Chapter 12 Famous People and Map Projections

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Abstract Map projections have been developed in parallel with the development of map production and cartography in general. Development of sciences, technical achievements and needs of everyday life have gradually increased demands for production of various topographic and thematic maps in various scales and for various purposes, which required continuous improvement of map projections and mathematical basis of maps. Beginnings of map projections date as far as two thousand years ago, when ancient Greek scientists applied mathematical principles to projecting Earth and the starry sky and started applying the graticule. Hundreds of map projections have been invented since the antique. Many people have been interested in the theory of map projections and have written about them. Since there are so many of them, we decided to make a narrower selection of abut 40 people. Those people are presented in chronological rather than alphabetical order in this chapter. Their most important contributions to map projections are described and illustrated.

Beginnings of map projections date as far as two thousand years ago, when ancient Greek scientists applied mathematical principles to projecting Earth and the starry sky and started applying the graticule. Hundreds of map projections have been invented since the antique. Development of sciences, technical achievements and needs of everyday life have gradually increased demands for production of various topographic and thematic maps in various scales and for various purposes, which required continuous improvement of map projections and mathematical basis of maps.

Many people dealt with theory of map projections and wrote about them (Snyder and Steward [1988\)](#page-67-0). Since there are too many of them to be dealt in details in one chapter, we decided to make a narrower selection. This selection is shown in Table [12.1](#page-1-0).

Small maps of the world in different projections have been prepared with the publicly-available resampled Shuttle Radar Topographic Mission (SRTM) digital elevation data to 250 m resolutions for the entire globe [\(http://srtm.csi.cgiar.org/](http://srtm.csi.cgiar.org/))

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for use in the sections below. We used either geoinformation software Grass GIS or map projection software G.Projector to transform an equirectangular map image into any of target map projections. Resulting images were further processed with vector graphics editor Inkscape and raster graphics editor GIMP, where the graticules have been drawn and the oceans lightened to enhance image legibility and contrast.

Hereafter follow basic information about persons from Table 12.1 in cronological order.

Table 12.1 Alphabetical list of persons presented in more detail in this chapter, which are extremely important for development of map projections

| Airy, George Biddell-British astronomer, mathematician and cartographer |
|---|
| Aitoff (Aitov, Aitow), David Aleksandrovich—Russian cartographer |
| Albers, Heinrich Christian-German cartographer |
| August, Friedrich Wilhelm Oscar-German professor of mathematics |
| Boggs, Samuel Whittemore—American geographer |
| Bonne, Rigobert-French engineer, mathematician and cartographer |
| Briesemeister, William A.-American cartographer |
| Bugayevskiy, Lev Moiseyevich—Russian cartographer professor, colonel |
| Cassini, César François Cassini de Thury—French astronomer and cartographer |
| Clarke, Alexander Ross-British geodesist, mathematician and officer |
| Eckert-Greifendorf, Max-German geographer and cartographer |
| Eisenlohr, Friedrich Eisenlohr-German mathematician |
| Euler, Leonhard—Swiss mathematician, physicist, astronomer, logician and engineer |
| Gall, James-Scottish clergyman, cartographer and astronomer |
| Gauss, Carl Friedrich—German mathematician, astronomer and geodesist |
| Gilbert, Edward Nelson-American mathematician and coding theorist |
| Goode, John Paul-American geographer and cartographer |
| Hammer, Ernst Hermann Heinrich von-German geodesist and cartographer |
| Hipparchus—Old Greek astronomer and mathematician |
| Jordan, Wilhelm-German geodesist and mathematician |
| Kavrayskiy, Vladimir Vladimirovich-Russian cartographer, professor, engineer-rear-admiral |
| Khristov (Hristov, Hristow), Vladimir Kirilov—Bulgarian geodesist, astronomer and cartographer |
| Krüger, Johann Heinrich Louis-German mathematician and geodesist |
| Lagrange, Joseph Louis—French mathematician and astronomer |
| Lambert, Johann Heinrich-German physicist, mathematician and astronomer and cartographer originating from France |
| Lee, Laurence Patrick—New Zealand surveyor and cartographer |
| McBryde, Felix Webster-American geographer, cartographic consultant and educator |
| Mercator, Gerhard Kremer-Flemish geographer, cartographer |
| Miller, Osborn Maitland-American cartographer and geographer |
| Mollweide, Karl Brandon-German mathematician and astronomer |
| Nicolosi, Giambattista—Italian priest and cartographer |
| Postel, Guillaume—French linguist, geographer, astronomer, diplomat, professor of mathematics |
| (continued) |

Table 12.1 (continued)

Thales (Greek Θαλής)

(Miletus, ca. 625. B.C.–ca. 547. B.C.) Old Greek philosopher and mathematician

Thales of Miletus was the first Greek philosopher, scientist and mathematician, one of seven Wise Greek Men. Unfortunately, no writings by Thales were preserved, so it is hard to determine his ideas or to be completely certain about his mathematical conclusions. In many books on the history of mathematics, Thales is credited with these theorems: a circle is bisected by any diameter, the base angles of an isosceles triangle are equal, the angles between two intersecting straight lines are equal (ceiling angles are considered), two triangles are congruent if they have two angles and one side equal, an angle in a semicircle is a right angle. Last mentioned theorem is today called Thales theorem. He used the properties of similarity and in that way he measured the height of pyramids and the distance of a boat on the open sea. It is reported that Thales predicted an eclipse of the Sun in 585 B.C. Prediction of Moon eclipse was well known at this time, but it was hard to say when the eclipse of Sun would occur, since this phenomenon could not have been seen from all the parts on Earth. Still, the most important thing mathematicians attribute to him is the fact that Thales was the first to give logical foundations for proving the theorems. In other words, he was first to emphasize that it is not sufficient just to observe the phenomena, but they must be proven.

He believed the Earth was a flat disc floating on water, i.e. on an infinite ocean and that all things come to be from water. Despite those, today unacceptable theses, Thales' greatness is that he was the first recorded person who tried to explain his attitudes by rational rather than by supernatural means, as many did before him.

It is believed that Thales made the first map in a projection 600 B.C. It was a map of the heavenly sphere in gnomonic projection.

Hipparchus (Greek Ίππαρχος, Hipparkos) (Nicaea, now Iznik, ca. 190. B.C.–Rhodes, ca. 120. B.C.) Old Greek astronomer and mathematician

Fig. 12.1 Celestial map of the northern sky by Albrecht Dürer, 1515. This, together with its southern sky companion, were the first printed star charts. National Gallery of Victoria, Melbourne, Felton Bequest, 1956

He is considered the greatest astronomical observer, and by some, the greatest overall astronomer of antiquity. He had been conducting very precise measurement of stars' positions and their apparent sizes, which he defined in a scale from 1 to 6. He measured the length of tropical year and synodic month; he discovered precession of equinoces and non-uniformities in lunar motions. He created the first big catalogue with 850 stars, was the first to determine positions on Earth with usage of geographical latitudes and longitudes, and he founded trigonometry. Ptolemy included Hipparchus' results in his works. Hipparchus' synthesis of astronomy excelled his work. Although he had written at least 14 books, only his comments on Arat's popular astronomic epic remained preserved from later scribes (Kovačec [2002\)](#page-67-0). Around 150 B.C., he used the stereographic and the orthographic projection, which belong to the oldest projections, to create a map of celestial sphere (Fig. [12.1\)](#page-3-0).

Claudius Ptolemy (Greek: Κλαύδιος Πτολεμαΐος, Klaúdios Ptolemaîos) (Alexandria, Egypt ca. 100 CE–Alexandria, Egypt, ca. 170 CE) Egyptian astronomer, mathematician, and geographer

He was a Greek-Roman citizen and lived in Alexandria, Egypt. In several fields his writings represent the culminating achievement of Greco-Roman science, particularly his geocentric (Earth-centred) model of the universe now known as the Ptolemaic system. Virtually nothing is known about Ptolemy's life except what can be inferred from his writings.

His first major astronomical work, the *Almagest*, was completed about 150 CE and contains reports of astronomical observations that Ptolemy had made over the preceding quarter of a century. The size and content of his subsequent literary production suggests that he lived until about 170 CE.

Ptolemy has a prominent place in the history of mathematics primarily because of the mathematical methods he applied to astronomical problems.

Ptolemy's fame as a geographer is hardly less than his fame as an astronomer. Geōgraphikē hyphēgēsis (Guide to Geography) provided all the information and techniques required to draw maps of the portion of the world known by Ptolemy's contemporaries. By his own admission, Ptolemy did not attempt to collect and sift all the geographical data on which his maps were based. Instead, he based them on the maps and writings of Marinus of Tyre (c. 100 CE), only selectively introducing more current information, chiefly concerning the Asian and African coasts of the Indian Ocean.

Ptolemy's most important geographical innovation was to record longitudes and latitudes in degrees for roughly 8000 locations on his world map, making it possible

Fig. 12.2 A printed map from the 15th century depicting Ptolemy's description of the *Ecumene*, (1482, Johannes Schnitzer, engraver)

to make an exact duplicate of his map. Hence, we possess a clear and detailed image of the inhabited world as it was known to a resident of the Roman Empire at its height—a world that extended from the Shetland Islands in the north to the sources of the Nile in the south, from the Canary Islands in the west to China and Southeast Asia in the east. Ptolemy's map is seriously distorted in size and orientation compared to modern maps, a reflection of the incomplete and inaccurate descriptions of road systems and trade routes at his disposal.

Ptolemy also devised two ways of drawing a grid of lines on a flat map to represent the circles of latitude and longitude on the globe (Snyder [1993](#page-67-0)). His grid gives a visual impression of Earth's spherical surface and also, to a limited extent, preserves the proportionality of distances. The more sophisticated of these map projections, using circular arcs to represent both parallels and meridians, anticipated later area-preserving projections. Ptolemy's geographical work was almost unknown in Europe until about 1300, when Byzantine scholars began producing many manuscript copies, several of them illustrated with expert reconstructions of Ptolemy's maps. The Italian Jacopo d'Angelo translated the work into Latin in 1406. The numerous Latin manuscripts and early print editions of Ptolemy's Guide to Geography, most of them accompanied by maps, attest to the profound impression this work made upon its rediscovery by Renaissance humanists (Fig. 12.2).

Gillaume Postel

(Barenton, France, 1510–Paris, 1581) French linguist, geographer, astronomer, diplomat, professor of mathematics

French linguist, adept at Semitic languages (Arabic, Hebrew and Syriac), as well as Classical languages (Ancient Greek and Latin), and in 1538 in his work Linguarum Duodecim Characteribus Differentium Alphabetum Introductio he gave an introduction to alphabetic characters of twelve different languages. In 1544, he published De orbis terrae concordia in which he advocated a universalist world religion. He was extremely tolerant to other religions at the time when such a tolerance was not common. It is believed that he spent the years 1548–1551 travelling to Israel and Syria, to collect manuscripts. After this trip, he earned the title of Professor of Mathematics and Oriental Languages at the Collège de France in Paris. After several years, Postel resigned his professorship and travelled all over central Europe, including Austria and Italy.

He was considered an originator of the equidistant azimuthal projection. Although this projection was possibly developed by the Egyptians for star charts, Postel was the first one to use it in 1581 and it was named after him the Postel projection (Figs. 12.3 and [12.4\)](#page-7-0).

Gerardus Mercator

Fig. 12.4 Map of the world in normal aspect equal-area azimuthal projection (the

from the North Pole to 60°

(Flanders, Belgium, 1512–Duisburg, Germany, 1594) Flemish geographer, cartographer

Out of need for more accurate mapping of larger Earth territories on marine charts and related to distortions appearing during this process, map projections and practical cartography developed in the 16th century. Mercator was educated in 's-Hertogenbosch in the Netherlands. He studied mathematics and astronomy at the Belgian University of Leuven and in 1532 he obtained the title of master. He was educated for engraver and globe maker. He travelled a lot and began to be interested in geography. He returned to Leuven and started to learn and work with Gemma Frisius (astronomer and mathematician) and Gaspar Myrica (engraver and goldsmith). They worked together to construct globes, maps and astronomical instruments.

The conformal cylindrical projection is named Mercator projection after him. The normal aspect of the Mercator projection has special importance in navigation, because the rhumb lines are represented as straight lines in this projection. The Transverse Mercator projection is used in many countries for official cartography. The Universal Transverse Mercator (UTM) coordinate system based on the transverse Mercator projection is used in military (NATO). The Web Mercator projection is a slight variant of the Mercator projection that is widely used on internet. Despite its obvious scale variation at small scales, this projection is well-suited as an interactive world map that can be zoomed seamlessly to large-scale (local) maps,

Fig. 12.5 Map of the world in the Mercator projection. It should not be used (although it frequently is) for depicting general information or any area-related subjects. Because of the excessive distortion of area it presents a misleading view of the world (Monmonier [2004\)](#page-67-0)

where there is relatively little distortion due to the variant projection's near-conformality (Figs. 12.5 and [12.6](#page-9-0)).

Nicolas Sanson

(Abbeville, 1600–Paris, 1667) French cartographer and geographer

He was born in Abbeville, where as a young man he studied history. He moved to Paris and founded craft in Rue d'el Arbe, St. Germain. He was a Royal Geographer from 1630 to 1665. He lectured geography both to Luis XIII and Luis XIV. In period from 1618 to 1667, he made atlases and illustrated texts.

Fig. 12.6 Marine charts are usually made in normal aspect Mercator projection. Sheet Split-Kaštelanski zaljev 47, publisher: Hydrographic Institute of the Republic of Croatia, 2002, original scale 1:15,000

He produced about 300 maps in total, out of which two maps of North America are particularly important: Septentrionale (1650) and Le Canada ou Nouvelle France (1656).

He proposed a new sinusoidal pseudocylindrical projection for world maps. This projection is named after him the Sanson projection, and it is an equal-area sinusoidal pseudocylindrical projection in which all the parallels and the central meridian are mapped in real size (Baily [1886;](#page-66-0) Curie [1900](#page-66-0)).

In fact, the projection was developed in the 16th century. It was used by J. Cossin in 1570 and by J. Hondius in Mercator atlases of the early 17th century. It is often called the sinusoidal or Sanson-Flamsteed projection after later users. This is the oldest current pseudocylindrical projection (Fig. [12.7\)](#page-10-0).

Giambattista Nicolosi

(Paternò, 1610–Roma, 1670)

Italian priest and cartographer

He was a priest and cartographer for Pope Gregory XV's Sacred Congregation for the Propagation of the Faith, established by the pope to promote missionary work.

In 1652 the *Propagation of the Faith* commissioned him to produce an atlas. The result, eight years later, was the very rare Dell'Hercole which contained maps of the world and the continents, the latter in four sheets each. It is based on Sanson's 1650

Fig. 12.7 Map of the world in the Sanson or Flamsteed-Sanson or sinusoidal projection

Fig. 12.8 Map of the world in Nicolosi globular projection

map, but follows Ramusio and earlier Italian cartographers in that south is on top. The geographical information is Ptolemaic and somewhat conservative for the time in that few place names are provided. The second state of the map (1670), with additional geographical information was published posthumously by Joanne Baptista Nicolosi.

The Nicolosi globular projection was used on hemispheric maps, common in atlases between 1850 and 1925 (Snyder [1993\)](#page-67-0). This projection was first presented by al-Biruni about A.D. 1000, but reinvented by Giambattista Nicolosi in 1660 (Fig. 12.8).

Arrowsmith A (1794) A companion to a map of the world: London, G. Bigg. [Relates to his Map of the world on a globular projection, exhibiting particularly the nautical researches of Capn. James Cook, with all the recent discoveries to the present time: London, 1794. Discusses al-Bīrūnī (or Nicolosi) Globular projection.]

Leonhard Euler

(Basel, Switzerland, 1707–Saint Petersburg, Russia, 1783) Swiss mathematician, physicist, astronomer, logician and engineer

Euler made important and influential discoveries in many branches of mathematics like infinitesimal calculus and graph theory while also making pioneering contributions to several branches such as topology and analytic number theory. He also introduced much of the modern mathematical terminology and notation, particularly for mathematical analysis, such as the notion of a mathematical function. He is also known for his work in mechanics, fluid dynamics, optics, astronomy, cartography, and music theory.

Euler was one of the most eminent mathematicians of the 18th century, and is held to be one of the greatest in history. He is also widely considered to be the most prolific mathematician of all times. His collected works fill 60–80 quarto volumes, more than anybody else in the field. He spent most of his adult life in St. Petersburg, Russia, and in Berlin, then the capital of Prussia.

The oldest proof that one can not map a sphere into a plane without distortion originates from Euler. He published three papers on map projections in 1777. Among them is the first formal proof about impossibility to map a sphere into a plane without any distortions. In the original form his papers are written in Latin, then published again in 1955. In Switzerland within Euler's Opera omnia, translated in German and published in 1898. And in Russia in 1959.

In order to decrease the angular distortions in map projections, Euler suggested the use of an equal-area projection with orthogonal graticule, known today as Euler projection.

Cartography was another area in which Euler became involved during his stay in St. Petersburg. In 1753, he was appointed director of the St. Petersburg Academy's geography section, and his task was to help French astronomer and cartographer Joseph Nicolas De l'Isle in preparing a map of the whole Russian Empire titled

Fig. 12.9 General map of the Russian Empire (Mappa Generalis Totius Imperii Russici). David Rumsey Map Collection www.davidrumsey.com

Mappa Generalis Totius Imperii Russici (1:8,9 Mill., Fig. 12.9). In his autobiographical writings, Euler says his eyesight problems began in 1738 with overstrain due to his cartographic work. However, it is believed that blindness was consequence of poisoning because of a boil. Despite everything, he continued with his work with great passion and devoted himself to the creation of an atlas. Atlas of the Russian Empire (De L'Isle – Atlas Rvssicvs … Vastissimvm Imperivm Rvssicvm cum adiacentibvs Regionibvs), consisting of 20 maps, was published in 1745 and in spite of several shortcomings (small number of astronomically determined points, low degree of accuracy of the maps, etc.); it represents an important contribution to Russian cartography.

Euler was involved in cartography during his stay at the Royal Academy of Sciences in Berlin. He is the author of a school atlas first published in 1753 under the title Atlas geographicus omnes orbis terrarum regiones in XLI tabulis. The next edition of the atlas with a title and foreword in German, French and Latin, was published in 1760 and it contained 44 maps (Fig. [12.10](#page-13-0)).

Euler continued his cartographic work during his second stay in St. Petersburg. In 1777, the St. Petersburg Academy of Sciences published three important Euler's works on cartography: De repraesentatione superficiei sphaericae super plano, De projectione geographica superficiei sphaericae and De projectione geographica de Lisliana in mappa generali Imperii Russici usitata. These works were later translated into German and were published in vol. 93 of Ostwald's Classics of Exact Sciences (Ostwald's Klassiker der Exakten Wissenschaften) in Leipzig in 1898 under the title Drei Abhandlungen über Kartenprojection (Three discussions on cartographic projections) and later on also into Russian (Eyler 1959).

Fig. 12.10 Map of the world and map of the Europe from Euler's Atlas, edition from 1760 www.vintage-maps.com

As it is well known, P. Chebyshev (1821–1894) was the first mathematician who investigated the problem of best uniform approximation in depth, and which can be expressed in classical notation in this way:

Let f be continuous function, $a, b \in R, n \in N$. Find a polynomial of degree at most n so that

$$
\max_{x \in [a,b]} |f(x) - p(x)|
$$

will be minimal for all polynomials of degree at most n .

The most important property of the solution of this problem is the fact that it gives an estimation of the error of approximation for every point of the interval $[a, b]$. But, due to the difficulty of this problem, the first results were presented only in 19th century. Chebyshev himself gave a necessary condition for the solution, stating that there must be at least $n + 2$ points where error function $f - p$ reaches its maximum, but he did not explicitly mention that these deviation points reach the maximum values with an alternating sign. So, the alternation theorem that characterizes the solution was not proven by Chebyshev. Using Chebyshev's results, Kirchberger made the first attempt that was completed by Borel and Young. An algorithm to calculate the best approximation was firstly presented by Remez only in 1934. Here, the alternation property is crucial for defining the iterating procedure that generates the solution.

Still, some special cases had been discussed before Chebyshev, and first of them was Euler's cartographic problem he dealt with during his second stay in St. Petersburg. Namely, Euler analysed local and global accuracy of the De l'Isle conic projection in 1777. This paper was the last of three papers dealing with cartography. It was published in 1777 and it seems that it was made based on his former work at the cartographic department of the Academy of Sciences in St. Petersburg. Euler first proved the fact that a part of the sphere cannot be projected onto a plane while preserving scale in both dimensions. Then he considered the issue of the most convenient projection for a map of all of Russia. Considering several kinds of

projections, for example stereographic and polar projections, he determined that De l'Isle projection should be used because of the following important properties:

- 1. Parallels and meridians intersect in the projection perpendicularly.
- 2. It gives a good approximation locally.

A map can be used for estimating the distance between any two points.

The mathematical problem that Euler solved was approximating the function cosx by a linear function. He noted that the best approximation can be characterized by the fact that there must be three points in which error $a - bx - c \cos x$ reaches the maximum value alternating in sign—the alternation theorem in its simplest form. With these settings, he was able to determine parameters for, in that sense, the best map. This work defined the Euler projection that was used for a map of whole Russia until the beginning of the 20th century (Grattan-Guinness and Pulte [2007;](#page-67-0) Steffens [2007\)](#page-67-0).

Euler's projections are equivalent mappings in which the graticule is being mapped to an orthogonal graticule in the projection plane. Of all equivalent projections, they are closest by character of deformation to conformal projections, so many scientists worked with them, like L. Euler, D.A. Grave, N.A. Urmayev, G.A. Meshcheryakov, K. Frankich, J. Györffy and others.

If we use R to denote the radius of a sphere, then Euler's projections can be described by following differential equations:

$$
\left| \frac{dx}{d\phi} \cdot \frac{dy}{d\lambda} - \frac{dy}{d\phi} \cdot \frac{dx}{d\lambda} \right| = R^2 \cos \phi \quad \text{(equivalency)}
$$
\n
$$
\frac{dx}{d\phi} \cdot \frac{dx}{d\lambda} + \frac{dy}{d\phi} \cdot \frac{dy}{d\lambda} = 0 \quad \text{(orthogonality of the gratitude)}
$$

where φ and λ are geographical coordinates of a point on the sphere, and x and y are corresponding coordinates in the projection plane. Different types of Euler's projections can be obtained by solving this system of partial differential equations. It is not hard to show that equivalent normal aspect cylindrical, conical and azimuthal projections are also Euler's projections.

Although interesting from a theoretical viewpoint, Euler's projections also attracted scientists to study these kinds of projections for specific areas. So, for example, Meshcheryakov [\(1958](#page-67-0), [1959,](#page-67-0) [1963,](#page-67-0) [1964](#page-67-0), [1968](#page-67-0)), Urmayev ([1947\)](#page-67-0) deals with Euler's projection for a world map, Frankich [\(1982](#page-67-0)) for Canada, and Györffy [\(2006](#page-67-0)) for Europe.

Euler's publications on map projections:

Euler L (1777) De repraesentatione superficiei sphaericae super plano: St. Petersburg, Academia Scientiarum Imperialis Petropolitanae, Acta, part 1, pp 107–132 [Latin. Translated into German as one of Drei Abhandlungen über Kartenprojection, in Ostwald's Klassiker der Exakten Wissenschaften, no. 93: Leipzig, Wilhelm Engelmann, 1898, with editing by Albert Wangerin, pp 3–37]

- Euler L (1777) De projectione geographica superficiei sphaericae: St. Petersburg, Academia Scientiarum Imperialis Petropolitanae, Acta, part 1, pp 133– 142 [Latin. Translated into German as one of Drei Abhandlungen über Kartenprojection, in Ostwald's Klassiker der Exakten Wissenschaften, no. 93: Leipzig, Wilhelm Engelmann, 1898, with editing by Albert Wangerin, pp 38–52]
- Euler L (1777) De projectione geographica de Lisliana in mappa generali Imperii Russici usitata: St. Petersburg, Academia Scientiarum Imperialis Petropolitanae, Acta, part 1, pp 143–153 [Latin. Equidistant conic projection used for map of Russia. Translated into German as one of Drei Abhandlungen über Kartenprojection, in Ostwald's Klassiker der Exakten Wissenschaften, no. 93: Leipzig, Wilhelm Engelmann, 1898, with editing by Albert Wangerin, pp 53–64]
- Eyler L (1959) Izbrannye kartograficheskiye stat'i. Tri stat'i po matematicheskoy kartografii. Perevod s nemetskim Bagratuni GV, Geodezizdat, Moscow 80 p. [Russian. Selected cartographic contributions: Three contributions on mathematical cartography. Translation from German.]

César-François Cassini de Thury (also called Cassini III)

(Thury-sous-Clermont, Oise, 1714–Paris, 1784) French astronomer and cartographer

In 1744, César-François Cassini de Thury began the construction of a great topographical map of France, one of the landmarks in the history of cartography. Completed by his son Jean-Dominique and published by the Académie des Sciences from 1744 to 1793, its 180 plates are known as the Cassini map.

His chief works are: La méridienne de l'Observatoire Royal de Paris (1744), a correction of the Paris meridian; Description géométrique de la terre (1775); and Description géométrique de la France (1784), which was completed by his son.

The Cassini projection is a map projection which he described in 1745. It is the transverse aspect of the equirectangular projection. In this projection, the globe is first rotated so the central meridian becomes the "equator", and then the normal equirectangular projection is applied. In practice, the projection has always been applied to models of the earth as an ellipsoid, which greatly complicates the mathematical development but is suitable for surveying. Nevertheless the use of the Cassini projection has largely been superseded by the Transverse Mercator projection, at least with central mapping agencies (Figs. [12.11](#page-16-0) and [12.12\)](#page-17-0).

Fig. 12.11 Map of the world in Cassini projection

Rigobert Bonne

(Raucourt, 1727–Paris, 1795)

French engineer, mathematician and cartographer

Rigobert Bonne was a hydrographer at the Royal Court in Paris (*Hydrographe*) du Roi a Paris). His main interest was the production of marine charts. He also published several atlases. Of significance is Atlas Encyclopédique, produced in collaboration with his son and with Nicolas Demarest and Bory de St. Vincent. In 1752, Bonne proposed a pseudoconical equal area map projection in which all the parallels and the central meridian are projected free of all distortion, for the map of France (Snyder [1993](#page-67-0)). Although the projection was developed in rudimentary form by Claudius Ptolemy (about year 100) and further developed by Bernardus Sylvanus (1511), it was named after Rigobert Bonne (Fig. [12.13](#page-18-0)).

Johann Heinrich Lambert

(Mülhausen, 1728–Berlin, 1777)

German physicist, mathematician, astronomer and cartographer originating from France

Johann Heinrich Lambert encompassed algebra, spherical geometry and perspective with his mathematical studies. He was the first to prove (1768) that π is an irrational number and made the first systematic use of hyperbolic functions. His

Fig. 12.12 The Carte de France was published by four generations of the Cassini family from 1750 to 1815. It consists of 182 sheets at the same scale 1:86,400, allowing the sheets to be joined together. The map was the first national survey completed systematically, relying on the latest science of its time. David Rumsey Map Collection www.davidrumsey.com

work on theory of parallel lines (1766) is of particular importance. In his work Photometry (Photometrie, 1760), he clearly distinguished concepts of brightness and luminance and in that way he set the foundations of photometry. Besides, he researched refraction of light in the atmosphere, comet paths and related to this, he discovered new properties of conics. In his astronomical works, there is first mention of double stars (Bollmann and Koch [2001\)](#page-66-0).

Lambert presented in 1772 a (conformal conical) projection which was named after him Lambert's conformal conic projection (Fig. [12.14](#page-18-0)). This projection is today still in use for the requirements of airplane navigation and in some countries as official state projection. In Croatia, this projection is the official map projection for general topographic maps (Fig. [12.15\)](#page-19-0). The equivalent azimuthal projection was also named after Lambert. The Transverse Mercator projection of sphere is also called the Lambert-Gauss projection.

Fig. 12.13 Map of the world in the Bonne projection

Fig. 12.14 Transverse Lambert azimuthal equal-area projection

Fig. 12.15 Rijeka and surroundings on the section of the topographic map in Lambert conformal conic projection, Republic of the Croatia, sheet 44 Zagreb-Zadar 54, publisher: Ministry of Defence of the Republic of Croatia, 1995, original scale 1:500,000

Lambert's theorem is known in the adjustment calculus. According to Frischauf [\(1905](#page-67-0)), the beginning of the theory of projecting one surface onto another belongs to J.H. Lambert, who dealt with generally given problem of projecting a sphere and spheroids into the plane in his Anmerkungen und Zusätze zur Entwerfung der Landund Himmelscharten (Remarks and Additions to the Establishment of Land and Sky Maps) in the third part of his Beyträge zum Gebrauche der Mathematik und deren Anwendung (Contributions to the Usage of Mathematics and its Application, 1772).

Lambert JH (1772) Beyträge zum Gebrauche der Mathematik und deren Anwendung, Part III, section 6: Anmerkungen und Zusätze zur Entwerfung der Land- und Himmelscharten. Berlin [Translated into English and introduced by W.R. Tobler as Notes and comments on the composition of terrestrial and celestial maps: Ann Arbor, Univ.

Michigan, 1972, Mich. Geographical Publication no. 8, 125 p. Also reprinted in German, 1894, Ostwald's Klassiker der Exakten Wissenschaften, no. 54: Leipzig, Wilhelm Engelmann, with editing by Albert Wangerin. Presents Conformal Conic, Equal-Area Conic, including "Isospheric Stenoteric," Transverse Mercator, Cylindrical Equal-Area, Transverse Cylindrical Equal-Area, "Lagrange," Azimuthal Equal-Area projections. See also review of Tobler translation by Wray Th, 1975: Canadian Cartographer, v. 12, no. 2, pp 231–233]

Joseph Louis Lagrange

(Torino, 1736–Paris, 1813) French mathematician and astronomer

He made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics.

In 1766, on the recommendation of Euler and d'Alembert, Lagrange succeeded Euler as the director of mathematics at the Prussian Academy of Sciences in Berlin, Prussia, where he stayed for over twenty years, producing volumes of work and winning several prizes of the French Academy of Sciences. Lagrange's treatise on analytical mechanics (Mécanique Analytique, 4. ed., 2 vols. Paris: Gauthier-Villars et fils, 1888–89), written in Berlin and first published in 1788, offered the most comprehensive treatment of classical mechanics since Newton and formed a basis for the development of mathematical physics in the nineteenth century.

In 1787, at age 51, he moved from Berlin to Paris and became a member of the French Academy. He remained in France until the end of his life. He was significantly involved in the decimalisation in Revolutionary France, became the first professor of analysis at the *École Polytechnique* upon its opening in 1794, founding member of the Bureau des Longitudes and Senator in 1799 (Bollmann and Koch [2001](#page-66-0)).

The *Lagrange projection* is a conformal projection of the Earth on a circle, except that angles are halved at the poles instead of being faithful there. This projection is named after Lagrange, who generalized Lambert's concept of pre-senting the world conformally in a circle (Fig. [12.16\)](#page-21-0).

Lagrange JL de (1779) Sur la construction des cartes géographiques: Nouveaux mémoires de l'Académie Royale des Sciences et Belles-lettres de Berlin, pp 161–210 [Also in Oeuvres de Lagrange, 1869:

Fig. 12.16 Lagrange projection of a sphere in a circle

Gauthier-Villars, Paris v. 4, pp 635–692. Also in German, 1894, as Ueber die Construction geographischer Karten: Ostwald's Klassiker der Exakten Wissenschaften, no. 55: Leipzig, Wilhelm Engelmann, pp 1–56, with editing by Albert Wangerin, pp 82– 97. Analyzes conformal projections with circular arcs for meridians and parallels, including "Lagrange projection" developed by Lambert (1772)]

Heinrich Christian Albers

(1773–1833)

German cartographer

Son of a merchant, native and lifelong resident of Lüneburg, Germany. He derived the formulas for the projection of the sphere using two standard parallels. He published this form of conical figure in a paper in the journal Monthly Zach's correspondence (Zachs Zeitschrift Monatliche Correspondenz) and has since been considered the inventor of the projection called Albers Equal-Area Conic Projection.

The Albers projection was used for a German map of Europe in 1817, but it was promoted for maps of the United States in the early part of 20th century by

Fig. 12.17 Map of the Europe in Albers projection (standard parallels: 40° and $66^{\circ}15'$)

Oscar S. Adams of the Coast and Geodetic Survey as an equal-area representation that is "as good as any other and in many respects superior to all others". It is also used and recommended for equal-area maps of regions that are predominantly eastwest in extent (Fig. 12.17).

Karl Brandan Mollweide

(Wolfenbüttel, 1774–Leipzig, 1825) German mathematician and astronomer

Mollweide was an observer at the Observatory of the Leipzig University until 1816. In 1812, he obtained the title of a full professor of astronomy, and from 1814, a full professor of mathematics. From 1820 to 1823, he was the Dean of the Faculty of Philosophy. He discovered spherical trigonometric formulae, which were named

Fig. 12.18 Map of the world in the Mollweide projection

Mollweide's formulae after him (Lapaine [2011](#page-67-0)). He discovered and published the pseudocylindrical equal-area projection (1805) that was, in his honor, named the Mollweide projection (Figs. 12.18 and 12.19).

Mollweide KB (1805) Ueber die vom Prof. Schmidt in Giessen in der zweyten Abtheilung seines Handbuchs der Naturlehre S. 595 angegeben Projection der Halbkugelfläche, Zach's Monatliche Correspondenz zur Beförderung der Erd- und Himmels-Kunde, vol. 12, Aug., pp 152–163

Carl Friedrich Gauss

(Braunschweig, 1777–Göttingen, 1855) German mathematician, astronomer and geodesist

A versatile mathematical genius and one of the greatest mathematicians of all time (princeps mathematicorum). He showed his great mathematical talent already in childhood, and he achieved first scientific results as a mathematics student in Göttingen. Relating to theory of circle division, he solved (1796) the problem of construction of regular polygons with a ruler and a compass. He was promoted in 1799 based on his doctoral thesis in which he gave a proof of extremely important, so called fundamental theorem of algebra. In his publication *Investigations in* Arithmetic (Disquisitiones arithmeicae, 1801) he set the foundations for the modern theory of numbers. His General Investigations of Curved Surfaces (Disquisitiones generales circa superficies curves, 1828) represent a new stage in the development of differential geometry and foundation of its progress until the present day. In this work he introduces systematic usage of parametrical representations of surfaces, two basic square forms, spherical projection, and based on this, the concept of curvature in the point of surface. The basic theorem about invariability of curvature of surface during its isometric projection was proved (Theorema egregium).

His contribution to theory of errors during measurement is also very important, and it was represented as the theory of least squares in the work Theory of the Combination of Observations Least Subject to Errors (Theoria combinationis observantium erroribus minimis obnoxiae, I–III, 1821–26), according to which the most adequate value of measured scale is the one in respect of which sum of errors' squares is minimal. His researches in the field of basic geometry are of particular importance, although he did not publish anything about it. He had been managing the observatory in Göttingen for a long time, and he had been calculating mathematical tables for the needs of astronomy for ten years, which were afterwards in use for decades.

Many things are named after him, like Gaussian curve, Gaussian elimination method during the solving of system of linear equations, Gaussian sum mark, Gaussian condition for tetragon with diagonals, Gauss-Krüger projection, etc. Between years 1821 and 1825, during the calculations of Hannover's triangulation for projection of ellipsoid into the plane, Gauss used a projection procedure which is today called the Gauss-Krüger projection. Professor Dr. Louis Krüger published a book about that projection in 1912, and in 1919 a collection of formulae for

Fig. 12.20 Section of the topographic map in Gauss-Krüger projection, sheet Varaždin, 271-2-3, publisher: State Geodetic Administration of the Republic of Croatia, 1997, original scale 1:25,000

practical usage. Since then, this projection had been called Gauss-Krüger. During the 20th century, this projection was the official map projection in many countries. It is also known as transverse Mercator projection (Fig. 12.20).

- Gauss CF (1816–1827) Conforme Abbildung des Sphäroids in der Ebene: in C. F. Gauss, Werke, Königlich Gesellschaft der Wissenschaften zu Göttingen, Abhandlungen, 1903, v. 9, pp 142–194
- Gauss CF (1825) Allgemeine Auflösung der Aufgabe: Die Theile einer gegebnen Fläche auf einer andern gegebnen Fläche so abzubilden, daß die Abbildung dem Abgebildeten in den kleinsten Theilen ähnlich wird: Preisarbeit der Kopenhagener Akademie 1822, Schumachers Astronomische Abhandlungen, Altona, no. 3, pp 5–30 [Reprinted, 1894, Ostwald's Klassiker der Exakten Wissenschaften, no. 55: Leipzig, Wilhelm Engelmann, pp 57–81, with editing by Albert

Wangerin, pp 97–101. Also in Herausgegeben von der Gesellschaft der Wissenschaften zu Göttingen in Kommission bei Julius Springer in Berlin, 1929, v. 12, pp 1–9]

George Biddell Airy

(Alnwick, 1801–Greenwich, 1892) British astronomer, mathematician and cartographer

He graduated at Trinity College, Cambridge, in 1823. Beginning in 1826, he became a professor of astronomy and mathematics at Cambridge, and in 1828 he became the director of the Cambridge observatory. His main interest was astrometry and optics, especially wave optics. In 1861, Airy suggested the arithmetic mean from the deformation on main directions, for the comparison of two projections to the middle square deformations on the whole area being projected. This criterion is named Airy's criterion after him. In the same year, he presented an azimuthal projection which is named Airy's projection after him (Fig. [12.21](#page-27-0)).

Airy GB (1861) Explanation of a projection by balance of errors for maps applying to a very large extent of the earth's surface and comparison of this projection with other projections, Philosophical Magazine and Journal of Science, 22, pp 409–421

James Gall

(1808–1895)

Scottish clergyman, cartographer and astronomer

James Gall was a Scottish clergyman, but his contribution to astronomy and cartography is also very important. He was a son of a famous publisher, raised in Edinburgh. He joined his father's business in 1838, but he quickly left it to pursue a religious career. He studied at the University in Edinburgh and then New College. He published several religious works, and he excelled with his works in astronomy: Easy Guide to the Constellations (1870) and People's Atlas of the Stars. Gall is the author of three map projections (Gall isographic, Gall stereographic and Gall orthographic projection), whose purpose was to reduce distortion on constellation maps. He presented his work in Glasgow, at the meeting of British Association for the Advancement of Science and explained it more fully in an article published in the Scottish Geographical Magazine entitled Use of Cylindrical Projections for Geographical, Astronomical and Scientific Purposes in 1885.

Fig. 12.21 Airy projection

Gall projection excels among perspective cylindrical projections that found greater use in practice (Snyder and Steward [1988](#page-67-0)). In this projection, the point of view is on the sphere's surface, and this is why it bears the name Gall stereographic projection, and the cylinder cuts the sphere alongside the parallels with the latitude $\varphi = \pm 45^{\circ}$ (Fig. [12.22\)](#page-28-0).

- Gall J (1855) On improved monographic projections of the world. British Assn. for the Advancement of Science, Sept., 25th mtg., Report, p. 148. [Two perspective secant cylindrical projections, orthographic and stereographic (the "Gall proj."), and an equirectangular.]
- Gall J (1871) On a new projection for a map of the world. Royal Geographical Society, Proceedings, v. 15, July 12, p. 159. [His stereographic projection.]
- Gall J (1885) Use of cylindrical projections for geographical, astronomical, and scientific purposes. Scottish Geographical Magazine, v. 1, no. 4, p. 119– 123. [Reports his 1855 projections.]

Nicolas Auguste Tissot

(Nancy, Meurthe-et-Moselle, France, 1824–Paris, 1897) French cartographer

Tissot was trained as an engineer in the French Army, from which he graduated as capitaine du génie. In the early 1860s he became an instructor in geodesy at the

Fig. 12.22 Map of the world in Gall Stereographic projection

well-reputed Ecole Polytechnique in Paris. Around the same time, he indulged a research program meant to determine the best way of cartographic projection for a particular region and presented his findings to the French Académie des Sciences. He published analyses of the distortion that occurs on map projections in 1859 and 1881. He devised the ellipse of distortion which indicates how the scale changes in every direction at a given point. This ellipse was named the Tissot indicatrix after him. Additionally, he pursued research studies of the projections which are best suited for representation of a certain part of the Earth's surface (Snyder and Steward [1988\)](#page-67-0). For the representation of relatively small parts of the Earth's surface in a plane, with minimal deformations of angles and lengths, Tissot proposed a specific projection, which was named after him the Tissot compensational projection (Fig. [12.23\)](#page-29-0).

- Tissot NA (1858) Sur le développement modifié de Flamsteed. Académie des Sciences, Comptes Rendus, v. 46, no. 13, pp 646–648 [Modification of Sinusoidal projection using hyperbolic curves.]
- Tissot NA (1859–60) Sur les cartes géographiques. Académie des Sciences, Comptes Rendus, 1859, v. 49 no. 19 pp 673–676; 1860, v. 50 no. 10 pp 474–476; 1860, v. 51 pp 964–969 [See also 1865, v. 60 pp 933– 934]

Fig. 12.23 Perspective projection of the Earth and Tissot's indicatrices

- Tissot NA (1878) Sur la représentation des surfaces et les projections, des cartes géographiques. Nouvelles Annales deMathématiques, 2nd series, v. 17, pp 49–55, 145–163, 351–366
- Tissot NA (1881) Mémoire sur la représentation des surfaces et les projections des cartes géographiques. Gauthier Villars, Paris, 337 p. +60 p. tables. [Details the use of his indicatrix, introduced earlier. Also presents his version of equal-area conic projection. Note: The preamble and first four chapters of this work, pp 1–126, first appeared in Nouvelles Annales de Mathématiques, series 2, 1878, v. 17; 1879, v. 18; 1880, v. 19.]
- Tissot NA (1887) Die Netzentwürfe geographischer Karten nebst Aufgaben über Abbildung beliebiger Flächen auf Einander. Autorisierte Deutsche Bearbeitung mit einigen Zusätzen bezorgt von E. Hammer, J. B. Metzlersche Buchhandlung, Stuttgart

Alexander Ross Clarke

(Reading, Berkshire, England, 1828–Strathmore, Reigate, Surrey, 1914) British geodesist, mathematician and officer

A geodesist whose work is primarily remembered for defining different reference ellipsoids. He was born in England, and he spent his childhood in Jamaica. He

Fig. 12.24 Map of the world in Clarke's Twilight general vertical perspective projection

returned to England and in 1847 joined the British army, and was assigned to the Royal Engineers. He served in Canada from 1851 to 1854. In the year 1856, he became the director of the measurement department, and in 1858 he published his first article on the history of land surveying in Great Britain. In June 1862, he was elected as a member of the Royal Society. His ellipsoids from 1858, 1866 and 1880, which are named after him (for example Clarke 1866 or Clarke 1880) are famous (Table 12.2). He received a gold medal from the Royal Society (1887) for his contribution of determining shape and size of Earth. A.R. Clarke was interested in map projections as well. In the year 1862, he used the least squares method for his perspective projection with minimal deformations for part of terrestrial sphere, margined with a determined spherical circle. He determined parameters for several continental areas, and he published his projection called Twilight as well, which presents most of the Earth's land. All these projections are classified as perspective projections with minimum or low-error distortions (Fig. 12.24).

Clarke AR (1879) Geography: mathematical geography. Encyclopædia Britannica, 9th ed., v. 10, pp 197–210 [Same text, vol., and page nos. in 10th ed., 1902]

Friedrich Eisenlohr

(Mannheim, 1831–Heidelberg, 1904) German mathematician

Eisenlohr studied in Göttingen, Berlin and Heidelberg, where he obtained his PhD in 1852. He finished his habilitation two years later and was appointed a professor in Heidelberg in 1872.

In 1870 he presented a conformal projection, which provides the minimum overall scale variation for a conformal world map, because the boundary of a world map is at a constant scale. Like the August Epicycloidal projection, the Eisenlohr has no "singular" points at which conformality fail. It is used for whole-world maps and was named Eisenlohr projection after him (Fig. 12.25).

- Eisenlohr F (1870) Ueber Flächenabbildung: Journal für die reine und angewandte Mathematik [Crelle's], v. 72, no. 2, pp 143–151 [His conformal projection of world with minimum overall distortion]
- Eisenlohr F (1875) Ueber Kartenprojection: Gesellschaft für Erdkunde zu Berlin, Zeitschrift, v. 10, no. 59, pp 321–334

Friedrich Wilhelm Oscar August

(Berlin, 1840–Berlin, 1900) German professor of mathematics

Fig. 12.25 Map of the world in Eisenlohr projection

Fig. 12.26 Map of the world in August epicycloidal projection

Dr. Friedrich August was professor of mathematics and son of the mathematician. His publications mainly relate to the field of geometry and mechanics since his position at the Royal Bavarian Artillery and Engineering School (Königlich Bayerische Artillerie- und Ingenieur-Schule) encouraged him to handle various questions from these fields.

In book form, he published Research on the imaginaries in Geometry (Untersuchungen über das Imaginäre in der Geometrie) in 1872 and the essay "A conformal mapping of the earth in epicycloidal projection" (Eine konforme Abbildung der Erde nach der epiex cloiden Projektion) in 1875.

August's epicycloidal map projection is a conformal map projection of the whole sphere into a two-cusped epicycloid. It was designed by Friedrich August and co-developed by Bellermann and published in 1874. In this projection, a world map is bounded by an epicycloid, the shape defined by a point on a circle rolling without sliding around another, fixed, circle (Fig. 12.26).

August F (1874) Ueber eine conforme Abbildung der Erde nach der epicycloidischen Projection: Gesellschaft für Erdkunde zu Berlin, Zeitschrift, v. 9, pp 1–22 [His conformal projection of world bounded by two-cusped epicycloid.]

Wilhelm Jordan

(Ellwangen, Germany, 1842–Hannover, 1899) German geodesist and mathematician

He worked as a professor at the polytechnic institute in Stuttgart (1865–68) and in Karlsruhe (1868–81). In 1847, Jordan took part in the expedition of Gerhard Rohlfs to Libya. As a member of the German Society of Geometers, he worked on restructuring the German geodetic school system, and in 1887 he founded the magazine Kalender für Vermessungswesen und Kulturtechnik, which was published regularly once a year till 1949. From 1881, he was professor of geodesy and practical geometry at the technical university Hannover and he had been working on his most significant work—handbook of geodesy which is today known under the title Jordan–Eggert–Kneissl: Handbuch der Vermessungskunde (Fig. [12.27\)](#page-34-0). He is remembered for his algorithm for bringing the matrix to reduced form, which is used for finding the inverse of a matrix. The method was named the Gauss-Jordan algorithm after him and Carl Friedrich Gauss. In 1896, Jordan suggested a formula for determination of mean square distortion in the given point in order to compare two projections according to mean square distortions in the whole area of projection. This criterion is named the Jordan criterion after him.

- Jordan W (1875) Zur Vergleichung der Soldner'schen rechtwinkligen sphärischen Coordinaten mit der Gauss'schen conformen Abbildungen des Ellipsoids auf die Ebene. Zeitschrift für Vermessungswesen, IV, pp 27–32
- Jordan, W. (1896) Der mittlere Verzerrungsfehler, Zeitschrift für Vermessungswesen, XXV, pp 249–252

Alphons J. Van der Grinten

(Kranenburg, Nordrhein-Westfalen, Germany, 1852–Chicago, USA, 1921) American cartographer

HANDRUCH DER VERMESSUNGSKUNDE

von

WEIL. DR. W. JORDAN

Professor an der Technischen Hochschule zu Hannover

Fortgesetzt von

WEIL. DR. C. REINHERTZ

Professor an der Technischen Hochschule zu Hannover

DRITTER BAND

LANDESVERMESSUNG UND GRUNDAUFGABEN DER ERDMESSIING

Mit zahlreichen Abbildungen

Siebente erweiterte Auflage

Bearbeitet

von

DR. O. EGGERT

Professor an der Landwirtschaftlichen Hochschule zu Berlin

Stuttgart J. B. Metzlersche Verlagsbuchhandlung 1923

Fig. 12.27 Title page of Jordan's work Handbuch der Vermessungskunde, published in 1923

He described, in patent specification from 1904, a graphical way of constructing a projection, which bears his name today (Snyder and Voxland [1989, 1994\)](#page-67-0). The Van der Grinten projection is the most famous projection in the group of circular projections (Fig. [12.28](#page-35-0)). According to distortion characteristics, it belongs to the group of arbitrary projections. Meridians are mapped as circles symmetrical in relation to the central meridian, which is mapped as a straight line. The Van der Grinten projection is often used for political world maps, although it is not suitable for this purpose because of large surface distortions. For example, Greenland is three times smaller than Australia, and in Grinten's projection it is larger than

Fig. 12.28 Map of the world in the Van der Grinten projection. Because of the great distortion of area near the poles, it is not recommended to be used (although it frequently is) for depicting the whole world on a map

Australia. The National Geographic Society used this projection for reference world maps from 1922 to 1988, afterwards it was replaced with the Robinson projection.

van der Grinten AJ (1904) Map: U.S. Patent 751,226, dated Feb. 2. [His best known circular map projection. Also patented in Canada, Great Britain, and France (van der Grinten (1905, New ….)).] van der Grinten AJ (1904) Darstellung der ganzen Erdoberfläche auf einer kreisförmigen Projektionsebene. Petermanns Mitteilungen, v. 50 no. 7 pp 155–159. [World map projections bounded by circles and having circles for meridians and parallels. See also corrections, no. 10, p. 250; 1905, v. 51, no. 2, p. 48. See also his supplementary comments, 1905, Zur Verebnung der ganzen Erdoberfläche. Petermanns Mitteilungen, v. 51, no. 10, p. 237; 1906, Zu der zweiten Notiz von van der Grinten über seine Weltprojektion. Petermanns Mitteilungen, v. 52, no. 2, p. 46]

van der Grinten AJ (1905) New circular projection of the whole earth's surface. Amer. Journal of Science, series 4, v. 19, no. 113, pp 357–366

David Aleksandrovich Aitoff (Aitov, Aitow)

(Orenburg, 1854–Paris, 1933) Russian cartographer

After his emigration to France in 1879 Aitoff worked as a cartographer at Hachette-Verlag, where he dealt mostly with the Atlas universel edited by Louis Vivien de Saint-Martin and after by Franz Schrader. Based on the general census of the Russian Empire in 1897 he published the ethnographic general map at the scale of 1:12,500,000 with an explanatory text.

He proposed a suitable for world map in 1889. This projection was named the Aitoff projection after him, and it was created by modification of transverse equidistant azimuthal projection. The modification of the projection is, that a perimeter circle of half-sphere map with longitude $\lambda = \pm 90^{\circ}$ is replaced with an ellipse, within which the whole terrestrial sphere will be mapped. In that projection, the pole is a point, the relation of length of the Equator to the length of central meridian is 2:1, and according to distortion characteristics, the projection is arbitrary.

Three years later, inspired by Aitoff's projection, professor of geodesy Ernst Hermann Heinrich von Hammer devised a projection called the Hammer-Aitoff projection, which was created by modification of the transverse equivalent azimuthal projection, in the same manner as the Aitoff projection was created by modification of the equidistant azimuthal projection. The projection is equal-area, the pole is a point, and relation of Equator length to the length of the central meridian is 2:1 (Fig. 12.29).

- Aitoff D (1889) Projections des cartes géographiques. In: Atlas de Géographie Moderne. Hachette, Paris. [Introduces his projection adapting the Azimuthal Equidistant projection.]
- Aitoff D (1892) Note sur la projection zenithale equidistante et sur les canevas qui en est dérivé. Nouvelles Géographiques, v. 6, June, pp 87–90
- Aitoff D (1893) Projection équivalente applicable au continent Américain. Nouvelles Géographiques, series 3, no. 5, pp 72–74

Fig. 12.29 Map of the world in the Aitoff projection

Aitoff D (1913) Projections des cartes géographiques. In: Schrader F et al., Atlas de Géographie Moderne, new ed. Hachette & cie, Paris

Vasiliy Vasil'evich Vitkovskiy

(Novogeorgievsk, 1856–Saint Petersburg, 1924)

Russian topographer and geodesist, cartographer, professor, lieutenant general

He worked in the military and as a lecturer of geodesy and astronomy at several military and civillian higher schools. He was responsible for introducing the Jäderin base-line apparatus to triangulation of Russia. He published scientific papers in Russian journals and translated Clarke's handbook Geodesy from English. In 1907 he became a distinguished full professor and had the reward of the Military-Topographic School named after him (Filatov and Potievskiy [2015](#page-67-0)).

In 1907 he published procedures for minimizing scale variation according to following criteria: the maximum scale error between the standard parallels equals (with opposite sign) the scale error at each of two limiting parallels. Once the standard parallels are selected, Vitkovskiy projection III is constructed by using the same formulas used for the Lambert Conformal Conic with two standard parallels.

Vitkovskiy VV (1911) Prakticheskaja geodezija. St. Petersburg

Johann Heinrich Louis Krüger

(Elze, 1857–Elze, 1923) German mathematician and geodesist

He finished the study of mathematics in Berlin. During his studies, he developed a passion for geodesy and at the age of 27, he became a doctor in that area. At Institute of Geodesy in Berlin he was firstly an assistant, then a professor and after that, the headmaster. The Gauss-Krüger projection (conformal transverse cylindrical projection of ellipsoid into the plane) was named after great German scientist Carl Friedrich Gauss. Professor Dr. Louis Krüger published a book on this projection in 1912, and in 1919 a collection of formulae for practical usage. Since then, this projection has been called Gauss-Krüger. In the Gauss-Krüger projection, the central meridian of the given area is mapped as a straight line and serves as the x axis of the rectangular coordinate system in the plane. The central meridian of the given area is mapped in its real length, i.e. without linear distortions or the linear scale along this meridian is constant, and the whole projection is conformal. This projection was used in many countries for official cartography (Fig. [12.30\)](#page-39-0).

- Krüger L (1903) Bemerkungen zu C.F. Gauss: Conforme Abbildungen des Sphäroides in der Ebene. In: C.F. Gauss, Werke, Königlich Gesellschaft der Wissenschaften, Göttingen, Abhandlungen, 1903, v. 9, pp 195–204
- Krüger L (1912) Konforme Abbildung des Erdellipsoids in der Ebene. Königlich Preußisches Geodätisches Institut, Potsdam, Veröffentlichung, new series, no. 52, 172 p. [He published formulas for the Prussian land survey as Formeln zur konformen Abbildung des Ellipsoids in der Ebene: Berlin, 1919]
- Krüger L (1914) Transformation der Koordinaten bei der konformen Doppelprojektion des Erdellipsoids auf die Kugel und die Ebene. Königlich Preußisches Geodätisches Institut, Potsdam, Veröffentlichung, new series, no. 60, 43 p.
- Krüger L (1922) Zur stereographischen Projektion. Preußisches Geodätisches Institut, Berlin, Veröffentlichung, new series, no. 80, 28 p.

Hermann Heinrich Ernst von Hammer

(Ludwigsburg, 1858–Stuttgart, 1925)

German geodesist and cartographer

He studied (1874–78) at high technical school (Technische Hochschule, now the Vienna University of Technology), where he worked from 1878 to 1884 as a teaching assistant, and from 1884 as a professor of geodesy. In the year 1885, he published his famous textbook of flat and spherical trigonometry with special accent on usage in geodesy and spherical astronomy. This book had several later editions. In 1887, he translated from French to German the famous work of A. Tissot on map projections. He created a modification of the transverse equivalent azimuthal projection in the same way as the Aitoff projection was created by modification of the transverse equidistant azimuthal projection, so it is known as the Hammer-Aitoff projection. The projection is equivalent, the pole is a point, and the relation of Equator length to the length of central meridian is 2:1 (Fig. [12.31\)](#page-40-0).

VERÖFFENTLICHUNG DES KÖNIGLICH PREUSZISCHEN GEODÄTISCHEN INSTITUTES NEUE FOLGE³N₅₂

KONFORME ABBILDUNG DES ERDELLIPSOIDS IN DER EBENE

VON

PROF. DR. L. KRÜGER ANTEILUNGSVORSTEHER IM KGL. PREUSZISCHEN GRODÄTISCHEN INSTITUT

POTSDAM

DRUCK UND VERLAG VON B. G. TEUBNER IN LEIPZIG 1912

Fig. 12.30 Title page of Kruger's work Konforme Abbildung des Erdellipsoids in der Ebene, published in 1912

Hammer E (1885) Lehr- und Handbuch der Ebenen und Sphärischen Trigonometrie, zum Gebrauch beim Selbstunterricht und in Schulen besonders als Vorbereitung auf Geodäsie und Sphärische Astronomie, 2nd ed. 1987, 3rd ed. 1907, 4th ed. 1916, J. B. Metzlersche Buchhandlung, Stuttgart

Fig. 12.31 Map of the world in Hammer-Aitoff projection

Tissot A (1887) Die Netzentwürfe geographischer Karten nebst Aufgaben über Abbildung beliebiger Flächen auf Einander, Autorisierte Deutsche Bearbeitung mit einigen Zusätzen bezorgt von E. Hammer, J. B. Metzlersche Buchhandlung, Stuttgart

John Paul Goode

(Stewartville, Minnesota, 1862–Little Point Sable, Michigan, 1932) American geographer and cartographer

He graduated at the University of Minnesota in 1889. He received his doctorate in economics in 1901 at the University of Pennsylvania, where he taught geography from 1901 to 1917, and from 1917 to 1928 he taught at the University of Chicago as well.

He is known as the inventor of asymmetrical interrupted projections. For about four centuries, interrupted projections were characterized by symmetry of representation. On the contrary, asymmetrical projections prevail today. In the year 1916, Goode suggested a way for reducing distortion in pseudocylindrical projections. By this way, any pseudocylindrical projection can be used for the creation of world maps by certain sections which are joined along the Equator. The central meridian is selected for each section. Its longitude should be selected in a way that distortions on that area are to be as small as possible (Fig. [12.32](#page-41-0)). Goode is the author of a great number of maps and books in the area of geography. Significant is his atlas Goode's School Atlas (1923; many later editions), which is known today as Goode's World Atlas.

Goode JP (1905) A new method of representing the earth's surface: The Van Der Grinten projection. Journal of Geography, v. 4, no. 9, pp 369–373

Goode JP (1919) Studies in projections: adapting the homalographic projection to the portrayal of the earth's surface entire. Geographical Society of Philadelphia, Bulletin, v. 17, no. 3, pp 103–113 [Pagination for no. 3 alone: pp 21–31]

Fig. 12.32 The Goode projection, developed as merging the Sanson and the Mollweide projection

- Goode JP (1925) The Homolosine projection: a new device for portraying the Earth's surface entire. Assn. of Amer. Geographers, Annals, v. 15, no. 3, pp 119–125
- Goode JP (1929) A new projection for the world map: the Polar Equal Area. Monthly Weather Review, v. 57, no. 4, pp 133–136 [Interrupted Werner projection.]
- Goode JP (1929) The Polar Equal Area, a new projection for the world map. Assn. of Amer. Geographers, Annals, v. 19, no. 3, pp 157–161 [Interrupted Werner projection.]

Max Eckert-Greifendorf

(Chemnitz, 1868–Aachen, 1938) German geographer and cartographer

Eckert studied geography and national economy in Leipzig. From 1907 to 1937, he worked as a professor in vocational university in Aachen, where he taught economic geography and cartography. After World War I, he dedicated himself to cartography. Famous is his work Die Kartenwissenschat (published in two volumes in Berlin 1921/25), in which he set the foundations of cartography as a scientific discipline. At the beginning of 20th century, Eckert proposed six new pseudocylindrical projections for the whole world map. The projections are known as Eckert's projections I–VI. In all six

Fig. 12.33 Eckert III projection

Fig. 12.34 Eckert IV projection

projections the pole is projected as a line half as long as the Equator. In the first two projections, meridians are broken at the Equator. This is their greatest flaw, so they are not used in practice. Eckert's projections III, IV, V and VI (Snyder and Voxland [1989,](#page-67-0) [1994](#page-67-0)) can be recommended for the creation of world maps (Figs. 12.33 and 12.34).

Oswald Winkel

(Leipzig, 1874–Leipzig, 1953) German cartographer

He assembled a great number of general and travelling maps for the guide and publisher Karl Beadeker. Oswald Winkel presented in 1921 a projection for the whole world map which was named after him the *Winkel Triple projection*. This arbitrary projection is created as an arithmetical mean between the Aitov and the equidistant cylindrical projection. Since the Aitov projection was created by modification of the transverse azimuthal equidistant projection, the Winkel projection is made of three projections, and this is why it is called triple (Frančula [1971;](#page-67-0) Fig. [12.35](#page-44-0)).

Winkel O (1909) Flächentreue, schiefachsige Zylinderprojektion mit längertreuem Grundkreis für eine Karte von Nord-, Mittel- und Südamerika. Petermanns Mitteilungen, v. 55, no. 11, pp 329–330 [Oblique Cylindrical Equal-Area projection. See also table of coordinates at 1:60M in no. 12, pp 379–380]

Fig. 12.35 Map of the world in the Winkel projection (standard parallel: 50°28′)

- Winkel O (1911) Flächentreue, zwischenständige, azimutale Projektion für eine Karte der Britischen Inseln als praktisches Beispiel eines Kartenentwurfes. Geographischer Anzeiger, v. 12, pp 30–31
- Winkel O (1913) Beitrag zur Entwicklung schiefachsigen, speziell zylindrischer Projektionen unter Annahme der Kugelgestalt der Erde. Petermanns Mitteilungen, v. 59–2, Dec., pp 304–306 [Oblique cylindrical projection.]
- Winkel O (1921) Neue Gradnetzkombinationen. Petermanns Mitteilungen, v. 67, Dec, pp 248–252 [Proposes three combinations of Equirectangular with Sinusoidal, Mollweide, and Aitoff projections, respectively.]
- Winkel O (1922) Die azimutaleschen Erdkartenentwürfe von D. Aitoff und E.v. Hammer. Geographische Zeitschrift, v. 28, pp 112–114 [Aitoff and Hammer projections.]
- Winkel O (1922) Allgemeine Betrachtungen über die Abbildung sehr breiter "Zonen". Geographische Zeitschrift, v. 28, no. 5–6, pp 177–181 [General projection considerations in representing latitude zones.]
- Winkel O (1927) Die Gewinnung geeigneter Landkartennetze. Geographische Zeitschrift, v. 33, no. 10, pp 599–604
- Winkel O (1928) Übersicht der Gradnetzkombinationen. Petermanns Mitteilungen, v. 74, no. 7–8, pp 201–204 [Pseudocylindrical projections.]
- Winkel O (1933) Flächentreue oder abstandstreue Projektionen? Geographische Wochenschrift, p. 207ff
- Winkel O (1933) Die Projektionswahl bei Erdkarten. Petermanns Mitteilungen, v. 79, no. 5–6, pp 125
- Winkel O (1939) 25 Jahre neue Netzkombinationen. Petermanns Geographische Mitteilungen, v. 85, no. 9, pp 278–280
- Winkel O (1951) Kurzgefaßte Kartenentwurfslehre. Atlantik-Verlag, Frankfurt-am-Main, 44 p.

Reinholds V. Putniņš

(Bērzpils parish, Balvi, Latvia, 1881–Riga, Latvia, 1934) Latvian mathematician

He was an associate professor of mathematics and natural sciences at Latvian University. He took part in organization of Latvian national meteorological office and was repeatedly elected as the president of the Latvian Association of Geographers.

In 1934 he presented twelve pseudocylindrical projections with simply described relationships resambling those of Eckert in 1906 and Wagner in 1932. On each projection, the central meridian is half the length of equador. Putninš distinguished them with P followed by subscripts 1–6, those without primes having pointed poles, and those with primes having poles half the length of equator (Bugayevskiy and Snyder [1995](#page-66-0)).

Putniņš P5 projection is pseudocylindrical projection with equally spaced parallels. In this projection scale is constant along any given latitude and the same for the latitude of opposite sign and also constant along the central meridian (Fig. 12.36).

Putniņš RV (1934) Jaunas projekcijas pasaules kartem. Geografiski raksti, Folia Geographica 3 and 4, pp 180–209 [Latvian with extensive French résumé. New projections for world maps. Describes 12 pseudocylindricals, half equal-area and half with equally spaced parallels, half pointed-polar and half with pole-lines, using elliptical, parabolic, or hyperbolic meridians.]

Fig. 12.36 Map of the world in Putniņš P5 projection

- Putniņš RV (1934) Application de la projection quadratique équivalente à l'étude des cartes anciennes. International Geographical Congress, Paris, 1931, Comptes Rendus, v. 3, pp 719–729
- Putniņš RV (1935) Sur quelques nouvelles projections à méridiens elliptiques. International Geographical Congress, Warsaw, 1934, Comptes Rendus, v. 1, pp 151–157

Vladimir Vladimirovich Kavrayskiy

(Zherebyatnikovo, 1884–Leningrad, 1954)

Russian cartographer, professor, engineer-rear-admiral

Kavrayskiy finished studying at the Kharkov university in 1916. He was employed at the Military-Maritime Academy beginning in 1921. Kavrayskiy was a State Prize winner of the USSR. He was an outstanding representative of the Kharkov scientific school in the field of astrometry and stellar astronomy, which was founded by Ludwig O. Struve (1858–1920). In 1936, Kavrayskiy proposed an equa-area sinusoidal pseudocylindrical projection for the world map (Bugayevskiy and Snyder [1995\)](#page-66-0). The pole in this projection is projected as a line whose length is equal to half of the Equator. The elliptical pseudocylindrical projection of Kavrayskiy is also named after him, and it is classified to the group of arbitrary projections (Fig. 12.37). His monograph in three volumes about the theory of map projections, which was published postmortem, is also well-known (Filatov and Potievskiy [2015\)](#page-67-0).

Kavrayskiy VV (1958) Izbrannye trudy, Tom II; Matematicheskaya kartografiya, Vyp. 1, Obshchaya teorija kartograficheskikh projektsij, Izdanie Upravlenija nachal'nika Gidrograficheskoj sluzhby VMF

Fig. 12.37 Elliptical pseudocylindrical projection of Kavrayskiy

Kavrayskiy VV (1959) Izbrannye trudy, Tom II; Matematicheskaya kartografiya, Vyp. 2, Konicheskie i tsilindricheskie proyekciji, ih primenenije, Izdanie Upravlenija nachal'nika Gidrograficheskoj sluzhby VMF Kavrayskiy VV (1960) Izbrannye trudy, Tom II; Matematicheskaya kartografiya, Vyp. 3, Perspektivye, krugovye i drugie vazhnejshie proyekcii. Navigacionnye zadachi, Izdanie Upravlenija nachal'nika Gidrograficheksoj sluzhby VMF

Samuel Whittemore Boggs

(Coolidge, Kansas, 1889–1954) American geographer

He was a long-time geographer to the United States Department of State, recognized as one of the world's leading experts on international boundaries and also as an authority on the subject of map compilation, editing, and cataloguing. He was a constant source of new ideas on cartographic techniques and through his imagination and enthusiasm he was able to contribute in a variety of ways to the development of the field of cartography.

In 1929 he presented a pseudocylindrical equal area projection, which he named the Eumorphic projection, today known as Boggs eumorphic projection. It is something of a compromise between the parabolic and sinusoidal projections, and was among other things an attempt to improve on the polar shapes of equal-area maps (Fig. [12.38](#page-48-0)). A political wall map of the world, based on this production, was subsequently marketed by the A.J. Nystrom Company.

Fig. 12.38 Map of the world in Boggs eumorphic projection

William A. Briesemeister

(1895–1967)

American cartographer

Prominent mapmaker, who retired as chief cartographer of the American Geographical Society in 1964, after more than 50 years working there.

Briesemeister Peak in Antarctica was named after him. This peak was photographed from the air by Sir Hubert Wilkins on December 20, 1928, and by the United States Antarctic Service in 1940, but Briesemeister recognised this peak on two photographs and established their continuity, an important clue to the identity and correct position of Stefansson Strait.

He supervised the preparation of maps of Antarctica for use during the International Geophysical Year (IGY) (1957–1958) and post-IGY programs of the United States Antarctic Research Program, including continental maps published at a scale of 1:6,000,000 (1956) and 1:5,000,000 (1962).

His many accomplishments include the *Briesemeister projection*, presented in 1953, which permits a flat map to show land areas in their true relative size (Fig. [12.39\)](#page-49-0).

- Briesemeister W (1953) A new oblique equal-area projection. Geographical Review, v. 43, no. 2, pp 260–261 [His oblique aspect of Hammer projection, with axes 1.75 to 1 instead of 2 to 1.
- Briesemeister W (1959) A world equal-area projection for the future. The selection of the most suitable equal-area projection for the purpose of plotting world wide statistics in this present day of super speed, jet planes and intercontinental missiles. Nachrichten aus dem Karten- und Vermessungswesen, series 2, no. 3, pp 60–63. [Published separately, 1958: Chicago, Rand McNally, 4 p. Translated to German as Eine flächentreue

Fig. 12.39 Map of the world in Briesemeister projection

Weltkarten-Projektion für die Zukunft. Nachrichten aus dem Karten- und Vermessungswesen, 1958, series 1, no. 7, pp 72– 74]

Briesemeister W (1959) A new equal area projection for the future. Second International Cartographic Conference, Chicago, 1958, Proceedings: Verlag des Instituts für Angewandte Geodäsie, pp 60–63

Osborn Maitland Miller

(Perth, Scotland, 1897–New York, USA, 1979) American cartographer and geographer

The Osborn Maitland Miller Cartographic Medal established in 1968 by the American Geographical Society Council was named after him, and it honors "outstanding contributions in the field of cartography or geodesy".

Miller's forty-six year career with the *American Geographical Society* was only one of the many accomplishments of his career. He headed the staff, researched, and taught at the American Geographical Society's School of Surveying, specializing in photogrammetry and cartography.

In 1941, in collaboration with William A. Briesemeister, he presented the bipolar oblique conic conformal projection, designed specifically for a low-error map of North and South America constructed by the American Geographical Society.

In 1942 he developed the Miller cylindrical projection. This projection is used for world maps in numerous American atlases and some other atlases, as a projection resembling the Mercator but having less distortion of area and scale, especially near the poles (Fig. [12.40](#page-50-0)).

Miller oblated stereographic projection was first applied by Miller to Africa and Europe in 1953 and to other Eastern Hemisphere regions in conjunction with

Fig. 12.40 Map of the world in Miller cylindrical projection

several nonconformal fill-in projections in 1955 (Snyder and Steward [1988\)](#page-67-0). This projection is used for an area that can be contained within an oval shape to minimize the scale distortions in the area (Fig. [12.41](#page-51-0)).

- Miller O M (1941) A conformal map projection for the Americas. Geographical Review, v. 31, no. 1, pp 100–104 [Presents Bipolar Oblique Conic Conformal projection.]
- Miller O M (1942) Notes on cylindrical world map projections. Geographical Review, v. 32, no. 3, pp 424–430 [Presents his cylindrical projection. See also 1943, v. 33, pp 328–329]
- Miller O M (1953) A new conformal projection for Europe and Asia [sic; should read Africa]. Geographical Review, v. 43, no. 3, pp 405–409 [Presents his Oblated Stereographic projection as applied to Europe and Africa only.]
- Miller O M (1955) Specifications for a projection system for mapping continuously Africa, Europe, Asia, and Australasia on a scale of 1:5,000,000. American Geographical Society. New York. Contract report to Army Map Service, May 1955. Also corrections in letter by Miller, June 2, 1955, and Supplement, January 31, 1956. [Extends his Oblated Stereographic projection to most landmasses of Eastern Hemisphere. See note in Geographical Review, 1956, v. 46, no. 2, p. 258]

- Miller O M (1962) Map projections. Encyclopædia Britannica, 15th ed., v. 14, pp 841–844 [Reprinted to 1972. Text less than one page by others in later editions.]
- Miller O M (1965) Some equivalent map projection transformations in the plane. Survey Review, v. 18, no. 136, pp 73–77

Vladimir Kirilov Khristov (Hristov, Hristow)

(Sofia, 1902–1979) Bulgarian geodesist, astronomer and cartographer

Khristov studied astronomy, mathematics and physics at Leipzig University, where he graduated in 1925 with his PhD in astronomy.

He worked in the Astronomical and Geographical Department of the State Institute of the Ministry of War from 1925 to 1948. The Department was assigned to organize and carry out necessary astronomical measurements for tasks such as: developing first class triangulation, choosing of a reference ellipsoid, creating working formulas, measuring the ellipsoid, making the orientation of the network, proposing selection of coordinates, and planning and selecting the map projection for topographic maps. Under these conditions, Khristov explored the extensive library of the Leipzig University and works of Helmert dealing with problems of size of the reference ellipsoid and its orientation. Khristov defined and solved the problem of map projection transformations using calculus. Development in series is the basis of his later works, such as the introduction of Gaussian coordinates and calculation of all necessary reductions and transformations in overlapping strips, putting the formulas for transition from one reference geodetic system to another and specific solutions introducing the Bulgarian system in 1950 with the ellipsoid of Krasovsky, such as transformation of the analogue system in 1930 defined on the ellipsoid of Hayford.

Transformation solutions by the method of Khristov were easily applied by using tables and such tables quickly entered the surveying practice.

In 1948, Khristov became Professor of Geodesy at the State Polytechnic (nowadays University of Architecture, Civil Engineering and Geodesy), where he remained until his retirement in 1970. At the University, he laid the foundations, created programs and issued the first Bulgarian textbooks on four main areas scientific disciplines that formed at that time the students' knowledge of higher geodesy: mathematical cartography, physical geodesy, geodetic astronomy and mathematical geodesy.

Khristov was a corresponding member from 1948, and from 1958 Academician of the Bulgarian Academy of Sciences. He headed the Central Laboratory of Geodesy from 1956 to 1970.

In 1959, academician Khristov managed to restore the Bulgarian National Committee of Geodesy and Geophysics and Bulgaria's membership in the International Union of Geodesy and Geophysics, which dates from 1932, but was suspended during World War II and the years of political isolation. As longtime chairman of this Committee, he developed a particularly fruitful scientific activity and created links with research institutions and communities around the world.

Khristov received recognition in the scientific community and a recognition from the Bulgarian government. Presented with all scientific and honorary titles, he won the highest state awards. He was elected an honorary member of the Hungarian Academy of Sciences in 1969.

Khristov's most important monographs (Snyder and Steward [1988](#page-67-0)):

- Khristov VK (1943) Die Gauß-Krügerschen Koordinaten auf dem Ellipsoid. B.G. Teubner, Leipzig and Berlin [Translated to English by the Army Map Service as The Gauss-Krüger coordinates on the ellipsoid; unpublished.]
- Khristov VK (1946) Gausz-Kryugerovite koordinati vyrhu rotacionniya elipszoid. Pecsatnicata na Dyrzsavniya Geografszki Insztitut, Sofia. [Bulgarian. Gauss-Krüger coordinates on the ellipsoid of rotation.]
- Khristov VK (1949) Kartii proyektsiy matematicheska kartografie. Sofia, 196 p. [Bulgarian. Map projections in mathematical cartography.]
- Khristov VK (1955) Die Gaußschen und geographischen Koordinaten auf dem Ellipsoid von Krassowsky. VEB Verlag Technik, Berlin
- Khristov VK (1957) Koordinaty Gaussa-Kryugera na ellipsoide vrashcheniya. Geodezizdat, Moscow 263 p. [Russian. Gauss-Krüger coordinates on the ellipsoid of rotation.]
- Khristov VK (1961) Matematicheska kartografiya. Tekhtsmka, Sofia, 155 p. [Bulgarian. Mathematical cartography.]

Karl Heinrich (Karlheinz) Wagner

(Leipzig, 1906–Berlin, 1985) German cartographer

Karl Heinrich Wagner was son of Eduard Wagner, co-owner of H. Wagner & E. Debes cartographic company in Leipzig, which predestined him for the vocation of geographer/cartographer. He majored in geography, and minored in oceanography, mathematics and geology.

He earned his PhD in 1931 with *Pseudocylindrical projection, their importance* and practical application. In the 1930's, he introduced the Wagner transformation (its original name is Umbeziffern vor Kartennetzen, which means renumbering of cartographic grids). This transformation uses a smaller part of a projection's grid, magnified to keep the original area of the projection.

In 1931, he started working as a cartographer for Wagner & Debes, where he worked until 1933. He then went to Barcelona and was employed in the cartographic department of one of the largest Spanish publishers of school books, Editorial Luis Vives. In 1937, he returned to Leipzig and became the manager of Wagner & Debes. The company was completely destroyed in an air strike in 1943, but it was rebuilt in 1945.

From 1944, Wagner also worked on his major book Kartografische Netzenwürfe (Map Projections). In his book, Wagner presented three different transformation methods that he applied to various types of map projections and obtained several new graticules.

In 1951, Wagner transferred the company's headquarters to Berlin and renamed it to Kartografische Anstalt Dr. Karl Heinrich Wagner. In 1958 and 1959, Wagner worked with the Bibliographische Institut on developing projections for a German world map and a German sea map. Based on these projections, Wagner developed

maps for Atlas zur Ozeanographie by G. Dietrich. Wagner also reworked Atlas zur physischen Geographie and developed relief representations for small scale maps (Ferschke [1966](#page-66-0); [1985](#page-66-0); Bosse [1976](#page-66-0); Mittelstaedt [1976\)](#page-67-0).

He developed several pseudocylindrical and modified azimuthal projections (Bugayevskiy and Snyder [1995](#page-66-0)), named after him Wagner projections.

The *Wagner IV projection* (presented in 1932) is pseudocylindrical equal area projection which has meridians that are only portions of semi ellipses, but parallels are also spaced for equal area.

The *Wagner VII projection* (presented in 1941) is modified azimuthal equal area projection, used for World maps, such as climatic maps prepared by the U.S. Department of Commerce (Figs. 12.42 and 12.43).

Fig. 12.42 Map of the world in Wagner IV projection

Fig. 12.43 Map of the world in Wagner VII projection

In 1976 he was appointed as Honorary Member of The German Cartographic Society e.V. (Deutsche Gesellschaft für Kartographie e.V.).

Wagner's main publications are:

- Wagner K (1932) Die unechten Zylinderprojektionen. Deutsche Seewarte, Archiv, v. 51, no. 4, 68 p. [Presents several pseudocylindrical projections with sinusoidal or elliptical meridians.]
- Wagner K (1941) Neue ökumenische Netzentwürfe für die kartographische Praxis. Jahrbuch der Kartographie, pp 176–202 [Presents modifications of Hammer projection with curved lines for poles.]
- Wagner K (1944) Umformung von Mercator-Netzen. Petermanns Geographische Mitteilungen, v. 90, no. 11–12, pp 299–306
- Wagner K (1949) Kartographische Netzentwürfe. Bibliographisches Institut, Leipzig, 262 p. [2nd ed. 1962. Presents new flat-polar pseudocylindrical projections with sinusoidal or elliptical meridians. Also combines Aitoff and equatorial Azimuthal Equidistant projections.]
- Wagner K (1966) Über das Zusammenfügen von geographischen Kartennetzen und die Netze der "Deutschen Weltkarte" und "Deutschen Meereskarte". Bibliographisches Institut, Hauszeitschrift, Wissenschaftliche Redaktion, no. 2, pp 89–117; no. 3, pp 7–55
- Wagner K (1973) Das neue Kartenbild des Herrn Peters. Kartographische Nachrichten, v. 23, no. 4, pp 162–163 [Peters projection.]
- Wagner K (1982) Bemerkungen zum Umbeziffern von Kartennetzen. Kartographische Nachrichten, v. 32, no. 6, pp 211–218

Felix Webster McBryde

(Lunenburg, Virginia, 1908–1995)

American geographer, cartographic consultant and educator

In 1943 he founded the American Society for Professional Geographers. Aside from his research of markets in villages in the Gautemalan Lake Atitlan area and his teaching geography at Ohio State University from 1937 to 1942, his career was primarily as a consultant.

During World War II he worked as a senior geographer in military intelligence in the War Department. After the war he became director of the Smithsonian Institution's Institute of Social Anthropology in Lima, Peru. After three years in that position he became chief geographer for the Latin American program of the United States Bureau of the Census that included his establishing the Ecuadorian Institute of Anthropology and Geography in Quito.

He also developed new world map projections (Snyder and Voxland [1989,](#page-67-0) [1994\)](#page-67-0):

- McBryde S3 projection, developed by F. Webster McBryde in 1977 as a merging of the Sinusoidal with the McBryde-Thomas flat-polar sinusoidal projection at the parallels of identical scale on the two projections, latitudes 55° 51′ N and S. U.S. Patent by McBryde
- McBryde-Thomas flat-polar sinusoidal projection, presented by F. Webster McBryde and Paul D. Thomas through the U.S. Coast and Geodetic Survey in 1949
- McBryde-Thomas flat-polar parabolic projection, presented by F. Webster McBryde and Paul D. Thomas through the U.S. Coast and Geodetic Survey in 1949
- McBryde-Thomas flat-polar quartic projection, presented by F. Webster McBryde and Paul D. Thomas through the U.S. Coast and Geodetic Survey in 1949 (Figs. 12.44, [12.45](#page-57-0), [12.46](#page-57-0) and [12.47](#page-58-0)).

McBryde FW (1977) New equal-area world map projections and base maps for biological data. Transemantics, Inc., Washington, 19 p. [Merges standard pseudocylindricals with his flat-polar forms.]

McBryde FW (1978) A new series of composite equalarea world map projections. International Cartographic Assn., 9th International Conference on Cartography, College Park, Maryland, Abstracts, pp 76–77

McBryde FW (1981) Pondering projections. Amer. Geophysical Union, Eos Transactions, v. 62, no. 48, p. 1162. [Proposes his interrupted equalarea pseudocylindricals for ocean maps.]

Fig. 12.44 Map of the world in interrupted McBryde S3 projection

Fig. 12.45 Map of the world in interrupted McBryde-Thomas flat-polar sinusoidal projection

Fig. 12.46 Map of the world in McBryde-Thomas flat-polar parabolic projection

Fig. 12.47 Map of the world in McBryde-Thomas flat-polar quartic projection

McBryde FW, Thomas PD (1949) Equal-area projections for world statistical maps. U.S. Coast and Geodetic Survey Spec. Pub. No. 245, 44 p. [Presents flat-polar forms of sinusoidal, quartic, and parabolic pseudocylindrical projections, and two other pseudocylindricals.]

Laurence Patrick Lee

(England, 1913–1985)

New Zealand surveyor and cartographer

He was educated at Sacred Heart College in Auckland and Auckland University where he undertook study towards a BSc degree. He was tutor in geodesy and astronomy for the Technical Correspondence Institute for many years and wrote the geodesy course for the survey examination in that subject.

His career from 1936 to retirement in 1974 was with New Zealand Department of Lands and Survey where he was chief computer during the last ten years and was involved with many major projects. Perhaps the most important were the completion of the First Order Geodetic Triangulation of New Zealand and the establishment of the Geodetic Datum 1949, the change to metric units and the computations for the International Geophysical Year latitude and longitude program during the period 1957–59.

He was deeply interested in map projections and became known worldwide for his work in this field and indeed was an authority on this subject He published many papers and developed many new ideas and methods of computation. He showed the conformal world map in an equilateral triangle and ellipse, a hemisphere in a rectangle, and the world conformally on the faces of all five of the regular Platonic polyhedral (tetrahedron, cube, etc.). Although these projections are truly conformal, they generally have a singular point at each vertex of the bounding polygon and often elsewhere as well, with wide ranges of scale. They principally remain admirable mathematical achievements and artistic curiosities (Fig. [12.48](#page-59-0)).

Fig. 12.48 Lee conformal tetrahedric projection of the world

Lee was elected an Honorary Member of the New Zealand Institute of Surveyors in 1971. He was also a foundation member and the first Life Member of the New Zealand Cartographic Society and a member of the Royal Astronomical Society of New Zealand.

New Zealand Cartographic Journal: Obituary: Laurence Patirck Lee, 1985, 15 (1):32

- Lee LP (1944) The nomenclature and classification of map projections. Empire Survey Review, v. 7, no. 51, pp 190–200
- Lee LP (1945) The transverse Mercator projection of the spheroid. Empire Survey Review, v. 8, no. 58, pp 142–152
- Lee LP (1946) The nomenclature of map projections. Empire Survey Review, v. 8, no. 60, pp 217–219
- Lee LP (1946) Marginal scales of latitude and longitude on transverse Mercator maps: Empire Survey Review, v. 8, no. 60, pp 220–221
- Lee LP (1946) The convergence of meridians. Empire Survey Review, v. 8, no. 61, pp 267–271 [See also 1954, v. 12, no. 91, pp 237–238]
- Lee LP (1947) A stereographic device for the solution of spherical triangles. Empire Survey Review, v. 9, no. 65, pp 123–131
- Lee LP (1947) The equidistant azimuthal projection of the sphere. New Zealand Geographer, v. 3, no. 1, pp 41–58
- Lee LP (1950) The planets 1950. New Zealand Astronomical Handbook, p. 22. [Oblique Mercator projection.]
- Lee LP (1953) A transverse Mercator projection of the spheroid alternative to the Gauss-Krüger form. Empire Survey Review, v. 12, no. 87, pp 12–17 [See also 1954, v. 12, no. 91, pp 237–238]
- Lee LP (1954) The oblique Mercator projection. New Zealand Geographer, v. 10, no. 2, pp 151–164 [Reprinted in Empire Survey Review, 1956, v. 13, no. 101, pp 321–335]
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Arthur H. Robinson

(Montreal, Canada, 1915–Madison, Wisconsin, 2004) American geographer and cartographer

He was educated at Miami University in Ohio and received a doctorate at Ohio State in 1947. After his service with military intelligence in the war, he joined the Wisconsin faculty, where he spent the rest of his career.

Robinson was a prolific writer and influential philosopher on cartography. In The Look of Maps (1952) he urged cartographers to consider the function of a map as an

integral part of the design process. In the publication The Nature of Maps (1976), Robinson and co-author Barbara Petchenik created the term map percipient, a map user who interacts with a map in a discerning way and not merely as a casual observer. Robinson also co-authored a widely used textbook, Elements of Cartography, the sixth and last edition of which was published in 1995.

He was appointed Professor Emeritus of the University of Wisconsin. He received, among others, the National Geographic society and the British geographical society medals and had the honor of chairing the International Cartographic Association (ICA) as his work turned out to be a great contribution to the development of cartographic science during the 20th century.

One of his most notable accomplishments is the Robinson projection. He presented this projection in 1963, at the request of Rand McNally and Company. It uses tabular coordinates rather than mathematical formulas to make the world map "look" right (Fig. 12.49).

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- Robinson AH (1974) A new map projection: Its development and characteristics: International Yearbook of Cartography, v. 14, p. 145–155. [His pseudocylindrical projection. See correction by Richardson, R.T., 1989.]
- Robinson AH (1990) Rectangular world maps no!: Professional Geographer, v. 42, no. 1, p. 101–104. [Opposes general use of cylindrical projections for world maps.]

Fig. 12.49 Map of the world in Robinson projection

Robinson AH (1953) Elements of Cartography, 1st ed., published by John Wiley & Sons, New York

Robinson AH, Morrison JL, Muehrcke PC, Kimerling AJ, Guptill SC (1995) Elements of Cartography, 6th ed. John Wiley and Sons, New York

Lev Moiseyevich Bugayevskiy

(Belaya Tserkv' Kievskoy oblasti, 1921–2010) Russian cartographer professor, colonel

After finishing high school, he enrolled the Leningrad Military-Topographic Higher School in 1939. He participated in defense of Leningrad, as well as in military operations in Leningrad, Belarus and Ukrainian fronts. He was awarded medals for his courage in military operations.

After World War II, he finished studying at the Faculty of Geodesy of the Military-Engineering Academy. He dedicated his entire career to mathematical cartography.

He finished the academy in 1954 and was sent to Irkutsk, where he was an editor and chief of map production department. He was transferred to Moscow in 1967 and continued his military service and scientific research in mathematical cartography. In 1971, he defended his doctoral thesis Problems of researching and using conformal and related projections in cartography and geodesy. He was demobilized as a colonel in 1976.

He continued working in the Chair of Map Planning and Composition in the MIIGAiK (Moscow Institute of Engineering Geodesy, Aerial Survey and Cartography), where he stayed until the end of his life. Bugayevskiy's years inMIIGAiK were the most prolific period of his scientific and pedagogical work. He published more than 130 scientific papers (Groshev [2012;](#page-67-0) Filatov and Potievskiy [2015](#page-67-0)).

In his monographs, Bugayevskiy was concerned with many theoretical and practical issues, such as:

- General map projection theory
- Distortion theory and mapping one surface onto another
- Theoretical bases of best projections
- Coordinate systems, especially isometric
- Projection transformations, double mapping
- Projection calculations, determining parameters of reference surfaces of celestial bodies
- Using projections to produce topographic, maritime and aerial navigation maps
- Design and nomenclature of map sheets in a particular scale
- Automation in mathematical cartography, etc.

His most famous monographs include:

Edward Nelson Gilbert

(Woodhaven, New York, 1923–Basking Ridge, New Jersey, 2013)

American mathematician and coding theorist

A longtime researcher at Bell Laboratories whose accomplishments include the Gilbert-Varshamov bound in coding theory, the Gilbert-Elliott model of bursty errors in signal transmission, and the Erdős-Rényi model for random graphs.

He invented and constructed a sphere (1973) that contained a conformal projected sphere on each of its hemispheres. This sphere is named after him Gilbert's globe. He had a real globe like this to play tricks on unsuspecting visitors. When people visit Gilbert's office, he liked to ask them what is wrong with his globe. If the visitor cannot see what is wrong, Gilbert gives the globe one slow, complete turn. "Even this hint", he writes, "does not always succeed". Gilbert's globe can be projected into a plane by means of perspective or orthographic projection and accordingly one can obtain the Gilbert projection. The Gilbert Two-World Perspective Projection is a useful visual illusion. It resembles the world as people will increasingly see it, from space and in the round (Gardner [1975;](#page-67-0) DeLucia and Snyder [1986](#page-66-0); Lapaine and Frančula [1993;](#page-67-0) Lapaine et al. [1994\)](#page-67-0). Also, the map communicates its message to the reader in a natural manner, by relying on each viewer's well-developed powers of perspective sight (Fig. [12.50\)](#page-64-0).

Gilbert EN (ca. 1973) An atlas of oddities. Murray Hill, N.J., Bell Laboratories, unpublished manuscript, 30 p. [Several existing or adapted projections.] Gilbert EN (1974) Distortion in maps. SIAM [Society for Industrial and Applied

Mathematics] Review, v. 16, no. 1, pp 47–62

Fig. 12.50 Map of the world in Gilbert projection

John Parr Snyder (1926–1997) American cartographer

He was president of the American Cartographic Association from 1990 to 1991 and also served as a secretary to the Washington Map Society. He also taught courses on map projection at George Mason University, but was most known for his work on map projections for the United States Geological Survey (USGS). Educated at Purdue University and Massachusetts Institute of Technology as a chemical engineer, he had a lifetime interest in map projections as a hobby, but found the calculations tedious without the benefit of expensive calculators or computers. At a cartography conference in 1976, he learned the need for a map

Fig. 12.51 GS50 projection encompassing the regions of the 50 states of the United States

projection that would suit the special needs of Landsat satellite imagery. He subsequently developed a solution with the Space Oblique Mercator projection, which he programmed on a pocket calculator.

He was subsequently offered a job within the USGS within two years, where his work apparently led him to the eventual publication of the definitive technical guide to map projections entitled Map Projections: A Working Manual among other works. He also authored Flattening the Earth: Two Thousand Years of Map Projections which details the historical development of hundreds of map projections.

Snyder developed conformal projection called GS50 projection, which uses a complex polynomial to project the 50 U.S. states with minimal distortion (Fig. 12.51).

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