Early Child Spatial Development: A Teaching Experiment with Programmable Robots

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Abstract This contribution addresses young children development of spatial competences, and investigates the didactic potentialities offered by a programmable robot. The theoretical framework addresses the delicate relationship between space as lived in everyday experience versus space as a mathematical notion, and takes a multimodal perspective on mathematics teaching and learning. An experimental study has been conducted in kindergarten school. The qualitative data analysis of video-recordings constitutes the background against which children spatial development is discussed.

Keywords Multimodality • Kindergarthen mathematics • Spatial thinking • Programmable robots

Introduction

Attention on early years mathematics is emerging in recent times in research, as witnessed by the new Thematic Working Group in CERME, (http://www.cerme8.metu.edu.tr/wgpapers/wg13_papers.html), and the ICMI Study 23 Conference (http://www.umac.mo/fed/ICMI23/). As in the latter case, the focus of attention is placed in particular on the development of whole numbers competences, which are fundamental steps for children mathematics education. Less attention is given to other competences, such as the spatial ones.

Spatial competences develop through a complex process, requiring long-time experiences in meaningful contexts. Kindergarten and the first year of primary school are the proper places for these experiences, constituting the base on which the learning of geometry can be grounded, first as modeling of spatial properties, and then as theoretical elaboration specific on the mathematics field. However, especially when starting primary school, spatial competences are often overlooked (at least in Italy, but this may not be an isolated case), being the major efforts put on numbers.

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This chapter focuses on children development of spatial competences, and explores the didactic potentialities offered by programmable robots. Cognitive aspects will be on the foreground, and in particular the delicate relationship between space as lived in everyday experience versus space as a mathematical notion will be addressed. On the background of psychological results on spatial conceptualisation in children, and taking a multimodal perspective on mathematics teaching and learning, an experimental study has been conducted in kindergarten school. The study was based on the teaching experiment methodology and explored the didactic potentialities offered by a programmable robot with a bee-shape, with respect to children development of spatial competences. In the following, after a theoretical discussion on young children spatial thinking development, the methodology of the teaching experiment will be described, and a case study data analysis will be provided, from video-recordings and collected written materials of the classroom activities in a kindergarten school.

The Development of Spatial Thinking in Early Years

The complexity of children spatial conceptualization processes has been pointed out by research in psychology and education for several years. Great differences in different theorizing in the field prevent researchers from reducing these processes to simple and linear models of learning, based on rigid pre-determined steps. Concerning spatial relationships, we can consider three different fields of experiences, which correspond to three different kinds of space, requiring each specific perceptive and exploration modalities (Bartolini Bussi 2008):

- The *body space*, that is the internal reference frame relative to the awareness of body movements, its parts, and to the construction of the body schema;
- *Specific external spaces*, including different kinds of living spaces (the house, the town, the school,. . .) and different representative spaces (the sheet of paper, squared papers, the computer screen, . . .);
- *Abstract spaces*, that are the geometrical models developed within mathematics science in its history.

The first two kinds of spaces refer to actual spaces in real world, the latter one belongs to the world of mathematics. Such a categorization must not be thought as a sort of hierarchical scale, or as a developmental sequence. On the contrary, according to Lurçat (1980): “it appears difficult to imagine a development in which the body schema is constructed before, to allow then the knowledge of external world” (p. 30, translation by the author). As a matter of fact, several studies agree in recognizing a fundamental role to the experiences that the child makes both in his/her family and in specific educational settings, and suggest to go beyond linear models, which position abstract space at the end of a developmental process (in the stage of formal operations, in the Piagetian case). A discussion in this direction may be found in Lurçat (1980), and in Donaldson (2010).

Recent strands in cognitive sciences place perception and everyday experiences with the body as grounding pillars for more abstract knowledge conceptualization, included the mathematical knowledge. In particular, the embodied cognition perspective (Lakoff and Núñez 2000) proposes a model for the “embodied mind”, as a radical criticism of the dualism between the mind and the body of classical cognitivist approaches.

If mathematics is no longer a purely “matter of head”, it becomes of paramount importance to carry out mathematical activities in suitable contexts in which children can interact with different kinds of space and spatial thinking. Concerning the external space, we can distinguish further between *macro-spaces* and *micro-spaces* (Bartolini Bussi 2008):

- *macro-spaces* are those in which the subject is embedded (the subject being part of the macro-space); their exploration is carried out through movement, and their perception is only local and partial, requiring usually to coordinate different points of views;
- *micro-spaces* are external to the subject; their exploration is carried out through manipulation, and their perception is global.

A park is an example of macro-space, whereas a sheet of paper and a book page are examples of micro-space. As an intermediate category, called *meso-space*, we can consider the big posters often used in classroom for group-work: children can enter into them, but also look at them at distance. The essential aspects in this distinction are the different modalities of perception and exploration: the school garden, for instance, can be an example of macro-space—when the child is playing within it—or of micro-space, when the child is observing it from a window above.

The body space and the external spaces share fundamental differences with respect to abstract spaces: as a matter of fact, they can be perceived and explored, and are featured by *fundamental directions* (vertical and horizontal) and by *typical objects* (e.g. a door in a room, a fridge in a kitchen). On the contrary, abstract spaces (like the geometrical ones) are isotropous and homogeneous, i.e. do not have any privileged directions, nor special points. These features may be sources for difficulties for students, when facing tasks with figures in non-prototypical positions, as in the assessing item reported in Fig. 1 from Italian National test INVALSI 2012–2013, grade 5): *Four isosceles triangles are cut from a paper sheet, with the same base and different heights. In each case, the height of the triangle is the double of the previous one. In triangle A the height measures 2 cm. Which is the total length of the paper sheet?*

Among the advantages of introducing reference systems like the Cartesian one in the geometrical space, we find the introduction of privileged points (in particular, the origin point) and special directions (those parallel to the axis).

In mathematics, reference systems are *objective* or *absolute*, in the sense that they do not depend of the position of the subject using them. Objective references are the product of the historical-cultural development of society and have to be introduced by the teacher starting from the subjective references (which depend on our position in the external space).

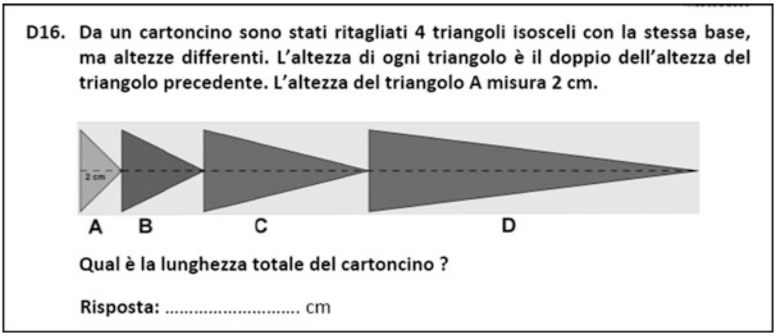

Fig. 1 Non prototypical positions in an Italian National Assessment item (INVALSI 2012–2013), grade 5

According to Lurçat (1980), our subjective references depend heavily not only to external objects (e.g. a door in a room), but also on our ways to project our body schema into objects. Subjective reference systems can be *egocentric*, if the description is provided according to the subject position (e.g. "to my left") or *allocentric*, when the reference is made with respect to another object or person (e.g. "to the left of the house"). Egocentric systems are the first to develop in children, but not the only ones. While Piaget and Inhelder (1956) claimed that children until 8–9 years of age are incapable of decentralize with imagination and so of correctly using allocentric references, following studies have refuted this conclusion, and proved that also children aged 3 are able to decentralize, if faced with problems comprehensible to them (for a discussion, see Donaldson 2010). Being able to coordinate egocentric and allocentric perspectives constitutes an important competence for spatial and geometrical development, and in Italian curriculum is placed as a goal for Primary school (MIUR 2012). An example of task requiring this competence is reported in Fig. 2, again from the Italian National Assessment test. Two children are looking at an object from different positions and the students are asked what the girl is seeing, thus activating an allocentric perspective:

On the base of this discussion of results from psychology, we can ground the hypothesis that the reality faced by young children (and indeed, by all of us) is full of *cognitively-different* spatial contexts, which require different related specific competences. In order to reach this goal, Lurçat (1980) underlines the importance of choosing carefully the requests to the child in the spatial activity:

> . . .not all spatial behaviours necessarily imply a knowledge on space. In order to have knowledge, a suitable activity is necessary: for instance, going in a place, locating objects, positioning in the space of places and objects [. . .]. As in other psychical fields, it does not exist an age for the development, which can be considered independent from the concrete conditions of existence (p. 16, translation by the author).

An educational implication of this perspective is that in order to develop the necessary different spatial competences, children need to be involved since their early childhood in dedicated activities with dedicated task design. For instance, in

Fig. 2 Allocentric perspective required in an Italian National Assessment item (INVALSI 2012–2013), grade 2

order to foster the passage from ego-based to allocentric references and lay the foundations of objective reference systems, activities on the change of points of view, such as the realization of maps of familiar places, can be proposed already at the kindergarten.

Along with meaningful experiences in different spaces, languages constitute a second fundamental set of sources of knowledge, including verbal and non-verbal means of communications.

The key role of verbalization not only as a communicative means but also for thinking processes has widely been discussed in Vygotskian studies (e.g. Vygotsky 1934), and stressed by Lurçat (1980) concerning spatial development:

> It seems hard to separate, in the appropriation of the environment realized by the young child, these two sources of knowledge, the one practice, the other verbal, since both converge early in the first months of life (pp. 15–16, translation by the author).

For mathematics, we know the importance of symbols and graphical representations of various kinds—in particular for geometry, of geometrical figures and Cartesian plane systems. Each of these representations situates in a specific way in the external space of the child: usually, school lessons heavily exploit bi-dimensional micro-spaces, such as the blackboard, the book sheet, or more recently the computer/tablet screen. The passage from experience and perception in the tri-dimensional (macro-) space to these representation spaces is a very complex process, so far little studied in literature. Also at primary school, this passage is often taken for granted and in many cases written representations are used but not problematized.

In such a passage, on the one hand the use of artefacts can be exploited as didactic resources in the development of children spatial competences, and on the other hand gestures and embodied means of expression may play an important role in synergy with verbal language, according to a multimodal perspective (Arzarello et al. 2009; Bazzini et al. 2010; Sabena et al. 2012). The role of artefacts will be discussed in the next section.

The role of embodied resources such as gestures, gazes, and body postures in thinking processes (and of course in communicative ones) has been pointed out in psychological literature with cognitive and linguistic focus (McNeill 1992, 2005). The study of gestures and embodied resources in synergy with verbal language has gained a certain attention also in mathematics education, in an increasing variety of contexts, such as: students solving problems (Radford 2010), students and teachers interacting (Arzarello et al. 2009; Bazzini et al. 2010; Bazzini and Sabena 2015), the teacher's lectures (Pozzer-Ardenghi and Roth 2010), considering not only the semantic but also the logical aspects of mathematical thinking (Arzarello and Sabena 2014). For what concerns spatial tasks, *iconic and pointing gestures* come to the fore: iconic gestures are those ones which resembling the semantic content they refer to, and pointing gestures are usually performed with the index forefinger and have the function of indicating something in the actual context.

The Teaching Experiment: Methodology

On the base of the outlined theoretical frame, an experimental study has been planned and carried out in a kindergarten school in Northern Italy, with the goal of studying the didactical possibilities for children spatial conceptualization offered by programmable robot toys.

The study is based on the teaching-experiment methodology. The activities have been organized around a programmable robot[1] with a bee-shape (Fig. 3a), a technological artefact new to the children. The robot is a kind of tri-dimensional and touchable version of the well-known Logo turtle by Papert (1984), and its movement can be programmed through buttons placed on the upper part (Fig. 3b): they are four arrows for onward and backward steps, right and left turns, and a pause of one second. The robot bee can move on a plane with 15 cm-long steps (the same measure of its length). Steps are marked by a quick stop, which creates a silent pause with respect to the noise of the movement, and by the lightening of its eyes (see Fig. 4b). Pushing the green button "GO", the robot executes the previously programmed sequence. A specific button ("clear") allows the user to clear the memory from past commands.

[1]Bartolini Bussi and Baccaglini-Frank (2015) carried out a study with the same artefact in primary school, about the introduction of the definition of rectangle.

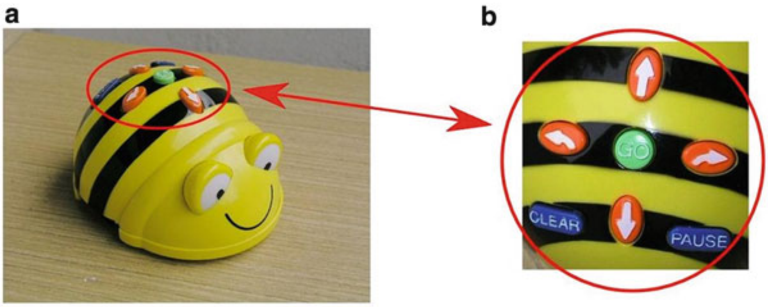

Fig. 3 (**a**, **b**) The programmable robot used in the teaching-experiment

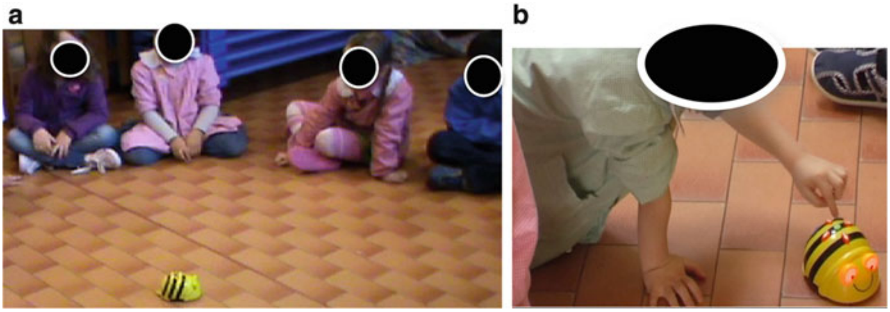

Fig. 4 (**a**, **b**) Initial exploration of the artefact in the meso-space

The teaching-experiments involved four classrooms of 5 years old children, and was carried out with the collaboration of the four teachers, four Master students in Primary school education, and the author. Being inserted in the usual school activities, the experiments had didactic as well as research goals.

From a didactic point of view, the activities had the general goals of promoting competences related to spatial thinking, but also problem-solving. These competences were linked to the use of a new artefact, in the context of exploring it through a playful environment. Concerning spatial thinking, the passage from egocentric to allocentric reference systems is particularly involved, in particular when the robot is not oriented parallel to and with the same orientation as the children. On the other hand, the activity of programming in advance the movements of the robot, and checking afterwards the consequences of the choices, by means of observing the obtained movement, offers a suitable context for stimulating and developing anticipation and control processes, which are at the base of successful problem-solving (Martignone and Sabena 2014).

The didactic dimension intertwines with the research one. The study had mainly an explorative character of the potentialities and the limits of the artefact-based activities with respect to the identified didactical goals. Such an analysis needs to consider the specific activities proposed to children, and the role of the teacher in their management.

The key-role of the teacher in using with success artefacts in the mathematics teaching and learning has been pointed out and stressed by Bartolini Bussi and Mariotti (2008):

> The role of the teacher is crucial, in fact the evolution of signs, principally related to the activity with artefacts, towards mathematics signs, is not expected to be neither spontaneous nor simple, and for this reason seems to require the guidance of the teacher (ibid., p. 755).

Adopting a Vygotskian perspective, Bartolini Bussi and Mariotti elaborate the Theory of Semiotic Mediation, according to which the fundamental elements of didactical activities involving artefacts are the signs that emerge when using the artefact, and above all the role of cultural mediator accomplished by the teacher when using the artefact as a *tool of semiotic mediation*: this expression refers to the fact that when using an artefact (for accomplishing a certain task) new meanings emerge. These meanings are linked to the use of the artefacts but can be general and can evolve under the guidance of the teacher:

> Any artefact will be referred to as a tool of semiotic mediation as long as it is (or it is conceived to be) intentionally used by the teacher to mediate a mathematical content through a design didactical intervention (ibid., p. 754).

An important didactic feature of this theory is the "mathematical discussion" (Bartolini Bussi 1998), in which the whole classroom is collectively engaged in discussing the personal meanings emerged from an activity, relating them—with the essential guidance of the teacher—to the mathematical signs.

The teaching experiment with the robot has been planned sharing the same Vygotskian view, assigning great relevance to the peer as well as teacher-students interaction, and focusing on the evolution of signs developed during the technology-based activities. Due to the young age, the specific mathematical contents have been limited, and the discussions have regarded more general competences, at the base of spatial and logical thinking.

The activities have been video-recorded and the obtained videos have been analysed in detail. Furthermore, children written drawings related to the activities have been collected and analysed.

The Teaching Experiment: Analysis

Children were organized in groups of about 10–12, with one or two bee-robots at disposal. For each group, the activities developed along 5–6 one-hour meetings,[2] for a period of about 1 month. Most of activities involved the whole group, with the

[2]In Italy, usually we use the term "lesson" starting from Primary school, were formal education begins (also with textbooks, notebooks, and so on). In kindergarten, activities unfold in a less formal way.

Fig. 5 Egocentric perspective kept during the comparison of steps lengths

coordination of the teacher, and only in some cases individual work was required (e.g. to produce a drawing).

The first meeting was always dedicated to the introduction of the new artefact. The initial exploration of the robot has been carried out letting the children play with the robot. In some groups, the activity was organized around a table, while in others children were sitting in a circle on the floor (see Fig. 4a): the resulting delimitation of space produced a sort of meso-space, since the children could globally perceive it with their sight, but also enter into it and explore it with their body.

One of the games played in this context was "sending the bee to my friend (name)". In this game, each child had to name a friend, and to program the bee so to be able to send it where stated. We observed that when programming, every child always started positioning herself/himself behind the robot (as in Fig. 4b). It is the most natural choice, since it keeps the cognitive burden low: in this way, in fact, the reference system introduced by the robot (allocentric system) is coincident with the child one (egocentric system). We kept therefore this choice in those activities focusing on more specific aspects of the artefact, such as estimating the length of the steps, compared with those of the teacher or of the children (see Fig. 5).

Other games required the imitation with one own body of some movements made by the robot, with or without verbal description. The imitation is simple if the child is oriented in the same way of the bee-robot (for instance, if the child is following the robot), because grounded on the ego-based reference system. When the robot is oriented differently with respect to the child, the task increases in difficulty, because it requires reproducing, during one's own movement, an external point of view. In other terms, it requires coordinating the egocentric system not only with an allocentric one, but with a mobile allocentric one: it is a coordination constantly in need of control and adjustments. In our experiences, verbalization has constituted an important supporting tool: when accompanying the bee-robot movement with a verbal description (such as 'onwards, onwards, onwards, turn right'), the task was more easily faced by children. However, verbal indications were of little help for children with difficulties in knowing right from left (a problem for which the bee-robot could not offer any support).

Fig. 6 'The bee-game': Ego-centric perspective to program the movement

With each group of children, at least a couple of meetings were dedicated to an activity on a poster showing a path to be travelled by the robot. The paths were all structured with lengths multiple of 15 cm (the exact dimension of the robot, and of its steps) and with right angle turns, so to be viable by the robot in an exact number of steps and rotations. These choices were meant to ask the children to program rotations, which were never activated in the explorative phase, avoiding problems provoked by non-perpendicular turnings—impossible to program with the bee-robot.

An example is the 'Bee game'[3] (Fig. 6), a sort of Snake and ladders game. The game setting facilitated the introduction of the rule of 'moving the bee only through its buttons' (and not pushing or rotating it with the hands, as the children were tempted to do. . .). In our intentions, the race setting would have also fostered the need of programming as many segments of the path as possible, in order to reach a farther place. For instance, if the first roll of the dice gives '3', the children have to program the sequence 'two onwards, turn left, one onward'. However, in our experiments the children did not fulfil this expectation. Indeed, in all groups children preferred to program one segment at a time: in the given example, programming two steps onwards, observing the robot movement, then programming one turn leftwards, observing the turn, and then programming the final two steps. Figure 6b shows a child while programming this last segment: again, the ego-based perspective is taken by the child in order to carry out the task.

Probably we missed the occasion of challenging the children, by introducing an additional rule, such as 'programming the robot sitting always in the black arrow place'. This request would have forced the children to coordinate their egocentric perspective with the moving perspective of the robot (allocentric for the children).

The activities were alternated with collective discussions, which constituted occasions for reflection on what happened. Discussion organized *before* to carry out new activities are of particular interest. In a group, a guided discussion

[3]In Italian the popular game Snake and ladders is called 'Gioco dell'oca' ('The goose game').

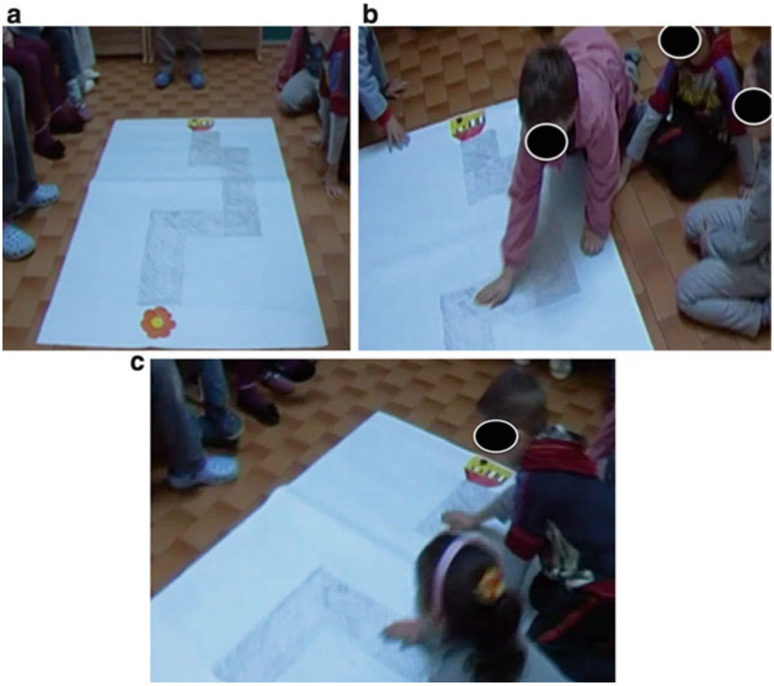

Fig. 7 (**a**–**c**) The setting of the activity “Let’s help the bee to reach the flower”

introduced the activity on the path. The bee-robot was not on the scene (Fig. 7a): the discussion constituted a moment of reflection for the children, during which the development of the spatial competences is realized by observing and describing the present scene, but also recalling past experiences with the artefact, and anticipating potential actions through imagination. We are going to analyse in greater details what happened.

The teacher guides the discussion with the goal of making the children to observe that the path is not linear. As a matter of fact, in the previous activity children moved the robot using only the “onward arrow”, without turns. The general goal of the activity is to make the students program more complex sequences involving turns, such as ‘forward-forward-turn left-forward’.

1. *Teacher*: Today we explore this (*looking at the poster*). What comes to your mind looking at this?
2. *Stefano*: It is a road
3. *Viviana*: A flower and a house
4. *Teacher*: And whose is the house?
5. *All the children*: The bees!
6. *Teacher*: And how is it this road? Is it straight?
7. *All children*: Noooo!
8. *Stefano*: It has some curves (*with his hand he is traveling the road,* Fig. 7b)

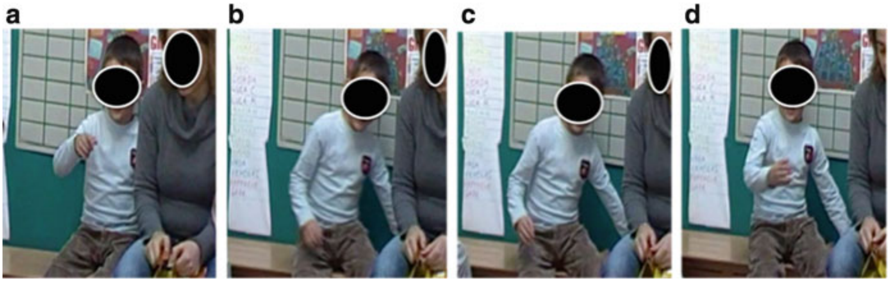

Fig. 8 (**a–d**) Fabio's gestures accompany the introduction of the terms « straight » and « turn ». In pictures **b–d** also the body rotation can be observed

9. *Cristina:* It makes like this and like this (*she travels the road with her hands, as Stefano is doing*)
10. *Other children do not make any verbal comment, but touch the entire path with their hands* (Fig. 7c).

The teacher's questions have the goal to help the children becoming aware of the characteristic of the road along which they will make the bee travel. Though not explicit, they play an important role with respect to the anticipatory thinking needed to program the robot. The children immediately answer to the question with very poor descriptions, made of the list of the elements of the poster, without relating them each other. They describe the road with deictic terms ("like this") that contain little information without the co-timed gesture. In order to push them to provide more suitable verbal descriptions, the teacher closes her eyes and asks them to better explain:

1. *Teacher*: And then? Let's do like this: I close my eyes and you tell me how is the road, because I do not know it…Is there a starting point? And an arrival? Explain to me.
2. *Fabio*: The start is in the house and maybe over there (*pointing gestures*) where there is the flower, it is the arrival.
3. *Teacher*: But in this way I would not be able to arrive: you must explain well.
4. *Fabio*: You must go straight (*pointing gesture,* Fig. 8a), then turn (*moving and turning his body,* Fig. 8b, c*, and making a turning gesture with right hand,* Fig. 8d), go still a bit straight, then turn again, go straight and you are arrived at the flower.

1. *Teacher*: But I don't know where to turn, how can I understand which part to turn…
2. *The children continue to explain mainly with deictic terms such as "here", "there", accompanied by gestures.*
3. *Teacher*: No, no, if you had to explain it only with words?
4. *Chiara*: Left and right

5. *Teacher*: Left and right, or towards. . .So, explain me better, you can do it: not like "I make some curves", but how many, I go straight and how far, or rightwards, or towards the benches, towards the door. . .

The first description provided in describing the poster referred to *static* elements: the house, the flower, and the road (lines 2–6). Soon after (from line 9), when pushed to better describe the path, a *dynamic* perspective is brought to the fore: children use dynamic pointing gestures (also materially touching the poster) and then words referring to the motion along the path (e.g. Fabio in line 13).

The teacher suggests some reference points, such as the starting and the arrival points (line 10), and insists on asking the children to provide a clear explanation ("explain well"). In line 13 Fabio introduces two verbs that characterize the movement of the robot: going straight, and turning. The introduction of these two terms is accompanied by two specific gestures: a deictic gesture made with the extended index (Fig. 8a), and a dynamic gesture, combined with the full-body rotation (Fig. 8b–d). The body movement and the hand gesture are the only semiotic resources that express the information about the direction of the rotation (leftwards). The teacher insists constantly about more accurate verbal descriptions, making this goal explicit to the children (line 15), and giving some indication on what aspects to mention: quantifying (line 17: "I go straight and how far"), subjective ("rightwards"), and objective references ("towards the benches, towards the door"). Analysing the following part of the video, we can see that children will seize only the subjective references, whereas for the quantification they will go by trial and error with the bee-robot.

The first path is run with the bee-robot programmed only with straight short traits, so the teacher asks to make more elaborate programs. But before asking to program the entire path, she sets an intermediate goal, consisting in programming until the third square, indicated with e deictic gesture on the poster (Fig. 9).

Fig. 9 The teacher indicates an intermediate goal to reach

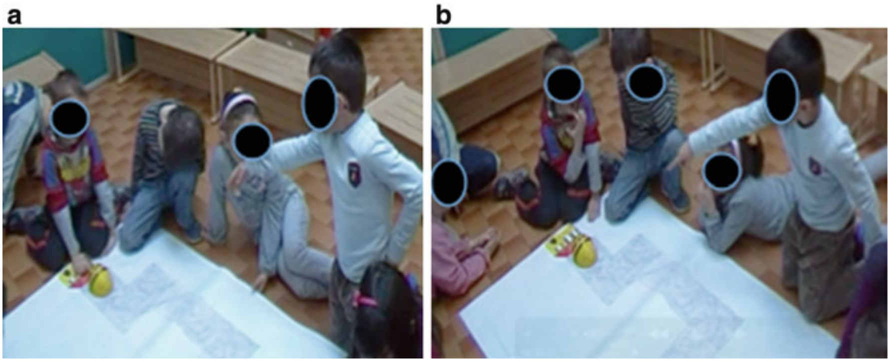

Fig. 10 (**a**, **b**) Fabio's gestures during his verbal instructions

The first attempts fails. Fabio then wants to give instructions to his mates, and the teacher pushes him to state precisely what he is saying:

1. *Teacher*: Think at which arrows you have to push.
2. *Fabio*: So, you have to push once the arrow, right
3. *Teacher*: To go forward, straight, backwards. . .how?
4. *Fabio*: Forward. Then you must do the. . .left. . .here (*he indicates the direction to the left of the robot,* Fig. 10a), then you must do again
5. *Anna*: Straight
6. *Fabio*: Straight and. . .and then we arrive here (*indicates on the path the third square, that is the arrival point stated by the teacher*).

The teacher asks to Fabio to repeat his proposal, so that all children can listen to it, before to check with the robot. Fabio would like to act directly on the robot, but the teacher insists that he gives the instructions from his place: the child accompanies then the verbal instructions with deictic gestures (Fig. 10b), and his mates follow them. We observe that Fabio is placed on the side with respect to the path direction: his egocentric reference system is therefore not aligned with that of the robot. Looking at the video we can clearly see that the child meets difficulties in accomplishing this task: to overcome them, he speaks slowly, and tries to incline his body so to position himself in the same direction as the robot (this can only be guessed by Fig. 10b, but is clearly visible in the video). The problem of *programming many steps consecutively*, when rotations are included, seems therefore strictly linked to the problem of *coordinating different reference systems*. The specific requests of giving instruction to others, while remaining far from the robot and in a different position, allowed Fabio to face the difficulties of the task, and to overcome them successfully, activating and developing his spatial competences, intertwined with the anticipatory thinking. As we can see from Chiara's intervention (line 22), also other children participated to Fabio's endeavour, either listening carefully, or pushing the robot buttons, or suggesting words, or checking in their mind his instructions: through the social interactive context, the request made by the teacher to an individual child, becomes a resource for making all children facing the complex task, each according to their actual capacities and specific attitudes.

Conclusion

Through teaching experiments in kindergarten, some potentialities and limits of robot-based activities for the development of spatial conceptualization were investigated. They were intertwined with anticipation and control competences, crucial to problem-solving in different fields.

Programming tasks, in fact, require children to imagine the consequences of their own actions, and allow later them to verify their correctness (in our case, through the observation of the robot actual motion). *Anticipatory processes*, that are cognitive processes carried out while imagining the consequences of our actions in a hypothetical future, are of paramount importance in problem-solving activities (Martignone and Sabena 2014). Their counterpart is *control processes*, which can be activated when checking if the actual robot motion does correspond to the programmed sequence of steps. In the light of our experimentation, we can affirm that robotic artefacts can offer great potentialities for the activation of these kinds of processes, but such activation requires an acute attention that in 5-years-old children is still in its initial development. Many children, in fact, showed great difficulty in keeping in mind even a small sequence of commands, and this difficulty made impossible to them to activate suitably control strategies.

For what concerns spatial conceptualization, robotic activities carried out in the material world can foster in children the intertwining and coordination between different reference systems. As discussed in the first part of the chapter, the coordination between different reference systems and points of view is necessary in order to face geometry problems.

A first remark regards the activations of different reference systems. In our observations, in order to face the proposed tasks, children always spontaneously took the egocentric perspective. Of course, to make sense of what their mates or the teacher were doing with the artefact, children were often in the need of coordinating their ego-based perspective with the allocentric one assumed by the robot. However, our findings suggest that specific constraints have to be set up on the task in order to ‘force’ children to actively work with allocentric perspective: for instance, have the children to imitate the movement of the robot when is not parallel to them, or to program it from a certain fixed position.

Both ego- and allo-centric perspectives are subjective reference systems, used in the space of reality. As discussed above, geometrical space requires the use of objective references. In the proposed activities, we did not focus on the passage from subjective to objective references. Some hints have been made by the teachers (as the one documented in the analysed episode), but with no success. Our

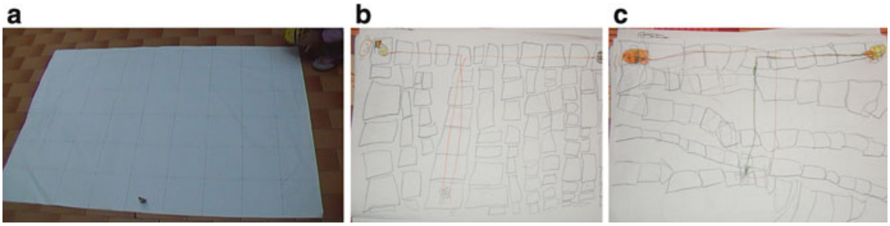

Fig. 11 (**a–c**) The grid used during the activities with the bee-robot and two drawings made afterwards by children

impression is that specific activities need to be designed in order to reach this goal, possibly in later age.

A second remark concerns two *different spatial conceptualizations* that emerged during the artefact-based activities: a *static and global* one, and a *dynamic and paths-based* one. The two perspectives do not constitute a dichotomy. For instance, in line 8 in the excerpt above Stefano is blending both of them: his words are referring to a global feature, and the gestures expressing dynamic ones (Fig. 7b). In the overall experimentations, gestures have often offered a window into the children's conceptualization of space, and new spatial terms have often been used the first time accompanied by corresponding gestures (as Fabio in line 13).

Evidence of how the experience with the robot paths has influenced the children's conceptualization of space can be seen also in several children drawings. Figure 11(b, c) reports the drawings made by two children, who had had the robot moving on a grid made by straight lines (Fig. 11a). In the children's drawings, the grid looses its global features and becomes a sequence of steps.

The paths-based perspective has been certainly fostered by the use of the bee-robot, and future research is needed to investigate its role in early spatial thinking. Studies in cognitive science within the embodied mind approach have shown that motion constitutes the source domain of many concepts, and that also static objects are often conceptualized in terms of motion[4] (Lakoff and Núñez 2000). Starting with motion activities seems thus promising for children spatial development.

The last remark concerns the crucial role of the specific requests made to the children. For instance, we encountered a great "resistance" from children to program sequences of steps that could include one or more turning: they preferred to divide the path in straight parts, and program each of them separately. Rotations in particular were never spontaneously linked to following onward steps. Probably programming an entire long sequence requires cognitive capacities still under construction by the children, but maybe the main difficulty lies in the fact that the goal of reaching a certain place through a single program sequence had not any understandable 'sense' for the children (Donaldson 2010). We could observe that even when this goal was proposed within a competitive setting (like a team

[4]Talmy (2000) has called 'fictive motion' the cognitive mechanism underlying the description of a static object (e.g. a path, in our example) in motion terms (e.g. 'it starts…it goes…').

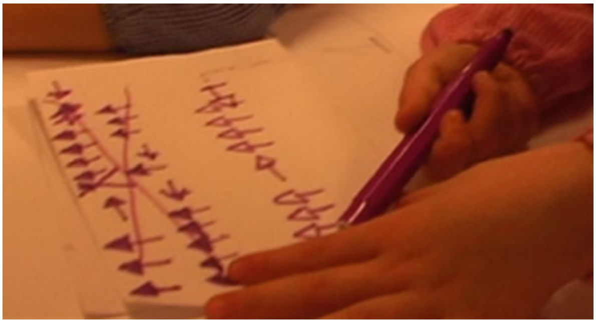

Fig. 12 Written signs representing the programmed sequence of steps

competition), the children did not undertake it. As a matter of fact, programming a certain artefact using less time as possible can be a goal for adults, which are often under time pressure. In the case of children, pleasure was given in using the robot as long as possible, because they liked it. In our task design, we initially underestimated this essential dimension, and not a few times the goals that we had chosen for the activities were completely neglected by the children.

The mediation of the teacher has therefore been necessary to introduce the possibility itself of articulated programs, and to make their benefits explicit to the children. The teacher mediation in the activities was accomplished through natural language, as well as embodied resources such as gestures, as in the analysed episode, but also through the introduction of written signs to register the commands given or to be given to the robot (see an example in Fig. 12).

The different resources (words, gestures, written signs) intertwined in complex interpretative processes of the programming code used by the robot, represented by the arrows buttons (Fig. 4b), and its actual movement. The introduction of written signs has not been here discussed, and requires further examinations. It has a limited scope for kindergarten level, but it constitutes an interesting didactic path for primary school, since it regards the delicate passage from experiences in macro-space of reality to the use of micro-space of representation, the fundamental background of much geometric activity.

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