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Fault Diagnosis and Fault-Tolerant Control Based on Adaptive Control Approach

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To our families for their love and support

Preface

With the development technology, modern control systems, such as flight control systems, become more and more complex and involve an increasing number of actuators and sensors. These physical components may become faulty which may cause system performance deterioration, may lead to instability that can further produce catastrophic accidents. In order to improve system reliability and to guarantee system stability in all situations, many effective fault-tolerant control (FTC) approaches including fault diagnosis (FD) have been proposed in literature. Among the faults occurred in the controlled systems, the actuator faults and sensor faults are common. Up to now, for the actuator faults and sensor faults, many relevant results have been obtained in the literature. However, these theoretical studies are not perfect, and there still are problems of actuator faults and sensor faults, which are worth to be further deeply investigated due to its academic meaning as well as its practical one:

1. **Motivation from academic research**

   Infinite-number-integrated-fault model. Most of the existing works on FD and FTC in literature only considered bias faults, while gain faults have not attracted enough attention. From the theoretical point of view, it is possible that bias and gain faults simultaneously occur in systems. Furthermore, the fault number may be infinite. Hence, it is necessary to propose a novel general fault model, which can describe infinite-number-faults and deal with time-varying bias fault and gain fault.

   Singularity of fault-tolerant controller. In order to compensate for actuator gain fault, the denominator of the fault-tolerant control input contains the estimation of the gain fault. If the denominator is equal to zero, a controller singularity occurs. Hence, a novel FTC scheme must be designed to avoid the controller's singularity.

   FTC against un-model fault. The actuator (sensor) bias and gain faults have an affine-like appearance of the control input (system output). The un-modeled faults have no traditional affine appearance. Furthermore, the existing results on the bias and gain faults in literature cannot be directly extended to FD and FTC against the un-modeled faults. Therefore, it is necessary to design novel FTC algorithm for the un-modeled faults.

Computation complexity in backstepping design procedure. To control including FTC for the unknown nonlinear systems in or transformable to parameter strict-feedback form, adaptive backstepping technique is a powerful tool. However, computation complexity caused by analytic computation of the higher derivatives of virtual control signals must be faced. Hence, how to reduce the computation becomes crucial issue in controller design.

2. **Motivation from practical application**

Decision threshold and algorithm. In some existing works, the asymptotic value of the state estimation error between the system state x and fault detection observer state $\hat{x}$, i.e., $\lim_{t\to\infty}(x(t) - \hat{x}(t)) = \lim_{t\to\infty} e_x(t) = e_x(\infty)$, is considered as an fault occurrence indicator. However, $e_x(\infty)$ is not available in practice, and $e_x(\infty) \neq 0$ cannot practically be considered as fault indicator. Hence, designing a more practical and efficient decision threshold and algorithm becomes more important and urgent.

Multi-type multi-fault isolation. In practical applications, multiple faults maybe simultaneously occur in the controlled systems. However, most of the results on FD in literature works under the restrictive condition that only one actuator or sensor fault occurs at one time, cannot be extended to the case where multiple actuator and sensor faults simultaneously occur. Therefore, it is a need for such case to design a novel FD observer to isolate multiple-type multiple faults occurred simultaneously.

Fault detection for time-delay systems. Most of fault detection observers of time-delay systems in literature contain time delay. If the time delay is unknown, then the observers are not reasonable and do not work in the practical applications. Hence, how to avoid the above shortcoming and design a proper observer for dynamical systems becomes important and practically useful.

Time delay due to fault diagnosis. There is always some level of time to detect, isolate and estimate the faults occurred in the systems. The time interval is called as the time delay due to fault diagnosis in this book. When faults occur, the faulty system works under the nominal control until the faults are diagnosed and fault accommodation is performed, which may cause severe loss of performance and stability. Hence, in the practical applications, the time delay due to FD should be derived strictly, and its adverse effect on the system performance should also be analyzed, and a proper solution is given to minimize its adverse effect.

This book provides recent theoretical results and applications of fault diagnosis and FTC for dynamic systems, including uncertain or certain systems, linear or nonlinear systems. Combining adaptive control technique with the other control technique or approaches, this book investigates the problem of FD and FTC for uncertain dynamic systems including linear and nonlinear systems with or without time delay.

This book intends to provide the readers a good understanding of FD and FTC based on adaptive control technology. The book can be used as a reference for the academic research on FD and FTC or used in Ph.D. study of control theory and engineering. The knowledge background for this monograph would be some undergraduate and graduate courses on linear system theory, nonlinear system theory, and FD and FTC control technology and theory.

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Contents

Acronyms

$\Re$	The field of real numbers
$\Re^r$	The r-dimensional real vector space
$\lvert \cdot \rvert$	The Euclidean norm
$\mathscr{L}_1$	$a(t) \in \mathscr{L}_1$ if $\int_0^\infty \lvert a(t) \rvert dt < \infty$
$\mathscr{C}^k$	The set of k times continuously differentiable functions
$\lVert \cdot \rVert_{[a,b]}$	The supremum norm of a signal on the time interval $[a,b]$
Class$\mathscr{K}$	A class of strictly increasing and continuous functions $[0,\infty) \to [0,\infty)$ which are zero at zero
Class $\mathscr{K}_\infty$	The subset of $\mathscr{K}$ consisting of all those functions that are unbounded
Class $\mathscr{K}\mathscr{L}$	$\beta : [0,\infty) \times [0,\infty) \to [0,\infty)$ belongs to class $\mathscr{K}\mathscr{L}$ if $\beta(\cdot,t)$ is of class $\mathscr{K}$ for each fixed $t \geq 0$ and $\beta(s,t) \to 0$ as $t \to \infty$ for each fixed $s \geq 0$
$\lambda_{\max}(\cdot)$	The maximal eigenvalue
$\lambda_{\min}(\cdot)$	The minimal eigenvalue
$L_g h$	The Lie derivative of h along a vector field g
t^-	The left limit time instant of t
$(\cdot)^\top$	The transposition
$\forall$	For all
$\exists$	There exists
$\subset$	Subset of
$\in$	Belongs to
$\triangleq$	Define
$\cap$	Intersect
$\cup$	Union
$\sum$	Sum
$\prod$	Product
FD	Fault diagnosis

FDI Fault detection and isolation
FTC Fault-tolerant control
MF Modeled fault
UMF Un-modeled fault

Chapter 1
Introduction

1.1 FD and FTC Against Actuator and/or Sensor Faults

With the development technology, modern control systems, such as flight control systems, become more and more complex and involve an increasing number of actuators and sensors. These physical components may become faulty which may cause system performance deterioration, may lead to instability that can further produce catastrophic accidents.

To improve system reliability and guarantee system stability in all situations, FD and fault accommodation methods have become attractive topics which received considerable attention during the past two decades as it can be attested by the abundant literatures [1–20].

Fault diagnosis including fault detection and isolation (FDI) [1, 6–9, 20] is used to detect faults and diagnosis their location and significance in a system [1]. It has the following tasks: fault detection, fault isolation and fault estimation. Fault detection is to make a decision, e.g., faults occur in the controlled systems or not. Fault isolation is used to determine the location of the faults, namely, which physical component has become faulty. The last task in FD is to estimate the size of the fault.

Fault tolerant control aims at preserving the functionalities of a faulty system with acceptable performances. FTC can be achieved in two ways, namely, passive and active ways [1]. Passive FTC uses feedback control law that is robust with respect to possible system faults [21–33]. Generally speaking, passive FTC is more conservative [1]. In order to relax the conservatism of the passive FTC approach, active FTC method is developed. Active FTC relies on a FD module process to monitor the performance of the controlled system, and to detect, isolate and estimate the faults in the controlled system [34–45]. Accordingly, the control law is reconfigured online.

In recent years, by using adaptive control technology, various FD and FTC approachers including passive and active FTC have been developed, and abundant results on adaptive FD and FTC can be found in literature [21–45].

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It is well known that, among the faults occurred in the controlled systems, actuator and sensor faults are common. In practical application, actuator and sensor faults have two kinds of faults, namely, bias faults and gain faults [46–52]. Bias fault model can be described as:

$$\begin{cases} Actuator\,fault: & u^f(t) = u(t) + f_u(x, u, t), \quad t \geq t_f \\ Sensor\,fault: & y^f(t) = y(t) + f_y(y, t), \quad t \geq t_f \end{cases} \tag{1.1}$$

where t_f is an unknown fault occurrence time; u and u_f denote actuator input and output, respectively; y and y_f denote system output and actual obtained system output, respectively; $f_u(x, u, t)$ and $f_y(y, t)$ denote actuator and sensor fault, respectively, which are commonly assumed to be unknown but bounded signal. Actuator and sensor gain faults have the following form,

$$\begin{cases} Actuator\,fault: & u^f(t) = (1 - \rho_u(x, u, t))u(t), \quad t \geq t_f \\ Sensor\,fault: & y^f(t) = (1 - \rho_y(y, t))y(t), \quad t \geq t_f \end{cases} \tag{1.2}$$

where t_f denotes an unknown fault occurrence time; u and u_f denote actuator input and output, respectively; y and y_f denote system output and actual obtained system out, respectively; $0 \leqslant \rho_u(x, u, t) \leqslant 1$ and $0 \leqslant \rho_y(y, t) \leqslant 1$ are unknown, which denote the remaining control rate and measurable part, respectively.

Recently, an integrated fault model is reported, which contains the above two kinds of faults [52, 53]. It can be uniformly described as:

$$\begin{cases} Actuator\,fault: & u^f(t) = (1 - \rho_u(x, u, t))u(t) + f_u(x, u, t), \quad t \geq t_f \\ Sensor\,fault: & y^f(t) = (1 - \rho_y(t))y(t) + f_y(t), \quad t \geq t_f \end{cases} \tag{1.3}$$

Very recently, a so-called infinite-number-faults model was reported [60], which can be described as follows:

$$\begin{cases} Actuator\,fault: & u^f(t) = (1 - \rho_u(x, u, t))u(t) + \sum_{j=1}^{p_u} f_{u,j}(x, u, t), \quad t \geq t_f \\ Sensor\,fault: & y^f(t) = (1 - \rho_y(y, t))y(t) + \sum_{j=1}^{p_y} f_{y,j}(y, t), \quad t \geq t_f \end{cases} \tag{1.4}$$

where $f_{u,j}(t)$ $(j = 1, \ldots, p_u)$ and $f_{y,j}(t)$ $(j = 1, \ldots, p_y)$ denote bounded signal, p_u and p_u are known positive constants.

From (1.1)–(1.4), it is easily seen that the actuator and sensor faults have an affine-like appearance of control input and/or system output. That is to say, the fault can be expressed explicitly as gain and/or bias fault [54–56], which is called *modeled fault* (MF) in this book. Unfortunately, there exist some cases in practical applications where the faults cannot be expressed in the above affine-like form [57–59]. The fault model can be described as follows:

$$\begin{cases} Actuator\,fault: & u^f = f(x,u), \quad t \geq t_f \\ Sensor\,fault: & y^f(t) = f(y), \quad t \geq t_f \end{cases} \tag{1.5}$$

where $f(x, u)$ and $f(y)$ are two unknown nonlinear smooth function, with t_f being unknown fault occurrence time. Obviously, fault model described by (1.5) has no the traditional affine-like appearance of control input and/or system output. The fault is called *un-modeled fault* (UMF).

Although abundant results on FD and FTC against actuator and/or sensor faults have been obtained in literature, FD and FTC for dynamic systems still need to be deeply investigated due to their academic meaning as well as practical one, and there exist many open problems to be solved, which is the topic of this book.

- FD and FTC against infinite-number-integrated-faults:
  In most of the existing works in literature only considered bias faults, while gain faults have not attracted enough attention. From the theoretical point of view, it is possible that time-vary bias fault and time-vary gain fault simultaneously occur in the controlled systems. Further, the number of the faults occurred in systems maybe infinite. Hence, it is necessary to propose a novel infinite-number-integrated-fault model and design corresponding FD and FTC algorithms. In addition, the denominator of the fault-tolerant control input contains the estimation of the gain fault. If the denominator is equal to zero, a controller singularity occurs. Hence, controller singularity should be considered, and a novel FTC scheme must be designed to avoid the singularity problem.
- Multi-type multi-fault isolation:
  In the practical applications, multiple type multiple faults maybe simultaneously occur in the controlled systems. However, most of the results on FD and FTC in literature works under the restrictive condition that only one actuator fault occurs at one time. What's more, the results cannot be easily extended to the case where multiple actuator faults simultaneously occur. Therefore, it is a need for such case to design a novel FD algorithm to isolate multiple faults occurred simultaneously.
- FD and FTC against un-modeled fault:
  Since un-modeled fault has no traditional affine-like appearance of control input or system output, the results concerning on MF cannot be extended directly to FTC against UMF. Under some restrictive conditions, some researchers investigated the problem of FTC against UMFs, and only a few results were obtained in literature. In [57], the problem of adaptive FTC for nonlinear systems with actuator MF was investigated. However, the results are only applicable to second-order nonlinear systems rather than more general high-order systems, which limit their practical applications. In [58, 59], robust detection and isolation schemes for UMFs were addressed. However, these FDI schemes worked under the condition that the system state variables and control inputs were bounded before and after the occurrence of a fault, which is too restrictive. In addition, the UMF was assumed to be a known function about control input and system state with an unknown gain.

Hence, how to control more general high-order nonlinear systems with UMFs is still an important and open problem, which motivates us for this study.

- Computation complexity in backstepping design procedure:
  For the unknown nonlinear systems in or transformable to parameter strict-feedback form, adaptive backstepping technique is a powerful tool. At standard backstepping design procedure, analytic computation of the higher derivatives of virtual control signals is necessary, which leads to a so-called computation complexity especially when the system dimension increases. Hence, how to reduce the computation becomes crucial issue in controller design.

1.2 Fault Detection for Time-Delay Systems

Time delay phenomenon often exists in the practical applications because of information transmission. It has been proven that such time delay will causes the performance degradation of the controlled systems, even instability. Hence, the control problem of time delay systems, including FTC, always is a hot topic Over the past decade [61–73]. Stability analysis of the time delay systems can be divided into two classes: time-dependent and time-independent results. The former is dependent on the size of time delay, while the latter does relay on thecite time delay. Generally speaking, time-dependent results are more conservative than the time-independent results.

Recently, FD and FTC for the time delay systems has drawn wide attentions [74–82, 87]. In order to compensate for these faults, various fault-tolerant control (FTC) methods are proposed. Among these FTC methods, active FTC methods is more common, important and useful. Fault detection [83–86] is the first and important step in active FTC method. For time delay systems, however, most of the FD observers proposed in literature have a major shortcoming that the fault detection observers contain the unknown time delay terms. For example, consider a simple system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu \\ y(t) = Cx(t) \end{cases} \tag{1.6}$$

where $x,\ y$ and u denote state, output and control input, respectively; $A,\ A_d,\ B$ and C are known real matrices with appropriate dimensions; time delay d is a constant. In most of the existing results such as [9], fault detection observer often is designed for (1.6) as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-d) + Bu + L(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{1.7}$$

where L is observer gain matrix, which will be designed. Notice that, the first equation in (1.7) contains time delay term $\hat{x}(t-d)$. Obviously, if d is unknown, then observer (1.7) is not reasonable and does not work in the practical applications.

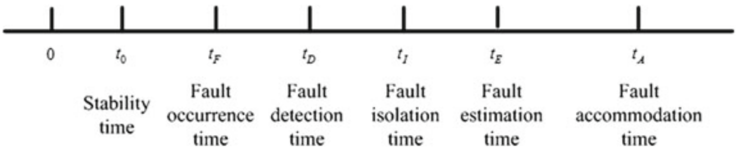

Fig. 1.1 The fault diagnosis and accommodation time sequence

Hence, how to avoid the above shortcoming and design a proper observer for time delay systems becomes important and practically useful.

1.3 Analysis of Time Delay Due to Fault Diagnosis

In general, active FTC framework includes the following steps: fault detection, fault isolation, fault estimation and fault accommodation. The fault diagnosis and accommodation time sequence can be seen in Fig. 1.1. From Fig. 1.1, it is easily seen that each step need some time. The time interval is called as *time delay due to FD* in this book.

Generally speaking, it is under the condition that the fault occurred in system can immediately be detected and isolated that an active fault tolerant control law is designed [91]. In fact, there is always some time delay. Furthermore, when a fault occurs, the faulty system works under the nominal control until the fault is detected, isolated and fault accommodation is performed. That is to say, the considered system is always controlled by the faulty actuators during $[t_F, t_A]$. Obviously, it will degrades the system performances even damage the system. However, its effect is not studied enough, and there are only few results reported in literature [88–91]. Hence, *the time delay due to FD* should be derived strictly, and its adverse effect on the system performance should also be analyzed and a proper solution to minimize its adverse effect is given.

1.4 Organization of the Book

This book presents several fundamental problems of FD and FTC for dynamic systems. Combining adaptive control technique with the other control techniques or approaches, a basic theoretical framework is formed towards the issues of FD and FTC of dynamic system. In order to conveniently reading this book, some preliminaries including same or similar lemmas are introduced in different chapters. This book contains ten chapters, which exploit several independent yet related topics in the detail.

Chapter 2 addresses the problem of fault tolerant control for T-S fuzzy systems with single actuator faults. A general actuator fault model with infinite number of faults is proposed, which integrates time-varying bias faults and time-varying gain faults. Then, sliding mode observers are designed to provide a bank of residuals for fault detection and isolation, and a novel fault diagnostic algorithm is proposed, which removes the classical assumption that the time derivative of the output errors should be known as in some existing work. Further, a novel fault estimation observer is designed. Utilizing the estimated actuator fault, an accommodation scheme is proposed to compensate for the effect of the fault.

Chapter 3 investigates the fault tolerant control problem of near space vehicle attitude dynamics with multiple actuator faults, which is described by a T-S fuzzy model. Firstly, an integrated state-dependant actuator fault model with infinite number of faults is proposed to simultaneously deal with state-dependent bias and gain faults. Then, sliding mode observers are designed to provide a bank of residuals for fault detection and isolation. Based on Lyapunov stability theory, a fault diagnostic strategy is proposed. Further, for the two cases where the state is available or not, two accommodation schemes are proposed to compensate for the effect of the faults.

Chapter 4 focuses on the problem of fuzzy adaptive tracking control for a class of uncertain nonlinear strict-feedback systems with actuator fault. The actuator fault is assumed to have not only time-varying gain fault but also time-varying bias fault. Combining command filtered backstepping design with the integral-type Lyapunov function and utilizing Nussbaum-type gain technique, an adaptive fuzzy fault-tolerant control scheme is proposed to guarantee that the resulting closed-loop system is asymptotically bounded with the tracking error converging to a neighborhood of the origin. The control scheme requires only virtual control and its first one derivative instead of them and their higher derivatives in backstepping design procedures.

In Chap. 5, we consider the problem of fault-tolerant dynamic surface control for a class of uncertain nonlinear systems with actuator faults and propose an active fault-tolerant control scheme. Using the DSC technique, a novel fault diagnostic algorithm is proposed, which removes the classical assumption that the time derivative of the output error should be known. Further, an accommodation scheme is proposed to compensate for both actuator time-varying gain and bias faults, and avoids the controller singularity. In addition, the proposed controller guarantees that all signals of the closed-loop system are semi-globally uniformly ultimately bounded, and converge to a small neighborhood of the origin.

Chapter 6 discusses the problem of fault-tolerant control for a class of uncertain nonlinear high-order systems with actuator faults, and propose an observer-based FTC scheme. Adaptive fuzzy observers are designed to provide a bank of residuals for fault detection and isolation. Using a backstepping approach, a novel fault diagnosis algorithm is proposed, which removes the classical assumption that the time derivative of the output error should be known. Further, an accommodation scheme is proposed to compensate for the effect of the fault, where it is not needed to know the bounds of the time derivative of the fault. The proposed controller guarantees that all signals of the closed-loop system are semi-globally uniformly ultimately bounded and converge to a small neighborhood of the origin by appropriately choosing designed parameters.

In Chap. 7, the problem of adaptive active fault-tolerant control for a class of nonlinear systems with unknown actuator fault is investigated. The actuator fault is assumed to have no traditional affine appearance of the system state variables and control input. The useful property of the basis function of the radial basis function neural network, which will be used in the design of the fault tolerant controller, is explored. Based on the analysis of the design of normal and passive fault tolerant controllers, by using the implicit function theorem, a novel neural networks-based active fault-tolerant control scheme with fault alarm is proposed. Comparing with results in literature, the fault-tolerant control scheme can minimize the time delay between fault occurrence and accommodation that is called the time delay due to fault diagnosis, and reduce the adverse effect on system performance. In addition, the FTC scheme has the advantages of a passive fault-tolerant control scheme as well as the traditional active fault-tolerant control scheme's properties. Furthermore, the fault-tolerant control scheme requires no additional fault detection and isolation model which is necessary in the traditional active fault-tolerant control scheme.

Chapter 8 discusses the problem of fault-tolerant control against actuator fault, derives the time spent at each steps in fault diagnosis which is called as the time delay due to fault diagnosis and quantitatively analyzes its effect on the faulty systems performance. A novel fault diagnosis algorithm is first proposed. The proposed fault tolerant controller guarantees that all signals in the closed-loop system are semi-globally uniformly ultimately bounded. What's more, the analytical expression of the time delay is derived strictly. Further, the quantitative analysis of system performance which is degraded by the time delay is developed, and the conditions that the magnitudes of the faults should be satisfied such that the faulty system controlled by the normal controller remains bounded even stable during the time delay are derived. In addition, the corresponding solution to the adverse effect of the time delay is proposed.

Chapter 9 investigates the fault detection of uncertain systems with unknown time-delay constant, and design a novel adaptive neural network-based fault detection observer, where not only the system states but also the unknown time delay can be estimated. Furthermore, comparing with the existing works where an asymptotic value is taken as an indicator to determine whether faults occur or not, a more efficient fault detection mechanism is proposed.

In Chap. 10, several future research directions are predicated.

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Chapter 2
Fault Tolerant Control for T-S Fuzzy Systems with Application to NSHV

2.1 Introduction

Modern control systems, such as NSHV that is considered in this chapter, become more and more complex and involve an increasing number of actuators and sensors. These physical components may become faulty which can cause system performance deterioration and lead to instability that can further produce catastrophic accidents. To improve system reliability and guarantee system stability in all situations, FDI and fault accommodation methods have become attractive topics which received considerable attention during the past two decades as it can be attested by the abundant literature [1–20]. Fault tolerant control (FTC) aims at preserving the functionalities of a faulty system with acceptable performances. FTC can be achieved in two ways namely passive and active ways. The former uses feedback control laws that are robust with respect to possible system faults. On the other hand, the latter uses a FDI module and accommodation techniques.

It is valuable to point out that, although there are abundant results in literature, most results concerning actuator faults reported in the literature only considered bias faults. Gain faults did not attract enough attention, which motivates this chapter. In addition, in some existing work, estimation error $\lim_{t\to\infty} e_x(t) = e_x(\infty)$ was considered as an indicator, by which the faulty system can be distinguished from the normal system. That is to say, if $e_x(\infty) = 0$, then the system is healthy; if $e_x(\infty) \neq 0$, the system is faulty. However, $e_x(\infty)$ is not available in practice, and $e_x(\infty) \neq 0$ can not practically be considered as fault indicator. Another motivation of this work is thus to provide a fault indicator with an associated decision algorithm which is efficient in practical application.

The concept of near space hypersonic vehicle was first proposed by American air force in a military exercise called "Schrieffer" in 2005. NSHV is a class of vehicle flying in near space which offers a promising and new, lower cost technology for future spacecraft. It can advance space transportation and also prompt global strike capabilities. Such complex technological system attracts considerable interests from

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the control research community and aeronautical engineering in the past couple of decades and significant results were reported [21–32]. For such high technological system, with great economical and societal issues, it is of course essential to maintain high reliability against possible faults. One of the difficulties to deal with FTC for NSHV is that the dynamics are complex nonlinear, multi-variable and strongly coupled ones. To solve the difficulties, T-S fuzzy system was used to describe the NSHV attitude dynamics [33]. During the past two decades, the stability analysis for Takagi-Sugeno (T-S) fuzzy systems has attracted increasing attention [34–42]. These studies combine the flexibility of fuzzy logic theory and rigorous mathematical theory of linear/nonlinear systems into a unified framework. The important advantage of a T-S fuzzy system is its universal approximation of any smooth nonlinear function by a "blending" of some local linear models, which greatly facilitates the analysis and synthesis of the complex nonlinear system. Lots of stability criteria of T-S fuzzy systems have been expressed in terms of linear matrix inequalities (LMIs) via various stability analysis methods (see [43–50] and the references therein). In [51], authors studied the problem of fault-tolerant tracking control for near-space-vehicle attitude dynamics with bias actuator fault, where the bias fault was assumed to be unknown constant. However, in practical application, the fault may be time-varying, which motivates this chapter.

In this chapter, we investigate the problem of fault tolerant control for T-S fuzzy systems with actuator time-varying faults, with the objective to provide an efficient solution for controlling NSHV in faulty situations. Compared with some existing work, there are four main contributions that are worth to be emphasized.

1. The actuator fault model presented in this chapter integrates not only time-varying gain faults, but also time-varying bias faults, which means that a wide class of faults can be handled. The theoretic developments and results of this chapter are thus valuable in a wide field of practical applications.
2. An adaptive fault estimation algorithm is proposed where the common assumption that the derivative of the output errors with respect to time should be known is removed and the parameter drift phenomenon is prevented even in the presence of bounded disturbances.
3. Compared with some results, a decision threshold for FDI is defined and applied on an online computable fault indicator and not on an asymptotic value of a criterion, which means the decision algorithm is thus more practical.
4. The proposed fault estimation observer is designed to online estimate not only bias faults but also gain faults.

The rest of the chapter of this chapter is organized as follows. In Sect. 2.2, the T-S fuzzy model is first briefly recalled. Actuator faults are integrated in such model and the FTC objective is formulated. In Sect. 2.3, the main technical results of this chapter are given, which include fault detection, isolation, estimation and fault-tolerant control scheme. The NSHV application is presented in Sect. 2.4. The T-S fuzzy model is employed to approximate the nonlinear NSHV attitude dynamics and simulation results of NSHV are presented to demonstrate the effectiveness of the proposed technique. Finally, Sect. 2.5 draws the conclusion.

2.2 Problem Statement and Preliminaries

Consider the following T-S fuzzy model composed of a set of fuzzy implications, where each implication is expressed by a linear state space model. The ith rule of this T-S fuzzy model is of the following form:

Plant Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{2.1}$$

where $i = 1, \ldots, r$, r is the number of the IF-THEN rules, $M_{ij}, j = 1, \ldots, q$ is the fuzzy set, $z(t) = [z_1(t), \ldots, z_q(t)]^T$ are the premise variables which are supposed to be known, $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$, $u(t) \in R^m$, $A_i \in R^{n\times n}$, and $B_i \in R^{n\times m}$.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \tag{2.2}$$

where $h_i(z(t))$ is defined as

$$h_i(z(t)) = \frac{\prod_{j=1}^{n} M_{ij}[z(t)]}{\sum_{i=1}^{r} \prod_{j=1}^{n} M_{ij}[z(t)]}, \quad i = 1, 2, \ldots, r \tag{2.3}$$

where $M_{ij}[z(t)]$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed in this chapter that $\prod_{j=1}^{n} M_{ij}[z(t)] \geqslant 0$ for all t. Therefore, we have $\sum_{i=1}^{r} h_i(z(t)) = 1, \quad 0 \leqslant h_i(z(t)) \leqslant 1$ for all t.

In this chapter, the state feedback control strategy is chosen as a parallel distributed compensation (PDC), which can be described as follows:

Control Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$u_i(t) = K_i x(t) \tag{2.4}$$

where K_i is the controller gain matrix to be determined later.

The overall fuzzy controller is given as follows:

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) K_i x(t) \tag{2.5}$$

The control objective under normal conditions is to design a proper state feedback control controller $u(t)$ such that the system (2.2) is stable.

However, in practical application, actuators may become faulty. Bias faults and gain faults are two kinds of actuator faults commonly occurring in practice. An actuator bias fault can be described as:

$$u_i^f(t) = u_i(t) + f_i(t), \quad i = 1, \ldots, m \tag{2.6}$$

where $f_i(t)$ denotes a bounded signal, and an actuator gain fault can be described as:

$$u_i^f(t) = (1 - \rho_i(t))u_i(t), \;\; i = 1, \ldots, m \tag{2.7}$$

where $0 \leqslant \rho_i(t) \leqslant 1$ which is supposed to be unknown, denotes the remaining control rate. Therefore, the above two kinds of actuator faults can be uniformly described as:

$$u_i^f(t) = (1 - \rho_i(t))u_i(t) + f_i(t) \tag{2.8}$$

Furthermore, a more general fault model can be given as:

$$u_i^f(t) = (1 - \rho_i(t))u_i(t) + \sum_{j=1}^{p_i} g_{i,j}f_{i,j}(t) \tag{2.9}$$

where $f_{i,j}(t), \;\; i = 1, \ldots, m, j = 1, \ldots, p_i$ denotes a bounded signal, p_i is a known positive constant. $g_{i,j}$ denotes an unknown constant. With no restriction, let suppose $p_1 = p_2 = \cdots = p_m = p$, with p a known positive constant. Consider the following notation: $[a_{i,j}(t) = g_{i,j}f_{i,j}(t)$. Then, (2.9) can be re-written as follows:

$$u_i^f(t) = (1 - \rho_i(t))u_i(t) + \sum_{j=1}^{p} a_{i,j}(t) \tag{2.10}$$

Denote

$$\Gamma(t) = diag(\rho_1(t), \ldots, \rho_m(t)) \tag{2.11}$$

$$F(t) = [f_1, f_2, \ldots, f_m]^T, f_i = \sum_{j=1}^{p} a_{i,j}(t) \tag{2.12}$$

Then, we have

$$u^f(t) = (I - \Gamma(t))(u(t) + F(t)), \;\; t > t_f \tag{2.13}$$

where the failure time instant t_j is unknown, and I denotes identity matrix with appropriate dimensions. In this chapter, both bias and gain faults are handled by considering the general fault model (2.13).

Notice that, in the following, just for the sake of notational simplicity, we will use h_i, ρ_i and $a_{i,j}$ to denote $h_i(z(t))$, $\rho_i(t)$ and $a_{i,j}(t)$.

Now, the control objective is re-defined as follows. An active fault tolerant control approach is proposed to make system (2.2) stable in normal and faulty conditions. Under normal condition (no fault), a state feedback control input $u(t)$ is designed, such that the system (2.2) is stable. Meanwhile, the FDI algorithm is working. As soon as an actuator fault is detected and isolated, the fault estimation algorithm is activated. The obtained fault estimation is used to design a proper control input$u(t)$, such that the system (2.2) is still maintained stable under faulty case.

Remark 2.1 In the literature, many chapters consider actuator faults. However, most of them only considered bias faults. Gain faults have not attracted enough attention. In [51], a class of bias fault was studied, where the fault was assumed to be an unknown constant. However, in practical application, the fault may be time-varying. Equation (2.10) is a deterministic but uncertain actuator model which represents a class of practical actuator faults such as actuator gain variations and measurement errors. In fact, the fault model in [51] can be described by (2.10). If $\rho_i(t) = 0$, then the model (10) becomes the bias fault model. If $\rho_i(t)$ is an unknown constant and $f_i(t) = 0$, then the model (2.10) denotes the constant bias faults model. Hence, the proposed actuator fault model (2.10) is more general and has wider practical use than the classical ones.

2.3 Fault Diagnosis and Accommodation

In this section, the main technical results of this chapter are given. We will first formulate the fault diagnosis and accommodation problem of the above T-S fuzzy system. We will then design a bank of SMOs to generate residuals, investigate the FDI algorithm based on the SMOs, and propose a FTC scheme to tolerate the fault using estimated fault information.

2.3.1 Preliminary

Consider the T-S fuzzy faulty system described in (2.2). We assume that only actuator faults occur and no sensor fault is involved. For simplicity, we consider the case that only one single actuator is faulty at one time. The actuator fault diagnosis problem is formulated as: with the available output$y$, we propose an observer based scheme to identify the faulty actuator, and then estimate the fault.

To solve the problem, we will design a bank of SMOs with desired actuator fault detection and fault estimation properties. Thus, the following assumptions are made in this chapter.

Assumption 2.1 Matrix B_i is of full column rank and the pair (A_i, C_i) is observable.

Assumption 2.2 There exist known positive constants $\bar{\rho}_i$, $\bar{\bar{\rho}}_i$, $\bar{\rho}_1, \bar{\rho}_2$, such that $|\rho_i(t)| \leqslant \bar{\rho}_i$ and $|\dot{\rho}_i(t)| \leqslant \bar{\bar{\rho}}_i$, $\bar{\rho}_1 = \max\{\bar{\rho}_1, \bar{\rho}_2, \ldots, \bar{\rho}_m\}$, $\bar{\rho}_2 = \max\{\bar{\bar{\rho}}_1, \bar{\bar{\rho}}_2, \ldots, \bar{\bar{\rho}}_m\}$, $i = 1, \ldots, m$.

Assumption 2.3 There exist known positive constants $\bar{a}_1$, $\bar{a}_2$, $\bar{a}_{i,j}$, $\bar{\bar{a}}_{i,j}$, such that $|a_{i,j}(t)| \leqslant \bar{a}_{i,j}$ and $|\dot{a}_{i,j}(t)| \leqslant \bar{\bar{a}}_{i,j}$, $\bar{a}_1 = \max\{\bar{a}_{1,1}, \ldots, \bar{a}_{i,p}, \ldots, \bar{a}_{m,1}, \ldots, \bar{a}_{m,p}\}$, $\bar{a}_2 = \max\{\bar{\bar{a}}_{1,1}, \ldots, \bar{\bar{a}}_{i,p}, \ldots, \bar{\bar{a}}_{m,1}, \ldots, \bar{\bar{a}}_{m,p}\}$, $i = 1, \ldots, m, j = 1, \ldots, p$.

Our actuator fault diagnosis and accommodation scheme consists of FDI and FTC. We first design the fault diagnosis observer utilizing SMOs to detect, isolate and estimate the fault, and then, propose a FTC method to compensate the fault.

2.3.2 *Fault Detection*

In order to detect the actuator faults, we design a fuzzy state-space observer for the system (2.8), which is described as:

Observer Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_i\hat{x}(t) \end{cases} \tag{2.14}$$

where L_i, $i = 1, \ldots, r$ is the observer gain for the ith observer rule.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}(t) \end{cases} \tag{2.15}$$

Denote

$$e_x = x(t) - \hat{x}(t), \quad e_y = y(t) - \hat{y}(t) \tag{2.16}$$

then the error dynamics is described by

$$\begin{cases} \dot{e}_x = \sum_{i=1}^{r} h_i(z(t))(A_i - L_iC_i)e_x(t)) \\ e_y = \sum_{i=1}^{r} h_i(z(t))C_ie_x(t) \end{cases} \tag{2.17}$$

Lemma 2.1 *The estimation error e_x converges asymptotically to zero if there exist matrices $P = P^T > 0$ and $Q_i > 0$ with appropriate dimensions such that the following linear matrix inequality is satisfied:*

$$P(A_i - L_iC_i) + (A_i - L_iC_i)^T P \leqslant -Q_i, \forall i = 1, 2, \ldots, r \tag{2.18}$$

Proof Consider the following Lyapunov function

$$V_1 = e_x^T(t)Pe_x(t)$$

Differentiating V_1 with respect to time t, one has

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{r} h_i(z(t))[e_x^T(t)(P(A_i - L_iC) + (A_i - L_iC)^T P)e_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[e_x^T(t)Q_ie_x(t)] \\ &\leqslant 0\end{aligned} \tag{2.19}$$

Because $V_1(t) \in L_\infty$ is a monotonous and non-increasing bounded function, $V_1(+\infty)$ exists. Hence, we have $V_1(0) - V_1(+\infty) \geqslant -\int_0^{+\infty} \sum_{i=1}^{r} h_i(z(t))[e_x^T(t)Q_ie_x(t)]$, i.e., $e_x(t) \in L_2$. And since $e_x(t), \dot{e}_x(t) \in L_\infty$, using the Lyapunov stability theory, we obtain $\lim_{t\to\infty} e_x(t) = 0$. Furthermore, we have $\lim_{t\to\infty} e_y(t) = 0$. The proof is completed.

From Lemma 1.1, we have

$$\begin{aligned}\dot{V}_1(t) &\leqslant -\sum_{i=1}^{r} h_i(z(t))[e_x^T(t)Q_ie_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[\lambda_{\min}(Q_i)e_x^T(t)e_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[\lambda_{\min}(Q_i)/\lambda_{\max}(P)e_x^T(t)Pe_x(t)] \\ &\leqslant -h_i(z(t))[\lambda_{\min}(Q_i)/\lambda_{\max}(P)]V(t) = -\kappa V(t)\end{aligned} \tag{2.20}$$

where $\kappa = \min(\frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P)}, \frac{\lambda_{\min}(Q_2)}{\lambda_{\max}(P)}, \ldots, \frac{\lambda_{\min}(Q_r)}{\lambda_{\max}(P)}) \in R$.

Hence,

$$V_1(t) \leqslant e^{-\kappa t}V(0) \tag{2.21}$$

Furthermore, we have

$$\lambda_{\min}(P)||e_x(t)||^2 \leqslant e^{-\kappa t}\lambda_{\max}(P)||e_x(0)||^2 \tag{2.22}$$

Therefore the norm of the error vector satisfies

$$\begin{aligned} ||e_x(t)|| &\leqslant \sqrt{\frac{e^{-\kappa t}\lambda_{\max}(P)}{\lambda_{\min}(P)}}||e_x(0)|| \\ &= \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_x(0)||e^{-\kappa t/2} \end{aligned} \tag{2.23}$$

Furthermore, the detection residual can be defined as:

$$J = ||y(t) - \hat{y}(t)|| \tag{2.24}$$

From (2.23), it can be seen that the following inequality holds in the healthy case:

$$J \leqslant \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||C_i||||e_x(0)||e^{-\kappa t/2} \tag{2.25}$$

Then, the fault detection can be performed using the following mechanism:

$$\begin{cases} J \leqslant T_d \text{ no fault occurred,} \\ J > T_d \text{ fault has occurred} \end{cases} \tag{2.26}$$

where threshold T_d is defined as follows:

$$T_d = \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||C_i||||e_x(0)||e^{-\kappa t/2}.$$

Remark 2.2 It is easy to find from (2.20) that, if no actuator fault occurs, we have $\lim_{t\to\infty} e_x = 0$. If there is an actuator fault, then $\lim_{t\to\infty} e_x \neq 0$. Therefore, in some existing work, the fault detection is carried out as:

$$\begin{cases} \lim_{t\to\infty} e_x = 0, \text{ no fault occurred} \\ \lim_{t\to\infty} e_x \neq 0, \text{ fault has occurred} \end{cases} \tag{2.27}$$

and the above observer given by (2.15) was referred to as the fault detection observer for the system described by (2.2). However, it is valuable to point out that $e_x(\infty)$ is not available in practice, thus $e_x(\infty) \neq 0$ cannot be considered as an indicator of fault occurrence. That is to say, the above fault detection (2.27) does not work in practical applications. Therefore, the mechanism (2.26) is more efficient for fault detection in practical cases.

2.3.3 Fault Isolation

Since the system has m actuators and it is assumed that only one single fault occurs at one time, we have m possible faulty cases in total. When the sth ($1 \leqslant s \leqslant m$) actuator is faulty, the faulty model can be described as:

$$\begin{cases} \dot{x}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i x_s(t) + \sum_{i=1}^{r} h_i(z(t))B_i u(t) - \\ \qquad \sum_{i=1}^{r} h_i(z(t))b_{i,s}[\rho_s(t)u_s(t) - \sum_{j=1}^{p} a_{s,j}(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases} \tag{2.28}$$

where $B_i = [b_{i,1}, b_{i,2}, \ldots, b_{i,m}]$, $b_{i,l} \in R^{n\times 1}, 1 \leqslant l \leqslant m$. $\rho_s(t)$, $a_{s,j}(t), j = 1, 2, \ldots, p$ denote the time profiles of the sth actuator fault, which are described by (2.10), $u_s(t)$ is the desired controller when the sth actuator is healthy. Inspired by the SMOs in [52], we are ready to present one of the results of this chapter. It is assumed that fuzzy observer and fuzzy control systems have the same premise variables $z(t)$, then the following fuzzy observers are proposed to isolate the actuator fault.

Isolation Observer Rule i: IF $z_1(t)$is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}_{is}(t) = A_i\hat{x}_{is}(t) + L_i(y(t) - \hat{y}_{is}(t)) + B_i u(t) + b_{i,s}\mu_s[\bar{\rho}_s|u_s(t)| + \sum_{j=1}^{p} \bar{a}_{s,j}] \\ \hat{y}_{is}(t) = C_{is}\hat{x}_{is}(t) \end{cases} \tag{2.29}$$

where $\hat{x}_{is}(t), \hat{y}_{is}(t)$ are the sth fuzzy observer's state and output, respectively. L_i is the observer's gain matrix for ith observer. The global fuzzy observer is represented as:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i\hat{x}_{is}(t) + \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}_{is}(t)) + \\ \qquad \sum_{i=1}^{r} h_i(z(t))B_i u(t) + \sum_{i=1}^{r} h_i(z(t))b_{i,s}\mu_s[\bar{\rho}_s|u_s(t)| + \sum_{i=1}^{p} \bar{a}_{s,j}] \\ \hat{y}_s(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}_s(t) \\ \mu_s = -\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)/||\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)|| \end{cases} \tag{2.30}$$

where $F_{is} \in R^{1\times n}$ is the sth row of $F_i \in R^{m\times n}$, which will be defined later, $L_i \in R^{n\times n}$ is chosen such that $A_i - L_iC_i$ is Hurwitz, $e_{xs}(t) = x_s(t) - \hat{x}_s(t)$ and $e_{ys}(t) = y(t) - \hat{y}_s(t)$ are respectively the state error and output error between the plant and the sth SMO observer.

For $s = l$, the error dynamics is obtained from (2.28) and (2.30).

$$\begin{aligned}
\dot{e}_{xs}(t) = &\sum_{i=1}^{r} h_i(z(t))A_i e_{is}(t) - \sum_{i=1}^{r} h_i(z(t))L_i(y(t)- \\
&\hat{y}_{is}(t)) + \sum_{i=1}^{r} h_i(z(t))b_{i,s}[(-\rho_s(t)u_s(t) - \mu_s\bar{\rho}_s\cdot \\
&|u_s(t)|) + \sum_{j=1}^{p}(a_{s,j}(t) - \mu_s\bar{a}_{s,j})] \\
= &\sum_{i=1}^{r} h_i(z(t))\{(A_i - L_iC_i)e_{is}(t) + b_{i,s}[(-\rho_s(t)\cdot \\
&u_s(t) - \mu_s\bar{\rho}_s|u_s(t)|) + \sum_{j=1}^{p}(a_{s,j}(t) - \mu_s\bar{a}_{s,j})]\}
\end{aligned} \tag{2.31}$$

For $s \neq l$, we have

$$\begin{aligned}
\dot{e}_{xs}(t) = &\sum_{i=1}^{r} h_i(z(t))(A_i - L_iC_i)e_{is}(t)+ \\
&\sum_{i=1}^{r} h_i(z(t))[(-b_{i,l}\rho_l(t)u_l(t) - b_{i,s}\mu_s\bar{\rho}_s|u_s(t)|)+ \\
&\sum_{j=1}^{p}(b_{i,l}a_{l,j}(t) - b_{i,s}\mu_s\bar{a}_{s,j})]
\end{aligned} \tag{2.32}$$

The stability of the state error dynamics is guaranteed by the following theorem.

Theorem 2.1 *Under Assumptions 2.1–2.3, if there exist a common symmetric positive definite matrix P and matrices L_i, F_i, and $Q_i > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold,*

$$(A_i - L_iC_i)^T P + P(A_i - L_iC_i) \leqslant -Q_i, \tag{2.33}$$

$$PB_i = (F_iC_i)^T. \tag{2.34}$$

Then, when the lth actuator is faulty, for $s = l$, $\lim_{t\to\infty} e_{xs} = 0$, and for $s \neq l$, $\lim_{t\to\infty} e_{xs} \neq 0$.

Proof (1) For $s = l$, according to (2.31), we have

$$\dot{e}_{xs}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i - L_iC_i)e_{is}(t) + \sum_{i=1}^{r} h_i(z(t))b_{i,s}[(-\mu_s\bar{\rho}_s|u_s(t)| - \rho_s(t)u_s(t)) - \sum_{j=1}^{p} \mu_s\bar{a}_{s,j} + \sum_{j=1}^{p} a_{s,j}(t)]$$

Define the following Lyapunov function

$$V_2(t) = e_{xs}^T(t)Pe_{xs}(t) \tag{2.35}$$

Differentiating V_2 with respect to time t, and using (2.33), one has

$$\begin{aligned}\dot{V}_2(t) &= \dot{e}_{xs}^T(t)Pe_{xs}(t) + e_{xs}^T(t)P\dot{e}_{xs}(t) \\ &\leqslant -e_{xs}^T(t)Q_ie_{xs}(t) + 2e_{xs}^T(t)P\sum_{i=1}^{r} h_i(z(t))b_{i,s}\cdot \\ &\quad [(-\mu_s\bar{\rho}_s|u_s(t)| - \rho_s(t)u_s(t)) - \sum_{j=1}^{p} \mu_s\bar{a}_{s,j} + \sum_{j=1}^{p} a_{s,j}(t)]\end{aligned}$$

From $\mu_s = -\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)/||\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)||$ and (2.34), one has

$$2e_{xs}^T(t)P\sum_{i=1}^{r} h_i(z(t))b_{i,s}(-\mu_s\bar{\rho}_s|u_s(t)| - \rho_s(t)u_s(t)) \leqslant 0,$$

$$2e_{xs}^T(t)P\sum_{i=1}^{r} h_i(z(t))b_{i,s}(-\sum_{j=1}^{p} \mu_s\bar{a}_{s,j} + \sum_{j=1}^{p} a_{s,j}(t)) \leqslant 0.$$

Hence,

$$\dot{V}_2(t) \leqslant -e_{xs}^T(t)Q_ie_{xs}(t) \leqslant 0 \tag{2.36}$$

Because $V_2(t) \in L_\infty$ is a monotonous and non-increasing bounded function, $V_2(+\infty)$ exists. Hence, we have $V_2(0) - V_2(+\infty) \geqslant -\int_0^{+\infty} e_{xs}^T(t)Q_ie_{xs}(t)$, i.e. $e_{xs}(t) \in L_2$. Since $e_{xs}(t)$ and $\dot{e}_{xs}(t) \in L_\infty$, using the Lyapunov stability theory, we have $\lim_{t\to\infty} e_{xs}(t) = 0$. Thus, we have $\lim_{t\to\infty} e_{ys}(t) = 0$.

(2) For $s \neq l$, it follows from (2.28) and (2.30) that:

$$\begin{aligned}\dot{e}_{xs}(t) = & \sum_{i=1}^{r} h_i(z(t))(A_i - L_i C_i)e_{is}(t) + \\ & \sum_{i=1}^{r} h_i(z(t))[(-b_{i,l}\rho_l(t)u_l(t) - b_{i,s}\mu_s\bar{\rho}_s|u_s(t)|) + \\ & \sum_{j=1}^{p}(b_{i,l}a_{l,j}(t) - b_{i,s}\mu_s\bar{a}_{s,j})]\end{aligned}$$

Because matrix B_i is of full column rank (Assumption 2.1), we know that b_{is} and b_{il} are linearly independent. Therefore,

$$\lim_{t\to\infty}\sum_{i=1}^{r} h_i(z(t))[(-b_{i,l}\rho_l(t)u_l(t) - b_{i,s}\mu_s\bar{\rho}_s|u_s(t)|) + \sum_{j=1}^{p}(b_{i,l}a_{l,j}(t) - b_{i,s}\mu_s\bar{a}_{s,j}) \neq 0 \tag{2.37}$$

Thus, we have $\lim_{t\to\infty} e_{xs}(t) \neq 0$ and $\lim_{t\to\infty} e_{ys}(t) \neq 0$.

From (1) and (2), we obtain the conclusions. This ends the proof.

Now, we denote the residuals between the real system and SMOs as follows:

$$J_s(t) = \left\| e_{ys}(t) \right\| = \left\| \hat{y}_s(t) - y(t) \right\|, \quad 1 \leqslant s \leqslant m \tag{2.38}$$

According to Theorem 2.1, when the lth actuator is faulty, i.e., $s = l$, the residual $J_s(t)$ must tend to zero; while for any $s \neq l$, basically, $J_s(t)$ does not equal zero. Furthermore, from Lemma 2.1, we have, if $l = s$, then

$$J_s(t) \leqslant \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2} \tag{2.39}$$

and if $l \neq s$, then

$$J_s(t) > \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2} \tag{2.40}$$

Hence, the isolation law for actuator fault can be designed as

$$\begin{cases} J_s(t) \leqslant T_I, l = s \Rightarrow \text{the } l\text{th actuator is faulty} \\ J_s(t) > T_I, l \neq s \end{cases} \tag{2.41}$$

where threshold T_I is defined as follows:

$$T_I = \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2}.$$

Note that, $\mu_s = -\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)/||\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)||$ in (2.30), which denominator contains $e_{ys}(t)$. Just as pointed out in [52], the chattering phenomenon occurs when $e_{ys}(t) \to 0$ in practice. Inspired by [52], in order to reduce this chattering in practical applications, we modify SMOs (2.30) by introducing a positive constant δ as follows:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i\hat{x}_s(t) - \sum_{i=1}^{r} h_iL_i(\hat{y}_s(t) - y(t)) + \\ \qquad \sum_{i=1}^{r} h_i(z(t))B_iu(t) - \sum_{i=1}^{r} h_i(z(t))\mu'_s[\bar{\rho}_s|u_s(t)| + \sum_{j=1}^{p} \bar{a}_{s,j}] \\ \hat{y}_s(t) = \sum_{i=1}^{r} h_i(z(t))C_{is}\hat{x}_s(t) \\ \mu'_s = -\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)/(||\sum_{i=1}^{r} h_i(z(t))F_{is}e_{ys}(t)|| + \delta) \end{cases} \tag{2.42}$$

where $\delta > 0 \in R$ is a constant, $s = 1, 2, \ldots, m$. Obviously, the denominator of μ'_s will converge asymptotically to δ when $e_{ys}(t) \to 0$, which reduces this chattering phenomenon.

From the above analysis, it is easy to find that, a suitable threshold δ must be selected such that $J_s(s = l)$ tends to be very small when the lth actuator is faulty, while other residuals $J_s(s \neq l)$ are not equal to zero on any small time intervals. Thus, the modified SMOs can not only decrease the chattering problem in practice, but also can realize fault diagnosis successfully.

2.3.4 Fault Estimation

After fault isolation, we can estimate the fault. Assume the sth ($1 \leqslant s \leqslant m$) actuator is faulty, the faulty system can be described as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))A_ix(t) + \sum_{i=1}^{r} h_i(z(t))B_iu(t) - \sum_{i=1}^{r} h_i(z(t))b_{i,s}[\rho_s u_s(t) - \sum_{j=1}^{p} a_{s,j}(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_ix(t) \end{cases} \tag{2.43}$$

To estimate the fault, an observer is presented as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))A_i\hat{x}(t) + \sum_{i=1}^{r} h_i(z(t))B_iu(t) - \\ \qquad \sum_{i=1}^{r} h_i(z(t))b_{i,s}[\hat{\rho}_s u_s(t) - \sum_{j=1}^{p} \hat{a}_{s,j}] + \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))C_ix(t) \end{cases} \tag{2.44}$$

where $\hat{\rho}_s$, $\hat{a}_{s,j}$ are the estimate values of $\rho_s(t)$, $a_{s,j}(t)$ at time t.

Remark 2.3 Many results about observer design were reported in literature. For faulty systems with only bias fault f_a described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u(t) + f_a) \\ \hat{y}(t) = Cx(t) \end{cases}$$

an observer is classically designed in the following form of

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B(u(t) + \hat{f}_a) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

Let $e_x(t) = x(t) - \hat{x}(t)$, then the error dynamics is described by

$$\dot{e}_x(t) = (A - LC)e_x(t) + B(f_a - \hat{f}_a)$$

where $\hat{f}_a$ denotes the estimation of f_a. However, in this chapter, actuator bias faults and gain faults are both considered, the above observer does not work. The novel observer (2.44) is proposed in order to estimate the two kinds of faults.

Using (2.43) and (2.44), the error dynamics is obtained:

$$\dot{e}_x(t) = \sum_{i=1}^{r} h_i(z(t))[(A_i - L_iC_i)e_x(t))] - \sum_{i=1}^{r} h_i(z(t))b_{i,s}[\tilde{\rho}_s u_s - \sum_{j=1}^{p} \tilde{a}_{s,j}] \tag{2.45}$$

where $e_x(t) = x(t) - \hat{x}(t)$, $\tilde{\rho}_s = \rho_s(t) - \hat{\rho}_s$, $\tilde{a}_{s,j} = a_{s,j}(t) - \hat{a}_{s,j}$.

Now, an adaptive fault diagnostic algorithm is proposed to estimate the actuator fault. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 2.2 *Under Assumptions 2.1–2.3, if there exist a common symmetric positive definite matrix P, real matrices L_i and $Q_i > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold,*

$$P(A_i + L_iC_i) + (A_i + L_iC_i)^T P < -Q_i \tag{2.46}$$

$$PB_i = (F_iC_i)^T \tag{2.47}$$

$$\dot{\hat{\rho}}_i = \begin{cases} 0, \ \hat{\rho}_i = \bar{\rho}_1 \ and - 2\eta_1 F_{i,s}e_y > 0 \ or \ \hat{\rho}_i = -\bar{\rho}_1 \ and - 2\eta_1 F_{i,s}e_y < 0 \\ -2\eta_1 F_{i,s}e_y u_s, \ otherwise \end{cases} \tag{2.48}$$

$$\dot{\hat{a}}_{i,j} = \begin{cases} 0, \ \hat{a}_{i,j} > \bar{a}_1 \ and \ 2\eta_2 F_{i,s}e_y > 0 \ or \ \hat{a}_{i,j} < -\bar{a}_1 \ and \ 2\eta_2 F_{i,s}e_y < 0 \\ 2\eta_2 F_{i,s}e_y, \ otherwise \end{cases} \tag{2.49}$$

where $i = 1, \ldots, m,\ j = 1, \ldots, p$, $F_{is} \in R^{1\times n}$ *is the sth row of* $F_i \in R^{m\times n}$, $\eta_1 > 0$, $\eta_2 > 0$ *denote the adaptive rates, then the error system (2.45) is asymptotically stable. Moreover,* $e_x(t)$, $\tilde{\rho}_s$ *and* $\tilde{a}_{s,j}$ *are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely,* $|e_x| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, $|\tilde{\rho}_i| \leqslant \sqrt{2\eta_1\alpha}$, *and* $|\tilde{g}_{i,j}| \leqslant \sqrt{2\eta_2\alpha}$, *where*

$$\mu_0 = \sum_{l}^{r} h_l(z(t))\left(\frac{2\bar{\rho}_1(2\bar{\rho}_1 + \bar{\rho}_2)}{\eta_1} + \sum_{j=1}^{p} \frac{2\bar{a}_1(2\bar{a}_1 + \bar{a}_2)}{\eta_2}\right),$$

$$\lambda_0 = \min\{\frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P)}, \ldots, \frac{\lambda_{\min}(Q_r)}{\lambda_{\max}(P)}, 1\}$$

and $\alpha = \mu_0/\lambda_0 + V(0)$.

Proof Define the following smooth function

$$V = V_1 + V_2 + V_3 \tag{2.50}$$

$$V_1 = e_x^T(t)Pe_x(t) \tag{2.51}$$

$$V_2 = \sum_{i=1}^{r} h_i(z(t))\left(\frac{1}{2\eta_1}\tilde{\rho}_s^2(t)\right) \tag{2.52}$$

$$V_3 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\left(\frac{1}{2\eta_2}a_{s,j}^2(t)\right) \tag{2.53}$$

Differentiating $V,\ V_i, i = 1, 2, 3$ with respect to time t, leads to

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \tag{2.54}$$

$$\begin{aligned}\dot{V}_1 = &\sum_{i=1}^{r} h_i(z(t))[e_x^T(t)(P(A_i - L_iC_i) + (A_i - L_iC_i)^T P)e_x(t)] - \\ &\sum_{i=1}^{r} h_i(z(t))[2e_x^T(t)Pb_{i,s}\tilde{\rho}_s u_s - \sum_{j=1}^{p} 2e_x^T(t)Pb_{i,s}\tilde{a}_{s,j}]\end{aligned} \tag{2.55}$$

$$\begin{aligned}\dot{V}_2 &= \sum_{i=1}^{r} h_i(z(t))(\frac{1}{\eta_1}\tilde{\rho}_s\dot{\tilde{\rho}}_s) = \sum_{i=1}^{r} h_i(z(t))(\frac{1}{\eta_1}\tilde{\rho}_s(\dot{\rho}_s - \dot{\hat{\rho}}_s) \\ &= \sum_{i=1}^{r} h_i(z(t))\frac{1}{\eta_1}\tilde{\rho}_s\dot{\rho}_s - \sum_{i=1}^{r} h_i(z(t))\frac{1}{\eta_1}\tilde{\rho}_s\dot{\hat{\rho}}_s\end{aligned} \tag{2.56}$$

$$\begin{aligned}\dot{V}_3 &= \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{\tilde{a}_{s,j}\dot{\tilde{a}}_{s,j}}{\eta_2} = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{\tilde{a}_{s,j}(\dot{a}_{s,j} - \dot{\hat{a}}_{s,j})}{\eta_2} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{\tilde{a}_{s,j}\dot{a}_{s,j}}{\eta_2} - \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{\tilde{a}_{s,j}\dot{\hat{a}}_{s,j}}{\eta_2}\end{aligned} \tag{2.57}$$

Substituting (2.55–2.57) into (2.54), it yields

$$\begin{aligned}\dot{V} = &-\sum_{i=1}^{r} h_i(z(t))e_x^T Q_i e_x + \sum_{i=1}^{r} h_i(z(t))\frac{1}{\eta_1}\tilde{\rho}_s\dot{\rho}_s + \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{1}{\eta_2}\tilde{a}_{s,j}\dot{a}_{s,j} - \\ &\sum_{i=1}^{r} h_i(z(t))\tilde{\rho}_s(2e_x^T Pb_{i,s}u_s + \frac{1}{\eta_1}\dot{\hat{\rho}}_s) + \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\tilde{a}_{s,j}(2e_x^T Pb_{i,s} - \frac{1}{\eta_2}\dot{\hat{a}}_{s,j})\end{aligned} \tag{2.58}$$

Substituting (2.48, 2.49) into (2.58), it yields

$$\dot{V} = -\sum_{i=1}^{r} h_i(z(t))e_x^T Q_i e_x + \sum_{i=1}^{r} h_i(z(t))\frac{1}{\eta_1}\tilde{\rho}_s\dot{\rho}_s + \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{1}{\eta_2}\tilde{a}_{s,j}\dot{a}_{s,j} \tag{2.59}$$

Since

$$\begin{aligned}\frac{\tilde{\rho}_i\dot{\rho}_i}{\eta_1} &= -\frac{\tilde{\rho}_i^2}{\eta_1} + \frac{\tilde{\rho}_i(\tilde{\rho}_i + \dot{\rho}_i)}{\eta_1} = -\frac{\tilde{\rho}_i^2}{\eta_1} + \frac{(\rho_i - \hat{\rho}_i)(\rho_i - \hat{\rho}_i + \dot{\rho}_i)}{\eta_1} \\ &\leqslant -\frac{\tilde{\rho}_i^2}{\eta_1} + \frac{(|\rho_i| + |\hat{\rho}_i|)(|\rho_i| + |\hat{\rho}_i| + |\dot{\rho}_i|)}{\eta_1}\end{aligned}$$

$$\begin{aligned}\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}\dot{a}_{i,j}}{\eta_2} &= -\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{\eta_2}+\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{\eta_2}+\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}\dot{a}_{i,j}}{\eta_2}\\ &\leqslant -\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{\eta_2}+\sum_{j=1}^{p}\frac{(|a_{i,j}|+|\hat{a}_{i,j}|)(|a_{i,j}|+|\hat{a}_{i,j}|+|\dot{a}_{i,j}|)}{\eta_2}\end{aligned}$$

and $|\hat{\rho}_i(t)| \leqslant \bar{\rho}_1$ and $|\hat{a}_{i,j}(t)| \leqslant \bar{a}_1$, which can be guaranteed by using the adaptive laws (2.48) and (2.49), and Assumptions 2.2 and 2.3 (i.e., $|\rho_i(t)| \leqslant \bar{\rho}_1$, $|\dot{\rho}_i(t)| \leqslant \bar{\rho}_2$, $|a_{i,j}(t)| \leqslant \bar{a}_1$, and $|\dot{a}_i(t)| \leqslant \bar{a}_2$) are satisfied, one has

$$\frac{\tilde{\rho}_i\dot{\rho}_i}{\eta_1} \leqslant -\frac{\tilde{\rho}_i^2}{\eta_1}+\frac{2\bar{\rho}_1(2\bar{\rho}_1+\bar{\rho}_2)}{\eta_1}$$

$$\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}\dot{a}_{i,j}}{\eta_2} \leqslant -\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{\eta_2}+\sum_{j=1}^{p}\frac{2\bar{a}_1(2\bar{a}_1+\bar{a}_2)}{\eta_2}$$

Hence, from (2.59), one has

$$\begin{aligned}\dot{V} &\leqslant \sum_{l=1}^{r}h_l(z(t))[-e_x^TQ_ie_x-\frac{\tilde{\rho}_i^2}{\eta_1}-\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{\eta_2}+\frac{2\bar{\rho}_1(2\bar{\rho}_1+\bar{\rho}_2)}{\eta_1}+\sum_{j=1}^{p}\frac{2\bar{a}_1(2\bar{a}_1+\bar{a}_2)}{\eta_2}]\\ &\leqslant \sum_{l}^{r}h_l(z(t))[-e_x^TQ_ie_x-\frac{\tilde{\rho}_i^2}{\eta_1}-\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{\eta_2}+\frac{2\bar{\rho}_1(2\bar{\rho}_1+\bar{\rho}_2)}{\eta_1}+\sum_{j=1}^{p}\frac{2\bar{a}_1(2\bar{a}_1+\bar{a}_2)}{\eta_2}]\\ &\leqslant \sum_{l=1}^{r}h_l(z(t))[-e_x^TQ_ie_x-\frac{\tilde{\rho}_i^2}{\eta_1}-\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{\eta_2}+\mu]\\ &\leqslant \sum_{l=1}^{r}h_l(z(t))[-\lambda_{\min}(Q_i)e_x^Te_x-\frac{\tilde{\rho}_i^2}{2\eta_1}-\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{2\eta_2}+\mu]\\ &\leqslant \sum_{l=1}^{r}h_l(z(t))[-\frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P)}e_x^TPe_x-\frac{\tilde{\rho}_i^2}{2\eta_1}-\sum_{j=1}^{p}\frac{\tilde{a}_{i,j}^2}{2\eta_2}+\mu]\\ &\leqslant -\lambda_0V(t)+\mu_0\end{aligned} \tag{2.60}$$

where

$$\mu = \frac{2\bar{\rho}_1(2\bar{\rho}_1+\bar{\rho}_2)}{\eta_1}+\sum_{j=1}^{p}\frac{2\bar{a}_1(2\bar{a}_1+\bar{a}_2)}{\eta_2},$$

$$\mu_0 = \sum_{l}^{r}h_l(z(t))(\frac{2\bar{\rho}_1(2\bar{\rho}_1+\bar{\rho}_2)}{\eta_1}+\sum_{j=1}^{p}\frac{2\bar{a}_1(2\bar{a}_1+\bar{a}_2)}{\eta_2}),$$

$\lambda_0 = \min\{\frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P)}, \frac{\lambda_{\min}(Q_2)}{\lambda_{\max}(P)}, \ldots, \frac{\lambda_{\min}(Q_r)}{\lambda_{\max}(P)}, 1\}$. Then, one has, $\frac{d}{dt}(V(t)e^{\lambda_0 t}) \leqslant e^{\lambda_0 t}\mu_0$. Furthermore, $0 \leqslant V(t) \leqslant \frac{\mu_0}{\lambda_0} + [V(0) - \frac{\mu_0}{\lambda_0}]e^{-\lambda_0 t} \leqslant \frac{\mu_0}{\lambda_0} + V(0)$ Let $\alpha = \frac{\mu_0}{\lambda_0} + V(0)$, one has $|e_x| \leqslant \sqrt{\frac{\alpha}{\lambda_{\min}(P)}}$, $|\tilde{\rho}_i| \leqslant \sqrt{2\eta_1\alpha}$, and $|\tilde{a}_{i,j}| \leqslant \sqrt{2\eta_2\alpha}$. This ends the proof.

Remark 2.4 If there exist two known constants $f_{\min}, f_{\max}$ such that $f_{\min} \leqslant |f(t)| \leqslant f_{\max}$, then the fault $f(t)$ can be approximated by the following form

$$f(t) = \frac{1}{2}(f_{\max} - f_{\min})(1 - \tanh\zeta) + f_{\min} \tag{2.61}$$

where ζ is an unknown constant. Thus, the fault $f(t)$ is estimated through the estimation of $\hat{\zeta}$, namely

$$\hat{f}(t) = \frac{1}{2}(f_{\max} - f_{\min})(1 - \tanh\hat{\zeta}) + f_{\min} \tag{2.62}$$

This method prevents the phenomenon of parameter drift in the presence of bounded disturbances because of $|\tanh\hat{\varsigma}| < 1$, and ensures $f_{\min} \leqslant |\hat{f}(t)| \leqslant f_{\max}$.

2.3.5 Fault Accommodation

After that the fault information is obtained, we will consider the fault-tolerant control problem of system (2.2), and design a fault-tolerant control law to recover the control system's dynamics performance when an actuator fault occurs. Firstly, we consider the fuzzy control problem for the following nominal system without actuator faults:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases}$$

The parallel distributed compensation technique offers a procedure to design a fuzzy control law from a given T-S fuzzy model. In the PDC design, each control rule is designed from the corresponding rule of T-S fuzzy model. The designed fuzzy controller has the same fuzzy sets as the considered fuzzy system.

Control Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$u_i(t) = K_i x(t)$$

and the overall fuzzy controller is given as follows:

$$u(t) = \sum_{i=1}^{r} h_i(z(t))K_i x(t)$$

where the controller gain matrix K_i is determined by solving the following LMI:

$$P(A_i + B_i K_i) + (A_i + B_i K_i)^T P < -Q_i \tag{2.63}$$

where $P = P^T > 0$ and $Q_i > 0$ are matrices with appropriate dimensions.

On the basis of the estimated actuator fault, the fault tolerant controller is constructed as

$$u_s = \frac{(u_s^N - \sum_{j=1}^{p_i} \hat{a}_{i,j})}{(1 - \hat{\rho}_s)} \tag{2.64}$$

where u_s^N is the sth normal control input, $\hat{\rho}_s$, $\hat{a}_{i,j}$ are the estimations of ρ_s, $a_{i,j}$, which are used to compensate for the gain fault and bias fault.

Theorem 2.3 *Consider system (2.2) under Assumptions 2.1–2.3. If there exist a common symmetric positive definite matrix P, real matrices L_i and $Q_i > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$P(A_i - L_i C_i) + (A_i - L_i C_i)^T P < -Q_i \tag{2.65}$$

$$PB_i = (F_i C_i)^T \tag{2.66}$$

$$\dot{\hat{\rho}}_i = \begin{cases} 0, \quad \hat{\rho}_i = \bar{\rho}_1 \text{ and } -2\eta_1 F_{i,s} e_y > 0 \text{ or } \hat{\rho}_i = -\bar{\rho}_1 \text{ and } -2\eta_1 F_{i,s} e_y < 0 \\ -2\eta_1 F_{i,s} e_y u_s, \text{ otherwise} \end{cases} \tag{2.67}$$

$$\dot{\hat{a}}_{i,j} = \begin{cases} 0, \ \hat{a}_{i,j} > \bar{a}_1 \text{ and } 2\eta_2 F_{i,s} e_y > 0 \text{ or } \hat{a}_{i,j} < -\bar{a}_1 \text{ and } 2\eta_2 F_{i,s} e_y < 0 \\ 2\eta_2 F_{i,s} e_y, \text{otherwise} \end{cases} \tag{2.68}$$

where $i = 1, \ldots, m$, $j = 1, \ldots, p$, Then system (2.2) is asymptotically stable under the feedback FTC (2.65) and all signals involved in the closed-loop system are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely,

$$|e| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}, \ |\tilde{\rho}_i| \leqslant \sqrt{2\eta_1 \alpha}, \quad |\tilde{a}_{i,j}| \leqslant \sqrt{2\eta_2 \alpha},$$

where $\lambda_0 = \min\{\frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P)}, \ldots, \frac{\lambda_{\min}(Q_r)}{\lambda_{\max}(P)}, 1\}$, $\mu_0 = \sum_{l}^{r} h_l(z(t))[\frac{2\bar{\rho}_1(2\bar{\rho}_1 + \bar{\rho}_2)}{\eta_1} + \sum_{j=1}^{p} \frac{2\bar{a}_1(2\bar{a}_1 + \bar{a}_2)}{\eta_2}]$, $\alpha = V(0) + \mu_0/\lambda_0$.

Proof Similar to the proof of Theorem 2.2, it is easy to obtain the conclusions of Theorem 2.3. The detailed proof is thus omitted here.

2.4 Simulation Results

2.4.1 NSHV Modeling and Analysis

Considering the longitudinal flight mode of NSHV, a mathematical model for a generic NSHV developed at NASA Langley Research Center is presented in [53]. The longitudinal dynamics of NSHV can be described by a set of differential equations involving its velocity V, flight-path angle γ, altitude h, angle of attack α and pitch rate q as

$$\dot{V} = \frac{T\cos\alpha - D}{m} - \frac{u\sin\gamma}{r^2} \tag{2.69}$$

$$\dot{\gamma} = \frac{L + T\sin\alpha}{mV} + \frac{(\mu - Vr^2)\cos\gamma}{Vr^2} \tag{2.70}$$

$$\dot{h} = V\sin\gamma \tag{2.71}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{2.72}$$

$$\dot{q} = \frac{M_{yy}}{I_{yy}} \tag{2.73}$$

where $L = \bar{q}SC_L, D = \bar{q}SC_D, T = \bar{q}SC_T, r = h + R_e,\ \ M_{yy} = \ \bar{q}S\bar{c}[C_M(\alpha) + C_M(\delta_e) + C_M(q)],\ \ C_L = 0.6203\alpha,\ \ C_D = 0.6450\alpha^2 + 0.0043378\alpha + 0.003772,\ \ C_M(\delta_e) = c_e(\delta_e - \alpha),\ \ C_M(q) = (\bar{c}/2V)q(-6.796\alpha^2 + 0.3015\alpha - 0.2289), C_M(\alpha) = -0.035\cdot\alpha^2 + 0.036617(1 + \Delta C_{M\alpha})\alpha + 5.3261e - 06$, and

$$C_T = \begin{cases} 0.02576\delta_T\,, \text{ when } \ \delta_T < 1 \\ 0.0224 + 0.00336\delta_T, \quad \text{ when } \ \delta_T > 1 \end{cases}.$$

The parameters are the aircraft mass m, the gravitational constant μ, the moments of inertia I_{yy} and the pitch moment coefficients. The aerodynamic coefficients and inertia data are coupled with state variables and control inputs. The control input vector is $u(t) = [\delta_e, \delta_T]^T$, where δ_e is the elevator detection, and δ_T is the throttle setting, respectively. The longitudinal model of the NSHV described by (2.69–2.73) can be written in the following affine nonlinear form:

$$\begin{cases} \dot{x}(t) = f(x) + g(x)u(t) \\ y(t) = Cx(t) \end{cases} \tag{2.74}$$

where $x(t) = [V, \gamma, h, \alpha, q]^T \in R^n$ denotes state vector, $u(t) = [\delta_e, \delta_T]^T \in R^m$ denotes the control input vector, and $y(t)$ is the output vector.

In this section, some simulation results are presented to demonstrate the effectiveness of the proposed techniques. For the purpose of this study, the aerodynamic coefficients are simplified around the cruising flight mode. The nominal flight of NSHV is at a trimmed cruise conditions: $Mach = 15$, $V = 15060$ ft/s and $h = 110000$ ft/s.

If each state variable is selected as a premise variable, then the number of fuzzy rules will become too large. However, from the property of NSHV, we know that the angle of attack α is a key variable affecting the nonlinear character of NSHV, and the velocity V has constraint relationship to the altitude h, and the pitch angle $\theta = \alpha + \gamma$. Similar to [53], we select $\bar{x} = [V, \theta, q]^T$ as a new state vector. As a result, we denote $z_1 = V$, $z_2 = \alpha + \gamma$, $z_3 = q$, and select z_1, z_2 and z_3 as premise variables for the T-S fuzzy system model. Hence, it can not only reduce the number of fuzzy rules but also well approximate the nonlinear system and characterize the NSHV model [7]. Furthermore, we assume

$$z_1 \in (6000 \;\; 16000) \;\; \text{m/s}, \;\; z_2 \in (-0.5 \;\; 0.5) \;\; \text{rad/s}, \;\; z_3 \in (-0.5 \;\; 0.5) \;\; \text{rad/s}.$$

Suppose that each premise variable has two associated fuzzy sets:

$$\{z_1 = 6000, 16000\}; \;\; \{z_2 = -0.5, 0.5\}; \;\; \{z_3 = -0.5, 0.5\}$$

The corresponding fuzzy membership functions are defined as

$$M_{z_1=6000} = \exp[-(z_1/\varsigma_1)^2], M_{z_1=16000} = 1 - M_{z_1=6000}$$

$$M_{z_2=-0.5} = \frac{1}{1 + \exp[((z_2)^2 - \sigma)/\varsigma_2]}, \;\; M_{z_2=0.5} = 1 - M_{z_2=-0.5}$$

$$M_{z_3=-0.5} = \exp[-(\frac{z_3}{\varsigma_3} - \bar{\sigma})], M_{z_3=-0.5} = 1 - M_{z_3=-0.5}$$

where the unknown parameters $\sigma, \bar{\sigma}, \varsigma_1, \varsigma_2, \varsigma_3$ should be selected to symmetrically cover the space of the input variables.

We choose eight working points of NSHV as follows:

$$[z_1, z_2, z_3]^T =: \begin{cases} [6000, -0.5, 0.5], [6000, 0.5, 0.5], [6000, -0.5, -0.5] \\ [6000, 0.5, -0.5], [5000, -0.5, 0.5], [16000, 0.5, 0.5] \\ [16000, 0.5, -0.5], [6000, -0.5, 0.5] \end{cases}$$

The parameters of the membership are selected as: $\sigma = 0.15$, $\bar{\sigma} = 4$, $\varsigma_1 = 3200$, $\varsigma_2 = 0.05$, $\varsigma_3 = 0.4$.

Then, eight plant rules and corresponding control rules can be obtained. We give the first rule as an example, and the other rules have the similar form.

Rule 1: IF z_1 is about 6000 m/s and z_2 is about -0.5 rad/s and z_3 is about -0.5 rad/s , THEN

$$\dot{\bar{x}}(t) = A_1\bar{x}(t) + B_1u(t), y(t) = C\bar{x}(t)$$

where A_i and B_i, $i = 1, 2, \ldots, 8$ can be easily obtained by the substitution of each of the eight operating points to $f(x)$ and $g(x)$.

In this study, we assume that only an actuator is faulty at one time. We consider:

Case 1:

$$u_1^f(t) = u_1(t),$$

$$u_2^f(t) = \begin{cases} y_2(t), \ t < 5 \\ (1 - \rho_2(t))(y_2(t) + \sum_{j=1}^{p} g_{2,j}f_{2,j}(t)), \ t \geqslant 5 \end{cases}$$

where $\rho_2(t) = 0.4\sin(\pi t), p = 1, g_{2,1} = 0.4, f_{2,1}(t) = \cos(t)$.

In order to compare with the results in [6, 8], we consider the following cases.

Case 2 (Bias fault) [24]:

$$u_1^f(t) = u_1(t),$$

$$u_2^f(t) = u_2(t) + f_{2,1}(t), \ f_{2,1}(t) = \begin{cases} 0, \ t < 4s \\ 5, t \geqslant 4s \\ 5 + 2(t-7), \ t \geqslant 7s \end{cases}$$

where $\rho_2(t) = 0, p = 1, g_{2,1} = 1$.

Case 3 (Gain fault) [51]:

$$u_1^f(t) = u_1(t),$$

$$u_2^f(t) = (1 - \rho_2(t))u_2(t), \ \rho_2(t) = \begin{cases} 0, \ t < 2s \\ 0.4, \ t \geqslant 2s \end{cases}$$

where $\rho_2(t) = 0, p = 0, g_{2,1} = 0$.

Remark 2.5 If each state variable of the near space hypersonic vehicle (NSHV) model is selected as premise variable, then the number of fuzzy rules becomes too large, which leads to the increasing amount of computing and thus affects the setting time of the closed loop system. In order to reduce the number of fuzzy rules, taking into account the main characteristics of NSHV, we select $\bar{x} = [V, \theta, q]^T$ as premise variables where $\theta = \alpha + \gamma$. As pointed out in [52], it can not only reduce the number of fuzzy rules but it provides also a good approximation of the nonlinear system. As a result, it can achieve satisfactory accuracy and dynamic performance of the proposed fault tolerant control.

2.4.2 Simulation Results

By using Matlab toolbox to solve the matrices inequalities (2.18), one can obtain the fault diagnostic observer gains L_i. By solving (2.64) and (2.67), one can obtain the positive definite symmetric matrix P and the nominal controller gains K_i. Due to the space limitation, only the common matrix P, and the matrices Q_1, L_1, K_1 of the first working point of NSHV are given here. Therefore, one can design the fault-tolerant controller (2.65).

$$P = 1.0\mathrm{e}+005 *$$
$$\begin{bmatrix} 3.4852 & -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0000 & 3.4852 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 3.4852 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 3.4852 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 & -0.0000 & 3.4852 \end{bmatrix}$$

$$Q_1 = 1.0\mathrm{e}+005 *$$
$$\begin{bmatrix} 3.4852 & -0.0000 & -0.0000 & 0.0001 & -0.0006 \\ 0.0000 & 3.4852 & -0.0000 & -0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 3.4852 & -0.0000 & 0.0000 \\ -0.0001 & 0.0000 & 0.0000 & 3.4852 & 0.0001 \\ 0.0006 & -0.0000 & -0.0000 & -0.0001 & 3.4852 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 9.4165 & 44487.8491 & 0.8575 & 181.5760 & 1.6392 \\ 5.6423 & 18484.9800 & -0.5165 & 85.75630 & 0.7744 \end{bmatrix}$$

$$L_1 = 1.0\mathrm{e}+008 *$$
$$\begin{bmatrix} -0.0035 & -0.1100 & -0.0003 & 0.0354 & 0.0003 \\ -0.1100 & -0.0035 & 6.9356 & 0.0000 & 0.0000 \\ -0.0003 & 6.9356 & -0.0035 & -0.0000 & -0.0000 \\ 0.0354 & 0.0000 & 0.0000 & -0.0035 & -0.7706 \\ 0.0003 & -0.0000 & 0.0000 & -0.7706 & -0.7755 \end{bmatrix}$$

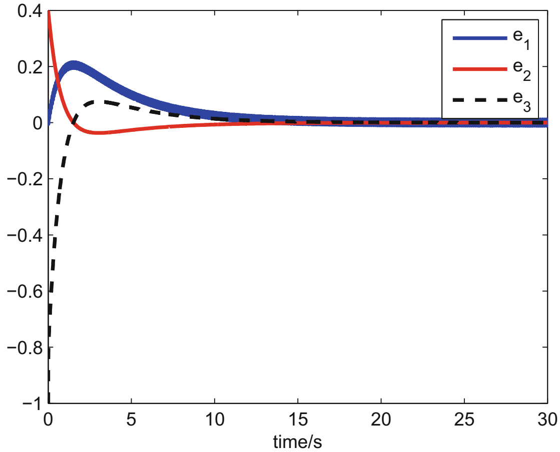

Fig. 2.1 The observer errors time responses: e_1, e_2, e_3 (healthy case)

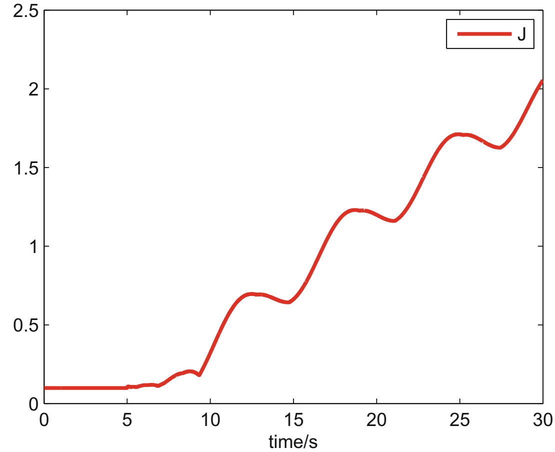

Fig. 2.2 Fault detection residual J with threshold

The simulation results are presented in Figs. 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 and 2.9. From Fig. 2.1, it is easy to see that, under normal operating condition, observation errors globally asymptotically converge to zero. In this chapter, it is assumed that the error system is stable before fault occurrence, namely, $e_x(0) = 0$, $\bar{e}_{xs}(0) = 0$, $||e_x(0)||e^{-\kappa t/2} * \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)} = 0$. Hence, in the ideal situation, the detection thresholdT_d and the isolation threshold T_I can be select as $T_d = T_I = 0$. However, there may exist noise and disturbance in practical situation. In the simulations, a white noise, with zero mean and standard deviation which is equal to 0.1, is added on each output. As a result, the detection threshold T_d and the isolation threshold T_I can be selected as $T_d = 0.1, T_I = 0.1$ according to the definition of detection residual and isolation residuals. Figure 2.2 shows that, when an actuator fault occurs in the system, an alarm is generated since the residual signal deviates significantly from zero. Meanwhile, the SMOs quickly isolate the fault, as shown in Fig. 2.3. From

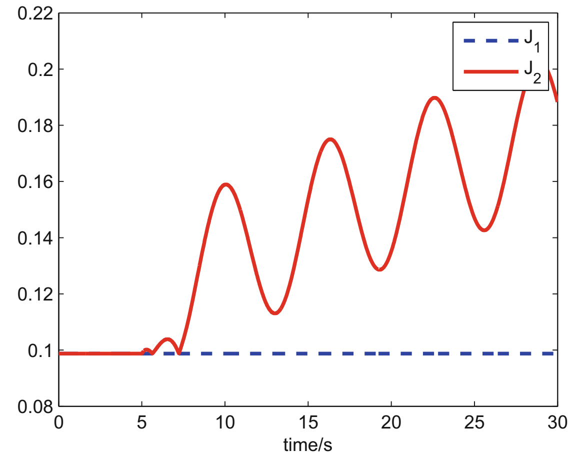

Fig. 2.3 Fault detection residuals J_1, J_2 with threshold

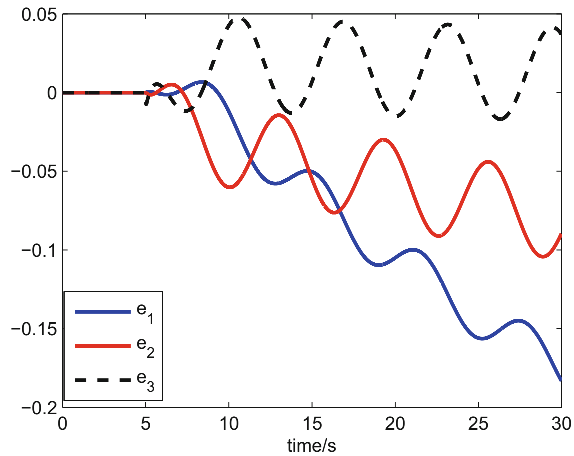

Fig. 2.4 Time responses of the observer errors: e_1, e_2, e_3 (no compensation for fault)

Fig. 2.4, we can see that, when an actuator fault occurs, with no fault compensation, the observation errors do not converge zero. However, compensating for the fault, the error system becomes stable, as shown in Fig. 2.5. From Figs. 2.6 and 2.7, we can clearly draw the conclusion that both gain faults and bias faults can be estimated accurately and promptly.

Compared with [24, 51], because a clear definition of threshold for fault detection and isolation is provided, it is easy to detect and isolate the faults. The fault estimation observer presented in this paper has the following two properties. On the one hand, differing from the classical fault estimation schemes in [24, 51, 52], where only bias faults or gain faults can be estimated, it is designed to estimate the two types of faults. On the other hand, from Figs. 2.8 and 2.9, it is obvious that it can estimate the

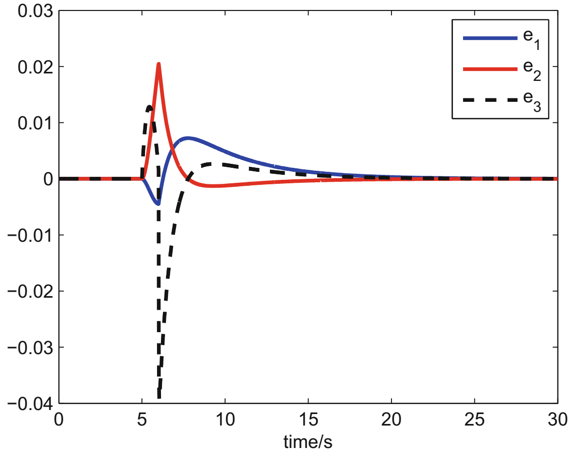

Fig. 2.5 Time responses of the observer errors: e_1, e_2, e_3 (with compensation for fault)

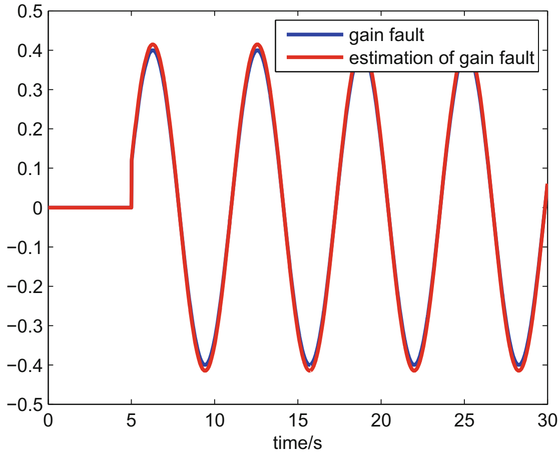

Fig. 2.6 The gain fault $\rho_2(t) = 0.4\sin(\pi t)$ and its estimation $\hat{\rho}_2(t)$

types of faults considered in [24, 51] and the fault estimation algorithm has better performances. From the above simulation results, it can be seen that, by the proposed fault detection and isolation observer, an actuator fault can be quickly detected and isolated, and using the fault estimation algorithm, the fault can be estimated online, which can be used to compensate for the fault and to ensure the stability of the closed-loop system in spite of actuator fault.

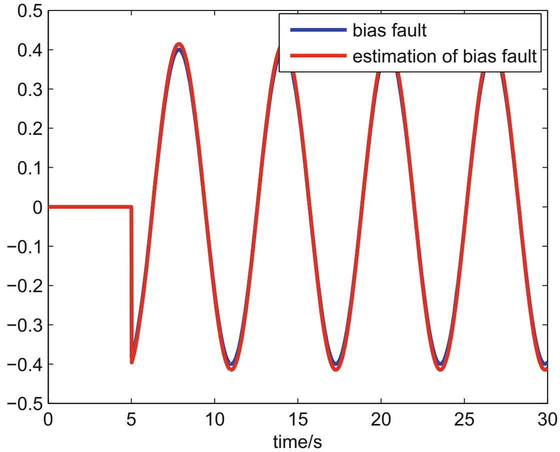

Fig. 2.7 The bias fault $f_2(t) = 0.4\cos(t)$ and its estimation $\hat{f}_2(t)$

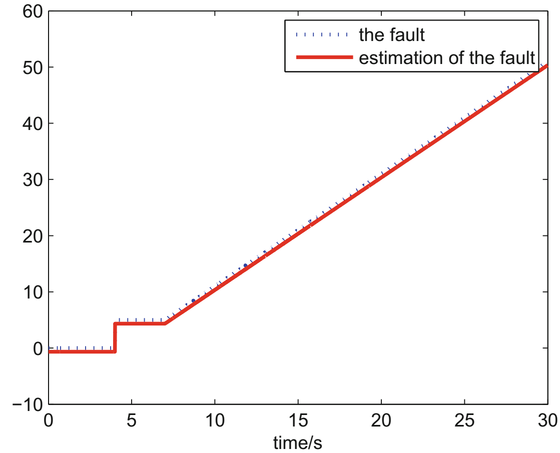

Fig. 2.8 The fault and its estimation (Case 2)

Remark 2.6 From the simulation results, it can be seen that (i) the proposed FDI/FTC scheme is effective because the fault can be detected, estimated and accommodated quickly, and (ii) the performance of our algorithm is better than that presented in the literature.

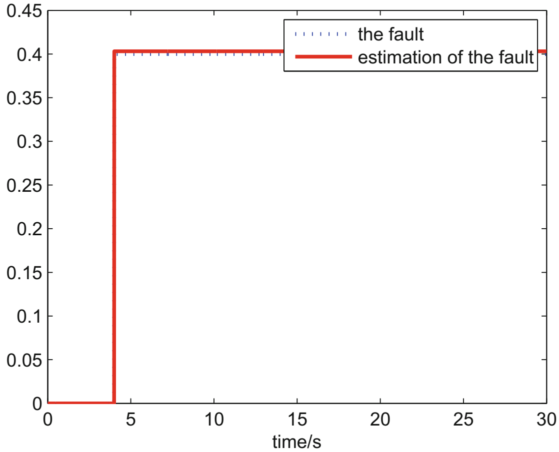

Fig. 2.9 The fault and its estimation (Case 3)

2.5 Conclusions

In this paper, the problem of fault tolerant control for T-S fuzzy systems with actuator faults is studied. We first design a bank of SMOs to detect and estimate the fault and a sufficient condition for the existence of SMOs is derived. Simulation results of NSHV show that the designed fault detection, isolation and estimation algorithms and fault-tolerant control scheme have good dynamic performances in the presence of actuator faults.

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Chapter 3
Fuzzy Logic System-Based Adaptive FC for NSV Attitude Dynamics with Multiple Faults

3.1 Introduction

It is well known that the controlled systems in practical applications may become faulty due to various reasons. Hence, FD and FTC have received considerable attention, and obtained significant results in the past decades, see [1–22] and the references therein. However, most of the existing results on FD and FTC work under the restrictive condition that only one actuator or sensor fault occurs at one time. In real applications, multiple types multiple faults may occur in the controlled system. The faulty cases include: multiple actuator faults, multiple sensors faults and multiple actuator and sensor faults. Up to now, few revelent results are reported in literature [23]. In [23], an actuator fault diagnosis scheme was proposed for a class of affine nonlinear systems with both known and unknown inputs, which was designed by making use of the derived input/output relation and the recently developed high-order sliding-mode robust differentiators. Hence, considering multiple type multiple faults simultaneously occurred in the controlled system is a motivation of this chapter.

Near space hypersonic vehicle, as a class of vehicle flying in near space which offers a promising and new, lower cost technology for future spacecraft. It can advance space transportation and also prompt global strike capabilities. Such complex technological system attracts considerable interests from the control research community and aeronautical engineering in the past couple of decades and significant results were reported [24–35]. For such high technological system, it is of course essential to maintain high reliability against possible faults [36–54].

Recently, T-S fuzzy system was used to describe the NSV attitude dynamics which are complex nonlinear, multi-variable and strongly coupled ones [24–35]. During the past two decades, the stability analysis for Takagi-Sugeno (T-S) fuzzy systems has attracted increasing attention [25–27]. In [55], the authors studied the problem of fault-tolerant tracking control for near-space-vehicle attitude dynamics with bias actuator fault, where the bias fault was assumed to be unknown constant. However, in practical application, the fault may be state-dependent, namely, it is a

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unknown function of system state. In this chapter, we will propose a more general FTC scheme that handles such state-dependent faults. On the other hand, as a universal approximation, fuzzy logic system (FLS) played an important role in modeling and controlling uncertain systems, see [56–61] and the references therein. In this chapter, we use the above FLSs to approximate the unknown state-dependent gain and bias faults.

In this chapter, we investigate the problem of fault tolerant control NSV with multiple state-dependent actuator faults, with the objective to provide an efficient solution for controlling NSV in faulty situations. Compared with existing literatures, the following contributions are worth to be emphasized.

(1) The actuator fault model presented in this chapter integrates state-dependent gain bias faults, which means that a wide class of faults can be handled. The theoretic developments and results of this chapter are thus valuable in a wide field of practical applications.
(2) Differing from some design scheme in literature, the fault-tolerant control scheme does not need the condition that the bounds of the time derivatives of the faults should be known constants, which thus enlarges the practical application scope.
(3) In general, the denominator of the fault-tolerant control input contains the estimation of the gain fault. If the denominator is equal to zero, a controller singularity occurs. In the proposed modified FTC scheme, the controller singularity is avoided without projection algorithm.

The rest of this chapter is organized as follows. In Sect. 3.2, the T-S fuzzy model for NSV attitude dynamics is first briefly recalled. Actuator faults are integrated in such model, and the FTC objective is formulated. In addition, mathematical description of fuzzy logic system is given. In Sect. 3.3, the main technical results of this chapter are given, which include fault detection, isolation, and fuzzy logic system-based fault accommodation in the two cases where system states are available or not. The NSV application is presented in Sect. 3.4. Simulation results of NSV are presented to demonstrate the effectiveness of the proposed technique. Finally, Sect. 3.5 draws the conclusion.

3.2 Problem Statement and Preliminaries

3.2.1 Problem Statement

In this chapter, a NSV attitude dynamics in re-entry phase is given as [62]:

$$\begin{cases} J\dot{\omega} = -\Omega J\omega + \delta \\ \dot{\gamma} = R(\cdot)\omega \end{cases} \tag{3.1}$$

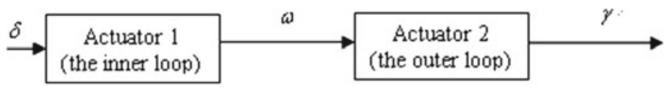

Fig. 3.1 The control diagram of NSV

where $J \in R^{3\times3}$ is the symmetric positive definite moment of inertia tensor, and $\omega = [p, q, r]^T = [\omega_1, \omega_2, \omega_3]^T$ is the angular rate vector composed of roll p, pitch q and yaw rate r, $\delta = [\delta_e, \delta_\alpha, \delta_r]^T \in R^{3\times1}$ is the control surface deflection, $\delta_e, \delta_\alpha, \delta_r$ are the elevator deflection, the aileron deflection, the rudder deflection, respectively. The skew symmetric matrix Ω is given by:

$$\Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ \omega_2 & \omega_1 & 0 \end{bmatrix} \tag{3.2}$$

In the re-entry phase, $R(\cdot)$ is defined as follows:

$$R(\cdot) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ \sin\alpha & 0 & -\cos\alpha \\ 0 & 1 & 0 \end{bmatrix} \tag{3.3}$$

where $\gamma = [\phi, \beta, \alpha]^T$ and ϕ, β, α are the bank, sideslip, and the attack angles, respectively. According to the singular perturbation theory, the above six equations can be expressed in the form of inner loop ω and outer loop γ; ω and γ are also respectively called fast loop and slow loop. The control diagram of NSV attitude dynamics is shown in Fig. 3.1. From the motion law of NSV, it is easy to find that, the response of the angular rate ω is faster than the one of the attitude angle γ. Based on time scale principle, we define ω as fast state and γ as slow state, thus system (3.1) can be divided into the following two subsystems: fast subsystem (3.4a) related to fast state ω and slow subsystem (3.4b) related slow state γ.

$$\begin{aligned} &\begin{cases} \dot{x}_\omega = f(x_\omega) + g(x_\omega)u(t) \\ y_\omega = x_\omega \end{cases} && (a) \\ &\begin{cases} \dot{x}_\gamma = f(x_\gamma, t)y_\omega \\ y_\gamma = x_\gamma \end{cases} && (b) \end{aligned} \tag{3.4}$$

where $f(x_\omega) = J^{-1}\Omega(\omega)Jx_\omega$, $g(x_\omega) = J^{-1}$, $f(x_\gamma) = R(\cdot)$, $x_\omega = \omega$, $x_\gamma = \gamma$.

The control objectives are,

(1) for the slow subsystem (the outer loop), to design the ideal angular rate $y_\omega(=\omega_d)$ such that subsystem output y_γ follows the desired reference signal y_d whose 1st derivative are available and bounded;

(2) for the fast subsystem (the inner loop), to design the control $u(t)$ such that the angular rate x_ω follows the ideal angular rate $y_\omega(=\omega_d)$.

That is to say, the main task is to design proper control input $u(t)$ such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0 \Rightarrow \lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$.

A fuzzy linear dynamic model has been proposed by Takagi and Sugeno in 1985 to represent a nonlinear system as an aggregation of local linear input/output relations. The fuzzy linear model is described by fuzzy IF-THEN rules and will be employed to deal with the fuzzy control problem for inner loop dynamics described by (3.4a) in this chapter.

Consider the following T-S fuzzy model composed of a set of fuzzy implications, where each implication is expressed by a linear state space model. The ith rule of this T-S fuzzy model is of the following form:

Plant rule i: IF $z_1(t)$ is M_{i1} and … $z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{3.5}$$

where $i = 1, \ldots, r$, r is the number of the IF-THEN rules, $M_{ij}, j = 1, \ldots, q$ is the fuzzy set, $z(t) = [z_1(t), \ldots, z_q(t)]^T$ are the premise variables which are supposed to be known, $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$ denotes state vector, $u(t) \in R^m$ denotes control input, $A_i \in R^{n\times n}$, and $B_i \in R^{n\times m}$ are local state and control matrices.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \tag{3.6}$$

where $h_i(z(t))$ is defined as

$$h_i(z(t)) = \frac{\prod_{j=1}^{n} M_{ij}[z(t)]}{\sum_{i=1}^{r}\prod_{j=1}^{n} M_{ij}[z(t)]}, \quad i = 1, 2, \ldots, r \tag{3.7}$$

where $M_{ij}[z(t)]$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed in this chapter that $\prod_{j=1}^{n} M_{ij}[z(t)] \geqslant 0$ for all t. Therefore, we have $\sum_{i=1}^{r} h_i(z(t)) = 1, \quad 0 \leqslant h_i(z(t)) \leqslant 1$ for all t.

In this chapter, the state feedback control strategy is chosen as a parallel distributed compensation (PDC), which can be described as follows:

Control rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$u_i(t) = K_i x(t) \tag{3.8}$$

where K_i is the controller gain matrix to be determined later.

The overall fuzzy controller is given as follows:

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) K_i x(t) \tag{3.9}$$

The control objective under normal conditions is to design a proper state feedback control input $u(t)$ such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0 \Rightarrow \lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$.

However, in practical application, actuators may become faulty. Two kinds of actuator faults are considered: loss of effectiveness of the actuators and actuator bias faults. The first kind of fault is modeled as follows.

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t),\ i = 1, \ldots, m,\ t \geqslant t_j \tag{3.10}$$

where $\rho_i^u(x)$ $(0 \leqslant \rho_i^u(x) < 1)$, which is supposed to be unknown, denotes the remaining control rate, t_j is unknown fault occurrence time. The second kind of fault, namely actuator bias fault, can be described as:

$$u_i^f(t) = u_i(t) + d_i^u(x),\quad i = 1, \ldots, m,\quad t \geqslant t_j \tag{3.11}$$

where $d_i^u(x)$ denotes a bounded signal. Therefore, the above two kinds of actuator faults can be uniformly described as

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t) + d_i^u(x),\quad t \geq t_f \tag{3.12}$$

Furthermore, a more general fault model can be given as:

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t) + \sum_{j=1}^{p_i^u} g_{i,j}^u d_{i,j}^u(x),\quad t \geq t_f \tag{3.13}$$

where $d_{i,j}^u(x), i = 1, \ldots, m, j = 1, \ldots, p_i^u$ denotes a bounded signal, p_i^u is a known positive constant. $g_{i,j}^u$ denotes an unknown constant. With no restriction, let suppose $p_1^u = \cdots = p_m^u = p$, with p a known positive constant. Consider the following notation $a_{i,j}^u(x) = g_{i,j}^u d_{i,j}^u(x)$, (3.13) can be re-written as follows:

$$u_i^f(t) = (1 - \rho_i^u(x))u_i(t) + \sum_{j=1}^{p} a_{i,j}^u(x),\quad t \geq t_f \tag{3.14}$$

where the nonlinear functions $\rho_i^u(x), a_{i,j}^u(x)$ and the failure time instant t_j are unknown. In this chapter, both bias and gain faults are handled by considering the general fault model (3.14).

Now, the control objective is re-defined as follows. An active fault tolerant control approach is proposed to obtain the above tracking objective in normal and faulty conditions, namely, $\lim_{t\to\infty}(x_\omega - \omega_d) = 0$. Furthermore, $\lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$. Under normal condition (no fault), a state feedback control input $u(t)$ is designed, such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0$. Meanwhile, the FDI algorithm is working. As soon as actuator faults are detected and isolated, the fault accommodation algorithm is activated and a proper fault-tolerant control input $u(t)$ is used such that the tracking performance ($\lim_{t\to\infty}(x_\omega - \omega_d) = 0$) is still maintained stable under faulty case.

3.2.2 Mathematical Description of Fuzzy Logic System

FLS consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

$$R^l : if\ x_1\ is\ A_1^l\ and\ x_2\ \ is\ A_2^l \ldots\ and\ x_n\ \ isA_n^l, \quad then\ y\ is\ B^l,\ \ l = 1, 2, \ldots, M$$

where $\underline{x} = [x_1, x_2, \ldots, x_n]^T \in U \subset R^n$ and y are the FLS input and output, respectively. Fuzzy sets A_i^l and B^l are associated with the fuzzy functions $\mu_{A_i^l}(x_i) = \exp(-(\frac{x_i - a_i^l}{b_i^l})^2)$ and $\mu_{B^l}(y^l) = 1$, respectively. M is the rules number. Through singleton function, center average defuzzification and product inference, the FLS can be expressed as

$$y(x) = \left\{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right\} / \left\{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right\} \tag{3.15}$$

where $\bar{y}^l = \max_{y\in R}\mu_{B^l}$. Define the fuzzy basis functions as

$$\xi^l(x) = \left[\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right] / \left[\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right] \tag{3.16}$$

and define $\theta^T = [\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^M] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi = [\xi^1, \xi^2, \ldots, \xi^M]^T$, then FLS (3.15) can be rewritten as

$$y(x) = \theta^T \xi(x) \tag{3.17}$$

Lemma 3.1 (Boulkroune et al. [60]) *Let* $f(x)$ *be a continuous function defined on a compact set* Ω*. Then for any constant* $\varepsilon > 0$*, there exists an FLS* (3.17) *such as*

$$\sup_{x\in\Omega} |f(x) - \theta^T \xi(x)| \leqslant \varepsilon \tag{3.18}$$

By Lemma 3.1, FLSs are universal approximations, i.e., they can approximate any smooth function on a compact space. Due to this approximation capability, we can assume that the nonlinear term $f(x)$ can be approximated as

$$f(x,\theta) = \theta^T \xi(x) \tag{3.19}$$

Define the optimal parameter vector θ^* as

$$\theta^* = \arg\min_{\theta\in\Omega}[\sup_{x\in U} |f(x) - f(x,\theta^*)|]$$

where Ω and U are compact regions for θ and x, respectively. Also the FLS minimum approximation error is defined as

$$\varepsilon = f(x) - \theta^{*T} \xi(x) \tag{3.20}$$

In this chapter, we use the above fuzzy logic system to approximate the unknown functions $\rho_i^u(x)$, $a_{i,j}^u(x)$, namely, there exist $\theta^*_{\rho,i}$, $\theta^*_{\alpha,i,j}$, $\varepsilon_{\rho,i}$, $\varepsilon_{\alpha,i,j}$ such that $\rho_i^u(x) = \theta^*_{\rho,i}\xi_{\rho,i}(x) + \varepsilon_{\rho,i} a_{i,j}^u(x) = \theta^*_{\alpha,i,j}\xi_{\alpha,i,j}(x) + \varepsilon_{\alpha,i,j}$. Now, the following assumptions are made.

Assumption 3.1 There exist unknown constants $\varepsilon^*_{\rho,i} > 0$, $\varepsilon^*_{\alpha,i,j} > 0$ and two known constants $\bar{M}_{\rho,s_k}$, $\bar{M}_{\alpha,s_k,j}$ such that $|\varepsilon_{\rho,i}| \leqslant \varepsilon^*_{\rho,i}$, $|\varepsilon_{\alpha,s_k,j}| \leqslant \varepsilon^*_{\alpha,i,j}$, $\varepsilon^*_{\rho,i} \leqslant \bar{M}_{\rho,s_k}$, $\varepsilon^*_{\alpha,i,j} \leqslant \bar{M}_{\alpha,s_k,j}$.

Assumption 3.2 There exist known constants M_{ρ,s_k}, $M_{\alpha,s_k,j}$ such that $||\theta^*_{\rho,s_k}|| \leqslant M_{\rho,s_k}$, $||\theta^*_{\alpha,s_k,j}|| \leqslant M_{\alpha,s_k,j}$.

3.3 Fault Diagnosis and FLS-Based Fault Accommodation

In this section, the main technical results of this chapter are given. We will first formulate the fault diagnosis and accommodation problems of the above T-S fuzzy system. We will then design a bank of SMOs to generate residuals, investigate the FDI algorithm based on the SMOs, and propose a FTC scheme to tolerate the faults by compensating for faults.

3.3.1 Preliminary

Consider the T-S fuzzy faulty system described in (3.6). We assume that only actuator faults occur and no sensor fault is involved. The following assumptions are considered.

Assumption 3.3 Matrix B_i is of full column rank and the pair (A_i, C_i) is observable.

We first design the fault diagnosis observers to detect and isolate the faults, and then, propose a FTC method to compensate the faults.

3.3.2 Fault Detection

In order to detect the actuator faults, we design a fuzzy state-space observer for the system (3.6), which is described as:

Observer rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_i\hat{x}(t) \end{cases} \tag{3.21}$$

where $L_i, i = 1, \ldots, r$ is the observer gain for the ith observer rule.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))] \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}(t) \end{cases} \tag{3.22}$$

Denote

$$e_x(t) = x(t) - \hat{x}(t), \quad e_y(t) = y(t) - \hat{y}(t) \tag{3.23}$$

then the error dynamics is described by

$$\begin{cases} \dot{e}_x(t) = \sum_{i=1}^{r} h_i(z(t))[(A_i - L_iC_i)e_x(t)] \\ e_y(t) = \sum_{i=1}^{r} h_i(z(t))C_ie_x(t) \end{cases} \tag{3.24}$$

Lemma 3.2 *The estimation error e_x converges asymptotically to zero if there exist common matrices $P = P^T > 0$ and $Q > 0$ with appropriate dimensions such that the following linear matrix inequality is satisfied:*

$$P(A_i - L_iC_i) + (A_i - L_iC_i)^T P \leqslant -Q, i = 1, 2, \ldots, r \tag{3.25}$$

Proof Consider the following Lyapunov function

$$V_D = e_x^T(t)Pe_x(t)$$

Differentiating V_1 with respect to time$t$, one has

$$\begin{aligned} \dot{V}_D(t) &= \sum_{i=1}^{r} h_i(z(t))[e_x^T(t)(P(A_i - L_iC) + (A_i - L_iC)^T P)e_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[e_x^T(t)Qe_x(t)] \\ &\leqslant 0 \end{aligned} \tag{3.26}$$

Because $V_D(t) \in L_\infty$ is a monotonous and non-increasing bounded function, $V_D(+\infty)$ exists. Hence, we have

$$V_D(0) - V_D(+\infty) \geqslant -\int_0^{+\infty} \sum_{i=1}^{r} h_i(z(t))[e_x^T(t)Qe_x(t)],$$

which means taht $e_x(t) \in L_2$. Since $e_x(t), \dot{e}_x(t) \in L_\infty$, using the Lyapunov stability theory, we obtain $\lim\limits_{t\to\infty} e_x(t) = 0$. Furthermore, we have $\lim\limits_{t\to\infty} e_y(t) = 0$. The proof is completed.

From Lemma 3.2, we have

$$\begin{aligned} \dot{V}_D(t) &\leqslant -\sum_{i=1}^{r} h_i(z(t))[e_x^T(t)Qe_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[\lambda_{\min}(Q)e_x^T(t)e_x(t)] \\ &\leqslant -\sum_{i=1}^{r} h_i(z(t))[\lambda_{\min}(Q)/\lambda_{\max}(P)e_x^T(t)Pe_x(t)] \\ &\leqslant -h_i(z(t))[\lambda_{\min}(Q)/\lambda_{\max}(P)]V(t) \\ &= -\kappa V_D(t) \end{aligned} \tag{3.27}$$

where $\kappa = \lambda_{\min}(Q)/\lambda_{\max}(P) \in R$. Hence,

$$V_D(t) \leqslant e^{-\kappa t} V(0) \tag{3.28}$$

Furthermore, we have

$$\lambda_{\min}(P)||e_x(t)||^2 \leqslant e^{-\kappa t}\lambda_{\max}(P)||e_x(0)||^2 \tag{3.29}$$

Therefore the norm of the error vector satisfies

$$||e_x(t)|| \leqslant \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_x(0)||e^{-\kappa t/2} \tag{3.30}$$

Furthermore, the detection residual can be defined as

$$J(t) = ||y(t) - \hat{y}(t)|| \tag{3.31}$$

From (3.30), it can be seen that the following inequality holds in the healthy case:

$$J(t) \leqslant \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||C_i||||e_x(0)||e^{-\kappa t/2} \tag{3.32}$$

Then, the fault detection can be performed using the following mechanism:

$$\begin{cases} J(t) \leqslant T_d \text{ no fault occurred,} \\ J(t) > T_d \text{ fault has occurred} \end{cases} \tag{3.33}$$

where threshold T_d is defined as follows.

$$T_d = \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||C_i||||e_x(0)||e^{-\kappa t/2} \tag{3.34}$$

3.3.3 Fault Isolation

Since the system has m actuators, which maybe become faulty, we have $C_m^1 + C_m^2 + \cdots + C_m^m$ possible faulty cases, where C_m^i denotes the number of faulty cases where there are i faulty actuators within m actuators. Let us define the following symbol, j_i^k $(i = 1, 2, \ldots, m;\ k = 1, 2, \ldots, i)$ which denotes the situation where the ith actuator fails when there are k possible faulty actuators among the m actuators. Fault patterns can be described in details as follows.

Case 1: only an actuator is faulty

$$\aleph_1 : \{\aleph_1^1, \aleph_1^2, \ldots, \aleph_1^{C_m^1}\} = \{\{j_1^1\}, \{j_2^1\}, \ldots, \{j_m^1\}\}$$

In this case, there are C_m^1 fault patterns.

Case 2: only two actuators are faulty

$$\aleph_2 : \{\aleph_2^1, \aleph_2^2, \ldots, \aleph_2^{C_m^2}\} = \left\{ \begin{array}{l} \{j_1^2, j_2^2\}, \{j_1^2, j_3^2\}, \ldots, \{j_1^2, j_m^2\}, \ldots, \\ \{j_2^2, j_3^2\}, \{j_2^2, j_4^2\}, \ldots, \{j_2^2, j_m^2\}, \ldots \{j_{m-1}^2, j_m^2\} \end{array} \right\}$$

where the number of fault patterns reached a total of C_m^2.

Case i: only i actuators are faulty

$$\aleph_i : \{\aleph_i^1, \aleph_i^2, \ldots, \aleph_i^{C_m^i}\} = \{\{j_1^i, j_2^i, \ldots, j_i^i\}, \ldots, \{j_{m-i+1}^2, \ldots, j_m^2\}\}$$

where the total fault pattern is C_m^i, $i = 1, 2, \ldots, m$.

Case m: all m actuators are faulty

$$\aleph_m : \{\aleph_m^1, \ldots, \aleph_m^{C_m^m}\} = \{\{j_1^i, j_2^i, \ldots, j_m^i\}\}$$

Here, there is only one fault pattern ($C_m^m = 1$).

Now, let $\aleph_m = \{\aleph_1^1, \ldots, \aleph_1^{C_m^1}, \ldots, \aleph_i^1, \ldots, \aleph_i^{C_m^i}, \ldots, \aleph_m^1, \ldots, \aleph_m^{C_m^m}\}$. Obviously, there are $C_m^1 + C_m^2 + \cdots + C_m^m$ possible fault patterns that are numbered as the 1st, 2nd, Nth fault pattern, where $N = C_m^1 + C_m^2 + \cdots + C_m^m$.

In this chapter, it is assumed that there d actuators became faulty whose pattern s is $\aleph_d^q$, namely, $s = \aleph_d^q$. We also assume that the d actuators are the s_1th, s_2th, ..., s_dth actuators, where $1 \leqslant s_1 < s_2 < \cdots < s_d \leqslant m$. Then the faulty model can be described as:

$$\begin{cases} \dot{x}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i x_s(t) + \sum_{i=1}^{r} h_i(z(t))B_i u(t) - \\ \qquad \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[\rho_{s_k}^u(x)u_{s_k}^s(t) - \sum_{j=1}^{p} a_{s_k j}^u(x)] \right\} \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases} \tag{3.35}$$

where $B_i = [b_{i,1}, b_{i,2}, \ldots, b_{i,m}]$, $b_{i,l} \in R^{n\times 1}, 1 \leqslant l \leqslant m$, $\rho_{s_k}^u(x)$, $a_{s,s_k}^u(x), j = 1, 2, \ldots, p$ denote the time profiles of the s_kth actuator fault, which are described by (3.14), $u_{s_k}^s(t)$ is the desired controller when the s_kth actuator is healthy.

Inspired by the SMOs in [63], we are ready to present one of the results of this chapter. It is assumed that fuzzy observer and fuzzy control systems have the same premise variables $z(t)$, then the following fuzzy observers are proposed to isolate the actuator fault.

Isolation Observer Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}_{is}(t) = A_i\hat{x}_{is}(t) + L_i(y(t) - \hat{y}_{is}(t)) + B_iu(t) + \\ \qquad \sum_{k=1}^{d} \left\{ b_{i,s_k}\mu_{s_k}[\bar{\rho}^u_{s_k} u^s_{s_k}(t) + \sum_{j=1}^{p} \bar{a}^u_{s_k,j}] \right\} \\ \hat{y}_{is}(t) = C_{is}\hat{x}_{is}(t) \end{cases} \tag{3.36}$$

where $\hat{x}_{is}(t), \hat{y}_{is}(t)$ are the sth fuzzy observer's state and output, respectively. L_i is the observer's gain matrix for ith observer. The global fuzzy observer is represented as:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i\hat{x}_{is}(t) + \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}_{is}(t)) + \sum_{i=1}^{r} h_i(z(t))B_iu(t) + \\ \qquad \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}\mu_{s_k}[\bar{\rho}^u_{s_k} u^s_{s_k}(t) + \sum_{j=1}^{p} \bar{a}^u_{s_k,j}] \right\} \\ \hat{y}_s(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}_s(t) \\ \mu_{s_k} = -\dfrac{\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)}{||\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)||} \end{cases} \tag{3.37}$$

where $F_{is_k} \in R^{1\times n}$ is the s_kth row of $F_i \in R^{m\times n}$, which will be defined later, $L_i \in R^{n\times n}$ is chosen such that $A_i - L_iC_i$ is Hurwitz, $e_{xs}(t) = x_s(t) - \hat{x}_s(t)$ and $e_{ys}(t) = y(t) - \hat{y}_s(t)$ are respectively the state error and output error between the plant and the sth SMO observer. Let l denotes the practical fault pattern where the faulty actuators are the l_1th, l_2th, $\ldots$, l_{d^*}th actuators, $1 \leqslant l_1 < l_2 < \cdots < l_{d^*} \leqslant m$.

For $s = l$, namely, $d = d^*$, $l_1 = s_1, l_2 = s_2, \ldots, l_{d^*} = s_d$, the error dynamics is obtained from (3.35) and (3.36).

$$\begin{aligned}\dot{e}_{xs}(t) &= \sum_{i=1}^{r} h_i(z(t))A_i e_{is}(t) - \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}_{is}(t)) + \\ &\quad \sum_{i=1}^{r} h_i(z(t)) \sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\ &\quad \sum_{j=1}^{p}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})] \\ &= \sum_{i=1}^{r} h_i(z(t))\{(A_i - L_iC_i)e_{is}(t) + \\ &\quad \sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\ &\quad \sum_{j=1}^{p}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})]\}\end{aligned} \tag{3.38}$$

For $s \neq 1$, namely, $d \neq d^* ord = d^*$ and at least there exists l_i such that $l_i \neq s_i, i = 1, 2, \ldots, d$, we have

$$\begin{aligned}\dot{e}_{xs}(t) &= \sum_{i=1}^{r} h_i(z(t))(A_i - L_iC_i)e_{is}(t) + \\ &\quad \sum_{i=1}^{r} h_i(z(t))[(-\sum\nolimits_{k=1}^{d^*} b_{i,l_k}\rho^u_{l_k}(x)u^s_{l_k}(t) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\ &\quad \sum_{j=1}^{p}(\sum\nolimits_{k=1}^{d^*} b_{i,l_k}a^u_{l_k,j}(x) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{a}^u_{s_k,j})]\end{aligned} \tag{3.39}$$

The stability of the state error dynamics is guaranteed by the following theorem.

Theorem 3.1 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P and matrices $L_i, F_i, Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$(A_i - L_iC_i)^T P + P(A_i - L_iC_i) \leqslant -Q \tag{3.40}$$

$$PB_i = (F_iC_i)^T \tag{3.41}$$

Then, when the lth pattern is the actual fault pattern i.e., $s = l$, we have $\lim_{t\to\infty} e_{xs} = 0$, and for $s \neq l$, we have $\lim_{t\to\infty} e_{xs} \neq 0$.

Proof (1) For $s = l$, according to (3.38), we have

$$\dot{e}_{xs}(t) = \sum_{i=1}^{r} h_i(z(t))\{(A_i - L_iC_i)e_{is}(t)+$$
$$\sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u_{s_k}(t)|)+$$
$$\sum_{j=1}^{p}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})]\}$$

Define the following Lyapunov function

$$V_I(t) = e^T_{xs}(t)Pe_{xs}(t) \tag{3.42}$$

Differentiating V_2 with respect to time t, and using (3.40), one has

$$\dot{V}_I(t) = \dot{e}^T_{xs}(t)Pe_{xs}(t) + e^T_{xs}(t)P\dot{e}_{xs}(t)$$
$$\leqslant -e^T_{xs}(t)Qe_{xs}(t) + 2e^T_{xs}(t)P\sum_{i=1}^{r} h_i(z(t))\sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) -$$
$$\mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \sum\nolimits_{k=1}^{d}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j})]$$

From $\mu_{s_k} = -\sum\limits_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys_k}(t)/||\sum\limits_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys_k}(t)||$ and (3.41), one has

$$2e^T_{xs}(t)P\sum_{i=1}^{r} h_i(z(t))\sum\nolimits_{k=1}^{d} b_{i,s_k}[(-\rho^u_{s_k}(x)u^s_{s_k}(t) - \mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) \leqslant 0,$$

$$2e^T_{xs}(t)P\sum_{i=1}^{r} h_i(z(t))\sum\nolimits_{k=1}^{d}(a^u_{s_k,j}(x) - \mu_{s_k}\bar{a}^u_{s_k,j}) \leqslant 0.$$

Hence,

$$\dot{V}_I(t) \leqslant -e^T_{xs}(t)Qe_{xs}(t) \leqslant 0 \tag{3.43}$$

Because $V_I(t) \in L_\infty$ is a monotonous and non-increasing bounded function, $V_I(+\infty)$ exists. Hence, we have $V_I(0) - V_I(+\infty) \geqslant -\int_0^{+\infty} e^T_{xs}(t)Qe_{xs}(t)$, i.e., $e_{xs}(t) \in L_2$. Since $e_{xs}(t)$ and $\dot{e}_{xs}(t) \in L_\infty$, using the Lyapunov stability theory, we have $\lim\limits_{t\to\infty} e_{xs}(t) = 0$. Thus, we have $\lim\limits_{t\to\infty} e_{ys}(t) = 0$.

(2) For $s \neq l$, it follows from (3.35) and (3.39) that:

$$\dot{e}_{xs}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i - L_iC_i)e_{is}(t) + \\ \sum_{i=1}^{r} h_i(z(t))[(-\sum\nolimits_{k1=1}^{d^*} b_{i,l_{k1}}\rho^u_{l_{k1}}(x)u^s_{l_k}(t) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\ \sum_{j=1}^{p}(\sum\nolimits_{k1=1}^{d^*} b_{i,l_{k1}}a^u_{l_{k1}j}(x) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{a}^u_{s_kj})]$$

Because matrix B_i is of full column rank (Assumption 3.1), we know that b_{is_k} and $b_{il_{k1}}$ are linearly independent. Therefore,

$$\lim_{t\to\infty}\sum_{i=1}^{r} h_i(z(t))[(-\sum\nolimits_{k=1}^{d^*} b_{i,l_k}\rho^u_{l_k}(x)u^s_{l_k}(t) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{\rho}^u_{s_k}|u^s_{s_k}(t)|) + \\ \sum_{j=1}^{p}(\sum\nolimits_{k=1}^{d^*} b_{i,l_k}a^u_{l_kj}(x) - \sum\nolimits_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{a}^u_{s_kj})] \neq 0 \tag{3.44}$$

Thus, we have $\lim_{t\to\infty} e_{xs}(t) \neq 0$ and $\lim_{t\to\infty} e_{ys}(t) \neq 0$.

From (1) and (2), we obtain the conclusion. This ends the proof.

Now, we denote the residuals between the real system and SMOs as follows:

$$J_s(t) = \left\|e_{ys}(t)\right\| = \left\|\hat{y}_s(t) - y(t)\right\|, \quad 1 \leqslant \mathrm{s} \leqslant m \tag{3.45}$$

According to Theorem 3.1, when the actual fault pattern is $s = l$, the residual $J_s(t)$ will tend to zero; while for any $s \neq l$, $J_s(t)$ does not equal zero. Furthermore, from Lemma 3.2, we have, if $l = s$,

$$J_s(t) \leqslant \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2} \tag{3.46}$$

and if $l \neq s$, then

$$J_s(t) > \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2} \tag{3.47}$$

Hence, the isolation law for actuator fault can be designed as

$$\begin{cases} J_s(t) \leqslant T_I, l = s \Rightarrow \text{the } l_1\text{th}, l_2\text{th}, \ldots, l_d\text{th actuators are faulty} \\ J_s(t) > T_I, l \neq s \end{cases} \tag{3.48}$$

where threshold T_I is defined as follows.

$$T_I = \sum_{i=1}^{r} h_i(z(t))\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{ys}(0)||e^{-\kappa t/2}$$

Notice that, the denominator of $\mu_{s_k} = -\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys_k}(t)/||\sum_{i=1}^{r} h_i(z(t))F_{is_k} e_{ys_k}(t)||$ in (3.37), contains $e_{ys}(t)$. Just as pointed out in [63], the chattering phenomenon occurs when $e_{ys}(t) \to 0$ in practice. Inspired by [63], in order to reduce this chattering in practical applications, we modify SMOs (3.37) by introducing a positive constantδ as follows:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r} h_i(z(t))A_i\hat{x}_{is}(t) + \sum_{i=1}^{r} h_i(z(t))L_i(y(t) - \hat{y}_{is}(t)) + \sum_{i=1}^{r} h_i(z(t))B_iu(t) + \\ \qquad \sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}\mu_{s_k}[\bar{\rho}^u_{s_k} u^s_{s_k}(t) + \sum_{j=1}^{p} \bar{a}^u_{s_k,j}] \right\} \\ \hat{y}_s(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}_s(t) \\ \mu'_{s_k} = -\dfrac{\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)}{||\sum_{i=1}^{r} h_i(z(t))F_{is_k}e_{ys}(t)|| + \delta} \end{cases} \tag{3.49}$$

where $\delta > 0 \in R$ is a constant. Obviously, the denominator of μ'_{s_k} will converge asymptotically to δ when $e_{ys} \to 0$, which reduces this chattering phenomenon.

3.3.4 Fuzzy Logic Systems-Based Fault Accommodation with Available System State

After fault isolation, the next task is fault accommodation. Before this task, we investigate firstly the following normal systems (fault-free), and drive the ideal control $u^s(t)$ when all actuators are healthy.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_ix(t) + B_iu^s(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_ix(t) \end{cases} \tag{3.50}$$

The parallel distributed compensation (PDC) technique offers a procedure to design a fuzzy control law from a given T-S fuzzy model. In the PDC design, each control rule is designed from the corresponding rule of T-S fuzzy model. The designed fuzzy controller has the same fuzzy sets as the considered fuzzy system.

Control Rule i: IF $z_1(t)$ is M_{i1} and $\ldots z_q(t)$ is M_{iq}, THEN

$$u_i^s(t) = K_i x(t)$$

and the overall fuzzy controller is given as follows:

$$u^s(t) = \sum_{i=1}^{r} h_i(z(t)) K_i x(t) \tag{3.51}$$

where the controller gain matrix K_i is determined by solving the following condition:

$$P(A_i+K_iB_i)+(A_i+K_iB_i)^T P+(A_i+K_iB_i)^T PS_1P(A_i+K_iB_i)+PS_2P \leqslant -Q \tag{3.52}$$

where $P = P^T > 0, Q > 0, S_1 > 0, S_2 > 0$ are matrices with appropriate dimensions.

Define tracking error $\bar{e} = y - \omega_d$. The tracking error dynamics is obtained from the above equations,

$$\dot{\bar{e}} = \dot{y} - \dot{\omega}_d = C_i\dot{x} - \dot{\omega}_d = \sum_{i=1}^{r} h_i(z(t))[C_iA_ix(t) + C_iB_iu^s(t)] - \dot{\omega}_d$$

Because all the states are supposed to be available, we have $C_i = I_{m\times m}$. The tracking error dynamics can be simplified as follows:

$$\begin{aligned}
\dot{\bar{e}} = \dot{x} - \dot{\omega}_d &= \sum_{i=1}^{r} h_i(z(t))[A_ix(t) + B_iK_ix(t) - \dot{\omega}_d] \\
&= \sum_{i=1}^{r} h_i(z(t))[(A_i + B_iK_i)x(t) - \dot{\omega}_d] \\
&= \sum_{i=1}^{r} h_i(z(t))[(A_i + B_iK_i)e(t) + \omega_d - \dot{\omega}_d]
\end{aligned} \tag{3.53}$$

Define the following Lyapunov function

$$V_0 = \bar{e}^T P \bar{e}$$

where $P = P^T > 0$.

Differentiating V_0 with respect to time t, leads to

$$\begin{aligned}\dot{V}_0 = &\sum_{i=1}^{r} h_i(z(t))[e^T(t)(P(A_i + K_iB_i) + (A_i + K_iB_i)^T P)e(t)] - \\ &\sum_{i=1}^{r} h_i(z(t))[2e^T(t)(A_i + K_iB_i)^T P(\omega_d - \dot{\omega}_d)] - \\ &\sum_{i=1}^{r} h_i(z(t))[2e^T(t)P(\omega_d - \dot{\omega}_d)] + \sum_{i=1}^{r} h_i(z(t))[2(\omega_d - \dot{\omega}_d)^T P(\omega_d - \dot{\omega}_d)]\end{aligned} \tag{3.54}$$

Since

$$\begin{aligned}-2\bar{e}^T(A_i + K_iB_i)^T P(\omega_d - \dot{\omega}_d) \leqslant &\bar{e}^T(t)(A_i + K_iB_i)^T PS_1P(A_i + K_iB_i)\bar{e} + \\ &(\omega_d - \dot{\omega}_d)^T S_1^{-1}(\omega_d - \dot{\omega}_d)\end{aligned}$$

$$-2\bar{e}^T P(\omega_d - \dot{\omega}_d) \leqslant \bar{e}^T(t)PS_2P\bar{e} + (\omega_d - \dot{\omega}_d)^T S_2^{-1}(\omega_d - \dot{\omega}_d)$$

(3.54) can be re-written as follows:

$$\begin{aligned}\dot{V}_0 \leqslant &\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)\Delta_1\bar{e}(t)] + \\ &\sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T(S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d)]\end{aligned}$$

where $\Delta_1 = P(A_i + K_iB_i) + (A_i + K_iB_i)^T P + (A_i + K_iB_i)^T PS_1P(A_i + K_iB_i) + PS_2P$.

Obviously, if

$$\Delta_1 = P(A_i + K_iB_i) + (A_i + K_iB_i)^T P + (A_i + K_iB_i)^T PS_1P(A_i + K_iB_i) + PS_2P \leqslant -Q,$$

then

$$\dot{V}_0 \leqslant -\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0 \leqslant -\lambda_0 V_0 + \mu_0,$$

where $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T(S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d)]$, $\lambda_0 = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$, $Q = Q^T > 0$.

Then, one has $\frac{d}{dt}(V_0(t)e^{\lambda_0 t}) \leqslant e^{\lambda_0 t}\mu_0$. Furthermore,

$$0 \leqslant V_0(t) \leqslant \frac{\mu_0}{\lambda_0} + [V_0(0) - \frac{\mu_0}{\lambda_0}]e^{-\lambda_0 t} \leqslant \frac{\mu_0}{\lambda_0} + V_0(0).$$

Therefore, the error system (3.53) is asymptotically stable. Moreover, $\bar{e}(t)$ is semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $|\bar{e}| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, where $\alpha = \mu_0/\lambda_0 + V_0(0)$.

After obtaining the desired control $u^s(t)$, we will design fault-tolerant control $u(t)$ such that the same control objective can be achieved in spite of actuator faults.

On the basis of the desired control $u^s(t)$, the fault tolerant controller is constructed as

$$u_{s_k} = \frac{u^s_{s_k} - \sum_{j=1}^{p} \hat{a}_{s_k,j}(x, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j}}{1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}} \tag{3.55}$$

where $\hat{\theta}_{\rho,s_k}$, $\hat{\theta}_{\alpha,s_k,j}$, $\hat{\rho}_{\rho,s_k}(x, \hat{\theta}_{\rho,s_k})$, $\hat{a}_{i,j}(x, \hat{\theta}_{\alpha,s_k,j})$ are the estimations of θ^*_{ρ,s_k}, $\theta^*_{\alpha,s_k,j}$, $\rho_{s_k}(x, \theta^*_{\rho,s_k})$, $a_{s_k,j}(x, \theta^*_{\alpha,s_k,j})$, which are used to compensate for the gain and bias faults $\rho_{s_k}(x)$, $\alpha_{s_k,j}(x)$, and $\rho_{s_k}(x) = \rho_{s_k}(x, \theta^*_{\rho,s_k}) + \varepsilon_{s_k}$, $a_{s_k,j}(x) = a_{s_k,j}(x, \theta^*_{\alpha,s_k,j}) + \varepsilon_{s_k,j}$, ε_{s_k}, $\varepsilon_{s_k,j}$ are approximation errors, θ^*_{ρ,s_k}, $\theta^*_{\alpha,s_k,j}$ are optimal vectors.

Consider the following faulty system

$$\begin{aligned}\dot{x}_s(t) = &\sum_{i=1}^{r} h_i(z(t))A_i x_s(t) + \sum_{i=1}^{r} h_i(z(t))B_i u^s(t) - \\ &\sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[\rho^u_{s_k}(x)u^s_{s_k}(t) - \sum_{j=1}^{p} a^u_{s_k,j}(x)] \right\}\end{aligned} \tag{3.56}$$

Submitting the fault-tolerant control law (3.55) to the faulty system (3.56), it yields

$$\begin{aligned}\dot{x}_S(t) = &\sum_{i=1}^{r} h_i(z(t))A_i x_S(t) + \sum_{i=1}^{r} h_i(z(t))B_i u^S(t) + \\ &\sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[(\tilde{\theta}^T_{\rho,s_k}\xi_{\rho,s_k}(x) + \tilde{\varepsilon}_{\rho,s_k})\kappa_k - \sum_{j=1}^{p} \tilde{\theta}^u_{\alpha,s_k,j}\xi_{\alpha,s_k,j}(x) - \tilde{\varepsilon}_{\rho,s_k}] \right\}\end{aligned} \tag{3.57}$$

where $\tilde{\theta}_{\alpha,s_k,j} = \hat{\theta}_{\alpha,s_k,j} - \theta^*_{\alpha,s_k,j}$, $\kappa_k = (\frac{u^s_{s_k} - \sum_{j=1}^{p}[\hat{\alpha}^u_{\alpha,s_k,j}(x,\hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j}]}{1 - \hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}})$, $\tilde{\theta}_{\rho,s_k} = \hat{\theta}_{\rho,s_k} - \theta^*_{\rho,s_k}$. Further, the error dynamics is obtained:

$$\begin{aligned}\dot{e} = &\sum_{i=1}^{r} h_i(z(t))[(A_i + B_i K_i)e(t) + \omega_d - \dot{\omega}_d] + \\ &\sum_{i=1}^{r} h_i(z(t)) \sum_{k=1}^{d} \left\{ b_{i,s_k}[(\tilde{\theta}^T_{\rho,s_k}\xi_{\rho,s_k}(x) + \tilde{\varepsilon}_{\rho,s_k})\kappa_k - \sum_{j=1}^{p} \tilde{\theta}^u_{\alpha,s_k,j}\xi_{\alpha,s_k,j}(x) - \tilde{\varepsilon}_{\rho,s_k}] \right\}\end{aligned} \tag{3.58}$$

Now, an adaptive fault accommodation algorithm is proposed to control the faulty system. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 3.2 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P, real matrices K_i and $Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$\begin{aligned} &P(A_i + K_iB_i) + (A_i + K_iB_i)^T P + \\ &\quad (A_i + K_iB_i)^T PS_1P(A_i + K_iB_i) + PS_2P \leqslant -Q \end{aligned} \tag{3.59}$$

Consider the control law (3.55) *and the adaptive laws given as follows:*

$$\dot{\hat{\theta}}_{\rho,s_k} = \begin{cases} -\eta_1\bar{e}^T Pb_{i,s_k}\xi^u_{\rho,s_k}(x)\kappa_k, \ \ if\, ||\hat{\theta}_{\rho,s_k}|| < M_{\rho,s_k} \ or \\ \quad ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \ and \ \eta_1\bar{e}^T Pb_{i,s_k}\xi^u_{\rho,s_k}(x)\kappa_k \geqslant 0; \\ -\eta_1\bar{e}^T Pb_{i,s_k}\xi^u_{\rho,s_k}(x)\kappa_k + \eta_1\bar{e}^T Pb_{i,s_k}\kappa_k \dfrac{\theta_{\rho,s_k}\theta^T_{\rho,s_k}}{||\hat{\theta}_{if}||^2}\xi^u_{\rho,s_k}(x), \\ \quad if\, ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \ and \ \eta_1\bar{e}^T Pb_{i,s_k}\xi^u_{\rho,s_k}(x)\kappa_k < 0 \end{cases} \tag{3.60}$$

$$\dot{\hat{\theta}}_{\alpha,s_k,j} = \begin{cases} \eta_2\bar{e}^T Pb_{i,s_k}\xi^u_{\alpha,s_k,j}(x), \ \ if\, ||\hat{\theta}_{\alpha,s_k,j}|| < M_{\alpha,s_k,j} \\ \quad or\ ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \ and \ -\eta_2\bar{e}^T Pb_{i,s_k}\xi^u_{\alpha,s_k,j}(x) \geqslant 0; \\ \eta_2\bar{e}^T Pb_{i,s_k}\xi^u_{\alpha,s_k,j}(x) + \eta_2\bar{e}^T Pb_{i,s_k}\dfrac{\hat{\theta}_{\alpha,s_k,j}\hat{\theta}^T_{\alpha,s_k,j}}{||\hat{\theta}_{\alpha,s_k,j}||^2}\xi^u_{\alpha,s_k,j}(x), \\ \quad if\ ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \ and \ -\bar{e}^T Pb_{i,s_k}\xi^u_{\alpha,s_k,j}(x) < 0 \end{cases} \tag{3.61}$$

$$\dot{\hat{\varepsilon}}_{\rho,s_k} = \begin{cases} 0, \ if\, \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\rho,s_k} \ and \ -\eta_3\bar{e}^T Pb_{i,s_k}\kappa_k > 0 \\ \quad or\ \hat{\varepsilon}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \ and \ -\eta_3\bar{e}^T Pb_{i,s_k}\kappa_k < 0; \\ -\eta_3\bar{e}^T Pb_{i,s_k}\kappa_k, \ otherwise \end{cases} \tag{3.62}$$

$$\dot{\hat{\varepsilon}}_{\alpha,s_k,j} = \begin{cases} 0, \ if\, \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\alpha,s_k,j} \ and \ \eta_4\bar{e}^T Pb_{i,s_k} > 0 \\ \quad \hat{\varepsilon}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \ and \ \eta_4\bar{e}^T Pb_{i,s_k} < 0; \\ \eta_4\bar{e}^T Pb_{i,s_k}, \ otherwise \end{cases} \tag{3.63}$$

where $\eta_i > 0, i = 1, \ldots, 4$ denote the adaptive rates, then the error system (3.59) is asymptotically stable. Moreover, $\bar{e}(t)$, $\tilde{\theta}_{\rho,s_k}$ and $\tilde{\theta}_{\alpha,s_k,j}$ are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $||\bar{e}|| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_{\rho,s_k}|| \leqslant \sqrt{2\eta_1\alpha}$, and $||\tilde{\theta}_{\alpha,s_k,j}|| \leqslant \sqrt{2\eta_2\alpha}$, where $\alpha = \frac{\mu_0}{\lambda_0} + V(0)$, $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$, $\mu = \sum_{i=1}^{r} h_i(z(t))(\frac{2}{\eta_2}\bar{\theta}^2_{\rho,s_k} + \sum_{j=1}^{p}\frac{2}{\eta_2}\theta^2_{\alpha,s_k,j}) + \mu_0$, and $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T(S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d)]$.

Proof Define the following smooth function

$$V = V_1 + V_2 + V_3 + V_4 + V_5$$

where

$$V_1 = \bar{e}^T P \bar{e}, \ V_2 = \sum_{i=1}^{r} h_i(z(t))(\frac{1}{2\eta_1}\tilde{\theta}^T_{\rho,s_k}\tilde{\theta}_{\rho,s_k})$$

$$V_3 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{2\eta_2}\tilde{\theta}^T_{\alpha,s_k,j}\tilde{\theta}_{\alpha,s_k,j}), \ V_4 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{2\eta_3}\tilde{\varepsilon}^2_{\rho,s_k})$$

$$V_5 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{2\eta_4}\varepsilon^2_{\alpha,s_k,j})$$

Differentiating V with respect to time t, it leads to

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + \dot{V}_5$$

where

$$\begin{aligned}\dot{V}_1 \leqslant & -\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0 + \\ & \sum_{i=1}^{r} h_i(z(t))2\bar{e}^T P \sum_{k=1}^{d} b_{i,s_k}[(\tilde{\theta}^T_{\rho,s_k}\xi^u_{\rho,s_k}(x) + \tilde{\varepsilon}_{\rho,s_k})\kappa] + \\ & \sum_{i=1}^{r} h_i(z(t))2\bar{e}^T P \sum_{k=1}^{d} b_{i,s_k}[\sum_{j=1}^{p}\tilde{\theta}^T_{\alpha,s_k,j}\xi^u_{\alpha,s_k,j}(x) + \tilde{\varepsilon}_{\alpha,s_k,j}]\end{aligned}$$

$$\dot{V}_2 = \sum_{i=1}^{r} h_i(z(t))\frac{1}{\eta_1}\tilde{\theta}^T_{\rho,s_k}\dot{\tilde{\theta}}_{\rho,s_k}, \ \dot{V}_3 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))\frac{\tilde{\theta}^T_{\alpha,s_k,j}\dot{\tilde{\theta}}_{\alpha,s_k,j}}{\eta_2}$$

$$\dot{V}_4 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{\eta_3}\tilde{\varepsilon}_{\rho,s_k}\dot{\hat{\varepsilon}}_{\rho,s_k}), \ \dot{V}_5 = \sum_{i=1}^{r}\sum_{j=1}^{p} h_i(z(t))(\frac{1}{\eta_4}\tilde{\varepsilon}_{\alpha,s_k,j}\dot{\hat{\varepsilon}}_{\alpha,s_k,j})$$

Since $u_{s_k} = \frac{(u^s_{s_k} - \sum_{j=1}^{p}\hat{a}_{s_k,j}(x,\hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j})}{1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}}$, then

$$
\begin{aligned}
u^f_{s_k} &= (1-\rho_{s_k}(x))u_{s_k} + \sum\nolimits_{j=1}^{p} a_{s_k,j}(x) \\
&= u^s_{s_k} - \sum\nolimits_{j=1}^{p} \tilde{\theta}_{\alpha,s_k,j}\xi_{\alpha,s_k,j}(x) - \sum\nolimits_{j=1}^{p} \tilde{\varepsilon}_{\alpha,s_k,j} + \tilde{\theta}_{\rho,s_k}\Delta + \varepsilon_{\rho,s_k}\Delta \\
&= u^s_{s_k} - \sum\nolimits_{j=1}^{p} \tilde{\theta}_{\alpha,s_k,j}\xi_{\alpha,s_k,j} + \sum\nolimits_{j=1}^{p} \tilde{\varepsilon}_{\alpha,s_k,j} + \tilde{\theta}_{\rho,s_k}\Delta + \tilde{\varepsilon}_{\rho,s_k}\Delta
\end{aligned}
$$

where $\Delta = \frac{u^s_{s_k} - \sum_{j=1}^{p} \hat{a}_{s_k,j}(x,\hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\rho,s_k,j}}{1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}}$. Furthermore, one has

$$
\begin{aligned}
\dot{V} \leqslant &- \sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0 + \\
&\sum_{i=1}^{r} h_i(z(t))2\bar{e}^T P \sum_{k=1}^{d} b_{i,s_k}[\tilde{\theta}^T_{\rho,s_k}(\xi^u_{\rho,s_k}(x)\Delta + \frac{1}{\eta_1}\dot{\tilde{\theta}}_{\rho,s_k}) - \tilde{\varepsilon}_{\rho,s_k}(\Delta + \frac{1}{\eta_3}\dot{\tilde{\varepsilon}}_{\rho,s_k}) - \\
&\sum_{j=1}^{p} \tilde{\theta}^T_{\alpha,s_k,j}(\xi^u_{\alpha,s_k,j}(x) - \frac{\dot{\tilde{\theta}}_{\alpha,s_k,j}}{\eta_2}) - \sum_{j=1}^{p} \tilde{\varepsilon}_{\alpha,s_k,j}(1 - \frac{\dot{\tilde{\theta}}_{\alpha,s_k,j}}{\eta_2})]
\end{aligned}
$$

Substituting the adaptive laws (3.60–3.63) into the above equation, it yields

$$
\dot{V} \leqslant -\sum_{i=1}^{r} h_i(z(t))[\bar{e}^T(t)Q\bar{e}(t)] + \mu_0
$$

Since $||\hat{\theta}_{\rho,s_k}|| \leqslant M_{\rho,s_k}$, $||\hat{\theta}_{\alpha,s_k,j}|| \leqslant M_{\alpha,s_k,j}$, which can be guaranteed by using the adaptive laws (3.60) and (3.61), when Assumptions 3.1 and 2.2 (i.e., $||\theta^*_{\rho,s_k}|| \leqslant M_{\rho,s_k}$, $||\theta^*_{\alpha,s_k,j}|| \leqslant M_{\alpha,s_k,j}$) are satisfied, one has

$$
\dot{V} \leq= \lambda V(t) + \mu
$$

where $\mu = \sum_{i=1}^{r} h_i(z(t))[\frac{4}{\eta_1}M^2_{\rho,s_k} + \sum_{j=1}^{p} \frac{4}{\eta_2}M^2_{\alpha,s_k,j} + \frac{4}{\eta_3}\bar{M}^2_{\rho,s_k} + \sum_{j=1}^{p} \frac{4}{\eta_4}\bar{M}^2_{\alpha,s_k,j})] + \mu_0$, $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$.

Then, one has, $\frac{d}{dt}(V(t)e^{\lambda t}) \leqslant e^{\lambda t}\mu$. Furthermore,

$$
0 \leqslant V(t) \leqslant \frac{\mu}{\lambda} + [V(0) - \frac{\mu}{\lambda}]e^{-\lambda t} \leqslant \frac{\mu}{\lambda} + V(0)
$$

Let $\alpha = \frac{\mu}{\lambda} + V(0)$, one has $|\bar{e}| \leqslant \sqrt{\frac{\alpha}{\lambda_{\min}(P)}}$, $|\tilde{\theta}_{\rho,s_k}| \leqslant \sqrt{2\eta_1\alpha}$, and $|\tilde{\theta}_{\alpha,s_k,j}| \leqslant \sqrt{2\eta_2\alpha}$. This ends the proof.

3.3.5 Modified Fault Accommodation with Available System State

In the above subsection, the fault tolerant controller was constructed as

$$u_{s_k} = \frac{u_{s_k}^s - \sum_{j=1}^p \hat{a}_{s_k,j}(x, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j}}{1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k}}$$

Unfortunately, there may exist controller singularity when $1-\hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k} = 0$.

In order to avoid such singularity, the fault tolerant controller is modified as follows

$$u_{s_k} = \frac{(1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})(u_{s_k}^s - \sum_{j=1}^p \hat{a}_{s_k,j}(x, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j})}{(1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})^2 + \varepsilon} \tag{3.64}$$

where $\varepsilon > 0 \in R$ is a design constant. Correspondingly, the adaptive laws in Theorem 3.2 are re-designed as follows

$$\dot{\hat{\theta}}_{\rho,s_k} = \begin{cases} -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa', \text{ if } ||\hat{\theta}_{\rho,s_k}|| < M_{\rho,s_k} \text{or} \\ \quad ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa' \geqslant 0; \\ -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa' + \eta_1 \bar{e}^T P b_{i,s_k} \kappa' \dfrac{\theta_{\rho,s_k}\theta^T_{\rho,s_k}}{||\hat{\theta}_{if}||^2} \xi^u_{\rho,s_k}(x), \\ \text{if } ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(x)\kappa' < 0 \end{cases} \tag{3.65}$$

$$\dot{\hat{\theta}}_{\alpha,s_k,j} = \begin{cases} \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x), \text{ if } ||\hat{\theta}_{\alpha,s_k,j}|| < M_{\alpha,s_k,j} \text{ or} \\ \quad ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -s_i \hat{\theta}^T_{if} \xi_{if} \geqslant 0; \\ \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x) + \eta_2 \bar{e}^T P b_{i,s_k} \dfrac{\hat{\theta}_{\alpha,s_k,j}\hat{\theta}^T_{\alpha,s_k,j}}{||\hat{\theta}_{\alpha,s_k,j}||^2} \xi^u_{\alpha,s_k,j}(x), \\ \quad \text{if } ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -\bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(x) < 0, \end{cases} \tag{3.66}$$

$$\dot{\hat{\varepsilon}}_{\alpha,s_k,j} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 \bar{e}^T P b_{i,s_k} > 0 \\ \quad \text{or } \hat{\varepsilon}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 \bar{e}^T P b_{i,s_k} < 0 \\ \eta_4 \bar{e}^T P b_{i,s_k}, \text{ otherwise} \end{cases} \tag{3.67}$$

$$\dot{\hat{\varepsilon}}_{\rho,s_k} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \kappa' > 0 \\ \quad \text{or } \hat{\varepsilon}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} \kappa' < 0, \\ -\eta_3 \bar{e}^T P b_{i,s_k} \kappa', \text{ otherwise} \end{cases} \tag{3.68}$$

where $\kappa' = \frac{(1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})[u^s_{s_k}-\sum_{j=1}^{p}\hat{\theta}^T_{\alpha,s_k,j}\xi^u_{\alpha,s_k,j}(x)]}{(1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})^2+\varepsilon}$, $\eta_l > 0, l = 1, \ldots, 4$ denote the adaptive rates.

Now, a modified adaptive fault accommodation algorithm is proposed to control the faulty system. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 3.3 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P, real matrices K_i and $Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$\begin{aligned} P(A_i + K_iB_i) + (A_i + K_iB_i)^T P + \\ (A_i + K_iB_i)^T PS_1P(A_i + K_iB_i) + PS_2P \leqslant -Q \end{aligned} \tag{3.69}$$

when the control law (3.64) and adaptive laws (3.65–3.68) are applied, the error system (3.58) is asymptotically stable. Moreover $\bar{e}(t)$, $\tilde{\theta}_{\rho,s_k}$ and $\tilde{\theta}_{\alpha,s_k,j}$ are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $||\bar{e}|| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_{\rho,s_k}|| \leqslant \sqrt{2\eta_1\alpha}$, and $||\tilde{\theta}_{\alpha,s_k,j}|| \leqslant \sqrt{2\eta_2\alpha}$ where $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$, $\mu = \sum_{i=1}^{r} h_i(z(t))(\frac{2}{\eta_2}\bar{\theta}^2_{\rho,s_k} + \sum_{j=1}^{p}\frac{2}{\eta_2}\bar{\theta}^2_{\alpha,s_k,j}) + \mu_0$, and $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T(S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d) + \omega]$, $\alpha = \frac{\mu}{\lambda} + V(0)$.

Proof Similar to the proof of Theorem 3.2, it is easy to obtain the conclusion. The detailed proof is omitted.

3.3.6 FLSs-Based Fault Accommodation with Unavailable System State

Notice that, the FTC (3.55) and the modified FTC (3.64) are designed under the condition that system states are measurable. In fact, in some situations, system state may be unavailable, and the above FTC (3.55) and (3.64) do not work. In this case, observers (3.21) and (3.22) may be used to obtain the estimation $\hat{x}$ of system state x, and design the following observer-based FTC.

$$u_{s_k} = \frac{(1 - \hat{\rho}_{s_k}(\hat{x}, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})(u^s_{s_k} - \sum_{j=1}^{p}\hat{a}_{s_k,j}(\hat{x}, \hat{\theta}_{\alpha,s_k,j}) - \hat{\varepsilon}_{\alpha,s_k,j})}{(1 - \hat{\rho}_{s_k}(\hat{x}, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k})^2 + \varepsilon} \tag{3.70}$$

Correspondingly, the adaptive laws in Theorem 3.3 are re-designed as follows:

$$\dot{\hat{\theta}}_{\rho,s_k} = \begin{cases} -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega, \text{ if } ||\hat{\theta}_{\rho,s_k}|| < M_{\rho,s_k} \text{ or} \\ \quad ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega \geqslant 0; \\ -\eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega + \\ \quad \eta_1 \bar{e}^T P b_{i,s_k} \omega \dfrac{\theta_{\rho,s_k}\theta^T_{\rho,s_k}}{||\hat{\theta}_{if}||^2} \xi^u_{\rho,s_k}(\hat{x}), \\ \text{if } ||\hat{\theta}_{\rho,s_k}|| = M_{\rho,s_k} \text{ and } \eta_1 \bar{e}^T P b_{i,s_k} \xi^u_{\rho,s_k}(\hat{x})\omega < 0 \end{cases} \tag{3.71}$$

$$\dot{\hat{\theta}}_{\alpha,s_k,j} = \begin{cases} \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}), \text{ if } ||\hat{\theta}_{\alpha,s_k,j}|| < M_{\alpha,s_k,j} \text{ or} \\ \quad ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -\bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}) \geqslant 0; \\ \eta_2 \bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}) + \eta_2 \bar{e}^T P b_{i,s_k} \dfrac{\hat{\theta}_{\alpha,s_k,j}\hat{\theta}^T_{\alpha,s_k,j}}{||\hat{\theta}_{\alpha,s_k,j}||^2} \xi^u_{\alpha,s_k,j}(\hat{x}), \\ \quad \text{if } ||\hat{\theta}_{\alpha,s_k,j}|| = M_{\alpha,s_k,j} \text{ and } -\bar{e}^T P b_{i,s_k} \xi^u_{\alpha,s_k,j}(\hat{x}) < 0 \end{cases} \tag{3.72}$$

$$\dot{\hat{\varepsilon}}_{\rho,s_k} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k}\omega > 0 \\ \quad \text{or } \hat{\varepsilon}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k}\omega < 0; \\ -\eta_1 \bar{e}^T P b_{i,s_k}\omega, \text{ otherwise} \end{cases} \tag{3.73}$$

$$\dot{\hat{\varepsilon}}_{\alpha,s_k,j} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 \bar{e}^T P b_{i,s_k} > 0 \text{ or} \\ \quad \hat{\varepsilon}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} < 0; \\ \eta_4 \bar{e}^T P b_{i,s_k}, \text{ otherwise} \end{cases} \tag{3.74}$$

where $\omega = \frac{(1-\hat{\rho}_{s_k}(x,\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})[u^s_{s_k} - \sum_{j=1}^{p} \hat{\theta}^T_{\alpha,s_k,j}\xi^u_{\alpha,s_k,j}(\hat{x})]}{(1-\hat{\rho}_{s_k}(\hat{x},\hat{\theta}_{\rho,s_k})-\hat{\varepsilon}_{\rho,s_k})^2+\varepsilon}$, $\eta_l > 0, l = 1, \ldots, 4$ denote the adaptive rates.

Now, an observer-based adaptive fault accommodation algorithm is proposed to control the faulty system. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 3.4 *Under Assumptions 3.1–3.3, if there exist a common symmetric positive definite matrix P, real matrices K_i and $Q > 0$, $i = 1, 2, \ldots, r$ with appropriate dimensions, such that the following conditions hold*

$$P(A_i+K_iB_i)+(A_i+K_iB_i)^T P+(A_i+K_iB_i)^T PS_1P(A_i+K_iB_i)+PS_2P \leqslant -Q \tag{3.75}$$

when the control law (3.70) and adaptive laws (3.71–3.74) are applied, then the error system (3.58) is asymptotically stable. Moreover $\bar{e}(t)$, $\tilde{\theta}_{\rho,s_k}$ and $\tilde{\theta}_{\alpha,s_k,j}$

are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $||\bar{e}|| \leqslant \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_{\rho,s_k}|| \leqslant \sqrt{2\eta_1\alpha}$, *and* $||\tilde{\theta}_{\alpha,s_k,j}|| \leqslant \sqrt{2\eta_2\alpha}$, *where* $\alpha = \mu/\lambda + V(0)$, $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{1}{2\eta_1}, \frac{1}{2\eta_2}\}$, $\mu = \sum_{i=1}^{r} h_i(z(t))(\frac{2}{\eta_2}\bar{\theta}^2_{\rho,s_k} + \sum_{j=1}^{p}\frac{2}{\eta_2}\bar{\theta}^2_{\alpha,s_k,j}) + \mu_0$, *and* $\mu_0 = \sum_{i=1}^{r} h_i(z(t))[(\omega_d - \dot{\omega}_d)^T \cdot (S_1^{-1} + S_2^{-1} + 2P)(\omega_d - \dot{\omega}_d) + \omega]$.

Proof Similar to the proof of Theorem 3.2, it is easy to obtain the conclusion. The detailed proof is omitted.

3.4 Simulation Results

To verify the effectiveness of the proposed method, we consider the re-entry phase of a NSV with the altitude $H = 40$ km and speed $V = 2500$ m/s as the initial states. The symmetric, positive definite moment of inertia tensor is given as follows:

$$J = \begin{bmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 0 & 1376852 \end{bmatrix}$$

Consider that the nonlinearity of NSV re-entry attitude dynamics mainly comes from attack angle α and attitude angular velocity ω. In NSV re-entry phase $\alpha \in [0, \pi/4]$, we assume that α has two related fuzzy sets $\{\alpha = 0\}$ and $\{\alpha = \pi/4\}$, the corresponding membership functions are given by:

$$M_{\omega=0} = (1 - \frac{1}{1 + \exp[-6 - 28\omega)]})\frac{1}{1 + \exp[6 - 28\omega)]}$$

$$M_{\omega=-0.5} = (\frac{1}{1 + \exp[6 + 28\omega)]}), M_{\omega=0.5} = (1 - \frac{1}{1 + \exp[-6 + 28\omega)]})$$

We choose six operating points:

$$[\alpha, \omega] \in \{[0, -0.5], [0, 0], [0, 0.5], [\pi/4, -0.5], [\pi/4, 0], [\pi/4, 0.5]\}$$

Under the membership functions and the six operating points, six plant rules and six control rules can be defined. All A_i and B_i can be obtained by substituting the six operating points to $f(x_\omega)$, $g(x_\omega)$. The detailed matrix parameters are given in [62].

Rule 1: IF ω is about -0.5 rad/s and α is about 0 rad, THEN
$$\dot{x}(t) = A_1 x(t) + B_1 u, \ \ y(t) = C_1 x(t)$$
Rule 2: IF ω is about -0.5 rad/s and α is about $\pi/4$ rad, THEN
$$\dot{x}(t) = A_2 x(t) + B_2 u, \ \ y(t) = C_2 x(t)$$
Rule 3: IF ω is about 0 rad/s and α is about 0 rad, THEN
$$\dot{x}(t) = A_3 x(t) + B_3 u, \ \ y(t) = C_3 x(t)$$
Rule 4: IF ω is about 0 rad/s and α is about $\pi/4$ rad, THEN
$$\dot{x}(t) = A_4 x(t) + B_4 u, \ \ y(t) = C_4 x(t)$$
Rule 5: IF ω is about 0.5 rad/s and α is about 0 rad, THEN
$$\dot{x}(t) = A_5 x(t) + B_5 u, \ \ y(t) = C_5 x(t)$$
Rule 6: IF ω is about0.5 rad/s and α is about $\pi/4$ rad, THEN
$$\dot{x}(t) = A_6 x(t) + B_6 u, \ \ y(t) = C_6 x(t)$$

The initial conditions are taken as follows: $\omega(0) = [0, 0, 0]^T, \gamma(0) = [0, 0, 0]^T$ and the tracking command is chosen as $\omega_d = [0, 0, 0]^T, \gamma_d = [1, 0, 2]^T$ during the re-entry phase. The parameters are taken as in [62] and will not be described in detail here. We consider the case where only two actuators fail at one time:

$$u_1^f(t) = \begin{cases} u_1(t), \ t < 5s \\ (1 - \rho_1(x))(u_1(t) + \sum_{j=1}^{p} g_{1,j} f_{1,j}(x)), \ t \geqslant 5s \end{cases}$$

$$u_2^f(t) = \begin{cases} u_2(t), \ t < 5 \\ (1 - \rho_2(x))(u_2(t) + \sum_{j=1}^{p} g_{2,j} f_{2,j}(x)), \ t \geqslant 5 \end{cases}$$

$$u_3^f(t) = u_3(t)$$

where $\rho_1(x) = 0.4\cos(x_1), p = 1, g_{1,1} = 0.4, f_{1,1}(x) = \cos(x_3), \ \rho_2(x) = 0.4\sin(x_2), g_{2,1} = 0.4, \ f_{2,1}(x) = \cos(x_3)$. By using Matlab toolbox to solve the matrices inequalities (3.25), one can obtain the fault diagnostic observer gains L_i. By solving (3.52), one can obtain the positive definite symmetric matrix P and the nominal controller gains K_i. Therefore, one can design the ideal control (3.51). Using the ideal control input (3.51), we can design fault-tolerant controller (3.55), the modified fault-tolerant (3.64) and the observer-based fault-tolerant control (3.70). In this example, we assume that the system state is not fully measured and thus the observer (3.22) is used to estimate the system state. Consequently, the observer-based fault-tolerant control input (3.70) is used to control the faulty system. The simulation results are presented in Figs. 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8 and 3.9. From Fig. 3.2,

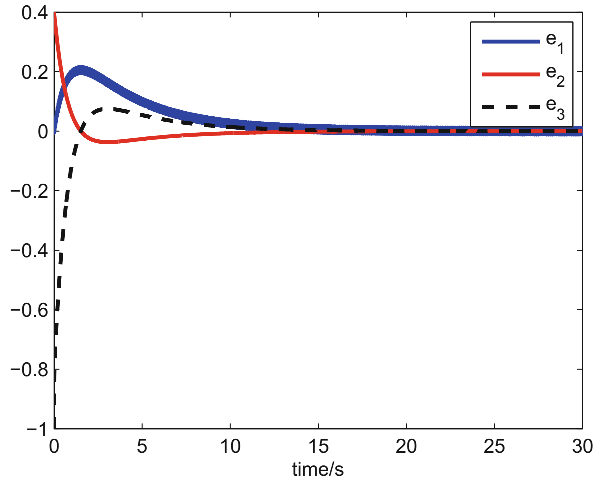

Fig. 3.2 The observer errors time responses: $e_1,\ e_2,\ e_3$

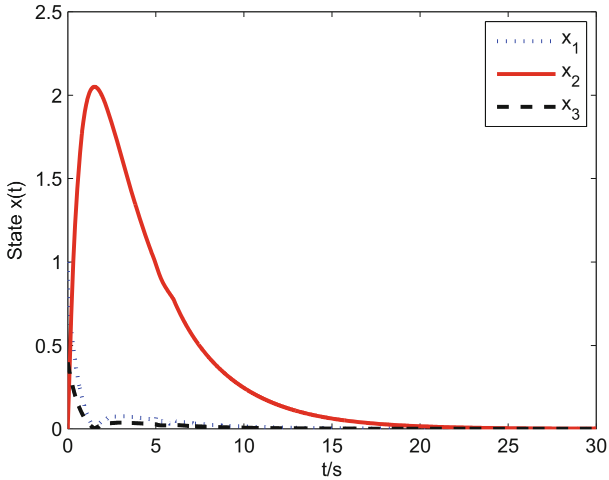

Fig. 3.3 State responses of NSV attitude dynamics under normal conditions

it is seen that, under normal operating condition, observation errors globally asymptotically converge to zero. If no actuator fails, the system states globally asymptotically converge to zero, as shown in Fig. 3.3. Figure 3.4 shows that, when an actuator fault occurs, when keeping the normal controller, the system states deviate significantly from zero. However, as shown in Fig. 3.5, using the proposed FTC (3.70), the

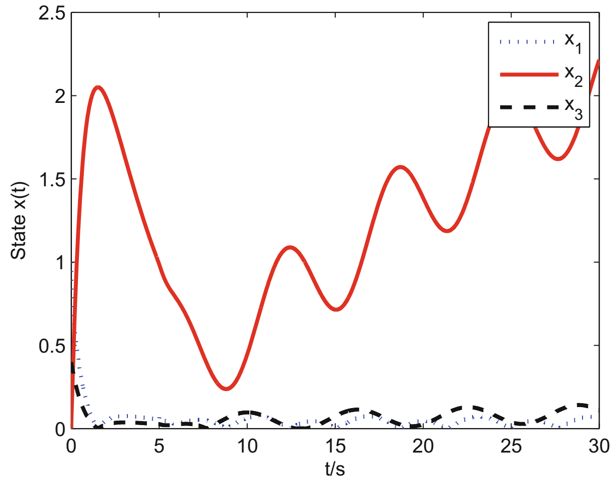

Fig. 3.4 State responses of NSV attitude dynamics without FTC

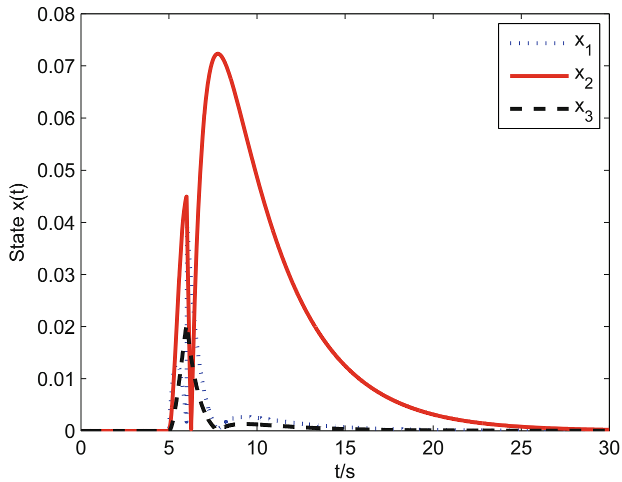

Fig. 3.5 State responses of NSV attitude dynamics with observer-based FTC (3.70)

system states globally asymptotically converge to zero. From Figs. 3.6, 3.7, 3.8 and 3.9, we can clearly draw the conclusion that both gain faults and bias faults can be approximated accurately and promptly by FLSs.

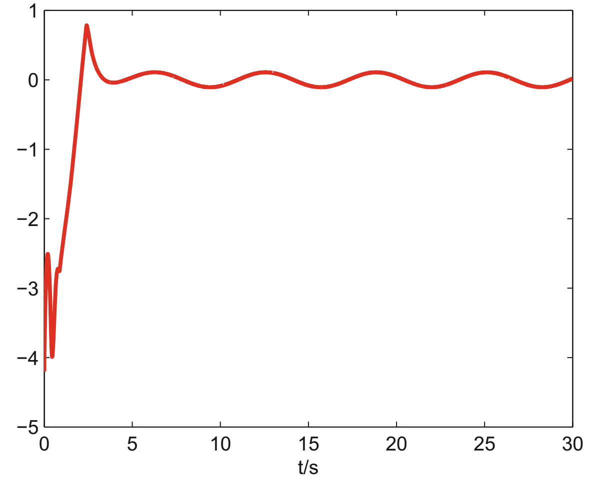

Fig. 3.6 The estimation error of bias fault $g_{1,1}f_{1,1}(x)$

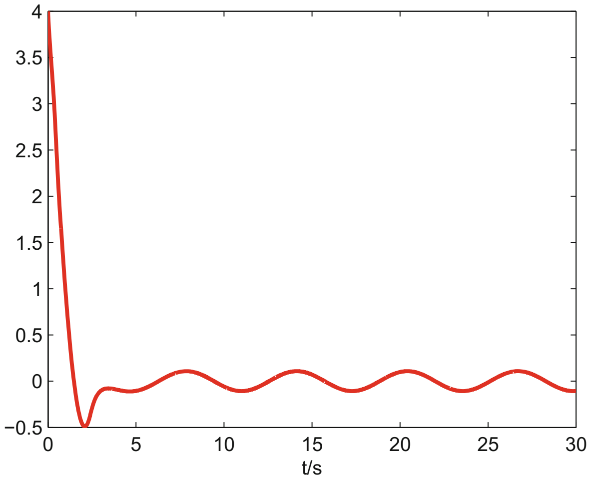

Fig. 3.7 The estimation error of gain fault $\rho_1(x)$

3.5 Conclusions

In this chapter, the problem of fault tolerant control for NSV with multiple state-dependent faults was studied. We first designed a bank of SMOs to detect and estimate the fault. Compared with some results in literature, the proposed fault accommodation scheme is designed to online approximate not only bias faults but also gain faults. Moreover, it can accommodate multiple actuator faults simultaneously. In addition, the adaptive fault accommodation algorithm removes the classical assumption that the time derivative of the output errors should be known. Simulation results of NSV

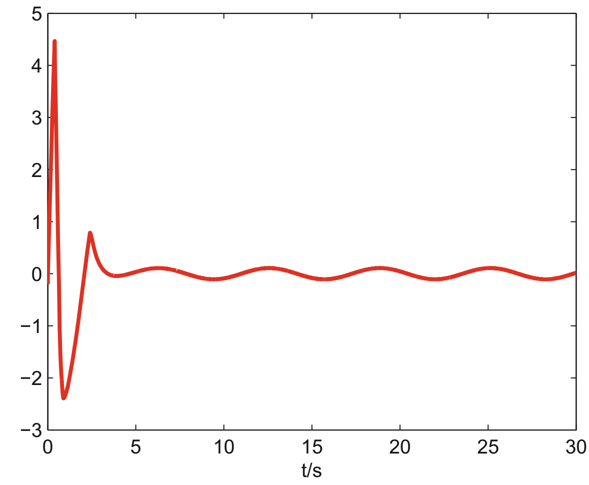

Fig. 3.8 The estimation error of bias fault $g_{2,1}f_{2,1}(x)$

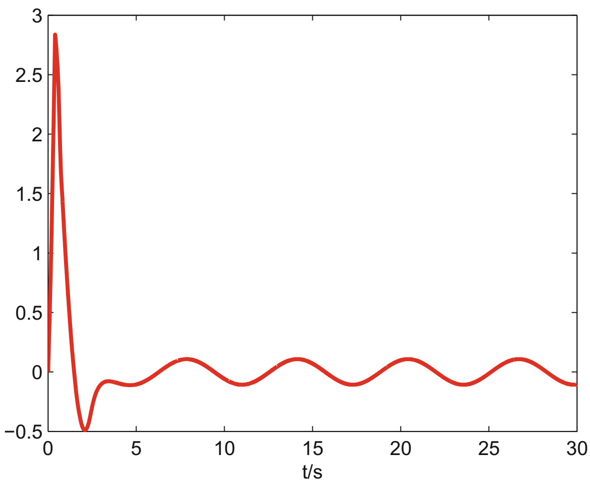

Fig. 3.9 The estimation error of gain fault $\rho_2(x)$

show that the designed fault detection, isolation and estimation algorithms as well as the fault-tolerant control scheme have good dynamic performances in the presence of multiple actuator faults.

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Chapter 4
Command Filtered Adaptive Fuzzy Backstepping FTC Against Actuator Fault

4.1 Introduction

Fuzzy control has found extensive applications for modeling nonlinear systems in the past 10 years. According to the fuzzy approximation theorem of the fuzzy logic systems (FLSs) [1–6], researchers proposed many approximation-based adaptive fuzzy control design methods for nonlinear systems (see, e.g., [7–12] and the references therein).

It has been proved that adaptive backstepping technique is a powerful tool to solve tracking or regulation control problems of unknown nonlinear systems in or transformable to parameter strict-feedback form [13]. For such systems, many adaptive fuzzy backstepping controllers have been developed (see, e.g., [14–19] and the references therein), where FLSs or neural networks are used to approximate unknown nonlinear smooth functions. It is well known that, however, in standard backstepping design procedure, analytic computation of the first derivatives of virtual control signals α_i $(i = 1, 2, \ldots, n-1)$, i.e., $\dot{\alpha}_i$, is necessary. Note that, the computation of $\dot{\alpha}_i$ requires the higher derivatives of $\dot{\alpha}_j$, $j = 0, 1, \ldots, i-1$. Obviously, as system dimension, i.e., n, increases, the computation of $\dot{\alpha}_i$ becomes increasingly complicated. This limits the theoretical results' field of practical applications. Hence, how to reduce the computation of $\dot{\alpha}_i$ is crucial issue in controller design, which is a motivation of this chapter. In addition, the aforementioned approaches required the knowledge of the desired trajectory $y_d(t)$ and the first n derivatives, i.e., $y_d^{(i)}(t), i = 1, 2, \ldots, n$ should be available. It is important to note that in some important applications (e.g., land vehicle or aircraft) the desired trajectory may be generated by a planner, an outer-loop, or a user input device that does not provide higher derivatives. Relaxing the assumption motivates us for this work.

On the other hand, actuators, sensors or other system components in practical engineering fail frequently, which can cause system performance deterioration and lead to instability that can further produce catastrophic accidents. Thus, many effective fault tolerant control (FTC) approaches have been proposed to improve system reliability and to guarantee system stability in all situations [20–39].

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In this chapter, a bank of command filters (see, e.g., [40, 41] and the references therein) are proposed to respectively generate the first derivations of the desired trajectory and virtual control signals. Then, by using backstepping technique, a robust adaptive fuzzy controller is proposed to guarantee that the tracking error converges to a neighborhood of the origin, where FLSs are utilized to approximate the unknown functions. The contributions form our work are generalized the following aspects:

(1) The desired trajectory and only its first derivative are necessary for the control scheme presented in this chapter, which is more reasonable in practical applications. The theoretic results of this chapter are thus valuable in a wide field of practical applications;
(2) Compared with the existing literatures concerning the standard backstepping design, the control scheme presented in this chapter does not need to compute the higher derivatives of virtual control signals in backstepping design procedures, which decreases the computation complexity;
(3) Different from some results in literature where all system functions are known, the system functions considered in this chapter are unknown. In particular, the signs of control gain functions are also unknown.
(4) The actuator fault model that is presented in this chapter integrates not only unknown gain faults, but also unknown bias faults,where both faults are dependent on the system state and will be approximated by FLSs.

The rest of this chapter is organized as follows. Section 4.2 formulates the problem under investigation. Nussbaum type gain and mathematical description of FLSs are also provided. In addition, some basic assumptions and preliminary results are given. In Sect. 4.3, the main technical results of this chapter are given, where command filters and adaptive fuzzy controller are designed, and the closed-loop system's stability analysis is developed. A numerical example is presented in Sect. 4.4. Simulation results are presented to demonstrate the effectiveness of the proposed technique. Finally, Sect. 4.5 draws the conclusion.

4.2 Problem Statement and Preliminaries

4.2.1 Problem Statement

Considers the following uncertain nonlinear systems:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(\bar{x}_{i+1}, t), \quad i = 1, 2, \ldots, n-1; \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u(t) + d_n(\bar{x}_n, t); \\ y \ = x_1 \end{cases} \tag{4.1}$$

where $\bar{x}_i = (x_1, \ldots, x_i)^T \in R^i, i = 1, \ldots, n$ is the state; y denotes the output; $u \in R$ is the input; $f_i(\cdot) \in R$ and $g_i(\cdot) \in R, i = 1, \ldots, n$ are the unknown smooth functions; $d_i(\cdot, t), i = 1, \ldots, n$, denote the unknown dynamic disturbances.

In practical applications, actuators may fail. The fault model considered in this chapter can be described as follows:

$$u^f = g_f(\bar{x}_n)u + b_f(\bar{x}_n), t > t_F \tag{4.2}$$

where $g_f(\bar{x}_n)$ and $b_f(\bar{x}_n)$ are smooth functions, which denote unknown gain fault and bias fault, respectively; t_F is an unknown fault occurrence time.

Control objective is to design an adaptive fuzzy controller by backstepping with command filter for system (4.1) such that output y can track accurately the desired trajectory y_d as possible regardless of actuator fault and unknown dynamic disturbances.

To design appropriate controller, the following lemma and some assumptions are given.

Lemma 4.1 ([42]) For $\forall x \in R$, $|x| - \tanh(x/\delta)x \leq 0.2785\delta$, where $\delta > 0 \in R$.

Assumption 4.1 There exist known constants $g_{i0} > 0 \in R$ and $g_{i1} > 0 \in R$ such that $g_{i1} \geqslant |g_i(\bar{x}_i)| \geqslant g_{i0} > 0, \forall \bar{x}_i \in R^i, i = 1, 2, \ldots, n$.

Assumption 4.2 There exist unknown constant p_i^* and known smooth positive function $\phi_i(\bar{x}_i)$ such that $|d_i(\cdot, t)| \leq p_i^* \phi_i(\bar{x}_i)$.

Assumption 4.3 The desired trajectory $y_d(t)$ and its first derivative are bounded and available.

Assumption 4.4 $g_f(\bar{x}_n)$ is bounded, i.e., there exist known constants $g_{f0} > 0 \in R$ and $g_1 > 0 \in R$ such that $g_{f1} \geq |g(\bar{x}_n)| \geq g_{f0}$.

Remark 4.1 In literature, the existing results concerning the trajectory tracking problems of the strict-feedback systems require the classical assumption that the desired trajectory $y_d(t)$ and the first n derivatives, i.e., $y_d^{(i)}(t), i = 0, 1, \ldots, n$ should be available. Just stated in Introduction, in some important applications (e.g., land vehicle or aircraft) the desired trajectory may be generated by a planner, an outer-loop, or a user input device that does not provide higher derivatives. Thus, in such case, these results do not work. Assumption 4.3 in this chapter is more reasonable in practical applications.

4.2.2 Nussbaum Type Gain

Any continuous function $N(s) : R \to R$ is a function of Nussbaum type if it has the following properties:

(1) $\lim\limits_{s\to+\infty} \sup \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = +\infty$;
(2) $\lim\limits_{s\to-\infty} \inf \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = -\infty$

For example, the continuous functions $\varsigma^2\cos(\varsigma)$, $\varsigma^2\sin(\varsigma)$, and $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ verify the above properties and are thus Nussbaum-type functions [43].The even Nussbaum function $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ is used throughout this chapter.

Lemma 4.2 ([44]) Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \leq c_0 + \int_0^t (\underline{g}N(\varsigma) + 1)\dot{\varsigma}d\tau, \forall t \in [0, t_f)$$

where $\underline{g} \neq 0$ is a constant, and c_0 represents a suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on$[0, t_f)$.

Lemma 4.3 ([45]) Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. For $\forall t \in [0, t_f)$, if the following inequality holds,

$$V(t) \leq c_0 + e^{-c_1 t}\int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}e^{c_1\tau}d\tau + e^{-c_1 t}\int_0^t \dot{\varsigma}e^{c_1\tau}d\tau$$

where constant $c_1 > 0$, $\underline{g}(\cdot)$ is a time-varying parameter which takes values in the unknown closed intervals $I := [l^{-1}, l^{+1}]$ with $0 \notin I$, and c_0 represents some suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0, t_f)$.

4.2.3 Mathematical Description of Fuzzy Logic Systems

A fuzzy logic system consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

$$\begin{aligned} R^l :\ & if\ x_1\ is\ A_1^l\ and\ x_2\ is\ A_2^l\ \ldots\ and\ x_n\ is\ A_n^l, \\ & then\ \ y\ is\ B^l,\ \ l = 1, 2, \ldots, M \end{aligned}$$

where $\underline{x} = [x_1, \ldots, x_n]^T \subset R^n$ and y are the FLS input and output, respectively. Fuzzy sets A_i^l and B^l are associated with the fuzzy functions $\mu_{A_i^l}(x_i) = \exp(-(\frac{x_i - a_i^l}{b_i^l})^2)$ and $\mu_{B^l}(y^l) = 1$, respectively. M is the rules number. Through singleton function, center average defuzzification and product inference, the FLS can be expressed as:

$$y(x) = \sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right) / \sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right)$$

where $\bar{y}^l = \max_{y \in R} \mu_{B^l}$. Define the fuzzy basis functions as:

$$\xi_l(x) = \prod_{i=1}^{n} \mu_{A_i^l}(x_i) \sum_{l=1}^{M} \left(\prod_{i=1}^{n} / \mu_{A_i^l}(x_i) \right)$$

and define $\theta^T = [\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^M] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi(x) = [\xi_1(x), \ldots, \xi_M(x)]^T$, then the above FLS can be rewritten as:

$$y(x) = \theta^T \xi(x)$$

Lemma 4.4 ([5, 6]) Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists a FLS such as

$$\sup_{x \in \Omega} |f(x) - \theta^T \xi(x)| \leq \varepsilon$$

By Lemma 4.4, we know, FLS can approximate any smooth function on a compact space. Due to this approximation capability, we can assume that the nonlinear function $f(x)$ can be approximated as

$$f(x, \theta) = \theta^T \xi(x)$$

Define the optimal parameter vector θ^* as

$$\theta^* = \arg \min_{\theta \in \Omega} [\sup_{x \in U} |f(x) - f(x, \theta^*)|]$$

where Ω and U are compact regions for θ and x, respectively. Also the FLS minimum approximation error is defined as:

$$\varepsilon = f(x) - \theta^{*T} \xi(x)$$

From Lemma 4.4, the following assumption is made.

Assumption 4.5 There exist an unknown real bounded constant $\varepsilon^* > 0$ such that $|\varepsilon| \leq \varepsilon^*$ on compact sets Ω and U.

In this chapter, we use the above FLS to approximate the unknown function $h_i(z_i)$, $(i = 1, \ldots, n)$ will defined later, namely, there exists θ_i^* and ε_i such that

$$h_i(z_i) = \theta_i^{*T} \xi_i(z_i) + \varepsilon_i$$

From Assumption 4.5, there exists an unknown positive real constant ε_i such that $|\varepsilon_i| \leq \varepsilon_i^*$.

For notational simplicity, we use $\bullet$ to denote $\bullet(\cdot)$. For example, f_i is the abbreviation of $f_i(\bar{x}_i)$.

4.3 Design of Adaptive Fuzzy Controller and Stability Analysis

Define

$$z_i = x_i - \alpha_{i-1}, \quad i = 1, 2, \ldots, n \tag{4.3}$$

where α_0=y_d, α_{i-1} $(i = 2, \ldots, n)$ is a virtual control which will be designed at each step, $\alpha_n = u$ is actual control input. The recursive design procedure contains n steps. From Step 1 to Step $n-1$, α_i $(i = 1, \ldots, n-1)$ is designed at each step. Finally an overall control law $u(\alpha_n)$ is constructed at Step n.

In order to estimate the virtual control α_{i-1} $(i = 2, \ldots, n)$, define the following command filter

$$\dot{\omega}_i = -\eta_\omega(\omega_i - \alpha_{i-1}), \quad i = 2, \ldots, n \tag{4.4}$$

where $\eta_\omega > 0$ is a design parameter. Let us define the estimation error signal v_i as

$$v_i = \omega_i - \alpha_{i-1}, \quad i = 2, \ldots, n$$

Remark 4.2 The command filter (4.4) is constructed to avoid the computation of the higher derivatives of α_{i-1}, $i = 2, \ldots, n$. It should be pointed out that the error v_i will be compensated at Step n in this chapter.

Step 1:

Now, consider z_1-subsystem: $z_1 = x_1 - \alpha_0$. Form (4.1) and (4.3), one has

$$\begin{aligned}\dot{z}_1 &= f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 + d_1(\bar{x}_2, t) - \dot{y}_d \\ &= f_1(\bar{x}_1) + g_1(\bar{x}_1)z_2 + g_1(\bar{x}_1)\alpha_1 + d_1(\bar{x}_2, t) - \dot{y}_d\end{aligned} \tag{4.5}$$

Define the following function

$$V_{z1} = \int_0^{z_1} \frac{\sigma}{|g_1(\sigma + y_d)|} d\sigma \tag{4.6}$$

From the integral-type mean value theorem, it can be known that, there exists a constant $\lambda_1 \in (0, 1)$ such that $V_{z_1} = {z_1}^2/2g(\lambda_1 z_1 + y_d)$. Hence, from Assumption 4.1, we have

$$\frac{z_1{}^2}{2g_{10}} \geqslant V_{z_1} \geqslant \frac{z_1{}^2}{2g_{11}} > 0$$

which means that, V_{z_1} is a positive definite function of variable z_1.

Since $\frac{\partial\left|g^{-1}(\sigma+y_d)\right|}{\partial y_d} = \frac{\partial\left|g^{-1}(\bar{x},\sigma+y_d)\right|}{\partial \sigma}$, we can obtain

$$\begin{aligned}
\dot{V}_{z_1} &= \frac{z_1}{|g_1(x_1)|}\dot{z}_1 + \int_0^{z_1} \sigma \left[\frac{\partial\left|g^{-1}(\sigma+y_d)\right|}{\partial y_d}\dot{y}_d\right] d\sigma \\
&= \frac{z_1}{|g_1(x_1)|}\dot{z}_1 + \dot{y}_d\left[\frac{z_1}{|g_1(x_1)|} - \int_0^{z_1}\left[\frac{1}{\left|g^{-1}(\sigma+y_d)\right|}d\sigma\right]\right] \\
&= \frac{z_1}{|g_1(x_1)|}[f_1(\bar{x}_1) + g_1(\bar{x}_1)z_2 + g_1(\bar{x}_1)\alpha_1 + d_1(\bar{x}_2, t) - \dot{y}_d] + \\
&\quad \dot{y}_d\left[\frac{z_1}{|g_1(x_1)|} - \int_0^{z_1}\frac{1}{\left|g^{-1}(\sigma+y_d)\right|}d\sigma\right]
\end{aligned} \tag{4.7}$$

Let $\bar{z}_1 = (x_1, \omega_1, \dot{\omega}_1)^{\mathrm{T}}$ and

$$h_1(\bar{z}_1) = \frac{f_1(x_1)}{|g_1(x_1)|} + \frac{\dot{\omega}_1}{z_1}\int_0^{z_1}\left[\frac{1}{\left|g^{-1}(\sigma+\omega_1)\right|}d\sigma\right] \tag{4.8}$$

$$\Delta_1(\bar{z}_1, \alpha_0, \dot{\alpha}_0, \omega_1, \dot{\omega}_1) = \frac{\dot{y}_d}{z_1}\int_0^{z_1}\left[\frac{1}{\left|g^{-1}(\sigma+y_d)\right|}d\sigma\right] - \frac{\dot{\omega}_1}{z_1}\int_0^{z_1}\left[\frac{1}{\left|g^{-1}(\sigma+\omega_1)\right|}d\sigma\right] \tag{4.9}$$

Note that, $h_i(\bar{z}_1)$ will be approximated by FLSs on a compact set Ω_{z_1} as: $h_1(z_1) = \theta_1^{*T}\xi_1(\bar{z}_1) + \varepsilon_1(\bar{z}_1)$. From Assumption 4.5, we know, there exists an unknown constant ε_1^* such that $|\varepsilon_1(\bar{z}_1)| \leq \varepsilon_1^*$.

Then, we have

$$\dot{V}_{z_1} = z_1[\frac{g_1(\bar{x}_1)}{|g_1(x_1)|}z_2 + \frac{g_1(\bar{x}_1)}{|g_1(x_1)|}\alpha_1 + \frac{d_1(\bar{x}_2, t)}{|g_1(x_1)|} + h_1(\bar{z}_1)] + \Delta_1(\bar{z}_1, \alpha_0, \dot{\alpha}_0, \omega_1, \dot{\omega}_1) \tag{4.10}$$

Virtual control α_1 is defined as follows:

$$\alpha_1 = N(\varsigma_1)[k_1 z_1 + h_1(z_1, \hat{\theta}_1) + \hat{b}_1\bar{\varphi}_1(x_1)\tanh(\frac{z_1\bar{\varphi}_1(\bar{x}_1)}{\eta_1})] \tag{4.11}$$

$$\dot{\varsigma}_1 = k_1 z_1^2 + h_1(z_1, \hat{\theta}_1)z_1 + \hat{b}_1\bar{\varphi}_1(x_1)z_1\tanh(\frac{z_1\bar{\varphi}_1(\bar{x}_1)}{\eta_1}) \tag{4.12}$$

where $k_1 > 1$ is a design parameter; $h_1(z_1, \hat{\theta}_1) = \hat{\theta}_1^T \xi_1(\bar{z}_1)$ and $\hat{\theta}_1$ are estimates of $\theta_1^{*T} \xi_1(\bar{z}_1)$ and θ_1^*, respectively; $\hat{b}_1$ is an estimate of $b_1^* = \max\{\varepsilon_1^*, \frac{p_1^*}{g_{10}}\}$, $\bar{\varphi}_1(\bar{x}_1) = 1 + \varphi_1(\bar{x}_1)$.

Hence, from Lemma 4.1 and Assumptions 4.1 and 4.2, (4.7) can be further developed as follows:

$$\begin{aligned}
\dot{V}_{z_1} \leqslant & \frac{g_1(\bar{x}_1)}{|g_1(x_1)|} z_1 z_2 + \frac{g_1(\bar{x}_1)}{|g_1(x_1)|} z_1 N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1 - \dot{\varsigma}_1 + \frac{p_1^* \varphi_1(\bar{x}_1)}{g_{10}} |z_1| + h_1(\bar{z}_1) z_1 \\
= & -k_1 z_1^2 + \frac{g_1(\bar{x}_1)}{|g_1(x_1)|} z_1 z_2 + \frac{g_1(\bar{x}_1)}{|g_1(x_1)|} z_1 N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1 + h_1(\bar{z}_1) z_1 - h_1(z_1, \hat{\theta}_1) z_1 - \\
& \hat{b}_1 \bar{\varphi}_1(x_1) z_1 \tanh(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1}) + \frac{p_1^* \varphi_1(\bar{x}_1)}{g_{10}} |z_1| \\
\leqslant & -k_1 z_1^2 + \frac{1}{4} z_2^2 + z_1^2 + \frac{g_1(\bar{x}_1)}{|g_1(x_1)|} z_1 N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1 - \tilde{\theta}_1 \xi_1(\bar{z}_1) z_1 + \\
& b_1^*[|z_1| \bar{\varphi}_1(\bar{x}_1) - z_1 \bar{\varphi}_1(\bar{x}_1) \tanh(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1})] - \tilde{b}_1 \bar{\varphi}_1(x_1) z_1 \tanh(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1}) \\
= & -(k_1 - 1) z_1^2 + \frac{1}{4} z_2^2 + \frac{g_1(\bar{x}_1)}{|g_1(x_1)|} z_1 N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1 - \tilde{\theta}_1 \xi_1(\bar{z}_1) z_1 + b_1^*[|z_1| \bar{\varphi}_1(\bar{x}_1) - \\
& z_1 \bar{\varphi}_1(\bar{x}_1) \tanh(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1})] - \tilde{b}_1 \bar{\varphi}_1(x_1) z_1 \tanh(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1}) + \Delta_1
\end{aligned} \tag{4.13}$$

where $\tilde{\theta}_1 = \theta_1^* - \theta_1$, $\tilde{b}_1 = b_1^* - b_1$.

Consider the following function

$$V_1(t) = V_{z_1} + \frac{1}{2} \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1 + \frac{1}{2\lambda_1} \tilde{b}_1^2 \tag{4.14}$$

Adaptive laws are defined as follows:

$$\dot{\hat{\theta}}_1 = \Gamma_1[z_1 \xi_1(\bar{z}_1) - \sigma_1 \hat{\theta}_1] \tag{4.15}$$

$$\dot{\hat{b}}_1 = \lambda_1[z_1 \bar{\varphi}_1(\bar{x}_1) \tanh(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1}) - \sigma_{b1} \hat{b}_1] \tag{4.16}$$

where Γ_1 is a positive matrix with appropriate dimensions, $\sigma_1 > 0$, $\sigma_{b1} > 0$, $\eta_1 > 0$ and $\lambda_1 > 0$ are design parameters.

Differentiating V_1 with respect to time t and considering (4.9)–(4.12), we have

$$\begin{aligned}
\dot{V}_1 \leqslant & -(k_1 - 1) z_1^2 + \frac{1}{4} z_2^2 + \frac{g_1(\bar{x}_1)}{|g_1(x_1)|} z_1 N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1 + \\
& 0.2785 \eta_1 b_1^* - \sigma_1 \tilde{\theta}_1^T \hat{\theta}_1 - \sigma_{b1} \tilde{b}_1 \hat{b}_1 + \Delta_1
\end{aligned} \tag{4.17}$$

where Lemma 4.1 is used, namely, $0 \leqslant |x| - x \tanh(\frac{x}{\varepsilon}) \leqslant 0.2785\varepsilon$, $\forall \varepsilon > 0, \forall x \in R$.

Since

$$\sigma_1\tilde{\theta}_1^{\mathrm{T}}\hat{\theta}_1 \leqslant -\frac{\sigma_1\left\|\tilde{\theta}_1\right\|^2}{2}+\frac{\sigma_1\left\|\theta_1^*\right\|^2}{2},\quad \sigma_{b1}\tilde{b}_1\hat{b}_1 \leqslant -\frac{\sigma_{b1}\tilde{b}_1^2}{2}+\frac{\sigma_{b1}b_1^{*2}}{2} \tag{4.18}$$

then (4.17) can be derived as

$$\dot{V}_1 \leqslant -c_1V_1+\frac{1}{4}z_2^2+\frac{g_1(\bar{x}_1)}{|g_1(x_1)|}z_1N(\varsigma_1)\dot{\varsigma}_1+\dot{\varsigma}+c_{\varepsilon1}+\Delta_1 \tag{4.19}$$

where

$$c_{\varepsilon1}=0.2785\eta_1b_1^*+\frac{\sigma_1\left\|\theta_1^*\right\|^2}{2}+\frac{\sigma_{b1}b_1^{*2}}{2}$$

$$c_1=\min\{2(k_1-1)g_{10},\frac{\sigma_1}{\lambda_{\min}(\Gamma_1^{-1})},\frac{\sigma_{b1}}{\lambda_1}\}$$

Further, we have

$$\frac{d}{dt}(V_1(t)e^{c_1t}) \leqslant \frac{1}{4}e^{c_1t}z_2^2+\frac{g_1(x)}{|g_1(x)|}N(\varsigma_1)\dot{\varsigma}_1e^{c_1t}+\dot{\varsigma}_1e^{c_1t}+c_{\varepsilon1}e^{c_1t}+\Delta_1e^{c_1t} \tag{4.20}$$

Let $\rho_1 = c_{\varepsilon1}/c_1$, and integrating both the sides of the above inequality (4.20), it yields

$$\begin{aligned}V_1(t) &\leqslant \rho_1+[V_1(0)-\rho_1]e^{-c_1t}+e^{-c_1t}\int_0^t\frac{1}{4}e^{c_1t}z_2^2d\tau+\\&\quad e^{-c_1t}\int_0^t(\frac{g_1(x)}{|g_1(x)|}N(\varsigma_1)+1)e^{c_1t}\dot{\varsigma}_1d\tau+e^{-c_1t}\int_0^te^{c_1t}\Delta_1d\tau\\&\leqslant \rho_1+V_1(0)+e^{-c_1t}\int_0^t\frac{1}{4}e^{c_1t}z_2^2d\tau+\\&\quad e^{-c_1t}\int_0^t(\frac{g_1(x)}{|g_1(x)|}N(\varsigma_1)+1)e^{c_1t}\dot{\varsigma}_1d\tau+e^{-c_1t}\int_0^te^{c_1t}\Delta_1d\tau\end{aligned} \tag{4.21}$$

Obviously, if there are not $e^{-c_1t}\int_0^t\frac{1}{4}e^{c_1t}z_2^2d\tau$ and $e^{-c_1t}\int_0^te^{c_1t}\Delta_1d\tau$ in (4.21), then, from Lemmas 4.2 and 4.3, it can be obtained that $V_1(t),\varsigma_1,\hat{\theta}_1,\hat{b}_1$ are bounded in $[0,t_f)$. On the other hand, if it can be proved that $z_2(t)$ is bounded in $[0,t_f)$, from the following inequality

$$e^{-c_1t}\int_0^t\frac{1}{4}e^{c_1t}z_2^2d\tau \leqslant \frac{1}{4}e^{-c_1t}\sup_{\tau\in[0,t]}[z_2^2(\tau)]\int_0^te^{c_1t}d\tau \leqslant \frac{1}{4c_1}e^{-c_1t}\sup_{\tau\in[0,t]}[z_2^2(\tau)] \tag{4.22}$$

we can obtain that $e^{-c_1 t}\int_0^t \frac{1}{4}e^{c_1 t}z_2^2 d\tau$ is bounded. From Lemmas 2 and 3, we further obtain that $V_1(t)$, ς_1, $\hat{\theta}_1$, $\hat{b}_1$ also are bounded in $[0, t_f)$.

Furthermore, from [43], the same results can be obtained when $t_f = +\infty$.

Notice that, the boundedness of z_2 will be considered in the next step, and the error $e^{-c_1 t}\int_0^t e^{c_1 t}\Delta_1 d\tau$ will be compensated in Step n.

Remark 4.3 In [41], the error between $\omega - 1$ and α_0 is not considered in the stability analysis of the overall closed-loop system. Since there exists a difference between them, the effect of the error should be considered in the closed-loop system stability analysis. If not, the stability analysis is not complete.

Remark 4.4 It is valuable to point out, the signs of the control gain functions considered in this chapter are unknown as well as the control coefficients, which means that the system model is more general and the results obtained in this chapter thus have a great significance both on theory and on practical implication.

Step i $(i = 2, 3, \ldots, n-1)$:

In this step, consider the subsystem: $z_i = x_i - \alpha_{i-1}$. From (4.1) and (4.3), we have

$$\dot{z}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)z_{i+1} + g_i(\bar{x}_i)\alpha_i + d_1(\bar{x}_2, t) - \dot{\alpha}_{i-1} \tag{4.23}$$

Define the following Lyapunov function

$$V_{z_i} = \int_0^{z_i} \frac{\sigma}{|g_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})|} d\sigma \tag{4.24}$$

Similar to the analysis in the first step, it can be easily seen that V_{z_i} is a positive definite function of z_i. Since

$$\frac{\partial \left|g_i^{-1}(\bar{x}_{i-1}, \sigma + \alpha_{i-1})\right|}{\partial \alpha_{i-1}} = \frac{\partial \left|g_i^{-1}(\bar{x}_{i-1}, \sigma + \alpha_{i-1})\right|}{\partial \sigma} \tag{4.25}$$

and from the derivation rule of compound function, we have

$$\begin{aligned}
\dot{V}_{z_i} = & \frac{z_i}{|g_i(\bar{x}_i)|}\dot{z}_i + \\
& \int_0^{z_i} \sigma \left[\frac{\partial \left|g_i^{-1}(\bar{x}_{i-1}, \sigma + \alpha_{i-1})\right|}{\partial \bar{x}_{i-1}} \dot{\bar{x}}_{i-1} + \frac{\partial \left|g_i^{-1}(\bar{x}_{i-1}, \sigma + \alpha_{i-1})\right|}{\partial \alpha_{i-1}} \dot{\alpha}_{i-1} \right] d\sigma \\
= & \frac{z_i}{|g_i(\bar{x}_i)|}\dot{z}_i + \int_0^{z_i} \sigma \left[\frac{\partial \left|g_i^{-1}(\bar{x}_{i-1}, \sigma + \alpha_{i-1})\right|}{\partial \bar{x}_{i-1}} \dot{\bar{x}}_{i-1} d\sigma \right] + \\
& \dot{\alpha}_{i-1} \int_0^{z_i} \sigma \left[\frac{\partial \left|g_i^{-1}(\bar{x}_{i-1}, \sigma + \alpha_{i-1})\right|}{\partial \alpha_{i-1}} d\sigma \right] \\
= & \frac{z_i}{|g_i(\bar{x}_i)|}\dot{z}_i + \int_0^{z_i} \sigma \left[\frac{\partial \left|g_i^{-1}(\bar{x}_{i-1}, \sigma + \alpha_{i-1})\right|}{\partial \bar{x}_{i-1}} \dot{\bar{x}}_{i-1} d\sigma \right] +
\end{aligned}$$

$$
\begin{aligned}
&\dot{\alpha}_{i-1}\int_0^{z_i}\sigma\left[\frac{\partial\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\alpha_{i-1})\right|}{\partial\sigma}d\sigma\right]\\
&=\frac{z_i}{|g_i(\bar{x}_i)|}\dot{z}_i+\int_0^{z_i}\sigma\left[\frac{\partial\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\alpha_{i-1})\right|}{\partial\bar{x}_{i-1}}\dot{\bar{x}}_{i-1}d\sigma\right]+\\
&\frac{\dot{\alpha}_{i-1}z_i}{|g(x)|}+\dot{\alpha}_{i-1}\int_0^{z_i}\frac{1}{\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\alpha_{i-1})\right|}d\sigma
\end{aligned}
\tag{4.26}
$$

From the definition of the error between the command filter's state and virtual control, we know, $\alpha_{i-1}=\omega_i-v_i$. Replacing α_{i-1} in (4.26) by ω_i-v_i, from (4.1) and (4.26), we have

$$
\begin{aligned}
\dot{V}_{z_i}=&\frac{z_i}{|g_i(\bar{x}_i)|}(f_i(\bar{x}_i)+g_i(\bar{x}_i)z_{i+1}+g_i(\bar{x}_i)\alpha_i+d_1(\bar{x}_2,t)-\dot{\alpha}_{i-1})+\\
&\int_0^{z_i}\sigma\left[\frac{\partial\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\alpha_{i-1})\right|}{\partial\bar{x}_{i-1}}\dot{\bar{x}}_{i-1}d\sigma\right]+\frac{\dot{\alpha}_{i-1}z_i}{|g_i(\bar{x}_i)|}+\\
&\dot{\alpha}_{i-1}\int_0^{z_i}\frac{1}{\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\alpha_{i-1})\right|}d\sigma\\
=&\frac{z_i}{|g_i(\bar{x}_i)|}(g_i(\bar{x}_i)z_{i+1}+g_i(\bar{x}_i)\alpha_i+d_1(\bar{x}_2,t))+h_i(\bar{z}_i)z_i+\Delta_i
\end{aligned}
\tag{4.27}
$$

where $\bar{z}_i=(\bar{x}_i^T,\omega_i,\dot{\omega}_i)^{\mathrm{T}}\in\Omega_{\bar{z}_i}\subset R^{i+2}$,

$$
\begin{aligned}
h_i(\bar{z}_i)=&\frac{f_i(\bar{x}_i)}{|g_i(\bar{x}_i)|}+\frac{1}{z_i}\int_0^{z_i}\sigma\left[\frac{\partial\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\omega_i)\right|}{\partial\bar{x}_{i-1}}\dot{\bar{x}}_{i-1}d\sigma\right]+\\
&\frac{\dot{\omega}_i}{z_i}\int_0^{z_i}\frac{1}{\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\omega_i)\right|}d\sigma
\end{aligned}
\tag{4.28}
$$

$$
\begin{aligned}
\Delta_i=&\int_0^{z_i}\sigma\left[\frac{\partial\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\alpha_{i-1})\right|}{\partial\bar{x}_{i-1}}\dot{\bar{x}}_{i-1}d\sigma\right]+\\
&\dot{\alpha}_{i-1}\int_0^{z_i}\frac{1}{\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\alpha_{i-1})\right|}d\sigma-\\
&\frac{1}{z_i}\int_0^{z_i}\sigma\left[\frac{\partial\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\omega_i)\right|}{\partial\bar{x}_{i-1}}\dot{\bar{x}}_{i-1}d\sigma\right]-\frac{\dot{\omega}_i}{z_i}\int_0^{z_i}\frac{1}{\left|g_i^{-1}(\bar{x}_{i-1},\sigma+\omega_i)\right|}d\sigma
\end{aligned}
\tag{4.29}
$$

Note that, $h_i(\bar{z}_i)$ will be approximated by FLSs on a compact set Ω_{z_i} as: $h_i(z_i)=\theta_i^{*T}\xi_i(\bar{z}_i)+\varepsilon_i(\bar{z}_i)$. From Assumption 4.5, we know, there exists an unknown constant ε_i^* such that $|\varepsilon_i(\bar{z}_i)|\le\varepsilon_i^*$.

The following virtual control is designed as follows:

$$\alpha_i = N(\varsigma_i)[k_i z_i + h_i(\bar{z}_i, \hat{\theta}_i) + \hat{b}_i \bar{\varphi}(\bar{x}_i) \tanh(\frac{z_i \bar{\varphi}(\bar{x}_i)}{\eta_i})] \tag{4.30}$$

$$\dot{\varsigma}_i = k_i z_i^2 + h_i(\bar{z}_i, \hat{\theta}_i) z_i + \hat{b}_i \bar{\varphi}(\bar{x}_i) z_i \tanh(\frac{z_i \bar{\varphi}(\bar{x}_i)}{\eta_i})] \tag{4.31}$$

where $k_i > 1\frac{1}{4}$ is a design parameter; $h_i(\bar{z}_i, \hat{\theta}_i) = \hat{\theta}_i^T \xi_i(\bar{z}_i)$ is an estimate of $\theta_i^{*T} \xi_i(\bar{z}_i)$; $\hat{b}_i$ is an estimate of b_i^*, $b_i^* = \max\{\varepsilon_i^*, \frac{p_i^*}{g_{10}}\}$, $\bar{\varphi}_i(\bar{x}_i) = 1 + \varphi_i(\bar{x}_i)$.

Remark 4.5 It seems strange that k_i is set to be $k_i > 1\frac{1}{4}$. The purpose of "$\frac{1}{4}$" is to compensate for the term $\frac{1}{4} z_i^2$ which derived in the previous step.

Similar to (4.13), substituting (4.30) and (4.31) into (4.27) and re-arranging it, we have

$$\begin{aligned} \dot{V}_{z_i} \leqslant &-(k_1 - 1) z_i^2 + \frac{1}{4} z_{i+1}^2 + \frac{g_i(\bar{x}_i)}{|g_i(\bar{x}_i)|} z_i N(\varsigma_i) \dot{\varsigma}_i + \dot{\varsigma}_i - \tilde{\theta}_i \xi_i(\bar{z}_i) z_i + \\ & b_i^* [|z_i| \, \bar{\varphi}_i(\bar{x}_i) - z_i \bar{\varphi}_i(\bar{x}_i) \tanh(\frac{z_i \bar{\varphi}_i(\bar{x}_i)}{\eta_i})] - \tilde{b}_i \bar{\varphi}_i(x_i) z_i \tanh(\frac{z_i \bar{\varphi}_i(\bar{x}_i)}{\eta_i}) + \Delta_i \end{aligned} \tag{4.32}$$

where $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$ and $\tilde{b}_i = b_i^* - \hat{b}_i$.

Consider the following Lyapunov function

$$V_i(t) = V_{i-1} + V_{z_i} + \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{1}{2\lambda_i} \tilde{b}_i^2 \tag{4.33}$$

The following adaptive laws are designed as follows:

$$\dot{\hat{\theta}}_i = \Gamma_i [z_i \xi_i(\bar{z}_i) - \sigma_i \hat{\theta}_i] \tag{4.34}$$

$$\dot{\hat{b}}_i = \lambda_i [z_i \bar{\varphi}_i(\bar{x}_i) \tanh(\frac{z_i \bar{\varphi}_i(\bar{x}_i)}{\eta_i}) - \sigma_{bi} \hat{b}_i] \tag{4.35}$$

where Γ_i is a positive definite matrix, and $\eta_i > 0, \sigma_i > 0, \sigma_{bi} > 0$ and $\lambda_i > 0$ are design parameters.

Similar Step 1, differentiating V_i with respect to time t and considering (4.34) and (4.35), from Lemma 4.1, one has

$$\begin{aligned} \dot{V}_i \leqslant &\dot{V}_{i-1} - (k_i - 1\frac{1}{4}) z_i^2 + \frac{1}{4} z_{i+1}^2 + \frac{g_i(\bar{x}_i)}{|g_i(\bar{x}_i)|} z_i N(\varsigma_i) \dot{\varsigma}_i + \dot{\varsigma}_i + \\ & 0.2785 \eta_i b_i^* - \sigma_i \tilde{\theta}_i^T \hat{\theta}_i - \sigma_{bi} \tilde{b}_i \hat{b}_i + \Delta_i \end{aligned} \tag{4.36}$$

Since $\sigma_i \tilde{\theta}_i^{\mathrm{T}} \hat{\theta}_i \leqslant -\frac{\sigma_i \|\tilde{\theta}_i\|^2}{2} + \frac{\sigma_i \|\theta_i^*\|^2}{2}$ and $\sigma_{bi} \tilde{b}_i \hat{b}_i \leqslant -\frac{\sigma_{bi} \tilde{b}_i^2}{2} + \frac{\sigma_{bi} b_i^{*2}}{2}$, then let $c_{\varepsilon i} = (0.2785\eta_i b_i^* + \frac{\sigma_i \|\theta_i^*\|^2}{2} + \frac{\sigma_{bi} b_i^{*2}}{2}$, $c_i = \min\{2(k_i - 1\frac{1}{4}) g_{i0}, \frac{\sigma_i}{\lambda_{\min}(\Gamma_i^{-1})}, \frac{\sigma_{bi}}{\lambda_i}\}$ and considering (4.17), then (4.36) can be developed as follows:

$$\dot{V}_i \leqslant \sum_{j=1}^{i} (-c_j V_j + \frac{g_j(\bar{x}_j)}{|g_j(\bar{x}_j)|} z_j N(\varsigma_j) \dot{\varsigma}_j + \dot{\varsigma}_j + c_{\varepsilon j}) + \sum_{j=1}^{i} \Delta_j \tag{4.37}$$

Further, we have

$$\frac{d}{dt}(V_i(t) e^{c_i t}) \leqslant \frac{1}{4} e^{c_i t} z_{i+1}^2 + [\sum_{j=1}^{i} (\frac{g_j(\bar{x}_j)}{|g_j(\bar{x}_j)|} z_j N(\varsigma_j) \dot{\varsigma}_j + \dot{\varsigma}_j + c_{\varepsilon j})] e^{c_i t} + \sum_{j=1}^{i} \Delta_j e^{c_i t} \tag{4.38}$$

As doing in the first step, integrating both the sides of (4.38), we have

$$\begin{aligned} V_i(t) \leqslant & \rho_i + V_i(0) + e^{-c_i t} \int_0^t \frac{1}{4} e^{c_i t} z_{i+1}^2 d\tau + \\ & e^{-c_i t} \sum_{j=1}^{i} \int_0^t (\frac{g_j(\bar{x}_j)}{|g_j(\bar{x}_j)|} N(\varsigma_j) + 1) e^{c_i t} \dot{\varsigma}_j d\tau + e^{-c_i t} \sum_{j=1}^{i} \int_0^t e^{c_i t} \Delta_j d\tau \end{aligned} \tag{4.39}$$

where $\rho_i = \frac{\sum_{j=1}^{i} c_{\varepsilon j}}{c_i}$.

Similar to step 1, if z_{i+1} is proved to be bounded and $\sum_{j=1}^{i} \Delta_j = 0$, then, from Lemmas 4.2 and 4.3, one has, $e^{-c_i t} \int_0^t \frac{1}{4} e^{c_i t} z_{i+1}^2 d\tau$ is bounded, and $V_i(t)$, ς_i, $\hat{\theta}_i$, $\hat{b}_i$ further are bounded in $[0, +\infty)$.

Note that, the boundedness of z_{i+1} will be considered in the next step while $\sum_{j=1}^{i} \Delta_j = 0$ will be compensated in the last step.

Remark 4.6 From the aforementioned analysis, it is easily seen that virtual control laws α_i are continuous functions of variables $\bar{x}_i$, $\bar{z}_i$, ω_1, $\dot{\omega}_1$ and $\hat{\theta}_i$. Since these variables are available, the first derivative of α_i, i.e., $\dot{\alpha}_i$, can be obtained by analytical computation. However, just stated in Introduction section, as system dimension, i.e., n, increases, the computation of the higher derivatives of α_i becomes increasingly complicated. In this chapter, by using command filter (4.4), only its first derivative is utilized, which reduce such computation complexity.

Step n:

Now, consider z_n-subsystem: $z_n = x_n - \alpha_{n-1}$. Form (4.1)–(4.3), one has

$$\begin{aligned} \dot{z}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n) g_f(\bar{x}_n) u + g_n(\bar{x}_n) b_f(\bar{x}_n) - \dot{\alpha}_{n-1} \\ &= \bar{f}_n(\bar{x}_n) + \bar{g}_n(\bar{x}_n) u - \dot{\alpha}_{n-1} \end{aligned} \tag{4.40}$$

where $\bar{f}_n(\bar{x}_n) = f_n(\bar{x}_n) + g_n(\bar{x}_n) b_f(\bar{x}_n)$ and $\bar{g}_n(\bar{x}_n) = g_n(\bar{x}_n) g_f(\bar{x}_n)$.

Define the following Lyapunov function

$$V_{z_n} = \int_0^{z_n} \frac{\sigma}{|\bar{g}_n(\bar{x}_{n-1}, \sigma + \alpha_{n-1})|} d\sigma \tag{4.41}$$

From the analysis in the previous step, V_{z_n} is a positive definite function of z_n.

Similar to the previous steps, differentiating V_{z_n} with respect to time t, one has

$$\dot{V}_{z_n} \leqslant \frac{z_n}{|\bar{g}_n(\bar{x}_n)|}(\bar{g}_n(\bar{x}_n)u + d_n(\bar{x}_n, t)) + h_n'(\bar{z}_n)z_n + \Delta_n \tag{4.42}$$

where

$$\begin{aligned} h_n'(\bar{z}_n) =& \frac{\bar{f}_n(\bar{x}_n)}{|\bar{g}_n(\bar{x}_n)|} + \frac{1}{z_n}\int_0^{z_n} \sigma \left[\frac{\partial \left|\bar{g}_n^{-1}(\bar{x}_n, \sigma + \omega_n)\right|}{\partial \bar{x}_n} \dot{\bar{x}}_n d\sigma \right] + \\ & \frac{\dot{\omega}_n}{z_n} \int_0^{z_n} \frac{1}{\left|\bar{g}_n^{-1}(\bar{x}_n, \sigma + \omega_n)\right|} d\sigma \end{aligned} \tag{4.43}$$

$$\begin{aligned} \Delta_n =& \int_0^{z_n} \sigma \left[\frac{\partial \left|\bar{g}_n^{-1}(\bar{x}_n, \sigma + \alpha_{n-1})\right|}{\partial \bar{x}_n} \dot{\bar{x}}_n d\sigma \right] + \dot{\alpha}_{n-1} \int_0^{z_i} \frac{1}{\left|\bar{g}_n^{-1}(\bar{x}_{n-1}, \sigma + \alpha_{n-1})\right|} d\sigma - \\ & \frac{1}{z_n}\int_0^{z_i} \sigma \left[\frac{\partial \left|\bar{g}_n^{-1}(\bar{x}_n, \sigma + \omega_n)\right|}{\partial \bar{x}_n} \dot{\bar{x}}_n d\sigma \right] - \frac{\dot{\omega}_n}{z_n}\int_0^{z_n} \frac{1}{\left|\bar{g}_n^{-1}(\bar{x}_n, \sigma + \omega_n)\right|} d\sigma \end{aligned} \tag{4.44}$$

Adding and subtracting $\sum_{j=1}^{n-1} \Delta_j$ in the right side of (4.42), we have

$$\dot{V}_{z_n} \leqslant \frac{z_n \bar{g}_n(\bar{x}_n)}{|\bar{g}_n(\bar{x}_n)|} u + |z_n| \rho^* + \frac{|z_n|}{g_{n0}} p_n^* \varphi_n(x_n) + h_n'(\bar{z}_n)z_n + \sum_{j=1}^{n} \Delta_j - \sum_{j=1}^{n-1} \Delta_j \tag{4.45}$$

Remark 4.7 The purpose of "adding and subtracting $\sum_{j=1}^{n-1} \Delta_j$" is to remove the error terms $\sum_{j=1}^{n-1} \Delta_j$ (4.37), which is introduced by command filter (4.4) in the previous $n-1$ steps.

It is easily seen that $\Delta_j (j = 1, \ldots, n)$ is a function of variables $\bar{x}_j$, $\bar{z}_j$, $\bar{\alpha}_j$, $\dot{\bar{\alpha}}_j$, $\bar{\omega}_j$ and $\dot{\bar{\omega}}_j$, where $\bar{x}_j = (x_1, \ldots, x_j)^T$, $\bar{z}_j = (z_1, \ldots, z_j)^T$, $\bar{\alpha}_j = (\alpha_0, \ldots, \alpha_{j-1})^T$, $\dot{\bar{\alpha}}_j = (\dot{\alpha}_0, \ldots, \dot{\alpha}_{j-1})^T$, $\bar{\omega}_j = (\omega_1, \ldots, \omega_j)^T$, $\dot{\bar{\omega}}_j = (\dot{\omega}_1, \ldots, \dot{\omega}_j)^T$. Let

$$h(\bar{Z}_n) = h'(\bar{Z}_n) + \sum_{j=1}^{n-1} \Delta_j$$

where $\bar{Z}_n = (\bar{x}_n^T, \bar{z}_n^T, \bar{\alpha}_n^T, \dot{\bar{\alpha}}_n^T, \bar{\omega}_n^T, \dot{\bar{\omega}}_n^T)^T$.

From the previous analysis, it is seen that $h'(\bar{Z}_n)$ and Δ_j are smooth, which means that $h(\bar{Z}_n)$ also is smooth. Hence, FLSs can be utilized to approximate it in the form: $h(\bar{Z}_n) = \theta_n^{*T}\xi_n(\bar{Z}_n) + \varepsilon_n(\bar{Z}_n)$. From Assumption 5, we know, there exists an unknown constant ε_n^* such that $|\varepsilon_n(\bar{Z}_n)| \leq \varepsilon_n^*$.

The actual control is defined as follows:

$$u = N(\varsigma_n)[k_n z_n + h_n(\bar{Z}_n, \hat{\theta}_n) + \hat{b}_n\bar{\varphi}(\bar{x}_n)\tanh(\frac{z_n\bar{\varphi}(\bar{x}_n)}{\eta_n})] \tag{4.46}$$

$$\dot{\varsigma}_n = k_n z_n^2 + h_n(\bar{Z}_n, \hat{\theta}_n)z_n + \hat{b}_n\bar{\varphi}(\bar{x}_n)z_n\tanh(\frac{z_n\bar{\varphi}(\bar{x}_n)}{\eta_n})] \tag{4.47}$$

where $k_n > \frac{1}{4}$ is a design parameter; $h_n(\bar{Z}_n, \hat{\theta}_n) = \hat{\theta}_n^T\xi_n(\bar{Z}_n)$ is an estimate of $\theta_n^{*T}\xi_n(\bar{Z}_n)$; $\hat{b}_n$ is an estimate of $b_n^* = \max\{\varepsilon_n^*, \frac{p_n^*}{g_{10}}\}$; $\bar{\varphi}_n(\bar{x}_n) = 1 + \varphi_n(\bar{x}_n)$.

Substituting (4.46) and (4.47) into (4.45), it yields

$$\dot{V}_{z_n} \leqslant -k_n z_n^2 + \frac{\bar{g}_n(\bar{x}_n)}{|\bar{g}_n(\bar{x}_n)|}z_n N(\varsigma_n)\dot{\varsigma}_n + \dot{\varsigma}_n - \tilde{\theta}_n\xi_n(\bar{z}_n)z_n - \sum_{j=1}^{n-1}\Delta_j +$$
$$b_n^*[|z_n|\,\bar{\varphi}_n(\bar{x}_n) - z_n\bar{\varphi}_n(\bar{x}_n)\tanh(\frac{z_n\bar{\varphi}_n(\bar{x}_n)}{\eta_n})] - \tilde{b}_n\bar{\varphi}_n(x_n)z_n\tanh(\frac{z_n\bar{\varphi}_n(\bar{x}_n)}{\eta_n}) \tag{4.48}$$

where $\tilde{\theta}_n = \theta_n^* - \hat{\theta}_n$ and $\tilde{b}_n = b_n^* - \hat{b}_n$.

Define the following Lyapunov function

$$V_n(t) = V_{n-1} + V_{z_n} + \frac{1}{2}\tilde{\theta}_n^T\Gamma_n^{-1}\tilde{\theta}_n + \frac{1}{2\lambda_n}\tilde{b}_n^2 \tag{4.49}$$

The following adaptive laws are defined as:

$$\dot{\hat{\theta}}_n = \Gamma_n[z_n\xi_n(\bar{Z}_n) - \sigma_n\hat{\theta}_n] \tag{4.50}$$

$$\dot{\hat{b}}_n = \lambda_n[z_n\bar{\varphi}_n(\bar{x}_n)\tanh(\frac{z_n\bar{\varphi}_n(\bar{x}_n)}{\eta_n}) - \sigma_{bn}\hat{b}_n] \tag{4.51}$$

where Γ_n is a positive definite matrix, $\eta_n > 0, \sigma_n > 0, \sigma_{bn} > 0$ and $\lambda_n > 0$ are design parameters.

Differentiating V_n with respect to time t and considering (4.50), (4.51) and Lemma 4.1, similar to the previous steps, one has

$$\dot{V}_n \leqslant \dot{V}_{n-1} - k_n z_n^2 + \frac{\bar{g}_n(\bar{x}_n)}{|\bar{g}_n(\bar{x}_n)|}N(\varsigma_n)\dot{\varsigma}_n + \dot{\varsigma}_n + 0.2785\eta_n b_n^* - \sigma_n\tilde{\theta}_n^T\hat{\theta}_n - \sigma_{bn}\tilde{b}_n\hat{b}_n \tag{4.52}$$

From Young's inequality, we have

$$\sigma_n \tilde{\theta}_n^{\mathrm{T}} \hat{\theta}_n \leqslant -\frac{\sigma_n \left\| \tilde{\theta}_n \right\|^2}{2} + \frac{\sigma_n \left\| \theta_n^* \right\|^2}{2}, \sigma_{bn} \tilde{b}_n \hat{b}_n \leqslant -\frac{\sigma_{bn} \tilde{b}_n^2}{2} + \frac{\sigma_{bn} b_n^{*2}}{2} \tag{4.53}$$

Let $c_{\varepsilon n} = 0.2785 \eta_n b_n^* + \frac{\sigma_n \|\theta_n^*\|^2}{2} + \frac{\sigma_{bn} b_n^{*2}}{2}$, then (4.52) can be derived as

$$\begin{aligned} \dot{V}_n \leqslant & \dot{V}_{n-1} - 2k_n \left| \bar{g}_n(\bar{x}_n) \right| V_n + \frac{\bar{g}_n(\bar{x}_n)}{\left| \bar{g}_n(\bar{x}_n) \right|} m v(t) N(\varsigma_n) \dot{\varsigma}_n + \dot{\varsigma}_n + \\ & c_{\varepsilon n} - \frac{\sigma_n \left\| \tilde{\theta}_n \right\|^2}{2} - \frac{\sigma_{bn} \left\| \tilde{b}_n \right\|^2}{2} \end{aligned} \tag{4.54}$$

Let

$$c_n = \min\{2k_n \bar{g}_{n0}, \frac{\sigma_n}{\lambda_{\min}(\Gamma_n^{-1})}, \frac{\sigma_{bn}}{\lambda_n}\}$$

from the analysis in the previous steps, then (4.54) can be further developed as follows:

$$\dot{V}_n \leqslant \sum_{i=1}^{n} [\frac{\bar{g}_i(\bar{x}_i)}{\left| \bar{g}_i(\bar{x}_i) \right|} N(\varsigma_i) \dot{\varsigma}_i + \dot{\varsigma}_i + c_{\varepsilon i}] \tag{4.55}$$

Further, we have

$$\frac{d}{dt}(V_n(t) e^{c_n t}) \leqslant e^{c_n t} \sum_{i=1}^{n} [\frac{\bar{g}_i(\bar{x}_i)}{\left| \bar{g}_i(\bar{x}_i) \right|} N(\varsigma_i) \dot{\varsigma}_i + \dot{\varsigma}_i + c_{\varepsilon i}] \tag{4.56}$$

where $\bar{g}_i(\cdot) = g_i(\cdot), i = 1, \ldots, n-1$.

Let $\rho_n = \frac{\sum_{j=1}^{n} c_{\varepsilon j}}{c_n}$. Similar to the previous steps, integrating both the sides of the above inequality, we have

$$\begin{aligned} V_n(t) \leqslant & \rho_n + [V_n(0) - \rho_n] e^{-c_n t} + e^{-c_n t} \int_0^t [e^{c_n t} \sum\nolimits_{i=1}^{n} (\frac{\bar{g}_i(\bar{x}_i)}{\left| \bar{g}_i(\bar{x}_i) \right|} N(\varsigma_i) + 1) \dot{\varsigma}_i] d\tau \\ \leqslant & \rho_n + V_n(0) + e^{-c_n t} \int_0^t [e^{c_n t} \sum\nolimits_{i=1}^{n} (\frac{\bar{g}_i(\bar{x}_i)}{\left| \bar{g}_i(\bar{x}_i) \right|} N(\varsigma_i) + 1) \dot{\varsigma}_i] d\tau \end{aligned} \tag{4.57}$$

From Lemmas 4.2 and 4.3, it is easily seen that $V_n(t), \varsigma_n, \hat{\theta}_n, \hat{b}_n$ are bounded in $[0, t_f)$. From [43], the same results can be obtained in $[0, +\infty)$. Thus, it can be obtained that z_n is bounded in $[0, +\infty)$, which means that z_{n-1} in $(n-1)$th step is bounded. Doing the same reasoning, we finally obtained that all $z_i(t), i = 1, 2, \ldots n$ are bounded.

From the definitions of V_{z_i} and V_i, $i = 1, \ldots, n$, we known

$$V_n(t) = \sum\nolimits_{i=1}^{n} [V_{z_i} + \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{1}{2\lambda_i} \tilde{b}_i^2] \tag{4.58}$$

From the previous analysis, we have

$$\frac{z_i^2}{2g_{i1}} \le V_{z_i} = \int_0^{z_i} \frac{\sigma}{|g_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})|} d\sigma \le \frac{z_i^2}{2g_{i0}} \tag{4.59}$$

Hence, from (4.57–4.59), we have

$$|\bar{z}_i| \leqslant \sqrt{\mu}, \quad \|\theta_i\|^2 \leqslant \frac{\mu}{\lambda_{\min}(\Gamma_i^{-1})}, \quad b_i^2 \leqslant \lambda_i \mu^2, \quad i = 1, 2, \ldots, n, \quad \forall t \geqslant 0$$

where $\mu = 2\bar{g}_{\max}(\rho_n + V_n(0) + N_n)$, $\tilde{g}_{\max} = \max\limits_{1 \leqslant i \leqslant n} \bar{g}_{i1} > 0$, $\bar{g}_{i1} = g_{i1}$, $i = 1, \ldots, n-1$, $\bar{g}_{n1} = g_{n1} g_{f1}$,

$$N_n = \lim_{t \to +\infty} \sum\nolimits_{i=1}^{n} \left[e^{-c_n t} \int_0^t \left(\frac{\bar{g}_i(\bar{x}_i)}{|\bar{g}_i(\bar{x}_i)|} N(\varsigma_i) + 1 \right) e^{c_n t} \dot{\varsigma}_n d\tau \right] \tag{4.60}$$

The above design procedures and analysis are summarized in the following theorem.

Theorem 4.1 *Consider system (4.1) and fault (4.2). If Assumptions 4.1–4.5 hold, command filters (4.4), actual control defined by (4.46) and (4.47), and the adaptation laws (4.15), (4.16), (4.34), (4.35), (4.50) and (4.51) are employed, then the closed-loop system is asymptotically bounded with the tracking error converging to a neighborhood of the origin.*

Proof From the aforementioned analysis, it is easy to obtain the conclusion. The detailed proof is omitted here.

4.4 Illustrative Example

In this example, a class of nonlinear systems are described as follows:

$$\begin{cases} \dot{x}_1 = x_1 + (1 + 0.5\sin(x_1^2))x_2 + 0.2x_1 \sin(x_2 t) \\ \dot{x}_2 = x_1 x_2 + (3 - \cos(x_1 x_2))u + 0.1\cos(0.5 x_2 t) \\ y = x_1 \end{cases} \tag{4.61}$$

From (4.61), it is easily seen that, $g_{10} = 0.5$, $g_{11} = 1.5$, $g_{20} = 2$, $g_{21} = 4$, $p_1^* = 0.2$, $\varphi_1 = x_1$, $p_2^* = 0.1$ and $\varphi_2 = 1$, which means that Assumptions 4.1 and 4.2 hold. In this work, the desired trajectory $y_d = 0.1\sin(t)$. Obviously, Assumption 4.3 holds. The actuator fault considered in this simulation research is described as follows:

$$u^f = (1 - 0.5\sin(x_2))u + \cos(x_1 x_2)$$

Obviously, $g_{f0} = 0.5$ and $g_{f1} = 1.5$, which means that Assumption 4.4 holds.

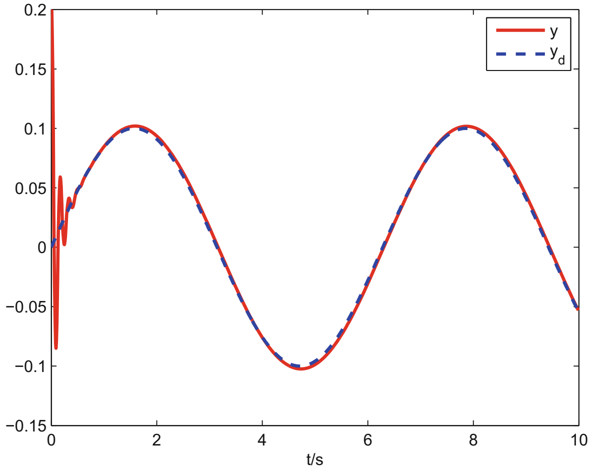

Fig. 4.1 The time profiles of system output y and desired signal y_d

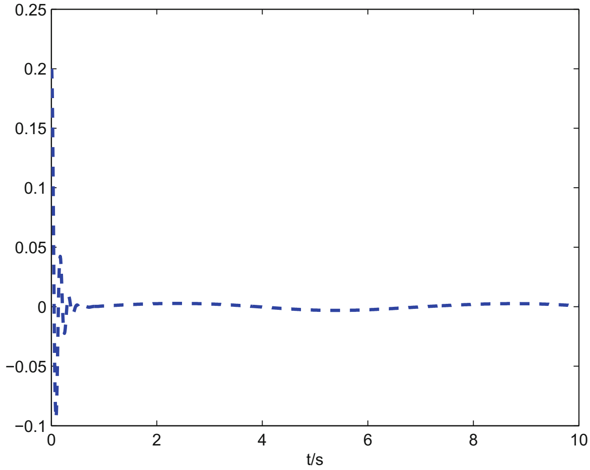

Fig. 4.2 The time profiles of tracking error

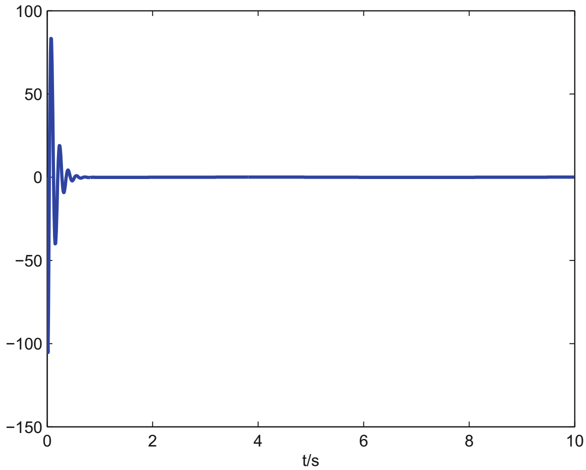

Fig. 4.3 The time profiles of control input signal

The control objective is to construct an adaptive state feedback controller for nonlinear system (4.61) such that the system output y tracks the desired reference signal y_d with all the signals in the resulting closed-loop system being asymptotically bounded.

For this work, the following parameters are given as follows: $k_1 = k_2 = 3$, $\Gamma_1 = \Gamma_2 = diag 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, \lambda_1 = \lambda_2 = 1, \eta_1 = \eta_2 = 0.01, \sigma_{b1} = \sigma_{b1} = 0.1$, $\theta_i \in R^{10}$, $i = 1, 2$ are taken randomly in interval (0,1]. Initial state $x(0)$ is set as $(0.2, 0.1)^T$. The sample time is 0.08s.

Simulation results are shown in Figs. 4.1, 4.2 and 4.3. From Fig. 4.1, we can find that system (4.61) has good tracking performance. Figure 4.2 shows that the tracking error converges to a neighborhood of the origin. Meanwhile, the boundedness of control input signal is shown in Fig. 4.3.

4.5 Conclusions

In this chapter, an adaptive fuzzy tracking fault-tolerant control problem of a class of uncertain strict-feedback nonlinear systems with actuator fault has been investigated. FLSs are used to approximate the unknown nonlinear functions. By applying adaptive command filtered backstepping recursive design, integral-type Lyapunov function method and Nussbaum-type gain technique, an adaptive fuzzy control scheme is proposed to guarantee that the closed-loop system is asymptotically bounded with the tracking error converging to a neighborhood of the origin.

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Chapter 5
Adaptive Fuzzy Fault-Tolerant DSC for a Class of Nonlinear Systems

5.1 Introduction

For strict-feedback systems, backstepping technique is commonly used to solve tracking or regulation control problem, and various adaptive backstepping control approaches have been developed for controlling uncertain nonlinear systems [1–13]. As stated in the Chap. 4, there exists a so-called computation complexity problem in convenient backstpping design procedures. Especially, the increasing of the system dimension produces a complexity explosion in traditional backstepping design methods. In order to overcome this problem, an original DSC scheme was proposed in [14–18], where the complexity was reduced by introducing the first-order filter in each step of the backstepping design. However, if actuator faults occur, then the control schemes in [14–18] do not guarantee the closed-loop system stability or correct tracking performances. One motivation of our work is thus to provide an active fault-tolerant control scheme which guarantees the closed-loop system stability and maintains satisfactory control performances in all situations. Another motivation is also to provide a control scheme that is applicable in practical applications where both the values and signs of control gain are not known. In addition, investigating both actuator time-varying bias and gain faults motivates this chapter.

In this chapter, we investigate the problem of tracking control for a class of nonlinear uncertain systems with complete unknown control gains and propose an active FTC against actuator faults. Compared with existing works, the following main contributions are worth to be emphasized. (1) The proposed FTC scheme considers both gain and bias faults simultaneously and does not need the conditions that the bounds of the varying faults and their time derivatives are known constants, which thus enlarges the practical application range of the method. (2) The proposed adaptive fault accommodation algorithm does not need the classical assumption that the time derivative of the output errors must be known. (3) A decision threshold for FDI is defined and applied on an online computable fault indicator and not on an asymptotic value of a criterion which is not available in practice. The decision algorithm is thus

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more practical. (4) In general, the denominator of the fault tolerant control input contains the estimate of the gain fault. If the denominator is equal to zero, a controller singularity occurs. In the proposed FTC scheme, the controller singularity is avoided. (5) The proposed active FTC scheme does not require the a priori knowledge of this sign.

The rest of this chapter is organized as follows. In Sect. 5.2, the problem formulation, Nussbaum-type function and mathematical description of FLS are introduced. Actuator faults are integrated in such problem and the FTC objective is formulated. In Sect. 5.3, the main technical results of this chapter are given, which include fault detection, isolation, estimation and fault-tolerant control scheme. An aircraft control application is presented in Sect. 5.4. These simulation results demonstrate the effectiveness of the proposed technique. Finally, Sect. 5.5 draws the conclusion.

5.2 Problem Statement and Mathematical Description of FLS

5.2.1 Problem Statement

Consider the following nonlinear system

$$\begin{cases} \dot{\bar{x}} = \bar{f}_0(\bar{x}) + \sum_{i=1}^{l} \theta_i \bar{f}_i(\bar{x}) + \sum_{j=1}^{m} \mu_j \bar{g}_j(\bar{x}) u_j \\ \bar{y} = h(\bar{x}) \end{cases} \tag{5.1}$$

where $\bar{x} \in R^n$ is the state, $\bar{y} \in R$ is the output, and $u_j \in R, j = 1, 2, \ldots, m$ are the plant control signals, $\bar{f}_i(\cdot) \in R^n$, $i = 0, 1, \ldots, l$, $\bar{g}_j(\cdot) \in R^n, j = 1, \ldots, m$ and $h(\cdot) \in R$ are smooth functions, $\theta_i \in R$, $i = 1, \ldots, l$ and $\mu_j \neq 0, j = 1, \ldots, m$ are unknown constants.

Control objective is to design adaptive controllers for system (1) to guarantee boundedness of the closed-loop signals and asymptotic tracking of a given reference output signal $y_d \in R$ by $\bar{y}$, $y_d \in \prod\{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le B_0\}$, $B_0 > 0 \in R$ denotes a known constant.

Actuator fault model considered can be described as follows:

$$u_j^f = \rho_j(\bar{x}) u_j + f_j^u(\bar{x}),\ t \ge t_j,\ j = 1, \ldots, m \tag{5.2}$$

where unknown functions $\rho_j(\bar{x}) \in [0, 1]$ and $f_j^u(\bar{x})$ denote the remaining control rate and a bounded signal, respectively, t_j is unknown fault occurrence time. Denote $u = [u_1, \ldots, u_m]^T$, $u^f = [u_1^f, \ldots, u_m^f]^T$, $\rho^u(\bar{x}) = diag(\rho_1(\bar{x}), \ldots, \rho_m(\bar{x}))$, $F^u(x) = [f_1^u(\bar{x}), \ldots, f_m^u(\bar{x})]^T$, then, on has

$$u^f = \rho^u(\bar{x}) u + F^u(\bar{x}) \tag{5.3}$$

Now, the control objective is re-defined as follows: An active FTC approach is proposed to obtain the above tracking objective in healthy and faulty conditions. Under healthy condition, control input u is designed, such that the system output $\bar{y}$ can track asymptotically the reference signal y_d. Meanwhile, the FDI algorithm is working. As soon as an actuator fault is detected and isolated, the fault accommodation algorithm is activated and a proper fault-tolerant control input u is used such that the tracking performance is still maintained stable.

Assumption 5.1 [19] $\bar{g}_j(\bar{x}) \in span\{\bar{g}_0(\bar{x})\}$, $\bar{g}_0(\bar{x}) \in R^n$, for $j = 1, \ldots, m$, and the nominal system $\dot{\bar{x}} = \bar{f}_0(\bar{x}) + \bar{F}(\bar{x})\theta + \bar{g}_0(\bar{x})u_0, \bar{y} = h(\bar{x})$ with $u_0 \in R$, is transformable into the parametric-strict-feedback (PSF) form with relative degree ρ, where $\bar{F}(\bar{x}) = [\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_l]^T$, $\theta = [\theta_1, \theta_2, \ldots, \theta_l]^T$.

As presented in [19], based on Assumption 5.1, there exists a diffeomorphism $Tr : [x^T, \eta^T]^T = Tr(\bar{x}), x \in R^\rho$ and $\eta \in R^\gamma$, $\rho + \gamma = n$ such that system (1) can be transformed into the following form.

$$\begin{cases} \dot{x}_i = x_{i+1} + \varphi_1(x_{[i]})\theta, \ i = 1, 2, \ldots, \rho - 1 \\ \dot{x}_\rho = \varphi_0(x, \eta) + \varphi_\rho^T(x, \eta)\theta + \beta^T(x, \eta)\mu u \\ \dot{\eta} = \psi_1(x, \eta) + \psi_2(x, \eta)\theta \\ y = x_1 \end{cases} \tag{5.4}$$

where $x_{[i]} = [x_1, x_2, \ldots, x_i]^T, x = [x_1, x_2, \ldots, x_\rho]^T$ and $u = [u_1, \ldots, u_m]^T$ denote the measurable state vector and input, $\mu = diag(\mu_1, \ldots, \mu_m)$, $\mu_j, j = 1, \ldots, m$ denote unknown constants, $\beta^T = [\beta_1, \ldots, \beta_m]^T$, $\beta_i = L_{\bar{g}_i(\bar{x})} L^{\rho-1}_{\bar{f}_0(\bar{x})} h(\bar{x}), i = 1, \ldots, m$, where $L_f p$ is the Lie derivative of a scalar function $p(x)$ along the vector field $f(x) = [f_1(x), \ldots, f_n(x)]^T$, defined as $L_f p = \sum_{i=1}^n (\frac{\partial p}{\partial x_1}) f_i$,

$$\varphi_i = [L_{f_1(\bar{x})} L^{i-1}_{f_0(\bar{x})} h(\bar{x}), \ldots, L_{f_i(\bar{x})} L^{i-1}_{f_0(\bar{x})} h(\bar{x})]^T, \ i = 1, \ldots, \rho - 1,$$

$$\varphi_\rho = [L_{f_1(\bar{x})} L^{\rho-1}_{f_0(\bar{x})} h(\bar{x}), \ldots, L_{f_i(\bar{x})} L^{\rho-1}_{f_0(\bar{x})} h(\bar{x})]^T,$$

$$\varphi_0 = L^\rho_{f_0(\bar{x})} h(\bar{x}), \ \psi_1 = \frac{\partial T_z}{\partial x} f_0(\bar{x}), \psi_2 = \frac{\partial T_z}{\partial x} \bar{F}(\bar{x}).$$

In order to solve the actuator failure compensation problem, the following assumptions are needed:

Assumption 5.2 [19] The zero dynamics $\dot{\eta} = \psi_1(x, \eta) + \psi_2(x, \eta)\theta$ is input-to-state stable with respect to x as the input, η is measurable and $\beta_j(x, \eta) \neq 0, j = 1, \ldots, m$.

Assumption 5.3 The signs of $\mu_j, j = 1, \ldots, m$ are unknown.

Let $f_i(x_{[i]}) = \varphi_i(x_{[i]})\theta$, $f_\rho(x,\eta) = \varphi_0(x,\eta) + \varphi_\rho^T(x,\eta)\theta$ $g^T(x,\eta) = \beta^T(x,\eta)$ $\mu = [g_1(x,\eta), \ldots, g_m(x,\eta)]^T$, then (5.4) can be rewritten as

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x_{[i]}),\ i = 1, \ldots, \rho - 1 \\ \dot{x}_\rho = f_\rho(x,\eta) + g^T(x,\eta)u \\ \dot{\eta} = \psi_1(x,\eta) + \psi_2(x,\eta)\theta \\ y = x_1 \end{cases}$$

Further, one has

$$\begin{cases} \dot{x} = Ax + Hy + f + Bg^T(x,\eta)u \\ \dot{\eta} = \psi_1(x,\eta) + \psi_2(x,\eta)\theta \\ y = x_1 \end{cases} \tag{5.5}$$

where $A = \begin{bmatrix} -h_1 & & \\ \vdots & I_{\rho-1} & \\ -h_\rho & 0 \ \ldots & 0 \end{bmatrix}$, $H = \begin{bmatrix} h_1 \\ \vdots \\ h_\rho \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$, $f = [f_1, \ldots, f_\rho]^T$, $h_i \in R$, $i = 1, \ldots, \rho$ are chosen such that A is a strict Hurwitz matrix.

Considering fault model (5.3), the faulty system can be described as

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x_{[i]}),\ i = 1, 2, \ldots, \rho - 1 \\ \dot{x}_\rho = f_\rho(x,\eta) + g^T(x,\eta)\rho^u(x)u(t) + g^T(x,\eta)F^u(x) \\ \dot{\eta} = \psi_1(x,\eta) + \psi_2(x,\eta)\theta \\ y = x_1 \end{cases} \tag{5.6}$$

5.2.2 Nussbaum Type Gain

Any continuous function $N(s) : R \to R$ is a function of Nussbaum type if it has the following properties:

(1) $\lim\limits_{s\to+\infty} \sup \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = +\infty$,
(2) $\lim\limits_{s\to-\infty} \inf \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = -\infty$

For example, the continuous functions $\varsigma^2\cos(\varsigma)$, $\varsigma^2\sin(\varsigma)$, and $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ verify the above properties and are thus Nussbaum-type functions [20]. The even Nussbaum function $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ is used throughout this chapter.

Lemma 5.1 [21, 22] *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:*

$$V(t) \leq c_0 + \int_0^t (\underline{g}N(\varsigma) + 1)\dot{\varsigma}d\tau, \forall t \in [0, t_f)$$

where $\underline{g} \neq 0$ is a constant, and c_0 represents a suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0, t_f)$.

Lemma 5.2 [22] *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. For $\forall t \in [0, t_f)$, if the following inequality holds,*

$$V(t) \leq c_0 + e^{-c_1 t} \int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}e^{c_1\tau}d\tau + e^{-c_1 t} \int_0^t \dot{\varsigma}e^{c_1\tau}d\tau$$

where constant $c_1 > 0$, $\underline{g}(\cdot)$ is a time-varying parameter which takes values in the unknown closed intervals $I := [l^{-1}, l^{+1}]$ with $0 \notin I$, and c_0 represents some suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0, t_f)$.

5.2.3 Mathematical Description of Fuzzy Logic Systems

A FLS consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

$$R^l: \text{ if } x_1 \text{ is } A_1^l \ldots \text{ and } x_n \text{ is } A_n^l, \text{ then } y \text{ is } B^l$$

where A_i^l, $i = 1, 2, \ldots, n$, $l = 1, 2, \ldots, M$ are fuzzy sets and B^l is the fuzzy singleton for the output in the lth rule, M is the rules number. Through singleton fuzzifier, center average defuzzification and product inference [23], the FLS output can be expressed as

$$y(x) = \sum_{l=1}^{M} y^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right) / \sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)$$

where $\mu_{A_i^l}(x_i)$ is the membership function of the fuzzy set A_j^l.

Define the fuzzy basis functions as

$$\xi^l(x) = \left[\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right] / \sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)$$

Define $\theta^T = [y^1, y^2, \ldots, y^M] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi = [\xi^1, \xi^2, \ldots, \xi^M]^T$, then the FLS output can be rewritten as

$$y(x) = \theta^T \xi(x) \tag{5.7}$$

The stability results obtained in FLS control literature are semi-global in the sense that, as long as the input variables x of the FLS remain within some pre-fixed compact set Ω, where the compact set can be made as large as desired, there exist controllers with sufficiently large number of FLS rules such that all the signals in the closed-loop remain bounded.

Lemma 5.3 [24, 25] *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists a FLS (5.7) such as* $\sup_{x\in\Omega} |f(x) - \theta^T\xi(x)| \le \varepsilon$.

By Lemma 5.3, the FLS (5.7) can approximate any smooth function on a compact set to any degree of accuracy. Similar to [7], by the FLS (5.7), $f_i(x)$, $i = 1, \ldots, \rho - 1, f_\rho(x)$, $g_j(x, \eta)$, $\rho_{gj}(x, \eta) = g_j(x, \eta)\rho_j(x)$ and $\rho_{fj}(x, \eta) = g_j(x, \eta) f_j^u(x)$, $j = 1, \ldots, m$ are approximated as:

$$\hat{f}_i(\hat{x}, \hat{\theta}_{fi}) = \hat{\theta}_{fi}^T \xi_{fi}(\hat{x}_{[i]}), \hat{f}_\rho(\hat{x}, \hat{\theta}_{f\rho}) = \hat{\theta}_{f\rho}^T \xi_{f\rho}(\hat{x}, \eta),$$

$$\hat{\rho}_{fj}(\hat{x}, \eta, \hat{\theta}_{f\rho j}^T) = \hat{\theta}_{f\rho j}^T \xi_{f\rho j}(\hat{x}, \eta),$$

$$\hat{g}_j(\hat{x}, \eta) = \hat{\theta}_{gj}^T \xi_{gj}(\hat{x}, \eta), \hat{\rho}_{gj}(\hat{x}, \eta, \hat{\theta}_{g\rho j}^T) = \hat{\theta}_{g\rho j}^T \xi_{g\rho j}(\hat{x}, \eta),$$

where $\hat{x}$, $\hat{\theta}_{fi}$, $\hat{\theta}_{gj}$, $\hat{\theta}_{f\rho j}$, $\hat{\theta}_{g\rho j}$ are the estimates of x, θ_{fi}^*, θ_{gj}^*, $\theta_{f\rho j}^*$, $\theta_{g\rho j}^*$, respectively. Let us define the optimal parameter vectors θ_{fi}^*, $i = 1, \ldots, \rho, \theta_{gj}^*, \theta_{f\rho j}^*$ and $\theta_{g\rho j}^*, j = 1, \ldots, m$ as

$$\theta_{fi}^* = \arg \min_{\theta_{fi}\in\Omega_f} [\sup_{x\in U, \hat{x}\in\hat{U}} |f_i(x_{[i]}) - \hat{f}_i(\hat{x}_{[i]}, \hat{\theta}_{fi}^T)|]$$

$$\theta_{f\rho}^* = \arg \min_{\theta_{f\rho}\in\Omega_f} [\sup_{x\in U, \hat{x}\in\hat{U}} |f_\rho(x) - \hat{f}(\hat{x}, \hat{\theta}_{f\rho}^T)|]$$

$$\theta_{gj}^* = \arg \min_{\theta_{gj}\in\Omega_g} [\sup_{x\in U, \hat{x}\in\hat{U}} |g_j(x, \eta) - \hat{g}_{gj}(\hat{x}, \eta, \hat{\theta}_{gj}^T)|],$$

$$\theta_{f\rho j}^* = \arg \min_{\theta_{f\rho j}\in\Omega_{f\rho}} [\sup_{x\in U, \hat{x}\in\hat{U}} |\rho_{f\rho j}(x, \eta) - \hat{\rho}_{f\rho j}(\hat{x}, \eta, \hat{\theta}_{f\rho j}^T)|]$$

$$\theta_{g\rho j}^* = \arg \min_{\theta_{g\rho j}\in\Omega_{g\rho}} [\sup_{x\in U, \hat{x}\in\hat{U}} |\rho_{gj}(x, \eta) - \hat{\rho}_{gj}(\hat{x}, \eta, \hat{\theta}_{g\rho j}^T)|]$$

where Ω_f, Ω_g, $\Omega_{g\rho}$, $\Omega_{f\rho}$, U and $\hat{U}$ are compact regions for $\hat{\theta}_{fi}$, $\hat{\theta}_{gj}$, $\hat{\theta}_{f\rho j}$, $\hat{\theta}_{g\rho j}$, x and $\hat{x}$. The FLS minimum approximation errors are defined as

$$\varepsilon_{fi} = f(x_{[i]}) - \theta_{fi}^{*T} \xi_{fi}(\hat{x}_{[i]}), \ \varepsilon_{f\rho} = f_\rho(x) - \theta_{f\rho}^{*T} \xi_{f\rho}(\hat{x}, \eta)$$

$$\varepsilon_{gj} = g_j(x, \eta) - \theta_{gj}^{*T} \xi_{gj}(\hat{x}, \eta), \ \varepsilon_{g\rho j} = \rho_{gj}(x, \eta) - \theta_{g\rho j}^{*T} \xi_{g\rho j}(\hat{x}, \eta),$$

$$\varepsilon_{f\rho j} = \rho_{fj}(x,\eta) - \theta_{f\rho j}^{*T}\xi_{f\rho j}(\hat{x},\eta),\ \delta_{gj} = g_j(x,\eta) - \hat{\theta}_{gj}^T\xi_{gj}(\hat{x},\eta),$$

$$\delta_{fi} = f(x_{[i]}) - \hat{\theta}_{fi}^T\xi_{fi}(\hat{x}_{[i]}),\ \delta_{f\rho} = f_\rho(x) - \hat{\theta}_{f\rho}^T\xi_{f\rho}(\hat{x},\eta)$$

$$\delta_{g\rho j} = \rho_{gj}(x,\eta) - \hat{\theta}_{g\rho j}^T\xi_{g\rho j}(\hat{x},\eta),\ \delta_{f\rho j} = \rho_{fj}(x,\eta) - \hat{\theta}_{f\rho j}^T\xi_{f\rho j}(\hat{x},\eta)$$

In order to simplify the notations in the following, let $\xi_{fi}, \xi_{f\rho}, \xi_{gj}, \xi_{g\rho j}$ and $\xi_{f\rho j}$ denote $\xi_{fi}(\hat{x}_{[i]}), \xi_{f\rho}(\hat{x},\eta), \xi_{gj}(\hat{x},\eta), \xi_{g\rho j}(\hat{x},\eta)$ and $\xi_{f\rho j}(\hat{x},\eta)$, respectively. Now, the following assumptions are made.

Assumption 5.4 [23, 26, 27] There exist known positive real constants $\bar{M}_{\varepsilon fi}$, $\bar{M}_{\varepsilon gj}, \bar{M}_{\varepsilon f\rho j}, \bar{M}_{\varepsilon g\rho j}, M_{fi}, M_{gj}, M_{f\rho j}, M_{g\rho j}, \bar{M}_{\delta fi}, \bar{M}_{\delta gj}, \bar{M}_{\delta f\rho j}$ and $\bar{M}_{\delta g\rho j}$ such that $|\varepsilon_{fi}| \leq \bar{M}_{\varepsilon fi}, |\varepsilon_{gj}| \leq \bar{M}_{\varepsilon gj}, |\varepsilon_{f\rho j}| \leq \bar{M}_{\varepsilon f\rho j}, |\varepsilon_{g\rho j}| \leq \bar{M}_{\varepsilon g\rho j}, ||\theta_{fi}^*|| \leq M_{fi}, ||\theta_{gj}^*|| \leq M_{gj}, ||\theta_{g\rho j}^*|| \leq M_{g\rho j}, ||\theta_{f\rho j}^*|| \leq M_{f\rho j}, |\delta_{fi}| \leq \bar{M}_{\delta fi}, |\delta_{gi}| \leq \bar{M}_{\delta gj}, |\delta_{f\rho j}| \leq \bar{M}_{\delta f\rho j}$ and $|\delta_{g\rho j}| \leq \bar{M}_{\delta g\rho j}$, where $i = 1, \ldots, \rho$ and $j = 1, \ldots, m$.

5.3 Main Results

5.3.1 Stability Control in Fault-Free Case and Fault Detection

Since the system states are not all measured, the following observer is constructed to estimate the system states.

$$\dot{\hat{x}} = A\hat{x} + Hy + \hat{f} + B\hat{g}^T u,\ \hat{y} = C\hat{x} \tag{5.8}$$

where $\hat{f}^T = [\hat{f}_1, \ldots, \hat{f}_\rho], \hat{g}^T = [\hat{g}_1, \ldots, \hat{g}_m], \hat{\varepsilon}_g^T = [\hat{\varepsilon}_{g1}, \ldots, \hat{\varepsilon}_{gm}], \hat{f}_i, \hat{\varepsilon}_{fi}, i = 1, \ldots, \rho$ and $\hat{g}_j, \hat{\varepsilon}_{gj}, j = 1, \ldots, m$ denote the estimates of f_i and g_j, $C = [1\ 0 \ldots 0]$. Let $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_\rho]^T$ and $e = x - \hat{x}$, the error dynamics can be written as:

$$\dot{e} = Ae + d + Bd_g,\ e_y = Ce \tag{5.9}$$

where $d = [d_1, \ldots, d_\rho]^T$, $d_i = \delta_{fi} = f_i - \hat{f}_i$, $d_g = \sum_{j=1}^m \delta_{gj}u_j = \sum_{j=1}^m (g_j - \hat{g}_j)$ u_j.

In the following, based on the previous section, we will incorporate the DSC technique into an adaptive fuzzy control design scheme for the ρ-order system described by (5.8). Similar to the traditional backstepping design method, the recursive design procedure contains ρ steps. From Step 1 to Step ρ, virtual control laws $\alpha_{i-1}, i = 2, \ldots, \rho$ are designed at each step. Finally overall control laws u_j, $j = 1, \ldots, m$ are constructed at step ρ.

Step 1: Let $S_1 = \hat{x}_1 - y_d$. Then, it follows form (5.8) that

$$\dot{S}_1 = \dot{\hat{x}}_1 - \dot{y}_d = \hat{x}_2 + \hat{\theta}_{f1}^T \xi_{f1} + \tilde{\theta}_{f1}^T \xi_{f1} + \varepsilon_{f1} - \delta_{f1} - \dot{y}_d \tag{5.10}$$

Choose a virtual control α_1 as follows:

$$\alpha_1 = -k_1 S_1 - [\hat{f}_1 - \dot{y}_d + (\bar{M}_{\varepsilon f1} + \bar{M}_{\delta f1}) \tanh(S_1(\bar{M}_{\varepsilon f1} + \bar{M}_{\delta f1})/w)] \tag{5.11}$$

Here and in the following, $k_i > 0 \in R$, $i = 1, \ldots, \rho$ are design parameters, $w > 0 \in R$ is a constant. Introduce a new state variable z_2 and let α_1 pass through a first-order filter with time constant ε_2 to obtain z_2,

$$\varepsilon_2 \dot{z}_2 + z_2 = \alpha_1, z_2(0) = \alpha_1(0) \tag{5.12}$$

Here and in the following, $\varepsilon_i > 0 \in R$, $i = 1, \ldots, \rho - 1$ are design parameters.

Step i $(i = 2, \ldots, \rho - 1)$: Consider $\dot{\hat{x}}_i = \hat{x}_{i+1} + \hat{f}_i + h_i e_1$. Define the ith error surface S_i to be $S_i = \hat{x}_i - z_i$, then

$$\dot{S}_i = \dot{\hat{x}}_i - \dot{z}_i = \hat{x}_{i+1} + \hat{\theta}_{fi}^T \xi_{fi} + h_i e_1 - \dot{z}_i + \tilde{\theta}_{fi}^T \xi_{fi} + \varepsilon_{fi} - \delta_{fi} \tag{5.13}$$

Choose a virtual control α_i as follows:

$$\alpha_i = -k_i S_i - [\hat{f}_i - \dot{z}_i + (\bar{M}_{\varepsilon fi} + \bar{M}_{\delta fi}) \tanh(S_i(\bar{M}_{\varepsilon fi} + \bar{M}_{\delta fi})/w)] \tag{5.14}$$

Introduce a new state variable z_{i+1} and let α_i pass through a first-order filter with constant ε_{i+1} to obtain z_{i+1}

$$\varepsilon_{i+1} \dot{z}_{i+1} + z_{i+1} = \alpha_i, \; z_{i+1}(0) = \alpha_i(0) \tag{5.15}$$

Step ρ: Consider $\dot{\hat{x}}_\rho = \hat{f}_\rho + h_\rho e_1 + \hat{g}^T u$.

Define the ρth error surface S_ρ to be $S_\rho = \hat{x}_\rho - z_\rho$, then

$$\begin{aligned} \dot{S}_\rho &= \dot{\hat{x}}_\rho - \dot{z}_\rho = \hat{f}_\rho + h_\rho e_1 + \hat{g}^T u - \dot{z}_\rho \\ &= \hat{\theta}_{f\rho}^T \xi_{f\rho} + h_\rho e_1 + \hat{g}^T u - \dot{z}_\rho + \tilde{\theta}_{f\rho}^T \xi_{f\rho} + \varepsilon_{f\rho} - \delta_{f\rho} + \hat{g}^T u \end{aligned} \tag{5.16}$$

Finally, let the final control $\alpha_{\rho j}, j = 1, \ldots, m$ be as follows:

$$\alpha_{\rho j} = u_j = [N(\varsigma)(k_\rho S_\rho + \frac{\Delta}{S_\rho})]/m, \; \dot{\varsigma} = -k_\rho S_\rho^2 - \Delta \tag{5.17}$$

where

$$\Delta = \sum\nolimits_{i=1}^{\rho} [|S_i|(\bar{M}_{\varepsilon fi} + \bar{M}_{\delta fi}) + \frac{\eta_{fi}}{2\eta_1} \theta_{fi}^{*T} \theta_{fi}^* + \frac{\eta_{gj}}{2\eta_2} \theta_{gj}^{*T} \theta_{gj}^*] +$$
$$\mu_e + (\rho - 1)\sigma_1/2 + S_\rho \hat{\theta}_{f\rho}^T \xi_{f\rho} + \sum\nolimits_{i=1}^{\rho-1} S_{i+1} k_{i+1} e_1$$

Just as pointed out in [22], for the above control (5.17), controller singularity may occur since Δ/S_ρ is not well defined at $S_\rho = 0$. Similar to [22], let define $\Omega_{c_{S_\rho}} \subset \Omega$ and $\Omega^0_{c_{S_\rho}}$ s.t. $\Omega_{c_{S_\rho}} := \{S_\rho \big| |S_\rho| < c_{S_\rho}\}$, $\Omega^0_{c_{S_\rho}} := \Omega - \Omega_{c_{S_\rho}}$, where $c_{S_\rho} > 0$ is a constant that can be chosen arbitrarily small and "−" is used to denote the complement of set B in set A as $A - B := \{x|x \in A \text{ and } x \notin B\}$. Thus, the final control $\alpha_{\rho j} = u_j, j = 1, \ldots, m$ can be modified as

$$u_j = \begin{cases} N(\varsigma)[k_\rho S_\rho + \dfrac{\Delta}{S_\rho}]/m,\ \dot{\varsigma} = k_\rho S_\rho^2 + \Delta,\ S_\rho \in \Omega^0_{c_{S_\rho}} \\ 0,\ S_\rho \in \Omega_{c_{S_\rho}} \end{cases} \tag{5.18}$$

In the following, we will give the closed-loop system stability analysis. The closed-loop system in the new coordinates S_i, z_i can be expressed as follows:

$$\dot{S}_1 = \hat{x}_2 + \hat{\theta}_{f1}^T \xi_{f1} + \tilde{\theta}_{f1}^T \xi_{f1} + \varepsilon_{f1} - \delta_{f1} - \dot{y}_d$$

$$\dot{S}_i = \hat{x}_{i+1} + \hat{\theta}_{fi}^T \xi_{fi} + h_i e_1 - \dot{z}_i + \tilde{\theta}_{fi}^T \xi_{fi} + \varepsilon_{fi} - \delta_{fi}$$

$$\dot{S}_\rho = \tilde{\theta}_{f\rho}^T \xi_{f\rho} + h_\rho e_1 + \hat{g}^T u - \dot{z}_\rho + \tilde{\theta}_{f\rho}^T \xi_{f\rho} + \varepsilon_{f\rho} - \delta_{f\rho}$$

$$\varepsilon_2 \dot{z}_2 + z_2 = \alpha_1,\ z_2(0) = \alpha_1(0)$$

$$\varepsilon_{i+1}\dot{z}_{i+1} + z_{i+1} = \alpha_i,\ z_{i+1}(0) = \alpha_i(0),\ i = 2, \ldots, \rho - 1$$

Define

$$\begin{aligned} y_2 &= z_2 - \alpha_1 \\ &= k_1 S_1 + \hat{f}_1 + (\bar{M}_{\varepsilon f1} + \bar{M}_{\delta f1}) \tanh(S_1/w) + z_2 - \dot{y}_d \end{aligned} \tag{5.19}$$

$$\begin{aligned} y_{i+1} &= z_{i+1} - \alpha_i \\ &= k_i S_i + \hat{f}_i - \dot{z}_i - (\bar{M}_{\varepsilon fi} + \bar{M}_{\delta fi}) \tanh(\frac{S_i(\bar{M}_{\varepsilon fi} + \bar{M}_{\delta fi})}{w}) + z_{i+1} + \frac{y_i}{\varepsilon_i} \end{aligned} \tag{5.20}$$

where $i = 2, \ldots, \rho - 1$. From (5.11), (5.14), (5.18)–(5.20), one has

$$\dot{S}_1 = \dot{\hat{x}}_1 - \dot{y}_d = S_2 - k_1 S_1 + y_2 + \tilde{\theta}_{f1}^T \xi_{f1} + \varepsilon_{f1} - \delta_{f1} \tag{5.21}$$

$$\dot{S}_i = \dot{\hat{x}}_i - \dot{z}_i = S_{i+1} - k_i S_i + h_i e_1 + y_{i+1} + \tilde{\theta}_{fi}^T \xi_{fi} + \varepsilon_{fi} - \delta_{fi} \tag{5.22}$$

$$\dot{S}_\rho = \hat{\theta}_{f\rho}^T \xi_{f\rho} + h_\rho e_1 + \hat{g}^T u - \dot{z}_\rho + \tilde{\theta}_{f\rho}^T \xi_{f\rho} + \varepsilon_{f\rho} - \delta_{f\rho} \tag{5.23}$$

Since $\dot{z}_i = (\alpha_i - z_{i+1})/\varepsilon_i = -y_i/\varepsilon_i,\ i = 2, \ldots, \rho - 1$, it gives

$$\dot{y}_2 = \dot{z}_2 - \dot{\alpha}_1 = -y_2/\varepsilon_2 + B_2 \tag{5.24}$$

where

$$\chi_1 = \frac{\partial\left(2\tanh(\frac{S_1(\bar{M}_{\varepsilon f1}+\bar{M}_{\delta f1})}{w})(\bar{M}_{\varepsilon f1} + \bar{M}_{\delta f1})\right)}{\partial(\hat{x}_1, \ldots, \hat{x}_\rho)} \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_\rho \end{bmatrix}$$

$$B_2 = k_1\dot{S}_1 + \dot{\hat{\theta}}_{f1}^T\xi_{f1} + \hat{\theta}_{f1}^T\frac{\partial\xi_{f1}}{\partial\hat{x}_1} + \chi - \ddot{y}_d$$

which is a continuous function. Similarly, for $i = 2, \ldots, \rho - 1$,

$$\dot{y}_{i+1} = -y_{i+1}/\varepsilon_{i+1} + B_{i+1} \tag{5.25}$$

where

$$B_{i+1} = k_i\dot{S}_i + \dot{\hat{\theta}}_{fi}^T\xi_{fi} + \hat{\theta}_{fi}^T\frac{\partial\xi_{fi}}{\partial(\hat{x}_1, \ldots, \hat{x}_i)}\begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_i \end{bmatrix} + \frac{\dot{y}_i}{\varepsilon_i} + \chi_i$$

is a continuous function, $\chi_i = \frac{\partial\left(2\tanh(\frac{S_i(\bar{M}_{\varepsilon fi}+\bar{M}_{\delta fi})}{w})(\bar{M}_{\varepsilon fi}+\bar{M}_{\delta fi})\right)}{\partial(\hat{x}_1,\ldots,\hat{x}_\rho)}\begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_\rho \end{bmatrix}$.

Differentiating $V_e = e^TPe$ with respect to time t and considering (5.9) and Assumption 5.4, it leads to

$$\dot{V}_e \le e^T(PA + A^TP + 2\lambda PP)e + \mu_e \tag{5.26}$$

where $\lambda > 0 \in R$ is a design parameter, and it is assumed that $|u_j| \le \bar{u}_j$, $\bar{u}_j > 0 \in R$, $\mu_e = (\sum_{i=1}^{\rho}\bar{M}_{\delta fi}^2 + \sum_{j=1}^{m}\bar{M}_{\delta gj}\bar{u}_j^2)/(4\lambda)$. Notice that, this assumption seems to be strict. However, in many practical systems, such as flight control systems considered in this chapter, control input is bounded. Hence, this assumption is reasonable in some case. In addition, λ can be chosen to be a larger constant such that $\mu_e \le \bar{\mu}_e$, $\bar{\mu}_e > 0 \in R$.

If for a given constant λ matrices $P = P^T > 0$, $Q > 0$ are chosen appropriately such that $PA + A^TP + 2\lambda PP \le -Q$, then,

$$\dot{V}_e = -e^TQe + \mu_e = -g_eV_e + \mu_e \tag{5.27}$$

where $g_e = \lambda_{\min}(Q)/\lambda_{\max}(P)$.

Consider the following Lyapunov function

$$V_1 = V_e + \frac{1}{2}[\sum_{i=1}^{\rho}(S_i^2 + \frac{1}{\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} + \frac{1}{\eta_2}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj}) + \sum_{k=1}^{\rho-1} y_{k+1}^2]$$

where $\eta_1 > 0, \eta_2 > 0$ are design constants, $\tilde{\theta}_{fi} = \theta_{fi}^* - \hat{\theta}_{fi}, \tilde{\theta}_{gj} = \theta_{gj}^* - \hat{\theta}_{gj}$.

Differentiating V_1 with respect to time t, it leads to

$$\begin{aligned} \dot{V}_1 \leq & \sum_{i=1}^{\rho-1}(S_iS_{i+1} - k_iS_i^2 + S_iy_{i+1}) + S_\rho\hat{\theta}_{f\rho}^T\xi_{f\rho} + \dot{V}_e + \\ & \sum_{k=1}^{\rho-1}(-\frac{y_{k+1}^2}{\varepsilon_{k+1}} + |y_{k+1}B_{k+1}|) + \sum_{i=1}^{\rho-1} S_{i+1}k_{i+1}e_1 + \\ & \sum_{i=1}^{\rho} S_i\tilde{\theta}_{fi}^T\xi_{fi} + \sum_{i=1}^{\rho} S_i(\varepsilon_{fi} - \delta_{fi}) + \\ & S_\rho\sum_{j=1}^{m}(\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j + S_\rho\sum_{j=1}^{m}\tilde{\theta}_{gj}^T\xi_{gj}u_j - \\ & \sum_{i=1}^{\rho}\frac{1}{2\eta_1}\tilde{\theta}_{fi}^T\dot{\hat{\theta}}_{fi} - \sum_{j=1}^{m}\frac{1}{2\eta_2}\tilde{\theta}_{gj}^T\dot{\hat{\theta}}_{gj} \end{aligned} \tag{5.28}$$

Define $\dot{\hat{\theta}}_{fi}, i = 1, \ldots, \rho, \dot{\hat{\theta}}_{gj}, j = 1, \ldots, m$ as follows

$$\dot{\hat{\theta}}_{fi} = \begin{cases} 2\eta_1S_i\xi_{fi} - \eta_{fi}\hat{\theta}_{fi}, \ \text{if } ||\hat{\theta}_{fi}|| < M_{fi} \text{ or } ||\hat{\theta}_{fi}|| = M_{fi} \\ \ \text{and } 2\eta_1S_i\xi_{fi} - \eta_{fi}\hat{\theta}_{fi} \geq 0; \\ 2\eta_1S_i\xi_{fi} - \eta_{fi}\hat{\theta}_{fi} + (2\eta_1S_i\frac{\hat{\theta}_{fi}\hat{\theta}_{fi}^T}{||\hat{\theta}_{fi}||^2}\xi_{fi} - \eta_{fi}\frac{\hat{\theta}_{fi}\hat{\theta}_{fi}^T}{||\hat{\theta}_{fi}||^2}\hat{\theta}_{fi}) \\ \ \text{if } ||\hat{\theta}_{fi}|| = M_{fi} \text{ and } 2\eta_1S_i\xi_{fi} - \eta_{fi}\hat{\theta}_{fi} < 0 \end{cases} \tag{5.29}$$

$$\dot{\hat{\theta}}_{gj} = \begin{cases} 2\eta_2S_\rho\xi_{gj}u_j - \eta_{gj}\hat{\theta}_{gj}, \ \text{if } ||\hat{\theta}_{gj}|| < M_{gj} \text{ or } ||\hat{\theta}_{gj}|| = M_{gj} \\ \ \text{and } 2\eta_2S_\rho\xi_{gj}u_j - \eta_{gj}\hat{\theta}_{gj} \geq 0; \\ 2\eta_2S_\rho\xi_{gj}u_j - \eta_{gj}\hat{\theta}_{gj} + (2\eta_2S_\rho\frac{\hat{\theta}_{gj}\hat{\theta}_{gj}^T}{||\hat{\theta}_{gj}||^2}\xi_{gj}u_j - \eta_{gj}\cdot \\ \frac{\hat{\theta}_{gj}\hat{\theta}_{gj}^T}{||\hat{\theta}_{gj}||^2}\hat{\theta}_{gj}), \ \text{if} |\hat{\theta}_{gj}|| = M_{gj} \text{and} 2\eta_2S_\rho\xi_{gj}u_j - \eta_{gj}\hat{\theta}_{gj} < 0 \end{cases} \tag{5.30}$$

where $\eta_{fi} > 0, \eta_{gj} > 0$ are design constants, u_j is a bounded control input which is applied simultaneously to the jth actuator in the system (5.5) and the observer (5.8). Applying Young's Inequality, one has

$$\eta_{fi}\tilde{\theta}_{fi}^T\hat{\theta}_{fi}/(\eta_1) \leq -\eta_{fi}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi}/(2\eta_1) + \eta_{fi}\theta_{fi}^{*T}\theta_{fi}^*/(2\eta_1)$$

$$\eta_{\varepsilon fi}\tilde{\varepsilon}_{fi}\hat{\varepsilon}_{fi}/\eta_1 \leq -\eta_{\varepsilon fi}\tilde{\varepsilon}_{fi}^2/(2\eta_1) + \eta_{\varepsilon fi}(\varepsilon_{fi}^*)^2/(2\eta_1)$$

$$\eta_{g,i}\tilde{\theta}_{gj}^T\hat{\theta}_{gj}/\eta_2 \leq -\eta_{gj}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj}/(2\eta_2) + \eta_{gj}\theta_{gj}^{*T}\theta_{gj}^*/(2\eta_2)$$

$$\eta_{\varepsilon gj}\tilde{\varepsilon}_{gj}\hat{\varepsilon}_{gj}/\eta_2 \leq -\eta_{\varepsilon gj}\tilde{\varepsilon}_{gj}^2/(2\eta_2) + \eta_{\varepsilon gj}(\varepsilon_{gj}^*)^2/(2\eta_2)$$

Substituting the above inequalities into (5.28), it yields

$$\begin{aligned}\dot{V}_1 \leq & \sum\nolimits_{i=1}^{\rho-1}(S_iS_{i+1} - k_iS_i^2 + S_iy_{i+1}) + S_\rho\hat{\theta}_{f\rho}^T\xi_{f\rho} + \\ & \sum\nolimits_{k=1}^{\rho-1}(-\frac{y_{k+1}^2}{\varepsilon_{k+1}} + |y_{k+1}B_{k+1}|) + S_\rho\hat{\theta}_{f\rho}^T\xi_{f\rho} + \\ & \sum\nolimits_{i=1}^{\rho-1}S_{i+1}k_{i+1}e_1 + \dot{V}_e + \sum\nolimits_{i=1}^{\rho}S_i(\varepsilon_{fi} - \delta_{fi}) + \\ & S_\rho\sum\nolimits_{j=1}^{m}(\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j - \sum\nolimits_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} - \\ & \sum\nolimits_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} - \sum\nolimits_{j=1}^{m}\frac{\eta_{gj}}{2\eta_2}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \\ & \sum\nolimits_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\theta_{fi}^{*T}\theta_{fi}^* + \sum\nolimits_{j=1}^{m}\frac{\eta_{gj}}{2\eta_2}\theta_{gj}^{*T}\theta_{gj}^*\end{aligned} \tag{5.31}$$

From Young's Inequality, one has

$$\begin{aligned}S_iS_{i+1} \leq S_i^2 + S_{i+1}^2/4,\ \ S_iy_{i+1} \leq S_i^2 + y_{i+1}^2/4 \\ |y_{k+1}B_{k+1}| \leq y_{k+1}^2B_{k+1}^2/\sigma_1 + \sigma_1/2,\ \ \forall\sigma_1 > 0 \in R\end{aligned} \tag{5.32}$$

where $\sigma_1 > 0 \in R$ is a design parameter. Substituting (5.32) to (5.31), yields

$$\begin{aligned}\dot{V}_1 \leq & \sum\nolimits_{i=1}^{\rho-1}[\frac{1}{4}S_{i+1}^2 + (2-k_i)S_i^2] + \frac{(\rho-1)\sigma_1}{2} + S_\rho\hat{\theta}_{f\rho}^T\xi_{f\rho} + \\ & \sum\nolimits_{i=1}^{\rho-1}S_{i+1}k_{i+1}e_1 + \dot{V}_e + \sum\nolimits_{k=1}^{\rho-1}(\frac{1}{4} - \frac{1}{\varepsilon_{k+1}} + \frac{B_{k+1}^2}{2\sigma_1})y_{k+1}^2 + \\ & \sum\nolimits_{i=1}^{\rho}S_i(\varepsilon_{fi} - \delta_{fi}) + S_\rho\sum\nolimits_{j=1}^{m}(\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j - \\ & \sum\nolimits_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} - \sum\nolimits_{j=1}^{m}\frac{\eta_{gj}}{2\eta_2}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \\ & \sum\nolimits_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\theta_{fi}^{*T}\theta_{fi}^* + \sum\nolimits_{j=1}^{m}\frac{\eta_{gj}}{2\eta_2}\theta_{gj}^{*T}\theta_{gj}^*\end{aligned} \tag{5.33}$$

As pointed out in [15], since for any $B_0 > 0$ and $p > 0$ the sets

$$\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\}$$

$$\Pi_i := \{\sum_{l=1}^{\rho} (S_l^2(0) + \tilde{\theta}_{fl}^T(0)\tilde{\theta}_{fl}(0)) + \sum_{j=1}^{m} \tilde{\theta}_{gj}^T(0)\tilde{\theta}_{gj}(0) +$$
$$\sum_{k=1}^{i} y_{k+1}^2(0) + 2e^T(0)e(0) \leq 2vp\}, i = 1, \dots, \rho$$

are compact, $\Pi_0 \times \Pi_i$ is also compact. Thus, $|B_{i+1}|$ has a maximum M_{i+1} on $\Pi_0 \times \Pi_i$, $v = \max\{1, \eta_1, \eta_2, 1/\lambda_{\min}(P)\}$.

Choose

$$k_1 = 2 + \beta_0,\ k_i = 2\frac{1}{4} + \beta_0,\ k_\rho = 1\frac{1}{4} + \beta_0,\ \frac{1}{\varepsilon_{k+1}} = \frac{1}{4} + \frac{M_{k+1}^2}{2\sigma_1} + \beta_0 \tag{5.34}$$

where $k = 1, \dots, \rho - 1, i = 2, \dots, \rho - 1 \beta_0 > 0 \in R$ is a constant. Thus, from (5.27), (5.33) and (5.34), one has

$$\dot{V}_1 \leq -\sum_{i=1}^{\rho} \beta_0 S_i^2 - \sum_{k=1}^{\rho-1} \beta_0 y_{k+1}^2 - g_e V_e - \sum_{i=1}^{\rho} \frac{\eta_{fi}}{2\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} +$$
$$\sum_{j=1}^{m} \frac{\eta_{gj}}{2\eta_2}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \frac{(\rho-1)\sigma_1}{2} + S_\rho\hat{\theta}_{f\rho}^T\xi_{f\rho} +$$
$$\sum_{i=1}^{\rho-1} S_{i+1}k_{i+1}e_1 + \sum_{i=1}^{\rho} |S_i|(\bar{M}_{\varepsilon fi} + \bar{M}_{\delta fi}) +$$
$$\sum_{i=1}^{\rho} \frac{\eta_{fi}}{2\eta_1}\theta_{fi}^{*T}\theta_{fi}^* + \sum_{j=1}^{m} \frac{\eta_{gj}}{2\eta_2}\theta_{gj}^{*T}\theta_{gj}^* +$$
$$\mu_e + S_\rho \sum_{j=1}^{m} (\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j$$

Define $\Delta = \mu_e + \frac{(\rho-1)\sigma_1}{2} + S_\rho\hat{\theta}_{f\rho}^T\xi_{f\rho} + \sum_{i=1}^{\rho-1} S_{i+1}k_{i+1}e_1 + \sum_{i=1}^{\rho} [|S_i|(\bar{M}_{\varepsilon fi} - \bar{M}_{\delta fi}) + \frac{\eta_{fi}}{2\eta_1}\theta_{fi}^{*T}\theta_{fi}^*] + \sum_{j=1}^{m} \frac{\eta_{gj}}{2\eta_2}\theta_{gj}^{*T}\theta_{gj}^*$, then, one has

$$\dot{V}_1 \leq -\sum_{i=1}^{\rho} \beta_0 S_i^2 - \sum_{k=1}^{\rho-1} \beta_0 y_{k+1}^2 - g_e V_e - \sum_{i=1}^{\rho} \frac{\eta_{fi}}{2\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} -$$
$$\sum_{j=1}^{m} \frac{\eta_{gj}}{2\eta_2}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \Delta + S_\rho \sum_{j=1}^{m} (\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j \tag{5.35}$$

Substituting control laws (5.18) into (5.35), it leads to

$$\dot{V}_1 \leq -gV_1 + \sum_{j=1}^{m} (h_{Dj}N(\varsigma) + 1)\dot{\varsigma} \tag{5.36}$$

where $g = \min\{\beta_0, g_e, \frac{\eta_{f1}}{2\eta_1}, \dots, \frac{\eta_{f\rho}}{2\eta_1}, \frac{\eta_{g1}}{2\eta_2}, \dots, \frac{\eta_{gm}}{2\eta_2}\}$ and $h_{Dj} = (\hat{g}_j + \varepsilon_{gj} - \delta_{gj})$. Applying Lemma 5.2, we can conclude that, $V_1(t)$, $\int_0^t \sum_{j=1}^{m} (h_{Dj}N(\varsigma) + 1)e^{-g\tau}\dot{\varsigma}d\tau$ and $\varsigma(t)$ are SGUUB on $[0, t_f)$. According to Proposition 2 in [20], if the solution of the closed-loop system is bounded, then $t_f = +\infty$. Let $\bar{\mu}_1$ be the upper bound of $\int_0^t \sum_{j=1}^{m} (h_{Dj}N(\varsigma) + 1)e^{-g\tau}\dot{\varsigma}d\tau$, we have the following inequalities:

$$\int_0^t \sum_{j=1}^m e^{-gt}(h_{Dj}N(\varsigma)+1)e^{-g\tau}\dot{\varsigma}d\tau \le \int_0^t \sum_{j=1}^m (h_{Dj}N(\varsigma)+1)e^{-g\tau}\dot{\varsigma}d\tau \le \bar{\mu}_1.$$

Thus, (5.36) becomes

$$\dot{V}_1 \le -gV_1 + \bar{\mu}_1 \tag{5.37}$$

Solving inequality (5.37) gives

$$0 \le V_1(t) \le \frac{\bar{\mu}_1}{g} + [V_1(0) - \frac{\bar{\mu}_1}{g}]e^{-gt} \le \frac{\bar{\mu}_1}{g} + V_1(0) = \mu_1 \tag{5.38}$$

which means that $V_1(t)$ is bounded by μ_1. Thus, all signals of the closed-loop system, i.e., $S_i(t), \tilde{\theta}_{fi}, \tilde{\theta}_{gj}, \tilde{\varepsilon}_{fi}, \tilde{\varepsilon}_{gj}$ and y_i are uniformly ultimately bounded, i.e. for $i = 1, \ldots, \rho, j = 1, \ldots, m$, $\sqrt{2\mu_1}$, $|y_i| \le \sqrt{2\mu_1}$, $||\tilde{\theta}_{fi}|| \le \sqrt{2\eta_1\mu_1}$, $|\tilde{\varepsilon}_{fi}| \le \sqrt{2\eta_1\mu_1}$, $||\tilde{\theta}_{gj}|| \le \sqrt{2\eta_2\mu_1}$, $|\tilde{\varepsilon}_{gj}| \le \sqrt{2\eta_2\mu_1}$, $||e|| \le \sqrt{\mu_1/\lambda_{\min}(P)}$.

Now, the following theorem guarantees the existence of the observer (5.8) and the corresponding tracking performance.

Theorem 5.1 *Consider system (5.5) and observer (5.8) under Assumptions 5.1–5.4, the virtual control (5.11), (5.14) and (5.18), the adaptive laws (5.29) and (5.30). If matrices* $H, Q > 0$, $P = P^T > 0$ *and constant* $\lambda > 0 \in R$ *are chosen such that*

$$PA + A^TP + 2\lambda PP \le -Q \tag{5.39}$$

for all initial conditions satisfying

$$\varPi_i := \left\{ \begin{array}{l} \sum_{l=1}^{\rho} (S_l^2(0) + \tilde{\theta}_{fl}^T(0)\tilde{\theta}_{fl}(0)) + \\ \sum_{j=1}^{m} \tilde{\theta}_{gj}^T(0)\tilde{\theta}_{gj}(0) + \sum_{k=1}^{i} y_{k+1}^2(0) + 2e^T(0)e(0) \le 2vp \end{array} \right\},$$

$i = 1, \ldots, \rho$, k_i, ε_k *are chosen as (5.34), then we can guarantee the following properties under bounded initial conditions: (i) All signals in the closed-loop system are globally uniformly ultimately bounded (ii) The vectors* S_i, $\tilde{\theta}_{fi}$, $\tilde{\varepsilon}_{fi}$, $i = 1, \ldots, \rho$, *and* $\tilde{\theta}_{gj}$, $\tilde{\varepsilon}_{gj}$, $j = 1, \ldots, m$ *remain in the compact set* $\varOmega_1$, *specified as*

$$\varOmega_1 := \left\{ \begin{array}{l} (S_i, y_i, \tilde{\theta}_{fi}, \tilde{\varepsilon}_{fi}, \tilde{\theta}_{gj}, \tilde{\varepsilon}_{gj}) \Big| \sum_{i=1}^{\rho} (S_i^2 + \dfrac{\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} + \tilde{\varepsilon}_{fi}^2}{\eta_1}) + \\ \sum_{j=1}^{m} \dfrac{(\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \tilde{\varepsilon}_{gj}^2)}{\eta_2} + \sum_{k=1}^{\rho-1} y_{k+1}^2 + 2\lambda_{\min}(P)e^Te \le 2\mu_1 \end{array} \right\}$$

where μ_1 *can be adjusted by appropriately choosing the design parameter such as* $\eta_1, \eta_2, \eta_{fi}, \eta_{\varepsilon fi}, \eta_{gj}, \eta_{\varepsilon gj}$ *and* σ_1, w, β_0.

Proof From the above analysis, it is easy to obtain the conclusion. The detailed proof is omitted here.

From Theorem 5.1, all signals of the closed-loop system belong to the following set Ω_1. Therefore, the detection residual can be defined as $J = |y_d(t) - \hat{x}_1(t)| = |S_1|$. Obviously, it is seen that the following inequality holds in the healthy case: $J \leq \sqrt{2\mu_1}$. Then, the fault detection can be performed using the following mechanism:

$$\begin{cases} J \leq T_d & \textit{no fault occurred}, \\ J > T_d & \textit{fault has occurred} \end{cases}$$

where threshold $T_d = \sqrt{2\mu_1}$.

5.3.2 Fault Isolation and Estimation

Since the system has m actuators and it is assumed that only one single fault occurs at one time, we have m possible faulty cases in total. When the sth $(1 \leq s \leq m)$ actuator is faulty, the faulty model can be described as: $u_s^f = \rho_s(x)u_s + f_s^u(x)$. The faulty system can be described as follows:

$$\begin{cases} \dot{x}_i = \dot{x}_{i+1} + f_i, i = 1, 2, \ldots, \rho - 1 \\ \dot{x}_\rho = f_\rho + \sum_{j=1, j\neq s}^{\rho} g_j u_j + g_s \rho_s u_s + g_s f_s^u \\ \dot{\eta} = \psi_1(x, \eta) + \psi_2(x, \eta)\theta \\ y = x_1 \end{cases} \tag{5.40}$$

where $f_i = f_i(x_{[i]})$, $i = 1, 2, \ldots, \rho - 1$, $f_\rho = f_\rho(x, \eta)$, $g_i = g_i(x, \eta)$, u_s is the sth actuator's desired control input when the sth actuator is healthy, t_s is the unknown fault occurrence time.

After a fault has been detected, the isolation scheme is activated. Inspired by [28], the following m nonlinear adaptive fuzzy observers are considered:

$$\dot{\hat{x}}_s = A\hat{x}_s + Hy_s + \hat{f} + B(\sum_{j=1, j\neq s}^{m} \hat{g}_j u_j + \hat{g}_{\rho s} u_s + \hat{f}_{\rho s}) \tag{5.41}$$

where $\hat{x}_s = [\hat{x}_{s1}, \ldots, \hat{x}_{s\rho}]^T$ is the observer state; $\hat{f}_i = \hat{\theta}_{fi}^T \xi_{fi}$, $i = 1, \ldots, \rho$ and $\hat{g}_j = \hat{\theta}_{gj}^T \xi_{gj}, j = 1, \ldots, m, j \neq r$, which are the estimates of f_i and g_j; $\hat{g}_{\rho s} = \hat{\theta}_{g\rho s}^T \xi_{g\rho s}$ and $\hat{f}_{\rho s} = \hat{\theta}_{f\rho s}^T \xi_{f\rho s}$ are the estimates of $g_s \rho_s$ and $g_s f_s^u$.

It is assumed that r ($1 \leq r \leq m$) is the practical fault pattern where the faulty actuators are the rth actuator.

Let $e_s(t) = x - \hat{x}_s$ is the output error and state error between the faulty plant and the sth observer, then the error dynamics can be written as follows:

$$\dot{e}_s = Ae_s + d + B(d_{sg} + d_{g\rho} + d_{f\rho}) \tag{5.42}$$

where $d_{g\rho} = g_{\rho s} - \hat{g}_s + g_r - \hat{g}_{\rho r}, d_{f\rho} = f_{\rho s} - \hat{f}_s + f_r - \hat{f}_{\rho r}, \hat{g}_{\rho r}$. In the following, similar to the previous section, stability analysis will be conducted using DSC method. The case ($s = r$) is first considered.

(1) $s = r$: Similar to the previous section, we will propose an adaptive DSC scheme for system (5.41). The recursive design procedure contains ρ steps. From Step 1 to Step ρ, virtual control laws $\alpha_{i-1}, i = 2, \ldots, \rho$ are designed at each step. Finally overall control laws $u_j, j = 1, \ldots, m$ are constructed at step ρ. Let us define dynamic surfaces S_i, $i = 1, \ldots, \rho$, virtual control laws α_i, first-order filters $\varepsilon_{i+1}\dot{z}_{i+1} + z_{i+1} = \alpha_i$ and y_{i+1}, $i = 1, \ldots, \rho - 1$ as in the previous section. Note that, the difference between the observer (5.8) and (5.41) lies in the last equality, i.e.,

$$\dot{\hat{x}}_\rho = \hat{f}_\rho + h_\rho e_1 + \sum\nolimits_{j=1}^{m} \hat{g}_j u_j$$

$$\dot{\hat{x}}_{s\rho} = \hat{f}_{s\rho} + h_{s\rho} e_1 + \sum\nolimits_{j=1, j\neq s}^{m} \hat{g}_j u_j + \hat{g}_{\rho s} u_s + \hat{f}_{\rho s}$$

Thus, $\dot{S}_i$, $i = 1, \ldots, \rho - 1$ have same expressions as in the previous section, only $\dot{S}_\rho$ is different, which is expressed as follows.

$$\begin{aligned} \dot{S}_\rho &= \dot{\hat{x}}_{s\rho} - \dot{z}_\rho \\ &= \hat{\theta}_{f\rho}^T \xi_{f\rho} + h_\rho e_1 + \sum\nolimits_{j=1, j\neq s}^{m} \hat{g}_j u_j + \hat{g}_{\rho s} u_s + \hat{f}_{\rho s} - \dot{z}_\rho + \tilde{\theta}_{f\rho}^T \xi_{f\rho} + \varepsilon_{f\rho} - \delta_{f\rho} \end{aligned} \tag{5.43}$$

Hence, $\alpha_{\rho j}, j = 1, \ldots, m$ are defined as follows:

$$\alpha_{\rho j} = u_j = [N(\varsigma)(k_\rho S_\rho + \Delta_s / S_\rho)]/m, \ \dot{\varsigma} = -k_\rho S_\rho^2 - \Delta_s \tag{5.44}$$

where

$$\begin{aligned} \Delta_s =& \mu_e + \frac{(\rho - 1)\sigma_1}{2} + S_\rho(\hat{f}_\rho + \hat{f}_{\rho s}) + \sum\nolimits_{i=1}^{\rho - 1} S_{i+1} k_{i+1} e_1 + \\ & \sum\nolimits_{i=1}^{\rho} [|S_i|(\bar{M}_{\varepsilon fi} + \bar{M}_{\delta fi}) + \frac{\eta_{fi}}{2\eta_1} \theta_{fi}^{*T} \theta_{fi}^*] + \bar{M}_{\varepsilon f\rho s} + \bar{M}_{\delta f\rho s} + \\ & \sum\nolimits_{j=1}^{m} \frac{\eta_{gj}}{2\eta_2} \theta_{gj}^{*T} \theta_{gj}^* + \frac{\eta_{f\rho s}}{2\eta_3} \theta_{f\rho s}^{*T} \theta_{f\rho s}^* + \frac{\eta_{g\rho s}}{2\eta_4} \theta_{g\rho s}^{*T} \theta_{g\rho s}^* \end{aligned}$$

Differentiating $V_{se} = e_s^T P e_s$ with respect to time t and considering (5.42) and Assumption 5.4, it leads to

$$\dot{V}_{se} \leq -e^T Q e + \mu_e = -g_e V_e + \mu_{se} \tag{5.45}$$

where $g_e = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$, $\mu_{se} = [\sum_{i=1}^{\rho} \bar{M}_{\delta fi}^2 + \bar{M}_{\delta f\rho s}^2 + \sum_{j=1}^{m} \bar{M}_{\delta gj}^2 \bar{u}_j^2 + \bar{M}_{\delta g\rho s}^2 \bar{u}_s^2]/\lambda$.

Consider the following Lyapunov function

$$V_2 = \sum_{i=1}^{\rho} S_i^2/2 + \sum_{k=1}^{\rho-1} y_{k+1}^2/2 + V_{se} + \sum_{i=1}^{\rho} \tilde{\theta}_{fi}^T \tilde{\theta}_{fi}/(2\eta_1) + \sum_{j=1}^{m} \tilde{\theta}_{gj}^T \tilde{\theta}_{gj}/(2\eta_2) + \tilde{\theta}_{f\rho s}^T \tilde{\theta}_{f\rho s}/(2\eta_3) + \tilde{\theta}_{g\rho s}^T \tilde{\theta}_{g\rho s}/(2\eta_4)$$

where $\eta_3 > 0$, $\eta_4 > 0$ are design parameters, $\tilde{\theta}_{f\rho s} = \theta_{f\rho s}^* - \hat{\theta}_{f\rho s}$, $\tilde{\theta}_{g\rho s} = \theta_{g\rho s}^* - \hat{\theta}_{g\rho s}$.

Differentiating V_2 with respect to time t, it leads to

$$\begin{aligned}
\dot{V}_2 \leq & \sum_{i=1}^{\rho-1} (S_i S_{i+1} - k_i S_i^2 + S_i y_{i+1}) + \sum_{i=1}^{\rho-1} S_{i+1} k_{i+1} e_1 + \dot{V}_{se} + \\
& S_\rho (\hat{\theta}_{f\rho}^T \xi_{f\rho} + \hat{\theta}_{f\rho}^T \xi_{f\rho}) + \sum_{k=1}^{\rho-1} (-\frac{y_{k+1}^2}{\varepsilon_{k+1}} + |y_{k+1} B_{k+1}|) + \\
& \sum_{i=1}^{\rho} S_i \tilde{\theta}_{fi}^T \xi_{fi} + \sum_{i=1}^{\rho} S_i (\varepsilon_{fi} - \delta_{fi}) + S_\rho (\varepsilon_{f\rho s} - \delta_{f\rho s}) + \\
& S_\rho \sum_{j=1, j\neq s}^{m} (\hat{g}_j + \varepsilon_{gj} - \delta_{gj} + \tilde{\theta}_{gj}^T \xi_{gj}) u_j + S_\rho \tilde{\theta}_{g\rho s}^T \xi_{g\rho s} u_s - \\
& S_\rho (\hat{g}_s + \varepsilon_{g\rho s} - \delta_{g\rho s}) u_s - \sum_{i=1}^{\rho} \frac{1}{2\eta_1} \tilde{\theta}_{fi}^T \dot{\hat{\theta}}_{fi} - \\
& \sum_{j=1, j\neq s}^{m} \frac{1}{2\eta_2} \tilde{\theta}_{gj}^T \dot{\hat{\theta}}_{gj} - \frac{1}{2\eta_3} \tilde{\theta}_{f\rho s}^T \dot{\hat{\theta}}_{f\rho s} - \frac{1}{2\eta_4} \tilde{\theta}_{g\rho s}^T \dot{\hat{\theta}}_{g\rho s}
\end{aligned} \tag{5.46}$$

Adaptive laws $\dot{\hat{\theta}}_{fi}, i = 1, \ldots, \rho$, $\dot{\hat{\theta}}_{gj}, j = 1, \ldots, m, j \neq s$ are defined as (5.29) and (5.30) in the previous section. Here, only $\dot{\hat{\theta}}_{f\rho s}$ and $\dot{\hat{\theta}}_{g\rho s}$ are defined as follows:

$$\dot{\hat{\theta}}_{f\rho s} = \begin{cases} 2\eta_3 S_\rho \xi_{f\rho s} - \eta_{f\rho s} \hat{\theta}_{f\rho s}, \text{ if} ||\hat{\theta}_{f\rho s}|| < M_{f\rho s} \text{or} \\ \quad |\hat{\theta}_{f\rho s}|| = M_{f\rho s} \text{ and } 2\eta_3 S_\rho \xi_{f\rho s} - \eta_{f\rho s} \hat{\theta}_{f\rho s} \geq 0; \\ 2\eta_3 S_\rho \xi_{f\rho s} - \eta_{f\rho s} \hat{\theta}_{f\rho s} + (2\eta_3 S_\rho \frac{\hat{\theta}_{f\rho s} \hat{\theta}_{f\rho s}^T}{||\hat{\theta}_{f\rho s}||^2} \xi_{f\rho s} - \\ \eta_{f\rho s} \frac{\hat{\theta}_{f\rho s} \hat{\theta}_{f\rho s}^T}{||\hat{\theta}_{f\rho s}||^2} \hat{\theta}_{f\rho s}), \text{ if } ||\hat{\theta}_{f\rho s}|| = M_{f\rho s} \text{ and} \\ \quad 2\eta_3 S_\rho \xi_{f\rho s} - \eta_{f\rho s} \hat{\theta}_{f\rho s} < 0 \end{cases} \tag{5.47}$$

$$\dot{\hat{\theta}}_{g\rho s} = \begin{cases} 2\eta_4 S_\rho \xi_{g\rho s} u_s - \eta_{g\rho s}\hat{\theta}_{g\rho s}, \text{ if } ||\hat{\theta}_{g\rho s}|| < M_{g\rho s} \text{ or} \\ \quad ||\hat{\theta}_{g\rho s}|| = M_{g\rho s} \text{ and } 2\eta_4 S_\rho \xi_{g\rho s} u_s - \eta_{g\rho s}\hat{\theta}_{g\rho s} \geq 0; \\ 2\eta_4 S_\rho \xi_{g\rho s} u_s - \eta_{g\rho s}\hat{\theta}_{g\rho s} + (2\eta_4 S_\rho \frac{\hat{\theta}_{g\rho s}\hat{\theta}_{g\rho s}^T}{||\hat{\theta}_{g\rho s}||^2}\xi_{g\rho s} u_s - \\ \eta_{g\rho s}\frac{\hat{\theta}_{g\rho s}\hat{\theta}_{g\rho s}^T}{||\hat{\theta}_{g\rho s}||^2}\hat{\theta}_{g\rho s}), \text{ if } ||\hat{\theta}_{g\rho s}|| = M_{g\rho s} \\ \quad \text{and } 2\eta_4 S_\rho \xi_{g\rho s} u_s - \eta_{g\rho s}\hat{\theta}_{g\rho s} < 0 \end{cases} \tag{5.48}$$

where $\eta_{f\rho s} > 0$, $\eta_{g\rho s} > 0$ are design constants.

Applying Young's inequality, one has

$$\eta_{f\rho s}\tilde{\theta}_{f\rho s}^T\hat{\theta}_{g\rho s}/\eta_3 \leq -\eta_{f\rho s}(\tilde{\theta}_{f\rho s}^T\tilde{\theta}_{f\rho s} - \theta_{f\rho s}^{*T}\theta_{f\rho s}^*)/(2\eta_3)$$

$$\eta_{g\rho s}\tilde{\theta}_{g\rho s}^T\hat{\theta}_{g\rho s}/\eta_4 \leq -\eta_{g\rho s}(\tilde{\theta}_{g\rho s}^T\tilde{\theta}_{g\rho s} - \theta_{g\rho s}^{*T}\theta_{g\rho s}^*)/(2\eta_4)$$

Substituting the above inequalities into (5.46), yields

$$\begin{aligned} \dot{V}_2 \leq & \sum\nolimits_{i=1}^{\rho-1}(S_i S_{i+1} - k_i S_i^2 + S_i y_{i+1}) + S_\rho(\hat{\theta}_{f\rho}^T\xi_{f\rho} + \hat{\theta}_{f\rho}^T\xi_{f\rho}) + \\ & \sum\nolimits_{k=1}^{\rho-1}(-\frac{y_{k+1}^2}{\varepsilon_{k+1}} + |y_{k+1}B_{k+1}|) + \sum\nolimits_{i=1}^{\rho-1} S_{i+1}k_{i+1}e_1 + \dot{V}_{se} + \\ & \sum\nolimits_{i=1}^{\rho} S_i(\varepsilon_{fi} - \delta_{fi}) + S_\rho(\varepsilon_{f\rho s} - \delta_{f\rho s}) + S_\rho(\hat{g}_s + \varepsilon_{g\rho s} - \\ & \delta_{g\rho s})u_s + S_\rho\sum\nolimits_{j=1, j\neq s}^{m}(\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j - \\ & \sum\nolimits_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}(\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} - \theta_{fi}^{*T}\theta_{fi}^*) - \frac{1}{2\eta_3}\tilde{\theta}_{f\rho s}^T\tilde{\theta}_{f\rho s} - \\ & \sum\nolimits_{j=1, j\neq s}^{m}\frac{\eta_{gj}}{2\eta_2}(\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} - \theta_{gj}^{*T}\theta_{gj}^*) - \frac{1}{2\eta_4}\tilde{\theta}_{g\rho s}^T\tilde{\theta}_{g\rho s} + \\ & \frac{\eta_{f\rho s}}{2\eta_3}\theta_{f\rho s}^{*T}\theta_{f\rho s}^* + \frac{\eta_{g\rho s}}{2\eta_4}\theta_{g\rho s}^{*T}\theta_{g\rho s}^* \end{aligned} \tag{5.49}$$

From Young's Inequality, one has

$$S_i S_{i+1} \leq S_i^2 + S_{i+1}^2/4,\ S_i y_{i+1} \leq S_i^2 + y_{i+1}^2/4,$$
$$|y_{k+1}B_{k+1}| \leq y_{k+1}^2 B_{k+1}^2/\sigma_1 + \sigma_1/2,\ \forall\sigma_1 > 0 \in R$$

where $\sigma_1 > 0 \in R$ is a design parameter. Thus, one has

$$\dot{V}_2 \le \sum_{i=1}^{\rho-1}[\frac{1}{4}S_{i+1}^2 + (2-k_i)S_i^2] + \sum_{k=1}^{\rho-1}(\frac{1}{4} - \frac{1}{\varepsilon_{k+1}} + \frac{B_{k+1}^2}{2\sigma_1})y_{k+1}^2 + \frac{(\rho-1)\sigma_1}{2} + S_\rho(\hat{\theta}_{f\rho}^T\xi_{f\rho} + \hat{\theta}_{f\rho}^T\xi_{f\rho}) + \sum_{i=1}^{\rho-1} S_{i+1}k_{i+1}e_1 + \dot{V}_{se} + \sum_{i=1}^{\rho} S_i(\varepsilon_{fi} - \delta_{fi}) + S_\rho(\varepsilon_{f\rho s} - \delta_{f\rho s}) + S_\rho\sum_{j=1,j\neq s}^{m}(\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j + S_\rho(\hat{g}_s + \varepsilon_{g\rho s} - \delta_{g\rho s})u_s - \sum_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} - \sum_{j=1,j\neq s}^{m}\frac{\eta_{gj}}{2\eta_2}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \sum_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\theta_{fi}^{*T}\theta_{fi}^* + \sum_{j=1,j\neq s}^{m}\frac{\eta_{gj}}{2\eta_2}\theta_{gj}^{*T}\theta_{gj}^* - \frac{1}{2\eta_3}\tilde{\theta}_{f\rho s}^T\tilde{\theta}_{f\rho s} - \frac{1}{2\eta_4}\tilde{\theta}_{g\rho s}^T\tilde{\theta}_{g\rho s} + \frac{\eta_{f\rho s}}{2\eta_3}\theta_{f\rho s}^{*T}\theta_{f\rho s}^* + \frac{\eta_{g\rho s}}{2\eta_4}\theta_{g\rho s}^{*T}\theta_{g\rho s}^*$$

As pointed out in [15], since for any $B_0 > 0$ and $p > 0$ the sets

$$\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le B_0\}$$

$$\Pi_i := \{\sum_{l=1}^{\rho}(S_l^2(0) + \tilde{\theta}_{fl}^T(0)\tilde{\theta}_{fl}(0)) + \tilde{\theta}_{f\rho s}^T(0)\tilde{\theta}_{f\rho s}(0) + \sum_{j=1}^{m}\tilde{\theta}_{gj}^T(0)\tilde{\theta}_{gj}(0) + \tilde{\theta}_{f\rho s}^T(0)\tilde{\theta}_{f\rho s}(0) + \sum_{k=1}^{i} y_{k+1}^2(0) +, i = 1, \ldots, \rho \quad 2e^T(0)e(0) \le 2\upsilon_s p\}$$

are compact, $\Pi_0 \times \Pi_i$ is also compact. Thus, $|B_{i+1}|$ has a maximum M_{i+1} on $\Pi_0 \times \Pi_i$, $\upsilon_s = \max\{1/\lambda_{\min}(P), 1, \eta_1, \eta_2, \eta_3, \eta_4\}$.

Choose $k_1 = 2 + \beta_0$, $k_i = 2\frac{1}{4} + \beta_0$, $k_\rho = 1\frac{1}{4} + \beta_0$, $\frac{1}{\varepsilon_{k+1}} = \frac{1}{4} + \frac{M_{k+1}^2}{2\sigma_1} + \beta_0$, $k = 1, \ldots, \rho - 1$, $i = 2, \ldots, \rho - 1$, where $\beta_0 > 0 \in R$ is a constant. Thus, from (5.45), one further has

$$\dot{V}_2 \le -\sum_{i=1}^{\rho}\beta_0 S_i^2 - \sum_{k=1}^{\rho-1}\beta_0 y_{k+1}^2 - g_e V_e - \sum_{i=1}^{\rho}\frac{\eta_{fi}}{2\eta_1}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} - \sum_{j=1,j\neq s}^{m}\frac{\eta_{gj}}{2\eta_2}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} - \frac{1}{2\eta_3}\tilde{\theta}_{f\rho s}^T\tilde{\theta}_{f\rho s} - \frac{1}{2\eta_4}\tilde{\theta}_{g\rho s}^T\tilde{\theta}_{g\rho s} + \Delta_s + S_\rho\sum_{j=1,j\neq s}^{m}(\hat{g}_j + \varepsilon_{gj} - \delta_{gj})u_j + S_\rho(\hat{g}_s + \varepsilon_{g\rho s} - \delta_{g\rho s})u_s$$

Substituting (5.44) into the above inequality, it leads to

$$\dot{V}_2 \le -g_s V_1 + (h_s N(\varsigma) + 1)\dot{\varsigma} \tag{5.50}$$

where

$$h_s = \sum_{j=1, j\neq s}^{m} (\hat{g}_j + \varepsilon_{gj} - \delta_{gj}) + \hat{g}_{\rho s} + \varepsilon_{g\rho s} - \delta_{g\rho s},$$

$$g_s = \min\{\beta_0, g_{se}, \frac{\eta_{f1}}{2\eta_1}, \ldots, \frac{\eta_{f\rho}}{2\eta_1}, \frac{\eta_{g1}}{2\eta_2}, \ldots, \frac{\eta_{gm}}{2\eta_2}, \frac{\eta_{f\rho s}}{2\eta_3}, \frac{\eta_{g\rho s}}{2\eta_4}\}.$$

Applying Lemma 5.2, we conclude that, $\int_0^t (h_s N(\varsigma) + 1)e^{-g\tau}\dot{\varsigma} d\tau$, $V_2(t)$ and $\varsigma(t)$ are SGUUB on $[0, t_f)$. According to Proposition 2 in [20], if the solution of the closed-loop system is bounded, then $t_f = +\infty$. Let $\bar{\mu}_2$ be the upper bound of $\int_0^t (h_s N(\varsigma) + 1) \cdot e^{-g\tau}\dot{\varsigma} d\tau$, we have the following inequalities:

$$\int_0^t e^{-g\tau}(h_s N(\varsigma) + 1)e^{-g\tau}\dot{\varsigma} d\tau \leq \int_0^t (h_s N(\varsigma) + 1)e^{-g\tau}\dot{\varsigma} d\tau \leq \bar{\mu}_2$$

Thus, (5.50) becomes $\dot{V}_2 \leq -g_s V_2 + \bar{\mu}_2$. Further, one has

$$0 \leq V_2(t) \leq \frac{\bar{\mu}_2}{g_s} + [V_2(0) - \frac{\bar{\mu}_2}{g_s}]e^{-g_s t} \leq \frac{\bar{\mu}_2}{g_s} + V_2(0) = \mu_2 \tag{5.51}$$

which means that $V_2(t)$ is bounded by μ_2. Thus, all signals of the closed-loop system are uniformly ultimately bounded, i.e. $|S_i| \leq \sqrt{2\mu_2}$, $|y_i| \leq \sqrt{2\mu_2}$, $||\tilde{\theta}_{fi}|| \leq \sqrt{2\eta_1\mu_2}$, $||\tilde{\theta}_{gj}|| \leq \sqrt{2\eta_2\mu_2}$, $||\tilde{\theta}_{f\rho s}|| \leq \sqrt{2\eta_3\mu_2}$, $||\tilde{\theta}_{g\rho s}|| \leq \sqrt{2\eta_4\mu_2}$, $||e|| \leq \sqrt{\mu_2/\lambda_{\min}(P)}$. That is to say, $S_i(t)$, $\tilde{\theta}_{fi}$, $\tilde{\theta}_{gj}$, $\tilde{\theta}_{g\rho s}$, $\tilde{\theta}_{f\rho s}$ belong to Ω_2 defined as:

$$\Omega_2 := \left\{ \begin{array}{l} (S_i, y_i, \tilde{\theta}_{fi}, \tilde{\theta}_{gj}, \tilde{\theta}_{f\rho s}, \tilde{\theta}_{g\rho s}) \Big| \sum_{i=1}^{\rho} (S_i^2 + \tilde{\theta}_{fi}^T\tilde{\theta}_{fj}/\eta_1) + \\ \sum_{j=1, j\neq s}^{m} \tilde{\theta}_{gj}^T\tilde{\theta}_{gj}/\eta_2 + \tilde{\theta}_{f\rho s}^T\tilde{\theta}_{f\rho s}/\eta_3 + \tilde{\theta}_{g\rho s}^T\tilde{\theta}_{g\rho s}/\eta_4 + \\ \sum_{k=1}^{\rho-1} y_{k+1}^2 + 2\lambda_{\min}(P)e^T e \leq 2\mu_2 \end{array} \right\}$$

(2) For $s \neq r$, one has $d_{g\rho} = (g_{\rho s} - \hat{g}_s)u_s + (g_r - \hat{g}_{\rho r})u_r$, $d_{f\rho} = g_s f_s^u - \hat{\theta}_{f\rho r}^T \xi_{f\rho r}$. From adaptive laws (5.29), (5.30), (5.47) and (5.48), it is found out that, $\dot{\hat{\theta}}_{g\rho r} \neq \dot{\hat{\theta}}_{g\rho s}$, $\dot{\hat{\theta}}_{f\rho r} \neq \dot{\hat{\theta}}_{f\rho s}$. Thus, both $(g_{\rho s} - \hat{g}_s)u_s + (g_r - \hat{g}_{\rho r})u_r$ and $g_s f_s^u - \hat{\theta}_{f\rho r}^T \xi_{f\rho r}$ do not converge to zero, i.e., $\lim_{t\to\infty}[(g_{\rho s} - \hat{g}_s)u_s + (g_r - \hat{g}_{\rho r})u_r] \neq 0$ and $\lim_{t\to\infty}(g_s f_s^u - \hat{\theta}_{f\rho r}^T \xi_{f\rho r}) \neq 0$. As a result, basically, all signals of the closed-loop systems such as S_i do not remain in Ω_2 using the above control law and adaptive laws. Therefore, from the above analysis, we can not obtain (5.50). Furthermore, we can not obtain (5.51). Hence, we can draw a conclusion that all signals involved in the closed-loop systems do not converge to the set Ω_2, i.e., $S_i(t)$, $\tilde{\theta}_{fi}$, $\tilde{\theta}_{gj}$, $\tilde{\theta}_{g\rho s}$, $\tilde{\theta}_{f\rho s}$ do not belong to Ω_2.

Now, the control procedures are ended. The above design procedures are summarized in the following theorem.

Theorem 5.2 *Consider the faulty system (5.40) and observers (5.41) under Assumptions 5.1–5.4, fault model (5.3) adaptive laws (5.29), (5.30), (5.47) and (5.48) and control law (5.44), If matrices* $H, Q > 0$ *and* $P = P^T > 0$ *are such that* $PA + A^T P + 2\lambda PP \leq -Q$, *for all initial conditions satisfying*

$$\Pi_i := \left\{ \begin{array}{c} \sum_{l=1}^{\rho} (S_l^2(0)+\tilde{\theta}_{fl}^T(0)\tilde{\theta}_{fl}(0)) + \tilde{\theta}_{g\rho s}^T(0)\tilde{\theta}_{g\rho s}(0) + \sum_{k=1}^{i} y_{k+1}^2(0) + \\ \sum_{j=1}^{m} \tilde{\theta}_{gj}^T(0)\tilde{\theta}_{gj}(0) + 2e^T(0)e(0) + \tilde{\theta}_{f\rho s}^T(0)\tilde{\theta}_{f\rho s}(0) \leq 2v_s p, \end{array} \right\} \tag{5.52}$$

k_i, ε_i *are chosen as follows:* $k_1 = 2 + \beta_0$, $k_i = 2\frac{1}{4} + \beta_0$, $k_\rho = 1\frac{1}{4} + \beta_0$, $\frac{1}{\varepsilon_{k+1}} = \frac{1}{4} + \frac{M_{k+1}^2}{2\sigma_1} + \beta_0$, $k = 1, \ldots, \rho - 1$, $i = 2, \ldots, \rho - 1$, *then, when the sth actuator is faulty, for* $s = r$, *the closed-loop system is semi-globally uniformly ultimately stable and all signals involved in the closed-loop systems converge to a small neighborhood of the origin* Ω_2, *and for* $s \neq r$, *all signals involved in the closed-loop systems do not converge to the set* Ω_2.

Now, we denote the residuals between the real system and isolation estimators as follows:

$$J_s(t) = \left\|\hat{y}_s(t) - y(t)\right\| = |S_1|, \quad 1 \leq \mathrm{s} \leq m$$

According to Theorem 5.2, when the rth actuator is faulty, i.e., $s = r$, the residual $J_s(t)$ must tend to Ω_2, while for any $s \neq r$, basically, $J_s(t)$ does not belong to Ω_2.

Hence, the isolation law for actuator fault can be designed as

$$\begin{cases} J_s(t) \leq T_I, l = s \Rightarrow \text{the } l\text{th actuator is faulty} \\ J_s(t) > T_I, l \neq s \end{cases}$$

where threshold T_I is defined as $T_I = \sqrt{2\mu_2}$.

5.3.3 Fault Accommodation

After that the fault information is obtained, we will consider the fault-tolerant control problem, and design a fault-tolerant control law to recover the control system's dynamics performance when an actuator fault occurs. Firstly, we consider the fuzzy control problem for the following nominal system without actuator faults:

$$\begin{cases} \dot{x}_i = \dot{x}_{i+1} + f_i(x_{[i]}), \ i = 1, 2, \ldots, \rho - 1 \\ \dot{x}_\rho = f(x, \eta) + g^T(x, \eta)u \\ \dot{\eta} = \psi_1(x, \eta) + \psi_2(x, \eta)\theta \\ y = x_1 \end{cases} \tag{5.53}$$

From Theorem 5.1, we can see that, under Assumptions 5.1–5.4, if matrices $Q > 0$ and $P = P^T > 0$ are chosen such that $PA + A^T P + 2\lambda PP \leq -Q$, and virtual control law (5.11), (5.14) and (5.17), and adaptive laws (5.29) and (5.30) are adopted, then, the closed-loop system is SGUUB stable, and all signals involved in the closed-loop systems converge to a small neighborhood of the origin Ω_1, which can be adjusted by appropriately choosing the design parameter. On the basis of the estimated actuator fault, the fault tolerant controller is constructed as

$$u_s = \hat{\rho}_{gs}(u_s^N - \hat{\rho}_{fs}(x))/(\hat{\rho}_{gs}^2 + \varepsilon_u) \tag{5.54}$$

where $\hat{\rho}_{gs}$ and $\hat{\rho}_{fs}$ are the estimates of $\rho_{gs} = g_s(x, \eta)\rho_s(x)$ and $\rho_{fs} = g_s(x, \eta)f_s^u(x)$, $\varepsilon_u > 0 \in R$ is a design parameter, u_s^N is the sth desired control input under the healthy condition.

Theorem 5.3 *Consider faulty system (5.52) under Assumptions 5.1–5.4, fault model (5.3), virtual control law (5.11), (5.14) and (5.17), and adaptive laws (5.29) and (5.30). If there exists a matrix $P = P^T > 0$ with appropriate dimensions, such that*

$$PA + A^T P + 2\lambda PP \leq -Q$$

then, system (5.52) is asymptotically stable under the feedback FTC (5.53) and all signals involved in the closed-loop system are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of the origin.

Proof Similar to the proof of Theorem 5.1, it is easy to obtain the conclusions of Theorem 3. The detailed proof is thus omitted here.

5.4 Application to Aircraft Longitudinal Motion Dynamics

In this section, for the purpose of demonstrating the application of the proposed fault tolerant control scheme, we apply it to accommodate failure for an aircraft longitudinal motion dynamics. The aircraft longitudinal motion dynamics of the twin otter can be described as follows:

$$\begin{cases} \dot{V} = [F_x \cos(\alpha) + F_z \sin(\alpha)]/m \\ \dot{\alpha} = q + [-F_x \sin(\alpha) + F_z \cos(\alpha)]/(mV) \\ \dot{\theta} = q \\ \dot{q} = M/I_y \end{cases} \tag{5.55}$$

where V is the velocity, α is the attack angle, θ is the pitch angle and q is the pitch rate, m is the mass, I_y is the moment of inertia, $F_x = \bar{q}SC_x(\alpha, q, \delta_{e1}, \delta_{e2}) + T_1 \cos\gamma_1 + T_2 \cos\gamma_2 - mg\sin(\theta)$, $F_z = \bar{q}SC_z(\alpha, q, \delta_{e1}, \delta_{e2}) + T_1 \sin\gamma_1 + T_2 \sin\gamma_2 - mg\cos(\theta)$, $M = \bar{q}cSC_m(\alpha, q, \delta_{e1}, \delta_{e2})$. For which $\bar{q} = \rho V^2/2$ is the dynamic pressure, ρ is the air density, S is the wing area, c is the mean chord, T_1 and T_2 are independent thrusts with corresponding thrust misalignments γ_1 and γ_2. C_x, C_z, C_m are of the polynomial form defined as in (5.3–5.6), δ_{e1} and δ_{e2} are the elevator angles of an augmented two-pieces elevator used as two actuators for failure compensation study. The notations through the model (5.54) are illustrated as [1]. Choosing V, α, θ and q as the states x_1, x_2, x_3 and x_4, and $\delta_{e1}, \delta_{e2}, T_1, T_2$ as the inputs u_1, u_2, u_3, u_4, as shown in [29, 30], there exists a diffeomorphism $[\eta^T, x^T]^T = T(\chi) = [T_1(\chi), T_2(\chi), x_3, x_4]^T$ such that (5.55) can be transform into the PSF form, i.e.,

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = \vartheta^T \phi(x) + \sum_{i=1}^{2} b_i x_1^2 u_i \end{cases}$$

and the zero dynamics $\dot{\xi} = \phi(\xi, \chi) + \Phi(\xi, \chi)\vartheta$, where $\vartheta \in R^4$ is an unknown constant vector. Relative degree is $o = 2$. The parameters in the simulation study are set based on the data sheet in [2, 3]. The fault case considered in this example is modeled as

$$u_1^f(t) = \begin{cases} u_1(t), \ t < 2 \\ (1 - \rho_1(x))u_1(t) + f_1^u(x), \ t \geq 2 \end{cases}, \quad u_2^f(t) = u_2(t)$$

where $\rho_1(x) = 0.4\cos(x_3)$, $f_1^u(x) = 0.2 + \sin(x_2)$. Initial values of system state are chosen as $x_1(0) = 0.1$, $x_2(0) = -0.1$, $x_3(0) = 0.1$, $x_4(0) = -0.1$. Firstly, Matlab LMI control toolbox is used to solve the matrix inequality (5.39). Therefore, one can design the desired control (5.18) and further design the fault-tolerant controller (5.53). Consequently, the observer-based fault-tolerant control input (5.53) is used to control the faulty system. Figure 5.1 shows that the tracking errors can asymptotically converge to a small neighborhood of the origin. When an actuator fault occurs at 2s, keeping the normal controller, both tracking errors deviate significantly from zero as shown in Fig. 5.2. However, as shown in Fig. 5.3, when the proposed FTC (5.53) is activated at about 2.2 s, the better convergence performance is obtained, which illustrates the effectiveness of the proposed FTC scheme.

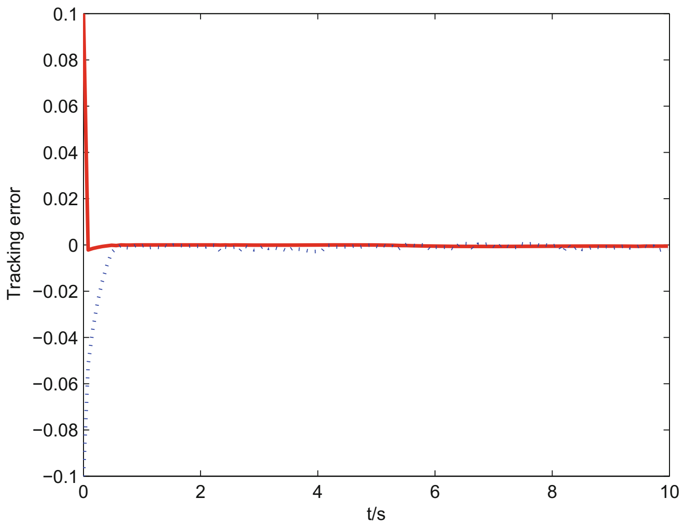

Fig. 5.1 Tracking error under normal condition

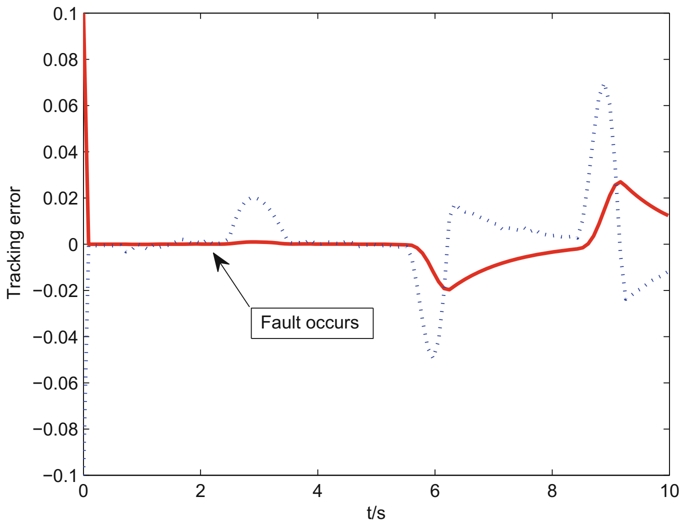

Fig. 5.2 Tracking error under faulty condition without FTC

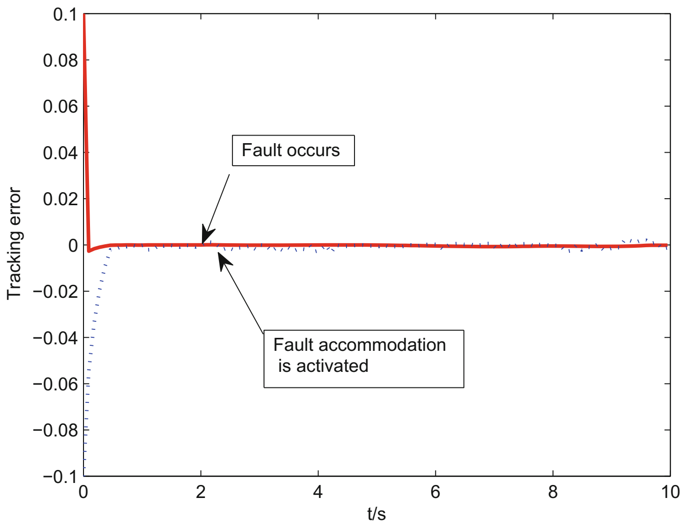

Fig. 5.3 Tracking error under faulty condition without FTC

5.5 Conclusions

In this chapter, an adaptive fuzzy tracking fault-tolerant control problem of a class of uncertain strict-feedback nonlinear systems with actuator fault has been investigated. FLSs are used to approximate the unknown nonlinear functions. By applying adaptive command filtered backstepping recursive design, integral-type Lyapunov function method and Nussbaum-type gain technique, an adaptive fuzzy control scheme is proposed to guarantee that the closed-loop system is asymptotically bounded with the tracking error converging to a neighborhood of the origin.

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Chapter 6
Adaptive Fault Tolerant Backstepping Control for High-Order Nonlinear Systems

6.1 Introduction

It is well known that system physical components may become faulty which may cause system performance deterioration or worth, may lead to instability that can further produce catastrophic accidents. The fault effects require to be compensated to enhance the reliability and safety of the system. Accommodating faults to maintain acceptable system performances is particularly important for life-critical systems. In order to improve system reliability and to guarantee system stability in all situations, many effective FTC approaches have been proposed the literature.

Fuzzy logic systems (FLSs), as universal function approximators, have been widely used to model the nonlinearities with arbitrary preciseness. Due to the capability, fuzzy logic systems are also adopted to solve identification and control problems in nonlinear systems [1–6]. Various adaptive fuzzy control approaches, based on the feedback linearization, were developed for controlling uncertain nonlinear systems. Robust adaptive backstepping control [1, 5–10] and observer-based backstepping control [11–13] attracted much attention from many researchers, and many excellent results were obtained during the past decades.

Recently, stable control problems of high-order systems attracted the interest of many researchers [14–19]. In [14], the authors presented a continuous feedback solution to the problem of global strong stabilization, for genuine nonlinear systems that may not be stabilized, even locally, by a smooth feedback. The same authors extended their results in [15], where they investigated the reference tracking problem in nonlinear systems with disturbances. However, the control schemes in [14, 15] do not guarantee the closed-loop systems' stability or better tracking performance under faulty conditions.

In this chapter, we investigate the problem of active FTC for a class of high-order nonlinear uncertain systems with actuator gain faults. Compared with some existing works, the following main contributions are worth to be emphasized:

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(1) In literature, results concerning FTC in the literature like [20–31] consider the 1-order systems. This chapter extends the results to the more general systems, i.e., so-called high-order systems as [32–37], and an observer-based active fault-tolerant backstepping control scheme is proposed.

(2) Differing from the classical backstepping technology, our fault-tolerant control scheme does not need computing the high order derivatives of virtual control signal at each step of backstepping design procedure, which thus reduces the computation complexity.

(3) In general, the denominator of the fault-tolerant control law contains the estimate of the gain fault. If the denominator equals zero, a singularity occurs. In the proposed FTC scheme, the controller singularity is avoided without using a projection algorithm.

(4) In contrast with [20–25], the proposed FTC scheme does not require the a priori knowledge of the signs of control gain terms.

The rest of this chapter is organized as follows. In Sect. 6.2, the problem formulation, Nussbaum-type function and mathematical description of FLS, are introduced. Actuator faults are described and the FTC objectives are formulated. In Sect. 6.3, the main technical results of this chapter are given, which include fault detection, isolation, estimation and fault-tolerant control scheme design. The aircraft control application is presented in Sect. 6.4 and simulation results are given and demonstrate the effectiveness of the proposed technique. Finally, Sect. 6.5 draws the conclusion.

6.2 Problem Formulation and Mathematical Description of FLSs

In this section, we will formulate control problem. Then, the FLS description is introduced.

6.2.1 Problem Statement

Considers the following nonlinear systems:

$$\begin{cases} \dot{x}_i = x_{i+1}^p, \ i = 1, \ldots, n-1 \\ \dot{x}_n = f(x) + \sum_{j=1}^{m} g_j(x) u_j^p \\ y = x_1 \end{cases} \tag{6.1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ denotes the state vector, $y = x_1$ denotes the system output, $u_j \in R$, $j = 1, 2, \ldots, m$ denote control inputs, $p \geq 1$ is a known positive odd number, $f(x) \in R$ denotes an unknown continuous smooth function,

$g_j(x) \in R$, $j = 1, \ldots, m$ are complete unknown control gain functions, i.e., the value and sign of $g_j(x)$ are both unknown.

Remark 6.1 System (6.1) is more general than the considered system in [18] which was described as $\dot{x}_i = x_{i+1}^p$, $i = 1, \ldots, n-1$ and $\dot{x}_n = u^p$. In addition, since actuator faults were not considered in [18], only one actuator was used. In this chapter, the FTC problem will be considered. In order to ensure the dependability of the controlled system, redundant actuators are added which leads to an over-actuated system.

In practical application, actuators may become faulty. In this chapter, actuator loss-of-effectiveness failures are considered, which can be modeled as follows.

$$u_j^f = k_j(x)u_j, \quad j = 1, \ldots, m, \quad t \geq t_j \tag{6.2}$$

where unknown function $k_j(x)$ denotes the remaining control rate, t_j is unknown fault occurrence time.

The control objectives, which are valid in normal (no fault) and faulty conditions, are to design the proper control inputs $u = [u_1, \ldots, u_m]^T$ which ensure that the system output can track asymptotically the reference model signal y_d with the tracking error converging to a small neighborhood of the origin and the closed-loop system is uniformly ultimately bounded (SGUUB). Under normal condition (no fault), u is designed to ensure boundedness of the closed-loop signals and asymptotic stability. Meanwhile, the FDI algorithm is working. As soon as actuator faults are detected and isolated, the fault accommodation algorithm is activated and a proper FTC input u is used such that the tracking performance is still maintained stable under faulty situation.

In order to design an appropriate controller, the following lemmas are introduced.

Lemma 6.1 ([38]) $\forall q > 1$, *being an odd integer,* $a, b \in R$, *the following inequality holds:*

$$|a+b|^q \leq (|a|+|b|)^q \leq 2^{q-1}|a^q + b^q| \tag{6.3}$$

Lemma 6.2 ([38]) $\forall m > 0 \in R, \forall n > 0 \in R$ *and* $r(x, y) > 0 \in R$, *the following inequality holds:*

$$|x|^m|y|^n \leq \frac{m}{m+n}r(x, y)|x|^{m+n} + \frac{n}{m+n}r^{-\frac{m}{n}}(x, y)|y|^{m+n} \tag{6.4}$$

Lemma 6.3 ([11]) *For* $\alpha \in R^{n_a}$, $\beta \in R^{n_b}$, $M \in R^{n_a \times n_b}$, *and arbitrary matrices* $X \in R^{n_a \times n_a}$, $Y \in R^{n_a \times n_b}$, $Z \in R^{n_b \times n_b}$, *if* $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$, *then*

$$-2\alpha^T M\beta \leq \begin{bmatrix} \alpha \\ \beta \end{bmatrix}^T \begin{bmatrix} X & Y-M \\ Y^T - M^T & Z \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{6.5}$$

6.2.2 Nussbaum Type Gain

Any continuous function $N(s) : R \to R$ is a function of Nussbaum type if it has the following properties:

$$\lim_{s\to+\infty} \sup \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = +\infty \tag{6.6}$$

$$\lim_{s\to-\infty} \inf \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = -\infty \tag{6.7}$$

For example, the continuous functions $\varsigma^2\cos(\varsigma)$, $\varsigma^2\sin(\varsigma)$, and $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ verify the above properties and are thus Nussbaum-type functions [39]. The even Nussbaum function $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ is used throughout this chapter.

Lemma 6.4 ([40, 41]) *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:*

$$V(t) \leq c_0 + \int_0^t (\underline{g}N(\varsigma) + 1)\dot{\varsigma}d\tau, \forall t \in [0, t_f) \tag{6.8}$$

where $\underline{g} \neq 0$ is a constant, and c_0 represents a suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0, t_f)$.

Lemma 6.5 ([41]) *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. For $\forall t \in [0, t_f)$, if the following inequality holds,*

$$V(t) \leq c_0 + e^{-c_1 t}\int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}e^{c_1\tau}d\tau + e^{-c_1 t}\int_0^t \dot{\varsigma}e^{c_1\tau}d\tau \tag{6.9}$$

where constant $c_1 > 0$, $\underline{g}(\cdot)$ is a time-varying parameter which takes values in the unknown closed intervals $I := [l^{-1}, l^{+1}]$ with $0 \notin I$, and c_0 represents some suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0, t_f)$.

6.2.3 Mathematical Description of FLSs

A fuzzy logic system consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

$$\begin{aligned} R^l :\ & if\ x_1\ is\ A_1^l\ and\ x_2\ \ is\ A_2^l\ \cdots\ and\ x_n\ \ is\ A_n^l, \\ & then\ \ y\ is\ B^l,\ \ l = 1, 2, \ldots, M \end{aligned} \tag{6.10}$$

where $\underline{x} = [x_1, \ldots, x_n]^T \subset R^n$ and y are the FLS input and output, respectively. Fuzzy sets A_i^l and B^l are associated with the fuzzy functions $\mu_{A_i^l}(x_i) = \exp(-(\frac{x_i - a_i^l}{b_i^l})^2)$ and $\mu_{B^l}(y^l) = 1$, respectively. M is the rules number. Through singleton function, center average defuzzification and product inference [42], the FLS can be expressed as:

$$y(x) = \sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right) \Big/ \sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right) \tag{6.11}$$

where $\bar{y}^l = \max_{y \in R} \mu_{B^l}$. Define the fuzzy basis functions as:

$$\xi_l(x) = \prod_{i=1}^{n} \mu_{A_i^l}(x_i) \sum_{l=1}^{M} \left(\prod_{i=1}^{n} / \mu_{A_i^l}(x_i) \right)$$

and define $\theta^T = [\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^M] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi(x) = [\xi_1(x), \ldots, \xi_M(x)]^T$, then the above FLS can be rewritten as:

$$y(x) = \theta^T \xi(x) \tag{6.12}$$

The stability results obtained in FLS control literature are semi-global in the sense that, as long as the input variable of the FLS remains within some pre-fixed compact set, where the compact set can be made as large as desired, there exist controllers with sufficiently large number of FLS rules such that all the signals in the closed-loop remain bounded.

Lemma 6.6 ([5, 6]) *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists a FLS such as*

$$\sup_{x \in \Omega} |f(x) - \theta^T \xi(x)| \leq \varepsilon$$

In this chapter, using FLS, the unknown functions $f(x)$, $g_j(x)$ and $g_{kj}(x)$, $j = 1, 2, \ldots, m$, are approximated as

$$\hat{f}(x) = \hat{\theta}_f^T \xi_f(x), \quad \hat{f}(\hat{x}) = \hat{\theta}_f^T \xi_f(\hat{x})$$

$$\hat{g}_j(x) = \hat{\theta}_{gj}^T \xi_{gj}(x), \quad \hat{g}_j(\hat{x}) = \hat{\theta}_{gj}^T \xi_{gj}(\hat{x})$$

$$\hat{g}_{kj}(x) = \hat{\theta}_{gkj}^T \xi_{gkj}(x), \quad \hat{g}_{kj}(\hat{x}) = \hat{\theta}_{gkj}^T \xi_{gkj}(\hat{x})$$

Let define the optimal parameter vector θ_f^*, θ_{gj}^* and θ_{gkj}^* as

$$\theta_f^* = \arg \min_{\theta \in \Omega_f} [\sup_{x \in U, \hat{x} \in \hat{U}} |f(x) - \hat{f}(\hat{x})|]$$

$$\theta_{gj}^* = \arg \min_{\theta_{gj} \in \Omega_{gj}} [\sup_{x \in U, \hat{x} \in \hat{U}} |g_j(x) - \hat{g}_{gj}(\hat{x},)|]$$

$$\theta_{gkj}^* = \arg \min_{\theta_{gkj} \in \Omega_{gkj}} [\sup_{x \in U, \hat{x} \in \hat{U}} |g_{kj}(x) - \hat{g}_{kj}(\hat{x})|]$$

where Ω_f, Ω_{gj}, Ω_{gkj}, U and $\hat{U}$ are compact regions for $\hat{\theta}_f$, $\hat{\theta}_{gj}$, $\hat{\theta}_{gkj}$, x and $\hat{x}$, respectively; $\hat{\theta}_f$, $\hat{\theta}_{gj}$, $\hat{\theta}_{gkj}$ and $\hat{x}$ are the estimates of θ_f^*, θ_{gj}^*, θ_{gkj}^* and x, respectively. Similar to [11–13], The FLS minimum approximation errors and actual approximation errors are defined as

$$\varepsilon_f = f(x) - \theta_f^{*T} \xi_f(\hat{x}), \quad \delta_f = f(x) - \hat{\theta}_f^T \xi_f(\hat{x})$$

$$\varepsilon_{gj} = g_j(x) - \theta_{gj}^{*T} \xi_{gj}(\hat{x}), \quad \delta_{gj} = g_j(x) - \hat{\theta}_{gj}^T \xi_{gj}(\hat{x})$$

$$\varepsilon_{gkj} = g_{kj}(x) - \theta_{gkj}^{*T} \xi_{gkj}(\hat{x}), \quad \delta_{gkj} = g_{kj}(x) - \hat{\theta}_{gkj}^T \xi_{gkj}(\hat{x})$$

Now, the following assumptions are made.

Assumption 6.1 There exist unknown positive real constants ε_f^*, δ_f^*, ε_{gj}^*, δ_{gj}^*, ε_{gkj}^*, δ_{gkj}^* and known positive real constants $\bar{M}_{\varepsilon f}$, $\bar{M}_{\delta f}$, $\bar{M}_{\varepsilon gj}$, $\bar{M}_{\varepsilon gkj}$, such that $|\varepsilon_f| \le \varepsilon_f^*$, $\varepsilon_f^* \le \bar{M}_{\varepsilon f}$, $|\delta_f| \le \delta_f^*$, $\delta_f^* \le \bar{M}_{\delta f}$, $|\varepsilon_{gj}| \le \varepsilon_{gj}^*$, $\varepsilon_{gj}^* \le \bar{M}_{\varepsilon gj}$, $|\varepsilon_{gkj}| \le \varepsilon_{gkj}^*$, $\varepsilon_{gkj}^* \le \bar{M}_{gkj}$.

Assumption 6.2 There exist known positive real constants $M_{\theta f}$, $M_{\theta gj}$ and M_{gkj} such that $||\theta_f^*|| \le M_{\theta f}$, $||\theta_{gj}^*|| \le M_{\theta gj}$ and $||\theta_{gkj}^*|| \le M_{\theta gkj}$.

In order to facilitate the descriptions, in the following, $f(x)$, $g(x)$, $g_{kj}(x)$, $\hat{f}(\hat{x})$, $\hat{g}(\hat{x})$, $\hat{g}_{kj}(\hat{x})$, $\xi_f(\hat{x})$, $\xi_{gj}(\hat{x})$ and $\xi_{gkj}(\hat{x})$ are abbreviated to f, g, g_{kj}, $\hat{f}$, $\hat{g}$, $\hat{g}_{kj}$, ξ_f, ξ_{gj} and ξ_{gkj}, respectively.

6.3 Main Results

In this section, the main technical results of this chapter are given. We will first consider the stability control problem of system (6.1) under normal conditions, design a bank of observers to generate residuals, investigate the FDI algorithm based on the observers, and propose a FTC scheme to tolerate the fault using estimated fault information.

6.3.1 Fault Detection

In order to detect the fault, the following observer is constructed.

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1}^p + l_i(y - \hat{y}),\ i = 1, \ldots, n-1 \\ \dot{\hat{x}}_n = \hat{f} + \sum_{j=1}^{m} [\hat{g}_j + \hat{\varepsilon}_{gj}] u_j^p + l_n(y - \hat{y}) \\ \hat{y} = \hat{x}_1 = C\hat{x} \end{cases} \tag{6.13}$$

where $l_i, i = 1, \ldots, n$ are constant parameters that will be designed later.

Let $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T$ and define observer errors $e_i = x_i - \hat{x}_i, i = 1, \ldots, n$, then observer error dynamics can be described as follows:

$$\begin{cases} \dot{e}_i = x_{i+1}^p - \hat{x}_{i+1}^p \\ \quad = (e_{i+1} + \hat{x}_{i+1})^p - \hat{x}_{i+1}^p - l_i(y - \hat{y}) \\ \quad = e_{i+1}^p - l_i(y - \hat{y}) + \sum_{l=1}^{p} C_p^l e_{i+1}^l \hat{x}_{i+1}^{p-l} \\ \dot{e}_n = f - \hat{f} + \sum_{j=1}^{m} (g_j - \hat{g}_j - \hat{\varepsilon}_{gj}) u_j^p - l_n(y - \hat{y}) \end{cases} \tag{6.14}$$

Using the notation $e = x - \hat{x}$, the above error dynamics can be re-written as:

$$\dot{e} = Ae_p + Re_p - L(y - \hat{y}) + d + B(d_f + d_g) \tag{6.15}$$

where $e_p = [e_1^p, \ldots, e_n^p]^T$, $d_i = \sum_{l=1}^{p} C_p^l e_{i+1}^l \hat{x}_{i+1}^{p-l}$, $i = 1, \ldots n-1$, $d_f = f - \hat{f} = \delta_f$, $d_g = \sum_{j=1}^{m} (g_j - \hat{g}_j - \hat{\varepsilon}_{gj}) u_i^p$, and

$$A = \begin{bmatrix} -r_1 & & \\ \vdots & & I \\ -r_n & 0 & \cdots & 0 \end{bmatrix}, R = \begin{bmatrix} r_1 \\ \vdots \\ r_n \end{bmatrix}, L = \begin{bmatrix} l_1 \\ \vdots \\ l_n \end{bmatrix}, C = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}^T, d = \begin{bmatrix} d_1 \\ \vdots \\ 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

In the following we will use the backstepping technique to design the fault-tolerant controller.

Define

$$z_1 = \hat{x}_1 - y_d, z_i = \hat{x}_i - \alpha_{i-1}(\hat{x}_1, \ldots, \hat{x}_{i-1}), i = 2, 3, \ldots, n \tag{6.16}$$

where $\alpha_0 = 0$, $z_{n+1} = 0$, and $\alpha_{i-1}, i = 1, \ldots, n-1$ are virtual controls which will be designed at each step, $\alpha_n = u$ is the actual control input. The recursive design procedure contains n steps. From Step 1 to Step $n-1$, virtual control α_{i-1} is designed at each step. Finally an overall control law u is constructed at step n.

Step 1:
From $z_1 = \hat{x}_1 - y_d$, one has

$$\begin{aligned}\dot{z}_1 = \dot{\hat{x}}_1 - \dot{y}_d = \hat{x}_2^p = (z_2 + \alpha_1)^p + l_1(y - \hat{y}) - \dot{y}_d \\ = \alpha_1^p + \sum_{j=1}^{p} C_p^j z_2^j \alpha_1^{p-j} + l_1(y - \hat{y}) - \dot{y}_d\end{aligned} \tag{6.17}$$

Define

$$V_1 = V_{11} + V_e, \quad V_{11} = \frac{1}{2} z_1^2, V_e = e^T P e$$

where $P = P^T > 0$ denotes a matrix with appropriate dimensions. Differentiating V_{11} with respect to time t leads to

$$\dot{V}_{11} = z_1 \dot{z}_1 = z_1 \alpha_1^p + z_1 \sum_{j=1}^{p} C_p^j z_2^j \alpha_1^{p-j} + z_1 l_1(y - \hat{y}) - z_1 \dot{y}_d \tag{6.18}$$

Notice that, $p + 1 \geq 2$ is an even number. Differentiating V_e with respect to time t, from Lemma 6.3, it leads to

$$\begin{aligned}\dot{V}_e = 2e^T[P(A + K) + (A + K)^T P]e_p + 2e^T Pd + 2e^T PBd_f - e^T(PLC + C^T L^T P)e \\ \leq \begin{bmatrix} e \\ e_p \end{bmatrix}^T \begin{bmatrix} X - PLC - C^T L^T P & Y + P(A + R) \\ Y^T + (A + R)^T P & Z \end{bmatrix} \begin{bmatrix} e \\ e_p \end{bmatrix} + 2e^T P(d + Bd_f + Bd_g)\end{aligned} \tag{6.19}$$

where X, Y, Z denote matrices with appropriate dimensions, and $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$.

From Lemma 6.2, one has

$$\begin{aligned}\sum_{k=1}^{p} C_p^k e_2^k \hat{x}_2^{p-k} \leq \sum_{k=1}^{p} C_p^k \frac{k}{p} |e_2|^p \cdot \sigma + \sum_{k=1}^{p} C_p^k \frac{p-k}{p} |\hat{x}_2|^p \cdot \sigma^{-\left(\frac{k}{p-k}\right)} \\ = \left[\sum_{k=1}^{p} C_p^k \frac{k}{p} \sigma\right] \cdot |e_2|^p + \left[\sum_{k=1}^{p} C_p^k \frac{p-k}{p} \sigma^{-\left(\frac{k}{p-k}\right)}\right] \cdot |\hat{x}_2|^p \\ = w_{e1}|e_2|^p + w_{e2}|\hat{x}_2|^p\end{aligned} \tag{6.20}$$

where $w_{e1} = \left[\sum_{k=1}^{p} C_p^k \frac{k}{p} \sigma\right]$, $w_{e2} = \left[\sum_{k=1}^{p} C_p^k \frac{p-k}{p} \sigma^{-\left(\frac{k}{p-k}\right)}\right]$.

Define

$$\sigma = \frac{p}{\lambda \sum_{k=1}^{p} C_p^k k}$$

where $\lambda > 1$ is a design parameter. Since $0 < \sigma \leq 1$, one has $w_{e1}|e_2|^p \leq \frac{1}{\lambda}|e_2|^p$. Therefore,

$$\sum_{k=1}^{p} C_p^k e_2^k \hat{x}_2^{p-k} \leq \frac{1}{\lambda}|e_2|^p + w_{e2}|\hat{x}_2|^p.$$

Further one has

$$(\sum_{k=1}^{p} C_p^k e_2^k \hat{x}_2^{p-k})^2 \leq \frac{2}{\lambda^2}(|e_2|^p)^2 + 2(w_{e2})^2(|\hat{x}_2|^p)^2.$$

Similarly, one has

$$(\sum_{k=1}^{p} C_p^k e_i^k \hat{x}_i^{p-k})^2 \leq \frac{2}{\lambda^2}|e_i|^p + 2(w_{e2})^2(|\hat{x}_i|^p)^2, i = 2, \ldots, n$$

Hence,

$$\begin{aligned}
d^T d &\leq [\frac{1}{\lambda}|e_2|^p + w_{e2}|\hat{x}_2|^p, \ldots, \frac{1}{\lambda}|e_n|^p + w_{e2}|\hat{x}_n|^p, 0] \begin{bmatrix} \frac{1}{\lambda}|e_2|^p + w_{e2}|\hat{x}_2|^p \\ \vdots \\ \frac{1}{\lambda}|e_n|^p + w_{e2}|\hat{x}_n|^p \\ 0 \end{bmatrix} \\
&= \sum_{i=2}^{n} \frac{2}{\lambda^2}(|e_i|^p)^2 + 2(w_{e2})^2(|\hat{x}_i|^p)^2 \\
&= \frac{2}{\lambda^2} \sum_{i=2}^{n} (|e_i|^p)^2 + \sum_{i=2}^{n} 2(w_{e2})^2(|\hat{x}_i|^p)^2 \\
&= [|e_2|^p, \ldots, |e_n|^p, 0] \begin{bmatrix} |e_2|^p \\ \vdots \\ |e_n|^p \\ 0 \end{bmatrix} + \sum_{i=2}^{n} 2(w_{e2})^2(|\hat{x}_i|^p)^2 \\
&= [|e_1|^p, |e_2|^p, \ldots, |e_n|^p] \begin{bmatrix} |e_1|^p \\ |e_2|^p \\ \vdots \\ |e_n|^p \end{bmatrix} - (e_1{}^p)^2 + \sum_{i=2}^{n} 2(w_{e2})^2(|\hat{x}_i|^p)^2 \\
&= e_p^T e_p - (e_1{}^p)^2 + \sum_{i=2}^{n} 2(w_{e2})^2(|\hat{x}_i|^p)^2
\end{aligned}$$

From Young's inequality, one has

$$e^T P d \leq e^T P P^T e + d^T d \leq e^T P P e + e_p^T e_p - (e_1{}^p)^2 + \sum_{i=2}^{n} 2(w_{e2})^2(|\hat{x}_i|^p)^2$$

$$2e^T B P d_f = e^T P B \delta_f \leq e^T P P e + \delta_f^2 \leq e^T P P^T e + (\delta_f^*)^2 \leq e^T P P e + (\bar{M}_{\delta f})^2.$$

Further, one has

$$\dot{V}_e \leq \begin{bmatrix} e \\ e_p \end{bmatrix}^T \begin{bmatrix} X - PLC - C^T L^T P + 2PP & Y + P(A+R) \\ Y^T + (A+R)^T P & Z + I \end{bmatrix} \begin{bmatrix} e \\ e_p \end{bmatrix} + \bar{\Delta}_0 + 2e^T P B d_g$$

where $\bar{\Delta}_0 = -(e_1{}^p)^2 + \sum_{i=2}^n 2(w_{e2})^2(|\hat{x}_i|^p)^2 + (\bar{M}_{\delta f})^2$, I denotes identity matrix with appropriate dimensions.

Hence, one has

$$\dot{V}_1 \leq z_1\alpha_1^p + z_1\sum_{j=1}^p C_p^j z_2^j \alpha_1^{p-j} + z_1 l_1(y-\hat{y}) - z_1\dot{y}_d + \Delta_0 + 2e^T PBd_g + \begin{bmatrix} e \\ e_p \end{bmatrix}^T \begin{bmatrix} X - PLC - C^T L^T P + 2PP & Y + P(A+R) \\ Y^T + (A+R)^T P & Z + I \end{bmatrix} \begin{bmatrix} e \\ e_p \end{bmatrix}$$

Obviously, if matrices $X, Y, Z, Q > 0$ and $P = P^T > 0$ are chosen appropriately such that $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$ and

$$\begin{bmatrix} X - PLC - C^T L^T P + 2PP & Y + P(A+R) \\ Y^T + (A+R)^T P^T & Z + I \end{bmatrix} \leq -Q$$

where I denotes identity matrix with appropriate dimensions, then,

$$\begin{aligned} \dot{V}_1 &\leq z_1\alpha_1^p + z_1\sum_{j=1}^p C_p^j z_2^j \alpha_1^{p-j} + z_1 l_1(y-\hat{y}) - z_1\dot{y}_d + \bar{\Delta}_0 + \\ &\quad 2e^T PBd_g - \begin{bmatrix} e \\ e_p \end{bmatrix}^T Q \begin{bmatrix} e \\ e_p \end{bmatrix} \\ &\leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T Pe + z_1\alpha_1^p + z_1\sum_{j=1}^p C_p^j z_2^j \alpha_1^{p-j} + \\ &\quad z_1 l_1(y-\hat{y}) - z_1\dot{y}_d + \bar{\Delta}_0 + 2e^T PBd_g \end{aligned} \tag{6.21}$$

Let $\Delta_0 = z_1 l_1(y-\hat{y}) - z_1\dot{y}_d + \bar{\Delta}_0$, one has

$$\dot{V}_1 \leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T Pe + z_1\alpha_1^p + z_1\sum_{j=1}^p C_p^j z_2^j \alpha_1^{p-j} + \Delta_0 + 2e^T PBd_g \tag{6.22}$$

Thus, virtual control α_1 can be modified as

$$\alpha_1 = \begin{cases} \sqrt[p]{(-\frac{1}{2}z_1 - \frac{\Delta_0}{z_1}}, \; z_1 \in \Omega_{c_{z_1}}^0 \\ 0, \; z_1 \in \Omega_{c_{z_1}} \end{cases} \tag{6.23}$$

Remark 6.2 In general, virtual control α_1 can be chosen as follows

$$\alpha_1 = \sqrt[p]{(-\frac{1}{2}z_1 - \frac{\Delta_0}{z_1}} \tag{6.24}$$

Just as pointed out in [41], for the above virtual control (6.23), controller singularity may occur since $\frac{\Delta_0}{z_1}$ is not well defined at $z_1 = 0$. Therefore, care must be taken to guarantee the boundedness of the control. It is noted that the controller singularity takes place at the point $z_1 = 0$, where the control objective is supposed to be achieved. From a practical point of view, once the system reaches its origin, no control action should be taken for less power consumption. As $z_1 = 0$ is hard to detect owing to the existence of measurement noise, it is more practical to relax our control objective of convergence to a "ball" rather than to the origin.

Similar to [41], let define $\Omega_{c_{z_i}} \subset \Omega$ and $\Omega^0_{c_{z_i}}$ s.t.

$$\Omega_{c_{z_i}} := \{z_i | \, |z_i| < c_{z_i}\} \Omega^0_{c_{z_i}} := \Omega - \Omega_{c_{z_i}}, \; i = 1, \ldots, m$$

where $c_{z_i} > 0$ is a constant that can be chosen arbitrarily small and "-" is used to denote the complement of set B in set A as $A - B := \{x | x \in A \text{ and } x \notin B\}$. Thus, virtual control α_1 can be modified as (6.23).

Step 2.

Since $z_2 = \hat{x}_2 - \alpha_1$, one has

$$\begin{aligned}
\dot{z}_2 &= \dot{\hat{x}}_2 - \frac{\partial \alpha_1}{\partial \hat{x}_1}(\hat{x}_2^p + l_i(y - \hat{y})) = \hat{x}_3^p - \frac{\partial \alpha_1}{\partial \hat{x}_1}\hat{x}_2^p - \frac{\partial \alpha_1}{\partial \hat{x}_1} l_1(y - \hat{y}) \\
&= (z_3 + \alpha_2)^p - \frac{\partial \alpha_1}{\partial \hat{x}_1}(z_2 + \alpha_1)^p - \frac{\partial \alpha_1}{\partial \hat{x}_1} l_i(y - \hat{y}) \\
&= \alpha_2^p + \sum\nolimits_{j=1}^{p} C_p^j z_3^j \alpha_2^{p-j} - \frac{\partial \alpha_1}{\partial \hat{x}_1} \sum\nolimits_{j=0}^{p} C_p^j z_2^j \alpha_1^{p-j} - \frac{\partial \alpha_1}{\partial \hat{x}_1} l_1(y - \hat{y})
\end{aligned} \tag{6.25}$$

Define

$$V_2 = V_1 + \frac{1}{2} z_2^2$$

Differentiating V_2 with respect to time t, leads to

$$\begin{aligned}
\dot{V}_2 \leq &\dot{V}_1 + \dot{z}_2 z_2 = -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T P e - \frac{1}{2} z_1^2 + z_1 \sum\nolimits_{j=1}^{p} \left[C_p^j \alpha_1^{p-j} z_2^j \right] + z_2 \alpha_2^p + \\
&z_2 \sum\nolimits_{j=1}^{p} C_p^j z_3^j \alpha_2^{p-j} - z_2 \frac{\partial \alpha_1}{\partial \hat{x}_1} \sum\nolimits_{j=0}^{p} C_p^j z_2^j \alpha_1^{p-j} - z_2 \frac{\partial \alpha_1}{\partial \hat{x}_1} l_1(y - \hat{y}) + 2e^T P B d_g
\end{aligned}$$

Let

$$\Delta_1 = \left\{ z_1 \sum\nolimits_{j=1}^{p} \left[C_p^j |\alpha_1^{p-j} z_2^{j-1}| \right] + \frac{\partial \alpha_1}{\partial \hat{x}_1} \sum\nolimits_{j=0}^{p} C_p^j |z_2^j \alpha_1^{p-j}| + |z_2| \frac{\partial \alpha_1}{\partial \hat{x}_1} l_1(y - \hat{y})| \right\} \tag{6.26}$$

$$\dot{V}_2 \leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T P e - \frac{1}{2} z_1^2 + \Delta_1 + z_2 \alpha_2^p + z_2 \sum\nolimits_{j=1}^{p} C_p^j z_3^j \alpha_2^{p-j} + 2e^T P B d_g \tag{6.27}$$

Similarly, choose a virtual control as follows

$$\alpha_2 = \begin{cases} \sqrt[p]{(-\frac{1}{2}z_2 - \frac{\Delta_1}{z_2}}, \ z_2 \in \Omega^0_{c_{z_2}} \\ 0, \ z_2 \in \Omega_{c_{z_2}} \end{cases} \tag{6.28}$$

Substituting α_2 into (6.27), it yields

$$\dot{V}_2 \le -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T Pe - \frac{1}{2}z_1^2 - \frac{1}{2}z_2^2 + z_2 \sum\nolimits_{j=1}^{p} C_p^j z_3^j \alpha_2^{p-j} + 2e^T PBd_g \tag{6.29}$$

Step k:
Since $z_k = \hat{x}_k - \alpha_{k-1}$, one has

$$\begin{aligned} \dot{z}_k &= \dot{\hat{x}}_k - \sum\nolimits_{l=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y - \hat{y})) \\ &= \hat{x}_{k+1}^p - \sum\nolimits_{l=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y - \hat{y})) \\ &= (z_{k+1} + \alpha_k)^p - \sum\nolimits_{l=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y - \hat{y})) \\ &= \alpha_k^p + \sum\nolimits_{j=1}^{p} C_p^j z_{k+1}^j \alpha_k^{p-j} - \sum\nolimits_{l=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y - \hat{y})) \end{aligned} \tag{6.30}$$

Define

$$V_k = V_{k-1} + \frac{1}{2}z_k^2$$

Differentiating V_k with respect to time t, leads to

$$\begin{aligned} \dot{V}_k \leqslant &-\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T Pe - \frac{1}{2}\sum\nolimits_{i=1}^{k-1} z_i^2 + \Delta_{k-1} + z_k \alpha_k^p + \\ &z_k \sum\nolimits_{j=1}^{p} C_p^j z_{k+1}^j \alpha_k^{p-j} + 2e^T PBd_g \end{aligned} \tag{6.31}$$

where

$$\Delta_{k-1} = \left\{ \begin{aligned} &z_{k-1} \sum\nolimits_{j=1}^{p} \left[C_p^j |\alpha_{k-1}^{p-j} z_k^{j-1}| \right] + \\ &\sum\nolimits_{i=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{x}_i} [\sum\nolimits_{l=0}^{p} C_p^l |z_k^l \alpha_{k-1}^{p-l}| + |l_i(y - \hat{y})|] \end{aligned} \right\}.$$

Just as α_{k-1}, virtual control α_k is chosen as follows

$$\alpha_k = \begin{cases} \sqrt[p]{(-\frac{1}{2}z_k - \frac{\Delta_{k-1}}{z_k}}, \ z_k \in \Omega^0_{c_{zk}} \\ 0, \ z_k \in \Omega_{c_{z_k}} \end{cases} \tag{6.32}$$

Substituting α_k into (6.28), yields

$$\dot{V}_k \leqslant -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T P e - \frac{1}{2}\sum_{i=1}^{k} z_i^2 + \sum_{j=1}^{p} C_p^j z_{k+1}^j \alpha_k^{p-j} + 2e^T P B d_g \quad (6.33)$$

Step n:
Since $z_n = \hat{x}_n - \alpha_{n-1}$, one has

$$\begin{aligned}
\dot{z}_n &= \dot{\hat{x}}_n - \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y-\hat{y})) \\
&= \hat{f} + \sum_{j=1}^{m} (\hat{g}_j + \varepsilon_j) u_j^p + l_n(y-\hat{y}) - \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y-\hat{y})) \\
&= f - \delta_f + \sum_{j=1}^{m} (\hat{g}_j + \varepsilon_j) u_j^p + l_n(y-\hat{y}) - \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y-\hat{y})) \\
&= \tilde{\theta}_f^T \xi_f + \hat{\theta}_f^T \xi_f + \gamma_f + \sum_{j=1}^{m} (\hat{g}_j + \varepsilon_j) u_j^p + l_n(y-\hat{y}) - \\
&\quad \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_l}(\hat{x}_{l+1}^p + l_{l+1}(y-\hat{y}))
\end{aligned} \quad (6.34)$$

Define

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2\eta_1} \tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\eta_2} \tilde{\gamma}_f^2 + \frac{1}{2\eta_3} \sum_{j=1}^{m} (\tilde{\theta}_{gj}^T \tilde{\theta}_{gj} + \tilde{\varepsilon}_{gj}^2) \quad (6.35)$$

where $\gamma_f^* = \varepsilon_f^* + \delta_f^*$, $\tilde{\gamma}_f = \gamma_f^* - \hat{\gamma}_f$, $\tilde{\theta}_f = \theta_f^* - \hat{\theta}_f$, $\tilde{\gamma}_f = \gamma_f^* - \hat{\gamma}_f$, $\tilde{\theta}_{gj} = \theta_{gj}^* - \hat{\theta}_{gj}$, $\tilde{\varepsilon}_{gj} = \varepsilon_{gj}^* - \hat{\varepsilon}_{gj}$, $\hat{\theta}_f$, $\hat{\gamma}_f$, $\hat{\theta}_{gj}$, $\hat{\varepsilon}_{gj}$ are the estimates of θ_f^*, γ_f^*, θ_{gj}^*, ε_{gj}^*, and $\eta_1 > 0$, $\eta_2 > 0$, $\eta_3 > 0$ are adaptive rates.

Differentiating V_n with respect to time t, leads to

$$\begin{aligned}
\dot{V}_n &\leqslant -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T P e - \frac{1}{2}\sum_{i=1}^{n-1} z_i^2 + z_n \tilde{\theta}_f^T \xi_f + |z_n| \tilde{\gamma}_f + z_n \sum_{j=1}^{m} (\hat{g}_j + \hat{\varepsilon}_j) u_j^p + \\
&\quad \Delta_{n-1} - \frac{1}{\eta_1} \tilde{\theta}_f^T \dot{\hat{\theta}}_f - \frac{1}{\eta_2} \tilde{\gamma}_f \dot{\hat{\gamma}}_f + 2e^T P d_g - \frac{1}{\eta_3} \sum_{j=1}^{m} (\tilde{\theta}_{gj}^T \dot{\hat{\theta}}_{gj} + \tilde{\varepsilon}_{gj} \dot{\hat{\varepsilon}}_{gj}) \\
&\leqslant -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T P e - \frac{1}{2}\sum_{i=1}^{n-1} z_i^2 + z_n \sum_{j=1}^{m} (\hat{g}_j + \hat{\varepsilon}_j) u_j^p + \Delta_{n-1} + 2e^T P d_g - \\
&\quad \frac{1}{\eta_3} \sum_{j=1}^{m} (\tilde{\theta}_{gj}^T \dot{\hat{\theta}}_{gj} + \tilde{\varepsilon}_{gj} \dot{\hat{\varepsilon}}_{gj}) + \tilde{\theta}_f^T (z_n \xi_f - \frac{1}{\eta_1} \dot{\hat{\theta}}_f) + \tilde{\gamma}_f (|z_n| - \frac{1}{\eta_2} \dot{\hat{\gamma}}_f)
\end{aligned}$$

where

$$\Delta_{n-1} = \left\{ \begin{array}{l} z_{n-1} \sum_{j=1}^{p} \left[C_p^j |\alpha_{n-1}^{p-j} z_n^{j-1}| \right] + z_n(\hat{\theta}_f^T \xi_f(\hat{x}, v) + l_n(y - \hat{y}) + \\ \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_i} [\sum_{j=0}^{p} C_p^j |z_k^j \alpha_{k-1}^{p-j}| + |l_i(y - \hat{y})|] - \\ \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_j} (\hat{x}_{j+1}^p + l_{j+1}(y - \hat{y})) + |z_n| \hat{\gamma}_f \end{array} \right\}.$$

Since

$$\begin{aligned} 2e^T PBd_g &= \sum_{j=1}^{m} 2e^T P_n (g_j - \hat{g}_j - \hat{\varepsilon}_{gj}) u_j^p \\ &= \sum_{j=1}^{m} 2e^T P_n (\theta_{gj}^{*T} \xi_{gj} + \varepsilon_{gj} - \hat{\theta}_{gj}^T \xi_{gj} - \hat{\varepsilon}_{gj})) u_j^p) \\ &= \sum_{j=1}^{m} 2e^T P_n \tilde{\theta}_{gj}^T \xi_{gj} u_j^p + \\ &\quad \sum_{j=1}^{m} 2e^T P_n (\varepsilon_{gj}^* - \hat{\varepsilon}_{gj}) u_j^p + \sum_{j=1}^{m} 2e^T P_n (\varepsilon_{gj} - \varepsilon_{gj}^*) u_j^p \\ &= \sum_{j=1}^{m} 2e^T P_n \tilde{\theta}_{gj}^T \xi_{gj} u_j^p + \sum_{j=1}^{m} 2e^T P_n \tilde{\varepsilon}_{gj} u_j^p + \\ &\quad \sum_{i=1}^{m} 2e^T P_n (\varepsilon_{gj} - \varepsilon_{gj}^*) u_j^p \end{aligned}$$

from the above inequality, one has

$$\begin{aligned} \dot{V}_n \leqslant & -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T Pe - \frac{1}{2} \sum_{i=1}^{n-1} z_i^2 + z_n \tilde{\theta}_f^T \xi_f + |z_n| \tilde{\gamma}_f + z_n \sum_{j=1}^{m} (\hat{g}_j + \hat{\varepsilon}_j) u_j^p + \\ & \Delta_{n-1} + \sum_{j=1}^{m} 2e^T P_n (\tilde{\theta}_{gj}^T \xi_{gj} + \tilde{\varepsilon}_{gj}) u_j^p + \sum_{i=1}^{m} 2e^T P_n (\varepsilon_{gj} - \varepsilon_{gj}^*) u_j^p - \\ & \frac{1}{\eta_1} \tilde{\theta}_f^T \dot{\hat{\theta}}_f - \frac{1}{\eta_2} \tilde{\gamma}_f \dot{\hat{\gamma}}_f - \frac{1}{\eta_3} \sum_{j=1}^{m} (\tilde{\theta}_{gj}^T \dot{\hat{\theta}}_{gj} + \tilde{\varepsilon}_{gj} \dot{\hat{\varepsilon}}_{gj}) \\ = & -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} e^T Pe + \sum_{j=1}^{m} [z_n(\hat{g}_j + \hat{\varepsilon}_{gj}) + 2e^T P_n (\varepsilon_{gj} - \varepsilon_{gj}^*)] u_j^p - \\ & \sum_{j=1}^{m} [\tilde{\theta}_{gj}^T (2e^T P_n \xi_{gj} u_j^p - \frac{\dot{\hat{\theta}}_{gj}}{\eta_3}) + \tilde{\varepsilon}_{gj} (2e^T P_n u_j^p - \frac{\dot{\hat{\varepsilon}}_{gj}}{\eta_3})] + \\ & \tilde{\theta}_f^T (z_n \xi_f - \frac{1}{\eta_1} \dot{\hat{\theta}}_f) + \tilde{\gamma}_f (|z_n| - \frac{1}{\eta_2} \dot{\hat{\gamma}}_f) - \frac{\sum_{i=1}^{n-1} z_i^2}{2} + \Delta_{n-1} \end{aligned} \tag{6.36}$$

Choose control law $\alpha_{n,i}$, $i = 1, 2, \ldots, m$ and adaptation functions $\dot{\hat{\theta}}_f$, $\dot{\hat{\gamma}}_f$, $\dot{\hat{\theta}}_{gj}$, $\dot{\hat{\varepsilon}}_{gj}$ as follows:

$$\alpha_{n,i} = u_i = \alpha_k = \left\{ \begin{array}{l} \sqrt[p]{\dfrac{N(\varsigma)(-\frac{1}{2} z_n - \frac{\Delta_{n-1}}{z_n})}{m}}, \ z_k \in \Omega_{c_{z_n}}^0 \\ 0, \ z_k \in \Omega_{c_{z_n}} \end{array} \right. \tag{6.37}$$

where $\dot{\varsigma} = -\frac{1}{2}z_n^2 - \Delta_{n-1}$,

$$\dot{\hat{\theta}}_f = \eta_1 z_n \xi_f - \eta_f \hat{\theta}_f \tag{6.38}$$

$$\dot{\hat{\gamma}}_f = \eta_2 |z_n| - \eta_\gamma \hat{\gamma}_f \tag{6.39}$$

$$\dot{\hat{\theta}}_{gj}^T = 2\eta_3 e^T P_n \xi_{gj} u_j^p + \eta_{gj} \hat{\theta}_{gj} \tag{6.40}$$

$$\dot{\hat{\varepsilon}}_{gj} = 2\eta_3 e^T P_n u_j^p + \eta_{gj} \hat{\varepsilon}_{gj} \tag{6.41}$$

and $\eta_f > 0, \eta_\gamma > 0, \eta_{gj} > 0$ are design parameters, u_j is a bounded control input which is applied simultaneously to the ith actuator in the system (6.1) and the observer (6.13).

Applying Young's inequality, one has

$$\frac{\eta_f}{\eta_1}\tilde{\theta}_f^T \hat{\theta}_f = \frac{\eta_f}{\eta_1}\tilde{\theta}_f^T(\theta_f^* - \tilde{\theta}_f) = -\frac{\eta_f}{\eta_1}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{\eta_f}{\eta_1}\tilde{\theta}_f^T\theta_f^* \leqslant -\frac{\eta_f}{2\eta_1}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{\eta_f}{2\eta_1}\theta_f^{*T}\theta_f^*,$$

$$\frac{\eta_\gamma}{\eta_2}\tilde{\gamma}_f\hat{\gamma}_f = \frac{\eta_\gamma}{\eta_2}\tilde{\gamma}_f(\gamma_f^* - \tilde{\gamma}_f) = -\frac{\eta_\gamma}{\eta_2}\tilde{\gamma}_f^2 + \frac{\eta_\gamma}{\eta_2}\tilde{\gamma}_f\gamma_f^* \leqslant -\frac{\eta_\gamma}{2\eta_2}\tilde{\gamma}_f^2 + (\frac{\eta_\gamma}{2\eta_2}\gamma_f^*)^2,$$

$$\frac{\eta_{gj}}{\eta_1}\tilde{\theta}_{gj}^T\hat{\theta}_{gj} = \frac{\eta_{gj}}{\eta_1}\tilde{\theta}_{gj}^T(\theta_{gj}^* - \tilde{\theta}_{gj}) = -\frac{\eta_{gj}}{\eta_1}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \frac{\eta_{gj}}{\eta_1}\tilde{\theta}_{gj}^T\theta_{gj}^* \leqslant -\frac{\eta_{gj}}{2\eta_1}\tilde{\theta}_{gj}^T\tilde{\theta}_{gj} + \frac{\eta_{gj}}{2\eta_1}\theta_{gj}^{*T}\theta_{gj}^*$$

$$\frac{\eta_{gj}}{\eta_3}\tilde{\varepsilon}_{gj}\hat{\varepsilon}_{gj} = \frac{\eta_{gj}}{\eta_3}\tilde{\varepsilon}_{gj}(\varepsilon_{gj}^* - \tilde{\varepsilon}_{gj}) = -\frac{\eta_{gj}}{\eta_3}\tilde{\varepsilon}_{gj}^2 + \frac{\eta_{gj}}{\eta_3}\tilde{\varepsilon}_{gj}\varepsilon_{gj}^* \leqslant -\frac{\eta_{g,i}}{2\eta_3}\tilde{\varepsilon}_{gj}^2 + \frac{\eta_{gj}}{2\eta_3}(\varepsilon_{gj}^*)^2$$

Substituting the above inequalities into (6.36), it yields

$$\begin{aligned}
\dot{V}_n \leqslant & -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}e^T Pe - \frac{1}{2}\sum\nolimits_{i=1}^n z_i^2 + \\
& \sum\nolimits_{i=1}^m \{z_n(\hat{g}_j + \hat{\varepsilon}_{gj}) + 2e^T P_n(\varepsilon_{gj} - \varepsilon_{gj}^*)]N(\varsigma)\dot{\varsigma} + \dot{\varsigma}\} \\
\leqslant & -gV_n + \frac{\eta_f}{2\eta_1}\theta_f^{*T}\theta_f^* + (\frac{\eta_\gamma}{2\eta_2}\gamma_f^*)^2 + \sum\nolimits_{j=1}^m (\frac{\eta_{gj}}{2\eta_1}\theta_{gj}^{*T}\theta_{gj}^* + \frac{\eta_{gj}}{2\eta_3}(\varepsilon_{gj}^*)^2) + \\
& \sum\nolimits_{i=1}^m \{z_n(\hat{g}_j + \hat{\varepsilon}_{gj}) + 2e^T P_n(\varepsilon_{gj} - \varepsilon_{gj}^*)]N(\varsigma)\dot{\varsigma} + \dot{\varsigma}\} \\
\leqslant & -gV_n + \frac{\eta_f}{2\eta_1}M_{\theta f}^2 + \frac{\eta_\gamma}{2\eta_2}(\bar{M}_{\varepsilon f} + \bar{M}_{\delta f})^2 + \sum\nolimits_{j=1}^m (\frac{\eta_{gj}}{2\eta_1}M_{\theta gj}^2 + \frac{\eta_{gj}}{2\eta_3}\bar{M}_{\varepsilon gj}^2) \\
& \sum\nolimits_{j=1}^m \{z_n(\hat{g}_j + \hat{\varepsilon}_{gj}) + 2e^T P_n(\varepsilon_{gj} - \varepsilon_{gj}^*)]N(\varsigma)\dot{\varsigma} + \dot{\varsigma}\} \\
\leqslant & -gV_n + \mu + hN(\varsigma) + 1)\dot{\varsigma}
\end{aligned} \tag{6.42}$$

where

$$g = \min\{\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}, \frac{1}{2}, \frac{\eta_f}{2\eta_1}, \frac{\eta_\gamma}{2\eta_2}, \frac{\eta_{g1}}{2\eta_3}, \ldots, \frac{\eta_{gm}}{2\eta_3}\},$$

$$\mu = \frac{\eta_f}{2\eta_1} M_{\theta f}^2 + \frac{\eta_\gamma}{2\eta_2}(\bar{M}_{\varepsilon f} + \bar{M}_{\delta f})^2 + \sum\nolimits_{j=1}^{m} \frac{\eta_{gj}}{2\eta_3}(\bar{M}_{\theta gj}^2 + \bar{M}_{\varepsilon gj}^2),$$

$$h = \sum\nolimits_{j=1}^{m} [z_n(\hat{g}_j + \hat{\varepsilon}_{gj}) + 2e^T P_n(\varepsilon_{gj} - \varepsilon_{gj}^*)].$$

The above control design procedures and analysis are summarized in the following theorem.

Theorem 6.1 *Consider nonlinear system (6.1) under Assumptions 6.1 and 6.2, control law (6.37) and adaptive laws (6.38–6.41). If matrices X, Y, Z, $Q > 0$ and $P = P^T > 0$ are such that $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$ and*

$$\begin{bmatrix} X - PLC - C^T L^T P + 2PP & Y + P(A + R) \\ Y^T + (A + R)^T P & Z + I \end{bmatrix} \leqslant -Q \tag{6.43}$$

we can guarantee the following properties under bounded initial conditions

(1) all signals in the closed-loop system are semi-globally uniformly ultimately bounded;

(2) the vectors z_i remain in the compact set $\Omega_{z_i}^0$, $i = 1, 2, \ldots, n$ specified as

$$\Omega_{z_i}^0 := \left\{ \begin{array}{l} (z_i, \tilde{\theta}_f, \tilde{\gamma}_f, \tilde{\theta}_{gj}, \tilde{\varepsilon}_{gj}, e) \Big| \, |z_i| \leqslant \sqrt{2\bar{\mu}}, ||\tilde{\theta}_f|| \leqslant \sqrt{2\eta_1\bar{\mu}}, \\ |\tilde{\gamma}_f| \leqslant \sqrt{2\eta_2\bar{\mu}}, ||\tilde{\theta}_{gj}|| \leqslant \sqrt{2\eta_3\bar{\mu}}, \\ |\tilde{\varepsilon}_{gj}| \leqslant \sqrt{2\eta_3\bar{\mu}}, ||e|| \leqslant \sqrt{\dfrac{\bar{\mu}}{\lambda_{\min}(P)}} \end{array} \right\}$$

whose size is $\bar{\mu} = \frac{\mu}{g} + c_g + V_n(0) > 0$, which can be adjusted by appropriately choosing the design parameters $\eta_1, \eta_2, \eta_3, \eta_f, \eta_\gamma, \eta_{g,1}, \ldots, \eta_{g,m}$.

Proof Since $\dot{V}_n \leqslant -gV_n + \mu + hN(\varsigma) + 1)\dot{\varsigma}$, one has

$$\begin{aligned} V_n(t) &\leqslant \frac{\mu}{g} + [V_n(0) - \frac{\mu}{g}]e^{-gt} + e^{-gt}\int_0^t (hN(\varsigma) + 1)e^{-g\tau}\dot{\varsigma}d\tau \\ &\leqslant \frac{\mu}{g} + V_n(0)e^{-gt} + e^{-gt}\int_0^t (hN(\varsigma) + 1)e^{-g\tau}\dot{\varsigma}d\tau \end{aligned} \tag{6.44}$$

Applying Lemma 6.5, we can conclude that, $V_n(t)$, $\int_0^t (hN(\varsigma)+1)e^{-g\tau}\dot{\varsigma}d\tau$ and $\varsigma(t)$ are SGUUB on $[0, t_f)$. According to Proposition 2 in [39], if the solution of the closed-loop system is bounded, then $t_f = +\infty$. Let c_g be the upper bound of $\int_0^t (hN(\varsigma)+1)e^{-g\tau}\dot{\varsigma}d\tau$, we have the following inequalities:

$$e^{-gt}\int_0^t (hN(\varsigma)+1)e^{-g\tau}\dot{\varsigma}d\tau \leqslant \int_0^t (hN(\varsigma)+1)e^{-g\tau}\dot{\varsigma}d\tau \leqslant c_g$$

Thus, (6.44) becomes

$$V_n(t) \leqslant \frac{\mu}{g} + c_g + V_n(0) = \bar{\mu} \tag{6.45}$$

Hence, if matrices X, Y, Z, Q and positive definite symmetric matrices P are chosen appropriately such that $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$ and (6.38) holds, then, the proposed control input (6.37) can ensure that $V_n(t)$ is bounded, namely, the closed-loop system is semi-globally uniformly ultimately bounded. Noting the definitions of $V_n(t)$ and $z_i, i = 1, 2, \ldots, n$, we have $\frac{1}{2}z_i^2 \leqslant V_n(t) \leqslant \bar{\mu}$ and $\frac{1}{2\eta_1}\tilde{\theta}_f^T\tilde{\theta}_f \leqslant \bar{\mu}$. Furthermore, we have $|z_i| \leqslant \sqrt{2\bar{\mu}}$, $||\tilde{\theta}_f|| \leqslant \sqrt{2\eta_1\bar{\mu}}$. Similarly, we have $|\tilde{\gamma}_f| \leqslant \sqrt{2\eta_2\bar{\mu}}$, $||\tilde{\theta}_{g,i}|| \leqslant \sqrt{2\eta_3\bar{\mu}}$, $|\tilde{\varepsilon}_{g,i}| \leqslant \sqrt{2\eta_3\bar{\mu}}$, $||e|| \leqslant \sqrt{\frac{\bar{\mu}}{\lambda_{\min}(P)}}$. From the above analysis, we can conclude that there do exist compact sets $\Omega_{z_i}^0$ such that $z_i \in \Omega_{z_i}^0, \forall t \geqslant 0$. The proof is completed.

From Theorem 6.1, one has

$$||e|| \leqslant \sqrt{\frac{2\bar{\mu}}{\lambda_{\min}(P)}} \tag{6.46}$$

Furthermore, the detection residual can be defined as

$$J = ||y(t) - \hat{y}(t)|| \tag{6.47}$$

From (6.46), it can be seen that the following inequality holds in the healthy case:

$$J \leqslant ||C||\sqrt{\frac{2\bar{\mu}}{\lambda_{\min}(P)}} \tag{6.48}$$

Then, the fault detection can be performed using the following mechanism:

$$\begin{cases} J \leqslant T_d \text{ no fault occurred,} \\ J > T_d \text{ fault has occurred} \end{cases} \tag{6.49}$$

where threshold T_d is defined as follows:

$$T_d = ||C||\sqrt{\frac{2\bar{\mu}}{\lambda_{\min}(P)}} \tag{6.50}$$

6.3.2 *Fault Isolation and Estimation*

Since the system has m actuators and it is assumed that only one actuator becomes faulty at one time, we have m possible faulty cases in total. When the sth $(1 \leqslant s \leqslant m)$ actuator is faulty, the faulty model can be described as:

$$u_s^f = \rho_s(x)u_s \tag{6.51}$$

The faulty system (6.1) can be described as follows:

$$\begin{cases} \dot{x}_{s,i} = x^p_{s,i+1}, \ i = 1, \ldots, n-1 \\ \dot{x}_{s,n} = f + \sum_{\substack{j=1 \\ j \neq s}}^{m} g_j u_j^p - g_s \rho_s^p u_s^p \\ y_s = x_{s,1} \end{cases} \tag{6.52}$$

After a fault has been detected, the isolation scheme is activated. Now, the following m nonlinear fault isolation observers are designed as follows:

$$\begin{cases} \dot{\hat{x}}_{s,i} = \hat{x}^p_{s,i+1} + l_{si}(y_s - \hat{y}_s), \ i = 1, \ldots, n-1 \\ \dot{\hat{x}}_{s,n} = \hat{\theta}_f^T \xi_f + \sum_{j=1, j\neq r}^{m} [\hat{g}_g + \hat{\varepsilon}_{gj}]u_j^p + l_{sn}(y - \hat{y}) + (\hat{\theta}^T_{gkr}\xi + \hat{\varepsilon}_{gkr})u_r^p \\ \hat{y}_s = \hat{x}_{s,1} = C\hat{x}_s \end{cases} \tag{6.53}$$

where $l_{si}, i = 1, 2, \ldots, n, s = 1, 2 \cdots .m$ are constants, which will be designed later, $\hat{\theta}^T_{g\rho,r}\xi_{g\rho,r}(\hat{x}_s, v)$ is the estimate of $g_r(x, v)\rho_r^p(x_r)$, $r = 1, \ldots, m$.

Let $\hat{x}_s = [\hat{x}_{s,1}, \hat{x}_{s,2}, \ldots, \hat{x}_{s,n}]^T$, the error terms $e_s = x_s - \hat{x}_s$ and $e_{ys} = y_s - \hat{y}_s$ are respectively the state error and output error between the faulty plant and the sth observer. The above error dynamics can be re-written as:

$$\dot{e}_s = A_s e_s^p + R_s e_s^p - L_s(y_s - \hat{y}_s) + d_s + B_s(d_f + d_g + \rho_s) \tag{6.54}$$

where $e_{sp} = [e_{s,1}^p, \ldots, e_{s,n}^p]^T$, $d_f = f - \hat{\theta}_f^T \xi_f$, $\rho_s = g_s k_s^p u_s^p - [\hat{\theta}_{gkr}^T \xi_{gkr} + \hat{\varepsilon}_{gkr}] u_r^p$, $d_g = \sum_{\substack{i=1 \\ i \neq s, i \neq r}}^{m} (g_j - \hat{g}_j - \hat{\varepsilon}_{gj}) u_j^p$ and

$$A = \begin{bmatrix} -r_1 & \\ \vdots & I \\ -r_n & 0 \cdots 0 \end{bmatrix}, R_s = \begin{bmatrix} r_1 \\ \vdots \\ r_n \end{bmatrix}, L_s = \begin{bmatrix} l_{s1} \\ \vdots \\ l_{sn} \end{bmatrix}, C_s = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}^T,$$

$$d_s = \begin{bmatrix} \sum_{k=1}^{p} C_p^k e_2^k \hat{x}_{s,2}^{p-k} \\ \vdots \\ 0 \end{bmatrix}, B_s = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

Similar to the previous subsection, differentiating $V_{se} = e_s^T P_s e_s$ with respect to time t and using (6.20) and (6.54), it leads to

$$\begin{aligned} \dot{V}_{se} &= e_s^T P_s \dot{e}_s + \dot{e}_s^T P_s e_s \\ &= 2e_s^T [P_s(A_s + R_s) + (A_s + R_s)^T P_s] e_s^p + \\ &\quad 2e_s^T P_s (d + B_s d_f + B_s d_g + B_s \rho_s) - e_s^T (P_s L_s C_s + C_s^T L_s^T P_s) e_s \end{aligned}$$

From Young's inequality, one has

$$\begin{aligned} e_s^T P_s d &\le e_s^T P_s P_s e_s + d^T d \\ &\le e_s^T P_s P_s e_s + e_{sp}^T e_{sp} - (e_{s,1}{}^p)^2 + \sum\nolimits_{i=2}^{n} 2(w_{e2})^2 (|\hat{x}_{s,i}|^p)^2 \end{aligned}$$

$$\begin{aligned} 2e_s^T P_s B_s d_f &= e_s^T P_s B_s d_f \le e_s^T P_s P_s e_s + d_f^2 \le e_s^T P_s P_s e_s + (\delta_f^*)^2 \\ &\le e_s^T P_s P_s e_s + (\bar{M}_{\delta f})^2 \end{aligned}$$

Furthermore, one has

$$\begin{aligned} \dot{V}_{se} \le &\begin{bmatrix} e_s \\ e_s^p \end{bmatrix}^T \begin{bmatrix} X_s - P_s L_s C_s - C_s^T L_s^T P_s + P_s P_s & Y_s + P_s(A_s + R_s) \\ Y_s^T + (A_s + R_s)^T P_s & Z_s + I \end{bmatrix} \begin{bmatrix} e_s \\ e_s^p \end{bmatrix} + \\ &\Delta_0 + 2e_s^T P_s B_s (d_g + \rho_s) \end{aligned} \tag{6.55}$$

where $\Delta_0 = -(e_{s,1}^p)^2 + \sum_{i=2}^{n} 2(w_{e2})^2 (|\hat{x}_{s,i}|^p)^2 + (\bar{M}_{\delta f})^2$.

In the following, stability analysis will be given at two cases, i.e., $s = r$ or $s \neq r$.

Case 1: $s = r$

Since

$$
\begin{aligned}
2e_s^T P_s B_s(\rho_s + d_g) &= 2e_s^T P_{sn}[\sum_{j=1, j\neq s}^{m} (g_j - \hat{g}_j - \hat{\varepsilon}_{gj})u_j^p + (g_{ks} - \hat{g}_{ks} - \hat{\varepsilon}_{gks})u_s^p] \\
&= 2e_s^T P_{sn}(\sum_{j=1, j\neq s}^{m} (\theta_{gj}^{*T}\xi_{gj} + \varepsilon_{gj} - \hat{\theta}_{gj}^T\xi_{gj} - \hat{\varepsilon}_{gj})u_j^p + \\
&\quad (\theta_{gks}^{*T}\xi_{gks} + \varepsilon_{gks} - \hat{\theta}_{gks}^T\xi_{gks} - \hat{\varepsilon}_{gks})u_s^p) \\
&= \sum_{j=1, j\neq s}^{m} 2e_s^T P_{sn}\tilde{\theta}_{gj}^T\xi_{gj}u_j^p + \sum_{j=1, j\neq s}^{m} 2e_s^T P_{sn}\tilde{\varepsilon}_{gj}u_j^p + \\
&\quad \sum_{j=1, j\neq s}^{m} 2e_s^T P_{sn}(\varepsilon_{gj} - \hat{\varepsilon}_{gj}^*)u_j^p + \\
&\quad 2e_s^T P_{sn}\tilde{\theta}_{gks}^T\xi_{gks}u_s^p + 2e_s^T P_{sn}\tilde{\varepsilon}_{gks}u_s^p + 2e_s^T P_{sn}(\varepsilon_{gks} - \hat{\varepsilon}_{gks}^*)u_s^p
\end{aligned}
\tag{6.56}
$$

$$
\begin{aligned}
\dot{V}_{se} \leq &\begin{bmatrix} e_s \\ e_s^p \end{bmatrix}^T \begin{bmatrix} X_s - P_s L_s C_s - C_s^T L_s^T P_s + 2P_s P_s^T & Y_s + P_s(A_s + R_s) \\ Y_s^T + (A_s + R_s)^T P_s & Z_s + I \end{bmatrix} \begin{bmatrix} e_s \\ e_s^p \end{bmatrix} + \\
&\Delta_0 + \sum_{j=1, j\neq s}^{m} 2e_s^T P_{sn}\tilde{\theta}_{gj}^T\xi_{gj}u_j^p + \sum_{j=1, j\neq s}^{m} 2e_s^T P_{sn}\tilde{\varepsilon}_{gj}u_j^p + \\
&\sum_{j=1, j\neq s}^{m} 2e_s^T P_{sn}(\varepsilon_{gj} - \hat{\varepsilon}_{gj}^*)u_j^p + \\
&2e_s^T P_{sn}\tilde{\theta}_{gks}^T\xi_{gks}u_s^p + 2e_s^T P_{sn}\tilde{\varepsilon}_{gks}u_s^p + 2e_s^T P_{sn}(\varepsilon_{gks} - \hat{\varepsilon}_{gks}^*)u_s^p
\end{aligned}
\tag{6.57}
$$

where P_{sn} is the nth column of P_s.

Similar to the above subsection, define

$$z_{s,1} = x_{s,1} = y_s$$

$$z_{s,i} = \hat{x}_{s,i} - \alpha_{s,i-1}(\hat{x}_{s,1}, \ldots, \hat{x}_{s,i-1}), i = 2, 3, \ldots, n$$

$$V_{s,1} = V_{s,11} + V_{se}, \quad V_{s,11} = \frac{1}{2}z_{s,1}^2$$

$$V_{s,i} = V_{s,i-1} + \frac{1}{2}z_{s,i-1}^2, i = 2, 3, \ldots, n$$

and choose a virtual control $\alpha_{s,i}$, $i = 1, 2, \ldots, n-1$ and practical control $\alpha_{s,nj}$, $j = 1, \ldots, m$ as follows

$$
\alpha_{s,1} = \begin{cases} \sqrt[p]{(-\frac{1}{2}z_{s,1} - \frac{\Delta_0}{z_{s,1}}}, & z_{s,1} \in \Omega_{c_{s,z_{s1}}}^0 \\ 0, & z_{s,1} \in \Omega_{c_{s,zs_1}} \end{cases}
\tag{6.58}
$$

$$
\alpha_{s,2} = \begin{cases} \sqrt[p]{(-\frac{1}{2}z_{s,2} - \frac{\Delta_1}{z_{s,2}}}, & z_{s,2} \in \Omega_{c_{s,zs_2}}^0 \\ 0, & z_{s,2} \in \Omega_{c_{s,zs_2}} \end{cases}
\tag{6.59}
$$

$$\alpha_{s,k} = \begin{cases} \sqrt[p]{(-\frac{1}{2}z_{s,k} - \frac{\Delta_{k-1}}{z_{s,k}}}, \ z_{s,k} \in \Omega^0_{c_{s,z_k}} \\ 0, \ z_{s,k} \in \Omega_{c_{s,z_k}} \end{cases} \tag{6.60}$$

$$\alpha_{s,nj} = u_j = \begin{cases} \sqrt[p]{\frac{N(\varsigma)(-\frac{1}{2}z_{s,n} - \frac{\Delta_{n-1}}{z_n})}{m}}, \ z_{s,k} \in \Omega^0_{c_{z_{s,zn}}} \\ 0, \ z_{s,k} \in \Omega_{c_{s,zn}} \end{cases} \tag{6.61}$$

where $\dot{\varsigma} = -\frac{1}{2}z^2_{s,n} - \Delta_{n-1}$, $\Omega_{c_{s,z_i}}$, $i = 1, \ldots, n$ are defined as $\Omega_{c_{z_k}}$ in the previous subsection. The adaptive laws are designed as follows:

$$\dot{\hat{\theta}}_f = 2\eta_1 e_s^T P_n \xi_f + \eta_f \hat{\theta}_f \tag{6.62}$$

$$\dot{\hat{\gamma}}_f = \eta_2 |z_n| + \eta_\gamma \hat{\gamma}_f \tag{6.63}$$

$$\dot{\hat{\theta}}_{gj} = 2\eta_3 e_s^T P_n \xi_{gj} u_j^p + \eta_{gj} \hat{\theta}_{gj} \tag{6.64}$$

$$\dot{\hat{\varepsilon}}_{gj} = 2\eta_3 e_s^T P_n u_j^p + \eta_{gj} \hat{\varepsilon}_{gj} \tag{6.65}$$

$$\dot{\hat{\theta}}_{gks} = 2\eta_4 e_s^T P_n \xi_{gks} u_s^p + \eta_{gks} \hat{\theta}_{gks} \tag{6.66}$$

$$\dot{\hat{\varepsilon}}_{gks} = 2\eta_4 e_s^T P_n u_s^p + \eta_{gks} \hat{\varepsilon}_{gks} \tag{6.67}$$

where u_j is a bounded control input which is applied simultaneously to the jth actuator in the system (6.1) and the observer (6.53), and $\eta_1 > 0$, $\eta_2 > 0$, $\eta_3 > 0$, $\eta_4 > 0$, $\eta_f > 0$, $\eta_\gamma > 0$, $\eta_{gks} > 0$, $\eta_{gj} > 0$, $\eta_{gks} > 0$ are design parameters.

Define

$$\begin{aligned} V_s =& V_{s,n} + \frac{1}{2\eta_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\eta_2}\tilde{\gamma}_f^2 + \\ & \frac{1}{2\eta_3}\sum\nolimits_{j=1, j\neq s}^m (\tilde{\theta}_{gj}^T \tilde{\theta}_{gj} + \tilde{\varepsilon}_{gj}^2) + \frac{1}{2\eta_4}(\tilde{\theta}_{gks}^T \tilde{\theta}_{gks} + \varepsilon_{gks}^2) \end{aligned} \tag{6.68}$$

Similar to the previous subsection, differentiating V_s with respect to time t, one has

$$\begin{aligned} \dot{V}_s \leq& \dot{V}_{s,n} + \frac{1}{\eta_1}\tilde{\theta}_f^T \dot{\hat{\theta}}_f + \frac{1}{\eta_2}\tilde{\gamma}_f \dot{\hat{\gamma}}_f + \frac{1}{\eta_4}(\tilde{\theta}_{gks}^T \dot{\hat{\theta}}_{gks} + \tilde{\varepsilon}_{gks}\dot{\hat{\varepsilon}}_{gks}) + \\ & \frac{1}{\eta_3}\sum\nolimits_{j=1, j\neq s}^m [\tilde{\theta}_{gj}^T \dot{\hat{\theta}}_{gj} + \tilde{\varepsilon}_{gj}\dot{\hat{\varepsilon}}_{gj}] \end{aligned} \tag{6.69}$$

It is obvious that if

$$\begin{bmatrix} X_s - P_s L_s C_s - C_s^T L_s^T P_s + 2P_s P_s & Y_s + P_s(A_s + R_s) \\ Y_s^T + (A_s + R_s)^T P_s & Z_s + I \end{bmatrix} < -Q_s \tag{6.70}$$

where X, Y, Z denote matrices with appropriate dimensions, respectively, and $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$, matrix $Q_s > 0$, then from (6.69), one has

$$\begin{aligned} \dot{V}_s \leq & \dot{V}_{s,n} - \begin{bmatrix} e_s \\ e_s^p \end{bmatrix}^T Q_s \begin{bmatrix} e_s \\ e_s^p \end{bmatrix} + \Delta_0 + \sum\nolimits_{j=1, j\neq s}^m 2e_s^T P_{sn}\tilde{\theta}_{gj}^T \xi_{gj} u_j^p + \\ & \sum\nolimits_{j=1, j\neq s}^m 2e_s^T P_{sn}\tilde{\varepsilon}_{gj} u_j^p + \sum\nolimits_{j=1, j\neq s}^m 2e_s^T P_{sn}(\varepsilon_{gj} - \hat{\varepsilon}_{gj}^*) u_j^p + \\ & 2e_s^T P_{sn}\tilde{\theta}_{gks}^T \xi_{gks} u_s^p + 2e_s^T P_{sn}\tilde{\varepsilon}_{gks} u_s^p + 2e_s^T P_{sn}(\varepsilon_{gks} - \hat{\varepsilon}_{gks}^*) u_s^p + \\ & \frac{\tilde{\theta}_f^T \dot{\hat{\theta}}_f}{\eta_1} + \frac{\tilde{\gamma}_f \dot{\hat{\gamma}}_f}{\eta_2} + \frac{\sum_{j=1, j\neq s}^m [\tilde{\theta}_{gj}^T \dot{\hat{\theta}}_{gj} + \tilde{\varepsilon}_{gj} \dot{\hat{\varepsilon}}_{gj}]}{\eta_3} + \frac{\tilde{\theta}_{gks}^T \dot{\hat{\theta}}_{gks} + \tilde{\varepsilon}_{gks} \dot{\hat{\varepsilon}}_{gks}}{\eta_4} \end{aligned} \tag{6.71}$$

Similar to (6.42) in the above subsection, considering (6.62–6.67), from (6.71), one has

$$\dot{V}_s \leq -g_s V_s + \bar{\mu}_s + (\bar{h}(\hat{x}) N(\varsigma)\dot{\varsigma} + \dot{\varsigma}) \tag{6.72}$$

where

$$\begin{aligned} \mu_s = & \frac{\eta_f}{2\eta_1} M_{\theta f}^2 + \frac{\eta_f}{2\eta_1} (\bar{M}_{\varepsilon f} + \bar{M}_{\delta f})^2 + \sum\nolimits_{j=1, j\neq s}^m \frac{\eta_{gj}}{2\eta_3} (\bar{M}_{\theta gj}^2 + \bar{M}_{\varepsilon gj}^2) + \\ & \frac{\eta_{gks}}{2\eta_4} (\bar{M}_{\theta gks}^2 + \bar{M}_{\varepsilon gks}^2) \end{aligned}$$

$$g_s = \min\{\frac{1}{2}, \frac{\eta_f}{2\eta_1}, \frac{\eta_\gamma}{2\eta_2}, \frac{\eta_{g1}}{2\eta_3}, \ldots, \frac{\eta_{gm}}{2\eta_3}, \frac{\eta_{gks}}{2\eta_4}, \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}\}$$

$$\begin{aligned} \bar{h}(\hat{x}) = & \sum\nolimits_{j=1, j\neq s}^m [z_n(\hat{g}_j + \hat{\varepsilon}_{gj}) + 2e^T P_n(\varepsilon_{gj} - \varepsilon_{gj}^*)] + \\ & z_n(\hat{g}_{ks} + \hat{\varepsilon}_{gks}) + 2e^T P_n(\varepsilon_{gks} - \varepsilon_{gks}^*) \end{aligned}$$

Since $\dot{V}_s \leq -g_s V_s + \bar{\mu}_s + (\bar{h}(\hat{x}) N(\varsigma)\dot{\varsigma} + \dot{\varsigma})$, one has

$$\begin{aligned} V_s(t) \leq & \frac{\bar{\mu}_s}{g_s} + [V_s(0) - \frac{\mu_{\cdot s}}{g_s}] e^{-g_s t} + e^{-gt} \int_0^t (\bar{h}(\hat{x}) N(\varsigma) + 1)\dot{\varsigma} e^{-g_s \tau} \dot{\varsigma} d\tau \\ \leq & \frac{\bar{\mu}_s}{g_s} + V_s(0) e^{-g_s t} + e^{-g_s t} \int_0^t (\bar{h}(\hat{x}) N(\varsigma) + 1)\dot{\varsigma} e^{-g_s \tau} \dot{\varsigma} d\tau \end{aligned} \tag{6.73}$$

Applying Lemma 6.5, we can conclude that, $V_n(t)$, $\int_0^t (\bar{h}(\hat{x})N(\varsigma)+1)\dot{\varsigma}e^{-g\tau}\dot{\varsigma}d\tau$ and $\varsigma(t)$ are SGUUB on $[0, t_f)$. According to Proposition 2 in [39], if the solution of the closed-loop system is bounded, then $t_f = +\infty$. Let c_g be the upper bound of $\int_0^t \bar{h}(\hat{x})(N(\varsigma)+1)\dot{\varsigma}e^{-g_s\tau}\dot{\varsigma}d\tau$, we have the following inequalities:

$$e^{-g_s t}\int_0^t (\bar{h}(\hat{x})N(\varsigma)+1)\dot{\varsigma}e^{-g_s\tau}\dot{\varsigma}d\tau \le c_g$$

Thus, (6.73) becomes

$$V_s(t) \le \frac{\bar{\mu}_s}{g_s} + c_g + V_s(0) = \mu_s \tag{6.74}$$

Hence, if matrices X_s, Y_s, Z_s, Q_s and the positive definite symmetric matrix P_s are chosen appropriately such that $\begin{bmatrix} X_s & Y_s \\ Y_s^T & Z_s \end{bmatrix} > 0$ and (6.74) holds, then, the proposed control input (6.61) and adaptive laws (6.62–6.67) can ensure that $V_s(t)$ is bounded, namely, the closed-loop system is semi-globally uniformly ultimately bounded. That is to say, all signals of the closed-loop system remain the following compact set Ω_1,

$$\Omega_1 := \left\{ \begin{array}{l} (z_i, \tilde{\theta}_f, \tilde{\gamma}_f, \tilde{\theta}_{gj}, \tilde{\varepsilon}_{gj}, \tilde{\theta}_{gks}, \tilde{\varepsilon}_{gks}, e) \Big| \, |z_i| \le \sqrt{2\mu_s}, ||\tilde{\theta}_f|| \le \sqrt{2\eta_1\mu_s}, \\ |\tilde{\gamma}_f| \le \sqrt{2\eta_2\mu_s}, ||\tilde{\theta}_{gj}|| \le \sqrt{2\eta_3\mu_s}, |\tilde{\varepsilon}_{gj}| \le \sqrt{2\eta_3\mu_s}, \\ ||\tilde{\theta}_{gks}|| \le \sqrt{2\eta_4\mu_s}, |\tilde{\varepsilon}_{gks}| \le \sqrt{2\eta_4\mu_s}, ||e|| \le \sqrt{\frac{\mu_s}{\lambda_{\min}(P_s)}} \end{array} \right\}$$

Case 2: $s \ne r$

Since $s \ne r$, from the faulty (6.52) and the observer (6.53), one has

$$2e_s^T P_s B_s \rho_s = 2e_s^T P_s B_s[(g_{ks} - \hat{g}_s - \hat{\varepsilon}_{gs})u_s^p + (g_r - \hat{g}_{kr} - \hat{\varepsilon}_{gkr})u_r^p] \tag{6.75}$$

From the adaptive laws (6.64–6.67), one has

$$\dot{\hat{\theta}}_{gs} \ne \dot{\hat{\theta}}_{gks}, \dot{\hat{\varepsilon}}_{gs} \ne \dot{\hat{\varepsilon}}_{gks}, \dot{\hat{\theta}}_{gr} \ne \dot{\hat{\theta}}_{gkr}, \dot{\hat{\varepsilon}}_{gr} \ne \dot{\hat{\varepsilon}}_{gkr}$$

It is noted that $2e_s^T P_s B_s[(g_{ks} - \hat{g}_s - \hat{\varepsilon}_{gs})u_s^p + (g_r - \hat{g}_{kr} - \hat{\varepsilon}_{gkr})u_r^p]$ varies infinitely since $\dot{\hat{\theta}}_{gs} \ne \dot{\hat{\theta}}_{gks}$, $\dot{\hat{\theta}}_{gr} \ne \dot{\hat{\theta}}_{gkr}$, $\dot{\hat{\varepsilon}}_{gs} \ne \dot{\hat{\varepsilon}}_{gks}$ and $\dot{\hat{\varepsilon}}_{gr} \ne \dot{\hat{\varepsilon}}_{gkr}$, which further cause that $V_s(t)$ varies infinitely. As a result, basically, all signals of the closed-loop systems such as e_{si} do not remain Ω_1 using the above control law and adaptive laws.

The above design procedure and analysis are summarized in the following theorem.

Theorem 6.2 *Consider the faulty system (6.52) under Assumptions 6.1 and 6.2, with virtual controls (6.58–6.60), control law (61) and adaptive laws (6.62–6.67). If matrices X_s, Y_s, Z_s, $Q_s > 0$ and $P_s = P_s^T > 0$ are such that $\begin{bmatrix} X_s & Y_s \\ Y_s^T & Z_s \end{bmatrix} > 0$ and*

$$\begin{bmatrix} X_s - P_s L_s C_s - C_s^T L_s^T P_s + 2P_s P_s & Y_s + P_s(A_s + R_s) \\ Y_s^T + (A_s + R_s)^T P_s & Z_s + I \end{bmatrix} < -Q_s \tag{6.76}$$

then, we can guarantee the following properties under bounded initial conditions, when the rth actuator is faulty,

(1) for $s = r$, the closed-loop system is semi-globally uniformly ultimately stable, and all signals involved in the closed-loop systems remain a small neighborhood of the origin, i.e., Ω_1 specified as

$$\Omega_1 := \left\{ \begin{array}{l} (z_i, \tilde{\theta}_f, \tilde{\gamma}_f, \tilde{\theta}_{gj}, \tilde{\varepsilon}_{gj}, \tilde{\theta}_{gks}, \tilde{\varepsilon}_{gks}, e) \Big| \, |z_i| \le \sqrt{2\mu_s}, ||\tilde{\theta}_f|| \le \sqrt{2\eta_1 \mu_s}, \\ |\tilde{\gamma}_f| \le \sqrt{2\eta_2 \mu_s}, ||\tilde{\theta}_{gj}|| \le \sqrt{2\eta_3 \mu_s}, |\tilde{\varepsilon}_{gj}| \le \sqrt{2\eta_3 \mu_s}, \\ ||\tilde{\theta}_{gks}|| \le \sqrt{2\eta_4 \mu_s}, |\tilde{\varepsilon}_{gks}| \le \sqrt{2\eta_4 \mu_s}, ||e|| \le \sqrt{\dfrac{\mu_s}{\lambda_{\min}(P_s)}} \end{array} \right\}$$

(2) $s \neq r$, all signals of the closed-loop systems do not remain the compact set Ω_1.

Remark 6.3 It is valuable to point out that, if the design parameters such as $\eta_i, i = 1, \ldots, 4, \eta_f, \eta_\gamma, \eta_{gks}, \eta_{gj}, \; j = 1, \ldots, m$ are appropriately chosen, μ_s is small enough, and all signals of the closed-loop system converge to a smaller neighborhood of the origin, which means that better control performance is obtained.

Now, we denote the residuals between the real system and isolation estimators as follows:

$$J_s(t) = \left\| \hat{y}_s(t) - y(t) \right\| = ||Ce(t)||, \quad 1 \le s \le m \tag{6.77}$$

According to Theorem 6.2, when the rth actuator is faulty, i.e., $s = r$, the residual $e_s(t)$ must tend to Ω_1; while for any $s \neq r$, basically, $e_s(t)$ does not belong to Ω_1. Hence, the isolation law for actuator fault can be designed as

$$\begin{cases} J_s(t) \le T_I, s = r \Rightarrow \text{the } r\text{th actuator is faulty} \\ J_s(t) > T_I, s \neq r \end{cases} \tag{6.78}$$

where threshold T_I is defined as follows.

$$T_I = ||C|| \sqrt{\frac{\mu_s}{\lambda_{\min}(P_s)}}$$

6.3.3 Fault Accommodation

After that the fault information is obtained, we will consider the fault-tolerant control problem of system (6.1), and design a fault-tolerant control law to recover the control system's dynamics performance when an actuator fault occurs. Firstly, we consider the fuzzy control problem for the following nominal system without actuator faults:

$$\begin{cases} \dot{x}_i = x_{i+1}^p, i = 1, \ldots, n-1 \\ \dot{x}_n = f(x) + \sum_{j=1}^m g_j(x) u_j^p \\ y = x_1 \end{cases}$$

Consider matrices $X, Y, Z, Q > 0$ and $P = P^T > 0$ such that $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$ and

$$\begin{bmatrix} X - PLC - C^T L^T P + 2PP & Y + P(A+R) \\ Y^T + (A+R)^T P & Z + I \end{bmatrix} \leq -Q$$

virtual control laws (6.58–6.60), control input (6.61) and adaptive laws (6.62–6.67).

From Theorem 6.1, under Assumptions 6.1 and 6.2, the closed-loop system is semi-globally uniformly ultimately stable, and all signals involved in the closed-loop systems converge to a small neighborhood of the origin.

On the basis of the estimated actuator fault, the fault tolerant controller is constructed as

$$u_s = \frac{\hat{\rho}_s u_s^N}{\hat{\rho}_s^2 + \varepsilon_u} \tag{6.79}$$

where $\varepsilon_u > 0$ is a design parameter, u_s^N is the sth desired control input under healthy condition, $\hat{\rho}_s$ is the estimate of $g_s k_s$, which is used to compensate for the gain fault k_s.

Theorem 6.3 *Consider the high-order system (6.1) under Assumptions 6.1 and 6.2, fault model (6.2), virtual and practical control laws (6.58–6.61) and adaptive laws (6.62–6.67). If there exist matrices* $X, Y, Z, Q > 0$ *and* $P = P^T > 0$ *with appropriate dimensions, such that* $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$ *and*

$$\begin{bmatrix} X - PLC - C^T L^T P + 2PP & Y + P(A+R) \\ Y^T + (A+R)^T P & Z + I \end{bmatrix} \leq -Q \tag{6.80}$$

then, the faulty system (6.1) is asymptotically stable under the feedback FTC (6.79) and all signals involved in the closed-loop system are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, i.e. $||\tilde{\theta}_f|| \leq \sqrt{2\eta_{sf}\mu_s}$, $||\tilde{\theta}_{gj}|| \leq \sqrt{2\eta_{gj}\mu_s}$, $||\tilde{\theta}_{g\rho,s}|| \leq \sqrt{2\eta_{gks}\mu_s}$, $||e|| \leq \sqrt{\frac{2\mu_s}{\lambda_{\min}(P_s)}}$, *where*

$$\mu_s = \frac{\bar{\mu}_s}{g_s} + c_g + V_s(0),\ g_s = \min\{\frac{1}{2}, \frac{\eta_f}{2\eta_1}, \frac{\eta_\gamma}{2\eta_2}, \frac{\eta_{g1}}{2\eta_3}, \ldots, \frac{\eta_{gm}}{2\eta_3}, \frac{\eta_{gks}}{2\eta_4}, \frac{\eta_{gks}}{2\eta_4}, \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\},$$

$$\bar{\mu}_s = \frac{\eta_f}{2\eta_1} M_{\theta f}^2 + \frac{\eta_\gamma}{2\eta_2}(\bar{M}_{\varepsilon f}^2 + \bar{M}_{\delta f}^2) + \sum_{j=1, j\neq s}^{m} \frac{\eta_{gj}}{\eta_3}[\bar{M}_{\theta gj}^2 + \bar{M}_{\varepsilon gj}^2] + \frac{\eta_{gks}}{2\eta_4} M_{\theta gks}^2 + \frac{\eta_{gks}}{2\eta_4} \bar{M}_{\varepsilon gks}^2$$

Proof Similar to the proof of Theorem 6.1, it is easy to obtain the conclusions of Theorem 3. The detailed proof is thus omitted here.

6.4 Simulation Results

In this section, a practical aircraft longitudinal motion dynamics, which can be described as a 1-order nonlinear system, namely $p = 1$, and a high-order numerical example where $p = 3$, are taken to show the effectiveness of the proposed fault tolerant control scheme.

6.4.1 *An Application to Aircraft Longitudinal Motion Dynamics*

In this subsection, we apply the proposed FTC scheme to diagnose and accommodate failures in an aircraft longitudinal motion dynamics. The aircraft longitudinal motion dynamics of the twin otter [43] can be described as 1-order nonlinear system as follows:

$$\begin{cases} \dot{V} = \dfrac{F_x \cos(\alpha) + F_z \sin(\alpha)}{m} \\ \dot{\alpha} = q + \dfrac{-F_x \sin(\alpha) + F_z \cos(\alpha)}{mV} \\ \dot{\theta} = q \\ \dot{q} = \dfrac{M}{I_y} \end{cases} \tag{6.81}$$

where V is the velocity, α is the angle of attack, θ is the angle of pitch and q is the pitch rate, m is the mass, I_y is the moment of inertia, and $F_x = \bar{q}SC_x(\alpha, q, \delta_{e1}, \delta_{e2}) + T_1 \cos\gamma_1 + T_2 \cos\gamma_2 - mg\sin(\theta)$, $F_z = \bar{q}SC_z(\alpha, q, \delta_{e1}, \delta_{e2}) + T_1 \cdot \sin\gamma_1 + T_2 \sin\gamma_2 - mg\cos(\theta)$, $M = \bar{q}cSC_m(\alpha, q, \delta_{e1}, \delta_{e2})$, where $\bar{q} = \frac{1}{2}\rho V^2$ is the dynamic pressure, ρ is the air density, S is the wing area, c is the mean chord, T_1 and T_2 are independent thrusts with corresponding thrust misalignments γ_1 and γ_2. The functions C_x, C_z, C_m are of the polynomial form: $C_x = C_{x1}\alpha + C_{x2}\alpha^2 + C_{x3} + C_{x4}(d_1\delta_{e1} + d_2\delta_{e2})$, $C_z = C_{z1}\alpha + C_{x2}\alpha^2 + C_{z3} + C_{z4}(d_1\delta_{e1} + d_2\delta_{e2}) + C_{x5}q$, $C_m = C_{m1}\alpha + C_{m2}\alpha^2 + C_{m3} +$

$C_{m4}(d_1\delta_{e1}+d_2\delta_{e2})+C_{m5}q$, where δ_{e1} and δ_{e2} are the elevator angles of an augmented two-pieces elevators used as two actuators u_1 and u_2 for failure compensation study. Choosing V, α, θ and q as the states x_1, x_2, x_3 and x_4, and δ_{e1}, δ_{e2}, T_1, T_2 as the inputs u_1, u_2, u_3, u_4, (6.81) will be put into the state form:

$$\begin{cases} \dot{x}_1 = (c_1^T\phi_0(x_2)x_1^2+\phi_1(x))\cos(x_2)+ \\ \quad +(c_2^T\phi_0(x_2)x_1^2+\phi_2(x))\sin(x_2)+ \\ \quad d_1g_1(x)u_1+d_2g_1(x)u_2+g_{31}(x)u_3+g_{41}(x)u_4 \\ \dot{x}_2 = x_4-(c_1^T\phi_0(x_2)x_1+\phi_1(x)\dfrac{1}{x_1})\sin(x_2)+ \\ \quad (c_2^T\phi_0(x_2)x_1+\phi_2(x)\dfrac{1}{x_1})\cos(x_2)+ \\ \quad d_1g_2(x)u_1+d_2g_2(x)u_2+g_{32}(x)u_3+g_{42}(x)u_4 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \theta^T\varphi(x)+b_1x_1^2u_1+b_2x_1^2u_2 \end{cases} \tag{6.82}$$

where

$$\phi_0(x_2)=[x_2, x_2^2, 1]^T, \quad \phi_1(x)=p_0\sin(x_3)$$

$$\phi_2(x)=p_1x_4x_1^2+p_0\sin(x_3),$$

$$g_1(x)=a_1x_1^2\cos(x_2)+a_2x_1^2\sin(x_2)$$

$$g_2(x)=-a_1x_1\sin(x_2)+a_2x_1\sin(x_2)$$

$$g_{31}(x)=\cos(\gamma_1)\cos(x_2)+\sin(\gamma_1)\sin(x_2)$$
$$g_{41}(x)=\cos(\gamma_2)\cos(x_2)+\sin(\gamma_2)\sin(x_2)$$

$$g_{32}(x)=-\cos(\gamma_1)\frac{\sin(x_2)}{x_1}+\sin(\gamma_1)\frac{\cos(x_2)}{x_1}$$
$$g_{42}(x)=-\cos(\gamma_2)\frac{\sin(x_2)}{x_1}+\sin(\gamma_2)\frac{\cos(x_2)}{x_1}$$
$$\varphi(x)=[x_1^2x_2, x_1^2x_2^2, x_1^2, x_1^2x_4]^T$$

and $\theta, p_1, a_1, a_2, , b_1, b_2, c_1, c_2, d_1, d_2, \gamma_1, \gamma_2$ are unknown constant parameters while p_0 is the gravity constant which is known. There exists a diffeomorphism $[\xi, x]^T = T(\chi) = [T_1(\chi), T_2(\chi), x_3, x_4]^T$ such that (6.82) can be transform into the parameter-strict-feedback form, where the positive odd number $p = 1$

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = \vartheta^T\phi(x)+\sum_{i=1}^{2} b_ix_1^2u_i \end{cases} \tag{6.83}$$

and the zero dynamics $\dot{\xi} = \phi(\xi, \chi) + \Phi(\xi, \chi)\vartheta$, where $\vartheta \in R^4$ is an unknown constant vector. The relative degree o equals 2. The aircraft parameters in the simulation study are chosen based on the data sheet in [44]: $m = 4600\,kg$, $I_y = 31027\,kg\,m^2$, $S = 39.2\,m^2$, $c = 1.98\,m$, $T_x = 4864\,N$, $T_z = 212\,N$, $\rho = 0.7377\,kg/m^3$ at the altitude of 5000 m, and for the 0° flap setting. In addition, $C_{x1} = 0.39$, $C_{x2} = 2.9099$, $C_{x3} = -0.0758$, $C_{x4} = 0.0961$, $C_{z1} = -7.0186$, $C_{z2} = 4.1109$, $C_{z3} = -0.3112$, $C_{z4} = -0.2340$, $C_{z5} = -0.1023$, $C_{m1} = -0.8789$, $C_{m2} = -3.852$, C_{m3} =-0.0108, $C_{m4} = -1.8987$, $C_{m5} = -0.6266$ are unknown constants. Reference signal $y_d$is set as $y_d = e^{-0.05t}\cdot\, \sin(0.2t)$. The initial states and estimates are set as $\chi(0) = [75, 0, 0, 15, 0]^T = e^{-0.05t}\sin(0.2t)$, $\hat{\vartheta}(0) = [0, 0, -0.004, 0]$. It is assumed that the zero dynamics $\dot{\xi} = \phi(\xi, \chi) + \Phi(\xi, \chi)\vartheta$ is input-to-state stable with respect to$x$ taken as the input. In addition, b_i, $i = 1, \ldots, m$ are assumed to be complete unknown, i.e., these values and signs are both unknown.

The fault case considered in this example is modeled as

$$u_1^f(t) = \begin{cases} u_1(t), \ t < 10 \\ (1 - \rho_1(x))u_1(t), \ t \geq 10 \end{cases}, \quad u_2^f(t) = u_2(t)$$

where $\rho_1(x) = 0.4\cos(x_3)$.

Firstly, the matrices inequality (6.43) are transformed to LMI, then by using Matlab toolbox to solve the matrices inequalities, one can obtain symmetric matrix $X, Y, Z, P, Q, X_s, Y_s, Z_s, P_s, Q_s$ and the nominal controller gains K_i. Therefore, one can design the desired control (6.37). Using this desired control, we can design fault-tolerant controller (6.79). In this example, we assume that the system state is not fully measured and thus the observer (6.53) is used to estimate the system state. Consequently, the observer-based fault-tolerant control (6.79) is used to control the faulty system. The simulation results are presented in Figs. 6.1, 6.2, 6.3, 6.4, 6.5 and 6.6. From Figs. 6.1 and 6.2, it is seen that, under normal operating condition, the system states globally asymptotically converge to a small neighborhood of the origin. Figures 6.3 and 6.4 show that, when an actuator fault occurs, when keeping the normal controller, the system states deviate significantly from the neighborhood. However, as shown in Figs. 6.5 and 6.6, using the proposed FTC (6.79), better tracking performance is obtained, again.

6.4.2 A High-Order Numerical Example

Consider the following high-order nonlinear system

$$\begin{cases} \dot{x}_1 = x_2^3 \\ \dot{x}_2 = u_1^3 + u_2^3 \end{cases} \tag{6.84}$$

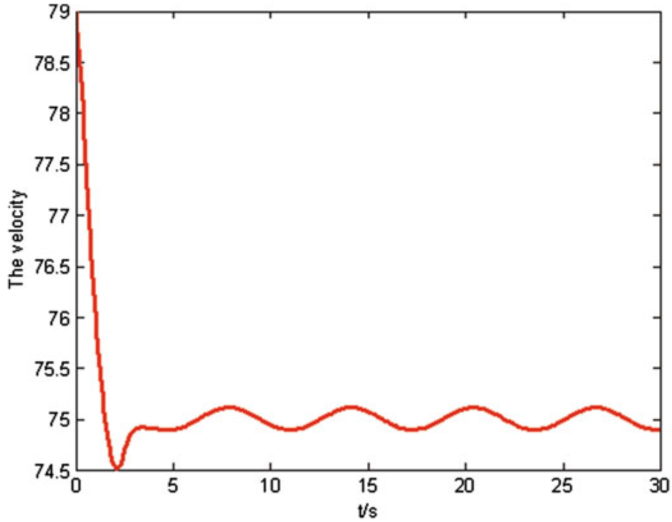

Fig. 6.1 Time response of the velocity without fault

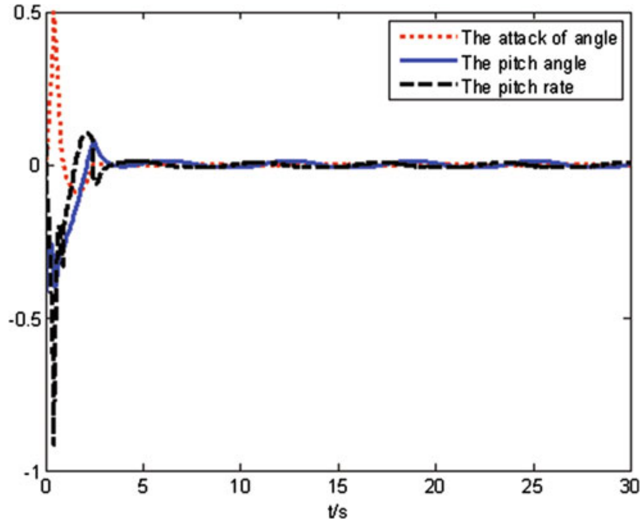

Fig. 6.2 Time response of the attack of angle, the pitch angle and the pitch rate without fault

The fault case considered in this example is modeled as

$$u_1^f(t) = \begin{cases} u_1(t), & t < 10 \\ (1 - \rho_1(x))u_1(t), & t \geq 10 \end{cases} \quad u_2^f(t) = u_2(t)$$

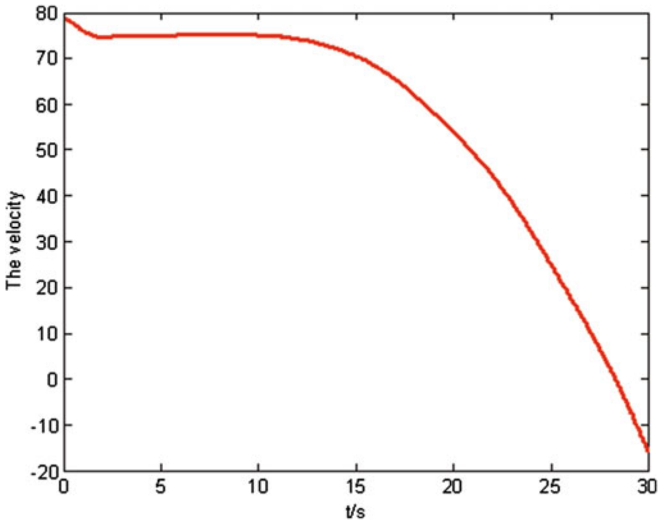

Fig. 6.3 Time response of the velocity without FTC

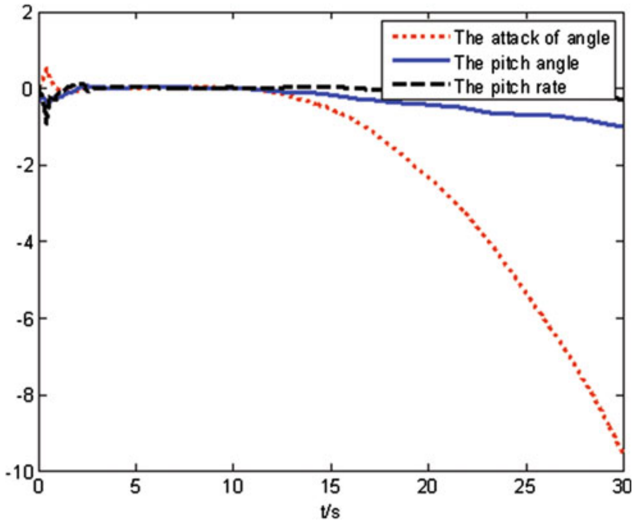

Fig. 6.4 Time response of the attack of angle, the pitch angle and the pitch rate without FTC

where $\rho_1(x) = 0.8\cos(2+x_1+x_2)$, the fault occurs at time $t = 10s$. As expected, we can find that system output y follows well $y_d = 0$ as shown in Fig. 6.7. Meanwhile, Figs. 6.8 and 6.9 illustrate that, under the faulty condition, the system output y does not converge to the desired reference signal without FTC, however, using FTC, the system has better tracking performance.

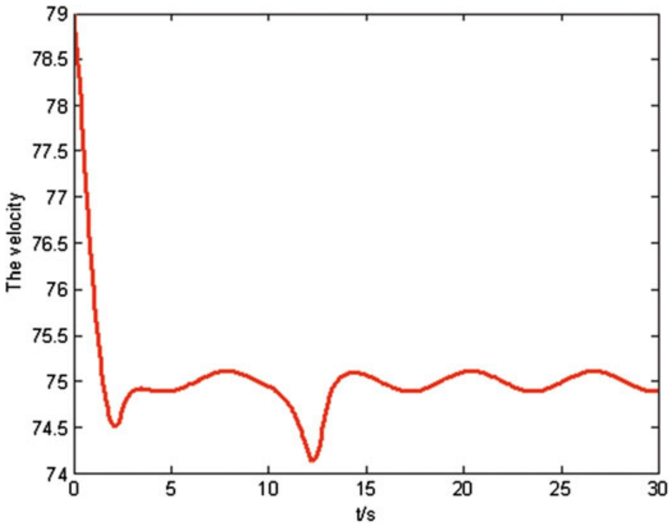

Fig. 6.5 Time response of the velocity with FTC

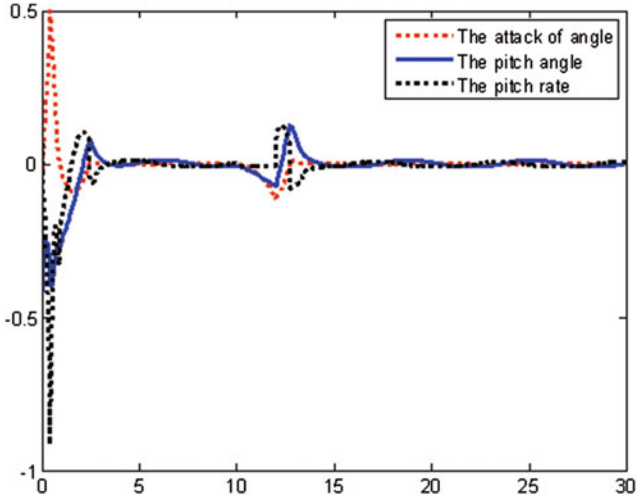

Fig. 6.6 Time response of the attack of angle, the pitch angle and the pitch rate with FTC

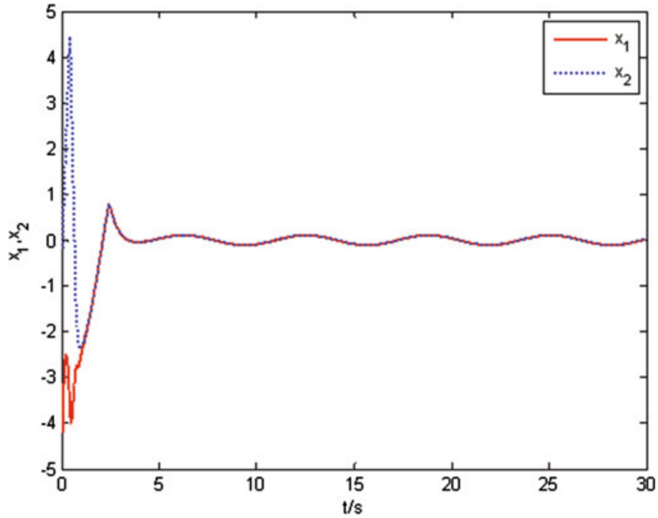

Fig. 6.7 State response under normal condition

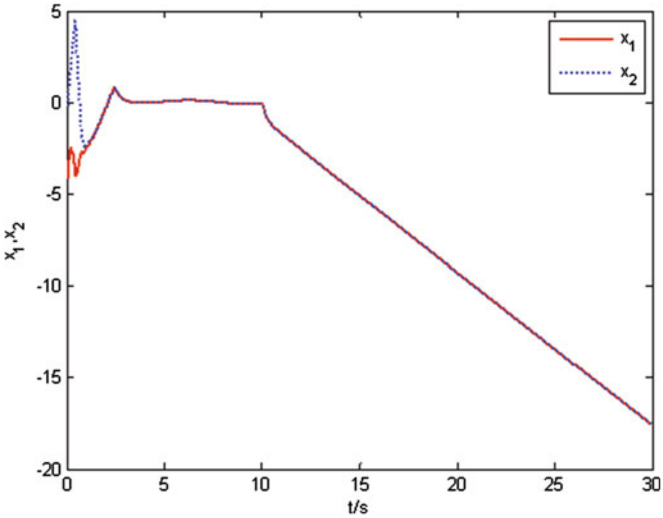

Fig. 6.8 State response under faulty condition without FTC

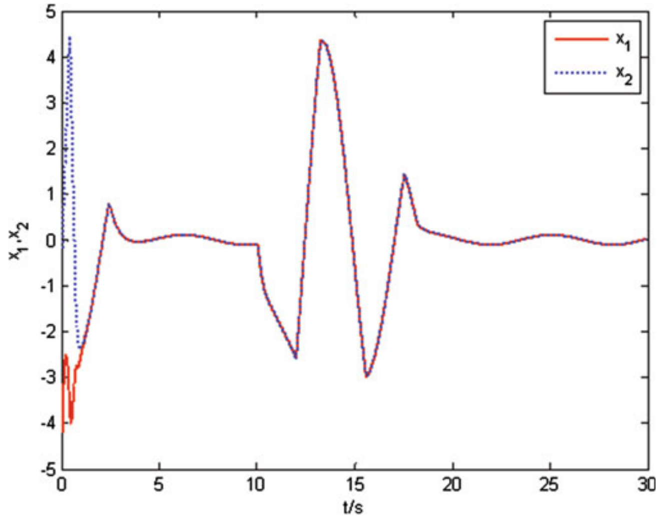

Fig. 6.9 State response under faulty condition with FTC

6.5 Conclusions

In this chapter, the fault-tolerant control problem for a class of uncertain nonlinear systems in presence of actuator faults is discussed. We first design a bank of observers to detect, isolate and estimate the fault. Then a sufficient condition for the existence of an FDI observer is derived. Simulation show that the designed fault detection, isolation and estimation algorithms and fault-tolerant control scheme have better dynamic performances in the presence of actuator faults.

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Chapter 7
Neural Network-Based Fault Tolerant Control Scheme Against Un-modeled Fault

7.1 Introduction

In control systems [1–34], actuator, sensor or component faults frequently occur, which can cause system performance deterioration and lead to instability that can further produce catastrophic accidents. Thus, to improve system reliability and to guarantee system stability in healthy and faulty situations, many effective fault-tolerant control (FTC) approaches have been proposed [1–3, 27–29]. In general, the FTC strategies can be categorized into two classes: passive approach and active approach. Passive FTC methods are robust control techniques with respect to an *a priori* fixed set of faults [21–33]. Active methods consist of online reconfiguring or reconstructing the controller to recover the stability and system performance as soon as a diagnostic algorithm has detected the presence of a fault [34, 35].

However, in most of the results about FTC or fault detection and isolation (FDI) in literature, the considered actuator or sensor faults are traditional affine appearances of the control input or system output. That is to say, the fault can be expressed explicitly as gain and/or bias fault, which is called modeled fault (MF) in this chapter. For example, the actuator fault can be described as: $u^f = g_f u + b_f$, where $g_f \in [0, 1)$ and $b_f \in R$ denote the remaining control rate and bias fault, respectively. Notice that, g_f and b_f may be constants or functions of time t or the system state x. Unfortunately, there exist some cases in practical applications where the fault cannot be expressed in the above form. This class of faults is called un-modeled fault (UMF) in this chapter. What's more, the results concerning on FTC against MF cannot be extended directly to FTC against UMF. Hence, the design of fault tolerant controller of systems with UMF is more challenging. To our best knowledge, up to now, there are only a few results reported in the literature. In [36], the problem of adaptive FTC for nonlinear systems with actuator NF was investigated. However, the results are only applicable to second-order nonlinear systems rather than more general high-order systems, which limit their practical applications. In [37, 38], robust detection and isolation schemes for nonlinear faults were addressed. However, these FDI schemes worked

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under the condition that the system state variables and control inputs were bounded before and after the occurrence of a fault, which is too restrictive. In addition, the UMF was assumed to be a known function about control input and system state with an unknown gain. Hence, how to control more general high-order nonlinear systems with actuator UMF still is an important and open problem, which motivates us for this study.

On the other hand, the active FTC scheme requires the fault information, which can be obtained by fault diagnosis (FD). However, FD takes time to performance, which means that there is some time delay between fault occurrence and fault accommodation. In this chapter, such time delay is called the time delay due to FD (TDDTFD). What is worst, during the time delay interval, the considered system is always controlled by the normal controller, which is designed under healthy conditions without considering any faults. As stated in [39], in general, an active FTC law is designed based on an open-loop system modeled as a function of fault parameters under the assumption that they are immediately identified by an FDI model. As previously pointed out, there is always some time needed to diagnose the fault occurring in the system. When a fault occurs, the faulty system works under the normal control until the fault is diagnosed and fault accommodation is performed, which may cause severe loss of performance and stability. Hence, it is important to analyze the effect of the time delay and to minimize its impact to the system. Unfortunately, only a few results have considered the TDDTFD's adverse effect on the stability of the system. The work in [37–41] investigated the effect and provided some effective approaches. However, the results in [37–41] worked well only under some restrictive conditions. Therefore, another motivation of our work in this chapter is to minimize TDDTFD and to reduce the adverse effect.

In this chapter, we investigate the problem of adaptive active FTC for a class of nonlinear systems with unknown actuator UMF. The design of the normal and fault tolerant controllers is first analyzed. Then a novel neural networks-based FTC scheme with fault alarm is proposed by using the implicit function theorem. Compared with existing results, this chapter makes the following contributions:

(1) The actuator fault considered in this chapter is assumed to have no-affine appearance of the system state variables and control input, which makes the control problem more challenging;

(2) Compared with [36] where only second-order systems without external disturbance were investigated, we study the adaptive FTC problem of a class of more general high-order nonlinear systems in this chapter;

(3) In [37, 38] the system state and control variables were bounded before and after the occurrence of a fault and the UMF was a known function of control input and system state with an unknown gain. The FTC scheme proposed in this chapter removes these assumptions. The theoretical developments and results of this chapter are thus valuable in wider practical applications; and

(4) The proposed scheme has the advantage of having the property of the passive FTC scheme as well as the traditional active FTC scheme that minimizes TDDTFD and reduces the adverse effect. Moreover, the FTC scheme doesn't require the FDI model which is necessary in the traditional active FTC scheme.

The rest of this chapter is organized as follows. In Sect. 7.2, the FTC objective is formulated. In addition, the actuator fault model and mathematical description of neural networks are given. In Sect. 7.3, the main technical results of this chapter are given, which include the designs of the normal controller, the traditional passive fault tolerant controller and novel adaptive fault tolerant controller. The example is presented in Sect. 7.4. Simulation results are presented to demonstrate the effectiveness of the proposed technique. Finally, Sect. 7.5 draws the conclusion.

7.2 Problem Statement and Description of NNs

In this section, we will first formulate the fault-tolerant control problem of a class of nonlinear systems. Then, neural networks (NNs) are introduced and their mathematical description is given.

7.2.1 Problem Statement

Consider the following nonlinear system

$$\begin{cases} \dot{x}_i = x_{i+1}, i = 1, 2, \ldots, n-1 \\ \dot{x}_n = f_0(x) + u + d(t) \\ y = x_1 \end{cases} \tag{7.1}$$

where $x = [x_1, \ldots, x_n]^T$, $u \in R$ and $y \in R$ are the state variables, system input and output, respectively. The nonlinear function $f_0(x) \in R$ is unknown and smooth, and $d(t)$ denotes the external bounded disturbance.

The control objective is to design an adaptive controller that generates u such that the system output y tracks as accurately a desired trajectory $y_d(t)$ as possible, regardless of the disturbance $d(t)$.

Define $x_d = [x_{d1}, \ldots, x_{dn}]^T$, where $x_{di} = y_d^{(i-1)}(t), i = 1, \ldots, n$.

The actuator failure model considered in this chapter can be described as

$$u^f = f(x, u), t \geq t_f \tag{7.2}$$

where $f(x, u)$ is a nonlinear smooth function, with t_f being unknown fault occurrence time.

Remark 7.1 In the literature, there are fruitful results about FD, FDI and FTC [42–48]. However, most of them only pay attention to MF. In general, two kinds of actuator explicit faults are considered: part loss of effectiveness of the actuators and actuator bias faults. They can be commonly described as

$$u^f = g_f u, u^f = u + b_f$$

and can be uniformly described as

$$u^f = g_f u + b_f$$

where $g_f (0 \le g_f < 1)$ and b_f denote, respectively, the remaining control rate and bias fault, which may be constants or functions about time t or system state x. In addition, they are assumed to be bounded. However, in some cases, the fault cannot always be described in the above affine form. The results as in [42–48] thus cannot be applied in such cases. Therefore, it becomes very necessary to investigate UMF.

Define the tracking errors e as follows

$$e = x - x_d = [e_1, \ldots, e_n]^T \tag{7.3}$$

Obviously, the control objective is that for any given target orbit $y_d(t)$, an adaptive fault tolerant controller u is designed to guarantee that the tracking error e is as small as possible despite both the actuator fault and the external disturbance.

Define the filtered function s as follows:

$$s = \sum\nolimits_{i=1}^{n-1} c_i e_i(t) + e_n(t) \tag{7.4}$$

where $c_i = C_{n-1}^{i-1} a^{n-i}$, $i = 1, 2, \ldots, n$, $C_{n-1}^{i-1} = \frac{(n-1)(n-2)\cdots(n-i+1)}{(i-1)(i-2)\cdots 1}$, and $a > 0 \in R$ denotes a design parameter.

To design an appropriate controller, for the system in (7.1) and the fault model in (7.2), the following lemma and some assumptions are used.

Lemma 7.1 *Let s be defined by (7.4).*
(1) If $s = 0$, then $\lim_{t\to\infty} e(t) = 0$;
(2) If $|s| \le a$ and $e(0) \in \Omega_a$, then $e(t) \in \Omega_a$, $\forall t \ge 0$;
(3) If $|s| \le a$ and $e(0) \notin \Omega_a$, then $\exists T = (m-1)\lambda$, $\ni \forall t \ge T$, $e(t) \in \Omega_a$, where $\Omega_a = \{e(t) | \, |e_i| \le 2^{(j-1)} \lambda^{j-m} a$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, with $\lambda > 0 \in R$ and $a > 0 \in R$ denoting the design parameters.

Assumption 7.1 There exists a known constant $\bar{d} > 0 \in R$, such that $|d(t)| \le \bar{d}$.

Assumption 7.2 There exists an unknown constant $M_d > 0 \in R$ such that $x_d \in \Omega_d = \{x_d | \, ||x_d|| \le M_d\} \subset \mathrm{R}^n$.

Assumption 7.3 The sign of $\frac{\partial f(x,u)}{\partial u}$ is known and there exist unknown constants $b_l > 0 \in R$, $l = 0, 1, 2$ such that $b_0 \le |\frac{\partial f(x,u)}{\partial u}| \le b_1$ and $|\frac{\partial^2 f(x,u)}{\partial u^2}| \le b_2$. Without loss of generality, it is assumed that $\frac{\partial f(x,u)}{\partial u} > 0$.

Remark 7.2 The purpose of Assumption 7.1 is to reduce the complexity of the normal controller. In fact, in the following FTC design in Sect. 7.2, Assumption 7.1 is removed where using an adaptive control technique. Assumption 7.3 seems to be restrictive. However, most of fault models in literature satisfy the assumption. In fact, for UMF with the boundary assumption, i.e.,

$$f(x, u) = g_f u + b_f$$

where $b_0 \leq g_f < b_1$, Assumption 7.3 is naturally satisfied, i.e.,

$$b_0 \leq |\frac{\partial f(x, u)}{\partial u}| = |g_f| \leq b_1, 0 = |\frac{\partial^2 f(x, u)}{\partial u^2}| \leq b_2$$

Obviously, if the assumption that the traditional MFs are bounded is extended to UMFs, then Assumption 7.3 is also needed. In addition, it is worth pointing out that b_l is only needed for analysis purpose, the exact value of b_l is not required in the controller design.

7.2.2 *Mathematical Description of Neural Networks*

Neural networks (NNs) have been widely used in modeling and controlling of nonlinear systems because of their capabilities of nonlinear function approximation, learning, and fault tolerance. The feasibility of applying NNs to unknown dynamic systems control has been demonstrated in many studies [49]. In this chapter, we use the radial basis function NNs presented in [49]

$$h(z, W) = W^T S(z)$$

to approximate a continuous function $h(z) : R^{n+2} \rightarrow R$, where the weight vector W, the basis function vector $S(z)$ are defined as follows: $W = [W_1, W_2, \ldots, W_N]^T$, $S(z) = [s_1(z), s_2(z), \ldots, s_N(z)]^T$ with N is the number of the NNs nodes. The function $s_i(z) = \exp(-(\sum_{j=1}^{p_i} (z - a_{ij})^2)/(\mu_i)^2)$, $\mu_i > 0$ is the center of the receptive field, and a_{ij} is the width of the Gaussian function. Let

$$\begin{aligned} \Omega_W &= \{W : \|W\| \leq w_m\}, W_i^* \\ &= \arg \min_{W \in \Omega_W} [\sup_{z \in \Omega_Z} |h(z, W) - h(z)|] \end{aligned}$$

where $w_m > 0$ is a design parameter. For a continuous function $h(z)$, we can obtain $h(z) = W^{*T} S(z) + \varepsilon^*(z)$, where W^* and $\varepsilon^*(z)$ denote the optimal weight vector and the optimal approximation error. Define two compact sets Ω_z and Ω_c as follows:

$$\begin{aligned} \Omega_z &= \left\{ \left[x^T, s, \gamma\right]^T |x \in \Omega \ , x_d \in \Omega_d \right\} \\ \Omega_c &= \{x \, ||s| \leq c_s, \ x_d \in \Omega_d\} \end{aligned}$$

where $\gamma = \sum_{i=1}^{n-1} c_i e_{i+1} - x_d^{(n)}$, $\Omega \in R^n$ is a enough large compact set, and $c_s > 0 \in R$ denotes a design parameter.

From the universal approximation results stated in [49], we know that NNs can approximate any continuous function to any accuracy on a compact set. Hence, the following assumption is made in this chapter.

Assumption 7.4 There exist an unknown constant $\varepsilon > 0 \in R$, such that $|\varepsilon^*(z)| \leq \varepsilon$.

For notational simplicity, we use $\bullet$ to denote $\bullet(\cdot)$. For example, d is the abbreviation of $d(t)$.

7.3 Design of Adaptive NNs-Based Fault Tolerant Controller

In this section, the main technical results are presented. First, the normal control scheme is examined in its healthy condition. Then, the passive FTC scheme is investigated to compensate for actuator faults. Finally, a novel FTC scheme is proposed to guarantee the control objective is met, despite the presence of actuator faults.

7.3.1 Design of Normal Controller (Fault-Free Condition)

In the healthy case, the system is described as follows:

$$\begin{cases} \dot{x}_i = x_{i+1}, i = 1, \ldots, n-1 \\ \dot{x}_n = f_0(x) + u + d(t) \end{cases}$$

From (7.4), one has

$$\dot{s} = f_0(x) + u + d(t) + \gamma$$

In the following, NNs are used to approximate the function $f_0(x)$ as $W_0^{*T} S_0(x) + \varepsilon_0^*(x)$, $\hat{W}_0$ and $\hat{\varepsilon}_0$ are the estimates of the optimal weight vector W_0^* and ε_0, respectively, where $|\varepsilon_0^*(x)| \leq \varepsilon_0$, $\varepsilon_0 > 0 \in R$ is an unknown constant.

Define the following function

$$V_0 = s^2/2 + (\tilde{W}_0^T \tilde{W} + \tilde{\varepsilon}_0^2)/(2\eta_0)$$

where $\tilde{W}_0 = W_0^* - \hat{W}_0$, $\tilde{\varepsilon}_0 = \varepsilon_0 - \hat{\varepsilon}_0$, and $\eta_0 > 0 \in R$ is an adaptive rate.

Differentiating V_0 with respect to time t, one has

$$\dot{V}_0 \leq s W_0^{*T} S_0(x) + |s|\varepsilon_0 + su + |s|(\bar{d} + |\gamma|) - \tilde{W}_0^T \dot{\hat{W}}_0/\eta_0 - \tilde{\varepsilon}_0 \dot{\hat{\varepsilon}}_0/\eta_0 \tag{7.5}$$

The control and adaptive laws are designed as follows:

$$u = -\hat{W}_0^T S_0(x) - |s|(\hat{\varepsilon}_0 + \bar{d} + |\gamma|) - s/2$$

$$\dot{\hat{W}}_0 = \eta_0 s S_0(x) + \eta_1 \hat{W}_0$$

$$\dot{\hat{\varepsilon}}_0 = \eta_0 |s| + \eta_1 \hat{\varepsilon}_0$$

where $\eta_1 > 0$ is a design parameter. Substituting the above control and adaptive laws into (7.5), yields

$$\dot{V}_0 \leq -\frac{1}{2}s^2 - \frac{\eta_1}{\eta_0}(\tilde{W}_0^T \hat{W}_0 + \tilde{\varepsilon}_0 \hat{\varepsilon}_0)$$

Since

$$-\frac{\eta_1}{\eta_0}\tilde{W}_0^T \hat{W}_0 \leq -\frac{\eta_1}{2\eta_0}\tilde{W}_0^T \tilde{W}_0 + \frac{\eta_1}{2\eta_0} w_m^2$$

$$-\frac{\eta_1}{\eta_0}\tilde{\varepsilon}_0 \hat{\varepsilon}_0 \leq -\frac{\eta_1}{2\eta_0}\tilde{\varepsilon}_0^2 + \frac{\eta_1}{2\eta_0}\varepsilon_0^2 \leq -\frac{\eta_1}{2\eta_0}\tilde{\varepsilon}_0^2 + \frac{\eta_1}{2\eta_0}\bar{\varepsilon}_0^2$$

one has

$$\begin{aligned} \dot{V}_0 &\leq -\frac{1}{2}s^2 - \frac{\eta_1}{2\eta_0}\tilde{W}_0^T \tilde{W}_0 - \frac{\eta_1}{2\eta_0}\tilde{\varepsilon}_0^2 + \frac{\eta_1}{2\eta_0} w_m^2 + \frac{\eta_1}{2\eta_0}\bar{\varepsilon}_0^2 \\ &\leq -\lambda_0 V_0 + \mu_0 \end{aligned}$$

where $\lambda_0 = \min\{1, \eta_1\}$, $\mu_0 = \frac{\eta_1}{2\eta_0} w_m^2 + \frac{\eta_1}{2\eta_0}\bar{\varepsilon}_0^2$, $\bar{\varepsilon}_0 > 0 \in R$ is the upper boundary of ε_0 and assumed to be known.

Remark 7.3 The assumption that there exists a known constant $\bar{\varepsilon}_0$ such that $\varepsilon_0 \leq \bar{\varepsilon}_0$ seems restrictive. However, we should point out that the main objective in this chapter is to control faulty systems with the fault given in (7.2), not the nominal systems (7.1). In order to reduce the normal controller's complexity, the assumption is made in this chapter. In fact, the assumption can be removed using such techniques as adaptive control.

Furthermore, we have the condition that

$$0 \leq V_0(t) \leq \mu_0/\lambda_0 + (V_0(0) - \mu_0/\lambda_0)e^{-\lambda_0 t} \leq \mu_0/\lambda_0 + V_0(0)$$

Since $s^2/2 \leq V_0$, then

$$|s(t)| \leq \sqrt{2(\mu_0/\lambda_0 + V_0(0))} = a$$

Similarly,

$$||\tilde{W}_0|| \leq \sqrt{\eta_0}a, \ |\tilde{\varepsilon}_0| \leq \sqrt{\eta_0}a$$

From Lemma 7.1, one has

$$e \in \Omega_a$$

where $\Omega_a = \{e(t)|\,|e_i| \leq 2^i \lambda^{i-n} a, i = 1, 2, \ldots, n\}$.

The above analysis is summarized in the following Theorem.

Theorem 7.1 *Consider the healthy nonlinear system (7.1) with Assumptions 7.1, 7.2 and 7.4. If the following control and adaptive laws are employed*

$$u = -\hat{W}_0^T S_0(x) - |s|(\hat{\varepsilon}_0 + \bar{d} + |\gamma|) - s/2 \tag{7.6}$$

$$\dot{\hat{W}}_0 = \eta_0 s S_0(x) + \eta_1 \hat{W}_0 \tag{7.7}$$

$$\dot{\hat{\varepsilon}}_0 = \eta_0 |s| + \eta_1 \hat{\varepsilon}_0 \tag{7.8}$$

then the closed-loop system is globally asymptotically bounded, all signals in the closed-loop system converge to the small neighborhood of origin Ω_0 *defined as* $\Omega_0 = \{(s, \tilde{W}_0, \tilde{\varepsilon}_0, e_i)|\,|s| \leq a, ||\tilde{W}_0|| \leq \sqrt{\eta_0} a, |\tilde{\varepsilon}_0| \leq \sqrt{\eta_0} a, |e_i| \leq 2^i \lambda^{i-n} a,$ $i = 1, \ldots, n\}$.

Proof From the above analysis, it is easy to obtain the conclusions.

From Theorem 7.1, one has

$$0 \leq s^2/2 = V_0(t) \leq \mu_0/\lambda_0 + (V_0(0) - \mu_0/\lambda_0)e^{-\lambda_0 t}$$

From the practical point of view, the tracking objective is obtained if $|s(t)| \leq \sqrt{2\delta_0}$, where $\delta_0 > 0 \in R$ is a designed parameter. Let

$$\delta_0 = V_0(t) \leq \mu_0/\lambda_0 + (V_0(0) - \mu_0/\lambda_0)e^{-\lambda_0 t_s}$$

one has

$$t_s = \frac{-\ln\left(\frac{\delta_0 - \frac{\mu_0}{\lambda_0}}{V_0(0) - \frac{\mu_0}{\lambda_0}}\right)}{\lambda_0} \tag{7.9}$$

We obtain from the above analysis that the tracking error e converges to a small neighborhood of the origin $\Omega'_a = \{e(t)|\,|e_i| \leq 2^i \lambda^{i-n} \sqrt{2\delta_0}, i = 1, 2, \ldots, n\}$, i.e., $e \in \Omega'_a$. Obviously, it is after t_s, i.e., $t \geq t_s$, that the tracking control objective is obtained. The time interval $[0, t_s]$ is spent to stabilize the tracking error dynamics, which is named as stabilization time (ST) in this chapter.

Remark 7.4 In the literature, it is not required generally to develop the analytical expression for ST. In this chapter, it is given because it will be used to design the FTC scheme with fault alarm.

7.3.2 Design of Passive Adaptive Fault Tolerant Controller

From (7.1), (7.2) and (7.4), one has

$$\dot{s} = f_0(x) + f(x, u) + d + \gamma \tag{7.10}$$

where $\gamma = \sum_{i=1}^{n-1} c_i e_{i+1} - x_d^{(n)}$.

Because of $\frac{\partial f(x,u)}{\partial u} \neq 0$, by the implicit function theorem, there exists an ideal control $u^*(x)$, such that

$$f_0(x) + f(x, u^*) = 0$$

By the mean value theorem, one has

$$f(x, u) = f(x, u^*) + \int_0^1 \frac{\partial f(x, u_\lambda)}{\partial u_\lambda} d\lambda (u - u^*)$$

where $\lambda \in [0, 1]$, and

$$u_\lambda = \lambda u + (1 - \lambda)u^* \tag{7.11}$$

Hence, (7.10) can be rewritten as follows:

$$\dot{s} = f_0(x) + b(x, u)[(u - u^*) + \frac{\gamma + d}{b(x, u)}] \tag{7.12}$$

where

$$b(x, u) = \int_0^1 \frac{\partial f(x, u_\lambda)}{\partial u_\lambda} d\lambda \tag{7.13}$$

From (7.3) and (7.4), one has

$$x_n = e_n + x_{dn} = s - \sum_{i=1}^{n-1} c_i e_i(t) + x_{dn}$$

Furthermore, $u^*(x)$ and $b(x, u)$ can be rewritten as follows:

$$u^*(x) = u^*(x_1, x_2, \ldots, x_{n-1}, s + \gamma_1)$$

$$b(x, u) = b(x_1, x_2, \ldots, x_{n-1}, s + \gamma_1, u)$$

where $\gamma_1 = -\sum_{i=1}^{n-1} c_i e_i(t) + x_{dn}$.

To obtain the above control objective, the desired control law is employed as follows:

$$u = -k(x_n, s)s + \hat{W}^T S(z) + u_r \tag{7.14}$$

where $z = [x_1, x_2, \ldots, x_{n-1}, sgn(|x_n|), s, 1]^T$, NNs are used to approximate a unknown continuous function $\varphi(z)$ defined later by (7.42) as $\varphi(z) = \hat{W}^{*T} S(z) + \varepsilon^*(z)$, $\hat{W}$ is the estimate of W^* at time t, $k(x_n, s)$ and u_r are defined by (7.45) and (7.46), respectively.

Define the following Lyapunov function

$$V_1(t) = \int_0^s \left[\int_0^1 \beta(u_{\lambda\sigma}) d\lambda\right]^{-1} \sigma d\sigma \tag{7.15}$$

where

$$\beta(u_{\lambda\sigma}) = \frac{\partial f(x, u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} \tag{7.16}$$

$$u_{\lambda\sigma} = \lambda u_\sigma + (1-\lambda) u^*(x_1, x_2, \ldots, x_{n-1}, s + \gamma_1) \tag{7.17}$$

$$u_\sigma = -k(\sigma + \gamma_1, \sigma)\sigma + \hat{W}^T S(z_\sigma) \tag{7.18}$$

$$z_\sigma = (x_1, \ldots, x_{n-1}, \sigma + \gamma_1, sgn(\sigma)|\sigma + \gamma_1|, \sigma, 1)^T \tag{7.19}$$

Since $0 < b_0 \le \frac{\partial f(x,u)}{\partial u} \le b_1$, then

$$V_1 \le \int_0^s \frac{1}{b_0} \sigma d\sigma = \frac{s^2}{2b_0},\ V_1 \ge \int_0^s \frac{1}{b_1} \sigma d\sigma = \frac{s^2}{2b_1} \tag{7.20}$$

It is obvious that V_1 is a positive function with the property: $V_1 \to 0$ as $s \to 0$ and $V_1 \to \infty$ as $s \to \infty$.

Differentiating V_1 with respect to time t, one has

$$\begin{aligned}\dot{V}_1 =& s\left[\int_0^1 \beta(u_{\lambda\sigma}) d\lambda\right]^{-1} \dot{s} + \dot{\gamma}_1 \int_0^s \frac{\partial\left[\int_0^1 \beta(u_{\lambda\sigma}) d\lambda\right]^{-1}}{\partial \gamma_1} \sigma d\sigma - \\ & \int_0^s \left\{\frac{\sum_{i=1}^{n-1} \int_0^1 \frac{\partial \beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} \frac{\partial u_{\lambda\sigma}}{\partial x_i} \dot{x}_i d\lambda}{\left[\int_0^1 \beta(u_{\lambda\sigma}) d\lambda\right]^2}\right\} \sigma d\sigma - \\ & \int_0^s \left\{\frac{\int_0^1 \frac{\partial \beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} (\frac{\partial u_{\lambda\sigma}}{\partial \hat{W}})^T \dot{\hat{W}} d\lambda}{\left[\int_0^1 \beta(u_{\lambda\sigma}) d\lambda\right]^2}\right\} \sigma d\sigma\end{aligned} \tag{7.21}$$

From $\frac{\partial(\sigma+\gamma_1)}{\partial \sigma} = \frac{\partial(\sigma+\gamma_1)}{\partial \gamma_1}$, it gives

$$\partial\left[\int_0^1 \beta(u_{\lambda\sigma}) d\lambda\right]^{-1} / \partial\gamma_1 = \partial\left[\int_0^1 \beta(u_{\lambda\sigma}) d\lambda\right]^{-1} / \partial\sigma \tag{7.22}$$

Furthermore, one has

$$\int_0^s \frac{\partial\left[\int_0^1 \beta(u_{\lambda\sigma})d\lambda\right]^{-1}}{\partial\sigma}\sigma d\sigma = \frac{s}{b(x,u)} - \int_0^s \left[\int_0^1 \beta(u_{\lambda\sigma})d\lambda\right]^{-1} d\sigma \tag{7.23}$$

From Assumption 7.3 and (7.16), one has

$$\beta(x, u_{\lambda\sigma}) = \frac{\partial f(x, u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} \geq b_0 \tag{7.24}$$

It follows that

$$\int_0^1 \beta(u_{\lambda\sigma})d\lambda \geq \int_0^1 b_0 d\lambda = b_0 \tag{7.25}$$

By (7.24) and (7.25) and $\dot{x}_i = x_{i+1}$, one has

$$\begin{aligned}\dot{V}_1 \leq & \frac{s\dot{s}}{b(x,u)} + \frac{\dot{\gamma}_1 s}{b(x,u)} + \left|\dot{\gamma}_1 \int_0^s \frac{1}{b_0}d\sigma\right| + \\ & \frac{1}{b_0^2}\int_0^s \sum_{i=1}^{n-1}\left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}\frac{\partial u_{\lambda\sigma}}{\partial x_i}x_{i+1}d\lambda\right|\sigma d\sigma + \\ & \frac{1}{b_0^2}\int_0^s \left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}\left(\frac{\partial u_{\lambda\sigma}}{\partial \hat{W}}\right)^T \dot{\hat{W}} d\lambda\right|\sigma d\sigma\end{aligned} \tag{7.26}$$

From (7.17) and (7.18), one has

$$\begin{aligned}\frac{\partial u_{\lambda\sigma}}{\partial x_i} &= \lambda\frac{u_\sigma}{\partial x_i} + (1-\lambda)\frac{\partial u^*(x)}{\partial x_i} \\ &= \lambda\frac{\partial\left[\hat{W}^T S(z_\sigma)\right]}{\partial x_i} + (1-\lambda)\frac{\partial u^*(x)}{\partial x_i}\end{aligned} \tag{7.27}$$

where $u^*(x) = u^*(x_1, x_2, \ldots, x_{n-1}, s + \gamma_1)$. Thus, one has

$$\begin{aligned}&\left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}\frac{\partial u_{\lambda\sigma}}{\partial x_i}x_{i+1}d\lambda\right| \leq \\ &\quad \left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}\lambda d\lambda\right|\left|\frac{\partial\left[\hat{W}^T S(z_\sigma)\right]}{\partial x_i}x_{i+1}\right| + \\ &\quad \left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}(1-\lambda)d\lambda\right|\cdot\left|\frac{\partial u^*(x)}{\partial x_i}x_{i+1}\right|\end{aligned} \tag{7.28}$$

From (7.16)–(7.18) and Assumption 7.3, one has

$$\left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}\lambda d\lambda\right| \leq \int_0^1 b_2\lambda d\lambda = \frac{b_2}{2} \tag{7.29}$$

$$\left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}(1-\lambda) d\lambda\right| \leq \int_0^1 b_2(1-\lambda) d\lambda = \frac{b_2}{2} \tag{7.30}$$

From (7.29) and (7.30), we can further derive (7.28) as

$$\begin{aligned} &\left|\int_0^1 \frac{\partial\beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}}\frac{\partial u_{\lambda\sigma}}{\partial x_i} x_{i+1} d\lambda\right| \leq \\ &\frac{b_2}{2}\left|\frac{\partial\left[\hat{W}^T S(z_\sigma)\right]}{\partial x_i} x_{i+1}\right| + \frac{b_2}{2}\cdot\left|\frac{\partial u^*(x)}{\partial x_i} x_{i+1}\right| \end{aligned} \tag{7.31}$$

From the basis function's definition, one has

$$\frac{\partial s_i(z_\sigma)}{\partial x_j} = \frac{-2(x_j - a_{ij})}{\mu_i^2}\exp\left(-\frac{\sum_{j=1}^N (z-a_{ij})^2}{(\mu_i)^2}\right) \tag{7.32}$$

Since

$$\exp\left[-\frac{\sum_{j=1}^N (z-a_{ij})^2}{(\mu_i)^2}\right] \leq \exp\left(\frac{-(x_j-a_{ij})^2}{(\mu_i)^2}\right)$$

then

$$\left|\frac{\partial s_i(z_\sigma)}{\partial x_j}\right| \leq \frac{2|x_j-a_{ij}|}{\mu_i^2}\exp\left(\frac{-(x_j-a_{ij})^2}{(\mu_i)^2}\right)$$

Obviously, if $\frac{2|x_j-a_{ij}|}{\mu_i} = \sqrt{\frac{1}{2}}$, then $\left|\frac{\partial s_i(z_\sigma)}{\partial x_j}\right|$ has the maximum value, i.e., $\left|\frac{\partial s_i(z_\sigma)}{\partial x_j}\right| \leq \frac{\sqrt{2}}{\mu_i}\exp(-\frac{1}{2})$. From the above analysis, we know that there exists a constant $w_{i0} > 0$, such that

$$\left|\frac{\partial\left[\hat{W}^T S(z_\sigma)\right]}{\partial x_i} x_{i+1}\right| \leq \left|\hat{W}^T\frac{\partial\left[S(z_\sigma)\right]}{\partial x_i}\right| |x_{i+1}| \leq w_{i0}|x_{i+1}| \tag{7.33}$$

where $w_{i0} = \sup_{||\hat{W}||\leq w_m}\{\sum_{i=1}^l (\sqrt{2}/\mu_i)\exp(-1/2)|\hat{w}_i|\}$. It is necessary to point out, $||\hat{W}|| \leq w_m$ is guaranteed in the following adaptive law (7.47). Furthermore, (7.28) can be re-expressed as follows:

$$\left|\int_0^1 \frac{\partial \beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} \frac{\partial u_{\lambda\sigma}}{\partial x_i} x_{i+1} d\lambda\right| \le \frac{b_2}{2}(w_{i0}|x_{i+1}| + \left|\frac{\partial u^*(x)}{\partial x_i} x_{i+1}\right|) \tag{7.34}$$

Therefore,

$$\begin{aligned} &\frac{1}{b_0^2}\int_0^s \sum\nolimits_{i=1}^{n-1} \left|\int_0^1 \frac{\partial \beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} \frac{\partial u_{\lambda\sigma}}{\partial x_i} x_{i+1} d\lambda\right| \sigma d\sigma \\ &\le \frac{1}{b_0^2}\int_0^s \sum_{i=1}^{n-1} (\frac{b_2}{2} w_{i0}|x_{i+1}| + \frac{b_2}{2}\left|\frac{\partial u^*(x)}{\partial x_i} x_{i+1}\right|)\sigma d\sigma \\ &= \frac{b_2 s}{4b_0^2}\sum\nolimits_{i=1}^{n-1}(w_{i0}|x_{i+1}|) + \frac{b_2}{2b_0^2}\int_0^s \sum\nolimits_{i=1}^{n-1}(\left|\frac{\partial u^*(x)}{\partial x_i} x_{i+1}\right|)\sigma d\sigma \end{aligned} \tag{7.35}$$

In addition, because

$$\frac{\partial u_{\lambda\sigma}}{\partial \hat{W}} = \lambda \frac{\partial u_\sigma}{\partial \hat{W}} = \lambda S(z_\sigma) \tag{7.36}$$

one has

$$\begin{aligned} &\left|\int_0^1 \frac{\partial \beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} (\frac{\partial u_{\lambda\sigma}}{\partial \hat{W}})^T \dot{\hat{W}} d\lambda\right| \\ &\le \left|\int_0^1 \frac{\partial \beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} \lambda d\right| \left|S^T(z_\sigma)\dot{\hat{W}}\right| \le \frac{b_2}{2}\left|S^T(z_\sigma)\dot{\hat{W}}\right| \end{aligned} \tag{7.37}$$

Furthermore,

$$\frac{1}{b_0^2}\int_0^s \left|\int_0^1 \frac{\partial \beta(u_{\lambda\sigma})}{\partial u_{\lambda\sigma}} (\frac{\partial u_{\lambda\sigma}}{\partial \hat{W}})^T \dot{\hat{W}} d\lambda\right| \sigma d\sigma \le \frac{b_2}{2b_0^2}\int_0^s \left|S^T(z_\sigma)\dot{\hat{W}}\right| \sigma d\sigma \tag{7.38}$$

From (7.12), (7.26), (7.35) and (7.38), one has

$$\begin{aligned} \dot{V}_1 \le & s[u-u^*] + \left|\frac{\gamma+d}{b_0} s\right| + \frac{b_2 s^2}{4b_0^2}\sum\nolimits_{i=1}^{n-1} w_{i0}|x_{i+1}| + \\ & \frac{b_2}{2b_0^2}(\int_0^s \sum\nolimits_{i=1}^{n-1}\left|\frac{\partial u^*(x)}{\partial x_i} x_{i+1}\right| \sigma d\sigma + \int_0^s \left|S^T(z_\sigma)\dot{\hat{W}}\right| \sigma d\sigma) \end{aligned} \tag{7.39}$$

Since $u^*(x) = u^*(x_1, \ldots, x_{n-1}, s + \gamma_1)$, one has

$$\begin{aligned} &\frac{b_2}{2b_0^2}\int_0^s \sum\nolimits_{i=1}^{n-1}\left|\frac{\partial u^*(x)}{\partial x_i} x_{i+1}\right| \sigma d\sigma \\ &= \frac{b_2 s^2}{2b_0^2}\int_0^1 \sum\nolimits_{i=1}^{n-1}\left|\frac{\partial u^*(x_1, \ldots, x_{n-1}, \lambda s + \gamma_1)}{\partial x_i} x_{i+1}\right| \lambda d\lambda \end{aligned} \tag{7.40}$$

Hence

$$\begin{aligned}\dot{V}_1 \leq & s(-k(x_n, s) + \hat{W}^T S(z) + u_r - \varphi(z)) + \left|\frac{\gamma + d}{b_0} s\right| + \\ & \frac{b_2 s^2}{4b_0^2} \sum_{i=1}^{n-1} w_{i0}|x_{i+1}| + \frac{b_2}{2b_0^2} \int_0^s \left|S^T(z_\sigma)\dot{\hat{W}}\right| \sigma d\sigma\end{aligned} \tag{7.41}$$

where

$$\varphi(z) = u^*(x) - \frac{b_2 s}{2b_0^2} \int_0^1 \sum_{i=1}^{n-1} \left|\frac{\partial u^*(x)}{\partial x_i} x_{i+1}\right| \lambda d\lambda \tag{7.42}$$

Since $\varphi(z)$ is a continuous function with respect to time t, it can be approximated by NNs as follows:

$$\varphi(z) = W^{*T} S(z) + \varepsilon^*(z)$$

Therefore,

$$\begin{aligned}\dot{V}_1 \leq & -s(k(x_n, s) - u_r) + k_1[\frac{s^2}{4} \sum_{i=1}^{n-1} w_{i0}|x_{i+1}| + \\ & \frac{1}{2} \int_0^s \left|S^T(z_\sigma)\dot{\hat{W}}\right| \sigma d\sigma] + k_2 |\gamma s| + k_3|s| + \\ & s\left(\hat{W}^T S(z) - W^{*T} S(z)\right) + |s|\varepsilon\end{aligned} \tag{7.43}$$

where $k_1 = b_2/b_0^2$, $k_2 = 1/b_0$, $k_3 = \bar{d}/b_0$.

Adopting the control law:

$$u = -k(x_n, s)s + \hat{W}^T S(z) + u_r \tag{7.44}$$

where

$$k(x_n, s) = s + \hat{k}_1(\eta_W N s^2 sgn(s) + \frac{s}{4} \sum_{i=1}^{n-1} w_{i0}|x_{i+1}|) \tag{7.45}$$

$$u_r = -\hat{k}_2 |\gamma s| - \hat{k}_3|s| - |s|\hat{\varepsilon} \tag{7.46}$$

where $\hat{k}_l, l = 1, 2, 3$ and $\hat{\varepsilon}$ are the estimates of $k_l, l = 1, 2, 3$ and ε at time t, respectively, $|\varepsilon^*(z)| \leq \varepsilon$, $\eta_W > 0 \in R$ will be defined later.

Employing the following adaptive laws:

$$\dot{\hat{W}} = \begin{cases} -\eta_W S(z)s, \quad \text{if} \quad \left\|\hat{W}\right\| < w_m \\ \quad \text{or} \left\|\hat{W}\right\| = w_m \text{ and } -\hat{W}^T \ S(z)s \le 0; \\ -\eta_W S(z)s + \eta_W \dfrac{\hat{W}\ \hat{W}^T S(z)s}{\left\|\hat{W}\right\|^2}, \\ \quad \text{if} \left\|\hat{W}\right\| = w_m \text{ and } -\hat{W}^T \ S(z)s > 0 \end{cases} \tag{7.47}$$

$$\dot{\hat{\varepsilon}} = \eta_\varepsilon \,|s| \tag{7.48}$$

$$\dot{\hat{k}}_1 = \eta_1(\eta_W N s^2 sgn(s) + \frac{s}{4}\sum\nolimits_{i=1}^{n-1} w_{i0}|x_{i+1}|) \tag{7.49}$$

$$\dot{\hat{k}}_2 = \eta_2 \,|\gamma s| \tag{7.50}$$

$$\dot{\hat{k}}_3 = \eta_3 \,|s| \tag{7.51}$$

where $\eta_l > 0 \in R, l = W, 1, 2, 3$ are the adaptive rates.

For system (7.1) with actuator fault (7.2), by using the above control and adaptive laws, we give the following theorem.

Theorem 7.2 *Consider nonlinear system (7.1) and the actuator fault (7.2) with Assumptions 7.1–7.4. If the control law (7.44) with (7.45) and (7.46) and the adaptive laws (7.47)–(7.51) are employed, then the closed-loop system is globally asymptotically stable, satisfying* $\lim_{t\to\infty}||e(t)|| = 0 \Leftrightarrow \lim_{t\to\infty}|e_k(t)| = 0$*,* $k = 1, 2, \ldots, n$.

Proof Before Theorem 7.2 is proven, we give the proof of the inequality $|S^T(z_\sigma)\dot{\hat{W}}| \le 2\eta_W N|s|$, where N is the numbers of RBF NNs nodes.

By (7.47), it is easy to find that $\dot{\hat{W}}$ depends on the states.

(1) If $||\hat{W}|| < w_m$ or $||\hat{W}|| = w_m$ and $\hat{W}^T\ S(z_\sigma)s \le 0$, then

$$|S^T(z_\sigma)\dot{\hat{W}}| \le \eta_W\ S^T(z_\sigma)S(z_\sigma)|s|$$

Since $|s_i(z_\sigma)| < 1$, $|S^T(z_\sigma)S(z_\sigma)| \le N$. Therefore, one has

$$|S^T(z_\sigma)\dot{\hat{W}}| \le \eta_W N|s| \le 2\eta_W N|s|$$

(2) If $||\hat{W}|| = w_m$ and $\hat{W}^T\ S(z_\sigma)s > 0$, then

$$|S^T(z_\sigma)\dot{\hat{W}}| \le \eta_W|[S^T(z_\sigma)S(z_\sigma) + \frac{S^T(z_\sigma)\hat{W}\hat{W}^T S(z_\sigma)}{||\hat{W}||^2}]s|$$

Since $|s_i(z_\sigma)| < 1$, $|S^T(z_\sigma)S(z_\sigma)| \le N$. Therefore,

$$S^T(\mathrm{z}_\sigma)\hat{W}\hat{W}^T S(z_\sigma) \le ||\hat{W}||_1$$

Noticing that $||\hat{W}||_1^2 \le N||\hat{W}||^2$, one has

$$|S^T(z_\sigma)\dot{\hat{W}}| \le \eta_W|s|(N + \frac{||\hat{W}||_1^2}{||\hat{W}||^2}) \le 2\eta_W N|s|$$

Hence, by (1) and (2), it results in

$$\begin{aligned} \frac{b_2}{2b_0^2}\int_0^s \left|S^T(z_\sigma)\dot{\hat{W}}\right| \sigma d\sigma &\le \frac{b_2}{2b_0^2}\int_0^s 2\eta_W N|s|\sigma d\sigma \\ &= \eta_W \frac{b_2 N}{b_0^2}|s^3| \end{aligned} \tag{7.52}$$

Now we give the proof of Theorem 7.2. Define

$$V = V_1 + \frac{1}{2\eta_W}\tilde{W}^T\tilde{W} + \frac{1}{2\eta_\varepsilon}\tilde{\varepsilon}^2 + \sum\nolimits_{i=1}^3 \frac{1}{2\eta_i}\tilde{k}_i^2$$

where $\tilde{W} = W^* - \hat{W}$, $\tilde{\varepsilon} = \varepsilon - \hat{\varepsilon}$, $\tilde{k}_i = k_i - \hat{k}_i$, $i = 1, 2, 3$.

Differentiating V with respect to time t, yields

$$\dot{V}(t) = \dot{V}_1 - \frac{1}{\eta_W}\tilde{W}^T\dot{\hat{W}} - \frac{1}{\eta_\varepsilon}\tilde{\varepsilon}\dot{\hat{\varepsilon}} - \sum\nolimits_{i=1}^3 \frac{1}{2\eta_i}\tilde{k}_i\dot{\hat{k}}_i \tag{7.53}$$

Substituting the control law (7.44) and the adaptive laws (7.47)–(7.51) into (7.53), yields

$$\dot{V} \le -s^2 \le 0 \tag{7.54}$$

Because $V(t)$ is a monotonous and non-increasing bounded function, $V(+\infty)$ exists, $\int_0^{+\infty} s^2(t)dt \le V(0) - V(+\infty)$, i.e., $s \in L_2$. And since $s, \dot{s} \in L_\infty$, by using Babalat's Lemma, the following result is obtained:

$$\lim_{t\to\infty} s(t) = 0$$

Furthermore, one has $\lim_{t\to\infty}|e_k(t)| = 0$, $k = 1, 2, \ldots, n$.

Remark 7.5 Regarding Assumption 7.1 and $k_3 = \bar{d}/b_0$, it is easily seen that Assumption 7.1 is not necessary for designing the passive FTC because $k_3 = \bar{d}/b_0$ can be estimated by (7.51).

Remark 7.6 The analysis in this subsection looks similar to that in [36], where second-order systems were considered. However, the systems considered in this chapter are more general and the design of the fault tolerant control scheme for such systems is more challenging than that in [36]. In addition, we will propose a novel active FTC scheme, which has wider application potentials.

7.3.3 Design of Novel Adaptive Fault Tolerant Controller

In the previously two subsections, the normal and passive fault tolerant controllers are designed. In this subsection, we will construct a novel FTC scheme, which contains both the advantages of the classical passive and active FTC schemes. Let us start by recalling the passive and active FTC's property.

Passive FTC is robust control techniques with respect to an *a priori* fixed set of faults. Compared with active FTC, this approach is simpler, but more conservative. In order to relax the conservatism of the passive FTC approach, active FTC method is developed. In general, active fault tolerant control framework includes the following steps: fault detection, isolation, estimation and accommodation. It is well known that each step in the framework takes some time to complete.

In this chapter, the time delay between fault occurrence and fault accommodation is called as TDDTFD. It is necessary to point out that, the considered system is always controlled by the faulty actuators during the time delay interval, which degrades the system performances and even damages the system. Hence, it is very important to minimize its adverse effort on the considered systems' performance when proposing a proper solution.

In the following, we will propose a novel fault tolerant controller. First, we assume that the fault occurrence time t_f is larger than the system stabilization time t_s, i.e., $t_f > t_s$. That is to say, the tracking objective has been obtained before the actuator fault occurrence.

Form Theorem 7.1, we know that it is after t_s, i.e., $t \geq t_s$, that the tracking error has converged to a small neighborhood of the origin Ω'_a, i.e., $|e_i(t)| \in \Omega'_a$, which means that the tracking control objective has been met. In other words, under the normal controller, $V_0(t) \leq \delta_0$, $t \geq t_s$ in the healthy case.

Define the following function

$$I(V_0) = \begin{cases} 1, & \text{if } V_0(t) > \delta_0 \text{ and } t \geq t_s; \\ 0, & \text{otherwise} \end{cases}$$

Now, based on Theorems 7.1 and 7.2, the fault tolerant controller is designed as follows:

$$u = (1 - I(V_0))u_{normal} + I(V_0)u_{PFTC} \tag{7.55}$$

where $u_{normal} = -\hat{W}_0^T S_0(x) - |s|\hat{\varepsilon}_0 - |s|(\bar{d} + |\gamma|) - \frac{1}{2}s$, $u_{PFTC} = -k(x_n, s)s + \hat{W}^T S(z)) + u_r$. Correspondingly, the adaptive laws should be modified as follows:

$$\dot{\hat{W}}_0 = (1 - I(V_0))(\eta_0 s S_0(x) + \eta_{10}\hat{W}_0) \tag{7.56}$$

$$\dot{\hat{\varepsilon}}_0 = (1 - I(V_0))(\eta_0 s + \eta_{10}\hat{\varepsilon}_0) \tag{7.57}$$

$$\dot{\hat{W}} = \begin{cases} -I(V_0)\eta_W S(z)s, \text{ if } ||\hat{W}|| < w_m \\ \quad \text{or } ||\hat{W}|| = w_m \text{ and } -\hat{W}^T S(z)s \le 0; \\ -I(V_0)(\eta_W S(z)s + \eta_W \dfrac{\hat{W}\,\hat{W}^T S(z)s}{||\hat{W}||^2}), \\ \quad \text{if } ||\hat{W}|| = w_m \text{ and } -\hat{W}^T S(z)s > 0 \end{cases} \tag{7.58}$$

$$\dot{\hat{\varepsilon}} = I(V_0)\eta_\varepsilon |s| \tag{7.59}$$

$$\dot{\hat{k}}_1 = I(V_0)\eta_1(\eta_W N s^2 sgn(s) + \frac{s}{4}\sum_{i=1}^{n-1} w_{i0}|x_{i+1}|) \tag{7.60}$$

$$\dot{\hat{k}}_2 = I(V_0)\eta_2 |\gamma s| \tag{7.61}$$

$$\dot{\hat{k}}_3 = I(V_0)\eta_3 |s| \tag{7.62}$$

From (7.55), we can see that only one of the two controllers, i.e., u_{normal} and u_{PFTC}, works at any one time. In fact, the fault occurred in the system can be detected and the switch between u_{normal} and u_{PFTC} can be decided by the value of the function $I(V_0)$. Therefore, $I(V_0)$ is called *fault alarm*.

The following theorem summarizes the aforementioned analysis.

Theorem 7.3 *Consider system (7.1) and actuator fault (7.2) with Assumptions 7.1–7.4. If the control law (7.55) and the adaptive laws (7.56)–(7.62) are employed, then the close-loop system is asymptotical stable with resulting tracking error converging to a small neighborhood of the origin.*

From the above analysis and the proofs of Theorems 1 and 2, it is easy to obtain the conclusion. The detailed proof is omitted here.

Remark 7.7 In fact, u_{normal} and u_{PFTC} denote the normal controller (7.6) and the passive fault tolerant controller (7.44), respectively. Obviously, the fault tolerant controller (7.55) can be constructed directly from (7.6) and (7.44) and there is no additional cost on the design and implementation of the controller (7.55).

Remark 7.8 Notice that, if $I(V_0) = 0$, which means that the actuator is healthy and the tracking error does not converge to a small neighborhood of the origin Ω_0, i.e., $|e_i(t)| \notin \Omega_0$, the normal controller (7.6) works while the fault tolerant controller (7.44) does not; if $I(V_0) = 1$, which means a fault has occurred, then the fault tolerant controller (7.44) is activated. By using (7.44), one can meet the control objective as shown in Theorem 7.2. Obviously, the function $I(V_0)$ can detect fault, which is why it is called as fault alarm.

7.4 Simulation

In this example, a system is described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1 - e^{-x_1}}{1 + e^{-x_1}} - (x_2^2 + 2x_1)\sin(x_2) + u + d \\ y = x_1 \end{cases} \tag{7.63}$$

where $d(t) = 0.5\sin(10t)$, $x_d = \sin t + \cos(0.5t)$. It is easily seen that $b_0 = 0.25$, $b_2 = 2025$, and $b_3 = 1.5$. For this work, we use the following parameters: $\eta_\varepsilon = \eta_W = \eta_i = 2$, $i = 1, 2, 3$, $x(0) = (0.6, 0.5)^T$, $W \in R^{10}$ are taken randomly in an interval $(0, 1]$, and the sample time is 0.08s.

In order to demonstrate the efficiency of the developed techniques, the following three fault models are considered. In addition, for comparing with the results obtained in this simulation, the fault occurrence time $T_f = 10$s.

Case 1: bias faults
Bias faults are common in practical applications. They can be described as follows:

$$u^f = \begin{cases} u, t < 10 \\ u + b_f, t \geq T_f \end{cases}$$

In general, depending on whether the value of b_f is constant or not, the faults can be categorized into two classes: constant bias faults and time-varying faults. In this simulation, it is assumed that $b_f = 2\cos(x_1 x_2)$.

Simulation results are shown in Figs. 7.1, 7.2, 7.3 and 7.4. From Fig. 7.1, using the proposed fault tolerant controller (7.55), the good tracking performance has been obtained while the tracking errors globally asymptotically converge to a small neighborhood of the origin shown in Fig. 7.2. Figures 7.3 and 7.4 show the simulation results using the fault tolerant control scheme in [50]. Comparing Figs. 7.1 and 7.2 with Figs. 7.3 and 7.4, it is clear that the time delay due to FD in the fault tolerant control scheme proposed in this chapter is smaller than that in [50].

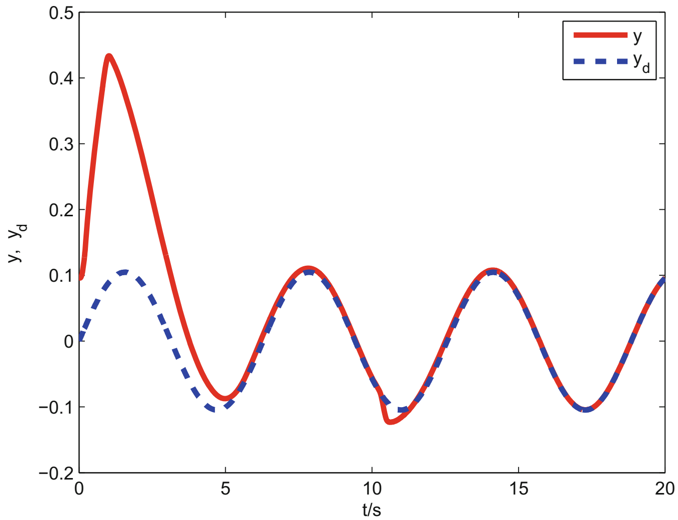

Fig. 7.1 The time profiles of y and y_d with (7.55)

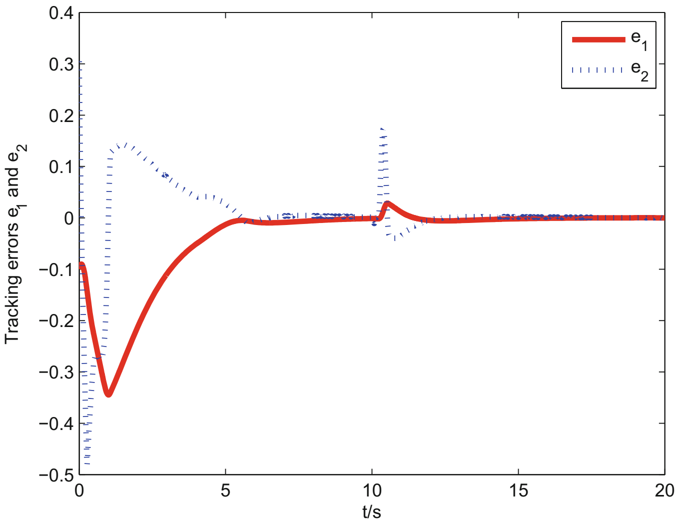

Fig. 7.2 The time profiles of tracking errors with (7.55)

Case 2: gain faults

Another class of common faults are gain faults, which are considered in [6, 16–18]. It is commonly described as:

$$u^f = \begin{cases} u, t < 10 \\ g_f u, t \geq T_f \end{cases}$$

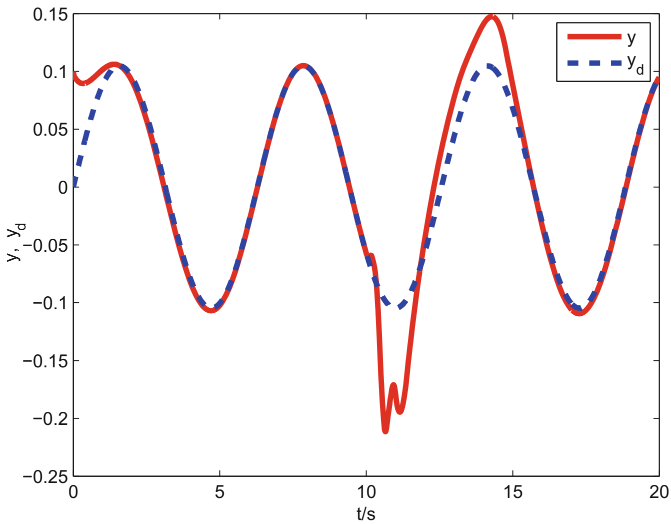

Fig. 7.3 The time profiles of y and y_d under the control scheme in [50]

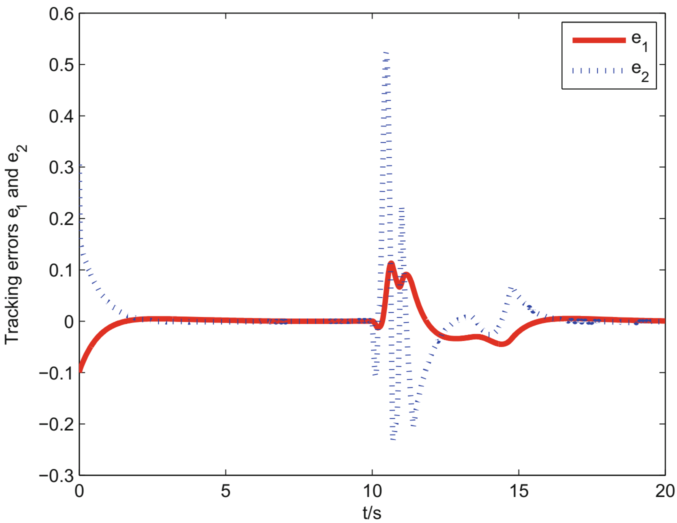

Fig. 7.4 The time profiles of tracking errors under the control scheme in [50]

where $0 < g_f \leq 1$. Similar to bias faults, according to whether g_f is constant or not, they can be categorized into two classes: constant gain faults and time-varying faults. In this simulation, we set $g_f = 1 + 0.2\sin(u)$.

Simulation results are shown in Figs. 7.5, 7.6, 7.7 and 7.8. From Fig. 7.5, using the proposed fault tolerant controller (7.55), good tracking performance has been obtained while the tracking errors globally asymptotically converge to a small neigh-

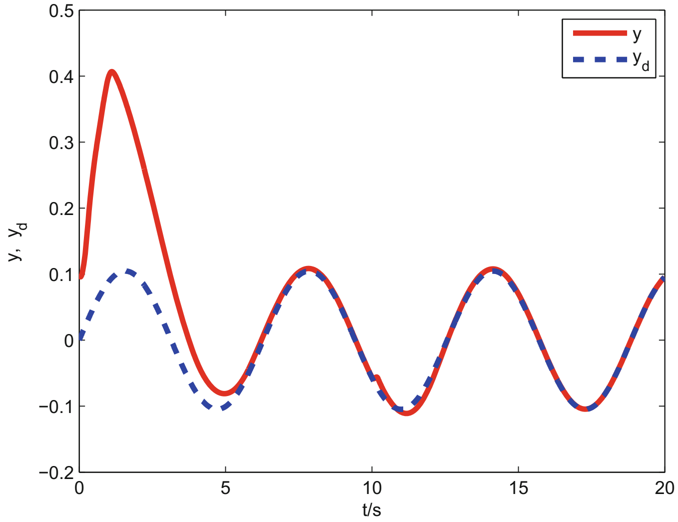

Fig. 7.5 The time profiles of y and y_d with (7.55)

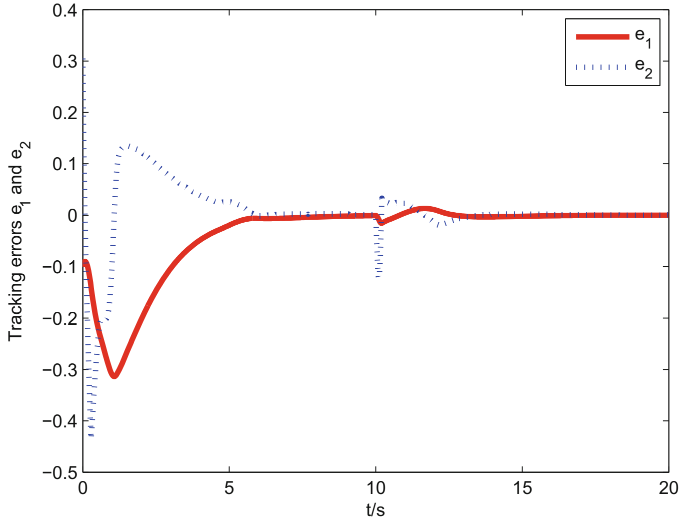

Fig. 7.6 The time profiles of tracking errors with (7.55)

borhood of the origin shown in Fig. 7.6. On the other hand, using the fault tolerant control scheme in [50], we obtain different results, which are shown in Figs. 7.7 and 7.8. Comparing with Figs. 7.5, 7.6, 7.7 and 7.8, it is easily seen that the time delay due to FD in the fault tolerant control scheme proposed in this chapter is smaller than that in [50].

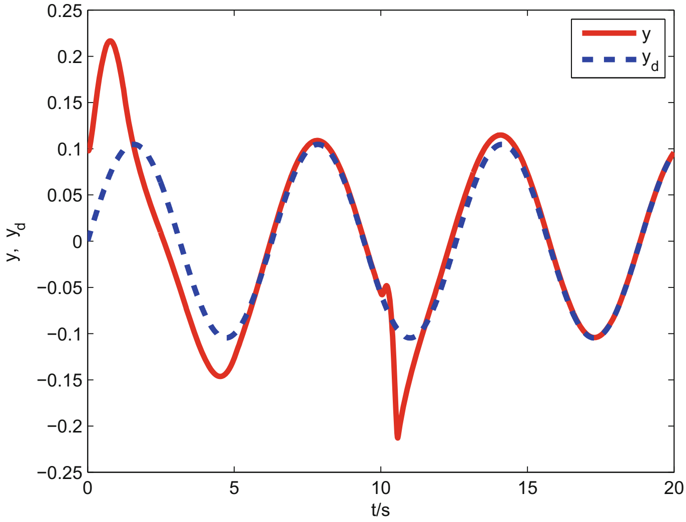

Fig. 7.7 The time profiles of y and y_d under the control scheme in [50]

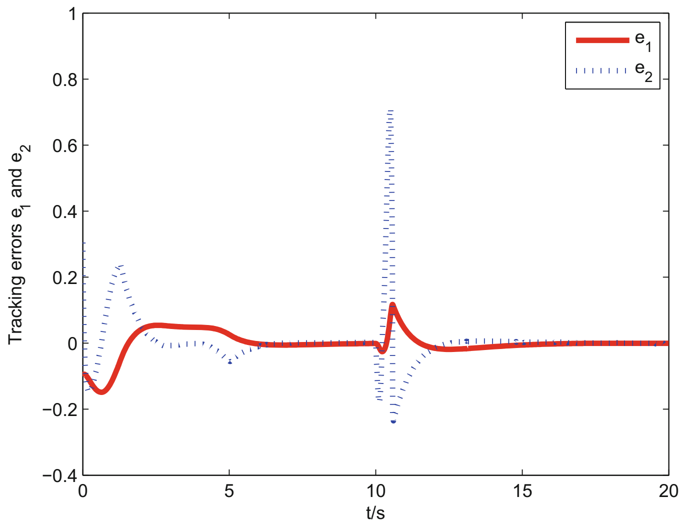

Fig. 7.8 The time profiles of tracking errors under the control scheme in [50]

Case 3: complex faults

In Cases 1 and 2, only gain faults or bias faults are considered. We now consider complex faults, which contain not only gain faults but also bias faults. The considered fault model can be described as follows:

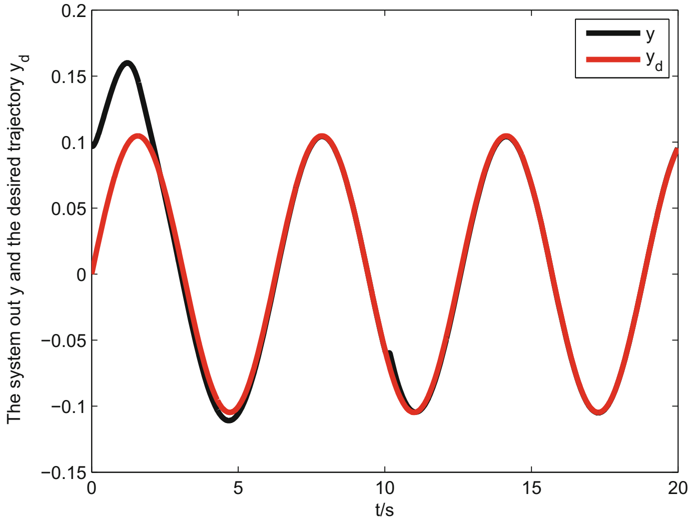

Fig. 7.9 The time profiles of y and y_d with (7.55)

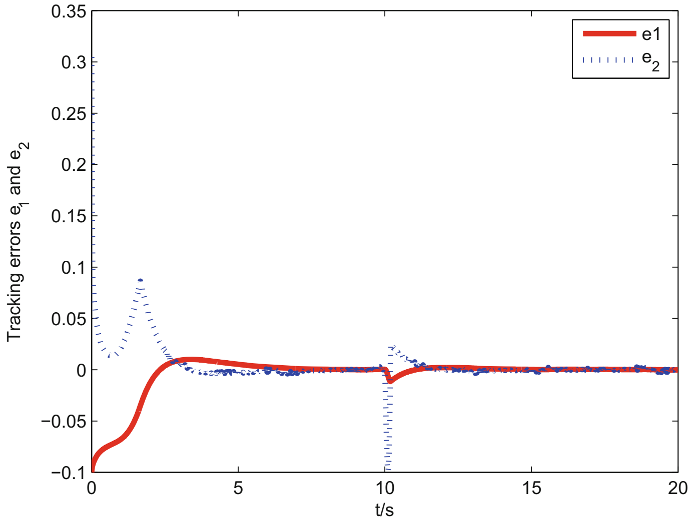

Fig. 7.10 The time profiles of tracking errors with (7.55)

$$u^f = \begin{cases} u, t < 10 \\ g_f u + b_f, t \geq T_f \end{cases}$$

In this simulation, we use $g_f = 1 + 0.2\sin(u)$ and $b_f = 2\cos(x_1 x_2)$.

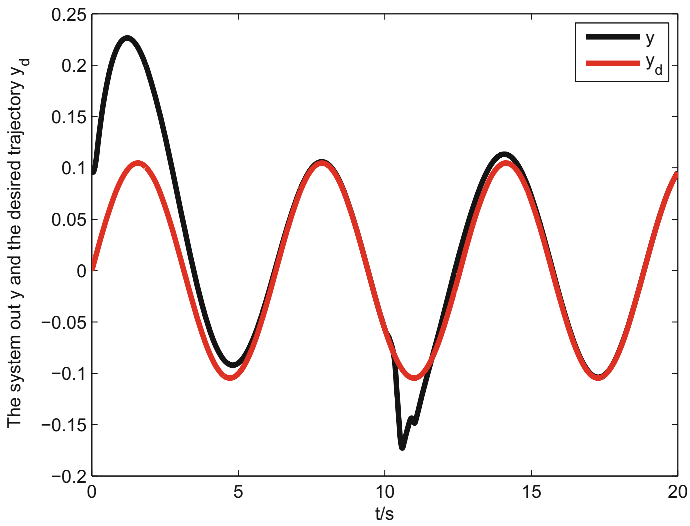

Fig. 7.11 The time profiles of y and y_d under the control scheme in [50]

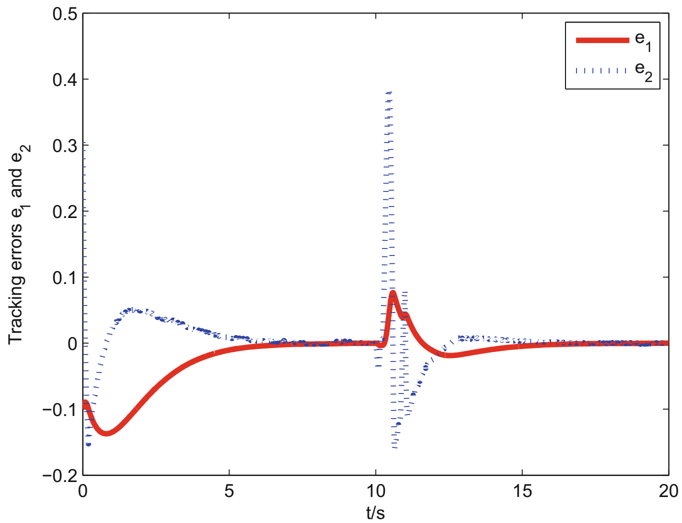

Fig. 7.12 The time profiles of tracking errors under the control scheme in [50]

Simulation results are shown in Figs. 7.9, 7.10, 7.11 and 7.12. Figure 6.9 shows the response of y and y_d using the proposed fault tolerant controller (7.55), good tracking performance has been obtained while the tracking errors globally asymptotically converge to a small neighborhood of the origin shown in Fig. 7.10. However, if the FTC scheme in [50] is used, then the simulation results shown in Figs. 7.11 and

7.12 are obtained. From Figs. 7.9, 7.10, 7.11 and 7.12, it can be easily seen that the time delay due to FD in the fault tolerant control scheme proposed in this chapter is smaller than that in [50].

From the simulation results in Cases 1–3, it can readily verify that the control scheme proposed in this chapter can minimize the time delay due to FD.

7.5 Conclusions

In this chapter, we have investigated the problem of adaptive FTC for a class of nonlinear systems with unknown actuator un-modeled actuator faults. The design of the normal and fault tolerant controller is first analyzed. Then a novel neural networks-based FTC scheme with fault alarm is proposed by using implicit function theorem. The proposed scheme has the advantage of passive FTC scheme as well as traditional active FTC scheme's property and minimizes TDDTFD and its the adverse effect. Moreover, the FTC scheme doesn't require the FDI model which is needed in the typical active FTC scheme.

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Chapter 8
Performance Analysis of the Effect of Time Delay Due to Fault Diagnosis

8.1 Introduction

In modern control mechanisms, various systems components such as actuators, sensors and processors may undergo abrupt failures during plant operation. To improve system reliability and to guarantee system stability in all situations, many effective fault-tolerant control (FTC) approaches including passive FTC and active FTC have been proposed in literature. Active FTC uses a fault detection and isolation (FDI) module and accommodation techniques. Generally speaking, there is always some level of time delay, which is called as *the time delay due to fault diagnosis (FD)* in this chapter, to detect, isolate and estimate the faults occurred in the systems [1]. When a fault occurs, the faulty system works under the nominal control until the fault is detected, isolated and fault accommodation is performed, which may cause severe loss of performance and stability. To our best knowledge, there are few chapters considering the time delay's adverse effect on the stability of the system. [2–5] tried to investigate the problem. However, the results in [2–5] were obtained under some restrictive conditions. Furthermore, the analytical expression of the time delay due to FD did not be given explicitly in [2–5], which motivates this chapter, again.

In this chapter, we investigate the problem of FTC for a class of uncertain systems with actuator time-varying faults. The time delay due to FD is derived strictly, and it‘s effect on system performance is analyzed. Compared with the existing results in the literatures, see for example, [2–5], the contribution from our work is as follows: (i) The time delay due to FD is derived strictly and its analytical expression is provided explicitly. In addition, its adverse effect on the system performance is analyzed and a proper solution to minimize its adverse effect is given; and (ii) The conditions that the magnitudes of the faults should be satisfied such that the faulty system controlled nominal controller maybe bounded even stable during the time delay interval are derived; and

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The rest of this chapter is organized as follows. In Sect. 8.2, Actuator faults are integrated in such model and the FTC objective is formulated. In Sect. 8.3, a FTC scheme is given, which include fault detection, isolation, estimation and fault accommodation. In Sect. 8.4, the analysis of the effect of the time delay due to FD is developed. Simulation results of a fourth-order lateral F-8 aircraft model are presented to demonstrate the effectiveness of the proposed technique in Sect. 8.5. Finally, Sect. 8.6 draws the conclusions.

8.2 Problem Statement

Consider the following system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + H(x(t)) \\ y(t) = Cx(t) \end{cases} \tag{8.1}$$

where $x(t) = [x_1(t), \cdots, x_n(t)]^T \in R^n$, $u(t) = [u_1(t), \cdots, u_m(t)]^T \in R^m$ and $y(t) = [y_1(t), \cdots, y_q(t)]^T \in R^q$ denote system measurable state vector, control input and system output, respectively, A, B and C are known real matrices with appropriate dimensions, $H(x(t)) = [h_1(x(t)), \cdots, h_n(x(t))]^T \in R^n$, $h_i(x(t)) \in R$ $(i = 1, \cdots, n)$ is an unknown nonlinear smooth function, which denotes the uncertainty and modelling error.

Actuators may fail. In this chapter, the considered fault model can be described as follows:

$$u_i^f(t) = u_i(t) + f_i(t), t > t_F \tag{8.2}$$

where f_i denotes an unknown but bounded signal, t_F is an unknown fault occurrence time.

The main task in this chapter is (i) to design FTC scheme for system (1) such that its states can follow those of a reference model under both normal and faulty conditions; (ii) to quantitatively analyze the time delay due to FD influence on the system performance. A fault tolerant scheme is first proposed to detect, isolate, estimate and accommodate faults occurred in the system controlled. Meanwhile, the time delay is derived strictly and its analytical expression is provided explicitly. Then, the analysis of system performance degraded by the time delay is developed, and the conditions that the magnitudes of the faults should be satisfied such that the faulty system controlled by the normal controller remains bounded even stable during the time delay interval are derived. In addition, the corresponding solution to the adverse effect of the time delay is proposed.

The reference model has the form as follows:

$$\begin{cases} \dot{x}_m(t) = A_m x(t) + Br(t) \\ y_m(t) = Cx(t) \end{cases} \tag{8.3}$$

where $x_m(t) = [x_{m1}(t), \cdots, x_{mn}(t)]^T \in R^n$, $r(t) \in R^m$ and $y_m(t) \in R^q$ denote the state vector, input and output of the reference model, respectively, A_m, B and C are known real matrices with appropriate dimensions.

Under normal system operation (fault-free), subtracting the reference model from the system (8.1), it follows that

$$\dot{\tilde{x}}(t) = A_m\tilde{x}(t) + (A - A_m)x(t) + B[u(t) - r(t)] + H(x) \tag{8.4}$$

where $\tilde{x}(t) = x(t) - x_m(t)$, representing the tracking error.

For the system (8.1) and fault model (8.2), the following assumptions are made in this chapter.

Assumption 8.1 Matrix B is of full column rank and the pair (A, B) is controllable and (A, C) is observable.

Assumption 8.2 $f_i(t)$ and $\dot{f}_i(t)$ are bounded, i.e., $|f_i(t)| \leq f_1$, $|\dot{f}_i(t)| \leq f_2$, where $f_1 > 0$ and $f_2 > 0$ are known real constants.

As a universal approximation, fuzzy logic systems (FLSs) have been widely used in control field. In this chapter, the unknown smooth function $h_i(x)$ will be approximated by FLSs as follows: $h_i(x, \theta_i) = \theta_i^{*T}\xi_i(x)$, where θ_i^* denotes the optimal parameter vector and $\xi_i(x)$ is the fuzzy basis function, which defined as in [6–11]. Optimal approximation error is defined as $\varepsilon_i = h_i(x) - {\theta_i}^{*T}\xi_i(x)$.

Assumption 8.3 ε_i and θ_i^* are bounded, i.e., $|\varepsilon_i| \leq \varepsilon_i^*$, $\varepsilon_i^* \leq M_{i\varepsilon}$, and $||\theta_i^*|| \leq M_{i\theta}$, where $\varepsilon_i^* > 0$ is an unknown real constant, $M_{i\varepsilon} > 0$ and $M_{i\theta} > 0$ are known real constants.

Remark 8.1 In general, $H(x(t))$ in (8.1) is assumed to be satisfied so-called matching condition in literature. This condition is strict and not always satisfied in practical applications. This chapter, however, removes this condition.

In the following, for the convenience of notation, $\bullet(\cdot)$ is simplified into $\bullet$. For example, x is the abbreviations of $x(t)$.

8.3 Main Results

In the section, a FTC framework, which includes the following steps: fault detection, isolation, estimation and accommodation, is proposed. Meanwhile, the time spent at each step in FD is derived strictly.

8.3.1 Fault Detection

A normal state feedback controller is firstly designed to guarantee the tracking error asymptotical converges to a compact set under the healthy condition (fault-free).

Take the tracking error as residual signal. After the tracking error has converged to a compact set, if the residual abruptly becomes large and does not belong to the compact set, then it can be concluded that a fault occurs in the system. The following result is given to explain the detailed design procedures.

Theorem 8.1 *Under Assumptions 8.1–8.3, if there exist matrices K, $P = P^T > 0$, F and $Q > 0$ with appropriate dimensions such that the following condition holds*

$$P(A_m + BK) + (A_m + BK)^T P \leq -Q \tag{8.5}$$

and the following adaptive and control laws are applied,

$$\dot{\hat{\theta}}_i = 2x_{pi}\xi_i(x) - \eta_\theta\hat{\theta}_i \tag{8.6}$$

$$\dot{\hat{\varepsilon}}_i = 2x_{pi}sgn(\tilde{x}_{pi}) - \eta_\varepsilon\hat{\varepsilon}_i \tag{8.7}$$

$$u = K\tilde{x} + r - B^+([A - A_m]x + \hat{H} + sgn(\tilde{x}^T P)\hat{\varepsilon}) \tag{8.8}$$

then, the error dynamics (8.4) is stable and all signals in the closed-loop system are asymptotically bounded belonging to a neighborhood of the origin, i.e., Ω defined as follows:

$$\Omega = \left\{ \begin{array}{l} (\tilde{x}, \tilde{\theta}_i, \tilde{\varepsilon}_i\ )|\ ||\tilde{x}|| \leq \sqrt{\alpha/\lambda_{\min}(P)},\ \|\tilde{\theta}_i\| \leq \\ \sqrt{2\alpha}|\tilde{\varepsilon}_i| \leq \sqrt{2\alpha}, i = 1, 2, \cdots, n \end{array} \right\}$$

where $\hat{H} = [\hat{h}_1, \cdots, \hat{h}_n]^T$, $\hat{\varepsilon} = [\hat{\varepsilon}_1, \cdots, \hat{\varepsilon}_n]^T$, $\hat{h}_i = \hat{\theta}_i^T\xi_i(x)$, $\hat{\theta}_i$ and $\hat{\varepsilon}_i$ are the estimate values of h_i, θ_i^* and ε_i^*, $sgn(\tilde{x}^T P) = diag\{sgn(\tilde{x}_{p1}), \cdots, sgn(\tilde{x}_{pn})\}$, $\tilde{x}_{pi}$ is the ith entry of $\tilde{x}^T P$, $\eta_\theta > 0 \in R$ and $\eta_\varepsilon > 0 \in R$ are design parameters, B^+ is the generalized inverse matrix of matrix B.

Proof Define $V = \tilde{x}^T P\tilde{x} + \sum_{i=1}^n (\tilde{\theta}_i^T\tilde{\theta}_i + \tilde{\varepsilon}_i^2)/2$, where $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$, $\tilde{\varepsilon}_i = \varepsilon_i^* - \hat{\varepsilon}_i$. Differentiating V with respect to time t, one has

$$\begin{aligned} \dot{V} =& \tilde{x}^T(P(A_m + BK) + (A_m + BK)^T P)\tilde{x} + \\ & 2\tilde{x}^T PH - 2\tilde{x}^T P(\hat{H} + sgn(\tilde{x}^T P)\hat{\varepsilon}) - \\ & \sum_{i=1}^n (\eta_\theta\tilde{\theta}_i^T\dot{\hat{\theta}}_i + \eta_\varepsilon\tilde{\varepsilon}_i\dot{\hat{\varepsilon}}_i) \end{aligned}$$

Since

$$\begin{aligned} & 2\tilde{x}^T PH - 2\tilde{x}^T P(\hat{H} + sgn(\tilde{x}^T P)\hat{\varepsilon}) \\ & \quad \leq \sum_{i=1}^n 2\tilde{x}_{pi}(\tilde{\theta}_i^T\xi_i(x) + sgn(\tilde{x}_{pi})\tilde{\varepsilon}_i) \end{aligned}$$

from (8.5)–(8.7), one has

$$\dot{V} \leq -\tilde{x}^T Q\tilde{x} + \sum_{i=1}^n (\eta_\theta\tilde{\theta}_i^T\hat{\theta}_i + \eta_\varepsilon\tilde{\varepsilon}_i\hat{\varepsilon}_i)$$

Since $\tilde{\theta}_i^T \hat{\theta}_i \leq -\tilde{\theta}_i^T \tilde{\theta}_i/2 + \theta_i^{*T} \theta_i^*/2$ and $\tilde{\varepsilon}_i \hat{\varepsilon}_i \leq -\tilde{\varepsilon}_i^2/2 + (\varepsilon_i^*)^2/2$, from Assumption 8.3, one further has

$$\dot{V} \leq -\tilde{x}^T Q \tilde{x} - \sum_{i=1}^{n} (\eta_\theta \tilde{\theta}_i^T \tilde{\theta}_i + \eta_\varepsilon \tilde{\varepsilon}_i^2)/2 + \mu = -\lambda V + \mu$$

where $\mu = \sum_{i=1}^{n} \frac{\eta_\theta M_{i\theta}^2 + \eta_\varepsilon M_{i\varepsilon}^2}{2}$ and $\lambda = \{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{\eta_\theta}{2}, \frac{\eta_\varepsilon}{2}\}$.

Since $\frac{d}{dt}(V(t)e^{\lambda t}) \leq e^{\lambda t}\mu$, one has

$$V(t) \leq \mu/\lambda + (V(0) - \mu/\lambda)e^{-\lambda t} \leq \mu/\lambda + V(0) \tag{8.9}$$

Let $\alpha = \mu/\lambda + V(0)$, one has $||\tilde{x}|| \leq \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_i|| \leq \sqrt{2\alpha}$, and $|\tilde{\varepsilon}_i| \leq \sqrt{2\alpha}$, which imply that all signals in the closed-loop system is asymptotical bounded belonging to the compact set Ω.

From (8.9), it is easy to find that $V(t)$ has the following property: It decreases while time t increases. Since $\lambda_{\min}(P)\tilde{x}^T\tilde{x} \leq \tilde{x}^T P \tilde{x} \leq V(t)$ one further has

$$||\tilde{x}|| \leq \sqrt{(\mu/\lambda + [V(0) - \mu/\lambda]e^{-\lambda t})/\lambda_{\min}(P)}$$

Now, the residual is chosen as $J_D = \|\tilde{x}\| = \|x - x_m\|$. Choose $\delta_D > 0 \in R$ and let $\delta_D = \sqrt{\frac{\mu/\lambda + [V(0) - \mu/\lambda]e^{-\lambda t_0}}{\lambda_{\min}(P)}}$, then one has

$$t_0 = -\left[\ln \frac{\lambda_{\min}(P)\delta_D - \mu/\lambda}{[V(0) - \mu/\lambda]}\right]/\lambda$$

It is necessary to indicate that, it is assumed that there is no any fault occurred in actuators in the early stage in this chapter. A fault detection can be performed conveniently utilizing the following mechanism:

$$\begin{cases} J_D > \delta_D \; and \; t > t_0 & \text{fault has occurred;} \\ \text{otherwise} & \text{no fault occurred} \end{cases}$$

Remark 8.2 Once faults occur at $t = t_F > t_0$, system state x varies at once, and the residual J_D thus has the same change, generally speaking, becomes greater than $J_D(t_0)$. Hence, the faults are almost immediately detected, where t_F is fault occurrence time. That is to say, fault detection time $t_D \approx t_F$. Therefore, using the mechanism, there is almost no additional time spent to detect fault.

8.3.2 Fault Isolation

Since it is assumed that only one single actuator fails at one time, there are m faulty cases. For each faulty case, we design an observer. The norm of the observer error between the actual system state and each observer state is defined as a residual signal. Hence, m residual signals are obtained. Then, residual signals are evaluated. For the residual signals, only one is smaller than a given value (called as threshold) while the others are larger than the threshold. Therefore, the fault is isolated. The details is given in the following.

When the sth $(1 \leq s \leq m)$ actuator is faulty, the faulty system can be described as:

$$\begin{cases} \dot{x}_s = Ax_s + Bu + b_s f_s + H \\ y_s = Cx_s \end{cases} \tag{8.10}$$

where $B = [b_1, \cdots, b_m]$, $b_i \in R^{n\times 1}$, $1 \leq i \leq m$, f_s is the time profiles of the sth actuator fault described by (8.2).

The following observers are constructed to isolate the fault.

$$\begin{cases} \dot{\hat{x}}_r = A\hat{x}_r + L(y_s - \hat{y}_r) + Bu + \quad r = 1, \cdots, m \\ \qquad b_r\mu_r f_1 + \hat{H} + sgn(e_{sr}^T P)\hat{M}_\xi \\ \hat{y}_r = C\hat{x}_r \end{cases} \tag{8.11}$$

where $\hat{x}_r$, $\hat{y}_r$ are the rth observer's state and output, respectively, $\mu_r = e_{sr}^T P/||e_{sr}^T P||$, $L \in R^{n\times n}$ is chosen such that $A - LC$ is Hurwitz, $\hat{H} = [\hat{h}_1, \cdots, \hat{h}_n]^T$, $\hat{h}_i = \hat{\theta}_i^T \xi_i(\hat{x})$, $\hat{M}_\xi = [\hat{M}_{1\xi}, \cdots, \hat{M}_{n\xi}]^T$, $\hat{M}_{i\xi}$ and $\hat{\theta}_i$ are the estimate values of H, h_i, $M_\xi = [M_{1\xi}, \cdots, M_{n\xi}]^T$, $M_{i\xi}$ and θ_i^*, $M_{i\xi}$ will be defined later, $sgn(e_{sr}^T P) = diag\{sgn(e_{p1}), \cdots, sgn(e_{pn})\}$, e_{pi} is the ith element of the vector $e_{sr}^T P$, $e_{sr} = x_s - \hat{x}_r$ denotes the state error between the faulty plant (10) and the rth observer (11), $P = P^T > 0$ denotes matrix, which will be defined later.

From (8.10) and (8.11), the error dynamics is obtained:

For $s = r$,

$$\begin{aligned} \dot{e}_{sr} =&(A - LC)e_{sr} + b_s(f_s - \mu_s f_1) + H- \\ &\hat{H} - sgn(e_{sr}^T P)\hat{M}_\xi \end{aligned} \tag{8.12}$$

For $s \neq r$,

$$\begin{aligned} \dot{e}_{sr} =&(A - LC)e_{sr} + b_s f_s - b_r\mu_r f_1 + H- \\ &\hat{H} - sgn(e_{sr}^T P)\hat{M}_\xi \end{aligned} \tag{8.13}$$

Theorem 8.2 *Under Assumptions 8.1–8.3, if there exist matrices L, $Q > 0$ and $P = P^T > 0$ with appropriate dimensions, such that the following conditions hold,*

$$P(A - LC) + (A - LC)^T P \leq -Q \tag{8.14}$$

and the following adaptive laws are applied,

$$\dot{\hat{\theta}}_i = 2e_{pi}\xi_i(\hat{x}) - \eta_\theta \hat{\theta}_i \tag{8.15}$$

$$\dot{\hat{M}}_{i\xi} = 2e_{pi} sgn(e_{pi}) - \eta_M \hat{M}_{i\xi} \tag{8.16}$$

then when the rth actuator is faulty, for $s = r$, $\lim_{t\to\infty} e_{sr} \in \Omega_I$, and for $s \neq r$, $\lim_{t\to\infty} e_{sr} \notin \Omega_I$, where $\eta_M > 0 \in R$ and

$$\Omega_I = \left\{ \begin{array}{l} (e_{sr}, \tilde{\theta}_i, \tilde{M}_{i\xi})| \, ||e_{sr}|| \leq \sqrt{\alpha_I/\lambda_{\min}(P)}, \\ ||\tilde{\theta}_i|| \leq \sqrt{2\alpha_I}, |\tilde{M}_{i\xi}| \leq \sqrt{2\alpha_I}, i = 1, \cdots, n \end{array} \right\}$$

Proof Define $V_I = e_{sr}^T P e_{sr} + \sum_{i=1}^n (\tilde{\theta}_i^T \tilde{\theta}_i + \tilde{M}_{i\xi}^2)/2$.

(1) For $s = r$, differentiating V_I with respect to time t, and using (8.12), one has

$$\begin{aligned} \dot{V}_I \leq & - e_{sr}^T Q e_{sr} + 2e_{sr}^T P b_s (f_s - \mu_s f_1) + \\ & 2e_{sr}^T P(H - \hat{H}) - \sum_{i=1}^n (\tilde{\theta}_i^T \dot{\hat{\theta}}_i + \hat{M}_{i\xi} \dot{\hat{M}}_{i\xi}) \end{aligned}$$

From $\mu_s = -e_{sr}^T P/||e_{sr}^T P||$ and Assumption 8.2, one has $2e_{sr}^T P b_s(-\mu_s f_1 + f_s) \leq 0$. Since the abstract value of each entry of $\xi_i^T(x)$ is less than 1 [11], one has

$$2e_{sr}^T P(H - \hat{H}) \leq \sum_{i=1}^n 2e_{pi}(\tilde{\theta}_i^T \xi_i(\hat{x}) + sgn(e_{pi})\tilde{M}_{i\xi})$$

where $M_{i\xi} = \sqrt{2N}||\theta_i^*|| + \sum_{i=1}^n \varepsilon_i^*$, N is the number of fuzzy rule. And since $2e_{sr}^T P b_s(f_s - \mu_s f_1) \leq 0$, one further has

$$\begin{aligned} \dot{V}_I \leq & - e_{sr}^T Q e_{sr} + \sum_{i=1}^n 2e_{pi}(\tilde{\theta}_i^T \xi_i(\hat{x}) + sgn(e_{pi})\tilde{M}_{i\xi}) - \\ & \sum_{i=1}^n (\tilde{\theta}_i^T \dot{\hat{\theta}}_i + \tilde{M}_{i\xi} \dot{\hat{M}}_{i\xi}) \end{aligned}$$

Similar to Theorem 8.1, substituting (8.15) and (8.16) into the above inequality, it yields

$$\begin{aligned} \dot{V}_I \leq & - e_{sr}^T Q e_{sr} - \sum_{i=1}^n (\tilde{\theta}_i^T \tilde{\theta}_i + \tilde{M}_{i\xi}^2)/2 + \\ & \sum_{i=1}^n (\eta_\theta \theta_i^{*T} \theta_i^* + \eta_M (M_{i\xi})^2)/2 \end{aligned}$$

From Assumption 8.3, one has

$$\dot{V}_I \leq -\lambda V_I + \mu$$

where

$$\lambda = \{\lambda_{\min}(Q)/\lambda_{\max}(P), \eta_\theta/2, \eta_\varepsilon/2\},$$

$$\mu = \sum\nolimits_{i=1}^{n} (\eta_\theta M_{i\theta}^2 + \eta_M(\sqrt{2N}M_{i\theta} + \sum\nolimits_{i=1}^{n} M_{i\varepsilon})^2)/2.$$

Since $\frac{d}{dt}(V(t)e^{\lambda t}) \leq e^{\lambda t}\mu$, one has

$$0 \leq V_I(t) \leq \frac{\mu}{\lambda} + [V_I(t_D) - \frac{\mu}{\lambda}]e^{-\lambda t} \leq \frac{\mu}{\lambda} + V_I(t_D) \tag{8.17}$$

Let $\alpha_I = \frac{\mu}{\lambda} + V_I(t_D)$, one has $||e_{sr}|| \leq \sqrt{\alpha_I/\lambda_{\min}(P)}$, $||\tilde{\theta}_i|| \leq \sqrt{2\alpha_I}$, and $|\tilde{M}_{i\xi}| \leq \sqrt{2\alpha_I}$, which imply that all signals in the closed-loop system is asymptotically bounded belonging to compact set Ω_I.

(2) For $s \neq r$, according to (8.13), one has

$$\dot{e}_{sr} = (A - LC)e_{sr} + b_s f_s - b_r \mu_r f_1 + H - \hat{H} - sgn(e_{sr}^T P)\hat{M}_\xi$$

Because matrix B is of full column rank (Assumption 8.1), b_s and b_r are linearly independent. Therefore, the following inequalities *do not* always hold

$$2e_{sr}^T P(-b_r\mu_r f_1 + b_s f_s) \leq 0$$

What's worst, $2e_{sr}^T P(-b_r\mu_r f_1 + b_s f_s)$ varies infinitely since b_s and b_r are linearly independent, which further cause that $V_I(t)$ varies infinitely. Thus, $\lim_{t\to\infty} e_{sr} \notin \Omega_I$.

From (1) and (2), we obtain the conclusions.

Now, denote the residuals between (8.10) and (8.11) as follows:

$$J_{sr} = \|e_{sr}\| = \left\|x_s - \hat{x}_r\right\|, \quad 1 \leq r \leq m \tag{8.18}$$

From Theorem 8.2, if $s = r$, then one has

$$\lambda_{\min}(P)||e_{sr}(t)||^2 \leq e^{-\lambda_I t}\lambda_{\max}(P)||e_{sr}(t_D)||^2$$

$$J_{sr} \leq \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{sr}(t_D)||e^{-\lambda t/2}$$

If $s \neq r$, from Theorem 8.1, one has

$$J_{sr} > \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{sr}(t_D)||e^{-\lambda t/2}$$

Therefore, fault isolation can be performed conveniently using the following mechanism:

$$\begin{cases} J_{sr} \leq T_I, r = s \Rightarrow \text{the } r\text{th actuator is faulty} \\ J_{sr} > T_I, r \neq s \end{cases}$$

where the threshold T_I is defined as $T_I = \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{sr}(t_D)||e^{-\lambda t/2}$. Further, let $\delta_I = \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{sr}(t_D)||e^{-\lambda t_I/2}$, one has

$$t_I = -\left(2\ln\frac{\delta_I}{\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}||e_{sr}(t_D)||}\right)/\lambda \tag{8.19}$$

From the above analysis, we can obtain, if fault occurred at $t = t_F$ in the sth actuators has been detected at $t = t_D$, the fault can be isolated at $t = t_I$. Obviously, the time interval $[t_D, t_I]$ is spent to isolate the fault, which is named as fault isolation time (FIT).

8.3.3 Fault Estimation

Assuming the sth $(1 \le s \le m)$ actuator is faulty. The faulty system can be described as:

$$\begin{cases} \dot{x} = Ax + Bu + b_s f_s + H \\ y = Cx \end{cases} \tag{8.20}$$

To estimate the fault, an observer is presented as follows:

$$\begin{cases} \dot{\hat{x}} = Ax + Bu + b_s \hat{f}_s + L(y - \hat{y}) + \hat{H} + sgn(e^T P)\hat{M}_\xi \\ \hat{y} = C\hat{x} \end{cases} \tag{8.21}$$

where $\hat{f}_s$ is the estimate values of the fault $f_s(t)$ at time t, $\hat{M}_\xi$ is defined as in (8.11). Define $e = x - \hat{x}$ and $\tilde{f}_s = f_s - \hat{f}_s$, then error dynamics is obtained:

$$\dot{e} = (A - LC)e + b_s \tilde{f}_s + H - \hat{H} - sgn(e^T P)\hat{M}_\xi \tag{8.22}$$

Now, the stability of the error dynamics is analyzed to obtain the fault estimation. The details are given by the following theorem.

Theorem 8.3 *Under Assumptions 8.1–8.3, if there exist real matrices $P = P^T > 0$, L, F and $Q > 0$ with appropriate dimensions, such that the following conditions hold*

$$P(A - LC) + (A - LC)^T P < -Q \tag{8.23}$$

and the adaptive laws are adopted

$$\dot{\hat{f}}_s = \begin{cases} 0, \ \textit{if}\ \hat{f}_s = f_1 \ \textit{and}\ 2\eta_1 e^T P b_s > 0 \ \textit{or} \\ \quad \hat{f}_s = -f_1 \ \textit{and}\ 2\eta_1 e^T P b_s < 0; \\ 2\eta_1 e^T P b_s, \ \textit{otherwise} \end{cases} \tag{8.24}$$

$$\dot{\hat{\theta}}_i = 2e_{pi}\xi_i(\hat{x}) - \eta_\theta \hat{\theta}_i \tag{8.25}$$

$$\dot{\hat{M}}_{i\xi} = 2e_{pi}sgn(e_{pi}) - \eta_M \hat{M}_{i\xi} \tag{8.26}$$

where b_s is the sth column of B, $\eta_1 > 0$ denote the adaptive rates, then, the error dynamics (8.22) is asymptotically stable and all signals involved in the closed-loop system are semi-globally uniformly ultimately bounded, converging asymptotically to a small neighborhood of the origin Ω_E defined as follows:

$$\Omega_E = \left\{ \begin{array}{l} (e, \tilde{\theta}_i, \tilde{M}_{i\xi}, \tilde{f}_s\ )|\ ||e|| \le \sqrt{\alpha/\lambda_{\min}(P)}, \\ |\tilde{f}_s| \le \sqrt{2\eta_1\alpha}, ||\tilde{\theta}_i|| \le \sqrt{2\alpha}, |\tilde{M}_{i\xi}| \le \sqrt{2\alpha} \end{array} \right\}$$

where $\alpha = \mu_E/\lambda_E + V_E(t_I)$, $\lambda_E = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)},\ \eta_\theta/2, \eta_\varepsilon/2, 1\}$, $\mu_E = 2f_1(2f_1 + f_2)/\eta_2 + \mu$ *and* $\mu = \sum_{i=1}^{n} (\eta_\theta M_{i\theta}^2 + \eta_M(\sqrt{2N}M_{i\theta} + \sum_{i=1}^{n} M_{i\varepsilon})^2)/2$.

Proof Define the following smooth function

$$V_E = e^T Pe + \frac{1}{2\eta_1}\tilde{f}_s^2 + \frac{1}{2}\sum_{i=1}^{n} (\tilde{\theta}_i^T \tilde{\theta}_i + \tilde{M}_{i\xi}^2) \tag{8.27}$$

Differentiating V_E with respect to time t, considering (8.23)–(8.26), similar to Theorem 8.2, yields

$$\dot{V}_E \le -e^T Qe + \frac{1}{\eta_1}\tilde{f}_s \dot{f}_s - \sum_{i=1}^{n} (\eta_\theta \tilde{\theta}_i^T \tilde{\theta}_i + \eta_\varepsilon \tilde{\varepsilon}_i^2)/2 + \mu \tag{8.28}$$

where $\mu = \sum_{i=1}^{n} (\eta_\theta M_{i\theta}^2 + \eta_M(\sqrt{2N}M_{i\theta} + M_{i\varepsilon})^2)/2$. Since $|\hat{f}_s| \le f_1$, which can be guaranteed by using the adaptive laws (8.24), and Assumption 8.2 (i.e., $|f_s(t)| \le f_1$, and $|\dot{f}_s(t)| \le f_2$) are satisfied, one has $\frac{\tilde{f}_s \dot{f}_s}{\eta_1} \le -\frac{\tilde{f}_s^2}{\eta_1} + \frac{2f_1(2f_1+f_2)}{\eta_1}$. Hence, (8.30) can been rewritten as follows

$$\dot{V}_E \le -\lambda_E V_E(t) + \mu_E$$

where $\lambda_E = \min\{\lambda_{\min}(Q)/\lambda_{\max}(P), \eta_\theta/2, \eta_M/2, 1/\eta_1\}$ and $\mu_E = 2f_1(2f_1 + f_2)/\eta_1 + \mu$. Then, one has, $\frac{d}{dt}(V_E(t)e^{\lambda_E t}) \le e^{\lambda_E t}\mu_E$. Furthermore,

$$0 \le V_E(t) \le \frac{\mu_E}{\lambda_E} + [V_E(t_I) - \frac{\mu_E}{\lambda_E}]e^{-\lambda_E t} \le \frac{\mu_e}{\lambda_E} + V_E(t_I)$$

Let $\alpha = \frac{\mu_E}{\lambda_E} + V_E(0)$, one has $||e_x|| \le \sqrt{\alpha/\lambda_{\min}(P)}$, $||\tilde{\theta}_i|| \le \sqrt{2\alpha}$, $|\tilde{M}_{i\xi}| \le \sqrt{2\alpha}$ and $|\tilde{f}_s| \le \sqrt{2\eta_2\alpha}$. This ends the proof.

From Theorem 8.3, one has

$$\lambda_{\min}(P)||e(t)||^2 \leq V_E(t) \leq \frac{\mu_E}{\lambda_E} + (V_E(t_I) - \frac{\mu_E}{\lambda_E})e^{-\lambda_E t}$$

$$||e(t)|| \leq \sqrt{\frac{\frac{\mu_E}{\lambda_E} + (\lambda_{\max}(P)||e_s(t_I)||^2 - \frac{\mu_E}{\lambda_E})e^{-\lambda_E t}}{\lambda_{\min}(P)}}$$

From the engineer point of view, the estimation objective is obtained when $||e(t)|| \leq \delta_E$, where $\delta_E > 0 \in R$. Let

$$\delta_E = \sqrt{\frac{\frac{\mu_E}{\lambda_E} + (\lambda_{\max}(P)||e_s(t_I)||^2 - \frac{\mu_E}{\lambda_E})e^{-\lambda_E t_E}}{\lambda_{\min}(P)}}$$

one has $t_E = -(\ln \frac{\lambda_{\min}(P)|\delta_E^2 - \frac{\mu_E}{\lambda_E}}{(\lambda_{\max}(P)||e_s(t_I)||^2 - \frac{\mu_E}{\lambda_E})})/\lambda_E$. From the above analysis, up to $t = t_E$, the fault occurred in the sth actuator which has been detected and isolated at the point-in-time $t = t_D$ and $t = t_I$, has been estimated. Obviously, the time interval $[t_I, t_E]$ is spent to estimate the fault, which is named as fault estimation time (FET).

8.3.4 Fault Accommodation

On the basis of the estimated actuator fault and Theorem 7.1, the fault tolerant controller is constructed as

$$u_s = u_s^N - \hat{f}_s \tag{8.29}$$

where u_s^N is the sth normal control input, $\hat{f}_s$ are the estimations of f_s, which are used to compensate for the fault.

Notice that, in this chapter, it is assumed that fault accommodation is activated immediately as soon as fault has been estimated. That is to say, controller switching is assumed to do not take any time.

8.4 Analysis of Time Delay's Effect on Systems Performance

In Sect. 8.3, the mathematic development of the time spent at each step of FTC has been derived strictly and its analytical expression has been given. In this section, the level of the adverse effect of the time delay due to FD is firstly analyzed during the time interval $[t_F, t_E]$, and the corresponding solutions are proposed to minimize the effect.

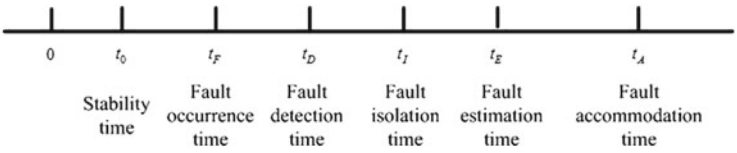

Fig. 8.1 The fault diagnosis and accommodation time sequence

In order to describe clearly the problem, let us recall the procedures of FTC. In general, FTC framework includes the following steps: fault detection, fault isolation, fault estimation and fault accommodation (Fig. 8.1). Each step need some time. Clearly, the time spent to FD contains not only FDI but also the other steps. Thus, in this chapter, the time delay is called as time delay due to FD. It is necessary to point out that, the considered system is always controlled by the faulty actuators during $[t_F, t_A]$, which degrades the system performances even damage the system. Hence, it is very important to analyze its adverse effort on the considered systems performance and to propose a proper solution.

It is assumed that only an actuator is faulty. Without lost of generality, it is assumed that the ith actuator is faulty ($u_i^f = u_i + f_i$). Then the faulty system can be described as follows:

$$\dot{x} = Ax + Bu + BF_i + H$$

where vector $F_i \in R^{m\times 1}$ is the vector whose ith component equals to $f_i(t)$ and the others equal to zero.

Similar to Theorem 8.1, differentiating $V = \tilde{x}^T P\tilde{x} + \sum_{i=1}^n (\tilde{\theta}_i^T\tilde{\theta}_i + \tilde{\varepsilon}_i^2)/2$ with respect to time t, and considering control law $u = K\tilde{x} + r - B^+([A - A_m]x + \hat{H} + sgn(\tilde{x}^T P)\hat{\varepsilon})$, inequality (8.5) and adaptive laws (8.6) and (8.7), one has

$$\begin{aligned}\dot{V} \leq & -\tilde{x}^T Q\tilde{x} + 2\tilde{x}^T PB_i f_i - \sum\nolimits_{i=1}^n (\eta_\theta\tilde{\theta}_i^T\tilde{\theta}_i + \\ & \eta_\varepsilon\tilde{\varepsilon}_i^2)/2 + \sum\nolimits_{i=1}^n (\eta_\theta\theta_i^{*T}\theta_i^* + \eta_\varepsilon(\varepsilon_i^*)^2)/2\end{aligned} \tag{8.30}$$

where B_i is the ith column of B.

Note that, compared with Theorem 8.1, there exists an additional term $2\tilde{x}^T PB_i f_i$ in (8.30) that degrades directly the system performance. In order to show the degree of its adverse effect on the system performance, the above inequality (8.30) is been further derived as follows:

$$\dot{V} \leq -\lambda V + \lambda_{\max}(S^{-1})||B_i f_i||^2 + \mu \tag{8.31}$$

where $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - \lambda_{\max}(PSP), \eta_\theta/2, \eta_\varepsilon/2, 1/2\}$, $\mu = \sum_{i=1}^n(\eta_\theta M_{i\theta}^2 + \eta_\varepsilon M_{i\varepsilon}^2)/2$, matrix $S = S^{-1} > 0$ with appropriate dimensions. Obviously, if the fault $f_i(t)$ satisfies the following property:

$$|f_i(t)| \leq f_1 < \sqrt{\frac{p(\lambda - \lambda_{\max}(PSP)) + \mu}{\lambda_{\max}(S^{-1})||B_i||^2}} \tag{8.32}$$

where $V(t_F) \leq p$, $p \geq 0 \in R$ denotes a constant, one has $\dot{V}(t) \leq 0$ on $V(t_F) = p$. Thus, $V(t) \leq p$ is an invariant set, i.e., if $V(t_F) \leq p$, then $V(t) \leq p$ for $\forall t \in [t_F, t_E]$. Further, from (8.32), it is easy to find that $V(t)$ is also bounded by $\lambda_{\max}(S^{-1})||B_i||^2 {f_1}^2 + \mu$.

On the other hand, if the fault $f_i(t)$ *does not* satisfy the property (8.32), then one *does not* have the following conclusion: $\dot{V}(t) \leq 0$ on $V(t_F) = p$. That is to say, if $f_1 > \sqrt{\frac{p(\lambda-\lambda_{\max}(PSP))+\mu}{\lambda_{\max}(S^{-1})||B_i||^2}}$, controlled by nominal controller defined by (8.8), the tracking error between the faulty system and the reference model does not belong to the compact set Ω during the time interval $[t_F, t_E]$.

Remark 8.3 It is necessary to point out that, it is during the time interval $[t_F, t_E]$ that the stability of the faulty system is investigated. The initiation condition is set at the point-in-time$t = t_F$, not at the time point $t = 0$. Thus, the conclusion obtained is, $V(t) \leq p$ for all$t \in [t_F, t_E]$, not for all $t \geq 0$.

In the following, it is assumed that the fault $f_i(t)$ satisfies the property (32), and we will try to seek a proper solution to guarantee the tracking error between the faulty system controlled by the normal controller and the reference model still belongs to an acceptable small neighborhood of the origin during the time interval $[t_F, t_E]$.

Integrating (8.31) on the time interval $[t_F, t_E]$, one has

$$V(t_E) \leq \triangle + (V(t_F) - \triangle)e^{-(\lambda-\lambda_{max}(PSP))(t_E-t_F)}$$

where $\triangle = \frac{\lambda_{\max}(S^{-1})||B_i||^2 {f_1}^2+\mu}{\lambda-\lambda_{\max}(PSP)}$. Since $\lambda_{\min}(P)||\tilde{x}||^2 = \lambda_{\min}(P)\tilde{x}^T\tilde{x} \leq \tilde{x}^T P\tilde{x}$, one has

$$\lambda_{\min}(P)||\tilde{x}||^2 \leq \frac{\mu_1}{\lambda_1} + [V(0) - \frac{\mu_1}{\lambda_1}]e^{-\lambda_1 t}$$

where $\mu_1 = \lambda_{\max}(S^{-1})||B_i||^2 {f_1}^2 + \mu$ and $\lambda_1 = \lambda - \lambda_{\max}(PSP)$.

Further, one has, for $\forall t \in [t_F, t_E]$,

$$||\tilde{x}|| \leq \sqrt{\frac{\triangle + (\lambda_{\max}(P)||x(t_F)||^2 - \triangle)e^{-(\lambda-\lambda_{\max}(PSP))(t-t_F)}}{\lambda_{\min}(P)}} \tag{8.33}$$

On the other hand, from Theorem 8.1, one has

$$V(t) \leq \mu/\lambda + (V(0) - \mu/\lambda)e^{-\lambda t} \leq \mu/\lambda + V(0)$$

Further, one has, for $\forall t \in [t_F, t_E]$,

$$||\tilde{x}(t)|| \leq \sqrt{\frac{\frac{\mu}{\lambda} + (\lambda_{\max}(P)||\tilde{x}(t_F)||^2 - \frac{\mu}{\lambda})e^{-\lambda(t-t_F)}}{\lambda_{\min}(P)}} \tag{8.34}$$

where

$$\alpha = \mu/\lambda + V(0), \quad \lambda = \{\lambda_{\min}(Q)/\lambda_{\max}(P), \eta_\theta/2, \eta_\varepsilon/2\}$$

$$\mu = \sum\nolimits_{i=1}^{n} (\eta_\theta M_{i\theta}^2 + \eta_\varepsilon M_{i\varepsilon}^2)/2.$$

Compared (8.33) with (8.34), one can find, the additional term $2x^T(t)PB_i f_i(t)$ not only decreases the convergence rate of the state tracking error, where the convergence rate $e^{-\lambda(t-t_F)}$ has decreased to $e^{-(\lambda-\lambda_{\max}(PSP))(t-t_F)}$, but also enlarges the bound of the convergence set Ω defined as follows: $\forall t \in [t_F, t_E]$,

$$\Omega = \left\{ \begin{array}{l} \tilde{x}(t)|\, ||\tilde{x}(t)|| \leq \\ \sqrt{\frac{\frac{\mu}{\lambda} + (\lambda_{\max}(P)||\tilde{x}(t_F)||^2 - \frac{\mu}{\lambda})e^{-\lambda(t-t_F)}}{\lambda_{\min}(P)}} \end{array} \right\}$$

which has been replaced by the set Ω' defined as follows: for $\forall t \in [t_F, t_E]$,

$$\Omega' = \left\{ \begin{array}{l} \tilde{x}(t)|\, ||\tilde{x}(t)|| \leq \\ \sqrt{\frac{\Delta + (\lambda_{\max}(P)||\tilde{x}(t_F)||^2 - \Delta)e^{-(\lambda-\lambda_{\max}(PSP))(t-t_F)}}{\lambda_{\min}(P)}} \end{array} \right\}$$

Therefore, the nominal control should be modified to guarantee the stability of the faulty system for all $t \in [t_F, t_E]$.

From the above analysis, if the fault satisfies the property (8.32) and matrices $P = P^T > 0$, $S = S^T > 0$, K and $Q > 0$ are chosen such that $P(A_m + BK) + (A_m + BK)^T P + PSP < -Q$, then, one has

$$\dot{V} \leq -\lambda_1 V(t) + \mu_1$$

where $\lambda_1 = \lambda - \lambda_{\max}(PSP)$, $\mu_1 = \lambda_{\max}(S^{-1})||B_i||^2 {f_1}^2 + \mu$. Further, one has, for $t \in [t_E, t_F]$

$$||\tilde{x}(t)|| \leq \sqrt{\frac{\frac{\mu_1}{\lambda_1} + (\lambda_{\max}(P)||\tilde{x}(t_F)||^2 - \frac{\mu_1}{\lambda_1})e^{-\lambda_1(t-t_F)}}{\lambda_{\min}(P)}}$$

Theorem 8.4 *Consider the tracking error dynamics (8.4) with actuator fault (8.2) satisfied (32), Assumptions 8.1–8.3. If matrices* $P = P^T > 0$, $S = S^T > 0$, K *and* $Q > 0$ *are chosen such that*

$$P(A_m + BK) + (A_m + BK)^T P + PSP < -Q \tag{8.35}$$

and the following adaptive and modified normal control laws are applied,

$$\dot{\hat{\theta}}_i = 2x_{pi}\xi_i(x) - \eta_\theta\hat{\theta}_i \tag{8.36}$$

$$\dot{\hat{\varepsilon}}_i = 2x_{pi}sgn(x_{pi}) - \eta_\varepsilon\hat{\varepsilon}_i \tag{8.37}$$

$$u = K\tilde{x} + r - B^+([A - A_m]x + \hat{H} + sgn(\tilde{x}^T P)\hat{\varepsilon}) \tag{8.38}$$

then, for all $t \in [t_F, t_E]$, the tracking error dynamics (4) still is stable with state tracking error asymptotically converging at the convergence rate $e^{-\lambda_1(t-t_F)}$ to the compact set Ω_1, specified as,

$$\Omega_1 = \left\{ \begin{array}{c} \tilde{x}(t) |\, ||\tilde{x}(t)|| \leq \\ \sqrt{\dfrac{\frac{\mu_1}{\lambda_1} + (\lambda_{\max}(P)||\tilde{x}(t_F)||^2 - \frac{\mu_1}{\lambda_1})e^{-\lambda_1(t_E-t_F)}}{\lambda_{\min}(P)}} \end{array} \right\}$$

Proof From the aforementioned analysis, it is easy to obtain the conclusion. The detailed proof is omitted here.

Remark 8.4 It should be mentioned that, if (8.35) holds, then the inequality (8.5) holds, too, which means that, the modified nominal controller which satisfies (8.35), still guarantees that the healthy system (1) has good tracking performance during $[0, t_F)$.

8.5 Experimental Results

To demonstrate the effectiveness of the proposed approach, an unmanned helicopter platform THeli260 shown in Fig. 8.2, is used in the experimental study. In general, unmanned helicopter includes four independent input signals, namely longitudinal, latitudinal, heave and heading inputs. The internal dynamics includes main rotor flapping dynamics, heading rate gyro dynamics, force/moment dynamics and translational dynamics. The mathematical description of helicopters is essentially nonlinear and time variant, which can be approximated in a linear form with nonlinear uncertain terms as in literatures such as [12] and [13]. The system can be divided into three subsystems, namely tip-path-plane subsystem, heave-heading subsystem, and position-velocity subsystem. The position-velocity subsystem can be described by (8.1), where the state $x = [x_1, x_2, x_3, x_4]^T = [p_x, p_y, v_x, v_y]^T$, the input $u = [u_1, u_2]^T = [\theta, \phi]^T$, and

Fig. 8.2 THeli260 unmanned helicopter

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & R_{vx} & 0 \\ 0 & 0 & 0 & R_{vy} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -g & 0 \\ 0 & g \end{bmatrix}, H = \begin{bmatrix} 0 \\ 0 \\ R_a a_s \\ R_b a_s \end{bmatrix}$$

These variables' specification is given as follows: p_x is latitudinal position, p_y is longitudinal position, v_x is latitudinal velocity, v_y is longitudinal velocity, θ is roll angle, ϕ is pitch angle, a_s Latitudinal flapping angle, b_s is longitudinal flapping angle, b_s is longitudinal flapping angle, and g is gravity constant.

THeli260's diameters of main rotor and tail rotor are 1.77 and 0.27 m respectively. In this study, an external avionics system is designed and installed to update the vehicle to an autonomous flight platform, which includes flight controller, flight computer, barometer, ultrasound unit, wireless router, et al. its total weight is about 10 kg. For this THeli260, $R_a = -9.81$, $R_b = 9.81$, $R_{vx} = -0.0076$ and $R_{vy} = -0.0093$.

The ultimate objective in this chapter is for the helicopter to track a predefined position trajectory $x_m(t)$, which are the states of the following model defined as (3) with

$$A_m = \begin{bmatrix} -3.598 & 14.8468 & -35.18 & -21.96 \\ -0.0377 & -0.1397 & 5.884 & -0.3269 \\ 0.0688 & -1.0011 & -0.2163 & 0.0814 \\ 0.9947 & 0.1027 & 0 & 0 \end{bmatrix}$$

$$B_m = B, C_m = C, r = [\sin(t) \quad \cos(t)]^T$$

We consider the case where only an actuator fails at one time:

$$u_1^f(t) = u_1(t), \; u_2^f(t) = \begin{cases} u_2(t), \; t < 20 \\ u_2(t) + 1.2\sin(t), \; t \geq 20 \end{cases}$$

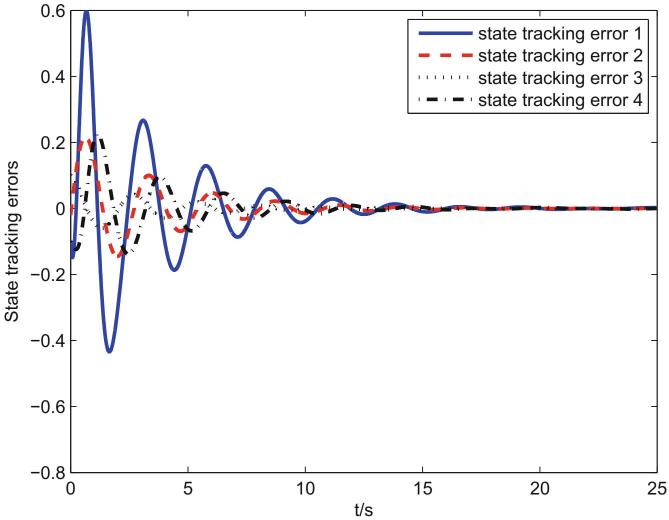

Fig. 8.3 State tracking error under the nominal condition

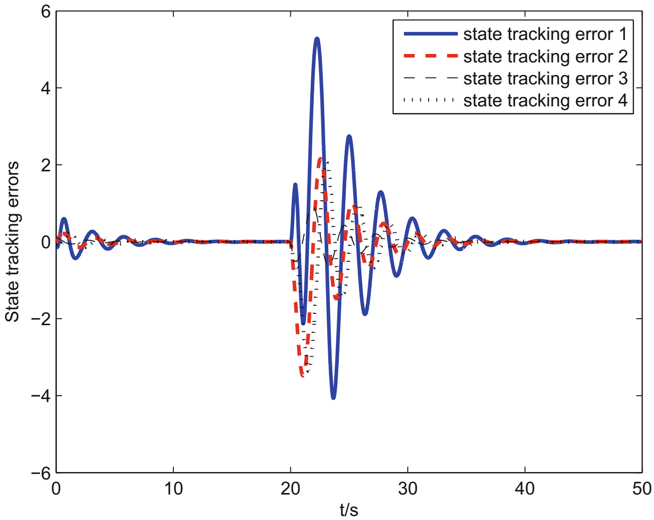

Fig. 8.4 State tracking error under the faulty condition with FTC

The experimental results are presented in Figs. 8.3, 8.4, 8.5 and 8.6. From Fig. 8.3, it is seen that, if no actuator fails, the tracking errors globally asymptotically converge to a small neighborhood of the origin. From Fig. 8.4, it is easy to find out that by using the proposed FTC (8.29), the state tracking errors become globally asymptotically bounded. Under the condition that the fault satisfies the condition (8.32), the modified normal control (8.38) is employed and the result is obtained shown

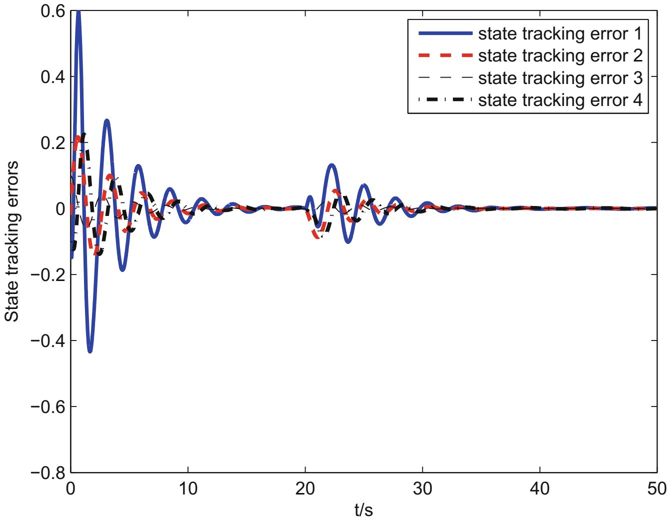

Fig. 8.5 State tracking errors with the modified controller (8.38)

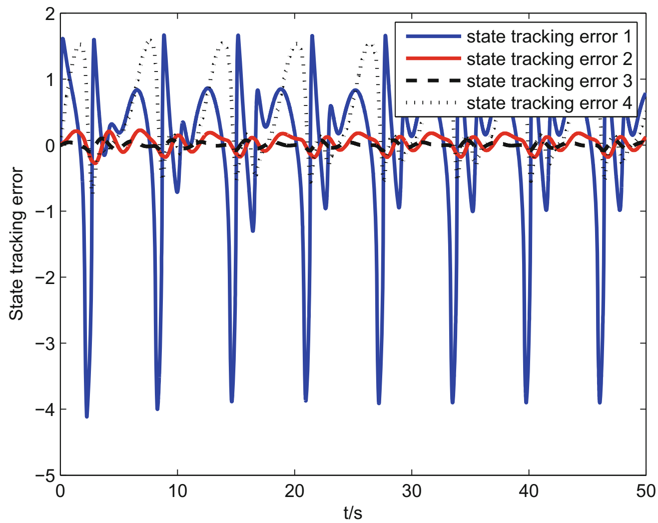

Fig. 8.6 State tracking error under the FTC approach in [14]

in Fig. 8.5. Comparing Fig. 8.4 with Fig. 8.5, it can been seen that better tracking performance during the time delay due to FD can be obtained by using the modified normal control (8.38), while the normal control (8.29) cannot guarantee the same better performance. This means that the time delay's adverse effect on the system performance is more serious, and further indicates the modified normal control (8.38) can alleviate the adverse effect of the time delay due to FD.

To compare with the works in [14], using the FTC approach in [14] where the nonlinear term $H(x(t))$ was not considered, the controlled system is *not* stable even in the fault-free case, as shown in Fig. 8.6. This further illustrates the effectiveness of the FTC approach presented in this chapter.

8.6 Conclusions

In this chapter, the problem of FTC against actuator time-varying faults is investigated, and an FTC scheme is proposed to guarantee that all signals in the closed-loop system are globally asymptotically bounded. At the same time, the time delay due to FD is derived strictly. Further, the analysis of control performance degraded by the time delay is developed, and the conditions that the magnitudes of the faults should be satisfied such that the faulty system controlled by the normal controller remains good tracking performance during the time delay interval are derived. In addition, the corresponding solution to the adverse effect of the time delay is proposed.

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Chapter 9
Adaptive Fault Detection for Uncertain Time-Delay Systems

9.1 Introduction

Time delay phenomenon often exists in the practical applications because of information transmission. It has been proven that such time delay will causes the performance degradation of the controlled systems, even instability. Hence, the research of such class of time delayed systems has become a hot issue on [1–6]. Design of observer including fault detection observer is an important and challenging problem. The main difficulty lies in handling the time delay [7]. For example, consider a simple system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu \\ y(t) = Cx(t) \end{cases} \tag{9.1}$$

where $x,\ y$ and u denote state, output and control input, respectively; $A,\ A_d,\ B$ and C are known real matrices; d is a constant. In most of the existing results such as [8], its observer often is designed as:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_d \hat{x}(t-d) + Bu + L(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{9.2}$$

where L is observer gain matrix. Notice that, the first equation in (9.2) contains time delay term $\hat{x}(t-d)$. Obviously, if d is unknown, then observer (9.2) is not reasonable and does not work in practical applications. Hence, how to avoid the above shortcoming and design a proper observer for dynamical systems becomes important and practically useful, which is the first motivation of our work.

On the other hand, faults/failures inevitable occur in the system parts such as actuators and sensors, which will lead to the decreasing of the system performance. In order to compensate for these faults/failures, various fault-tolerant control (FTC) methods are proposed [9–68]. Among these FTC methods, active FTC methods is more common and important useful [43–53]. Fault detect (FD) is the first and

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important step in active FTC method [9]. In general, the so-called FD observer is designed to detect the faults occurred in the system. Recently, the FD problem of time delay systems has drawn wide attentions. For time delay systems, however, most of the FD observers proposed in literature are similar to (9.2), which also have the same shortcoming, i.e., the FD observers contain the unknown time delay terms. In [8], an asymptotic value of the norm of state estimation error vector is taken as a fault indicator. However, the asymptotic value cannot be accessed in practical applications. Therefore, how to design an efficient FD mechanism is another motivation of this work.

Uncertainty/nonlinearity is common in the controlled systems. In general, as [8], the uncertainty is assumed to be known and to satisfy the so-called Lipschitz condition. Indeed, under the condition, control design and system stability analysis are simplified largely. It should be pointed out that, however, this condition could not be always satisfied in practical applications. Hence, how to efficiently detect the fault occurred in nonlinear systems where the uncertainties do not satisfy Lipschitz condition is particularly valuable and helpful, which also motivate us for this work.

In this chapter, based on the above-mentioned works, the FD problem of time delay systems is considered, where neural networks (NNs) [59, 69, 70] are used to approximate the unknown smooth functions. Compared with the existing results, the contributions of our work are as follows:

(1) First, a novel adaptive neural networks-based fault detection observer is constructed for a class of uncertain time delay systems. In the observer design, by using a suitable adaptation mechanism, the real value of time delay can be estimated online, which means that the conditions (the time delay should be known) and shortcoming (the fault detection observer contains the unknown time delay) are removed.

(2) Next, different from [8] where the uncertainty was assumed to satisfy the Lipschitz condition, the condition is relaxed in our work, and it is just required that the norm of the uncertainty is less than the sum of unknown functions. Thus, the algorithm proposed in this chapter can be used in the widespread practical applications.

(3) Furthermore, a novel fault detection mechanism is proposed, which is more efficient for FD under practical conditions.

The rest of this chapter is organized as follows. Section 9.2 gives the problem formulation and the preliminaries of neural networks are presented. In Sect. 9.3, a novel adaptive NNs-based fault detection observer is proposed. In Sect. 9.4, simulations are presented. Finally, Sect. 9.5 draws the conclusions.

9.2 Problem Statement and Description of NNs

In this section, we will first formulate the fault detection problem. Then, the mathematical description of NNs is introduced.

9.2.1 Problem Statement

Consider the time-delayed system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + g(x(t), x(t-d); t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\bar{d}, 0] \end{cases} \tag{9.3}$$

where $x(t) \in R^n$ is state, $u(t) \in R^m$ is input and $y(t)$ denote output; A, A_d, B and C are known real matrices with appropriate dimensions; $d \in R$ is unknown and satisfies $0 < d \leq \bar{d}$, $\bar{d}$ is a known real constant;

$$g(\cdot) = [g_1(\cdot), g_2(\cdot); \cdots, g_n(\cdot)]^T \in R^n,$$

$g_i(\cdot) = g_i(x(t), x(t-d); t) \in R$, $i = 1, 2, \ldots, n$ are the uncertainties, which denote model uncertainty, external disturbance, time-varying parameter variation, and system nonlinearity; $\varphi(t)$ is an arbitrarily known continuous bounded function.

Throughout this chapter, (A, C) is assumed to be observable and only system output y is measurable.

In this chapter, the faulty system can be described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + \\ \qquad\quad g(x(t), x(t-d); t) + f(x(t), u(t); t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \ t \in [-\bar{d}, 0] \end{cases} \tag{9.4}$$

where $f(\cdot) \in R^n$ denotes the unknown faults occurred in actuators or the other system components.

The aim of this chapter in this chapter is to design a suitable adaptive observer and more efficient fault detection mechanism for system (9.3) to detect the occurred faults.

For notational convenience, let us define the following notations: $g_i = g_i(\cdot)$ and $g = [g_1, g_2, \cdots, g_n]^T$. In addition, $\star(t)$ will be abbreviated as $\star$.

Assumption 9.1 There exist two unknown smooth functions $g_{i1}(x(t)) \geq 0 \in R$, $g_{i2}(x(t-d)) \geq 0 \in R$ and an unknown real constant $g_{i3} \geq 0$ satisfying

$$|g_i| \leq g_{i1}(x(t)) + g_{i2}(x(t-d)) + g_{i3}.$$

Assumption 9.2 Time delay d is bounded, namely, there exist two known real constants $\bar{d} > 0 \in R$ and $\underline{d} > 0 \in R$ such $\underline{d} < d \leq \bar{d}$.

Remark 9.1 In [8], the nonlinear function g_i was assumed to be known satisfying the Lipschitz condition. However, this condition could be not always satisfied in practical applications. In such case, the results in [8] would not work. In this chapter, the condition is replaced by Assumption 9.1. What's important, it is not necessary that g_{i1}, g_{i2} and g_{i3} are known, which relaxes largely the condition in [8]. Thus, the proposed method in this chapter can be used in the widespread practical applications.

9.2.2 Mathematical Description of NNs

NNs have been widely used in controlling of nonlinear systems due to their capabilities of nonlinear function approximation [69]. In this chapter, RBF NNs

$$h(Z, \theta) = \theta^T \xi(Z)$$

will be used to approximate a smooth function $h(Z)$, where the weight vector θ, the basis function vector $\xi(Z)$ are defined as follows:

$$\theta = (\theta_1, \theta_2, \ldots, \theta_N)^T,$$

$$\xi(Z) = (\xi_1(Z), \xi_2(Z), \ldots, \xi_N(Z))^T,$$

$$\theta_i(Z) = \exp(-(\sum\nolimits_{j=1}^{p} (z_j - a_{ij})^2)/(\mu_i)^2),$$

$\mu_i > 0$ denotes the width of the receptive field, and a_{ij} denotes the center of the Gaussian function, z_j denotes the jth element of Z, p denotes the dimension of Z, N is the number of the NNs nodes.

In this chapter, for $i = 1, \ldots, n$, $g_{i1}(x(t))$ and $g_{i2}(x(t-d))$ are approximated by NNs as:

$$\hat{g}_{i1}(\hat{x}(t), \hat{\theta}_{i1}) = \hat{\theta}_{i1}^T \xi_{i1}(\hat{x}(t))$$

$$\hat{g}_{i2}(\hat{x}(t-\hat{d}), \hat{\theta}_{i2}) = \hat{\theta}_{i2}^T \xi_{i2}(\hat{x}(t-\hat{d}))$$

Optimal parameter vectors θ_{gi1}^* and θ_{gi2}^* are defined as

$$\theta_{i1}^* = \arg \min_{\theta_{i1} \in \Omega_{i1}} [\sup_{x \in U, \hat{x} \in \hat{U}} |g_{i1}(x(t)) - \hat{\theta}_{i1}^T \xi_{i1}(\hat{x}(t))|]$$

$$\theta_{i2}^* = \arg \min_{\theta_{i2} \in \Omega_{i2}} [\sup_{x \in U, \hat{x} \in \hat{U}} |g_{i2}(x(t-d)) - \hat{\theta}_{i2}^T \xi_{i2}(\hat{x}(t-\hat{d}))|]$$

where Ω_{i1}, Ω_{i2}, U and $\hat{U}$ are compact regions for $\hat{\theta}_{i1}$, $\hat{\theta}_{i2}$, x and $\hat{x}$, $\hat{d}$, $\hat{\theta}_{i1}$ and $\hat{\theta}_{i2}$ are the estimates of d, θ_{i1}^* and θ_{i2}^*, respectively.

The NNs minimum approximation errors are defined as

$$\varepsilon_{i1} = g_{i1}(x(t)) - \theta_{i1}^{*T}\xi_{i1}(\hat{x}(t)),$$

$$\varepsilon_{i2} = g_{i2}(x(t-d)) - \theta_{i2}^{*T}\xi_{i2}(\hat{x}(t-\hat{d})).$$

Now, the following assumptions are made throughout this chapter.

Assumption 9.3 $|\varepsilon_{i1}| \le \varepsilon_{i1}^*$ and $|\varepsilon_{i2}| \le \varepsilon_{i2}^*$, where $\varepsilon_{i1}^* > 0 \in R$ and $\varepsilon_{i2}^* > 0 \in R$ are unknown constants.

9.3 Fault Detection Observer Design

For (9.3), the FD observer is designed as:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-\hat{d}) + A_d\Delta_1 + Bu(t) + \\ \qquad L(\hat{y}(t) - y(t)) + sgn(e_y^T F)\hat{g} + \Delta_2 \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{x}(t) = 0, \ t \in [-\bar{d}, 0] \end{cases} \tag{9.5}$$

where $\hat{x}(t) \in R^n$ is observer state, $u(t) \in R^m$ is observer control input, and $\hat{y}(t)$ is observer output;

$$sgn(e_y^T F) = diag\{sgn(e_y^T F_1), \ldots, sgn(e_y^T F_n)\}$$

$$\hat{g} = \hat{g}_1 + \hat{g}_2 + \hat{g}_3$$

$$\hat{g}_1 = [\hat{g}_{11}, \ldots, \hat{g}_{n1}]^T$$

$$\hat{g}_2 = [\hat{g}_{12}, \ldots, \hat{g}_{n2}]^T$$

$$\hat{g}_3 = [\hat{g}_{13}, \ldots, \hat{g}_{n3}]^T$$

$\hat{g}_{i1}$ $(= \hat{g}_{i1}(x(t)))$, $\hat{g}_{i2}$ $(= \hat{g}_{i2}(x(t-\hat{d})))$ and $\hat{g}_{i3}$ are the estimates of unknown smooth functions g_{i1}, g_{i2} and unknown constant g_{i3}, respectively; g_{i1}, g_{i2} and g_{i3} are defined in Assumption 9.1, F_i $(i = 1, 2, \ldots, n)$ is the ith column of matrix F, which satisfies the following condition

$$(F^T C)^T = P \tag{9.6}$$

real matrix $P = P^T > 0$ will be defined later, $e_y = y - \hat{y}$, $\hat{d}$ is an estimate of d, Δ_1 and Δ_2 are robust terms to be defined later.

Denote

$$e_x(t) = x(t) - \hat{x}(t), \quad e_d = x(t-d) - \hat{x}(t-\hat{d})$$

Then, from (9.3) and (9.5), the observer error dynamics can be described as follows:

$$\begin{aligned}\dot{e}_x(t) &= (A-LC)e_x(t) + A_d x(t-d) - A_d\hat{x}(t-\hat{d}) + \\ &\quad \tilde{g} - A_d\Delta_1 - \Delta_2 \\ &= (A-LC)e_x(t) + A_d x(t-d) - A_d\hat{x}(t-\hat{d}) - \\ &\quad A_d\hat{x}(t-d) + A_d\hat{x}(t-d) + \tilde{g} - A_d\Delta_1 - \Delta_2 \\ &= (A-LC)e_x(t) + A_d e_x(t-d) + A_d\hat{x}(t-d) - \\ &\quad A_d\hat{x}(t-\hat{d}) + \tilde{g} - A_d\Delta_1 - \Delta_2\end{aligned} \tag{9.7}$$

where $\tilde{g} = [\tilde{g}_1, \ldots, \tilde{g}_n]^T$ and

$$\tilde{g}_i = g_i - sgn(e_y^T F_i)(\hat{g}_{i1} + \hat{g}_{i2} + \hat{g}_{i3}),$$

Note that, $A_d\hat{x}(t-d)$ is added to and subtracted from the right side of (9.7).

Remark 3: Many researchers study the observer design of time-delayed systems in literature. For example, consider

$$\begin{cases}\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\bar{d}, 0]\end{cases}$$

where x, y and u denote the system state, output and input, $d > 0 \in R$ denotes the time delay. In general, as doing in [8], the FD observer was given as:

$$\begin{cases}\dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-d) + Bu(t) + L(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{x}(t) = 0, \; t \in [-\bar{d}, 0]\end{cases}$$

then we obtain the error dynamics

$$\dot{e}_x(t) = (A-LC)e_x(t) + A_d e_x(t-d)$$

However, as Jiang pointed out in [7], the shortcoming of the aforementioned observer is that d must be known. If not, the observer does not work in the practical applications. Hence, for avoiding the shortcoming, a novel fault detection observer (9.5) is designed in this chapter.

Define the following smooth function

$$V_{De_x} = e_x^T(t)Pe_x(t) \tag{9.8}$$

where $P = P^T > 0$ is defined as in (9.6).

Differentiating V_{De_x} with respect to time t, we have

$$\dot{V}_{De_x} = e_x^T(t)(P(A - LC) + (A - LC)^T P)e_x(t) + \\ 2e_x^T(t)PA_d e_x(t-d) + 2e_x^T(t)PA_d(\hat{x}(t-d) - \\ \hat{x}(t-\hat{d})) + 2e_x^T(t)P\tilde{g} - 2e_x^T(t)P(A_d\Delta_1 + \Delta_2) \tag{9.9}$$

From Young's inequality, we have

$$2e_x^T(t)PA_d e_x(t-d) \\ \leq e_x^T(t)PA_d S^{-1} A_d^T P e_x(t) + e_x^T(t-d)Se_x(t-d) \tag{9.10}$$

where real matrix $S = S^T > 0$.

From Assumption 9.1, it follows

$$2e_x^T(t)P\tilde{g} \\ = \sum_{i=1}^n 2e_x^T(t)P_i\tilde{g}_i \\ = \sum_{i=1}^n 2e_x^T(t)P_i(g_i - sgn(e_y^T F_i)\hat{g}_i) \\ \leq \sum_{i=1}^n (|2e_x^T(t)P_i||g_i| - sgn(e_y^T F_i)2e_x^T(t)P_i\hat{g}_i)$$

where P_i is the ith column of matrix P.

From (9.6), we know, $P = (F^T C)^T$ and $P = P^T > 0$. Further, we have

$$e_x^T(t)P_i = e_y^T(t)F_i$$

Hence, we have

$$2e_x^T(t)P\tilde{g} \\ \leq \sum_{i=1}^n |2e_y^T(t)F_i||g_i| - \sum_{i=1}^n |2e_y^T(t)F_i|\hat{g}_i \\ \leq \sum_{i=1}^n |2e_y^T(t)F_i|(g_{i1} + g_{i2} + g_{i3}) - \\ \sum_{i=1}^n |2e_y^T(t)F_i|(\hat{g}_{i1} + \hat{g}_{i2} + \hat{g}_{i3}) \\ = \sum_{i=1}^n |2e_y^T(t)F_i|(\tilde{g}_{i1} + \tilde{g}_{i2} + \tilde{g}_{i3}) \\ = \sum_{i=1}^n |2e_y^T(t)F_i|[\theta_{i1}^{*T}\xi_{i1}(\hat{x}(t)) + \varepsilon_{i1}(\hat{x}(t)) - \\ \hat{\theta}_{i1}\xi_{i1}(\hat{x}(t)) + \theta_{i2}^{*T}\xi_{i2}(\hat{x}(t-\hat{d})) + \\ \varepsilon_{i2}(\hat{x}(t-\hat{d})) - \hat{\theta}_{i2}\xi_{i2}(\hat{x}(t-\hat{d})) + \tilde{g}_{i3}] \\ \leq \sum_{i=1}^n |2e_y^T(t)P_i|(\tilde{\theta}_{i1}^T\xi_{i1} + \tilde{\theta}_{i2}^T\xi_{i2} + \tilde{g}_{i3}) + \\ \sum_{i=1}^n |2e_y^T(t)F_i|(\varepsilon_{i1}^* + \varepsilon_{i2}^*) \tag{9.11}$$

where $\tilde{\theta}_{i1} = \theta_{i1}^* - \hat{\theta}_{i1}, \tilde{\theta}_{i2} = \theta_{i2}^* - \hat{\theta}_{i2}$, ξ_{i1} and ξ_{i2} are the abbreviations of $\xi_i(\hat{x}(t))$ and $\xi_{i2}(\hat{x}(t-\hat{d}))$, respectively.

Substituting (9.10) and (9.11) into (9.9), it yields

$$\begin{aligned}\dot{V}_{De_x} \leq & e_x^T(t)(P(A-LC)+(A-LC)^T P)e_x(t)- \\ & 2e_x^T(t)P(A_d\Delta_1+\Delta_2)+ \\ & e_x^T(t)PA_dS^{-1}A_d^T Pe_x(t)+ \\ & e_x^T(t-d)Se_x(t-d)+ \\ & 2e_x^T(t)PA_d(\hat{x}(t-d)-\hat{x}(t-\hat{d}))+ \\ & \sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(\tilde{\theta}_{i1}^T\xi_{i1}+\tilde{\theta}_{i2}^T\xi_{i2}+\tilde{g}_{i3})+ \\ & \sum\nolimits_{i=1}^n |2e_y^T(t)F_i|(\varepsilon_{i1}^*+\varepsilon_{i2}^*)\end{aligned} \tag{9.12}$$

Define the following smooth function

$$\begin{aligned}V_{D1} = & V_{De_x} + \int_{t-d}^t e_x^T(s)Se_x(s)ds+ \\ & \sum\nolimits_{i=1}^n [\frac{1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1}+\frac{1}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2}]+ \\ & \sum\nolimits_{i=1}^n [\frac{1}{2\eta_3}\tilde{g}_{i3}^2+\frac{1}{2\eta_4}\tilde{\varepsilon}_i^2]\end{aligned} \tag{9.13}$$

where $\tilde{\varepsilon}_i = \varepsilon_i^* - \hat{\varepsilon}_i, \varepsilon_i^* = \varepsilon_{i1}^* + \varepsilon_{i2}^*, \hat{\varepsilon}_i$ is the estimate of $\varepsilon_i^*, \eta_l > 0 \in R, l = 1, 2, 3, 4$ are adaptive rates, I is an identity matrix.

Differentiating V_{D1} with respect to time t, it yields

$$\begin{aligned}\dot{V}_{D1} = & \dot{V}_{De_x} + e_x^T(t)Se_x(t)- \\ & e_x^T(t-d)(S+2I)e_x(t-d)- \\ & \sum\nolimits_{i=1}^n [\frac{1}{\eta_1}\tilde{\theta}_{i1}^T\dot{\hat{\theta}}_{i1}+\frac{1}{\eta_2}\tilde{\theta}_{i2}^T\dot{\hat{\theta}}_{i2}]- \\ & \sum\nolimits_{i=1}^n [\frac{1}{\eta_3}\tilde{g}_{i3}\dot{\hat{g}}_{i3}+\frac{1}{\eta_4}\tilde{\varepsilon}_i\dot{\hat{\varepsilon}}_i]\end{aligned} \tag{9.14}$$

Substituting (9.12) into (9.14), it yields

$$\begin{aligned}\dot{V}_{D1} \le\ & e_x^T(t)\Xi_1 e_x(t) - 2e_x^T(t)P(A_d\Delta_1 + \Delta_2)+\\ & 2e_x^T(t)PA_d(\hat{x}(t-d) - \hat{x}(t-\hat{d}))+\\ & \sum\nolimits_{i=1}^{n}[\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})]+\\ & \sum\nolimits_{i=1}^{n}[\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})]+\\ & \sum\nolimits_{i=1}^{n}[\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})]+\\ & \sum\nolimits_{i=1}^{n}[|2e_y^T(t)F_i|\varepsilon_i^* - \frac{1}{\eta_4}\tilde{\varepsilon}_i\dot{\hat{\varepsilon}}_i]\end{aligned} \tag{9.15}$$

where

$$\Xi_1 = (P(A-LC) + (A-LC)^T P + PA_d S^{-1} A_d^T P + S) \tag{9.16}$$

Now, Δ_1 and Δ_2 are designed as follows:

$$\begin{aligned}\Delta_1 &= sgn(e_y^T(t)F_{Ad})(|\hat{x}(t-\hat{d})| + |\hat{x}_m|)\\ \Delta_2 &= sgn(e_y^T(t)F)\hat{\varepsilon}\end{aligned} \tag{9.17}$$

where

$$sgn(e_y^T F_{Ad}) = diag\{sgn(e_y^T F_{Ad1}), \ldots, sgn(e_y^T F_{Adn})\},$$

$$sgn(e_y^T(t)F) = diag\{sgn(e_y^T F_1), \ldots, sgn(e_y^T F_n)\},$$

F_{Adi} and F_i, $i = 1, \ldots, n$, denote the ith column of matrix F_{Ad} and F, respectively,

$$|\hat{x}(t-\hat{d})| = [|\hat{x}_1(t-\hat{d})|, \cdots, |\hat{x}_n(t-\hat{d})|]^T,$$

$$|\hat{x}_m| = [\hat{x}_{m1}, \ldots, \hat{x}_{mn}]^T,$$

$$\hat{x}_{mi} = \max\nolimits_{0\le\tau\le\bar{d}}\{|\hat{x}_i(t-\tau)|\}, i = 1, \ldots, n,$$

matrix F satisfies (9.6), while F_{Ad} satisfies the following condition

$$PA_d = (F_{Ad}^T C)^T, \tag{9.18}$$

and

$$\hat{\varepsilon} = [\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_n]^T$$

From (9.6), (9.17) and (9.18), we have

$$-2e_x^T(t)PA_d\Delta_1 + 2e_x^T(t)PA_d(\hat{x}(t-d) - \hat{x}(t-\hat{d})) \le 0 \tag{9.19}$$

$$2e_x^T(t)P\Delta_2 = \sum\nolimits_{i=1}^{n} |2e_y^T(t)F_i|\hat{\varepsilon}_i \tag{9.20}$$

Substituting (9.19) and (9.20) into (9.15) and considering (9.6) and (9.18), one has

$$\begin{aligned}\dot{V}_{D1} \leq & e_x^T(t)\Xi_1 e_x(t)+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})]+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})]+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})]+ \\ & \sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_y^T(t)F_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i)\end{aligned} \tag{9.21}$$

In order to derive the adaptive law of $\hat{d}$, $\tilde{d}e_y^T(t)e_y(t)$ is added to and subtracted from the the right hand of (9.21), then, we have

$$\begin{aligned}\dot{V}_{D1} \leq & e_x^T(t)\Xi_1 e_x(t)+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})]+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})]+ \\ & \sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})]+ \\ & \sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_y^T(t)F_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i)+ \\ & \tilde{d}e_y^T(t)e_y(t) - \tilde{d}e_y^T(t)e_y(t)\end{aligned} \tag{9.22}$$

where $\tilde{d} = d - \hat{d}$.

Since $e_y = Ce_x$, we have

$$\tilde{d}e_y^T(t)e_y(t) = \tilde{d}e_x^T(t)C^TCe_x(t)$$

And since

$$\tilde{d} = d - \hat{d} \text{ and } e_x^T(t)C^TCe_x(t) \geq 0,$$

we have

$$\begin{aligned}\tilde{d}e_y^T(t)e_y(t) &= \tilde{d}e_x^T(t)C^TCe_x(t) \\ &= (d - \hat{d})e_x^T(t)C^TCe_x(t) \\ &= de_x^T(t)C^TCe_x(t) - \hat{d}e_x^T(t)C^TCe_x(t) \\ &\leq \bar{d}e_x^T(t)C^TCe_x(t)\end{aligned}$$

where the properties: $0 \leq d \leq \bar{d}$ (Assumption 9.2) and $0 \leq \hat{d}$, are used. Note that, $0 \leq \underline{d} \leq \hat{d} \leq \bar{d}$ is ensured by adaptive law (9.31). Further,

$$\begin{aligned}\dot{V}_{D1} \leq e_x^T(t)(\Xi_1 + \bar{d}C^T C)e_x(t)+ \\ \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})]+ \\ \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})]+ \\ \sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})]+ \\ \sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_x^T(t)P_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i)- \\ \tilde{d}e_y^T(t)e_y(t)\end{aligned} \tag{9.23}$$

If $Q > 0 \in R^{n\times n}$, $L \in R^{n\times n}$ and $P = P^T > 0 \in R^{n\times n}$ are chosen to satisfy the following inequality,

$$\begin{aligned}P(A - LC) + (A - LC)^T P+ \\ PA_d S^{-1} A_d^T P + S + \bar{d}C^T C \leq -Q\end{aligned} \tag{9.24}$$

then (9.23) can be developed as follows:

$$\begin{aligned}\dot{V}_{D1} \leq - e_x^T(t)Qe_x(t)+ \\ \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1} - \frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})]+ \\ \sum\nolimits_{i=1}^{n} [\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2} - \frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})]+ \\ \sum\nolimits_{i=1}^{n} [\tilde{g}_{i3}(|2e_y^T(t)F_i| - \frac{1}{\eta_3}\dot{\hat{g}}_{i3})]+ \\ \sum\nolimits_{i=1}^{n} \tilde{\varepsilon}_i(|2e_y^T(t)F_i| - \frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i)- \\ \tilde{d}e_y^T(t)e_y(t)\end{aligned} \tag{9.25}$$

Define the following Lyapunov function

$$V_D = V_{D1} + \frac{1}{2\eta_5}\tilde{d}^2$$

where $\eta_5 > 0$ is a design parameter.

Differentiating V_D with respect to time t and considering (9.25), it yields

$$\begin{aligned}\dot{V}_D \leq & -e_x^T(t)Qe_x(t)+ \\ & \sum\nolimits_{i=1}^{n}[\tilde{\theta}_{i1}^T(|2e_y^T(t)F_i|\xi_{i1}-\frac{1}{\eta_1}\dot{\hat{\theta}}_{i1})]+ \\ & \sum\nolimits_{i=1}^{n}[\tilde{\theta}_{i2}^T(|2e_y^T(t)F_i|\xi_{i2}-\frac{1}{\eta_2}\dot{\hat{\theta}}_{i2})]+ \\ & \sum\nolimits_{i=1}^{n}[\tilde{g}_{i3}(|2e_y^T(t)F_i|-\frac{1}{\eta_3}\dot{\hat{g}}_{i3})]+ \\ & \sum\nolimits_{i=1}^{n}\tilde{\varepsilon}_i(|2e_y^T(t)F_i|-\frac{1}{\eta_4}\dot{\hat{\varepsilon}}_i)- \\ & \tilde{d}(e_y^T(t)e_y(t)+\frac{1}{\eta_5}\dot{\hat{d}})\end{aligned} \tag{9.26}$$

Define the following adaptive laws

$$\dot{\hat{\theta}}_{i1}=\eta_1|2e_y^T(t)F_i|\xi_{i1}-\sigma_1\hat{\theta}_{i1} \tag{9.27}$$

$$\dot{\hat{\theta}}_{i2}=\eta_2|2e_y^T(t)F_i|\xi_{i2}-\sigma_2\hat{\theta}_{i2} \tag{9.28}$$

$$\dot{\hat{g}}_{i3}=\eta_3|2e_y^T(t)F_i|-\sigma_3\hat{g}_{i3} \tag{9.29}$$

$$\dot{\hat{\varepsilon}}_i=\eta_4|2e_y^T(t)F_i|-\sigma_4\hat{\varepsilon}_i \tag{9.30}$$

$$\dot{\hat{d}}=\begin{cases}\kappa,\ \text{if } \underline{d}\leq\hat{d}\leq\bar{d} \text{ or} & \\ \quad(\hat{d}=\bar{d} \text{ or } \hat{d}=\underline{d}) \text{ and } \hat{d}\kappa\leq 0 & ,\ \underline{d}<\hat{d}(0)<\bar{d} \\ 0,\ \text{if } (\hat{d}=\bar{d} \text{ or } \hat{d}=\underline{d}) \text{ and } \hat{d}\kappa>0 & \end{cases} \tag{9.31}$$

where $i=1,\ldots,n$, $\sigma_l>0, l=1,\ldots,5$ are design parameters, $\kappa=-\eta_5e_y^T(t)e_y(t)-\sigma_5\hat{d}$.

Note that, under the initial condition that $\underline{d}<\hat{d}(0)<\bar{d}$, the adaptive law (9.31) can guarantees that

$$\underline{d}\leq\hat{d}(t)\leq\bar{d},\quad \text{for } t\geq 0$$

In fact, it is easily derived by lyapunov stability theory. Let us define the following Lyapunov function

$$V_d=\frac{1}{2}\hat{d}^2$$

Differentiating V_d with respect to time t, we have

$$\dot{V}_d = \hat{d}\dot{\hat{d}}$$

The following analysis will be derived in two cases.

Case 1: the first condition of (9.31) holds

Since

$$\underline{d} \leq \hat{d} \leq \bar{d} \text{ or } (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa \leq 0$$

we have

$$\dot{V}_d = \hat{d}\kappa = \hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \leq 0$$

Case 2: the second condition of (9.31) holds

Because

$$(\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa > 0$$

we have

$$\dot{V}_d = \hat{d} \cdot 0 = 0$$

From Cases 1 and 2, using Lyapunov stability theory, we have the following results,

$$\underline{d} \leq \hat{d}(t) \leq \bar{d}, \quad \text{for } t \geq 0.$$

Note that,

$$0 = -\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d} - (-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d})$$

Thus, the adaptive law (9.31) can be rewritten as follows:

$$\dot{\hat{d}} = \begin{cases} \kappa, \text{ if } \underline{d} \leq \hat{d} \leq \bar{d} \text{ or} \\ \quad (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa \leq 0 \\ -\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d} - (-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}), \\ \quad \text{if } (\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa > 0 \end{cases}$$

Substituting adaptive laws (9.27)–(9.31) into (9.26), it yields

$$\begin{aligned} \dot{V}_D \leq & -e_x^T(t)Qe_x(t) + I\tilde{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) + \\ & \sum_{i=1}^{n}(\sigma_1\tilde{\theta}_{i1}^T\hat{\theta}_{i1} + \sigma_2\tilde{\theta}_{i2}^T\hat{\theta}_{i2}) + \\ & \sum_{i=1}^{n}(\sigma_3\tilde{g}_{i3}\hat{g}_{i3} + \sigma_4\tilde{\varepsilon}_i\hat{\varepsilon}_i) + \sigma_5\tilde{d}\hat{d} \end{aligned} \tag{9.32}$$

where $I = 0$ (or 1), if the first (second) condition of (9.31) holds.

If the second condition of (9.31) holds, namely,

$$(\hat{d} = \bar{d} \text{ or } \hat{d} = \underline{d}) \text{ and } \hat{d}\kappa > 0$$

then

$$\begin{aligned} & I\tilde{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \\ & = I\tilde{d}\frac{\hat{d}\hat{d}}{\hat{d}^2}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \end{aligned}$$

Note that,

$$\tilde{d}\hat{d} = \frac{1}{2}[d^2 - \hat{d}^2 - (d - \hat{d})^2] \tag{9.33}$$

If $\hat{d} = \bar{d}$ and $\hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) > 0$, then

$$\tilde{d}\hat{d} < 0$$

On the other hand, if $\hat{d} = 0$ and $\hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) > 0$, then

$$\tilde{d}\hat{d} = 0$$

Hence, we have

$$\tilde{d}\hat{d} \le 0$$

And since $\hat{d}\kappa = \hat{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) > 0$, we have

$$I\tilde{d}(-\eta_5 e_y^T(t)e_y(t) - \sigma_5\hat{d}) \le 0$$

Therefore, (9.32) can be further derived as

$$\begin{aligned} \dot{V}_D \le & -e_x^T(t)Qe_x(t) + \\ & \sum_{i=1}^{n}(\sigma_1\tilde{\theta}_{i1}^T\hat{\theta}_{i1} + \sigma_2\tilde{\theta}_{i2}^T\hat{\theta}_{i2}) + \\ & \sum_{i=1}^{n}(\sigma_3\tilde{g}_{i3}\hat{g}_{i3} + \sigma_4\tilde{\varepsilon}_i\hat{\varepsilon}_i) + \sigma_5\tilde{d}\hat{d} \end{aligned} \tag{9.34}$$

Since $\tilde{\theta}_{i1} = \theta_{i1}^* - \hat{\theta}_{i1}$, using Young's inequality, we have

$$\begin{aligned} \sigma_1\tilde{\theta}_{i1}^T\hat{\theta}_{i1} &= \sigma_1\tilde{\theta}_{i1}^T(\theta_{i1}^* - \tilde{\theta}_{i1}) \\ &= -\sigma_1\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \sigma_1\tilde{\theta}_{i1}^T\theta_{i1}^* \\ &\le -\frac{1}{2}\sigma_1\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{1}{2}\sigma_1\theta_{i1}^{*T}\theta_{i1}^* \end{aligned} \tag{9.35}$$

Similarly, we have

$$\sigma_2\tilde{\theta}_{i2}^T\hat{\theta}_{i2} \le -\frac{1}{2}\sigma_2\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} + \frac{1}{2}\sigma_2\theta_{i2}^{*T}\theta_{i2}^* \tag{9.36}$$

$$\sigma_3\tilde{g}_{i3}\hat{g}_{i3} \le -\frac{1}{2}\sigma_3\tilde{g}_{i3}^2 + \frac{1}{2}\sigma_3 g_{i3}^2 \tag{9.37}$$

$$\sigma_4\tilde{\varepsilon}_i\hat{\varepsilon}_i \le -\frac{1}{2}\sigma_4\tilde{\varepsilon}_i^2 + \frac{1}{2}\sigma_4\varepsilon_i^2 \tag{9.38}$$

$$\sigma_5\tilde{d}\hat{d} \le -\frac{1}{2}\sigma_5\tilde{d}^2 + \frac{1}{2}\sigma_5\bar{d}^2 \tag{9.39}$$

Since

$$\lambda_{min}(Q)e_x^T(t)e_x(t) \le e_x^T(t)Qe_x(t)$$

then substituting (9.34)–(9.38) into (9.33), it yields

$$\begin{aligned}\dot{V}_D \le & -\lambda_{min}(Q)e_x^T(t)e_x(t)- \\ & \sum\nolimits_{i=1}^n(\frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2})- \\ & \sum\nolimits_{i=1}^n(\frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2) - \frac{\sigma_5}{2\eta_5}\tilde{d}^2+ \\ & \sum\nolimits_{i=1}^n(\frac{\sigma_1}{2\eta_1}\theta_{i1}^{*T}\theta_{i1}^* + \frac{\sigma_2}{2\eta_2}\theta_{i2}^{*T}\theta_{i2}^*)+ \\ & \sum\nolimits_{i=1}^n(\frac{\sigma_3}{2\eta_3}g_{i3}^2 + \frac{\sigma_4}{2\eta_4}\varepsilon_i^{*2}) + \frac{\sigma_5}{2\eta_5}\bar{d}^2\end{aligned} \tag{9.40}$$

Let

$$\begin{aligned}\mu = & \sum\nolimits_{i=1}^n(\frac{\sigma_1}{2\eta_1}\theta_{i1}^{*T}\theta_{i1}^* + \frac{\sigma_2}{2\eta_2}\theta_{i2}^{*T}\theta_{i2}^*)+ \\ & \sum\nolimits_{i=1}^n(\frac{\sigma_3}{2\eta_3}g_{i3}^2 + \frac{\sigma_4}{2\eta_4}\varepsilon_i^{*2}) + \frac{\sigma_5}{2\eta_5}\bar{d}^2\end{aligned}$$

then (9.39) can be re-written as follows:

$$\begin{aligned}\dot{V}_D \le & -\lambda_{min}(Q)e_x^T(t)e_x(t)- \\ & \sum\nolimits_{i=1}^n(\frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2})- \\ & \sum\nolimits_{i=1}^n(\frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2) - \frac{\sigma_5}{2\eta_5}\tilde{d}^2 + \mu\end{aligned} \tag{9.41}$$

It can be seen from (9.40) that, if

$$\lambda_{min}(Q)e_x^T(t)e_x(t) + \frac{\sigma_5}{2\eta_5}\tilde{d}^2 + \\ \sum_{i=1}^{n}(\frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} + \frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2) \geq \mu$$

then $\dot{V}_D < 0$. Hence, set Ω defined as:

$$\Omega = \left\{ \begin{pmatrix} e_x, \\ \tilde{\theta}_{i1}, \\ \tilde{\theta}_{i2}, \\ \tilde{g}_{i3}, \\ \tilde{d} \end{pmatrix} \middle| \begin{pmatrix} \lambda_{min}(Q)e_x^T e_x + \frac{\sigma_5}{2\eta_5}\tilde{d}^2 + \\ \sum_{i=1}^{n} \frac{\sigma_1}{2\eta_1}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \\ \sum_{i=1}^{n} \frac{\sigma_2}{2\eta_2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2}) + \\ \sum_{i=1}^{n}(\frac{\sigma_3}{2\eta_3}\tilde{g}_{i3}^2 + \frac{\sigma_4}{2\eta_4}\tilde{\varepsilon}_i^2) \end{pmatrix} \leq \mu \right\}$$

is an invariable set. This implies that $e_x, \tilde{\theta}_{i1}, \tilde{\theta}_{i2}, \tilde{g}_{i3}$ and $\tilde{d}$ are asymptotically bounded, namely,

$$||\tilde{\theta}_{i1}|| \leq \sqrt{\frac{2\eta_1\mu}{\sigma_1}}, \quad ||\tilde{\theta}_{i2}|| \leq \sqrt{\frac{2\eta_2\mu}{\sigma_2}}, \ ||\tilde{g}_{i3}|| \leq \sqrt{\frac{2\eta_3\mu}{\sigma_3}},$$

$$||e_x|| \leq \sqrt{\frac{\mu}{\lambda_{min}(Q)}},$$

$$||\tilde{\varepsilon}_i|| \leq \sqrt{\frac{2\eta_4\mu}{\sigma_4}}, \ |\tilde{d}| \leq \sqrt{\frac{2\eta_5\mu}{\sigma_5}}$$

It is necessary to point out that the size of Ω can become arbitrarily small by adjusting the parameters: σ_i and η_i, $i = 1, 2, \ldots, 5$.

Now, the following theorem is given to summarize the above design procedures and analysis.

Theorem 9.1 *Consider system* (9.1) *and observer* (9.5) *with Assumptions 1 and 2, if there exist matrices* L, F, F_{Ad}, $Q > 0$, $S > 0$ *and* $P = P^T > 0$ *satisfying* (9.6), (9.18) *and* (9.24), *and adaptive laws* (9.27)–(9.31) *are used, then error dynamics* (9.7) *is asymptotically bounded with all the signals in the closed-systems converging to an adjustable neighborhood of the origin.*

Proof From the above analysis, it is easy to obtain the conclusions. The detailed proof is thus omitted here.

From Theorem 9.1, we have

$$||e_x|| \leq \sqrt{\frac{\mu}{\lambda_{min}(Q)}}$$

Let us define detection residual

$$R(t) = ||y(t) - \hat{y}(t)|| = ||Ce_x(t)||$$

Obviously, in the free-fault case, one has

$$R(t) \leq ||C||\sqrt{\frac{\mu}{\lambda_{min}(Q)}}$$

Hence, by using the following mechanism, fault detection can be performed,

$$\begin{cases} R(t) \leq T_d & \text{no fault occurred,} \\ R(t) > T_d & \text{fault has occurred} \end{cases} \tag{9.42}$$

where $T_d = ||C||\sqrt{\frac{\mu}{\lambda_{min}(Q)}}$.

Remark 9.2 It can been seen that, if there is no fault in the controlled system, then $\lim_{t\to\infty} e_x(t) = 0$. If some actuator faults occur in system, then $\lim_{t\to\infty} e_x(t) \neq 0$. Thus, in some existing works, the fault detection is designed as:

$$\begin{cases} \lim_{t\to\infty} e_x(t) = 0, & \text{no fault occurred} \\ \lim_{t\to\infty} e_x(t) \neq 0, & \text{fault has occurred} \end{cases}$$

observer (9.5) was taken to as the FD observer of system (9.1). However, $e_x(\infty)$ is not available in practice applications. Thus, $e_x(\infty) \neq 0$ cannot be seen as an indicator to detect fault occurrence or not. Hence, (9.32) is more efficient mechanism for FD in practical applications.

9.4 Simulation Results

The following time delayed system is considered:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + g \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\bar{d}, 0] \end{cases}$$

where

$$A = \begin{bmatrix} -4 & 0 \\ 1 & 0.5 \end{bmatrix}, \ A_d = \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.2 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \ g = \begin{bmatrix} x_1(t)sin(x_2) + x_2(t-d)sin(x_1) \\ x_2(t)cos(x_1) + x_1(t-d)cos(x_2) \end{bmatrix},$$

time delay $d = 0.5$, $\phi(t) = e^{-1} - 0.1e^{-t}$.

In this simulation, it is assumed that the fault occurs at 6s in the system.

Note that (9.19) can be transformed to the the following linear matrix inequality (LMI),

$$\begin{bmatrix} PA - YC + A^T P - C^T Y^T + S + Q & PA_d \\ A_d^T P & -S^{-1} \end{bmatrix} < 0$$

where $Y = PL$. By solving this LMI, we can have:

$$P = \begin{bmatrix} 1.7096 & 0.0590 \\ 0.0590 & 1.5033 \end{bmatrix}, \; Q = \begin{bmatrix} 1.7414 & 0 \\ 0 & 1.7414 \end{bmatrix},$$

$$Y = \begin{bmatrix} -5.9088 & 0.2191 \\ 1.0779 & 1.6224 \end{bmatrix}, \; L = \begin{bmatrix} -3.4856 & 0.0911 \\ 0.8537 & 1.0756 \end{bmatrix}$$

and

$$F = \begin{bmatrix} 2.5102 & 0.0590 \\ 0.1180 & 1.5033 \end{bmatrix}$$

The simulation results are shown in Figs. 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7 and 9.8. From Fig. 9.1, It can be seen that the state observe errors are bounded, which implies that the proposed observer has a better convergent property, while Fig. 9.2 shows the residual signal asymptotically converges to the small neighborhood of the origin. Figures 9.3, 9.4, 9.5 and 9.6 also show the closed-loop system signals are bounded.

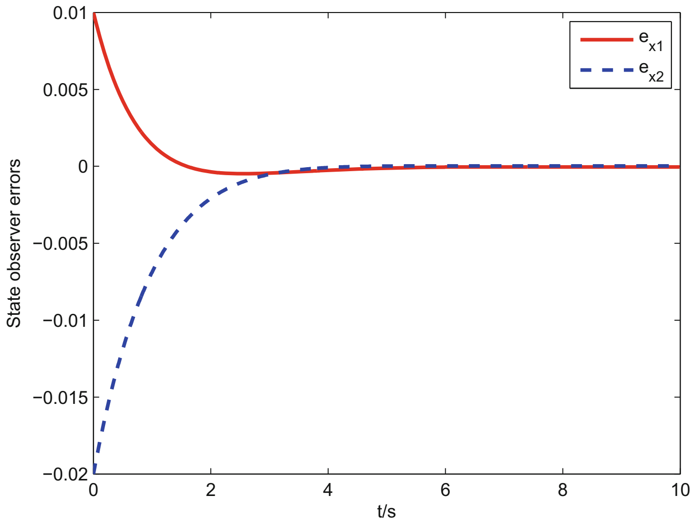

Fig. 9.1 The state observer errors (no fault)

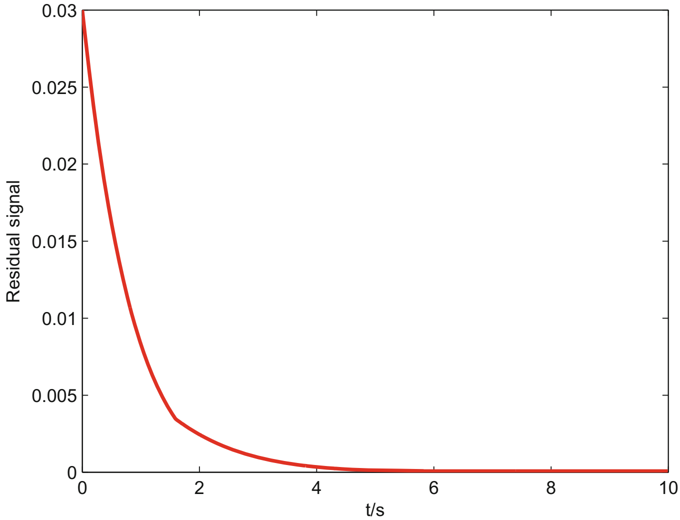

Fig. 9.2 The residual signal (no fault)

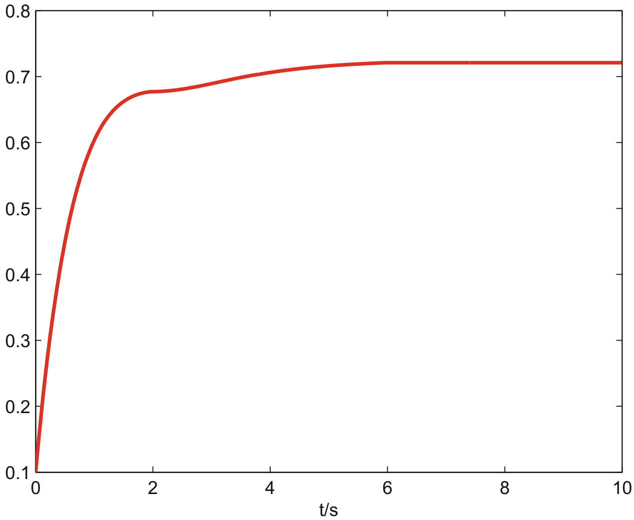

Fig. 9.3 The norm of $\hat{\theta}_{11}$ (no fault)

However, when a fault occurs in the system, Fig. 9.7 shows that, the residual signal significantly deviates from the origin, and the alarm occurs. Correspondingly, the state observe errors significantly deviates from the origin, too, shown in Fig. 9.8.

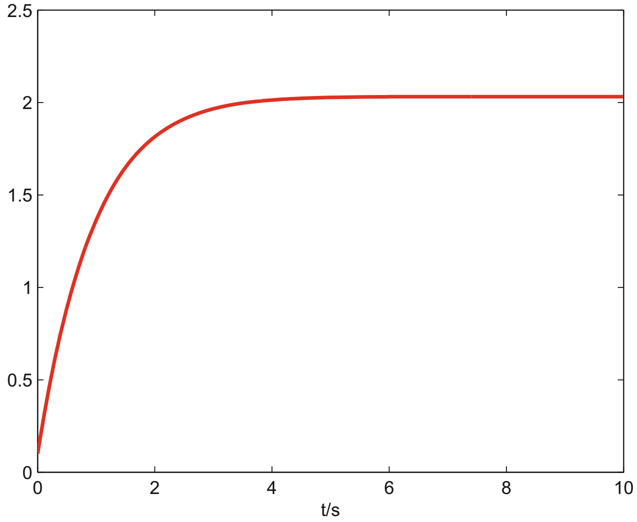

Fig. 9.4 The norm of $\hat{\theta}_{12}$ (no fault)

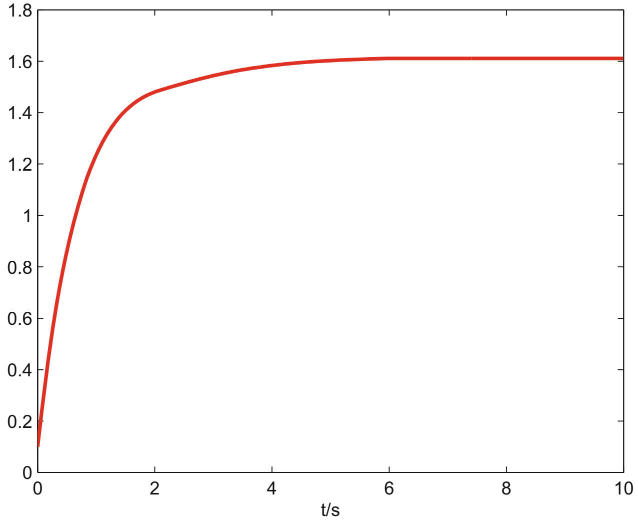

Fig. 9.5 The norm of $\hat{\theta}_{13}$ (no fault)

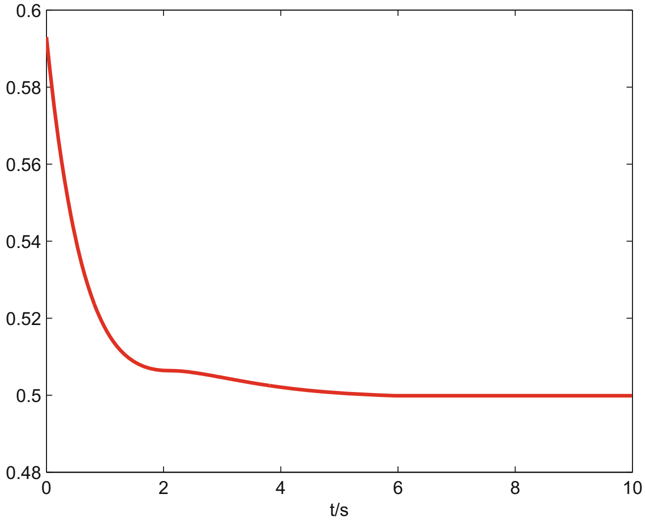

Fig. 9.6 Trajectory of $\hat{d}$ (no fault)

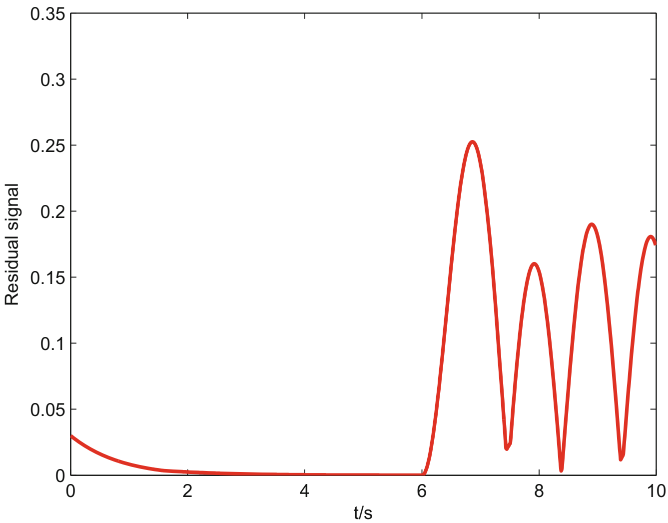

Fig. 9.7 The residual signal in faulty case

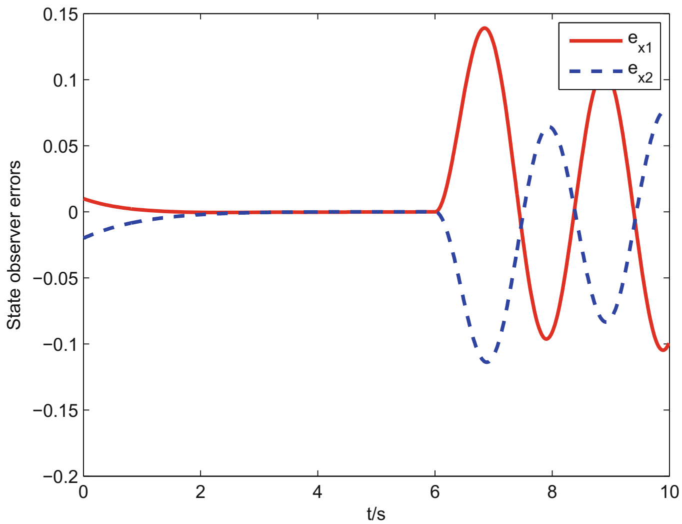

Fig. 9.8 The state observer errors in faulty case

9.5 Conclusions

In this chapter, the fault detection problem of uncertain time-delayed systems is studied. To overcome the shortcoming in existing works where the exact value of time delay needs to be known, a novel adaptive NNs-based fault detection observer is designed, which can estimate online the unknown time delay with system state. Simulation results show the effectiveness of the technique proposed in this chapter.

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Chapter 10
Conclusion and Future Research Directions

10.1 Conclusions

With the development technology, modern control systems, such as flight control systems, become more and more complex and involve an increasing number of actuators and sensors. These physical components may become faulty which may cause system performance deterioration, may lead to instability that can further produce catastrophic accidents. Hence, the study of fault diagnosis and fault tolerant control for dynamic systems has important theoretical and practical application significance. This book focuses on the issues of adaptive fault diagnosis and fault tolerant control for uncertain systems including linear and nonlinear systems. The main contributions presented in this book include

1. A general composite fault model with infinite number of faults is proposed, which can deals with both time-varying gain and bias faults, and FD and FTC for nonlinear systems with such faults occurred in one or multiple actuators are investigated, respectively.
2. The FD and FTC problem of uncertain strict-feedback systems is considered. Based on adaptive technology and other control techniques and/or methods, two modified backstepping FD and FTC schemes are proposed, where the computation complexity is significantly reduced.
3. By using the implicit function theorem and exploring the useful property of the basis function of the radial basis function neural network, FD and fault compensation for un-modeled faults are discussed.
4. The fault detection problem of uncertain time-delay systems is considered, and a novel adaptive fault detection observer is proposed, which can estimate the unknown time delay.
5. The time delay due to fault diagnosis and isolation (FDI) and its influence on the controlled systems performance are quantitatively analyzed.

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10.2 Future Research Directions

Fault diagnosis and fault tolerant control for uncertain systems is a hot research topic that has important both academic meaning as well as practical one. This book has presented several recent results and applications on this topic. There exist still a great number of crucial and fundamental issues to be exploited, which are open and challenging. To the best of our knowledge, some problems are listed as follows.

1. **Research on FD and FTC for nonlinear time-delay systems**
   Although Chap. 9 of this book has considered a class of time-delay systems and proposed some FD and FTC methods, the FD and FTC problem of time-delay systems, in particular, nonlinear time-delay systems is still challenging. How to design FD observer for the nonlinear time-delay systems is one of main difficulties, which deserves further research.
2. **Research on FD and FTC for networked multi-agent systems**
   Cooperative or distributed control of multi-agent systems has attracted extensive attention from the control community for the last decades. Although fruitful results are and obtained in literature, most of the existing works only focus on the healthy case, namely, there is no any faults in networked multi-agent systems. In fact, networked multi-agent systems are more complex than single-agent systems because they have a great number of physical components. Obviously, multi-agent systems are quite likely to be faulty. Hence, FD and FTC for networked multi-agent systems is necessary and important.
3. **Research on discrete-time systems**
   This book focuses on continuous-time systems. Discrete systems including certain and uncertain, linear and nonlinear systems are of interest in many practical systems, e.g., digital control systems, networked control systems, etc. Although fruitful results have been obtained for discrete systems, their stability analysis in the presence of faults would become much more difficult. The research on on FD and FTC for discrete-time systems is still interesting.
4. **Research on FD and FTC for switched nonlinear systems**
   Switched systems, in particular, switched nonlinear time-delay systems, are used to model many practical systems. Due to the existence of switching and a time delay, the stability analysis of switched time-delay systems has new challenges, especially when the systems are faulty. Although there are abundant results on FD and FTC for switched systems reported in literature, most of them only focus on actuator or sensor faults. The other faults, for example, the faults occurred in the switching single, are not investigated deeply.
5. **Research on FD and FTC using Filippov-framework**
   In control theory, system dynamics is usually described by the first order differential equation on system state and control input. Note that, it is under the assumption on the smoothness of the vector field that most of the existing results in analysis and control of dynamical systems are established, namely, the right hand side of the differential equation must be continuously differentiable.

However, when a fault occurs in system, the right hand side of the differential equation does not remain continuously differentiable, and becomes discontinuous. That is to say, the above assumption does not hold, and the systems controlled become discontinuous systems. Although there are many researchers begin to consider systematic design problem with the Filippov framework in recent years, they do not investigated the faults occurred systems. Therefore, it is still a challenging problem to establish an analysis approach with the Filippov framework for FTC systems.