Disclosing the “Ræmotionality” of a Mathematics Teacher Using Technology in Her Classroom Activity

Marina De Simone

Abstract In this chapter, I will focus on the relation between affect and technology during the classroom activity of a mathematics teacher. This constitutes a first approach for developing a new aspect of my PhD thesis where, in general, I tried to bring together cognitive and affective dimensions in the classroom behaviour of mathematics teachers, often considered separately. In particular, in this paper, I will focus on the practice of a teacher who routinely uses digital technologies in her mathematical activity, showing how her expectations on the use of technology are actually reflected in her classroom experiences and how these expectations inform us about the reasons of their actions.

Keywords Technology • Mathematics teaching • Emotional orientation • Linear equations

Introduction

Over the last two decades, mathematics education research has increasingly focused on the role of digital technologies in teaching/learning processes (Artigue 2007, 2010; Clark-Wilson et al. 2015; Gueudet et al. 2013). Many studies have documented how the use of ICT can enhance students’ learning (Artigue 2013; Buckingham 2013; Clark-Wilson et al. 2013). As a result, curriculum documents and professional development programmes commonly encourage teachers to employ technology in their practice. However, teachers using technology have to cope with factors of a different nature than they are used to. In particular, as presented in this chapter, I will discuss how cognitive and the affective factors are unavoidably intertwined in the practice of a teacher who uses digital technologies in her classroom practice. In fact, the teacher decides to employ technology not only on a rational level, but also on an affective one, because she has expectations toward students’ learning, and toward integrating ICT into her practice.

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Looking at this teacher's expectations for the use of technology, I could also infer numerous reasons for the decisions she makes in her classroom in general.

The core of my research is the decision-making of the teacher, because, as much research in mathematics education has consistently highlighted, decision-making has a very crucial role in teaching activity. For example, Bishop pointed out that decision-making is "at the heart of the teaching process" (Bishop 1976, p. 42).

From a theoretical point of view, I rely on the notion of "emotional orientation" (Brown and Reid 2006), locally analysing teacher's decisions drawing on the philosophical theory of rationality (Habermas 1998). In particular, I employ the neologism "ræmotionality" (De Simone 2015), which refers to the rationality and the emotions of the teacher as a *unicum*, that is, as a unique example or specimen.

Entering in the structure of this chapter: in the first section, I illustrate the theoretical perspective that contextualizes my work, explaining also the analytical tool I chose for analysing my data; in the second section, I present qualitative data analyses of five excerpts of the activity of a teacher, Silvia, who uses two kinds of technology while explaining linear equations (GeoGebra and two Java applets); in the third section, I make some concluding remarks, highlighting both points common to the two different types of technology, and how my theoretical framework allows me to make an in-depth analysis of a teacher who uses digital technologies in her practice. I would like to underline that my research is a qualitative study in which I attempt to construct theoretical concepts for analysing particular case studies. These theoretical concepts might be applicable to other cases, without the presumption of generalization.

Linear equations is a mathematical topic that is very interesting to analyse in terms of the coordination among different representation registers, especially using digital technologies. Thus, I choose to develop the analysis about examples concerning linear equations to study how multi-representations influence and intervene in the affective and rational decisions within the mathematics activity.

Theoretical Perspective and Methodology

As already anticipated in the introduction of the paper, my research interest is the study of the intertwinement between the emotional and the rational aspects in the decision-making processes of a mathematics teacher, who uses digital technologies in her practice.

In the mathematics education literature, several authors have focused on the decision-making of the teacher in classroom. For example, Schoenfeld (2010) offered a model for describing the decision-making of teachers according to three different elements: "their knowledge and other intellectual, social, and material resources; their goals; and their orientations (their beliefs, values, and preferences)" (Schoenfeld 2011, p. 1). As he pointed out, these three aspects are deeply related, and the third one, orientation, heavily affects the other two.

In my work described here, I combined two different theoretical perspectives: the philosophical speculation offered by Habermas (1998) and the concept of "emotional orientation" developed by Brown and Reid (2006). The integration of these two theories has produced a new possible theoretical lens, which I have called "ræmotionality", through which I tried to bring together the rationality and emotions of the mathematics teacher, often considered separately.

The Habermasian philosophical speculation has been re-elaborated and adjusted to mathematics education by many researchers (see, e.g., Boero and Planas 2014). Habermas centred his theory on the concept of discursive rationality proper of a rational being (e.g., the mathematics teacher) involved in a discursive activity. He explains that discursive rationality is constituted by three different components: the epistemic rationality, the teleological rationality, and the communicative rationality. These three components of rationality are always present and intertwined in the discursive activity of a rational being. In particular, we face an epistemic rationality when we can simultaneously give an account of the justification of the knowledge at play, the teleological rationality surfaces when "the actor has achieved this result on the basis of the deliberately selected and implemented means" (Habermas 1998, p. 313), and the communicative rationality "is expressed in the unifying force of speech oriented toward reaching understanding" (Habermas 1998, p. 315).

Within the mathematics-related affect research, Brown and Reid (2006) proposed the notion of emotional orientation in order to study the decision-making processes both of the teacher and the students. As the words themselves suggest, the "orientation" in a teacher's decision-making processes is "emotional", that is, affected by emotions in particular ways. Hence, this concept allows me to speak of the interconnection between rationality and emotion. For operationalizing the notion of emotional orientation, I propose an adaptation of the concept of the "emotional orientation" of a teacher in terms of her "set of expectations": the term "expectation" is connected to her "emotions of being right" when she uses specific criteria for accepting an explanation from the class rather than other ones (Ferrara and De Simone 2014).

My research questions are twofold: How does the use of emotional orientation help me understand the decisions of teachers relating to the use of technology, and thus complement the Habermasian rationality framework? And, How does ræmotionality help me understand why teachers make certain choices in their teaching with digital technologies and not others?

This chapter focuses on the work of an Italian teacher-researcher,[1] Silvia, while she explains linear equations in her grade 9 classroom, in a scientifically-oriented secondary school in the Piedmont region of Italy. The teacher was first interviewed, and the interview was transcribed for analysis. Teacher's usual lessons in the classroom were also videotaped. All voice and bodily movements during the classroom activities were recorded. The videos were transcribed for data analysis.

[1] In Italy, the teacher-researcher is a teacher of the school who participates to the research carried out within the academic research group.

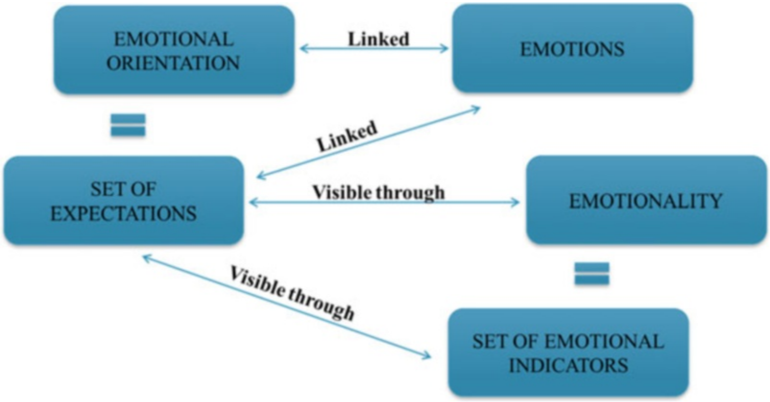


Fig. 1 Diagram about relations of different concepts

Concerning the structure of the analysis, I first considered the *a-priori* interview and, from what Silvia explicitly described to me, I was able to identify some of her expectations for the use of technology in terms of what she hoped for her students. At this stage these expectations were only potential, because they were not yet driving the action of Silvia in her classroom. Hence, I also looked at what actually happened in the classroom, in order to see if there was a correspondence between what the teacher stated *a-priori* and how she actually behaved in classroom. For determining this correspondence or non-correspondence, I looked at "emotional indicators", namely the gestures, facial expressions, word emphases, repetitions, rhetorical questions, pauses, the tone of voice, and so on. These emotional indicators informed me on the emotionality of the teacher, where the term "emotionality" is defined "in terms of *behaviours* that are *observable* and theoretically *linked to* the (hypothetical) underlying emotion" (Reber et al. 1995). Hence, the expectations of the teacher became visible through her emotionality. In this way, I outlined the emotional orientation for the teacher, intended as the set of her expectations. In the diagram below (Fig. 1), the relations among these different concepts are schematised.

Following the initial emotional schematization, I went deeper into the lessons of the teacher, in order to identify the intertwinement between her rationality and emotionality. In particular, I looked at her decisions related to the use of ICT, through the three components of rationality (epistemic, teleological and communicative); simultaneously looking at the emotional indicators and expressions of her expectations, I was able to say something about why she made linkedsome decisions and not others.

In particular, the emotionality will be always intertwined with the rationality of the teacher. For this reason, I describe the emotionality of the teacher using the adjectives of the Habermasian rationality, epistemic emotionality, of teleological emotionality and of communicative emotionality (De Simone 2015): these Habermasian adjectives constitute the different components of ræmotionality. For example, the *epistemic emotionality* surfaces when the teacher decides to draw on the properties of equations via the model of a virtual scale, and, at the same time, says, with a blaring tone of voice, the expressions "to put on the scale" and "to take away from the scale", manipulating the virtual scale. This way, it is not only which kind of knowledge she chooses to consider (properties of equations, epistemic rationality), but why she decides to focus on it in that way (through the virtual scale). The reason is connected to her expectation that using the virtual scale will help students understand the meaning of the properties of equations, by having the possibility of visually manipulating the scale. This expectation is made visible through the tone of voice of the words "to put on the scale" and "to take away from the scale". In other words, the *epistemic emotionality* is related to why the teacher uses that specific justification of the knowledge at play.

The *teleological emotionality* could be highlighted when, for example, the teacher decides to explain equations with the graph option of Geogebra for geometrically interpreting the solution and, simultaneously, with the highest pitch of her tone of voice, repeats many times the verb "to see", pointing to different elements on the graph. In addition to the action the teacher undertakes to accomplish a goal, namely interpreting geometrically the resolution of a linear equation, we can also observe that she expects GeoGebra to help students to reason about equations, "seeing" through the graphical register of GeoGebra. This expectation is made visible through the tone of voice of the verb "to see", while gesturing on the graph. Thus the *teleological emotionality* is related to why the teacher makes these actions to achieve a particular goal.

The *communicative emotionality* surfaces, for example, when the teacher has an insistent rhythm to her voice, as she directs the class to look at what happens both on the graph and on the "Algebra view" of GeoGebra. In this instance, there is not just the matter of her speech oriented towards reaching understanding within the classroom, but also the question of why she decides to communicate with an insistent rhythm. Her reason for this repetition is connected to her expectation that students are facilitated to connect different registers of representations through the use of technology. Hence, the *communicative emotionality* is related to why the teacher uses a particular type of speech during her discursive activity in the classroom.

For pragmatic necessities of analysis, these three types of emotionality could appear separated. Nevertheless, it is important to stress that they are always intertwined and present in the discursive activity of the teacher.

Data Analysis

From what Silvia explicitly described during the *a-priori* interview, I detected different expectations of the teacher, mostly concerning the role of technology. These expectations are actually reflected in her classroom activity and contribute to shaping her emotional orientation.

I will show five examples of Silvia's activity in the context of linear equations for shaping a convincing and representative outline of Silvia's ræmotionality in using two types of technology: first GeoGebra and then a Java applet.

Firstly, I will quote the passages of the interview from which I identified her expectations involved in these examples. Then, I will analyse her ræmotionality, looking at both the decisions of the teacher and, simultaneously, at the emotional indicators, expressions of her expectations. This way I can reveal the different components of her ræmotionality, and thus explain why Silvia takes those decisions and not others.

During the analysis of teacher's activity on linear equations, three different treatments of graphical representations developed by Duval in 1988 also emerge. In particular, Duval speaks of the "*démarche de pointage*", the "*démarche d'extension*" and the "*démarche d'interprétation globale*". The first approach, *dé marche de pointage*, concerns the focus on particular points of the graph. For example, it is related to the drawing of the graph of a first grade equation or to the reading of the coordinates of an interesting point of a graph. The "*démarche d'extension*" concerns the imagination of a set of infinitely potential points that have a particular property. The "*démarche d'interpretation globale*" is related to the association between what happens on the graph and on the algebraic representation of the graph.

GeoGebra

First Example

This example comes after two lessons in which Silvia had introduced the concept of equation as a mathematical statement that two expressions are equal. The solution of an equation is the value that, when substituted for the unknown, makes the equation a true statement. Then, in this excerpt, she begins to work on the dynamic geometric software GeoGebra for introducing the solution to an equation from a geometrical point of view. In this example, two of Silvia's expectations that I have extracted from the *a-priori* interview are involved. I quote the passages of the interview that allowed me to identify them.

In the middle of the *a-priori* interview, Silvia presents her way of introducing linear equations:

> I introduce linear equations through an activity of M@at.abel (M@at.abel is an Italian teacher education programme for in-service mathematics teacher supported by the Ministry of Education). In particular, we consider a pseudo-real situation of a boy who walks with constant velocity and we ask, knowing the velocity, how many kilometers he covers while the time passes. "How many kilometers while the time passes" is a linear function, then, on GeoGebra, we consider a table and we start to see after how much time he will cover 300m and then we go to see [she mimes the solution on the graph] the answer on the graph of GeoGebra. We start from that for talking of equations because, after we have the straight line [she mimes the straight line], we can read on the graph of GeoGebra and then we have the intersection between the oblique straight line that represents the velocity and the horizontal straight line that represents, for example, 300m. Then we are able to see the

> intersection point as the solution of an equation. Always working on the graph of GeoGebra, I try to highlight that if I translate the graph up or down [she mimes the translation] the solution is simply translated up or down. Hence, I can add or subtract the same term to both sides of the equation and I will obtain the same solution.

Hence, it can be plausible to think that Silvia has ***the expectation that students learn "to see" through the graphic register of GeoGebra in order to reason (think of) about the equations.*** I used the verb "to see" with the quotation marks, because, during the interview, Silvia herself used this verb for referring to the discussion of the equations.

Then, from the following of the interview, I infer that Silvia has ***the expectation that the use of GeoGebra helps students to pass from one representation register to another***. Indeed, as I am going to show, Silvia believes that GeoGebra could facilitate students in coordinating the different registers, stressing that, when she passes from one register to the other, they are all equivalent ways to speak about the same thing:

> Using GeoGebra, I expected that students are able to intertwine the graph, Algebra, numbers and words not just for the equations, but for all of the mathematical concepts. I believe that this is the power of GeoGebra. For example, yesterday, with my 11 grade classroom, when I spoke of the definition of the arithmetic progression, I remained astonished because they were able to see the graph of a straight line: we were in the classroom, within a totally numerical environment and they were quickly passed from the numbers to the graph without problems and in a fruitful way. I believe that this is a very important added value. I hope that GeoGebra helps students in thinking and searching for counterexamples even when they don't have GeoGebra at their disposal. In particular, for the equation I expected that they see the deep link among equations, inequalities, zero of a function: they are all equivalent different ways to speak of the same thing.

From the interviews' excerpts, there are initial hints of the fact that the teacher considers GeoGebra to be a useful tool for fostering students' imagination. In fact, firstly, GeoGebra allows students to actually see things, when they directly work with it. As time goes on, this thing gives an "added value" to the use of the software, namely the fact that GeoGebra supports students' imagination, even when they don't have it at their disposal.

The two expectations are actually reflected in her classroom activity. Indeed, I am going to make the analysis of two excerpts in which these expectations are present, surfacing the ræmotionality of the teacher referring to the use of GeoGebra. The first one refers to the discussion of a graphical solution for an equation. The class is working on the already quoted M@t.abel activity. They have to solve the equation $1/5x + 1/2 = 8$. The solution of it is the time Luca uses to cover 8 km, starting from 500 m from the starting line. Hence, knowing that he uses 15 min to cover 3 km, namely his velocity is 1/5 km/min, the time he uses is the solution of $1/5x + 1/2 = 8$.

Listen to Silvia as she explains the task to the classroom:

#1 T: In the activity we prepared a slider k that varies from −15 to 15 and (*blaring tone of voice*) then we considered two equations. [...] The equation previously solved was $1/5x + 1/2 = 8$. To solve this equation

Fig. 2 Pointing the abscissa of the intersection point (Here in after, the description of the rest of the figures is included in the correspondent analysis)

we have already said that, actually, we could work on two different (*pronouncing*) functions, precisely on two (*pronouncing*) straight lines: one was this straight line (*she draws on GeoGebra the function y = 1/5x +1/2*) (*pause and she looks at the screen*) and the other one was y = 8. Then, you have told me, if you remember, that the solution to the equation was the intersection point between these two straight lines. (*rhetorical question*) Do you remember it? There was Elena who said "(*highest pitch*) I go to see where I intersect and then I read the solution". Actually, the solution we have to read is not on the y-axis, but it is on the x-axis, because it is the value of x that is of interest to us as solution and (*blaring tone of voice*) then the fact of asking to draw the perpendicular line to the x-axis passing through A served simply to say that (*highest pitch*) I can go to read the solution. I can go to read the solution here (*she stands up and she goes on the screen, pointing to the abscissa of the intersection point and then she looks at the class as in* Fig. 2).

(*highest pitch*) Going to read this number or (*speeding up*) given that I cannot be sure of the value of this number because GeoGebra has limits, I can read it here (*she points to the "Algebra view" of GeoGebra*). In the "Algebra view" the point A has coordinates 37.5 and 8 and, then, the solution of the equation is the number 37.5. If instead of x I put 37.5 the two straight lines intersect and they have the same value (Fig. 3), ok? (*she nods and she returns to the pc*).

Then we were asked to draw another two straight lines: one is y = 0.2x +0.5+k (Fig. 4).

In this case, k is equal to 1 and we see that GeoGebra writes (*pointing to the "Algebra view"*) y = 0.2x + 1.5. Why 1.5? (*pause: a student tries to say something but he does not finish the sentence*) because k

#2 S1: It is 1

#3 T: It is 1and, then, 1 plus 0.5 is 1.5, it has already calculated (*referring to GeoGebra*). The other straight line is y = 8+k (*blaring tone of voice and*

Fig. 3 The two straight lines intersect

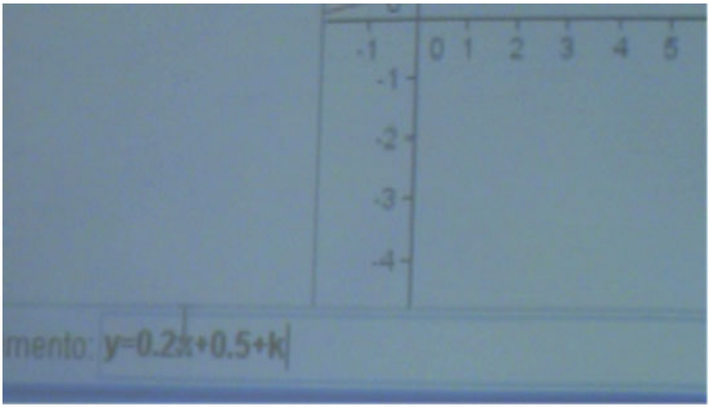


Fig. 4 Another straight line

she looks at the class) if I write y = 8+k, in the "Algebra view" it will write 8 + k? (*facial expression in* Fig. 4, *long pause and she continues to look at the class*) (*smiling*) I don't hear answers (*she looks at the class smiling*).

#4 Ss: Not

#5 T: Not, what will it write? (*same facial expression as in* Fig. 5)

#6 Ss: 9

Discussion

Silvia works with GeoGebra in order to talk about the graphical solution of an equation. She has already introduced it in the previous lesson, in fact the teacher

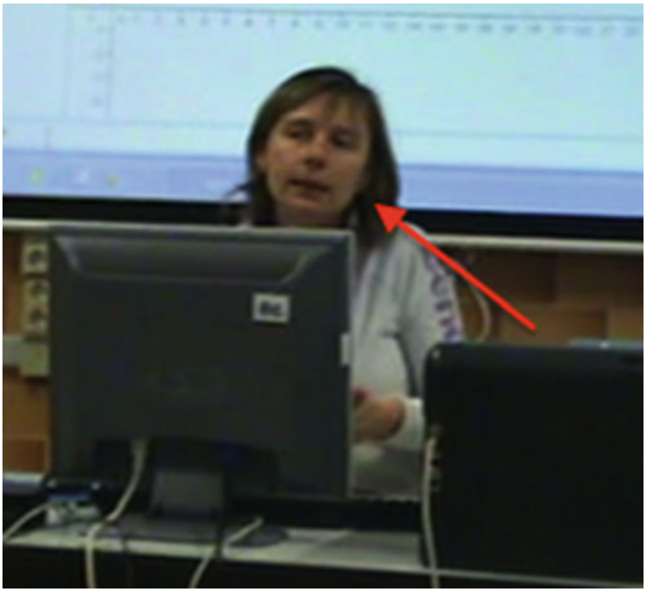


Fig. 5 Facial expression

reminds students that they could work on two straight lines: y = 1/5x + 1/2 and y = 8. She pronounces both “functions” and “straight lines” (#1), in order to recalling that the solution of the equation is related to the intersection point between them. Hence, she is working on the global interpretation of the straight lines (*dé marche d’interpretation globale*) for focusing on the intersection point between these two functions (*démarche de pointage*).

Moreover, she “plays” with two representations of the line: a geometrical representation of the straight line and the graphical representation of a function. Silvia seems quite sure that the students remember this, indeed, she rhetorically asks “Do you remember it?”. Then, she recalls what a student said in the previous lesson (#1). Furthermore, she clarifies that they have constructed the perpendicular line to the x−axis passing through A, because the solution of the equation can be “read” on the x−axis (*démarche de pointage*). Silvia accompanies this justification with a blaring tone of voice (#1), for stressing it as much as possible. She repeats this fact looking at the class for feedback and pointing to the solution (see Fig. 2). She uses many times the expressions “to read the solution” and “to go to see” on the graph.

Then, stressing that GeoGebra has limits, she invites students to read the solution, not directly on the graph, but on the “Algebra view” of it. This is interesting: she is saying that it has limits if we look visually, but not if we look numerically. From a didactical point of view, the teacher would like to present the calculus as something more rigorous than visual representation. From the other side, she wants to pass on the “Algebra view”, speeding up, because she surely has the expectation that students reason on equations visually on the graph; however, she needs students to coordinate the registers for institutionalizing the mathematics at play. In terms of Duval’s theory, this represents a global vision in which the teacher stresses the association between the graphical and the numerical solution (*demarche globale*).

After that, she continues with the activity in which they are requested to draw another couple of straight lines depending on k: $y = 0.2x + 0.5 + k$ and $y = 8 + k$. The teacher highlights that GeoGebra gives automatically the value of k to the first function ($y = 0.2x + 0.5 + k$), and then she seems to want from the students the response for the second equation ($y = 8 + k$). In fact, after asking, with a blaring tone of voice, what happens for $y = 8 + k$, she pauses, as in Fig. 5. The blaring tone of voice is probably intended to reveal how GeoGebra directly makes the addition both on the Algebra view and on the graph, according to the value of k. Then, she smiles when she says that she isn’t hearing any answers, perhaps to keep the mood light (#3).

Hence the *teleological emotionality* of Silvia is constituted by considering the two straight lines and their intersection point to find the solution (rational key). Moreover, the teleological involves the fact that she is expecting that students are used to “seeing” through the graphic register in order to find the solution. This emotional key is revealed, for example, by the fact that she often says, increasing her tone of voice, “to read the solution” and “to go to see” on the graph (#1, #2), possibly to draw the attention of the class to these important aspects of her lesson. This emotional aspect is shown also by the rhetorical question, “Do you remember it?” in #1. Finally, it is reveled by her gesture in Fig. 3 in which she mimes the intersection between the straight lines.

The *epistemic emotionality* of the teacher is, from one side, the geometrical interpretation of the solution of an equation, accompanied by how the software works (rational key). From the other side, it is related to her expectation that students know how to pass from one register of representation to another one (emotional key). This is strictly related to being able to “see” through the graphic register to reason about equations. For example, she hopes that students recognize the solution on the graph, pointing to it and maintaining a certain facial expression (Fig. 1), and seeking feedback from the class. Then, in #1, from her increasing velocity of speaking, her need to quickly pass from the graphical register to the algebraic one for formalizing the calculation becomes clearly visible. Moreover, she hopes that students are able to link how the algebraic and graphic registers of GeoGebra work together. In fact, after asking with the blaring tone of voice what happens for $y = 8+k$, she pauses with a facial expression as in Fig. 5. It is quite clear that the teacher is expecting an intervention from the students. Moreover, her smiling probably communicates a desire for more participation from the class (#3). Actually, this attitude did trigger several comments from the students.

Her speech is full of emotional hues because she has certain hopes and needs in relation to her students. She changes her tone of voice to emphasize what she is saying, such that students understand the importance of it. Especially in this part of the lesson, she seems like a soloist, because she speaks exclusively for most of the time. Because of this, her pausing is meaningful: when she stops, she has the need

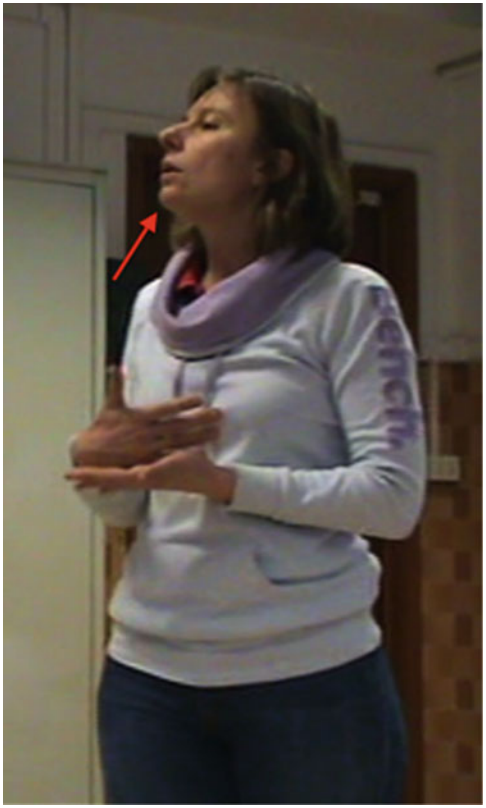


Fig. 6 Gesture moving her hand up an down for the translation

for students to speak. Being that her discourse is not neutral, we can speak of the *communicative emotionality* of Silvia.

Second Example

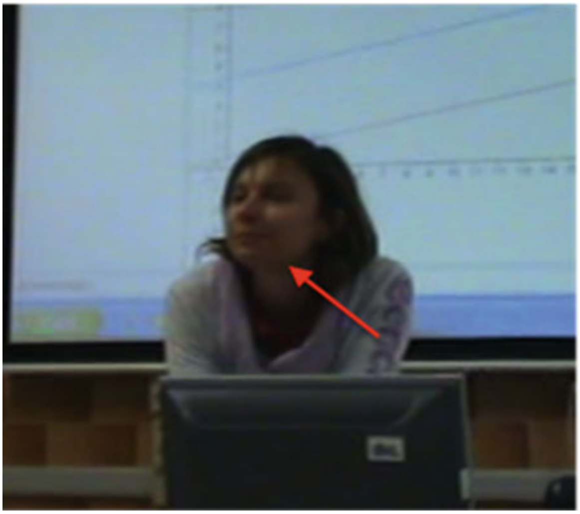
After discussing the graphical solution for an equation, Silvia continues to work on GeoGebra, because of her expectation that GeoGebra can help students to pass more easily from one representation register to another; and she aims to link the vertical translation of the straight lines to the concept of equivalent equations from an algebraic point of view:

#1 T: (*pronouncing*) What are we doing (Fig. 6: *gesture moving her hand up and down for the translation*)?

#2 S3: Equivalent equations

#3 T: (*repeating and nodding*) We are constructing many equivalent equations. You remember that in the previous lesson we have said that we have equivalent equations (*same gesture as in* Fig. 6), namely equations written (*pronouncing*) in a different way, but that they have (pronouncing) always (pausing) the same result. (*Highest pitch*) Do we

Fig. 7 Waiting for an answer



have equivalent equations just for k = 7.5, for k = 3 (*speeding up*) that are the equations we have seen? or do we have equivalent equations for many values of k (*she returns to the pc and she moves k, looking at the class and smiling waiting for an answer*, Fig. 7)?

#4 Ss: Many

#5 T: For many or for each value of k (*she continuously moves k*)?

#6 Ss: For all of them

#7 T: For each value of k. For each value of k I obtain however equivalent equations. The filling of the table was just to write equivalent equations. For example, when I write 0.2x + 1.5, what value has k to have 1.5? (*pause and she lifts up her chin*)

Confusion in the classroom

#8 S2: 1

#9 T: 1. Then, If I give the value 1 (*she returns on GeoGebra to put k equal to 1*) I see that the equation is (*pointing*) 0.2x + 1.5 = 9. (*Pronouncing*) What happened to the sides of the equations? What did we do to the sides of the equation (*she lifts up her chin*, Fig. 8)?

10 S1: We have added 1

#11 T: We have added 1 (*pausing*)

#12 S1: To both sides

#13 T: (*smirking*) We have added 1 to both sides. In the previous lesson, we have said that the first principle of equivalence said us that we could add the same number to both sides and that the result of the equation does not change, ok? then I could add or subtract the same number to both terms and have (*pronouncing*) always equivalent equations. Then, what does it mean (*returning on the "Algebra view"*)? It means that I can add to both sides (*moving k*), see that the blue straight lines have the

Fig. 8 Facial expression

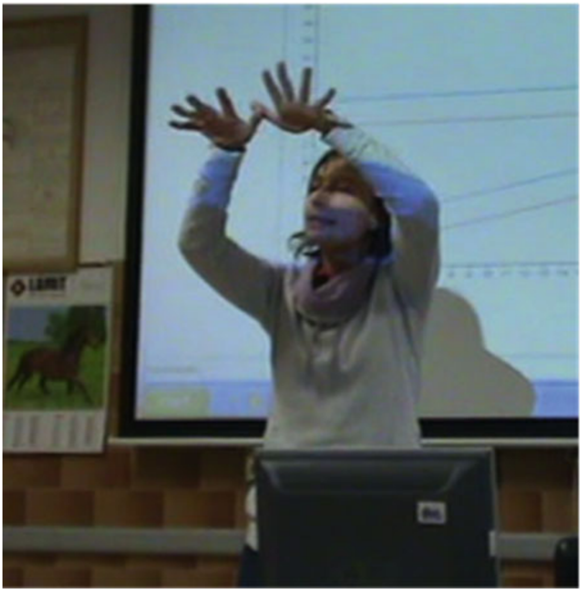


Fig. 9 A mime of the translation

same movement, they have the same translations (*she mimes the translation moving hands up and down:* Fig. 9), namely they have exactly the same movement, then we add or subtract to both sides exactly the same quantity, our result doesn't change. If I wanted to

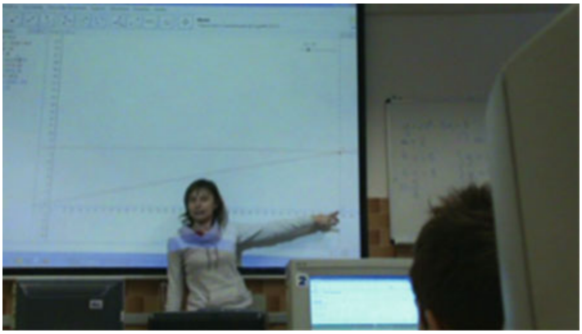


Fig. 10 Gesture along the x-axis

Fig. 11 Gesture to accompany the pronouncing

obtain the result of the equation, I would take k, I would do such that B coincide (*pronouncing*) exactly with the x-axis (*she is doing it on GeoGebra*). To let coincide B exactly with the x axis, what value I have to give to k?

#14 S11: −8

#15 T: −8. If I give −8 to k, what happens is that B belongs to the x axis (Fig. 10). The second side of the equation (*pronouncing*) takes the value 0. The first side of our equation has a certain expression and I, actually, go (*pronouncing*) to see where the blue equation intersects the x-axis (Fig. 11). I go to find what it is called the (*pronouncing*) zero of function (*gesture to accompany the pronouncing*) because it is the point in which the straight line touches the x axis, ok?

Discussion

At the beginning, Silvia explicitly asks to her class what they were doing in the previous part of the lesson (#1). This action comes along with her pronouncing and her posture of waiting (see Fig. 6) for having as much as possible the attention of the entire class, because they are about to construct an important link. Then, satisfied, she repeats, nodding, what a student answers (#3), remembering the definition of equivalent equations. She pronounces “in a different way” and “always” (#3) in order to focus the attention of students on these two crucial words of the definition.

Moreover, returning to GeoGebra, Silvia asks for how many values of k they can have equivalent equations. This question comes along with an increasing volume of voice and her emblematic posture in Fig. 6, in which she seems quite satisfied that students are able to answer. Actually, while Silvia moves the slider k, students become aware that they can have equivalent equations for infinite values of k, because the number of straight lines that could be potentially constructed moving k is also infinite.

The teacher exploits the opportunity of imaging something, without actually seeing it. In fact, she moves the slider k continuously, and this allows students to realise that they could potentially construct infinitely many straight lines, even if they cannot visualize all of them on the screen. Another interesting thing is that in #4 students just answer “many”, but then after seeing the teacher continuously moving the slider, say all together “for all of them” (#6). This has an important effect on the algebraic point of view: in fact, they become aware of the infinity of the straight lines on the number of equivalent equations.

In this brief moment, constructing in their minds the straight lines they could have sliding k, they are at the level of the “*démarche d’extension*” à la Duval that draws on infinite sets of potentially equivalent straight lines.

After, Silvia explains how the first principle works,[2] showing that if k is 1, GeoGebra automatically adds 1 on both sides (#7, #9). She accompanies this discussion with many questions for her students, pauses and facial expressions with the chin up (#7, #9). It is quite clear that she is waiting for answers from the class. This is also indicated by her smirk in #13 when a student says that they have added 1 to both sides. Then, she repeats what the first principle says, again pronouncing “always” (#13). In terms of what happens on the graph, she highlights that the straight lines are translated of the same value, hence the result doesn’t change. To explain what happens she uses a specific example: adding 0 to both sides. In fact, she invites her students to move the intersection point of the straight lines on the x-axis. She stresses this fact by pronouncing “exactly with the x-axis” (#13). At this moment, *exploiting the “démarche globale” that links the graphical and the numerical aspects*, Silvia introduces the concept of the zero of a function, stating both “zero of the function” and “because it is the point in which the straight line touches the x axis” (#15).

Hence, her *teleological emotionality* involves both the construction of the link between the concept of equivalent equations and the translation of the straight lines

[2] She speaks of the properties of the equations, that in Italy, we call “first principle of equivalence” and “second principle of equivalence”. The former says that adding or subtracting the same quantity to both sides of an equation produces an equivalent equation. The latter says that multiplying or dividing by a quantity ($\neq 0$) both sides of an equation produces an equivalent equation.

(rational key), and her expectation that, with GeoGebra, students are able to link these two different representation registers. Moreover, she hopes that it is not too difficult for students to see the translations of the straight lines as adding a quantity to both sides of the equations (emotional key). In fact, she wants to link the translation of the straight lines to the properties of equations they have already seen from an algebraic point of view. This emotional counterpart is revealed, for example, by her pausing as in Fig. 8, her satisfaction after the answer of a student, her pronouncing key-words (#1, #3), her gesture in Fig. 9 moving the hands up and down for miming what happens to the straight lines and, at the same time, for linking the movement the operations of addition and subtraction on the algebraic expressions.

She justifies how the first principle of equivalence functions using how GeoGebra works and she makes the specific example of adding 0 to both terms of the equation (rational key). At the same time, she is expecting that students are able to connect the algebraic register to the graphical one (emotional key). In particular, Silvia introduces the zero of the function as the intersection point of the straight line with the x-axis. The emotional key is revealed by pronouncing several times "exactly with the x-axis" and by her gesture to recall what they have already done with the scale. These two intertwined aspects form the *epistemic emotionality* of the teacher.

During all the activity, Silvia, being emotionally involved, cannot have a "plain" discourse, indeed she uses the language of the body, gestures, change of the tone of voice and so on. For this reason, it can be always highlighted the *communicative emotionality* of Silvia. With a pure rationality I could keep track just of what she is saying, but, looking at the emotional timbres, I could also say something about the fact that she hopes and needs in using the technology with her students.

Java Applets

In the *a-priori* interview, she anticipates that she will use two Java applets featuring virtual scales for reinforcing the meaning of the principles of equivalence. They have already experienced them from both the algebraic and the geometrical points of view.

From a mathematical point of view, through these scales, we can represent and solve simple linear equations. After having shaped the given equation using the unit-blocks and the x-boxes (whose weight is unknown) on each side of the scale, we can make arithmetical operations. The aim is to have one x-box on one side and the number of unit-blocks on the other one, from which we can read the solution. The user chooses the operation to be performed, and after each operation the new equation is updated so that both the original equation and the latest equivalent form are seen together. The first scale only allows working with positive whole numbers as coefficients, while the second one has been designed to work with negative numbers. In fact, in the latter, there are negative values for the unit-blocks and the x-boxes, represented by red balloons for thinking of something that lifts, conveying the idea of subtraction. One of the key ideas that should be highlighted is that no operation can be performed on just one side of the equation. Moreover, the virtual scale is a useful tool for visualizing the addition and subtraction of quantities (first principle of equivalence), but it is more obscure concerning the meaning of division and multiplication in terms of the scale. This limit will also surface in Silvia's lessons.

In the interview piece I'm going to quote, Silvia speaks about the metaphor of the virtual scale. I detected her ***expectation that students understand the meaning of the principles of equivalence through the direct manipulation of the virtual scale, drawing upon their arithmetical or algebraic knowledge for bypassing the technology's limit***:

> I will use the virtual scale for underlining the principles of equivalence, using a tool that students know [. . .] I stress the first principle of equivalence with another register, different from the words, namely the fact that if I add or subtract something from one side of the scale, I have to add or subtract the same thing from the other side of it [. . .] I will use it after doing the principles of equivalence first with GeoGebra and, then, from an algebraic point of view. Hence, if someone has not still got them, could catch their meaning through these scales [. . .] There are balloons for the negative values and this is again good for me, because it is not only an idea of adding, but also that of subtracting. In conclusion, the use of the virtual scale is an added support for understanding the first principle of equivalence. The choice of technology [. . .] is because all of the students can use the scale and this is an important value [. . .] it is one thing to use it and quite another to see someone else uses it; second, the students have the web link of the scales and they could use it also at home. Then, the most important thing is that I can physically manipulate it without seeing other people doing that. For example, through moving to the trash the small cubes or using the arithmetical operations, the students can physically see what happens, also seeing the direct result of the operation. On the contrary, when you are working from an algebraic point of view, removing 5 from both sides doesn't allow you to see an immediate effect. I just see a number that changes, while with the scale I see the scale that goes up and down: they are things for grasping the meaning of the principle of equivalence. For the second principle of equivalence the thing is more complicated: you cannot see the division with the metaphor of the scale, namely we cannot physically do the division on it. This is a scale limit, in fact it's interesting that the positive values are the small cubes, while the negative ones are the balloons. We do the division in a numerical way, it's not a visual thing. [. . .] From what I can see on the video, dividing means having a lower number of cubes, but that's all.

Third Example

In this example, she uses the virtual scale that works just with positive integers numbers:

#1 T: in this scale there is already written (*pronouncing*) an equation. Then, the equation that is written involves (*pronouncing*) to put (*she mimes the first member*, Fig. 12) on the first side the things on the first plate and (*she mimes the second member*, Fig. 13) in the second side the things on the second plate. For example, (*she looks at the screen of a student*), here there is written 3x+1, then it means that I put three small cubes (*she mimes again the first member*, Fig. 14) that correspond to the x, (*she is speaking with the same student*), (*with a sparky tone of voice and pronouncing*) take

Fig. 12 A mime of the first member of the equation

Fig. 13 A mime of the second member of the equation

Fig. 14 Another mime of the first member of the equation

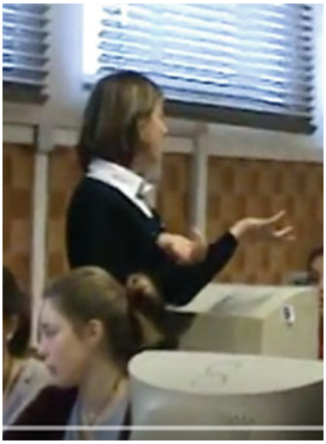


Fig. 15 Mime of the balance plate

Fig. 16 Mime of the balance

them and put them with the mouse, (*she remains as in posture of* Fig. 14) one, two, three and then one, namely one small cube. Then, on the other side (*she mimes the second plate*, Fig. 15) there are four, then (*pronouncing*) let's put 4 small cubes. When we have put four small cubes and even when we have put 4 of them, the scale remains in equilibrium (*she mimes the balance*, Fig. 16). The small cubes can be carried from one side to the other, obviously if I have a pan balance (*she mimes the balance again*, Fig. 16) in equilibrium, if I take a small cube from the left plate and I move it on the right plate, the scale will not continue of remaining in equilibrium. Actually, (*rhetorical question*) what happens? It happens that the right plate will weigh more than the left plate, then the scale will become in perfect unbalance (*she mimes the unbalance*, Fig. 17). For arriving to the equilibrium, what has to happen to the small cubes whose values are equal to 1?

#2 Ss: I have to remove it also from the other side.

Fig. 17 Mime of the unbalance

#3 T: exactly as we said above (*highest pitch*) for the equations, the first principle of equivalence says us that we can remove a small cube from one side and another small cube from the other side. If someone is lucky in "gaming" (*the applet on her computer does not work*), removing a small cube is possible because there is a trash and I can take the small cube and move it to the trash. If I move to the trash one small cube just from one side, the scale becomes unbalanced, but if I remove a small cube also from the other one, the scale is again in equilibrium. Then I also have some x, but if I remove one x, then I have to remove also from the other side the same value corresponding to x. Then I finished and I solved the equation when on one of the two plates I have one x and on the other plate I have just small cubes valuing 1, such that I can say which is the equivalent of the value of x.

Discussion

In the above excerpt, she is presenting the virtual scale in order to see the first principle of equivalence for the equations through the metaphor of the virtual scale (rational key). In particular, firstly, she explains how to set up the equation on the virtual scale (#1) and, then, she introduces how this Java applet works: it is possible to discuss the balance or the unbalance of the virtual scale moving the small cubes on the plates to the "Trash" or not (a button offered by the applet). She mimes the balance (Fig. 16) and the unbalance (Fig. 17) of the virtual scale when she is thinking of what happens if she removes "a small cube that values 1" just from "one side" of the scale (#1), without students actually setting up the equation on the applet. Evidently, in this case, the scale would be unbalanced. It is very clear in this moment that Silvia is again drawing on the fact that the virtual scale could also foster students' imaginations, without their actually acting on it. Moreover, from her gesturing in #1, it is also visible that Silvia is exploiting the potential of using a virtual scale: there is an intrinsic dynamism in the use of it (adding, moving to the trash, the balance, the unbalance) that constitutes an added value with respect to a static scale drawn on the blackboard, upon which students cannot practically work without visualizing the results of their actions on it.

Hence, we can deduce from the excerpt Silvia's expectation that students understand the meaning of the first principle of equivalence directly manipulating on the virtual scale and that students are able to translate into mathematical expression what is played at the metaphor level (emotional key). This emotional key is revealed by her use of numerous verbs that refer to the physical action on the scale: for example, she pronounces the verb "to put" as the action of student is actually happening ("put them with the mouse", "to put… on the first plate", "let's put 4 small cubes"); she invites a students using a sparky tone of voice to "take them (the small cubes) and put them with the mouse" as she actually places the cubes on the plate with her hands; the gestures of miming the balance and the unbalance that accompany the dynamism of the actions of carrying "from one side to the other", of "take away a small cube from the left plate and I move it on the right plate", for referring to the unbalance. Furthermore, the rhetorical question "What happens?": it is very immediate for the students to visualize that removing one weigh from one plate of a scale in balance provokes its unbalance, and that for returning to a state of equilibrium, it is sufficient to remove the same weight from the other plate. Hence, the *teleological emotionality* of Silvia is demonstrated here, because she not only invokes the action of considering the virtual scale for seeing in another way the principle of equivalence for the equations, but also indicates her hope that they become aware of the meaning of the first principle: she communicates implicitly via her lesson that each of them can directly act themselves on a virtual scale, immediately visualizing the results of their actions on the scale.

Furthermore, she justifies the balance and the unbalance of the virtual scale through making this model analogous to the algebraic form of the principle of equivalence, by stressing that there are two equivalent manners for explaining how the first principle of equivalence works (rational key). This rational key is accompanied by the fact that she expects again that it is useful to directly act on the virtual scale (emotional key). This emotional key is revealed in this excerpt by her exactly repeating for two times, at the end and at the beginning, the same words, explaining what happens when moving the small cubes on the scale in terms of the equilibrium or unbalance of the balance. Moreover, due to the fact that the applet sometimes could not work, she considers "lucky" the students who can use the applet, for actually becoming aware of the meaning of what they are doing. These latter emotional and rational keys shape together Silvia's *epistemic emotionality*, because it is not only the matter of constructing the equivalence between the algebraic form of the principle and the metaphor of the scale, but also her expectations for students become themselves aware of it through using the virtual scale, that create the possibility of physically manipulating the scale.

The prosody, the gestures, and the repetitions highlighted above, sketch the Silvia's emotional engagement, and accompany her verbal communication. This

Fig. 18 Miming the lifting

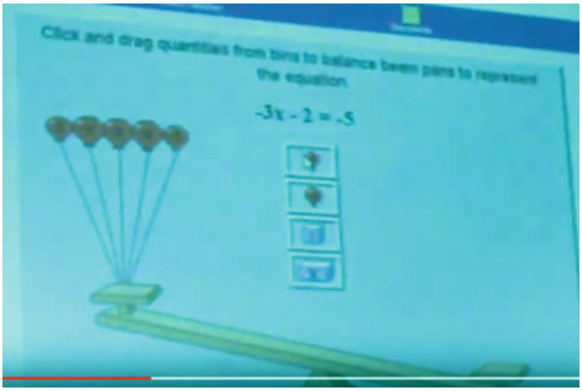


Fig. 19 Balloons that value -5

emotional involvement is unavoidable during her decision-making processes; it is for this reason that I speak of Silvia's *communicative emotionality*.

Fifth Example

In this example, the teacher presents the virtual scale that also works with negative numbers:

#1 T: This is an applet conceived for working with negative numbers (*she changes again the problem*) –3x–2 = −5, the principle is always the same, but now I have to remove (*pronouncing –3x and at the same time she raises one hand for miming the lifting*, Fig. 18) –3x, then I have to lift, then I can do –3x with some balloons that are –x and I can do −2 with some balloons that are −2 (Fig. 19) and I can do −5 with some balloons that value −5. Now if I have to solve this thing (*another time*

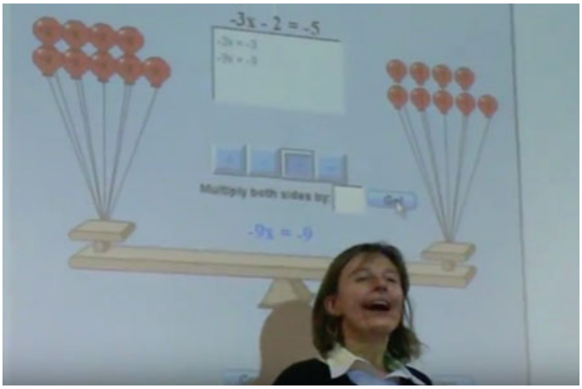


Fig. 20 Smiling: they tripled, right?

she is detaching from the screen) what can I do? (*pause and she looks at the class*)

#2 S3: Adding 2

#3 T: (nodding) Adding 2, if I add 2 to both sides, what happens? From one side 2 balloons explode and from the other side it happens the same. What can I still do?

#4 S11: Removing 3

#5 S17: Adding 3

#6 T: Then adding 3 and then multiplying by 3. If I multiply by 3 what happens? (the applet gives as result $-9x = -9$ and she smiles)?

#7 S4: They multiply each others

#8 T: (smiling, Fig. 20) They triple, right? The balloons triple. Then, for coming back to the previous thing, what can I do?

#9 Ss: Sividing by 3 (*the applet gives* $-3x = -3$)

#10 S6: And then adding 3 for knowing the value of x

#11 T: If I add 3 (*she writes 3, but she does not press on "Enter" and she asks at the class*) what happens on the left?

#12 S6: 3 balloons will explode

#13 T: (*nodding*) Three balloons and on the other plate?

#14 S5: There are no more balloons

#15 T: There are no more balloons then what will happen? (*She smiles and then she pushes on the "Enter" button and 3 small cubes of value 1 are created on the left plate*, Fig. 21). Here (*she refers to the right plate*) 3 balloons are exploded, while there (*she refers to the left plate*) the three units that we added are arrived. The equation is become this one (*she points to the rectangle of the equivalent equations*), namely $-3x+3=0$, 0 because $-3+3$ is 0 and then it becomes 0. Now, what is happening? (*again she detaches from the computer and she is going towards the class and she pauses*) I want the value of x, actually (*she turns towards the equation on the scale*) here I have some balloons with –x, how can I know the value of x? I want have some x and not –x.

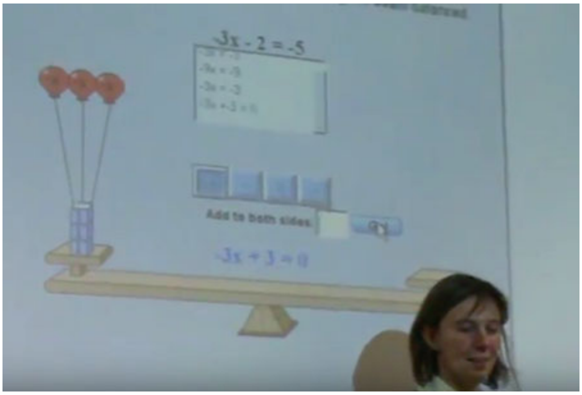


Fig. 21 Three new balloons on the right plate

Noise in the classroom

#16 S4: Divided by –x

#17 S9: You can explode the balloons

#18 T: How can we explode the balloons?

#19 S5: +3x

#20 T: (*nodding*) +3x, because it is dangerous divided by –x, because we don't know what is x, x could also be 0. If I do +3, what happens?

S8: it is –3x+6=6

#21 T: (*she returns on the screen and she points to the plates*) On one side I will have 3 units (*she mimes them on the virtual plate of the scale*).

On the other side I already have 3 of them and I will have three more. However, I want to explode the balloons, then I add 3x, because in this way if I add 3x, what happens? On one side the 3 balloons explode (*she points to the left plate of the scale*) and on this side (*right plate*) it comes 3 values of x and now I have to know the value of x, how can do it? (*She detaches from the computer and she looks at the class*)

#22 Ss: Divided by 3

#23 T: Divided by 3 and I obtain that 1 is equal to x, so which will be the solution?

#24 Ss: x equal to 1

#25 T: Not, the number 1 because yesterday we have said that the solution was the number that makes true our sentence. If I say x equal to 1 actually I'm saying the sentence, I say that (*pronouncing*) x is equal to 1, I'm not saying the solution, and it is exactly the same thing writing (*she points to the algebraic expression on the screen*, Fig. 22) 1 equal to x or x equal to 1: it is the same equality.

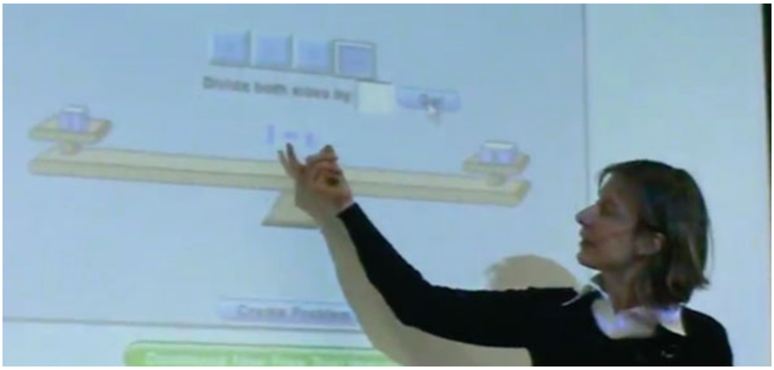


Fig. 22 Pointing the algebraic expression on the screen

Discussion

Silvia has explained that another Java applet exists which is designed to use a scale that also works with negative numbers. In the above excerpt she wants to solve the equation –3x–2 = −5, applying the principle of equivalences (rational key). She uses this particular virtual scale because she has the expectation that students can better understand the meaning of working with negative numbers through its modelling of the important principles. For example, she speaks of the monomial –3x (#1): actually, in the scale, the balloons could convey the sense of negative values, because there is something that "lifts" the plate of the scale (emotional key). The emotional key, for example, is evident from her pronouncing –3x, for focusing the attention of the students on the fact that they are going to speak of this –3x, because it has an important role in this context. Moreover, the expectation of better understanding the meaning of –3x through the use of the virtual scale is visible through her gesture of "lifting" in relation to the balloons (see Fig. 18), for better clarify this as something that has a negative weight.

In this brief passage we can also note the *teleological emotionality* of Silvia, because we find both actions for solving an equation and also her expectation that doing this with the virtual scale can assist students in better anchoring the meaning of operating with negative numbers to something that is very intuitive (such as the metaphor of the balloons). However, she detaches from the screen of the computer when she asks her students how to solve the equation –3x–2 = −5. It seems that she is on the algebraic level of solving the equation, not interpreting the resolution of it on the level of the metaphor of the scale (#1). Actually, one student answers on the same algebraic level (#2: "adding 2"), not referring to the scale. The teacher quickly returns to the scale for seeing the effects of having added 2 in terms of the balloons that are exploded. The interesting thing is that she uses "sides" instead of "plates". This is another hint of the fact they, firstly, worked on the numerical level. Again they return to the numerical level, in fact they arrive at the equation –9x = −9, working on the algebraic manipulation level. But again, she wants to focus the attention of the students on the effects of this operation in terms of the virtual scale, interpreting the multiplying by 3 as a tripling of the balloons. She smiles (see Fig. 20), perhaps because she is in an uncomfortable situation: actually, operating on the scale, they have just complicated the equation (# 8). Then, she actually wants to return to the previous equivalent equation, $-3x = -3$ (# 8). So she asks the students what they have to do on the mathematical level. After her question,"what happens on the left?" (#11), a student returns to the level of the metaphor, answering "3 balloons explode" (# 12). It is interesting that, yet again, Silvia has used the opportunity given by the applet of "reasoning without seeing". In fact, she poses this question in #11, but she does not press "Enter", expecting instead feedback from the class. Actually S6 answers on the level of imagination, thinking of what the applet would do. Silvia explains in response that, on the right plate, there are no more balloons, because $-3+3=0$ (she justifies it on the numerical level), while on the left plate, 3 small cubes appear; the equation becomes $-3x+3=0$, and she points to the rectangle in which the applet writes all the equivalent equations constructed during the resolution. Again, she smiles – probably because she knows that it is difficult to justify the things just remaining on the metaphor of the scale – and, at this point, she is considering the mathematical level (# 15). Concerning the emotional involvement, she actually detaches from the screen; but after, she turns towards the virtual scale. The interesting thing is that although one student answers on the algebraic level (#16: "divided by –x"), the other one would like to answer on the metaphor level (#18: "you can explode the balloons"). The teacher chooses to go more deeply into the metaphor level, asking how to explode the balloons. A student proposes to divide by 3x, but Silvia explains that would be dangerous because they don't know the value of x (# 20). Instead, she justifies the impossibility of adding 3x, from an algebraic point of view: actually it cannot be explained through the metaphor of the scale. This intertwining between the algebraic manipulation and the effects of it in terms of the metaphor of the scale continues, until the finding of the solution, namely the number 1. In particular, she explains that the solution is not $x=1$, because this is still a sentence with the verb "to be equal to", namely an equation, returning to the mathematical definition of equation. It seems that she is reviewing all the passages of solving the equation from the algebraic point of view, because it is not very simple directly working on the scale with the balloons and the small cube. But, after, she always returns to the metaphor for highlighting the results of what they did in terms of the balloons, that is, for linking the arithmetical operation to a possible meaning of it. There is a dialectic between the algebraic level and the metaphor level. The technology and Algebra complement each other: where the former does not explain, the latter intervenes, and where the latter lacks an actual meaning the former intervenes to make visible the results of the algebraic manipulation. In summary, in addition to the justifications of the passages within the resolution of the equation (rational key), the teacher expects that the virtual scale will help students to anchor the effects of their algebraic manipulations in terms of the scale metaphor, drawing upon their mathematical knowledge when it is not clearly evident what to do, simply by using technology (emotional key). This emotional key is demonstrated by her unconscious detaching from the screen of the computer when she clarifies what happens on the mathematical level, and again by her returning to the screen when trying to reinterpret it in terms of the metaphor. Moreover, she often smiles when she is uncomfortable, in becoming aware that reasoning on the level of technology has just complicated the problem. When she is uncomfortable in this way, she attempts to fix things by referring to the mathematics knowledge. Hence, Silvia's *epistemic emotionality* is revealed in the dialectics between the two levels.

Finally, her communication is both oriented towards reaching understanding by the class and also full of eloquent facial expressions, gestures and specific tone of voice; students grasp meanings conveyed by the combinations of these, and answer her questions, also influenced by how she is speaking. In other words, students understand what she is expecting from them because she cannot hide her hopes and needs. For this reason I further highlight Silvia's *communicative emotionality*.

Conclusion

As already described in the introduction to this chapter, my work concerns the analysis of a case study. For this reason, I cannot infer general conclusions. I show instead how the theoretical perspective enables a detailed analysis of the behaviour of the teacher. In particular, through the concept of emotional orientation, intended as the set of the expectations of the teacher, it is possible to outline the teacher's ræmotionality. As demonstrated in the data analysis, the components of the ræmotionality of the teacher (the epistemic emotionality, the teleological emotionality and the communicative emotionality) are always related to why she decides to put into play that specific knowledge related to the technology; to why she chooses to act in a specific manner for achieving a particular goal connecting to the use of digital technologies; and to why she speaks in that way for reaching understanding within the classroom. In this sense, complementing Habermasian rationality with the affective dimension allows me to detect reasons for the decisions of the teacher that occur in the moment.

Moreover, in the data analysis, I considered the use of two kinds of technology: GeoGebra and the applet virtual scale. Even if they are very different, there is at least one common point between them: the fact that both of them "feed" the creativity and the imagination of students. For example, in the case of GeoGebra, students see just some straight lines moving up and down, because of the constraints of the screen. But, thanks to the dynamic quality of the technological environment, and the illusion of "continuity" generated by the software, students can immediately imagine that they could potentially construct infinitely many straight lines, obtaining the same abscissa of the intersection point. This has an important effect from an algebraic point of view: in fact, the dynamic continuity reflects possibility of infinitely many different equivalent equations we can construct from an algebraic point of view. Hence, the fact that technology helps students in training their imagination facilitates also the coordination among different registers of representation. Furthermore, during the *a-priori* interview, Silvia declares that GeoGebra supports the imagination of students even when they don't have the technology at their disposal.

Concerning the use of the Java applet, very often the teacher and also the students imagine what might happen in terms of the metaphor evoked by the virtual scale, without actually acting on it. For example, in the third example, Silvia discusses the equilibrium and the unbalance of the scale using only one's imagination (#1), and the students do the same (#2). Moreover, Silvia asks students what the virtual scale is going to do, without press the button "Enter" for actually see the effect of the operation on it.

These concluding remarks evolved out of my analysis of the different excerpts I share above. I was curious to know if they were merely my conjectures or not. For this reason, I again interviewed Silvia, asking her a direct question about her expectation for technology in general. Look at what she answered:

> "In the last GeoGebra Day,[3] Barzel said that the technology can be used as a white box or as a black box: as the former when I know the theory and I want to know if it works; as the latter when I want to make conjectures, counterexamples. I am faced in this discourse [. . .] The technology helps you in introducing a difficult topic, because it allows you to see the things. Through the technology, it is clear the link among the different registers of representation. It is so much evident that you have not to explain things [. . .] For example, seeing the graphs in movement and making conjectures on them helps very much the imagination. It helps me also when I don't have the technology at my disposal, because in my head I can imagine the graph, the formula, the table of the points and so on very easily, having behind the support of the technology. Then, the technology helps you to imagine "what it would be, if". The technology does not flatten the thinking level, but it increases it."

Silvia describes her ***expectation that the use of technology in general fosters students' imagination for constructing mathematical concepts***. This is for me a way to "close the circle". From the *a-priori* interview, I identified the expectations on GeoGebra and the Java applet in a separated way; but after the analyses I became aware of the fact that I could infer something more generally on the use of technology. Actually there exists a deep link between the reasons of using different technologies. Moreover, it also becomes clearer why Silvia uses technology as often as she does in her classroom activity.

After having identified teacher's expectations on the use of digital technologies within the mathematics classroom, it is interesting to investigate if these expectations are really translated into student learning. During the experimentation, there are different episodes in which it is possible to verify such actual transfer from teacher's expectations to students' learning, but another more precise study would be necessary to pursue this related set of questions. I look forward to sharing my analysis as a future development of my research.

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[3]During the last GeoGebra Day (in the city of Torino in 2015), Barbel Barzel has made a conference, from the title: "From the value of teaching mathematics with technology: discovering, conceptualising, modelling".

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