Chapter 4
Model Detection

In this chapter, we first formulate the basic model detection task to discover among N given LTI models, that particular model which best matches the current plant behaviour. Then, the concept of model detectability is introduced and characterized. The exact and approximate model detection problems are formulated. These problems target the synthesis of a bank of N LTI model detection filters which generate a structured set of residuals allowing the discrimination of models in the case of absence or presence of noise inputs, respectively. The discussion of specific performance requirements for model detection and the selection of thresholds to be used for decision-making conclude the chapter.

4.1 Basic Model Detection Task

Multiple models which describe various fault situations have been frequently used for fault detection purposes. In such applications, the detection of the occurrence of a fault comes down to identifying, using the available measurements from the measurable outputs and control inputs, that model (from a collection of models) which best matches the dynamical behaviour of the faulty plant. Another typical application is the multiple-model-based adaptive control, where the adaptation of the control law (for example, by switching from one controller to another) is based on the recognition of that model which best approximates the current dynamical behaviour of the plant. In this book, we will use the (not yet standard) term *model detection* to describe the model identification task consisting of the selection of a model from a collection of N models, which best matches the current dynamical behaviour of a plant.

A related term used in the literature is *model validation*, which covers an arsenal of statistical methods to assess the adequacy of a model to a set of measurements. Often model validation also includes the identification of suitable uncertainty bounds which account for the unmodelled dynamics, initial condition uncertainty, and measurement noise. Strictly speaking, model validation is generally impossible (or at least very

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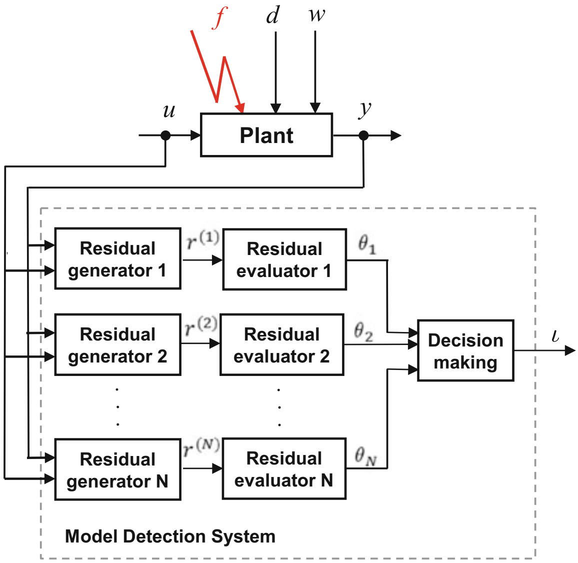

Fig. 4.1 Basic model detection setup

challenging), because it would involve to checking that the model can describe *any* input–output behaviour of the physical plant. Therefore, a closer related term to model detection is *model invalidation*, which relies on the trivial fact that a model can be invalidated (i.e., it does not fit with the input and output data) on the basis of a *single* input–output data set. In this sense, the invalidation of $N-1$ models can be seen as part of the model detection task.

A typical model detection setting is shown in Fig. 4.1. A bank of N residual generation filters (or residual generators) is used, with $r^{(i)}(t)$ being the output of the i-th residual generator. The i-th component θ_i of the N-dimensional evaluation vector θ usually represents an approximation of $\|r^{(i)}\|_2$, the $\mathcal{L}_2$- or ℓ_2-norm of $r^{(i)}$. The i-th component of the N-dimensional decision vector ι is set to 0 if $\theta_i \leq \tau_i$ and 1 otherwise, where τ_i is a suitable threshold. The j-th model is "detected" if $\iota_j = 0$ and $\iota_i = 1$ for all $i \neq j$. It follows that model detection can be interpreted as a particular type of week fault isolation with N signature vectors, where the N-dimensional j-th signature vector has all elements set to one, excepting the j-th entry which is set to zero. An alternative decision scheme can also be devised if θ_i can be associated with a distance function from the current model to the i-th model. In this case, ι is a scalar, set to $\iota = j$, where j is the index for which $\theta_j = \min_{i=1:N} \theta_i$. Thus, the decision scheme selects that model j which best fits with the current model characterized by the measured input and output data.

The underlying synthesis techniques of model detection systems rely on multiple-model descriptions of physical fault cases of the form (2.22). Since different degrees of performance degradations can be easily described via multiple models, model detection techniques have potentially the capability to address certain fault identification aspects too.

4.2 Residual Generation

Assume we have N LTI models of the form (2.22), where for $j = 1, ..., N$, the j-th model is specified in the input–output form

$$\mathbf{y}^{(j)}(\lambda) = G_u^{(j)}(\lambda)\mathbf{u}^{(j)}(\lambda) + G_d^{(j)}(\lambda)\mathbf{d}^{(j)}(\lambda) + G_w^{(j)}(\lambda)\mathbf{w}^{(j)}(\lambda) \,. \tag{4.1}$$

We further assume that the N models originate from a common underlying system with $y(t) \in \mathbb{R}^p$, the measurable output vector, and $u(t) \in \mathbb{R}^{m_u}$, the known control input. Therefore, $y^{(j)}(t) \in \mathbb{R}^p$ is the output vector of the j-th system with the control input $u^{(j)}(t) \in \mathbb{R}^{m_u}$, disturbance input $d^{(j)}(t) \in \mathbb{R}^{m_d^{(j)}}$, and noise input $w^{(j)}(t) \in \mathbb{R}^{m_w^{(j)}}$, respectively, and $G_u^{(j)}(\lambda)$, $G_d^{(j)}(\lambda)$, and $G_w^{(j)}(\lambda)$ are the TFMs from the corresponding plant inputs to outputs. We assume that all models are controlled with the same control inputs $u^{(j)}(t) := u(t)$, but the disturbance and noise inputs $d^{(j)}(t)$ and $w^{(j)}(t)$, respectively, may differ for each component model. For complete generality of our problem formulations, we will allow that these TFMs are general rational matrices (proper or improper) for which we will not a priori assume any further properties.

Residual generation for model detection is performed using N linear residual generators which process the measurable system outputs $y(t)$ and known control inputs $u(t)$ and generate N residual signals $r^{(i)}(t)$, $i = 1, \ldots, N$, which serve for decision-making on which one of the models best matches the current input–output measurement data. As already mentioned, model detection can be interpreted as a week fault isolation problem with an $N \times N$ structure matrix S having all its elements equal to one, excepting those on its diagonal which are zero. The task of model detection is thus to find out the model which best matches the measurements of outputs and inputs, by comparing the resulting decision vector ι with the set of signatures associated to each model and coded in the columns of S. The residual generation filters in their implementation form are described by the input–output relations

$$\mathbf{r}^{(i)}(\lambda) = Q^{(i)}(\lambda) \begin{bmatrix} \mathbf{y}(\lambda) \\ \mathbf{u}(\lambda) \end{bmatrix}, \; i = 1, \ldots, N \,, \tag{4.2}$$

where y is the *actual* measured system output, being one of the system outputs generated by the multiple model (4.1). The TFMs $Q^{(i)}(\lambda)$, for $i = 1, \ldots, N$, must be proper and stable. The overall model detection filter has the form

$$Q(\lambda) = \begin{bmatrix} Q^{(1)}(\lambda) \\ \vdots \\ Q^{(N)}(\lambda) \end{bmatrix} . \tag{4.3}$$

The dimension q_i of the residual vector component $r^{(i)}(t)$ can be chosen always one, but occasionally values $q_i > 1$ may provide better sensitivity to model mismatches.

Assuming $y(t) = y^{(j)}(t)$, the residual signal component $r^{(i)}(t)$ in (4.2) generally depends on all system inputs $u^{(j)}(t)$, $d^{(j)}(t)$, and $w^{(j)}(t)$ via the system output $y^{(j)}(t)$. The *internal form* of the i-th filter driven by the j-th model is obtained by replacing in (4.2) $\mathbf{y}(\lambda)$ with $\mathbf{y}^{(j)}(\lambda)$ from (4.1) and $\mathbf{u}(\lambda)$ with $\mathbf{u}^{(j)}(\lambda)$. To make explicit the dependence of $r^{(i)}$ on the j-th model, we will use $\widetilde{r}^{(i,j)}$, to denote the i-th residual output for the j-th model. After replacing in (4.2), $\mathbf{y}(\lambda)$ with $\mathbf{y}^{(j)}(\lambda)$ from (4.1), and $\mathbf{u}(\lambda)$ with $\mathbf{u}^{(j)}(\lambda)$, we obtain

$$\begin{aligned}\widetilde{\mathbf{r}}^{(i,j)}(\lambda) &:= R^{(i,j)}(\lambda)\begin{bmatrix}\mathbf{u}^{(j)}(\lambda)\\ \mathbf{d}^{(j)}(\lambda)\\ \mathbf{w}^{(j)}(\lambda)\end{bmatrix}\\ &= R_u^{(i,j)}(\lambda)\mathbf{u}^{(j)}(\lambda) + R_d^{(i,j)}(\lambda)\mathbf{d}^{(j)}(\lambda) + R_w^{(i,j)}(\lambda)\mathbf{w}^{(j)}(\lambda)\,,\end{aligned} \tag{4.4}$$

with $R^{(i,j)}(\lambda) := \left[\, R_u^{(i,j)}(\lambda) \,\middle|\, R_d^{(i,j)}(\lambda) \,\middle|\, R_w^{(i,j)}(\lambda) \,\right]$ defined as

$$\left[\, R_u^{(i,j)}(\lambda) \,\middle|\, R_d^{(i,j)}(\lambda) \,\middle|\, R_w^{(i,j)}(\lambda) \,\right] := Q^{(i)}(\lambda)\left[\begin{array}{c|c|c} G_u^{(j)}(\lambda) & G_d^{(j)}(\lambda) & G_w^{(j)}(\lambda)\\ I_{m_u} & 0 & 0\end{array}\right]. \tag{4.5}$$

For a successfully designed set of filters $Q^{(i)}(\lambda)$, $i = 1, \ldots, N$, the corresponding internal representations $R^{(i,j)}(\lambda)$ in (4.4) are also a proper and stable.

4.3 Model Detectability

The concept of model detectability concerns with the sensitivity of the components of the residual vector to individual models from a given collection of models. Assume that we have N models, with the j-th model specified in the input–output form (4.1). For the discussion of the model detectability concept, we will assume that no noise inputs are present in the models (4.1) (i.e., $w^{(j)} \equiv 0$ for $j = 1, \ldots, N$). For model detection purposes, N filters of the form (4.2) are employed. It follows from (4.4) that the i-th component $r^{(i)}$ of the residual r is sensitive to the j-th model provided

$$R^{(i,j)}(\lambda) := \left[\, R_u^{(i,j)}(\lambda)\ R_d^{(i,j)}(\lambda) \,\right] \neq 0\,. \tag{4.6}$$

We can associate to the $N \times N$ blocks $R^{(i,j)}(\lambda)$ defined in (4.6), the $N \times N$ structure matrix S_R with the (i, j)-th element set to 1 if $R^{(i,j)}(\lambda) \neq 0$ and set to 0 if $R^{(i,j)}(\lambda) = 0$. As already mentioned, model detection can be interpreted as a week fault isolation problem with an $N \times N$ structure matrix S having all its elements equal to one, excepting those on its diagonal which are zero. Having this analogy in mind, we introduce the following concept of model detectability.

Definition 4.1 The multiple model defined by the N component systems (4.1) with $w^{(j)} \equiv 0$ for $j = 1, \dots, N$, is *model detectable* if there exist N filters of the form (4.2), such that $R^{(i,j)}(\lambda)$ defined in (4.6) fulfills $R^{(i,i)}(\lambda) = 0$ for $i = 1, \dots, N$ and $R^{(i,j)}(\lambda) \neq 0$ for all $i, j = 1, \dots, N$ such that $i \neq j$.

The following result characterizes the model detectability property.

Theorem 4.1 *The multiple model defined by the N component systems (4.1) with $w^{(j)} \equiv 0$ for $j = 1, \dots, N$, is model detectable if and only if for $i = 1, \dots, N$*

$$\text{rank}\,[\, G_d^{(i)}(\lambda)\;\; G_d^{(j)}(\lambda)\;\; G_u^{(i)}(\lambda) - G_u^{(j)}(\lambda)\,] > \text{rank}\; G_d^{(i)}(\lambda)\;\; \forall j \neq i\,. \tag{4.7}$$

Proof For the proof of necessity, assume the model detectability of the multiple model (4.1) and, for $i = 1, \dots, N$, let $Q^{(i)}(\lambda)$ be a corresponding set of filters for model detection. Let us partition the columns of each $Q^{(i)}(\lambda)$ as

$$Q^{(i)}(\lambda) = \left[\, Q_y^{(i)}(\lambda)\;\; Q_u^{(i)}(\lambda)\,\right],$$

to correspond to the two filter inputs $y(t)$ and $u(t)$ in (4.2). The conditions to achieve the i-th specification are $R^{(i,i)}(\lambda) = 0$ and $R^{(i,j)}(\lambda) \neq 0$ for all $j \neq i$. With the above partitioning of $Q^{(i)}(\lambda)$, this comes down to

$$\begin{aligned} Q_y^{(i)}(\lambda)G_u^{(i)}(\lambda) + Q_u^{(i)}(\lambda) &= 0\,, \\ Q_y^{(i)}(\lambda)G_d^{(i)}(\lambda) &= 0 \end{aligned}$$

and

$$\left[\, Q_y^{(i)}(\lambda)G_u^{(j)}(\lambda) + Q_u^{(i)}(\lambda)\;\; Q_y^{(i)}(\lambda)G_d^{(j)}(\lambda)\,\right] \neq 0, \quad \forall j \neq i\,.$$

Since $Q_u^{(i)}(\lambda) = -Q_y^{(i)}(\lambda)G_u^{(i)}(\lambda)$, after some manipulations, we obtain the conditions to be satisfied by $Q_y^{(i)}(\lambda)$

$$\begin{aligned} Q_y^{(i)}(\lambda)G_d^{(i)}(\lambda) &= 0\,, \\ Q_y^{(i)}(\lambda)\left[\, G_u^{(j)}(\lambda) - G_u^{(i)}(\lambda)\quad G_d^{(j)}(\lambda)\,\right] &\neq 0, \quad \forall j \neq i\,. \end{aligned}$$

For each $j \neq i$, the second condition requires that there exists at least one column in $\left[\, G_u^{(j)}(\lambda) - G_u^{(i)}(\lambda)\quad G_d^{(j)}(\lambda)\,\right]$, say $g(\lambda)$, for which $Q_y^{(i)}(\lambda)g(\lambda) \neq 0$. This condition together with $Q_y^{(i)}(\lambda)G_d^{(i)}(\lambda) = 0$ is equivalent with the fault detectability condition (see Theorem 3.1)

$$\text{rank}\,[\, G_d^{(i)}(\lambda)\;\; g(\lambda)\,] > \text{rank}\; G_d^{(i)}(\lambda)\,.$$

It is easy to observe that this condition implies (4.7).

To prove the sufficiency of (4.7), we determine a bank of N filters $Q^{(i)}(\lambda)$, $i = 1, \ldots, N$ to solve the model detection problem. For this, we construct the i-th filter $Q^{(i)}(\lambda)$ such that the corresponding

$$R^{(i,j)}(\lambda) := Q^{(i)}(\lambda) \begin{bmatrix} G_u^{(j)}(\lambda) & G_d^{(j)}(\lambda) \\ I_{m_u} & 0 \end{bmatrix}$$

satisfies $R^{(i,i)}(\lambda) = 0$ and $R^{(i,j)}(\lambda) \neq 0 \; \forall j \neq i$. We show that we can determine $Q^{(i)}(\lambda)$ in the stacked form

$$Q^{(i)}(\lambda) = \begin{bmatrix} Q_1^{(i)}(\lambda) \\ \vdots \\ Q_N^{(i)}(\lambda) \end{bmatrix}, \tag{4.8}$$

where each row $Q_j^{(i)}(\lambda)$ is a stable scalar output filter which satisfies

$$Q_j^{(i)}(\lambda) \begin{bmatrix} G_u^{(i)}(\lambda) & G_d^{(i)}(\lambda) \\ I_{m_u} & 0 \end{bmatrix} = 0 \tag{4.9}$$

and, additionally for $j \neq i$

$$Q_j^{(i)}(\lambda) \begin{bmatrix} G_u^{(j)}(\lambda) & G_d^{(j)}(\lambda) \\ I_{m_u} & 0 \end{bmatrix} \neq 0\,. \tag{4.10}$$

For convenience, we set $Q_i^{(i)}(\lambda) = 0$ (a null row vector). This construction of $Q^{(i)}(\lambda)$ in (4.8), ensures with the help of the condition (4.10) that the corresponding $R^{(i,j)}(\lambda) \neq 0 \; \forall j \neq i$.

To determine $Q_j^{(i)}(\lambda)$ for $j \neq i$, we observe that the condition (4.7) can be interpreted as an extended fault detectability condition for (fictive) fault inputs corresponding to an input–output faulty system defined by the triple of TFMs

$$\left\{ G_u^{(i)}(\lambda), G_d^{(i)}(\lambda), \left[\, G_d^{(j)}(\lambda) \;\; G_u^{(i)}(\lambda) - G_u^{(j)}(\lambda) \,\right] \right\}$$

from suitably defined control, disturbance and fault inputs, respectively. It follows, that there exists $Q_j^{(i)}(\lambda)$ such that (4.9) is fulfilled and

$$Q_j^{(i)}(\lambda) \begin{bmatrix} G_d^{(j)}(\lambda) & G_u^{(i)}(\lambda) - G_u^{(j)}(\lambda) \\ 0 & 0 \end{bmatrix} \neq 0\,.$$

Taking into account (4.9), this condition can be rewritten in the equivalent form (4.10), which in turn implies that $R^{(i,j)}(\lambda) \neq 0$ for $j \neq i$. ■

4.4 Model Detection Problems

In this section we formulate the exact and approximate synthesis problems of model detection filters for the collection of N LTI systems (4.1). As in the case of the EFDIP or AFDIP, we seek N linear residual generators (or model detection filters) of the form (4.2), which process the measurable system outputs $y(t)$ and known control inputs $u(t)$ and generate the N residual signals $r^{(i)}(t)$ for $i = 1, \ldots, N$. These signals serve for decision-making by comparing the pattern of fired and not fired residuals with the signatures coded in the columns of the associated standard $N \times N$ structure matrix S with zeros on the diagonal and ones elsewhere. The standard requirements for the TFM of the overall filter $Q(\lambda)$ in (4.3) are *properness* and *stability*. For practical purposes, the order of the overall filter $Q(\lambda)$ must be as small as possible. A least-order $Q(\lambda)$ can be usually achieved by employing N scalar output least-order filters (see Sect. 6.2).

In analogy to the formulations of the EFDIP and AFDIP, we use the internal form of the i-th residual generator (4.4) to formulate the basic model detection requirements. Independently of the presence of the noise inputs $w^{(j)}$, we will target that the i-th residual is exactly decoupled from the i-th model if $w^{(i)} \equiv 0$ and sensitive to the j-th model, for all $j \neq i$. These requirements can be easily translated into algebraic conditions using the internal form (4.4) of the i-th residual generator:

$$\begin{array}{ll} (i) & [\, R_u^{(i,i)}(\lambda) \;\; R_d^{(i,i)}(\lambda)\,] = 0, \quad i = 1, \ldots, N\,, \\ (ii) & [\, R_u^{(i,j)}(\lambda) \;\; R_d^{(i,j)}(\lambda)\,] \neq 0, \quad \forall j \neq i, \text{ with } [\, R_u^{(i,j)}(\lambda) \;\; R_d^{(i,j)}(\lambda)\,] \text{ stable.} \end{array} \tag{4.11}$$

Here, (i) is the *model decoupling condition* for the i-th model in the i-th residual component, while (ii) is the *model sensitivity condition* of the i-th residual component to all models, excepting the i-th model. In the case when condition (i) cannot be fulfilled (e.g., due to lack of sufficient measurements), some (or even all) components of $d^{(i)}(t)$ can be redefined as noise inputs and included in $w^{(i)}(t)$.

In what follows, we formulate two model detection problems which are addressed in this book.

4.4.1 Exact Model Detection Problem

The standard requirement for solving the *exact model detection problem* (EMDP) is to determine for the multiple model (4.1), in the absence of noise input (i.e., $w^{(j)} \equiv 0$ for $j = 1, \ldots, N$), a set of N proper and stable filters $Q^{(i)}(\lambda)$ such that, for $i = 1, \ldots, N$, the conditions (4.11) are fulfilled. These conditions are similar to the model detectability requirement and lead to the following solvability condition:

Theorem 4.2 *For the multiple model (4.1) with $w^{(j)} \equiv 0$ for $j = 1, \ldots, N$, the EMDP is solvable if and only if the multiple model (4.1) is model detectable.*

Proof For each i, the conditions (4.11) can be fulfilled provided the multiple model (4.1) is model detectable. To ensure the stability of $Q^{(i)}(\lambda)$, $R_u^{(i,j)}(\lambda)$ and $R_d^{(i,j)}(\lambda)$, the filter with TFM $Q^{(i)}(\lambda)$ can be replaced by $M^{(i)}(\lambda)Q^{(i)}(\lambda)$, where

$$(M^{(i)}(\lambda))^{-1}N^{(i)}(\lambda) = [\, Q^{(i)}(\lambda) \;\; R_u^{(i,1)}(\lambda) \;\; R_d^{(i,1)}(\lambda) \;\; \cdots \;\; R_u^{(i,N)}(\lambda) \;\; R_d^{(i,N)}(\lambda)\,]$$

is a stable left coprime factorization. ■

4.4.2 Approximate Model Detection Problem

The effects of the noise input $w^{(i)}(t)$ can usually not be fully decoupled from the residual $r^{(i)}(t)$. In this case, the basic requirements for the choice of $Q^{(i)}(\lambda)$ can be expressed as achieving that the residual $r^{(i)}(t)$ is influenced by all models in the multiple model (4.1), while the influence of the i-th model is only due to the noise signal $w^{(i)}(t)$ and is negligible. For the *approximate model detection problem* (AMDP) the following additional conditions to (4.11) have to be fulfilled:

$$\begin{array}{ll} (iii) & R_w^{(i,i)}(\lambda) \approx 0, \quad \text{with} \;\; R_w^{(i,i)}(\lambda) \;\; \text{stable;} \\ (iv) & R_w^{(i,j)}(\lambda) \;\; \text{stable} \; \forall j \neq i. \end{array} \tag{4.12}$$

Here, (iii) is the *attenuation condition* of the noise input.

The solvability conditions of the formulated AMDP can be easily established:

Theorem 4.3 *For the multiple model (4.1) the AMDP is solvable if and only the EMDP is solvable.*

Proof We can always determine a solution of the EMDP with $Q(\lambda)$ in the form (4.3), such that additionally the resulting $R_w^{(i,j)}(\lambda)$ are stable for $i, j = 1, \ldots, N$. Moreover, by rescaling $Q^{(i)}(\lambda)$ with a constant factor γ_i, the norm of $R_w^{(i,i)}(\lambda)/\gamma_i$ can be made arbitrarily small. The necessity is trivial, because any solution of the AMDP is also a solution of the EMDP. ■

4.5 Threshold Selection

Similar to the performance requirements for FDD systems, a well-designed model detection system as that in Fig. 4.1, must fulfill standard performance requirements as timely and unequivocal identification of a *single* model out of N candidate models which best fits with the input–output measurements. Assume that we use N residual evaluation signals $\theta_i(t), i = 1, \ldots, N$, where $\theta_i(t)$ is an approximation of $\|r^{(i)}\|_2$ (see Sect. 3.6), and for each i let τ_i be the corresponding threshold. For the unequivocal identification of the i-th model, we must have $\theta_i(t) \leq \tau_i$ and $\theta_j(t) > \tau_j$ for all $j \neq i$,which corresponds to a binary signature with $N - 1$ ones and single zero in the i-th element. A false alarm occurs when, due to the effects of noise inputs, the j-th model (a "false" one) is identified as the best matching one instead the i-th model (the "true" one). A missed detection occurs, for example, when $\theta_i(t) > \tau_i$ for all $i = 1, \ldots, N$, or when the resulting binary signature contains several zero entries. In both of these cases, no unequivocal model identification can take place.

In what follows, we discuss the choice of the decision thresholds τ_i, $i = 1, \ldots, N$ to be used in the model detection schemes, such that false alarms and missed detections can be avoided. For $j = 1, \ldots, N$, let $\mathcal{U}^{(j)}$, $\mathcal{D}^{(j)}$ and $\mathcal{W}^{(j)}$ be the classes of control inputs $u^{(j)}$, disturbance inputs $d^{(j)}$ and noise inputs $w^{(j)}$, respectively, which are relevant for a model detection application. For example, $\mathcal{U}^{(j)}$ is the class of nonzero control inputs with bounded variations, $\mathcal{D}^{(j)}$ may be the class of disturbance signals with bounded variations for the j-th model, while $\mathcal{W}^{(j)}$ may be the class of white noise signals of given maximal amplitude and covariance for the j-th model. We consider the selection of the threshold τ_i, which is instrumental for the discrimination of the i-th model from the rest of models.

To account for the dependence of the evaluation signal $\theta(t)$ of the input variables $u^{(j)} \in \mathcal{U}^{(j)}$, $d^{(j)} \in \mathcal{D}^{(j)}$, and $w^{(j)} \in \mathcal{W}^{(j)}$ and of the corresponding time response of the output signal $y^{(j)}$ of the j-th model up to the time moment t, we will indicate this dependence explicitly as $\theta(t, u^{(j)}, d^{(j)}, w^{(j)}, y^{(j)})$. Assume that the i-th model is the current model (to be detected) and $y^{(i)}$ is the corresponding time response of the i-th model output. The requirement for no false alarms in recognizing the i-th model leads to a lower bound for τ_i, representing the i-th *false alarm bound*

$$\tau_f^{(i)} := \sup_{\substack{t\in[0,t_m] \\ u^{(i)}\in\mathcal{U}^{(i)} \\ d^{(i)}\in\mathcal{D}^{(i)} \\ w^{(i)}\in\mathcal{W}^{(i)}}} \theta_i(t, u^{(i)}, d^{(i)}, w^{(i)}, y^{(i)}) \,, \tag{4.13}$$

where t_m is the maximum signal monitoring time. We can define the i-th *detection bound* as the least of the $N - 1$ lower bounds of the evaluation signal for any other current model different of the i-th model:

$$\tau_d^{(i)} := \min_{j\neq i} \inf_{\substack{t\in[0,t_m] \\ u^{(j)}\in\mathcal{U}^{(j)} \\ d^{(j)}\in\mathcal{D}^{(j)} \\ w^{(j)}\in\mathcal{W}^{(j)}}} \theta_i(t, u^{(j)}, d^{(j)}, w^{(j)}, y^{(j)}) \,. \tag{4.14}$$

It is usually assumed, that the choice of the i-th filter $Q^{(i)}(\lambda)$, can be done such that $\tau_f^{(i)} < \tau_d^{(i)}$, which ensures that the threshold τ_i can be chosen such that

$$\tau_f^{(i)} < \tau_i \leq \tau_d^{(i)} \,.$$

With such a choice for all N threshold values τ_i, $i = 1, \ldots, N$, it is possible to guarantee the lack of false alarms and missed detections, and thus ensure the unequivocal identification of any of the N models. Note that, with a suitable rescaling of the N component filters $Q^{(i)}(\lambda)$, $i = 1, \ldots, N$, it is possible to arrange that all thresholds can be taken equal to a common value $\tau_i = \tau$, for $i = 1, \ldots, N$. If the condition $\tau_f^{(i)} < \tau_d^{(i)}$ cannot be enforced, then no unequivocal identification of the i-th model is possible. A possible remedy in such cases is to redefine the set of models, by including only models which are sufficiently "far" from each other.

Remark 4.1 In practical applications the chosen N models usually form a representative set of models, but frequently do not cover the entire set of possible models, which can even be infinite due to continuous ranges of variation of fault parameters (e.g., loss of efficiency degree). Thus, a typical operation mode for any model detection setup is with the current model lying "in between" two candidate models. To handle this situation and to avoid false alarms and missed detections, an alternative decision scheme can be employed, where the i-th model is selected, provided the corresponding evaluation signal $\theta_i < \theta_j$ for all $j \neq i$. Although this decision scheme "always" works, still wrong identifications may result, because of the difficulty to correctly map (via a set of N filters $Q^{(i)}(\lambda)$, $i = 1, \ldots, N$), the "nearness" of two models, as for example, the i-th and j-th models, into the "nearness" of the corresponding evaluations θ_i and θ_j. □

4.6 Notes and References

The term *model detection* has been apparently used for the first time in [142]. Model validation and also model invalidation have been discussed in [104, 111] in the context of model identification for robust control. The definition of *model detectability* appears to be new.

Two *model selection* problems have been formulated, in a stochastic setting, by Baram in [4], which are very similar to the model detection problem considered in this book. These problems consist of the selection of a model out of N given models, which is the closest to or exactly matches the "true" model. Stochastic measures of closeness are used to discriminate between two models. The use of Kalman filters to perform model selection has been discussed by Willsky [161]. The selection of adequate models for the purpose of *multiple-model adaptive control* (MMAC) is discussed in [41].

The use of multiple model techniques for fault detection and isolation has been considered in several publications, see—for example—[16, 158]. The exact model detection problem has been formulated and solved in [142]. The formulation of the approximate model detection problem is similar to several formulations based on the use of Kalman filters as model detection filters, where all unknown inputs (noise and disturbances) are assumed to be white noise signals [84, 90, 161]. The model detection approach discussed in this book is a viable alternative to Kalman filter-based approaches used for switching or interpolating among different controllers for MMAC (see—for example, [2]) or in *interacting multiple model* (IMM) Kalman filters-based reconfiguration schemes [70]. The main advantages of using model detection filters over various Kalman filter-based techniques are the ability of formers to exactly decouple the influence of nonstochastic disturbances from the residual signals and their significantly lower dynamical orders. The first of these advantages has been noted in a related approach based on unknown-input observers proposed in [158].

The decision scheme based on the choice of that model for which the corresponding evaluation signal has the least value among all evaluation signals has been advocated in [90], where the Narendra-type residual evaluation filter has also been introduced.