 0$$

we have

$$\dot{V}_d = \hat{d}\kappa = \hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \le 0$$

Case 2: the second condition of (9.31) holds

Because

$$(\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa > 0$$

we have

$$\dot{V}_d = \hat{d} \cdot 0 = 0$$

From Cases 1 and 2, using Lyapunov stability theory, we have the following results,

$$\underline{d} \le \hat{d}(t) \le \bar{d}, \quad \text{for } t \ge 0.$$

Note that,

$$0 = -\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d} - (-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d})$$

Thus, the adaptive law (9.31) can be rewritten as follows:

$$\dot{\hat{d}} = \begin{cases} \kappa, \text{ if } \underline{d} \le \hat{d} \le \bar{d} \text{ or} \\ \quad (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa \le 0 \\ -\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d} - (-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}), \\ \quad \text{if } (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa > 0 \end{cases}$$

Substituting adaptive laws (9.27)–(9.31) into (9.26), it yields

$$\begin{aligned} \dot{V}_D \le & - e_x^T(t)Qe_x(t) + I\tilde{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) + \\ & \sum_{i=1}^{n}(\sigma_1\tilde{\theta}_{i1}^T\hat{\theta}_{i1} + \sigma_2\tilde{\theta}_{i2}^T\hat{\theta}_{i2}) + \\ & \sum_{i=1}^{n}(\sigma_3\tilde{g}_{i3}\hat{g}_{i3} + \sigma_4\tilde{\varepsilon}_i\hat{\varepsilon}_i) + \sigma_5\tilde{d}\hat{d} \end{aligned} \tag{9.32}$$

where $I = 0$ (or 1), if the first (second) condition of (9.31) holds.

If the second condition of (9.31) holds, namely,

$$(\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa > 0$$

then

$$\begin{aligned} &I\tilde{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \\ &= I\tilde{d}\frac{\hat{d}\hat{d}}{\hat{d}^2}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \end{aligned}$$

Note that,

$$\tilde{d}\hat{d} = \frac{1}{2}[d^2 - \hat{d}^2 - (d - \hat{d})^2] \tag{9.33}$$

If $\hat{d} = \bar{d}$ and $\hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) > 0$, then

$$\tilde{d}\hat{d} < 0$$

On the other hand, if $\hat{d} = 0$ and $\hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) > 0$, then

$$\tilde{d}\hat{d} = 0$$

Hence, we have

$$\tilde{d}\hat{d} \le 0$$

And since $\hat{d}\kappa = \hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) > 0$, we have

$$I\tilde{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \le 0$$

Therefore, (9.32) can be further derived as

$$\begin{aligned} \dot{V}_D \le &- e_x^T(t)Qe_x(t) + \\ &\sum_{i=1}^{n}(\sigma_1\tilde{\theta}_{i1}^T\hat{\theta}_{i1} + \sigma_2\tilde{\theta}_{i2}^T\hat{\theta}_{i2}) + \\ &\sum_{i=1}^{n}(\sigma_3\tilde{g}_{i3}\hat{g}_{i3} + \sigma_4\tilde{\varepsilon}_i\hat{\varepsilon}_i) + \sigma_5\tilde{d}\hat{d} \end{aligned} \tag{9.34}$$

Since $\tilde{\theta}_{i1} = \theta_{i1}^* - \hat{\theta}_{i1}$, using Young's inequality, we have

$$\begin{aligned} \sigma_1\tilde{\theta}_{i1}^T\hat{\theta}_{i1} &= \sigma_1\tilde{\theta}_{i1}^T(\theta_{i1}^* - \tilde{\theta}_{i1}) \\ &= -\sigma_1\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \sigma_1\tilde{\theta}_{i1}^T\theta_{i1}^* \\ &\le -\frac{1}{2}\sigma_1\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{1}{2}\sigma_1\theta_{i1}^{*T}\theta_{i1}^* \end{aligned} \tag{9.35}$$

Similarly, we have

$$\sigma_2\tilde{\theta}_{i2}^T\hat{\theta}_{i2} \le -\frac{1}{2}\sigma_2\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} + \frac{1}{2}\sigma_2\theta_{i2}^{*T}\theta_{i2}^* \tag{9.36}$$

$$\sigma_3\tilde{g}_{i3}\hat{g}_{i3} \le -\frac{1}{2}\sigma_3\tilde{g}_{i3}^2 + \frac{1}{2}\sigma_3 g_{i3}^2 \tag{9.37}$$

$$\sigma_4\tilde{\varepsilon}_i\hat{\varepsilon}_i \le -\frac{1}{2}\sigma_4\tilde{\varepsilon}_i^2 + \frac{1}{2}\sigma_4\varepsilon_i^2 \tag{9.38}$$

$$\sigma_5\tilde{d}\hat{d} \le -\frac{1}{2}\sigma_5\tilde{d}^2 + \frac{1}{2}\sigma_5\bar{d}^2 \tag{9.39}$$

Since

$$\lambda_{min}(Q)e_x^T(t)e_x(t) \le e_x^T(t)Qe_x(t)$$

then substituting (9.34)–(9.38) into (9.33), it yields

$$\begin{aligned}\dot{V}_D \le & -\lambda_{min}(Q)e_x^T(t)e_x(t) - \\ & \sum\nolimits_{i=1}^n \left(\frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2}\right) - \\ & \sum\nolimits_{i=1}^n \left(\frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2\right) - \frac{\sigma_5}{2\eta_5}\tilde{d}^2 + \\ & \sum\nolimits_{i=1}^n \left(\frac{\sigma_1}{2\eta_1}\theta_{i1}^{*T}\theta_{i1}^* + \frac{\sigma_2}{2\eta_2}\theta_{i2}^{*T}\theta_{i2}^*\right) + \\ & \sum\nolimits_{i=1}^n \left(\frac{\sigma_3}{2\eta_3}g_{i3}^2 + \frac{\sigma_4}{2\eta_4}\varepsilon_i^{*2}\right) + \frac{\sigma_5}{2\eta_5}\bar{d}^2\end{aligned} \tag{9.40}$$

Let

$$\begin{aligned}\mu = & \sum\nolimits_{i=1}^n \left(\frac{\sigma_1}{2\eta_1}\theta_{i1}^{*T}\theta_{i1}^* + \frac{\sigma_2}{2\eta_2}\theta_{i2}^{*T}\theta_{i2}^*\right) + \\ & \sum\nolimits_{i=1}^n \left(\frac{\sigma_3}{2\eta_3}g_{i3}^2 + \frac{\sigma_4}{2\eta_4}\varepsilon_i^{*2}\right) + \frac{\sigma_5}{2\eta_5}\bar{d}^2\end{aligned}$$

then (9.39) can be re-written as follows:

$$\begin{aligned}\dot{V}_D \le & -\lambda_{min}(Q)e_x^T(t)e_x(t) - \\ & \sum\nolimits_{i=1}^n \left(\frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2}\right) - \\ & \sum\nolimits_{i=1}^n \left(\frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2\right) - \frac{\sigma_5}{2\eta_5}\tilde{d}^2 + \mu\end{aligned} \tag{9.41}$$

It can be seen from (9.40) that, if

$$\lambda_{min}(Q)e_x^T(t)e_x(t) + \frac{\sigma_5}{2\eta_5}\tilde{d}^2 + \sum_{i=1}^{n}(\frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} + \frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2) \geq \mu$$

then $\dot{V}_D < 0$. Hence, set Ω defined as:

$$\Omega = \left\{ \begin{pmatrix} e_x, \\ \tilde{\theta}_{i1}, \\ \tilde{\theta}_{i2}, \\ \tilde{g}_{i3}, \\ \tilde{d} \end{pmatrix} \middle| \begin{pmatrix} \lambda_{min}(Q)e_x^T e_x + \frac{\sigma_5}{2\eta_5}\tilde{d}^2 + \\ \sum_{i=1}^{n}\frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \\ \sum_{i=1}^{n}\frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2}) + \\ \sum_{i=1}^{n}(\frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2) \end{pmatrix} \leq \mu \right\}$$

is an invariable set. This implies that $e_x, \tilde{\theta}_{i1}, \tilde{\theta}_{i2}, \tilde{g}_{i3}$ and $\tilde{d}$ are asymptotically bounded, namely,

$$||\tilde{\theta}_{i1}|| \leq \sqrt{\frac{2\eta_1\mu}{\sigma_1}}, \quad ||\tilde{\theta}_{i2}|| \leq \sqrt{\frac{2\eta_2\mu}{\sigma_2}}, \; ||\tilde{g}_{i3}|| \leq \sqrt{\frac{2\eta_3\mu}{\sigma_3}},$$

$$||e_x|| \leq \sqrt{\frac{\mu}{\lambda_{min}(Q)}},$$

$$||\tilde{\varepsilon}_i|| \leq \sqrt{\frac{2\eta_4\mu}{\sigma_4}}, \; |\tilde{d}| \leq \sqrt{\frac{2\eta_5\mu}{\sigma_5}}$$

It is necessary to point out that the size of Ω can become arbitrarily small by adjusting the parameters: σ_i and η_i, $i = 1, 2, \ldots, 5$.

Now, the following theorem is given to summarize the above design procedures and analysis.

Theorem 9.1 *Consider system* (9.1) *and observer* (9.5) *with Assumptions 1 and 2, if there exist matrices* L, F, F_{Ad}, $Q > 0$, $S > 0$ *and* $P = P^T > 0$ *satisfying* (9.6), (9.18) *and* (9.24)*, and adaptive laws* (9.27)–(9.31) *are used, then error dynamics* (9.7) *is asymptotically bounded with all the signals in the closed-systems converging to an adjustable neighborhood of the origin.*

Proof From the above analysis, it is easy to obtain the conclusions. The detailed proof is thus omitted here.

From Theorem 9.1, we have

$$||e_x|| \leq \sqrt{\frac{\mu}{\lambda_{min}(Q)}}$$

Let us define detection residual

$$R(t) = ||y(t) - \hat{y}(t)|| = ||Ce_x(t)||$$

Obviously, in the free-fault case, one has

$$R(t) \leq ||C||\sqrt{\frac{\mu}{\lambda_{min}(Q)}}$$

Hence, by using the following mechanism, fault detection can be performed,

$$\begin{cases} R(t) \leq T_d & \text{no fault occurred,} \\ R(t) > T_d & \text{fault has occurred} \end{cases} \tag{9.42}$$

where $T_d = ||C||\sqrt{\frac{\mu}{\lambda_{min}(Q)}}$.

Remark 9.2 It can been seen that, if there is no fault in the controlled system, then $\lim_{t\to\infty} e_x(t) = 0$. If some actuator faults occur in system, then $\lim_{t\to\infty} e_x(t) \neq 0$. Thus, in some existing works, the fault detection is designed as:

$$\begin{cases} \lim_{t\to\infty} e_x(t) = 0, & \text{no fault occurred} \\ \lim_{t\to\infty} e_x(t) \neq 0, & \text{fault has occurred} \end{cases}$$

observer (9.5) was taken to as the FD observer of system (9.1). However, $e_x(\infty)$ is not available in practice applications. Thus, $e_x(\infty) \neq 0$ cannot be seen as an indicator to detect fault occurrence or not. Hence, (9.32) is more efficient mechanism for FD in practical applications.

9.4 Simulation Results

The following time delayed system is considered:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + g \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\bar{d}, 0] \end{cases}$$

where

$$A = \begin{bmatrix} -4 & 0 \\ 1 & 0.5 \end{bmatrix}, \; A_d = \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \; g = \begin{bmatrix} x_1(t)sin(x_2) + x_2(t-d)sin(x_1) \\ x_2(t)cos(x_1) + x_1(t-d)cos(x_2) \end{bmatrix},$$

time delay $d = 0.5$, $\phi(t) = e^{-1} - 0.1e^{-t}$.

In this simulation, it is assumed that the fault occurs at 6s in the system.

Note that (9.19) can be transformed to the the following linear matrix inequality (LMI),

$$\begin{bmatrix} PA - YC + A^T P - C^T Y^T + S + Q & PA_d \\ A_d^T P & -S^{-1} \end{bmatrix} < 0$$

where $Y = PL$. By solving this LMI, we can have:

$$P = \begin{bmatrix} 1.7096 & 0.0590 \\ 0.0590 & 1.5033 \end{bmatrix}, \; Q = \begin{bmatrix} 1.7414 & 0 \\ 0 & 1.7414 \end{bmatrix},$$

$$Y = \begin{bmatrix} -5.9088 & 0.2191 \\ 1.0779 & 1.6224 \end{bmatrix}, \; L = \begin{bmatrix} -3.4856 & 0.0911 \\ 0.8537 & 1.0756 \end{bmatrix}$$

and

$$F = \begin{bmatrix} 2.5102 & 0.0590 \\ 0.1180 & 1.5033 \end{bmatrix}$$

The simulation results are shown in Figs. 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7 and 9.8. From Fig. 9.1, It can be seen that the state observe errors are bounded, which implies that the proposed observer has a better convergent property, while Fig. 9.2 shows the residual signal asymptotically converges to the small neighborhood of the origin. Figures 9.3, 9.4, 9.5 and 9.6 also show the closed-loop system signals are bounded.

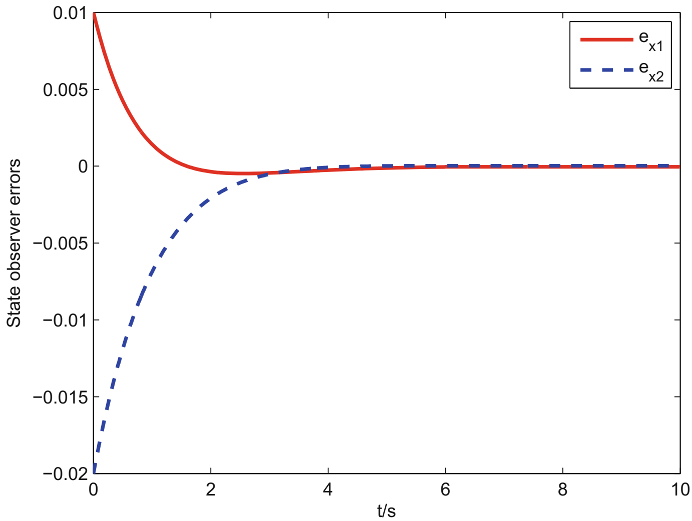

Fig. 9.1 The state observer errors (no fault)

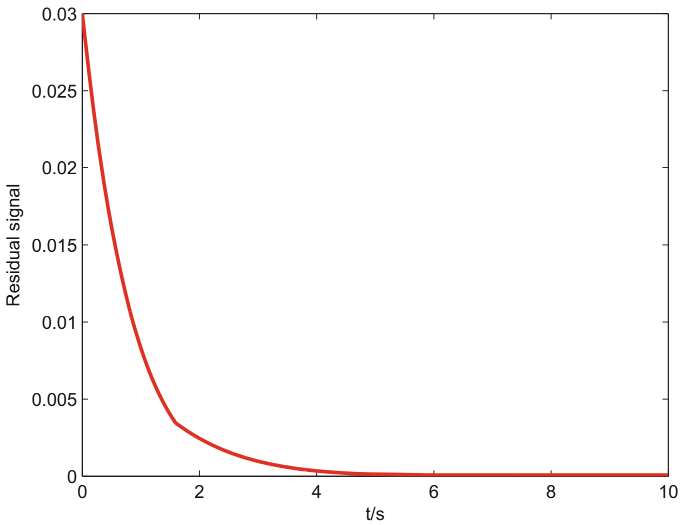

Fig. 9.2 The residual signal (no fault)

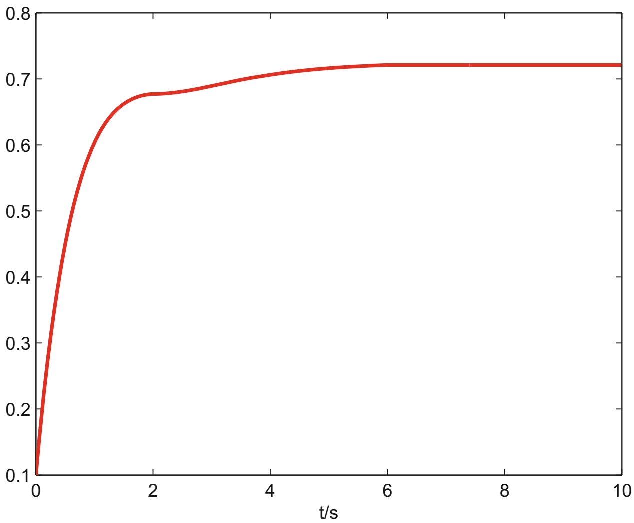

Fig. 9.3 The norm of $\hat{\theta}_{11}$ (no fault)

However, when a fault occurs in the system, Fig. 9.7 shows that, the residual signal significantly deviates from the origin, and the alarm occurs. Correspondingly, the state observe errors significantly deviates from the origin, too, shown in Fig. 9.8.

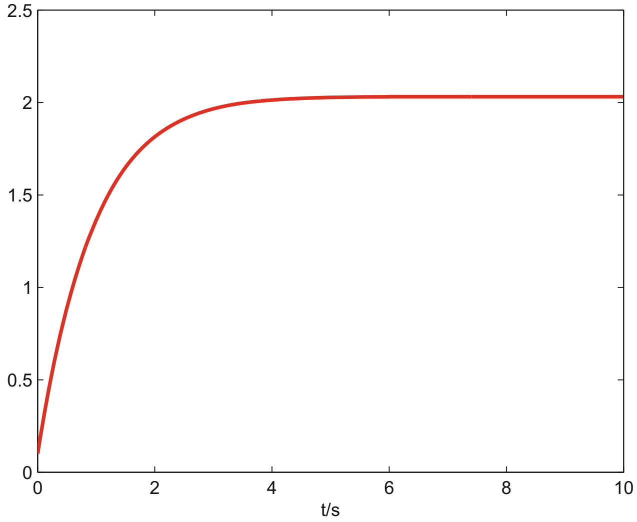

Fig. 9.4 The norm of $\hat{\theta}_{12}$ (no fault)

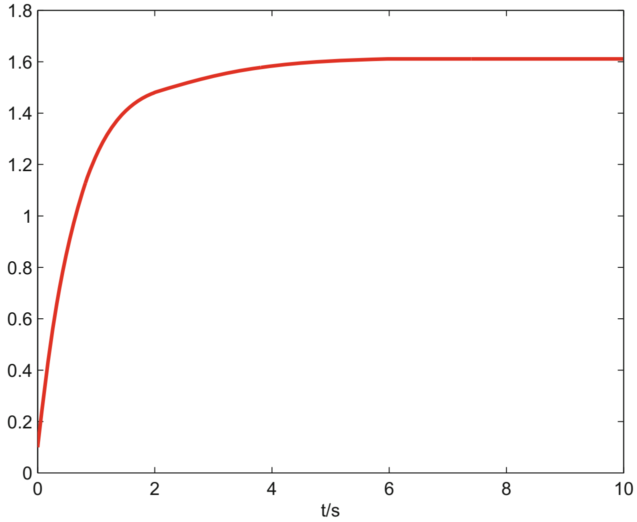

Fig. 9.5 The norm of $\hat{\theta}_{13}$ (no fault)

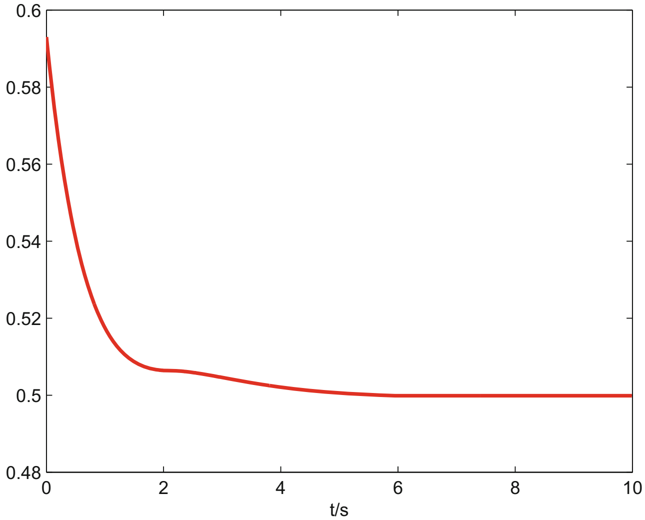

Fig. 9.6 Trajectory of $\hat{d}$ (no fault)

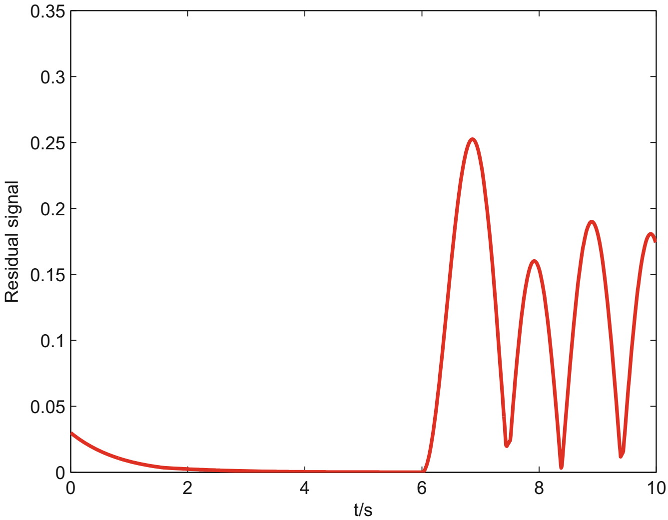

Fig. 9.7 The residual signal in faulty case

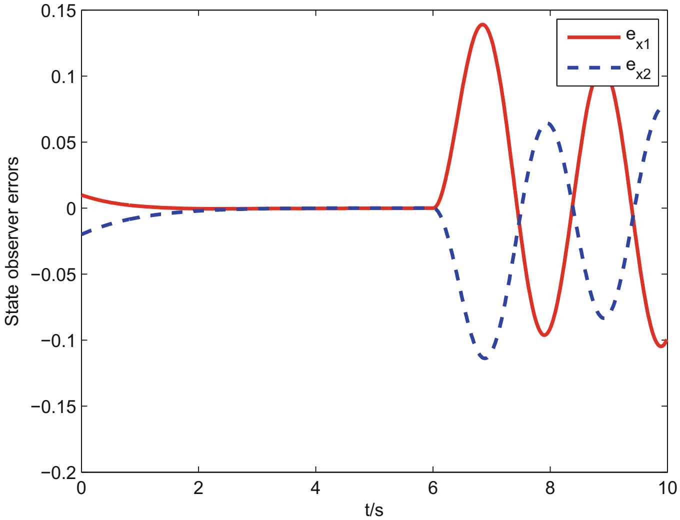

Fig. 9.8 The state observer errors in faulty case

9.5 Conclusions

In this chapter, the fault detection problem of uncertain time-delayed systems is studied. To overcome the shortcoming in existing works where the exact value of time delay needs to be known, a novel adaptive NNs-based fault detection observer is designed, which can estimate online the unknown time delay with system state. Simulation results show the effectiveness of the technique proposed in this chapter.

References

1. Kazantzis, N., Wright, R.A.: Nonlinear observer design in the presence of delayed output measurements.Systems Control & Letters. **54**(9), 877–886 (2005)
2. Ding, S. X., Zhong, M., Tang, B., Zhang, P.: An LMI approach to the design of fault detection filter for time-delay LTI systems with unknown inputs. In: Proceedings of the American Control Conferences, pp. 2137–2142 (2001)
3. Jiang, B., Staroswieck, M., Cocquempot, V.: H∞ fault detection filter design for linear discrete-time systems with multiple time delays. Int. J. Syst. Sci. **34**(5), 365–373 (2003)
4. Zhong, M., Ding, S.X., Lam, J., Zhang, C.: Fault detection filter design for LTI system with time delays. In: Proceedings of the 42nd IEEE Conference on Decision and Control, pp. 1467–1472 (2003)
5. Zhang, Y., Guo, L., Wang, H.: Robust filtering for fault tolerant control using output PDFs of non-Gaussian systems. IET Control Theor. Appl. **1**(3), 636–645 (2007)
6. Zhao, H., Zhong, M., Zhang, M. H∞ fault detection for linear discrete time-varying systems with delayed state. IET Control Theor. Appl. **4**(11), 2303–14, (2010)

7. Jiang, X.F., Xu, W.L., Han, Q.L.: Observer-based fuzzy control design with adaptation to delay parameter for time-delay systems. Fuzzy Sets Syst. **152**(3), 637–649 (2005)
8. Zhang, K., Jiang, B., Cocquempot, V.: Fast adaptive fault estimation and accommodation for nonlinear time-varying delay systems. Asian J. Control. **11**(6), 643–652 (2009)
9. Chen, J., Patton, R.J.: Robust Model-Based Fault Diagnosis For Dynamic Systems. Kluwer Academic, Boston (1999)
10. Mahmoud, M.M., Jiang, J., Zhang, Y.: Active Fault Tolerant Control Systems. Springer-Verlag, NewYork (2003)
11. Yang, H., Jiang, B., Cocquempot, V.: Fault Tolerant Control Design For Hybrid Systems. Springer, Berlin Heidelberg (2010)
12. Wang, D., Shi, P., Wang, W.: Robust filtering and fault detection of switched delay systems. Springer, Berlin Heidelberg (2013)
13. Du, D., Jiang, B., Shi, P.: Fault tolerant control for switched linear systems. Springer, Cham Heidelberg (2015)
14. Shen, Q., Jiang, B., Cocquempot, V.: Fault diagnosis and estimation for near-space hypersonic vehicle with sensor faults. In: Proc. Inst. mech. eng. part I-J. Syst. Control Eng. **226**(3), pp. 303–313 (2012)
15. Shen, Q., Jiang, B., Cocquempot, V.: Adaptive fault Tolerant synchronization with unknown propagation delays and actuator faults. Int. J. Control. Autom. Syst.**10**(5), pp. 883–889 (2012)
16. Shen, Q., Jiang, B., Cocquempot, V.: Fuzzy logic system-based adaptive fault tolerant control for near space vehicle attitude dynamics with actuator faults.IEEE Trans. Fuzzy Syst. **21**(2), 289–300 (2013)
17. Shen, Q., Jiang, B., Cocquempot, V.: Adaptive fault-tolerant backstepping control against actuator gain faults and its applications to an aircraft longitudinal motion dynamics. Int. J. Robust Nonlinear Control. **20**(10), 448–459 (2013)
18. Astrom, K.J.: Intelligent control. In: Proceedings of 1st European Control Conference, Grenoble, pp. 2328–2329 (1991)
19. Gertler, J.J.: Survey of model-based failure detection and isolation in complex plants. IEEE Control Syst. Mag. **8**(6), 3–11 (1988)
20. Frank, P.M., Seliger, R.: Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy: A survey and some new results. Automatica **26**(3), 459–474 (1990)
21. Frank, P.M.: Analytical and qualitative model-based fault diagnosis-a survey and some new results. European Journal of Control **2**(1), 6–28 (1996)
22. Garcia, E.A., Frank, P.M.: Deterministic nonlinear observer-based approaches to fault diagnosis: A survey. IFAC Control Engineering Pratice **5**(6), 663–670 (1997)
23. Patton, R.J.: Fault-tolerant control: The 1997 situation (survey). In: Proceeding IFAC Symposium. Fault Detection, Supervision and Safety for Technical Processes: SAFEPROCESS, pp. 1029–1052 (1997)
24. Isermann, R., Schwarz, R., Stolzl S.: Fault-tolerant drive-by-wire systems-concepts and realization. In: Procceedings IFAC Symposium Fault Detection, Supervision and Safety for Technical Processes: SAFEPROCESS, pp. 1–5 (2000)
25. Frank, P.M.: Online fault detection in uncertain nonlinear systems using diagnostic observers: a survey. Int. J. Syst. Sci. **25**(12), 2129–2154 (1994)
26. Patton, R.J.: Robustness issues in fault-tolerant control. In Proceedings of International Conference on Fault Diagnosis. Toulouse, France (1993)
27. Song, Q., Hu, W.J., Yin, L., Soh, Y.C.: Robust adaptive dead zone technology for fault- tolerant control of robot manipulators. J. Intell. Robot. Syst. **33**(1), 113–137 (2002)
28. Shen, Q.K.: Bin Jiang, and Peng Shi, Novel neural networks-based fault tolerant control scheme with fault alarm. IEEE Trans. Cybern. **44**(11), 2190–2201 (2014)
29. Vidyasagar, M., Viswanadham, N.: Reliable stabilization using a multi-controller configuration. Automatica **21**(4), 599–602 (1985)
30. Gundes, A.N.: Controller design for reliable stabilization. In: Procceeding of 12th IFAC World Congress, vol. 4, pp. 1–4, (1993)

31. Sebe, N., Kitamori, T.: Control systems possessing reliability to control. In: Procceeding of 12th IFAC World Congress, vol. 4, pp. 1–4, (1993)
32. Saeks, R., Murray, J.: Fractional representation, algebraic geometry, and the simultaneous stabilization problem. IEEE Trans. Autom. Control **24**(4), 895–903 (1982)
33. Kabamba, P.T., Yang, C.: Simultaneous controller design for linear time-invariant systems. IEEE Trans. Autom. Control **36**(1), 106–111 (1991)
34. Olbrot, A.W.: Fault tolerant control in the presence of noise: a new algorithm and some open problems. In: Procceding of 12th IFAC World Congress, vol. 7, pp. 467–470, (1993)
35. Morari, M.: Robust stability of systems with integral control. IEEE Trans. Autom. Control **30**(4), 574–588 (1985)
36. Shen, Q.K., Jiang, B.B., Zhang, T.P.: Adaptive fault-tolerant tracking control for a class of time-delayed chaotic systems with saturation input containing Sector. In: Proceedings of the 31th Chinese Control Conference, Hefei, pp. 5204–5208 (2012)
37. Shen, Q.K., Jiang, B. Zhang, T.P.: Fuzzy systems-based adaptive fault-tolerant dynamic surface control for a class of high-order nonlinear systems with actuator fault. In: Proceedings of the 10th World Congress on Intelligent Control and Automation, Beijing, pp. 3013–3018, (2012)
38. Shen, Q.K., Zhang, T.P., Zhou, C.Y.: Decentralized adaptive fuzzy control of time-delayed interconnected systems with unknown backlash-like hysteresis. J. Syst. Eng. Electron. **19**(6), 1235–1242 (2008)
39. Yu, C.C., Fan, M.K.H.: Decentralized integral controllability and D-stability. Chem. Eng. Sci. **45**(11), 3299–3309 (1990)
40. Bao, J., Zhang, W.Z., Lee, P.L.: Decentralized fault-tolerant control system design for unstable processes. Chem. Eng. Sci. **58**(22), 5045–5054 (2003)
41. Zhang, W.Z., Bao, J., Lee, P.L.: Decentralized unconditional stability conditions based on the Passivity Theorem for multi-loop control systems. Ind. Eng. Chem. Res. **41**(6), 1569–1578, (2002)
42. Saljak, D.D.: Reliable control using multiple control systems. Int. J. Control **31**(2), 303–329 (1980)
43. Kaminer, I., Pascoal, A.M., Khargonekar, P.P., Coleman, E.E.: A velocity algorithm for the implementation of gain-scheduled controllers. Automatica **31**(8), 1185–1192 (1995)
44. Li, W., Xu, W. Z., Wang, J.: Active fault tolerant control using BP network application in the temperature control of 3-layer PE steel pipe producing. In: Proceedings of the 5th World Congress on Intelligent Control and Automation, Hangzhou, China, pp. 1525–1529 (2004)
45. Moerder, D.D.: Application of pre-computed laws in a reconfigurable aircraft flight control system. J. Guid. Control Dyn. **12**(3), 325–333 (1989)
46. Huber, R.R., McCulloch, B.: Self-repairing flight control system. SAE Tech. Paper Series, pp. 1–20 (1984)
47. Srichande, R., Walker, B.K.: Stochastic stability analysis for continuous-time fault tolerant control systems. Int. J. Control **57**(3), 433–452 (1993)
48. Ranmamurthi, K., Agogino, A.M.: Real-time expert system for fault tolerant supervisory control. J. Dyn. Syst. Meas. Control **115**(3), 219–227 (1993)
49. Wu, N.E., Zhou, K., Salomon, G.: Control recongurability of linear time-invariant systems. Automatica **36**(12), 1767–1771 (2000)
50. Morse, W.D., Ossman, K.A.: Model-following reconfigurable flight control systems for the AFTI/F-16. J. Guid. Control Dyn. **13**(6), 969–976 (1990)
51. Huang, C.Y., Stengel, R.F.: Re-structurable control using proportional-integral implicit model following. J. Guid. Control Dyn. **13**(2), 303–309 (1990)
52. Napolitanob, M.R., Swaim, R.L.: New technique for aircraft flight control reconfiguration. J. Guid. Control Dyn. **14**(1), 184–190 (1991)
53. Kwong, W., Passino, K.M., Laukonen, E.G., Yurkovich, S.: Expert supervision of fuzzy learning systems for fault tolerant aircraft control. Proc. IEEE **83**(3), 466–483 (1995)
54. Tao, G., Chen, S., Joshi, S.M.: An adaptive actuator failure compensation controller using output feedback. IEEE Trans. Autom. Control **47**(3), 506–511 (2002)

55. Jin, X., Yang, G.: Robust adaptive fault-tolerant compensation control with actuator failures and bounded disturbances. Acta Autom. Sinic. **35**(3), 305–309 (2009)
56. Jiang, B., Staroswiecki, M., Cocquempot, V.: Fault accommodation for nonlinear dynamic systems. IEEE Trans. Autom. Control **51**(9), 1578–1583 (2006)
57. He, X., Wang, Z.D., Zhou, D.H.: Robust fault detection for networked systems with communication delay and data missing. Automatica **45**(11), 2634–2639 (2009)
58. Liu, Y.H., Wang, Z.D., Wang, W.: Reliable H∞ filtering for discrete time-delay systems with randomly occurred nonlinearities via delay-partitioning method. Signal Process. **91**, 713–727 (2011)
59. Dong, J.X. Yang, G.: Robust static output feedback control for linear discrete-time systems with time-varying uncertainties. Syst. Control Lett. **57**(2), pp. 123–131(2008)
60. Wang, Y., Zhou, D., Qin, S.J., Wang, H.: Active fault-tolerant control for a class of nonlinear systems with sensor faults. Int. J. Control Autom. Syst. **6**(3), 339–350 (2008)
61. Li, S., Tao, G.: Feedback based adaptive compensation of control system sensor uncertainties. Automatica **45**(2), 393–404 (2009)
62. Du, D., Jiang, B., Shi, P.: Active fault-tolerant control for switched systems with time delay. Int. J. Adapt. Control Signal Process. **25**(5), 466–480 (2011)
63. Wu, H.N.: Reliable LQ fuzzy control for continuous-time nonlinear systems with actuator faults, IEEE Trans. Syst. Man and Cybern., Part B. Cybern. **34**(4), 1743–1752 (2004)
64. Wang, W., Wen, C.: Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance. Automatica **46**(12), 2082–2091 (2010)
65. Zhang, T., Guay, M.: Adaptive control for a class of second-order nonlinear systems with unknown input nonlinearities. IEEE Trans. Syst. Man Cybern. Part B Cybern. **33**(1), 143–149 (2003)
66. Zhang, X., Parisini, T., Polycarpou, M.M.: Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach. IEEE Trans. Autom. Control **49**(8), 1259–1274 (2004)
67. Zhang, X., Polycarpou, M.M., Parisini, T.: A robust detection and isolation scheme for abrupt and incipient fault in nonlinear systems. IEEE Trans. Autom. Control **47**(4), 576–593 (2002)
68. Wang, W., Wen, C.Y.: Adaptive compensation for infinite number of actuator failures or faults. Automatica **47**(10), 2197–2210 (2011)
69. Ge, S.S., Hong, F., Lee, T.H.: Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients. IEEE Trans. Syst. Man Cybern. Part B Cybern **34**(1), 499–516 (2004)
70. Sanner, R.M., Slotine, J.E.: Gaussian networks for direct adaptive control. IEEE Trans. Neural Netw. **3**(6), 837–863 (1992)