Chapter 3
Fuzzy Logic System-Based Adaptive FC for NSV Attitude Dynamics with Multiple Faults

3.1 Introduction

It is well known that the controlled systems in practical applications may become faulty due to various reasons. Hence, FD and FTC have received considerable attention, and obtained significant results in the past decades, see [1–22] and the references therein. However, most of the existing results on FD and FTC work under the restrictive condition that only one actuator or sensor fault occurs at one time. In real applications, multiple types multiple faults may occur in the controlled system. The faulty cases include: multiple actuator faults, multiple sensors faults and multiple actuator and sensor faults. Up to now, few revelent results are reported in literature [23]. In [23], an actuator fault diagnosis scheme was proposed for a class of affine nonlinear systems with both known and unknown inputs, which was designed by making use of the derived input/output relation and the recently developed high-order sliding-mode robust differentiators. Hence, considering multiple type multiple faults simultaneously occurred in the controlled system is a motivation of this chapter.

Near space hypersonic vehicle, as a class of vehicle flying in near space which offers a promising and new, lower cost technology for future spacecraft. It can advance space transportation and also prompt global strike capabilities. Such complex technological system attracts considerable interests from the control research community and aeronautical engineering in the past couple of decades and significant results were reported [24–35]. For such high technological system, it is of course essential to maintain high reliability against possible faults [36–54].

Recently, T-S fuzzy system was used to describe the NSV attitude dynamics which are complex nonlinear, multi-variable and strongly coupled ones [24–35]. During the past two decades, the stability analysis for Takagi-Sugeno (T-S) fuzzy systems has attracted increasing attention [25–27]. In [55], the authors studied the problem of fault-tolerant tracking control for near-space-vehicle attitude dynamics with bias actuator fault, where the bias fault was assumed to be unknown constant. However, in practical application, the fault may be state-dependent, namely, it is a

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unknown function of system state. In this chapter, we will propose a more general FTC scheme that handles such state-dependent faults. On the other hand, as a universal approximation, fuzzy logic system (FLS) played an important role in modeling and controlling uncertain systems, see [56–61] and the references therein. In this chapter, we use the above FLSs to approximate the unknown state-dependent gain and bias faults.

In this chapter, we investigate the problem of fault tolerant control NSV with multiple state-dependent actuator faults, with the objective to provide an efficient solution for controlling NSV in faulty situations. Compared with existing literatures, the following contributions are worth to be emphasized.

(1) The actuator fault model presented in this chapter integrates state-dependent gain bias faults, which means that a wide class of faults can be handled. The theoretic developments and results of this chapter are thus valuable in a wide field of practical applications.
(2) Differing from some design scheme in literature, the fault-tolerant control scheme does not need the condition that the bounds of the time derivatives of the faults should be known constants, which thus enlarges the practical application scope.
(3) In general, the denominator of the fault-tolerant control input contains the estimation of the gain fault. If the denominator is equal to zero, a controller singularity occurs. In the proposed modified FTC scheme, the controller singularity is avoided without projection algorithm.

The rest of this chapter is organized as follows. In Sect. 3.2, the T-S fuzzy model for NSV attitude dynamics is first briefly recalled. Actuator faults are integrated in such model, and the FTC objective is formulated. In addition, mathematical description of fuzzy logic system is given. In Sect. 3.3, the main technical results of this chapter are given, which include fault detection, isolation, and fuzzy logic system-based fault accommodation in the two cases where system states are available or not. The NSV application is presented in Sect. 3.4. Simulation results of NSV are presented to demonstrate the effectiveness of the proposed technique. Finally, Sect. 3.5 draws the conclusion.

3.2 Problem Statement and Preliminaries

3.2.1 Problem Statement

In this chapter, a NSV attitude dynamics in re-entry phase is given as [62]:

$$\begin{cases} J\dot{\omega} = -\Omega J\omega + \delta \\ \dot{\gamma} = R(\cdot)\omega \end{cases} \tag{3.1}$$

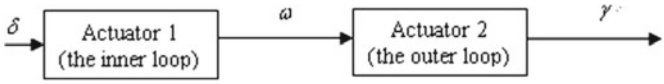

Fig. 3.1 The control diagram of NSV

where $J \in R^{3\times3}$ is the symmetric positive definite moment of inertia tensor, and $\omega = [p, q, r]^T = [\omega_1, \omega_2, \omega_3]^T$ is the angular rate vector composed of roll p, pitch q and yaw rate r, $\delta = [\delta_e, \delta_\alpha, \delta_r]^T \in R^{3\times1}$ is the control surface deflection, $\delta_e, \delta_\alpha, \delta_r$ are the elevator deflection, the aileron deflection, the rudder deflection, respectively. The skew symmetric matrix Ω is given by:

$$\Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ \omega_2 & \omega_1 & 0 \end{bmatrix} \tag{3.2}$$

In the re-entry phase, $R(\cdot)$ is defined as follows:

$$R(\cdot) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ \sin\alpha & 0 & -\cos\alpha \\ 0 & 1 & 0 \end{bmatrix} \tag{3.3}$$

where $\gamma = [\phi, \beta, \alpha]^T$ and ϕ, β, α are the bank, sideslip, and the attack angles, respectively. According to the singular perturbation theory, the above six equations can be expressed in the form of inner loop ω and outer loop γ; ω and γ are also respectively called fast loop and slow loop. The control diagram of NSV attitude dynamics is shown in Fig. 3.1. From the motion law of NSV, it is easy to find that, the response of the angular rate ω is faster than the one of the attitude angle γ. Based on time scale principle, we define ω as fast state and γ as slow state, thus system (3.1) can be divided into the following two subsystems: fast subsystem (3.4a) related to fast state ω and slow subsystem (3.4b) related slow state γ.

$$\begin{cases} \dot{x}_\omega = f(x_\omega) + g(x_\omega)u(t) \\ y_\omega = x_\omega \end{cases} \quad (a)$$
$$\begin{cases} \dot{x}_\gamma = f(x_\gamma, t)y_\omega \\ y_\gamma = x_\gamma \end{cases} \quad (b) \tag{3.4}$$

where $f(x_\omega) = J^{-1}\Omega(\omega)Jx_\omega$, $g(x_\omega) = J^{-1}$, $f(x_\gamma) = R(\cdot)$, $x_\omega = \omega$, $x_\gamma = \gamma$.

The control objectives are,

(1) for the slow subsystem (the outer loop), to design the ideal angular rate $y_\omega(=\omega_d)$ such that subsystem output y_γ follows the desired reference signal y_d whose 1st derivative are available and bounded;

(2) for the fast subsystem (the inner loop), to design the control $u(t)$ such that the angular rate x_ω follows the ideal angular rate $y_\omega(=\omega_d)$.

That is to say, the main task is to design proper control input $u(t)$ such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0 \Rightarrow \lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$.

A fuzzy linear dynamic model has been proposed by Takagi and Sugeno in 1985 to represent a nonlinear system as an aggregation of local linear input/output relations. The fuzzy linear model is described by fuzzy IF-THEN rules and will be employed to deal with the fuzzy control problem for inner loop dynamics described by (3.4a) in this chapter.

Consider the following T-S fuzzy model composed of a set of fuzzy implications, where each implication is expressed by a linear state space model. The ith rule of this T-S fuzzy model is of the following form:

Plant rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{3.5}$$

where $i = 1, \ldots, r$, r is the number of the IF-THEN rules, $M_{ij}, j = 1, \ldots, q$ is the fuzzy set, $z(t) = [z_1(t), \ldots, z_q(t)]^T$ are the premise variables which are supposed to be known, $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$ denotes state vector, $u(t) \in R^m$ denotes control input, $A_i \in R^{n\times n}$, and $B_i \in R^{n\times m}$ are local state and control matrices.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \tag{3.6}$$

where $h_i(z(t))$ is defined as

$$h_i(z(t)) = \frac{\prod_{j=1}^{n} M_{ij}[z(t)]}{\sum_{i=1}^{r}\prod_{j=1}^{n} M_{ij}[z(t)]}, \quad i = 1, 2, \ldots, r \tag{3.7}$$

where $M_{ij}[z(t)]$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed in this chapter that $\prod_{j=1}^{n} M_{ij}[z(t)] \geqslant 0$ for all t. Therefore, we have $\sum_{i=1}^{r} h_i(z(t)) = 1, \quad 0 \leqslant h_i(z(t)) \leqslant 1$ for all t.

In this chapter, the state feedback control strategy is chosen as a parallel distributed compensation (PDC), which can be described as follows:

Control rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$u_i(t) = K_i x(t) \tag{3.8}$$

where K_i is the controller gain matrix to be determined later.

The overall fuzzy controller is given as follows:

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) K_i x(t) \tag{3.9}$$

The control objective under normal conditions is to design a proper state feedback control input $u(t)$ such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0 \Rightarrow \lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$.

However, in practical application, actuators may become faulty. Two kinds of actuator faults are considered: loss of effectiveness of the actuators and actuator bias faults. The first kind of fault is modeled as follows.

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t),\ i = 1, \ldots, m,\ t \geqslant t_j \tag{3.10}$$

where $\rho_i^u(x)$ $(0 \leqslant \rho_i^u(x) < 1)$, which is supposed to be unknown, denotes the remaining control rate, t_j is unknown fault occurrence time. The second kind of fault, namely actuator bias fault, can be described as:

$$u_i^f(t) = u_i(t) + d_i^u(x),\quad i = 1, \ldots, m,\quad t \geqslant t_j \tag{3.11}$$

where $d_i^u(x)$ denotes a bounded signal. Therefore, the above two kinds of actuator faults can be uniformly described as

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t) + d_i^u(x),\quad t \geq t_f \tag{3.12}$$

Furthermore, a more general fault model can be given as:

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t) + \sum_{j=1}^{p_i^u} g_{i,j}^u d_{i,j}^u(x),\quad t \geq t_f \tag{3.13}$$

where $d_{i,j}^u(x), i = 1, \ldots, m, j = 1, \ldots, p_i^u$ denotes a bounded signal, p_i^u is a known positive constant. $g_{i,j}^u$ denotes an unknown constant. With no restriction, let suppose $p_1^u = \cdots = p_m^u = p$, with p a known positive constant. Consider the following notation $a_{i,j}^u(x) = g_{i,j}^u d_{i,j}^u(x)$, (3.13) can be re-written as follows:

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t) + \sum_{j=1}^{p} a_{i,j}^u(x),\quad t \geq t_f \tag{3.14}$$

where the nonlinear functions $\rho_i^u(x), a_{i,j}^u(x)$ and the failure time instant t_j are unknown. In this chapter, both bias and gain faults are handled by considering the general fault model (3.14).

Now, the control objective is re-defined as follows. An active fault tolerant control approach is proposed to obtain the above tracking objective in normal and faulty conditions, namely, $\lim_{t\to\infty}(x_\omega - \omega_d) = 0$. Furthermore, $\lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$. Under normal condition (no fault), a state feedback control input $u(t)$ is designed, such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0$. Meanwhile, the FDI algorithm is working. As soon as actuator faults are detected and isolated, the fault accommodation algorithm is activated and a proper fault-tolerant control input $u(t)$ is used such that the tracking performance ($\lim_{t\to\infty}(x_\omega - \omega_d) = 0$) is still maintained stable under faulty case.

3.2.2 *Mathematical Description of Fuzzy Logic System*

FLS consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

$$R^l : \textit{if } x_1 \textit{ is } A_1^l \textit{ and } x_2 \textit{ is } A_2^l \ldots \textit{ and } x_n \textit{ is} A_n^l, \quad \textit{then } y \textit{ is } B^l, \; l = 1, 2, \ldots, M$$

where $\underline{x} = [x_1, x_2, \ldots, x_n]^T \in U \subset R^n$ and y are the FLS input and output, respectively. Fuzzy sets A_i^l and B^l are associated with the fuzzy functions $\mu_{A_i^l}(x_i) = \exp(-(\frac{x_i - a_i^l}{b_i^l})^2)$ and $\mu_{B^l}(y^l) = 1$, respectively. M is the rules number. Through singleton function, center average defuzzification and product inference, the FLS can be expressed as

$$y(x) = \left\{ \sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right) \right\} / \left\{ \sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right) \right\} \tag{3.15}$$

where $\bar{y}^l = \max_{y\in R} \mu_{B^l}$. Define the fuzzy basis functions as

$$\xi^l(x) = \left[\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right] / \left[\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right) \right] \tag{3.16}$$

and define $\theta^T = [\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^M] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi = [\xi^1, \xi^2, \ldots, \xi^M]^T$, then FLS (3.15) can be rewritten as

$$y(x) = \theta^T \xi(x) \tag{3.17}$$

Lemma 3.1 (Boulkroune et al. [60]) *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists an FLS (3.17) such as*

$$\sup_{x\in\Omega} |f(x) - \theta^T \xi(x)| \leqslant \varepsilon \tag{3.18}$$

By Lemma 3.1, FLSs are universal approximations, i.e., they can approximate any smooth function on a compact space. Due to this approximation capability, we can assume that the nonlinear term $f(x)$ can be approximated as

$$f(x, \theta) = \theta^T \xi(x) \tag{3.19}$$

Define the optimal parameter vector θ^* as

$$\theta^* = \arg\min_{\theta\in\Omega}[\sup_{x\in U} |f(x) - f(x, \theta^*)|]$$

where Ω and U are compact regions for θ and x, respectively. Also the FLS minimum approximation error is defined as

$$\varepsilon = f(x) - \theta^{*T} \xi(x) \tag{3.20}$$

In this chapter, we use the above fuzzy logic system to approximate the unknown functions $\rho_i^u(x)$, $a_{i,j}^u(x)$, namely, there exist $\theta^*_{\rho,i}$, $\theta^*_{\alpha,i,j}$, $\varepsilon_{\rho,i}$, $\varepsilon_{\alpha,i,j}$ such that $\rho_i^u(x) = \theta^*_{\rho,i}\xi_{\rho,i}(x) + \varepsilon_{\rho,i} a_{i,j}^u(x) = \theta^*_{\alpha,i,j}\xi_{\alpha,i,j}(x) + \varepsilon_{\alpha,i,j}$. Now, the following assumptions are made.

Assumption 3.1 There exist unknown constants $\varepsilon^*_{\rho,i} > 0$, $\varepsilon^*_{\alpha,i,j} > 0$ and two known constants $\bar{M}_{\rho,s_k}$, $\bar{M}_{\alpha,s_k,j}$ such that $|\varepsilon_{\rho,i}| \leqslant \varepsilon^*_{\rho,i}$, $|\varepsilon_{\alpha,s_k,j}| \leqslant \varepsilon^*_{\alpha,i,j}$, $\varepsilon^*_{\rho,i} \leqslant \bar{M}_{\rho,s_k}$, $\varepsilon^*_{\alpha,i,j} \leqslant \bar{M}_{\alpha,s_k,j}$.

Assumption 3.2 There exist known constants M_{ρ,s_k}, $M_{\alpha,s_k,j}$ such that $||\theta^*_{\rho,s_k}|| \leqslant M_{\rho,s_k}$, $||\theta^*_{\alpha,s_k,j}|| \leqslant M_{\alpha,s_k,j}$.

3.3 Fault Diagnosis and FLS-Based Fault Accommodation

In this section, the main technical results of this chapter are given. We will first formulate the fault diagnosis and accommodation problems of the above T-S fuzzy system. We will then design a bank of SMOs to generate residuals, investigate the FDI algorithm based on the SMOs, and propose a FTC scheme to tolerate the faults by compensating for faults.

3.3.1 Preliminary

Consider the T-S fuzzy faulty system described in (3.6). We assume that only actuator faults occur and no sensor fault is involved. The following assumptions are considered.

Assumption 3.3 Matrix B_i is of full column rank and the pair (A_i, C_i) is observable.

We first design the fault diagnosis observers to detect and isolate the faults, and then, propose a FTC method to compensate the faults.

3.3.2 Fault Detection

In order to detect the actuator faults, we design a fuzzy state-space observer for the system (3.6), which is described as:

Observer rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_i\hat{x}(t) \end{cases} \tag{3.21}$$

where $L_i, i = 1, \ldots, r$ is the observer gain for the ith observer rule.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))] \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}(t) \end{cases} \tag{3.22}$$

Denote

$$e_x(t) = x(t) - \hat{x}(t), \quad e_y(t) = y(t) - \hat{y}(t) \tag{3.23}$$

then the error dynamics is described by

$$\begin{cases} \dot{e}_x(t) = \sum_{i=1}^{r} h_i(z(t))[(A_i - L_iC_i)e_x(t)] \\ e_y(t) = \sum_{i=1}^{r} h_i(z(t))C_ie_x(t) \end{cases} \tag{3.24}$$

Lemma 3.2 *The estimation error* e_x *converges asymptotically to zero if there exist common matrices* $P = P^T > 0$ *and* $Q > 0$ *with appropriate dimensions such that the following linear matrix inequality is satisfied:*

$$P(A_i - L_i C_i) + (A_i - L_i C_i)^T P \leqslant -Q, i = 1, 2, \ldots, r \tag{3.25}$$

Proof Consider the following Lyapunov function

$$V_D = e_x^T(t) P e_x(t)$$

Differentiating V_1 with respect to time$t$, one has

$$\begin{aligned} \dot{V}_D(t) &= \sum_{i=1}^{r} h_i(z(t))[e_x^T(t)(P(A_i - L_i C) + (A_i - L_i C)^T P)e_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[e_x^T(t) Q e_x(t)] \\ &\leqslant 0 \end{aligned} \tag{3.26}$$

Because $V_D(t) \in L_\infty$ is a monotonous and non-increasing bounded function, $V_D(+\infty)$ exists. Hence, we have

$$V_D(0) - V_D(+\infty) \geqslant -\int_0^{+\infty} \sum_{i=1}^{r} h_i(z(t))[e_x^T(t) Q e_x(t)],$$

which means taht $e_x(t) \in L_2$. Since $e_x(t), \dot{e}_x(t) \in L_\infty$, using the Lyapunov stability theory, we obtain $\lim_{t\to\infty} e_x(t) = 0$. Furthermore, we have $\lim_{t\to\infty} e_y(t) = 0$. The proof is completed.

From Lemma 3.2, we have

$$\begin{aligned} \dot{V}_D(t) &\leqslant -\sum_{i=1}^{r} h_i(z(t))[e_x^T(t) Q e_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[\lambda_{\min}(Q) e_x^T(t) e_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[\lambda_{\min}(Q)/\lambda_{\max}(P) e_x^T(t) P e_x(t)] \\ &\leqslant -h_i(z(t))[\lambda_{\min}(Q)/\lambda_{\max}(P)] V(t) \\ &= -\kappa V_D(t) \end{aligned} \tag{3.27}$$

where $\kappa = \lambda_{\min}(Q)/\lambda_{\max}(P) \in R$. Hence,

$$V_D(t) \leqslant e^{-\kappa t} V(0) \tag{3.28}$$

Furthermore, we have

$$\lambda_{\min}(P)||e_x(t)||^2 \leqslant e^{-\kappa t}\lambda_{\max}(P)||e_x(0)||^2 \tag{3.29}$$

Therefore the norm of the error vector satisfies

$$||e_x(t)|| \leqslant \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_x(0)||e^{-\kappa t/2} \tag{3.30}$$

Furthermore, the detection residual can be defined as

$$J(t) = ||y(t) - \hat{y}(t)|| \tag{3.31}$$

From (3.30), it can be seen that the following inequality holds in the healthy case:

$$J(t) \leqslant \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||C_i||||e_x(0)||e^{-\kappa t/2} \tag{3.32}$$

Then, the fault detection can be performed using the following mechanism:

$$\begin{cases} J(t) \leqslant T_d \text{ no fault occurred,} \\ J(t) > T_d \text{ fault has occurred} \end{cases} \tag{3.33}$$

where threshold T_d is defined as follows.

$$T_d = \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||C_i||||e_x(0)||e^{-\kappa t/2} \tag{3.34}$$

3.3.3 Fault Isolation

Since the system has m actuators, which maybe become faulty, we have $C_m^1 + C_m^2 + \cdots + C_m^m$ possible faulty cases, where C_m^i denotes the number of faulty cases where there are i faulty actuators within m actuators. Let us define the following symbol, j_i^k $(i = 1, 2, \ldots, m; k = 1, 2, \ldots, i)$ which denotes the situation where the ith actuator fails when there are k possible faulty actuators among the m actuators. Fault patterns can be described in details as follows.

Case 1: only an actuator is faulty

$$\aleph_1 : \{\aleph_1^1, \aleph_1^2, \ldots, \aleph_1^{C_m^1}\} = \{\{j_1^1\}, \{j_2^1\}, \ldots, \{j_m^1\}\}$$

In this case, there are C_m^1 fault patterns.

Case 2: only two actuators are faulty

$$\aleph_2 : \{\aleph_2^1, \aleph_2^2, \ldots, \aleph_2^{C_m^2}\} = \left\{ \begin{array}{l} \{j_1^2, j_2^2\}, \{j_1^2, j_3^2\}, \ldots, \{j_1^2, j_m^2\}, \ldots, \\ \{j_2^2, j_3^2\}, \{j_2^2, j_4^2\}, \ldots, \{j_2^2, j_m^2\}, \ldots \{j_{m-1}^2, j_m^2\} \end{array} \right\}$$

where the number of fault patterns reached a total of C_m^2.

Case i: only i actuators are faulty

$$\aleph_i : \{\aleph_i^1, \aleph_i^2, \ldots, \aleph_i^{C_m^i}\} = \{\{j_1^i, j_2^i, \ldots, j_i^i\}, \ldots, \{j_{m-i+1}^2, \ldots, j_m^2\}\}$$

where the total fault pattern is C_m^i, $i = 1, 2, \ldots, m$.

Case m: all m actuators are faulty

$$\aleph_m : \{\aleph_m^1, \ldots, \aleph_m^{C_m^m}\} = \{\{j_1^i, j_2^i, \ldots, j_m^i\}\}$$

Here, there is only one fault pattern ($C_m^m = 1$).

Now, let $\aleph_m = \{\aleph_1^1, \ldots, \aleph_1^{C_m^1}, \ldots, \aleph_i^1, \ldots, \aleph_i^{C_m^i}, \ldots, \aleph_m^1, \ldots, \aleph_m^{C_m^m}\}$. Obviously, there are $C_m^1 + C_m^2 + \cdots + C_m^m$ possible fault patterns that are numbered as the 1st, 2nd, Nth fault pattern, where $N = C_m^1 + C_m^2 + \cdots + C_m^m$.

In this chapter, it is assumed that there d actuators became faulty whose pattern s is $\aleph_d^q$, namely, $s = \aleph_d^q$. We also assume that the d actuators are the s_1th, s_2th, ..., s_dth actuators, where $1 \leqslant s_1 < s_2 < \cdots < s_d \leqslant m$. Then the faulty model can be described as:

$$\begin{cases} \dot{x}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i x_s(t) + \sum_{i=1}^{r} h_i(z(t))B_i u(t) - \\ \qquad \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[\rho_{s_k}^u(x)u_{s_k}^s(t) - \sum_{j=1}^{p} a_{s_k j}^u(x)] \right\} \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases} \tag{3.35}$$

where $B_i = [b_{i,1}, b_{i,2}, \ldots, b_{i,m}]$, $b_{i,l} \in R^{n\times 1}$, $1 \leqslant l \leqslant m$, $\rho_{s_k}^u(x)$, $a_{s,s_k}^u(x)$, $j = 1, 2, \ldots, p$ denote the time profiles of the s_kth actuator fault, which are described by (3.14), $u_{s_k}^s(t)$ is the desired controller when the s_kth actuator is healthy.

Inspired by the SMOs in [63], we are ready to present one of the results of this chapter. It is assumed that fuzzy observer and fuzzy control systems have the same premise variables $z(t)$, then the following fuzzy observers are proposed to isolate the actuator fault.

Isolation Observer Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}_{is}(t) = A_i\hat{x}_{is}(t) + L_i(y(t) - \hat{y}_{is}(t)) + B_iu(t) + \\ \qquad \sum_{k=1}^{d} \left\{ b_{i,s_k}\mu_{s_k}[\bar{\rho}^u_{s_k} u^s_{s_k}(t) + \sum_{j=1}^{p} \bar{a}^u_{s_k,j}] \right\} \\ \hat{y}_{is}(t) = C_{is}\hat{x}_{is}(t) \end{cases} \tag{3.36}$$

where $\hat{x}_{is}(t), \hat{y}_{is}(t)$ are the sth fuzzy observer's state and output, respectively. L_i is the observer's gain matrix for ith observer. The global fuzzy observer is represented as:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i\hat{x}_{is}(t) + \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}_{is}(t)) + \sum_{i=1}^{r} h_i(z(t))B_iu(t) + \\ \qquad \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}\mu_{s_k}[\bar{\rho}^u_{s_k} u^s_{s_k}(t) + \sum_{j=1}^{p} \bar{a}^u_{s_k,j}] \right\} \\ \hat{y}_s(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}_s(t) \\ \mu_{s_k} = -\dfrac{\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)}{||\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)||} \end{cases} \tag{3.37}$$

where $F_{is_k} \in R^{1\times n}$ is the s_kth row of $F_i \in R^{m\times n}$, which will be defined later, $L_i \in R^{n\times n}$ is chosen such that $A_i - L_iC_i$ is Hurwitz, $e_{xs}(t) = x_s(t) - \hat{x}_s(t)$ and $e_{ys}(t) = y(t) - \hat{y}_s(t)$ are respectively the state error and output error between the plant and the sth SMO observer. Let l denotes the practical fault pattern where the faulty actuators are the l_1th, l_2th, $\ldots$, l_{d^*}th actuators, $1 \leqslant l_1 < l_2 < \cdots < l_{d^*} \leqslant m$.

For $s = l$, namely, $d = d^*$, $l_1 = s_1, l_2 = s_2, \ldots, l_{d^*} = s_d$, the error dynamics is obtained from (3.35) and (3.36).

$$
\begin{aligned}
\dot{e}_{xs}(t) &= \sum_{i=1}^{r} h_i(z(t))A_i e_{is}(t) - \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}_{is}(t)) + \\
&\quad \sum_{i=1}^{r} h_i(z(t)) \sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\
&\quad \sum_{j=1}^{p} (a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})] \\
&= \sum_{i=1}^{r} h_i(z(t))\{(A_i - L_iC_i)e_{is}(t) + \\
&\quad \sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\
&\quad \sum_{j=1}^{p} (a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})]\}
\end{aligned}
\tag{3.38}
$$

For $s \neq 1$, namely, $d \neq d^* ord = d^*$ and at least there exists l_i such that $l_i \neq s_i, i = 1, 2, \ldots, d$, we have

$$
\begin{aligned}
\dot{e}_{xs}(t) =& \sum_{i=1}^{r} h_i(z(t))(A_i - L_iC_i)e_{is}(t) + \\
& \sum_{i=1}^{r} h_i(z(t))[(-\sum\nolimits_{k=1}^{d^*} b_{i,l_k}\rho^u_{l_k}(x)u^s_{l_k}(t) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\
& \sum_{j=1}^{p} (\sum\nolimits_{k=1}^{d^*} b_{i,l_k}a^u_{l_k,j}(x) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{a}^u_{s_k,j})]
\end{aligned}
\tag{3.39}
$$

The stability of the state error dynamics is guaranteed by the following theorem.

Theorem 3.1 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P and matrices $L_i, F_i, Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$(A_i - L_iC_i)^T P + P(A_i - L_iC_i) \leqslant -Q \tag{3.40}$$

$$PB_i = (F_iC_i)^T \tag{3.41}$$

Then, when the lth pattern is the actual fault pattern i.e., $s = l$, we have $\lim_{t\to\infty} e_{xs} = 0$, and for $s \neq l$, we have $\lim_{t\to\infty} e_{xs} \neq 0$.

Proof (1) For $s = l$, according to (3.38), we have

$$\dot{e}_{xs}(t) = \sum_{i=1}^{r} h_i(z(t))\{(A_i - L_i C_i)e_{is}(t)+ \sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u_{s_k}(t)|)+ \sum_{j=1}^{p}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})]\}$$

Define the following Lyapunov function

$$V_I(t) = e^T_{xs}(t)Pe_{xs}(t) \tag{3.42}$$

Differentiating V_2 with respect to time t, and using (3.40), one has

$$\dot{V}_I(t) = \dot{e}^T_{xs}(t)Pe_{xs}(t) + e^T_{xs}(t)P\dot{e}_{xs}(t) \leqslant -e^T_{xs}(t)Qe_{xs}(t) + 2e^T_{xs}(t)P\sum_{i=1}^{r} h_i(z(t))\sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \sum\nolimits_{k=1}^{d}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})]$$

From $\mu_{s_k} = -\sum\limits_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys_k}(t)/||\sum\limits_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys_k}(t)||$ and (3.41), one has

$$2e^T_{xs}(t)P\sum_{i=1}^{r} h_i(z(t))\sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) \leqslant 0,$$

$$2e^T_{xs}(t)P\sum_{i=1}^{r} h_i(z(t))\sum\nolimits_{k=1}^{d}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j}) \leqslant 0.$$

Hence,

$$\dot{V}_I(t) \leqslant -e^T_{xs}(t)Qe_{xs}(t) \leqslant 0 \tag{3.43}$$

Because $V_I(t) \in L_\infty$ is a monotonous and non-increasing bounded function, $V_I(+\infty)$ exists. Hence, we have $V_I(0) - V_I(+\infty) \geqslant -\int_0^{+\infty} e^T_{xs}(t)Qe_{xs}(t)$, i.e., $e_{xs}(t) \in L_2$. Since $e_{xs}(t)$ and $\dot{e}_{xs}(t) \in L_\infty$, using the Lyapunov stability theory, we have $\lim\limits_{t\to\infty} e_{xs}(t) = 0$. Thus, we have $\lim\limits_{t\to\infty} e_{ys}(t) = 0$.

(2) For $s \neq l$, it follows from (3.35) and (3.39) that:

$$\dot{e}_{xs}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i - L_i C_i)e_{is}(t) + \sum_{i=1}^{r} h_i(z(t))[(-\sum\nolimits_{k1=1}^{d^*} b_{i,l_{k1}} \rho^u_{l_{k1}}(x)u^s_{l_k}(t) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \sum_{j=1}^{p}(\sum\nolimits_{k1=1}^{d^*} b_{i,l_{k1}} a^u_{l_{k1}j}(x) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{a}^u_{s_k j})]$$

Because matrix B_i is of full column rank (Assumption 3.1), we know that b_{is_k} and $b_{il_{k1}}$ are linearly independent. Therefore,

$$\lim_{t\to\infty} \sum_{i=1}^{r} h_i(z(t))[(-\sum\nolimits_{k=1}^{d^*} b_{i,l_k} \rho^u_{l_k}(x)u^s_{l_k}(t) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \sum_{j=1}^{p}(\sum\nolimits_{k=1}^{d^*} b_{i,l_k} a^u_{l_k j}(x) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{a}^u_{s_k j})] \neq 0 \tag{3.44}$$

Thus, we have $\lim_{t\to\infty} e_{xs}(t) \neq 0$ and $\lim_{t\to\infty} e_{ys}(t) \neq 0$.

From (1) and (2), we obtain the conclusion. This ends the proof.

Now, we denote the residuals between the real system and SMOs as follows:

$$J_s(t) = \left\| e_{ys}(t) \right\| = \left\| \hat{y}_s(t) - y(t) \right\|, \quad 1 \leqslant s \leqslant m \tag{3.45}$$

According to Theorem 3.1, when the actual fault pattern is $s = l$, the residual $J_s(t)$ will tend to zero; while for any $s \neq l$, $J_s(t)$ does not equal zero. Furthermore, from Lemma 3.2, we have, if $l = s$,

$$J_s(t) \leqslant \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2} \tag{3.46}$$

and if $l \neq s$, then

$$J_s(t) > \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2} \tag{3.47}$$

Hence, the isolation law for actuator fault can be designed as

$$\begin{cases} J_s(t) \leqslant T_I, l = s \Rightarrow \text{the } l_1\text{th}, l_2\text{th}, \ldots, l_d\text{th actuators are faulty} \\ J_s(t) > T_I, l \neq s \end{cases} \tag{3.48}$$

where threshold T_I is defined as follows.

$$T_I = \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2}$$

Notice that, the denominator of $\mu_{s_k} = -\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys_k}(t)/||\sum_{i=1}^{r} h_i(z(t))F_{is_k} e_{ys_k}(t)||$ in (3.37), contains $e_{ys}(t)$. Just as pointed out in [63], the chattering phenomenon occurs when $e_{ys}(t) \to 0$ in practice. Inspired by [63], in order to reduce this chattering in practical applications, we modify SMOs (3.37) by introducing a positive constantδ as follows:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i\hat{x}_{is}(t) + \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}_{is}(t)) + \sum_{i=1}^{r} h_i(z(t))B_i u(t) + \\ \qquad \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}\mu_{s_k}[\bar{\rho}^u_{s_k} u^s_{s_k}(t) + \sum_{j=1}^{p} \bar{a}^u_{s_k,j}] \right\} \\ \hat{y}_s(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}_s(t) \\ \mu'_{s_k} = -\dfrac{\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)}{||\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)|| + \delta} \end{cases} \tag{3.49}$$

where $\delta > 0 \in R$ is a constant. Obviously, the denominator of μ'_{s_k} will converge asymptotically to δ when $e_{ys} \to 0$, which reduces this chattering phenomenon.

3.3.4 *Fuzzy Logic Systems-Based Fault Accommodation with Available System State*

After fault isolation, the next task is fault accommodation. Before this task, we investigate firstly the following normal systems (fault-free), and drive the ideal control $u^s(t)$ when all actuators are healthy.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u^s(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases} \tag{3.50}$$

The parallel distributed compensation (PDC) technique offers a procedure to design a fuzzy control law from a given T-S fuzzy model. In the PDC design, each control rule is designed from the corresponding rule of T-S fuzzy model. The designed fuzzy controller has the same fuzzy sets as the considered fuzzy system.

Control Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$u_i^s(t) = K_i x(t)$$

and the overall fuzzy controller is given as follows:

$$u^s(t) = \sum_{i=1}^{r} h_i(z(t)) K_i x(t) \tag{3.51}$$

where the controller gain matrix K_i is determined by solving the following condition:

$$P(A_i+K_iB_i)+(A_i + K_iB_i)^T P+(A_i + K_iB_i)^T PS_1P(A_i+K_iB_i)+PS_2P \leqslant -Q \tag{3.52}$$

where $P = P^T > 0, Q > 0, S_1 > 0, S_2 > 0$ are matrices with appropriate dimensions.

Define tracking error $\bar{e} = y - \omega_d$. The tracking error dynamics is obtained from the above equations,

$$\dot{\bar{e}} = \dot{y} - \dot{\omega}_d = C_i\dot{x} - \dot{\omega}_d = \sum_{i=1}^{r} h_i(z(t))[C_iA_ix(t) + C_iB_iu^s(t)] - \dot{\omega}_d$$

Because all the states are supposed to be available, we have $C_i = I_{m\times m}$. The tracking error dynamics can be simplified as follows:

$$\begin{aligned} \dot{\bar{e}} = \dot{x} - \dot{\omega}_d &= \sum_{i=1}^{r} h_i(z(t))[A_ix(t) + B_iK_ix(t) - \dot{\omega}_d] \\ &= \sum_{i=1}^{r} h_i(z(t))[(A_i + B_iK_i)x(t) - \dot{\omega}_d] \\ &= \sum_{i=1}^{r} h_i(z(t))[(A_i + B_iK_i)e(t) + \omega_d - \dot{\omega}_d] \end{aligned} \tag{3.53}$$

Define the following Lyapunov function

$$V_0 = \bar{e}^T P\bar{e}$$

where $P = P^T > 0$.

Differentiating V_0 with respect to time t, leads to

$$\dot{V}_0 = \sum_{i=1}^{r} h_i(z(t))[e^T(t)(P(A_i + K_iB_i) + (A_i + K_iB_i)^T P)e(t)] - \sum_{i=1}^{r} h_i(z(t))[2e^T(t)(A_i + K_iB_i)^T P(\omega_d - \dot{\omega}_d)] - \sum_{i=1}^{r} h_i(z(t))[2e^T(t)P(\omega_d - \dot{\omega}_d)] + \sum_{i=1}^{r} h_i(z(t))[2(\omega_d - \dot{\omega}_d)^T P(\omega_d - \dot{\omega}_d)] \quad (3.54)$$

Since

$$-2\bar{e}^T(A_i + K_iB_i)^T P(\omega_d - \dot{\omega}_d) \leqslant \bar{e}^T(t)(A_i + K_iB_i)^T PS_1P(A_i + K_iB_i)\bar{e} + (\omega_d - \dot{\omega}_d)^T S_1^{-1}(\omega_d - \dot{\omega}_d)$$

$$-2\bar{e}^T P(\omega_d - \dot{\omega}_d) \leqslant \bar{e}^T(t)PS_2P\bar{e} + (\omega_d - \dot{\omega}_d)^T S_2^{-1}(\omega_d - \dot{\omega}_d)$$

(3.54) can be re-written as follows:

$$\dot{V}_0 \leqslant \sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)\Delta_1\bar{e}(t)] + \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T(S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d)]$$

where $\Delta_1 = P(A_i + K_iB_i) + (A_i + K_iB_i)^T P + (A_i + K_iB_i)^T PS_1P(A_i + K_iB_i) + PS_2P$.

Obviously, if

$$\Delta_1 = P(A_i + K_iB_i) + (A_i + K_iB_i)^T P + (A_i + K_iB_i)^T PS_1P(A_i + K_iB_i) + PS_2P \leqslant -Q,$$

then

$$\dot{V}_0 \leqslant -\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0 \leqslant -\lambda_0 V_0 + \mu_0,$$

where $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T(S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d)]$, $\lambda_0 = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$, $Q = Q^T > 0$.

Then, one has $\frac{d}{dt}(V_0(t)e^{\lambda_0 t}) \leqslant e^{\lambda_0 t}\mu_0$. Furthermore,

$$0 \leqslant V_0(t) \leqslant \frac{\mu_0}{\lambda_0} + [V_0(0) - \frac{\mu_0}{\lambda_0}]e^{-\lambda_0 t} \leqslant \frac{\mu_0}{\lambda_0} + V_0(0).$$

Therefore, the error system (3.53) is asymptotically stable. Moreover, $\bar{e}(t)$ is semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $|\bar{e}| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, where $\alpha = \mu_0/\lambda_0 + V_0(0)$.

After obtaining the desired control $u^s(t)$, we will design fault-tolerant control $u(t)$ such that the same control objective can be achieved in spite of actuator faults.

On the basis of the desired control $u^s(t)$, the fault tolerant controller is constructed as

$$u_{s_k} = \frac{u^s_{s_k} - \sum_{j=1}^{p} \hat{a}_{s_k,j}(x, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j}}{1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}} \tag{3.55}$$

where $\hat{\theta}_{\rho,s_k}$, $\hat{\theta}_{\alpha,s_k,j}$, $\hat{\rho}_{\rho,s_k}(x, \hat{\theta}_{\rho,s_k})$, $\hat{a}_{i,j}(x, \hat{\theta}_{\alpha,s_k,j})$ are the estimations of θ^*_{ρ,s_k}, $\theta^*_{\alpha,s_k,j}$, $\rho_{s_k}(x, \theta^*_{\rho,s_k})$, $a_{s_k,j}(x, \theta^*_{\alpha,s_k,j})$, which are used to compensate for the gain and bias faults $\rho_{s_k}(x)$, $\alpha_{s_k,j}(x)$, and $\rho_{s_k}(x) = \rho_{s_k}(x, \theta^*_{\rho,s_k}) + \varepsilon_{s_k}$, $a_{s_k,j}(x) = a_{s_k,j}(x, \theta^*_{\alpha,s_k,j}) + \varepsilon_{s_k,j}$, ε_{s_k}, $\varepsilon_{s_k,j}$ are approximation errors, θ^*_{ρ,s_k}, $\theta^*_{\alpha,s_k,j}$ are optimal vectors.

Consider the following faulty system

$$\begin{aligned} \dot{x}_s(t) = & \sum_{i=1}^{r} h_i(z(t))A_i x_s(t) + \sum_{i=1}^{r} h_i(z(t))B_i u^s(t) - \\ & \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[\rho^u_{s_k}(x)u^s_{s_k}(t) - \sum_{j=1}^{p} a^u_{s_k,j}(x)] \right\} \end{aligned} \tag{3.56}$$

Submitting the fault-tolerant control law (3.55) to the faulty system (3.56), it yields

$$\begin{aligned} \dot{x}_s(t) = & \sum_{i=1}^{r} h_i(z(t))A_i x_s(t) + \sum_{i=1}^{r} h_i(z(t))B_i u^S(t) + \\ & \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[(\tilde{\theta}^T_{\rho,s_k}\xi_{\rho,s_k}(x) + \tilde{\varepsilon}_{\rho,s_k})\kappa_k - \sum_{j=1}^{p} \tilde{\theta}^u_{\alpha,s_k,j}\xi_{\alpha,s_k,j}(x) - \tilde{\varepsilon}_{\rho,s_k}] \right\} \end{aligned} \tag{3.57}$$

where $\tilde{\theta}_{\alpha,s_k,j} = \hat{\theta}_{\alpha,s_k,j} - \theta^*_{\alpha,s_k,j}$, $\kappa_k = \left(\frac{u^s_{s_k} - \sum_{j=1}^{p}[\hat{\alpha}^u_{\alpha,s_k,j}(x,\hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j}]}{1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}}\right)$, $\tilde{\theta}_{\rho,s_k} = \hat{\theta}_{\rho,s_k} - \theta^*_{\rho,s_k}$. Further, the error dynamics is obtained:

$$\begin{aligned} \dot{e} = & \sum_{i=1}^{r} h_i(z(t))[(A_i + B_i K_i)e(t) + \omega_d - \dot{\omega}_d] + \\ & \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[(\tilde{\theta}^T_{\rho,s_k}\xi_{\rho,s_k}(x) + \tilde{\varepsilon}_{\rho,s_k})\kappa_k - \sum_{j=1}^{p} \tilde{\theta}^u_{\alpha,s_k,j}\xi_{\alpha,s_k,j}(x) - \tilde{\varepsilon}_{\rho,s_k}] \right\} \end{aligned} \tag{3.58}$$

Now, an adaptive fault accommodation algorithm is proposed to control the faulty system. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 3.2 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P, real matrices K_i and $Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$\begin{aligned} &P(A_i + K_i B_i) + (A_i + K_i B_i)^T P + \\ &\quad (A_i + K_i B_i)^T P S_1 P(A_i + K_i B_i) + P S_2 P \leqslant -Q \end{aligned} \tag{3.59}$$

Consider the control law (3.55) and the adaptive laws given as follows:

$$\dot{\hat{\theta}}_{\rho,s_k} = \begin{cases} -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa_k, \ \textit{if } ||\hat{\theta}_{\rho,s_k}|| < M_{\rho,s_k} \textit{ or} \\ \quad ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \textit{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa_k \geqslant 0; \\ -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa_k + \eta_1 \bar{e}^T P b_{i,s_k} \kappa_k \dfrac{\theta_{\rho,s_k}\theta^T_{\rho,s_k}}{||\hat{\theta}_{if}||^2} \xi^u_{\rho,s_k}(x), \\ \quad \textit{if } ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \textit{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa_k < 0 \end{cases} \tag{3.60}$$

$$\dot{\hat{\theta}}_{\alpha,s_k,j} = \begin{cases} \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x), \ \textit{if } ||\hat{\theta}_{\alpha,s_k,j}|| < M_{\alpha,s_k,j} \\ \quad \textit{or } ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \textit{ and } -\eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x) \geqslant 0; \\ \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x) + \eta_2 \bar{e}^T P b_{i,s_k} \dfrac{\hat{\theta}_{\alpha,s_k,j}\hat{\theta}^T_{\alpha,s_k,j}}{||\hat{\theta}_{\alpha,s_k,j}||^2} \xi^u_{\alpha,s_k,j}(x), \\ \quad \textit{if } ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \textit{ and } -\bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x) < 0 \end{cases} \tag{3.61}$$

$$\dot{\hat{\varepsilon}}_{\rho,s_k} = \begin{cases} 0, \ \textit{if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\rho,s_k} \textit{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \kappa_k > 0 \\ \quad \textit{or } \hat{\varepsilon}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \textit{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \kappa_k < 0; \\ -\eta_3 \bar{e}^T P b_{i,s_k} \kappa_k, \textit{ otherwise} \end{cases} \tag{3.62}$$

$$\dot{\hat{\varepsilon}}_{\alpha,s_k,j} = \begin{cases} 0, \textit{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\alpha,s_k,j} \textit{ and } \eta_4 \bar{e}^T P b_{i,s_k} > 0 \\ \quad \hat{\varepsilon}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \textit{ and } \eta_4 \bar{e}^T P b_{i,s_k} < 0; \\ \eta_4 \bar{e}^T P b_{i,s_k}, \textit{ otherwise} \end{cases} \tag{3.63}$$

where $\eta_i > 0, i = 1, \ldots, 4$ denote the adaptive rates, then the error system (3.59) is asymptotically stable. Moreover, $\bar{e}(t)$, $\tilde{\theta}_{\rho,s_k}$ and $\tilde{\theta}_{\alpha,s_k,j}$ are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $||\bar{e}|| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_{\rho,s_k}|| \leqslant \sqrt{2\eta_1\alpha}$, and $||\tilde{\theta}_{\alpha,s_k,j}|| \leqslant \sqrt{2\eta_2\alpha}$, where $\alpha = \frac{\mu_0}{\lambda_0} + V(0)$, $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$, $\mu = \sum_{i=1}^{r} h_i(z(t))(\frac{2}{\eta_2}\bar{\theta}^2_{\rho,s_k} + \sum_{j=1}^{p} \frac{2}{\eta_2}\theta^2_{\alpha,s_k,j}) + \mu_0$, and $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T (S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d)]$.

Proof Define the following smooth function

$$V = V_1 + V_2 + V_3 + V_4 + V_5$$

where

$$V_1 = \bar{e}^T P \bar{e},\ \ V_2 = \sum_{i=1}^{r} h_i(z(t))(\frac{1}{2\eta_1}\tilde{\theta}^T_{\rho,s_k}\tilde{\theta}_{\rho,s_k})$$

$$V_3 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{2\eta_2}\tilde{\theta}^T_{\alpha,s_k,j}\tilde{\theta}_{\alpha,s_k,j}),\ \ V_4 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{2\eta_3}\tilde{\varepsilon}^2_{\rho,s_k})$$

$$V_5 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{2\eta_4}\varepsilon^2_{\alpha,s_k,j})$$

Differentiating V with respect to time t, it leads to

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + \dot{V}_5$$

where

$$\begin{aligned}\dot{V}_1 \leqslant & -\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0 + \\ & \sum_{i=1}^{r} h_i(z(t))2\bar{e}^T P \sum_{k=1}^{d} b_{i,s_k}[(\tilde{\theta}^T_{\rho,s_k}\xi^u_{\rho,s_k}(x) + \tilde{\varepsilon}_{\rho,s_k})\kappa] + \\ & \sum_{i=1}^{r} h_i(z(t))2\bar{e}^T P \sum_{k=1}^{d} b_{i,s_k}[\sum_{j=1}^{p}\tilde{\theta}^T_{\alpha,s_k,j}\xi^u_{\alpha,s_k,j}(x) + \tilde{\varepsilon}_{\alpha,s_k,j}]\end{aligned}$$

$$\dot{V}_2 = \sum_{i=1}^{r} h_i(z(t))\frac{1}{\eta_1}\tilde{\theta}^T_{\rho,s_k}\dot{\tilde{\theta}}_{\rho,s_k},\ \ \dot{V}_3 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{\tilde{\theta}^T_{\alpha,s_k,j}\dot{\tilde{\theta}}_{\alpha,s_k,j}}{\eta_2}$$

$$\dot{V}_4 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{\eta_3}\tilde{\varepsilon}_{\rho,s_k}\dot{\tilde{\varepsilon}}_{\rho,s_k}),\ \ \dot{V}_5 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{\eta_4}\tilde{\varepsilon}_{\alpha,s_k,j}\dot{\tilde{\varepsilon}}_{\alpha,s_k,j})$$

Since $u_{s_k} = \frac{(u^s_{s_k} - \sum_{j=1}^{p}\hat{a}_{s_k,j}(x,\hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j})}{1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}}$, then

$$\begin{aligned} u^f_{s_k} &= (1-\rho_{s_k}(x))u_{s_k} + \sum\nolimits_{j=1}^{p} a_{s_k,j}(x) \\ &= u^s_{s_k} - \sum\nolimits_{j=1}^{p} \tilde{\theta}_{\alpha,s_k,j}\xi_{\alpha,s_k,j}(x) - \sum\nolimits_{j=1}^{p} \tilde{\varepsilon}_{\alpha,s_k,j} + \tilde{\theta}_{\rho,s_k}\varDelta + \varepsilon_{\rho,s_k}\varDelta \\ &= u^s_{s_k} - \sum\nolimits_{j=1}^{p} \tilde{\theta}_{\alpha,s_k,j}\xi_{\alpha,s_k,j} + \sum\nolimits_{j=1}^{p} \tilde{\varepsilon}_{\alpha,s_k,j} + \tilde{\theta}_{\rho,s_k}\varDelta + \tilde{\varepsilon}_{\rho,s_k}\varDelta \end{aligned}$$

where $\varDelta = \frac{u^s_{s_k} - \sum_{j=1}^{p} \hat{a}_{s_k,j}(x,\hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\rho,s_k,j}}{1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}}$. Furthermore, one has

$$\begin{aligned} \dot{V} \leqslant & -\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0 + \\ & \sum_{i=1}^{r} h_i(z(t))2\bar{e}^T P \sum_{k=1}^{d} b_{i,s_k}[\tilde{\theta}^T_{\rho,s_k}(\xi^u_{\rho,s_k}(x)\varDelta + \frac{1}{\eta_1}\dot{\tilde{\theta}}_{\rho,s_k}) - \tilde{\varepsilon}_{\rho,s_k}(\varDelta + \frac{1}{\eta_3}\dot{\tilde{\varepsilon}}_{\rho,s_k}) - \\ & \sum_{j=1}^{p} \tilde{\theta}^T_{\alpha,s_k,j}(\xi^u_{\alpha,s_k,j}(x) - \frac{\dot{\tilde{\theta}}_{\alpha,s_k,j}}{\eta_2}) - \sum_{j=1}^{p} \tilde{\varepsilon}_{\alpha,s_k,j}(1 - \frac{\dot{\tilde{\theta}}_{\alpha,s_k,j}}{\eta_2})] \end{aligned}$$

Substituting the adaptive laws (3.60–3.63) into the above equation, it yields

$$\dot{V} \leqslant -\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0$$

Since $||\hat{\theta}_{\rho,s_k}|| \leqslant M_{\rho,s_k}$, $||\hat{\theta}_{\alpha,s_k,j}|| \leqslant M_{\alpha,s_k,j}$, which can be guaranteed by using the adaptive laws (3.60) and (3.61), when Assumptions 3.1 and 2.2 (i.e., $||\theta^*_{\rho,s_k}|| \leqslant M_{\rho,s_k}$, $||\theta^*_{\alpha,s_k,j}|| \leqslant M_{\alpha,s_k,j}$) are satisfied, one has

$$\dot{V} \leq= \lambda V(t) + \mu$$

where $\mu = \sum_{i=1}^{r} h_i(z(t))[\frac{4}{\eta_1}M^2_{\rho,s_k} + \sum_{j=1}^{p} \frac{4}{\eta_2}M^2_{\alpha,s_k,j} + \frac{4}{\eta_3}\bar{M}^2_{\rho,s_k} + \sum_{j=1}^{p} \frac{4}{\eta_4}\bar{M}^2_{\alpha,s_k,j})] + \mu_0$, $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$.

Then, one has, $\frac{d}{dt}(V(t)e^{\lambda t}) \leqslant e^{\lambda t}\mu$. Furthermore,

$$0 \leqslant V(t) \leqslant \frac{\mu}{\lambda} + [V(0) - \frac{\mu}{\lambda}]e^{-\lambda t} \leqslant \frac{\mu}{\lambda} + V(0)$$

Let $\alpha = \frac{\mu}{\lambda} + V(0)$, one has $|\bar{e}| \leqslant \sqrt{\frac{\alpha}{\lambda_{\min}(P)}}$, $|\tilde{\theta}_{\rho,s_k}| \leqslant \sqrt{2\eta_1\alpha}$, and $|\tilde{\theta}_{\alpha,s_k,j}| \leqslant \sqrt{2\eta_2\alpha}$. This ends the proof.

3.3.5 *Modified Fault Accommodation with Available System State*

In the above subsection, the fault tolerant controller was constructed as

$$u_{s_k} = \frac{u_{s_k}^s - \sum_{j=1}^{p} \hat{a}_{s_k,j}(x, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j}}{1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}}$$

Unfortunately, there may exist controller singularity when $1-\hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k} = 0$.

In order to avoid such singularity, the fault tolerant controller is modified as follows

$$u_{s_k} = \frac{(1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})(u_{s_k}^s - \sum_{j=1}^{p} \hat{a}_{s_k,j}(x, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j})}{(1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})^2 + \varepsilon} \tag{3.64}$$

where $\varepsilon > 0 \in R$ is a design constant. Correspondingly, the adaptive laws in Theorem 3.2 are re-designed as follows

$$\dot{\hat{\theta}}_{\rho,s_k} = \begin{cases} -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa', \text{ if } ||\hat{\theta}_{\rho,s_k}|| < M_{\rho,s_k} \text{or} \\ \quad ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa' \geqslant 0; \\ -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa' + \eta_1 \bar{e}^T P b_{i,s_k} \kappa' \dfrac{\theta_{\rho,s_k}\theta^T_{\rho,s_k}}{||\hat{\theta}_{if}||^2} \xi^u_{\rho,s_k}(x), \\ \text{if } ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa' < 0 \end{cases} \tag{3.65}$$

$$\dot{\hat{\theta}}_{\alpha,s_k,j} = \begin{cases} \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x), \text{ if } ||\hat{\theta}_{\alpha,s_k,j}|| < M_{\alpha,s_k,j} \text{ or} \\ \quad ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -s_i \hat{\theta}^T_{if} \xi_{if} \geqslant 0; \\ \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x) + \eta_2 \bar{e}^T P b_{i,s_k} \dfrac{\hat{\theta}_{\alpha,s_k,j}\hat{\theta}^T_{\alpha,s_k,j}}{||\hat{\theta}_{\alpha,s_k,j}||^2} \xi^u_{\alpha,s_k,j}(x), \\ \quad \text{if } ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -\bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x) < 0, \end{cases} \tag{3.66}$$

$$\dot{\hat{\varepsilon}}_{\alpha,s_k,j} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 \bar{e}^T P b_{i,s_k} > 0 \\ \quad \text{or } \hat{\varepsilon}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 \bar{e}^T P b_{i,s_k} < 0 \\ \quad \eta_4 \bar{e}^T P b_{i,s_k}, \text{ otherwise} \end{cases} \tag{3.67}$$

$$\dot{\hat{\varepsilon}}_{\rho,s_k} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \kappa' > 0 \\ \quad \text{or } \hat{\varepsilon}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \kappa' < 0, \\ -\eta_3 \bar{e}^T P b_{i,s_k} \kappa', \text{ otherwise} \end{cases} \tag{3.68}$$

where $\kappa' = \frac{(1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})[u^s_{s_k}-\sum_{j=1}^{p}\hat{\theta}^T_{\alpha,s_k,j}\xi^u_{\alpha,s_k,j}(x)]}{(1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})^2+\varepsilon}$, $\eta_l > 0, l = 1, \ldots, 4$ denote the adaptive rates.

Now, a modified adaptive fault accommodation algorithm is proposed to control the faulty system. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 3.3 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P, real matrices K_i and $Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$\begin{aligned} &P(A_i + K_iB_i) + (A_i + K_iB_i)^T P+ \\ &(A_i + K_iB_i)^T PS_1P(A_i + K_iB_i) + PS_2P \leqslant -Q \end{aligned} \tag{3.69}$$

when the control law (3.64) and adaptive laws (3.65–3.68) are applied, the error system (3.58) is asymptotically stable. Moreover $\bar{e}(t)$, $\tilde{\theta}_{\rho,s_k}$ and $\tilde{\theta}_{\alpha,s_k,j}$ are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $||\bar{e}|| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_{\rho,s_k}|| \leqslant \sqrt{2\eta_1\alpha}$, and $||\tilde{\theta}_{\alpha,s_k,j}|| \leqslant \sqrt{2\eta_2\alpha}$ where $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$, $\mu = \sum_{i=1}^{r} h_i(z(t))(\frac{2}{\eta_2}\bar{\theta}^2_{\rho,s_k} + \sum_{j=1}^{p}\frac{2}{\eta_2}\bar{\theta}^2_{\alpha,s_k,j}) + \mu_0$, and $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T(S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d) + \omega]$, $\alpha = \frac{\mu}{\lambda} + V(0)$.

Proof Similar to the proof of Theorem 3.2, it is easy to obtain the conclusion. The detailed proof is omitted.

3.3.6 FLSs-Based Fault Accommodation with Unavailable System State

Notice that, the FTC (3.55) and the modified FTC (3.64) are designed under the condition that system states are measurable. In fact, in some situations, system state may be unavailable, and the above FTC (3.55) and (3.64) do not work. In this case, observers (3.21) and (3.22) may be used to obtain the estimation $\hat{x}$ of system state x, and design the following observer-based FTC.

$$u_{s_k} = \frac{(1 - \hat{\rho}_{s_k}(\hat{x}, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})(u^s_{s_k} - \sum_{j=1}^{p} \hat{a}_{s_k,j}(\hat{x}, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j})}{(1 - \hat{\rho}_{s_k}(\hat{x}, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})^2 + \varepsilon} \tag{3.70}$$

Correspondingly, the adaptive laws in Theorem 3.3 are re-designed as follows:

$$\dot{\hat{\theta}}_{\rho,s_k} = \begin{cases} -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega, \text{ if } ||\hat{\theta}_{\rho,s_k}|| < M_{\rho,s_k} \text{ or} \\ \quad ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega \geqslant 0; \\ -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega + \\ \quad \eta_1 \bar{e}^T P b_{i,s_k} \omega \dfrac{\theta_{\rho,s_k}\theta^T_{\rho,s_k}}{||\hat{\theta}_{if}||^2} \xi^u_{\rho,s_k}(\hat{x}), \\ \text{if } ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega < 0 \end{cases} \tag{3.71}$$

$$\dot{\hat{\theta}}_{\alpha,s_k,j} = \begin{cases} \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}), \text{ if } ||\hat{\theta}_{\alpha,s_k,j}|| < M_{\alpha,s_k,j} \text{ or} \\ \quad ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -\bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}) \geqslant 0; \\ \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}) + \eta_2 \bar{e}^T P b_{i,s_k} \dfrac{\hat{\theta}_{\alpha,s_k,j}\hat{\theta}^T_{\alpha,s_k,j}}{||\hat{\theta}_{\alpha,s_k,j}||^2} \xi^u_{\alpha,s_k,j}(\hat{x}), \\ \quad \text{if } ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -\bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}) < 0 \end{cases} \tag{3.72}$$

$$\dot{\hat{\varepsilon}}_{\rho,s_k} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \omega > 0 \\ \quad \text{or } \hat{\varepsilon}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \omega < 0; \\ -\eta_1 \bar{e}^T P b_{i,s_k} \omega, \text{ otherwise} \end{cases} \tag{3.73}$$

$$\dot{\hat{\varepsilon}}_{\alpha,s_k,j} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 \bar{e}^T P b_{i,s_k} > 0 \text{ or} \\ \quad \hat{\varepsilon}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} < 0; \\ \eta_4 \bar{e}^T P b_{i,s_k}, \text{ otherwise} \end{cases} \tag{3.74}$$

where $\omega = \frac{(1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})[u^s_{s_k} - \sum_{j=1}^{p} \hat{\theta}^T_{\alpha,s_k,j} \xi^u_{\alpha,s_k,j}(\hat{x})]}{(1-\hat{\rho}_{s_k}(\hat{x},\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})^2+\varepsilon}$, $\eta_l > 0, l = 1, \ldots, 4$ denote the adaptive rates.

Now, an observer-based adaptive fault accommodation algorithm is proposed to control the faulty system. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 3.4 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P, real matrices K_i and $Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$P(A_i+K_iB_i)+(A_i+K_iB_i)^T P+(A_i+K_iB_i)^T PS_1P(A_i+K_iB_i)+PS_2P \leqslant -Q \tag{3.75}$$

when the control law (3.70) and adaptive laws (3.71–3.74) are applied, then the error system (3.58) is asymptotically stable. Moreover $\bar{e}(t)$, $\tilde{\theta}_{\rho,s_k}$ and $\tilde{\theta}_{\alpha,s_k,j}$

are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $||\bar{e}|| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_{\rho,s_k}|| \leqslant \sqrt{2\eta_1\alpha}$, *and* $||\tilde{\theta}_{\alpha,s_k,j}|| \leqslant \sqrt{2\eta_2\alpha}$, *where* $\alpha = \mu/\lambda + V(0)$, $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$, $\mu = \sum_{i=1}^{r} h_i(z(t))(\frac{2}{\eta_2}\bar{\theta}^2_{\rho,s_k} + \sum_{j=1}^{p}\frac{2}{\eta_2}\bar{\theta}^2_{\alpha,s_k,j}) + \mu_0$, *and* $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T \cdot (S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d) + \omega]$.

Proof Similar to the proof of Theorem 3.2, it is easy to obtain the conclusion. The detailed proof is omitted.

3.4 Simulation Results

To verify the effectiveness of the proposed method, we consider the re-entry phase of a NSV with the altitude $H = 40$ km and speed $V = 2500$ m/s as the initial states. The symmetric, positive definite moment of inertia tensor is given as follows:

$$J = \begin{bmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 0 & 1376852 \end{bmatrix}$$

Consider that the nonlinearity of NSV re-entry attitude dynamics mainly comes from attack angle α and attitude angular velocity ω. In NSV re-entry phase $\alpha \in [0, \pi/4]$, we assume that α has two related fuzzy sets $\{\alpha = 0\}$ and $\{\alpha = \pi/4\}$, the corresponding membership functions are given by:

$$M_{\omega=0} = (1 - \frac{1}{1 + \exp[-6 - 28\omega)]})\frac{1}{1 + \exp[6 - 28\omega)]}$$

$$M_{\omega=-0.5} = (\frac{1}{1 + \exp[6 + 28\omega)]}), M_{\omega=0.5} = (1 - \frac{1}{1 + \exp[-6 + 28\omega)]})$$

We choose six operating points:

$$[\alpha, \omega] \in \{[0, -0.5], [0, 0], [0, 0.5], [\pi/4, -0.5], [\pi/4, 0], [\pi/4, 0.5]\}$$

Under the membership functions and the six operating points, six plant rules and six control rules can be defined. All A_i and B_i can be obtained by substituting the six operating points to $f(x_\omega)$, $g(x_\omega)$. The detailed matrix parameters are given in [62].

Rule 1: IF ω is about $-$ 0.5 rad/s and α is about 0 rad, THEN
$\dot{x}(t) = A_1x(t) + B_1u, \ y(t) = C_1x(t)$
Rule 2: IF ω is about $-$ 0.5 rad/s and α is about $\pi/4$ rad, THEN
$\dot{x}(t) = A_2x(t) + B_2u, \ y(t) = C_2x(t)$
Rule 3: IF ω is about 0 rad/s and α is about 0 rad, THEN
$\dot{x}(t) = A_3x(t) + B_3u, \ y(t) = C_3x(t)$
Rule 4: IF ω is about 0 rad/s and α is about $\pi/4$ rad, THEN
$\dot{x}(t) = A_4x(t) + B_4u, \ y(t) = C_4x(t)$
Rule 5: IF ω is about 0.5 rad/s and α is about 0 rad, THEN
$\dot{x}(t) = A_5x(t) + B_5u, \ y(t) = C_5x(t)$
Rule 6: IF ω is about0.5 rad/s and α is about $\pi/4$ rad, THEN
$\dot{x}(t) = A_6x(t) + B_6u, \ y(t) = C_6x(t)$

The initial conditions are taken as follows: $\omega(0) = [0, 0, 0]^T, \gamma(0) = [0, 0, 0]^T$ and the tracking command is chosen as $\omega_d = [0, 0, 0]^T, \gamma_d = [1, 0, 2]^T$ during the re-entry phase. The parameters are taken as in [62] and will not be described in detail here. We consider the case where only two actuators fail at one time:

$$u_1^f(t) = \begin{cases} u_1(t), \ t < 5s \\ (1 - \rho_1(x))(u_1(t) + \sum_{j=1}^{p} g_{1,j}f_{1,j}(x)), \ t \geqslant 5s \end{cases}$$

$$u_2^f(t) = \begin{cases} u_2(t), \ t < 5 \\ (1 - \rho_2(x))(u_2(t) + \sum_{j=1}^{p} g_{2,j}f_{2,j}(x)), \ t \geqslant 5 \end{cases}$$

$$u_3^f(t) = u_3(t)$$

where $\rho_1(x) = 0.4\cos(x_1), p = 1, g_{1,1} = 0.4, f_{1,1}(x) = \cos(x_3), \ \rho_2(x) = 0.4\sin(x_2), g_{2,1} = 0.4, \ f_{2,1}(x) = \cos(x_3)$. By using Matlab toolbox to solve the matrices inequalities (3.25), one can obtain the fault diagnostic observer gains L_i. By solving (3.52), one can obtain the positive definite symmetric matrix P and the nominal controller gains K_i. Therefore, one can design the ideal control (3.51). Using the ideal control input (3.51), we can design fault-tolerant controller (3.55), the modified fault-tolerant (3.64) and the observer-based fault-tolerant control (3.70). In this example, we assume that the system state is not fully measured and thus the observer (3.22) is used to estimate the system state. Consequently, the observer-based fault-tolerant control input (3.70) is used to control the faulty system. The simulation results are presented in Figs. 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8 and 3.9. From Fig. 3.2,

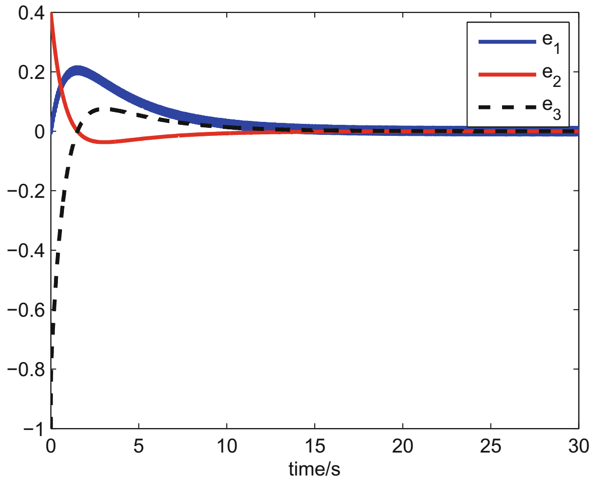

Fig. 3.2 The observer errors time responses: $e_1,\ e_2,\ e_3$

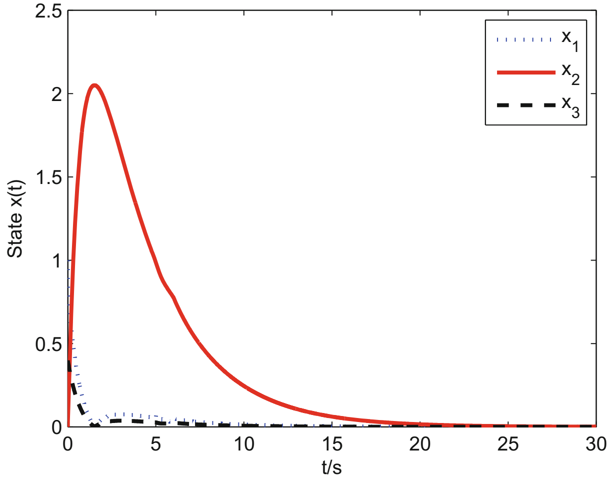

Fig. 3.3 State responses of NSV attitude dynamics under normal conditions

it is seen that, under normal operating condition, observation errors globally asymptotically converge to zero. If no actuator fails, the system states globally asymptotically converge to zero, as shown in Fig. 3.3. Figure 3.4 shows that, when an actuator fault occurs, when keeping the normal controller, the system states deviate significantly from zero. However, as shown in Fig. 3.5, using the proposed FTC (3.70), the

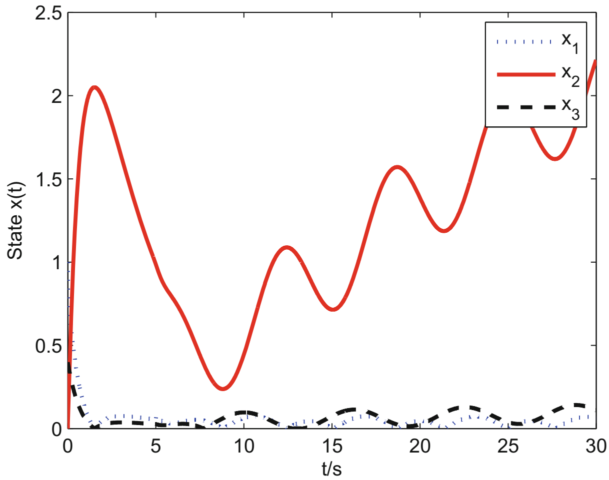

Fig. 3.4 State responses of NSV attitude dynamics without FTC

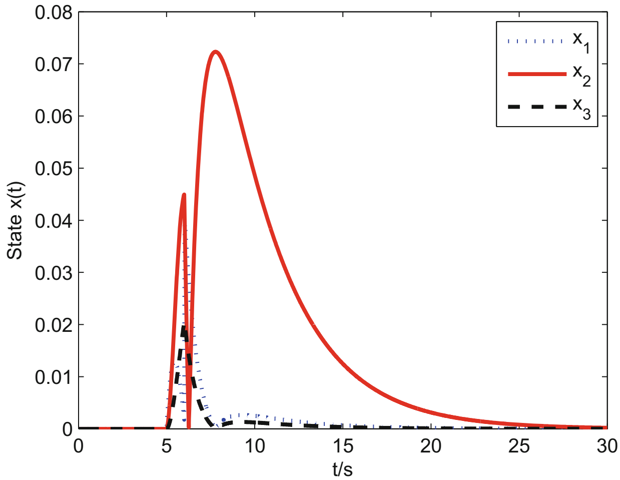

Fig. 3.5 State responses of NSV attitude dynamics with observer-based FTC (3.70)

system states globally asymptotically converge to zero. From Figs. 3.6, 3.7, 3.8 and 3.9, we can clearly draw the conclusion that both gain faults and bias faults can be approximated accurately and promptly by FLSs.

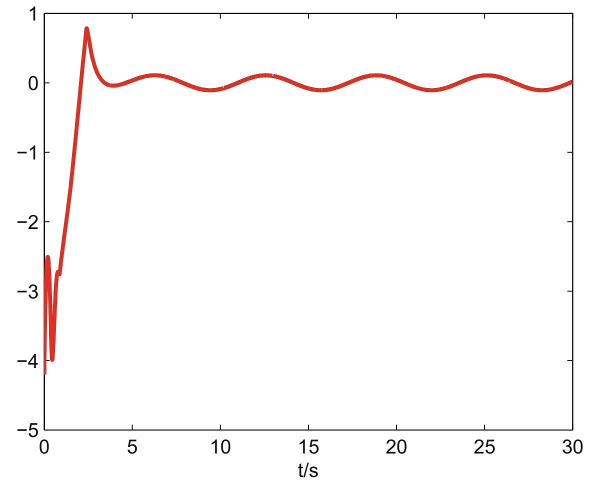

Fig. 3.6 The estimation error of bias fault $g_{1,1}f_{1,1}(x)$

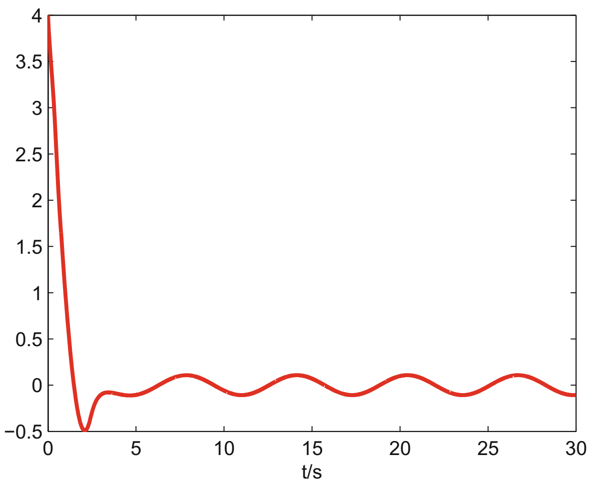

Fig. 3.7 The estimation error of gain fault $\rho_1(x)$

3.5 Conclusions

In this chapter, the problem of fault tolerant control for NSV with multiple state-dependent faults was studied. We first designed a bank of SMOs to detect and estimate the fault. Compared with some results in literature, the proposed fault accommodation scheme is designed to online approximate not only bias faults but also gain faults. Moreover, it can accommodate multiple actuator faults simultaneously. In addition, the adaptive fault accommodation algorithm removes the classical assumption that the time derivative of the output errors should be known. Simulation results of NSV

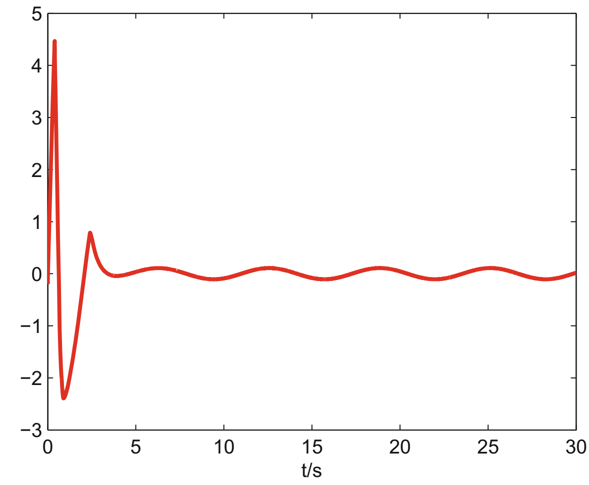

Fig. 3.8 The estimation error of bias fault $g_{2,1}f_{2,1}(x)$

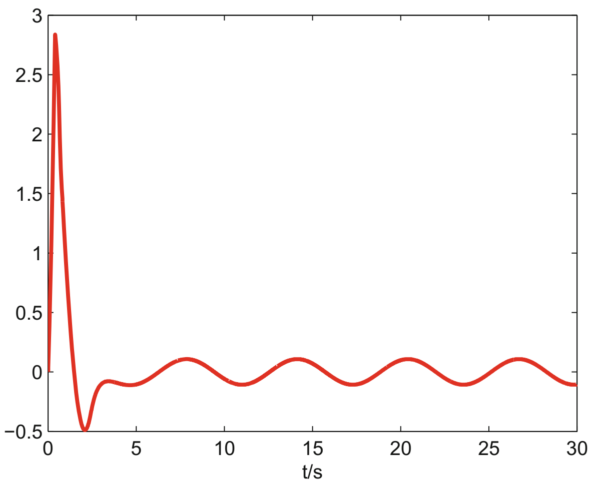

Fig. 3.9 The estimation error of gain fault $\rho_2(x)$

show that the designed fault detection, isolation and estimation algorithms as well as the fault-tolerant control scheme have good dynamic performances in the presence of multiple actuator faults.

References

1. Chen, J., Patton, R.J.: Robust Model-Based Fault Diagnosis for Dynamic Systems. Kluwer Academic, Boston (1999)
2. Mahmoud, M.M., Jiang, J., Zhang, Y.: Active Fault Tolerant Control Systems. Springer, NewYork (2003)

3. Yang, H., Jiang, B., Cocquempot, V.: Fault Tolerant Control Design for Hybrid Systems. Springer, Berlin (2010)
4. Wang, D., Shi, P., Wang, W.: Robust Filtering and Fault Detection of Switched Delay Systems. Springer, Berlin (2013)
5. Du, D., Jiang, B., Shi, P.: Fault Tolerant Control for Switched Linear Systems. Springer, Cham (2015)
6. Shen, Q., Jiang, B., Cocquempot, V.: Fault diagnosis and estimation for near-space hypersonic vehicle with sensor faults. Proc. Inst. Mech. Eng. Part I-J. Syst. Control Eng. **226**(3), 302–313 (2012)
7. Shen, Q., Jiang, B., Cocquempot, V.: Adaptive fault tolerant synchronization with unknown propagation delays and actuator faults. Int. J. Control Autom. Syst. **10**(5), 883–889 (2012)
8. Shen, Q., Jiang, B., Cocquempot, V.: Fuzzy logic system-based adaptive fault tolerant control for near space vehicle attitude dynamics with actuator faults. IEEE Trans. Fuzzy Syst. **21**(2), 289–300 (2013)
9. Shen, Q., Jiang, B., Cocquempot, V.: Adaptive fault-tolerant backstepping control against actuator gain faults and its applications to an aircraft longitudinal motion dynamics. Int. J. Robust Nonlinear Control **20**(10), 448–459 (2013)
10. Astrom, K.J.: Intelligent control. In: Proceedings of 1st European Control Conference, Grenoble, pp. 2328–2329 (1991)
11. Gertler, J.J.: Survey of model-based failure detection and isolation in complex plants. IEEE Control Syst. Mag. **8**(6), 3–11 (1988)
12. Frank, P.M., Seliger, R.: Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy—a survey and some new results. Automatica **26**(3), 459–474 (1990)
13. Frank, P.M.: Analytical and qualitative model-based fault diagnosis—a survey and some new results. Eur. J. Control **2**(1), 6–28 (1996)
14. Garcia, E.A., Frank, P.M.: Deterministic nonlinear observer-based approaches to fault diagnosis: a survey. IFAC Control Eng. Prat. **5**(6), 663–670 (1997)
15. Patton, R.J.: Fault-tolerant control: the 1997 situation (survey). In: Proceedings of IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes: SAFEPROCESS, pp. 1029–1052 (1997)
16. Isermann, R., Schwarz, R., Stolzl, S.: Fault-tolerant drive-by-wire systems-concepts and realization. In: Proceedings of IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes: SAFEPROCESS, pp. 1–5 (2000)
17. Frank, P.M.: Online fault detection in uncertain nonlinear systems using diagnostic observers: a survey. Int. J. Syst. Sci. **25**(12), 2129–2154 (1994)
18. Patton, R.J.: Robustness issues in fault-tolerant control. In: Proceedings of International Conference on Fault Diagnosis, Toulouse, France, pp. 1081–1117 (1993)
19. Song, Q., Hu, W.J., Yin, L., Soh, Y.C.: Robust adaptive dead zone technology for fault-tolerant control of robot manipulators. J. Intell. Robot. Syst. **33**(1), 113–137 (2002)
20. Shen, Q., Jiang, B., Shi, P.: Novel neural networks-based fault tolerant control scheme with fault alarm. IEEE Trans. Cybern. **44**(11), 2190–2201 (2014)
21. Wang, Y., Zhou, D., Qin, S.J., Wang, H.: Active fault-tolerant control for a class of nonlinear systems with sensor faults. Int. J. Control Autom. Syst. **6**(3), 339–350 (2008)
22. Escobar, R.F., Astorga-Zaragoza, C.M., Tllez-Anguiano, A.C., Jurez-Romero, D., Hernndez, J.A., Guerrero-Ramrez, G.V.: Sensor fault detection and isolation via high-gain observers: application to double-pipe heat exchanger. ISA Trans. **50**(3), 480–486 (2011)
23. Chenand, W., Saif, M.: Actuator fault diagnosis for a class of nonlinear systems and its application to a laboratory 3D crane. Automatica **47**(7), 1435–1442 (2011)
24. Li, H., Wu, L., Si, Y., Gao, H., Hu, X.: Multi-objective fault-tolerant output tracking control of a flexible air-breathing hypersonic vehicle. Proc. IMechE Part I: J. Syst. Control Eng. **224**(6), 647–667 (2010)
25. Gao, Z., Jiang, B.: Fault-tolerant control for a near space vehicle with a stuck actuator fault based on a Takagi-Sugeno fuzzy model. Proc. IMechE Part I: J. Syst. Control Eng. **224**(5), 587–598 (2010)

26. Xu, Y., Jiang, B., Gao, Z.: Fault tolerant control for near space vehicle: a survey and some new results. J. Syst. Eng. Electr. **22**(1), 88–94 (2011)
27. Gao, Z., Jiang, B., Shi, P., Xu, Y.: Fault accommodation for near space vehicle via T-S fuzzy model. Int. J. Innov. Comput. Inf. Control **6**(11), 4843–4856 (2010)
28. Hu, Q., Zhang, Y., Huo, X., Xiao, B.: Adaptive integral-type sliding mode control for spacecraft attitude maneuvering under actuator stuck failures. Chin. J. Aeronaut. **24**(1), 32–45 (2011)
29. Xu, D., Jiang, B., Shi, P.: Robust NSV fault-tolerant control system design against actuator faults and control surface damage under actuator dynamics. IEEE Trans. Ind. Electr. **62**(9), 5919–5928 (2015)
30. Zhao, J., Jiang, B., Shi, P., He, Z.: Fault tolerant control for damaged aircraft based on sliding mode control scheme. Int. J. Innov. Comput. Inf. Control **10**(1), 293–302 (2014)
31. Xu, D., Jiang, B., Liu, H., Shi, P.: Decentralized asymptotic fault tolerant control of near space vehicle with high order actuator dynamics. J. Frankl. Inst. **350**(9), 2519–2534 (2013)
32. Jiang, B., Xu, D., Shi, P., Lim, C.: Adaptive neural observer-based backstepping fault tolerant control for near space vehicle under control effector damage. IET Control Theory Appl. **8**(9), 658–666 (2014)
33. Hu, Q., Xiao, B.: Fault-tolerant sliding mode attitude control for flexible spacecraft under loss of actuator effectiveness. Nonlinear Dyn. **64**(1–2), 13–23 (2011)
34. Cai, X., Wu, F.: Multi-objective fault detection and isolation for flexible air-breathing hypersonic vehicle. J. Syst. Eng. Electr. **22**(1), 52–62 (2011)
35. Ye, D., Yang, G.H.: Adaptive fault-tolerant tracking control against actuator faults with application to flight control. IEEE Trans. Control Syst. Technol. **14**(6), 1088–1096 (2006)
36. Xu, H.J., Mirmirani, M.D., Ioannou, P.A.: Adaptive sliding mode control design for a hypersonic flight vehicle. J. Guid. Control Dyn. **27**(5), 829–838 (2004)
37. Zhang, K., Jiang, B., Staroswiecki, M.: Dynamic output feedback-fault tolerant controller design for Takagi-Sugeno fuzzy systems with actuator faults. IEEE Trans. Fuzzy Syst. **18**(1), 194–201 (2010)
38. Dong, J., Yang, G.H.: Control synthesis of TS fuzzy systems based on a new control scheme. IEEE Trans. Fuzzy Syst. **19**(2), 323–338 (2011)
39. Zhang, H., Xie, X.: Relaxed stability conditions for continuous-time TS fuzzy-control systems via augmented multi-indexed matrix approach. IEEE Trans. Fuzzy Syst. **19**(3), 478–492 (2011)
40. Ding, B.: Dynamic output feedback predictive control for nonlinear systems represented by a Takagi Sugeno model. IEEE Trans. Fuzzy Syst. **19**(5), 831–843 (2011)
41. Lam, H.K., Narimani, M.: Quadratic-stability analysis of fuzzy-model-based control systems using staircase membership functions. IEEE Trans. Fuzzy Syst. **18**(1), 125–137 (2010)
42. Lam, H.K.: LMI-based stability analysis for fuzzy-model-based control systems using artificial TS fuzzy model. IEEE Trans. Fuzzy Syst. **19**(3), 505–513 (2011)
43. Peng, C., Yang, T.C.: Communication-delay-distribution-dependent networked control for a class of T-S fuzzy systems. IEEE Trans. Fuzzy Syst. **18**(2), 326–335 (2010)
44. An, J., Wen, G., Lin, C., Li, R.: New results on a delay-derivative-dependent fuzzy H filter design for TS fuzzy systems. IEEE Trans. Fuzzy Syst. **19**(4), 770–779 (2011)
45. Lee, D.H., Park, J.B., Joo, Y.H.: A new fuzzy Lyapunov function for relaxed stability condition of continuous-time Takagi-Sugeno fuzzy systems. IEEE Trans. Fuzzy Syst. **19**(4), 785–791 (2011)
46. Zhou, S.S., Lam, J., Zheng, W.X.: Control design for fuzzy systems based on relaxed nonquadratic stability and performance conditions. IEEE Trans. Fuzzy Syst. **15**(2), 188–199 (2007)
47. Nguang, S.K., Shi, P.: H∞ fuzzy output feedback control design for nonlinear systems: an LMI approach. IEEE Trans. Fuzzy Syst. **11**(3), 331–340 (2003)
48. Zhang, H.G., Lun, S.X., Liu, D.R.: Fuzzy H∞ filter design for a class of nonlinear discrete-time systems with multiple time delays. IEEE Trans. Fuzzy Syst. **15**(3), 453–469 (2007)
49. Gao, H.J., Zhao, Y., Chen, T.W.: H∞ fuzzy control of nonlinear systems under unreliable communication links. IEEE Trans. Fuzzy Syst. **17**(2), 265–278 (2009)
50. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. Syst. Man Cybern. Part B: Cybern. **15**(1), 116–132 (1985)

51. Dong, H., Wang, Z., Daniel, W.C.H., Gao, H.: Robust H∞ fuzzy output-feedback control with multiple probabilistic delays and multiple missing measurements. IEEE Trans. Fuzzy Syst. **18**(4), 712–725 (2010)
52. Dong, H., Wang, Z., Gao, H.: H∞ fuzzy control for systems with repeated scalar nonlinearities and random packet losses. IEEE Trans. Fuzzy Syst. **17**(2), 440–450 (2009)
53. Zhang, J., Shi, P., Xia, Y.: Robust adaptive sliding-mode control for fuzzy systems with mismatched uncertainties. IEEE Trans. Fuzzy Syst. **18**(4), 700–711 (2010)
54. Jiang, B., Gao, Z., Shi, P., Xu, Y.: Adaptive fault-tolerant tracking control of near-space vehicle using Takagi-Sugeno fuzzy models. IEEE Trans. Fuzzy Syst. **18**(5), 1000–1007 (2010)
55. Soest, W.R.V., Chuand, Q.P., Mulder, J.A.: Combined feedback linearization and constrained model predictive control for entry flight. AIAA J. Guid. Control Dyn. **29**(2), 427–434 (2006)
56. Wang, L.X., Mendel, J.M.: Fuzzy basis functions, universal approximation and orthogonal least-squares learning. IEEE Trans. Neural Netw. **3**(5), 807–814 (1992)
57. Ying, H.: Sufficient conditions on general fuzzy systems as function approximators. Automatica **30**(3), 521–525 (1994)
58. Wang, L.X.: Stable adaptive fuzzy control of nonlinear system. IEEE Trans. Fuzzy Syst. **1**(2), 146–155 (1993)
59. Driankov, D., Hellendoom, H., Reinfrank, M.: An Introduction to Fuzzy Control. Springer, New York (1993)
60. Boulkroune, A., Tadjine, M., Saad, M.M., Farza, M.: How to design a fuzzy adaptive controller based on observers for uncertain affine nonlinear systems. Fuzzy Sets Syst. **159**(8), 926–948 (2008)
61. Wang, L.X.: Adaptive Fuzzy Systems and Control: Design and Stability Analysis. Prentice-Hall, Englewood Cliffs, NJ, USA (1994)
62. Wang, Y., Wu, Q.X., Jiang, C.-H., Huang, G.Y.: Reentry attitude tracking control based on fuzzy feedforward for reusable launch vehicle. Int. J. Control Autom. Syst. **7**(4), 503–511 (2009)
63. Xu, Y., Jiang, B., Tao, G., Gao, Z.: Fault accommodation for near space hypersonic vehicle with actuator fault. Int. J. Innov. Comput. Inf. Control **7**(5), 1054–1063 (2011)