

Chapter 8

Combining World Map Projections

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Abstract Cartographers have developed various techniques for deriving new projections from existing projections. The goal of these techniques is to substitute a disadvantageous trait of one of the source projections with the second source projection. This chapter discusses creating new projections by the juxtaposition and blending of two existing projections. It also presents a new approach for selectively combining projection characteristics. The emphasis in this chapter is on projections for world maps, as the described techniques are most useful for this scale.

8.1 Introduction

There are various techniques for creating new map projections from existing projections. These techniques are applied to substitute a disadvantageous trait of one of the source projections with the second source projection. This chapter discusses juxtaposition and blending, the two most commonly used techniques for combining two existing projections. A recent technique for combining selected projection characteristics is also presented. The techniques are useful for creating new projections for world maps.

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8.2 Compositing Projections Along Parallels

The first technique creates a new projection by the juxtaposition of two existing projections. This fusion technique is almost exclusively applied to create pseudocylindrical projections by aligning two pseudocylindrical source projections along two straight parallels. It is generally limited to projections with regularly spaced meridians, otherwise the longitudes would not align along the fusion parallel. Compositing two cylindrical projections with this technique is possible, but used rarely. While compositing azimuthal projections is a theoretical possibility that serves little purpose, the technique cannot generally be applied to conic projections (Fenna 2007).

The Goode homolosine is the best-known example of a combined projection (Goode 1925). It combines the Sanson sinusoidal and the Mollweide projection (Fig. 8.1). The projection is commonly used in interrupted form and is equal-area. The Goode homolosine, like most other combined projections, shows a discontinuity in the graticule where the sinusoidal and Mollweide projections join (Gede 2011). This crease, which may be visually disturbing, is a typical trait of combined projections.

Goode chose to combine the sinusoidal and the Mollweide projections along the parallel at $40^{\circ} 44' 12''\text{N}$ and S, since this is the latitude of equal length for both projections. The sinusoidal projection retains the scale along all parallels, and can therefore be combined with a variety of pseudocylindrical projections along the parallel of the second projection where true scale is retained (Fenna 2007, p. 174).

When none of the source projections is the sinusoidal, or when an arbitrary parallel is chosen to combine the two source projections, one of the two projections must be scaled. The scaling brings the parallels along which projections are fused to the same length. For example, Érdi-Krausz connected a transformed sinusoidal projection with the Mollweide projection (Érdi-Krausz 1968). To bring the fusion parallels to the same length, he scaled one of the source projections (Gede 2011).

McBryde (1978) has created combined projections called P3, S2, S3, and Q3. The S3, perhaps the best known of this group, is a fusion along $55^{\circ} 51'\text{N}$ and S of the McBryde-Thomas flat-polar and the sinusoidal projection.

A composition of two equal-area projections can still be equal-area as in the Goode homolosine. This can be achieved when the x coordinate is multiplied by a

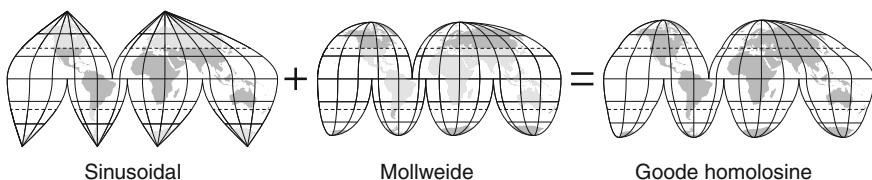


Fig. 8.1 The Goode homolosine combines the sinusoidal and the Mollweide projections. It is commonly used in interrupted form

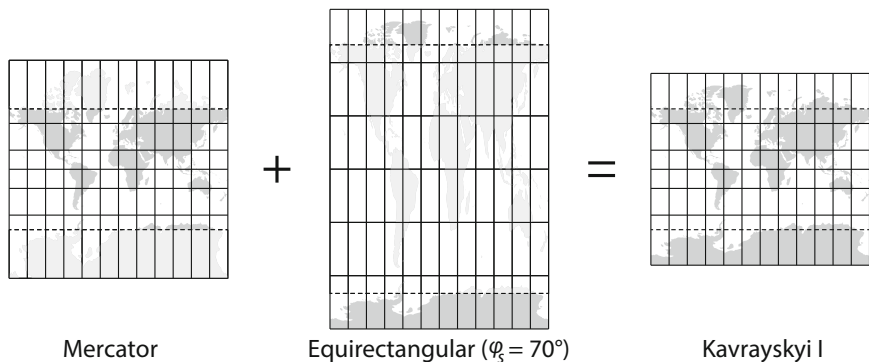


Fig. 8.2 The cylindrical Kavrayskiy I combines the Mercator and the equirectangular projections

scale factor that brings the length of the fusion parallels to the same length, and the y coordinate is divided by this same factor. This results in stretching and compressing one of the two source projections, but this transformation does not alter its equal-area property.

A rare example of a composite of two cylindrical projections is the Kavrayskiy I projection (Fig. 8.2). It combines the Mercator projection with the equirectangular projection along 70°N and S , thereby reducing the gross areal distortion of the Mercator projection. The equation for the x coordinate is $x = \lambda$ for the unary sphere. The vertical coordinate is computed with the equations below

$$y = \begin{cases} \frac{\varphi - \varphi_s}{\cos \varphi_s} + d_y & \text{for } \varphi > \varphi_s \\ \frac{\varphi + \varphi_s}{\cos \varphi_s} - d_y & \text{for } \varphi < -\varphi_s \\ \ln \tan(45^\circ + \frac{\varphi}{2}) & \text{otherwise} \end{cases}$$

where: $d_y = \ln \tan(45^\circ + \frac{\varphi_s}{2})$ and $\varphi_s = 70^\circ$.

8.3 Projection Blending

An alternative approach to combining two projections along parallels is computing the mean of two source projections. The spherical coordinates, longitude and latitude, are first converted with both projections, resulting in two pairs of Cartesian coordinates, x_1, y_1 and x_2, y_2 . The blended coordinates are then computed with an arithmetic mean: $x = w \cdot x_1 + (1-w) \cdot x_2$ and $y = w \cdot y_1 + (1-w) \cdot y_2$, with the weight $0 \leq w \leq 1$.

Any pair of projections can theoretically be blended, but a self-intersecting graticule may result if the projection geometries differ considerably. For example,

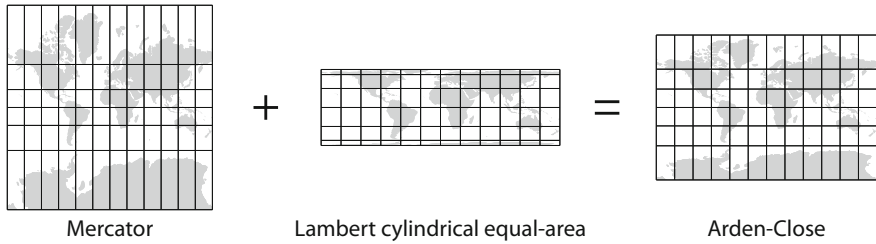


Fig. 8.3 The cylindrical Arden-Close is a blending of the Mercator and the Lambert cylindrical equal-area projection. The Mercator and the Arden-Close projections are truncated at 85°N and S

blending an azimuthal and a pseudocylindrical projection would result in a curvy and folded graticule (Jenny 2012).

The Arden-Close projection (Fig. 8.3) is a cylindrical projection (straight parallels and straight meridians), created by blending the Mercator and the Lambert cylindrical equal-area projections (Arden-Close 1947). It is a rare example of a cylindrical projection created by blending two cylindrical source projections. This combination compensates the gross enlargement of polar areas by the Mercator projection with the vertical compression of the Lambert cylindrical projection. However, since the Mercator projection has an infinite vertical extension, the blended Arden-Close projection also places the poles at infinite distance from the equator, which requires the graticule to be truncated along parallels close to the poles. The projection is not frequently used for this reason.

A series of more useful pseudocylindrical projections have been created in the past by blending two source projections with an arithmetic mean. The majority of these blended projections combine a cylindrical projection, such as the equirectangular projection, and a pseudocylindrical projection with meridians converging at pole points, such as the sinusoidal. For example, the Eckert V projection combines the sinusoidal and the plate carrée projections (Eckert 1906). Winkel generalized this approach by replacing the plate carrée with the equirectangular projection using any standard parallel (Winkel 1921). For his Winkel I projection (Fig. 8.4), he proposed standard parallels at $50^{\circ} 28'\text{N}$ and S such that the total area of the map is at correct scale (Snyder 1993, p. 195). Putniņš proposed two projections called P1' and P3', which are both arithmetic means of the plate carrée and projections created

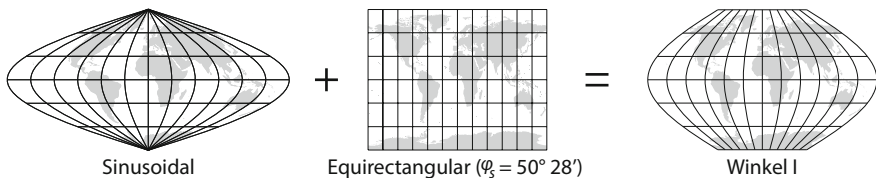


Fig. 8.4 The Winkel I is a blending of the sinusoidal and the equirectangular with standard parallels at $50^{\circ} 28'\text{N}$ and S

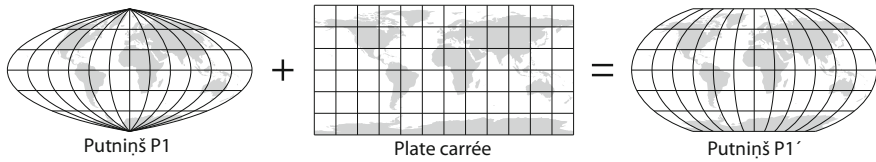


Fig. 8.5 The Putniņš P1' is a blending of the plate carrée and the Putniņš P1 projections

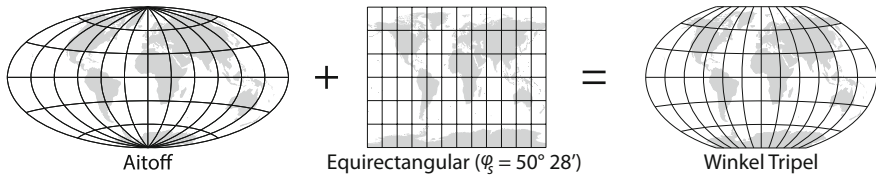


Fig. 8.6 The Winkel Tripel is a blending of the Aitoff and the equirectangular with standard parallels at 50° 28'N and S

by the same author (Putniņš 1934). Figure 8.5 shows the Putniņš P1' projection (name with apostrophe) with a polar line as a result of the arithmetic mean between the Putniņš P1 (name without apostrophe) that has pointed poles and the cylindrical plate carrée projection.

The Winkel Tripel projection (Fig. 8.6) combines the Aitoff and the cylindrical equirectangular projection with standard parallels at 50° 28'N and S (Winkel 1921). Because the Aitoff projection does not have straight parallels, the Winkel Tripel also has bent parallels and is not a pseudocylindrical projection.

Unlike the compositing technique discussed before, blending two source projections does not create an equal-area projection, even if the two source projections are equal-area. If the equal-area property is to be retained, only the *x* or the *y* coordinate can be a blend, and the other coordinate must be mathematically derived from the equal area condition (Tobler 1973). An example of a well known equal-area blend is the Boggs eumorphic projection (Fig. 8.7), which is the arithmetic mean of the sinusoidal and the Mollweide projections only for the *y* coordinate (Boggs 1929). The equation for the *x* coordinate is adjusted such that the resulting projection does not distort area. Foucaut (1862), Hammer (1900), and

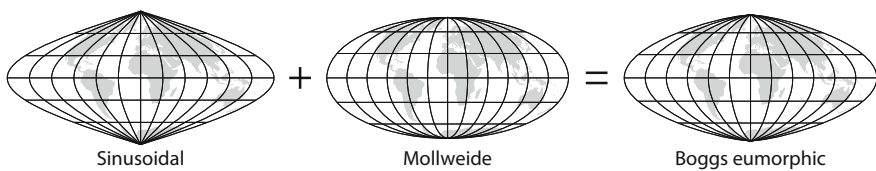


Fig. 8.7 The equal-area Boggs eumorphic projection blends the *y* coordinate of the sinusoidal and Mollweide projections

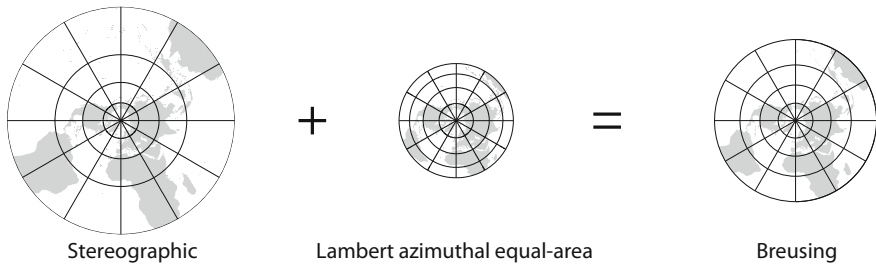


Fig. 8.8 The azimuthal Breusing projection uses a geometric means of the radii of the conformal stereographic and the equal-area azimuthal projections

Tobler (1973) have averaged the cylindrical equal-area and the sinusoidal projections to create other equal-area projections with similar techniques (after Snyder 1977).

The arithmetic blending technique, described above, uses constant weights w_1 and w_2 for the two projections. An extension to this approach is to vary weights with the location. A simple approach is to vary weights with the latitude, as described by Anderson and Tobler (2011): $w_1 = w(\varphi) \geq 0$, and $w_2 = 1 - w_1$, with one of the weights decreasing from one at the equator to zero at the poles.

Computing the geometric mean of two projections is an alternative to the arithmetic means. Tobler (1973) explores this method for creating pseudocylindrical projections. The projection by Breusing is a rare example of an azimuthal projection that blends two source projections involving a geometric mean (Snyder 1993, pp. 129–130). It combines the azimuthal equal-area projection and the conformal stereographic azimuthal projection (Fig. 8.8). The radius from the projection center is computed with a geometric mean of the radii of the two source projections.

8.4 Selective Combinations

When combining two source projections to create a new projection, the goal is to replace an unfavorable trait of one projection with the characteristics of another projection. Jenny and Patterson (2013) introduce an alternative approach to the blending and compositing techniques discussed above that aims at selectively combining the traits of two projections. This software-based technique first extracts the geometric characteristics of the two source projections and builds tables with according values, then lets the user selectively mix these tables, and finally converts the mixed tables back to a new projection. The tables encode (1) the horizontal length of parallels, (2) the vertical distance of parallels from the equator, (3) the distribution of meridians, and (4) the bending of parallels. The tables contain values for every 5° or 10° of increasing longitude and latitude. This selective combination

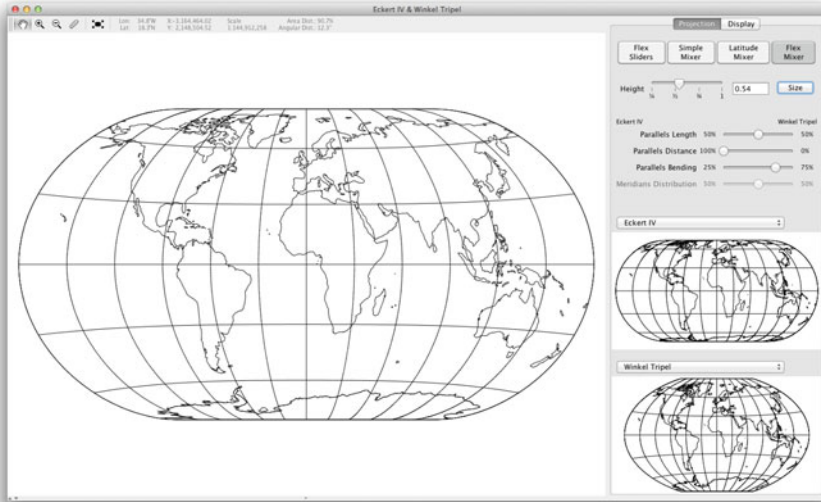


Fig. 8.9 Combination of selected projection traits of the Eckert IV and the Winkel Tripel projections with Flex Projector

technique is available in Flex Projector, a specialized software application for designing new world map projections (Jenny and Patterson 2007; Jenny et al. 2008). The user can adjust the geometry of a single projection, or combine two source projections using the methods described in this chapter.

The user can adjust weights using four sliders (Fig. 8.9, top right). For example, in Fig. 8.9, the Eckert IV and the Winkel Tripel (itself a blend of the Aitoff and the equirectangular) are combined. The length of the resulting parallels is a 50/50 blend of the two source projections. The parallels are vertically distributed as in the Eckert IV projection. Parallels are bent, but not as severely as in the Winkel Tripel projection, due to a 25/75 blend. With a height-to-width aspect set to 0.54, a compact graticule results with distinctly curved meridians near pole lines. The reader is referred to Jenny and Patterson (2013) and Jenny et al. (2010) for additional examples and more details on the algorithmic procedure.

8.5 Conclusion

The techniques for combining two source projections to create a new projection, outlined in this chapter, allow for the creation of a large variety of projections. The mentioned techniques can also be extended. For example, the Geocart software by Maphematics can blend projection parameters, such as the latitude of standard parallels, between two source projections (Strebe 2010). Alternatively, more than

two projections can be combined to form a new one. The extreme case would be an infinite number of differently parameterized projections, which is the concept behind polyconic (Fenna 2007) and polycylindric projections (Tobler 1986).

The designer of new map projections is not limited to the techniques discussed in this chapter. There are alternative methods for creating a new projection from scratch, deriving it from existing ones, or adjusting projection parameters to create a new one (Canters 2002; Snyder 1993). Some of these techniques are used in the adaptive composite projections for Web maps, a new field of map projection research (Jenny 2012). The goal of this research is to develop an alternative to the Web Mercator projection for small-scale Web maps, where maps automatically use an optimum projection depending on the map scale, the map's height-to-width ratio, and the central latitude of the displayed area. Multiple projections are combined with seamless transitions, using projection blending for compromise projections, or Wagner's method (Wagner 1949) to transform projections into each other by adjusting transformation parameters.

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