Chapter 1
What Can We Learn from Textbook Analysis?

Ji-Won Son and Jeri Diletti

Abstract As a fundamental resource, textbooks have the potential to shape the way we teach and learn mathematics. While a growing body of textbook analysis studies has sought better ways to improve students' mathematics achievement, no meta-analysis has yet summarized those studies and their methods. This chapter reviews international comparative studies that analyzed learning opportunities presented in mathematics textbooks in the USA and five high-achieving Asian education systems. We summarize what research studies say about learning opportunities presented in textbooks in connection to the theoretical frameworks used, and their plausible relationship with students' mathematics achievement. Following this description and analysis, we raise several questions and issues for mathematics education researchers to discuss, to promote a critical examination of what can be learned from the content of textbooks in other countries.

Keywords Textbook analysis • Literature review • Content analysis • Problem analysis

Introduction

Over the past 30 years, changes in mathematics classroom practices and teaching methodologies have led to concerns regarding the quality of mathematics textbooks. Because textbooks are often the curricular materials that are the most influential on what happens in classrooms (Kilpatrick, Swafford, & Findell, 2001), they have attracted more and more research attention from the international mathematics education community in the past three decades, particularly in connection with international assessment studies (Cai, 2010; Cai, Mok, Reedy, & Stacey, 2016). For instance, the results from international assessment studies of mathematics, including the *Trends in International Mathematics and Science Study*

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J.-W. Son et al. (eds.), *What Matters? Research Trends in International Comparative Studies in Mathematics Education*, Research in Mathematics Education, DOI 10.1007/978-3-319-51187-0_1

(TIMSS: 1995, 1999, 2003, 2007, and 2011) and the *Program for International Student Assessment* (PISA: 2003 to 2012), offer opportunities to compare the mathematics performance of US students with that of their peers in other countries. The recent 2011 TIMSS revealed that while US fourth and eighth graders scored above the international average of the 63 TIMSS countries, they fell significantly behind the students in five Asian education systems at both grade levels: Hong Kong, Singapore, Korea, China, and Japan (Mullis, Martin, Foy, & Arora, 2012). Accordingly, researchers have been looking into the varying reasons why students in Asian countries tend to outperform their US counterparts and have identified several important factors that are linked to student achievement (Cai & Howson, 2013; Wagemaker, 2003).

Factors that potentially impact student learning include the curriculum as a whole and the curricular materials available, including textbooks. Researchers have identified multiple factors that have an impact on student learning, including student-level factors (e.g., students' home background, their socioeconomic status, and gender differences) (Bos & Kuiper, 1999), teacher- or classroom-level factors (e.g., peer influence, teacher quality, and teachers' instructional approaches) (Cai, Ding, & Wang, 2014; Kupari, 2006; Papanastasiou, 2008), and contextual or school-level factors (e.g., the location of the school, the number of desks) (Creemers, 1994). In particular, with a focus on identifying curricular influence on students' academic achievement, researchers have come to the understanding that cross-system similarities and differences in curriculum can provide *partial* explanations for cross-national discrepancies in students' mathematics performance (Cai, Ni, & Lester, 2011; Cai, Wang, Moyer, Wang, & Nie, 2011; Schmidt et al., 2001; Silver, 2009). For example, when comparing the curriculum materials from several high-achieving countries, such as Japan, South Korea, and China, researchers noted that the US curriculum materials devoted more page space to student practice than to content instruction (e.g., Carter, Li, & Ferrucci, 1997; Kim, 2012) and failed to provide challenging mathematics content and problems (e.g., Cai, Ni, & Lester, 2011; Li, 2007; Schmidt, McKnight, & Raizen, 1997; Son & Hu, 2016). Thus, by changing content presentation and organization, researchers have attempted to improve students' mathematics achievement.

However, researchers use several different methods to study the important question of how mathematics textbooks in different countries structure learning opportunities for their students. A complete framework for textbook analysis remains unavailable (Charalambous, Delaney, Hsu, & Mesa, 2010; O'Keeffe & O'Donoghue, 2015). In addition, despite the important position of curriculum materials in mathematics classrooms worldwide, researchers have expressed contrasting views about what can be learned from analyzing mathematics textbooks. Some researchers claim that textbook analysis can explain differences in students' performance in international comparative studies (Cai, Ni, & Lester, 2011; Cai, Wang, Moyer, Wang, & Nie, 2011; Fuson, Stigler, & Bartsch, 1988; Li, 2002; Son & Senk, 2010). Other researchers, however, have argued that textbooks bear little influence on instruction and on what students learn (Freeman & Porter, 1989). This line of research has viewed textbooks as a potential source for teacher learning, a goal that is frequently unfulfilled (Newton & Newton, 2007; Remillard, 2005; Son & Kim, 2015).

We generally agree that the analyses of curriculum materials reveal *nuanced insights* into variations in what is made available to students and teachers in textbooks, and also how that content is made available (Silver, 2009). In this chapter, we take a more moderate viewpoint by suggesting that textbooks afford probabilistic rather than deterministic opportunities to learn mathematics (Mesa, 2004; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). While the deterministic perspective toward textbook analysis links students' mathematics achievement *directly* to the content of their textbooks, the probabilistic perspective acknowledges the possible mediated effect of teachers and students on the learning opportunities presented in textbooks, because the role of textbooks in instruction depends on how students and teachers interact with them (Remillard, 2005; Son & Kim, 2015). From the probabilistic perspective, textbook analysis can only reveal different *performance expectations* made of students in different countries, the extent to which a country's textbook series prioritizes conceptual understanding or procedural fluency, and how the treatment of mathematical content and problems differs among countries. With this view, we intend to review international comparative studies that analyzed learning opportunities presented in mathematics textbooks.

This chapter presents a survey study that aims to examine, analyze, and review relevant textbook research systematically. We focus on textbook analysis research studies that perform international mathematics assessments between and among the USA and five high-performing Asian education systems: Japan, China, Singapore, South Korea, and Taiwan. We first summarize what research studies say about variations or commonalities in terms of the learning opportunities presented in mathematics textbooks across different education systems, as this might account in part for disparities in students' mathematics achievement. In doing so, we specifically look at the theoretical frameworks used in textbook analysis studies and the findings drawn from each framework. Charalambous et al. (2010) defined a "textbook signature" as "the uniform distinctive features within a particular country." By summarizing the findings from prior research, this study reports whether any unique signature represents each education system's textbooks. We then raise questions and issues for mathematics education researchers in terms of conceptualization and methodological matters, leading to a critical examination of what can be learned from textbooks from other countries. In the next section, we define curriculum and textbooks, and discuss the data and coding framework used in this study.

Methods

Assumptions and Definition of Terms

In this study, by *textbook series* we mean a set of curricular resources that teachers use for day-to-day teaching, which includes student texts, workbooks, and the teacher's guide. Drawn from TIMSS' definition (Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999), we define the *intended curriculum* as the set of standards students are required to achieve. The *potentially implemented curriculum* may include teacher manuals, students' main textbooks, and supplemental materials such as student workbooks, review materials, and assessments. An examination of textbooks informs policymakers of how societal visions and educational objectives, seen in national policies and official documents as *the intended curriculum*, are *potentially* embodied in classrooms (Schmidt et al., 1999; Valverde et al., 2002). This study focuses on research studies that analyzed the content (e.g., content coverage) and/or problems of mathematics textbooks in international comparisons, while comparing the similarities and differences of two or more series of mathematics textbooks.

Selection Search Procedures

The research review presented here is based on an analysis of peer-reviewed research articles that focused on learning opportunities presented in mathematics textbooks between and among the USA, Japan, China, Singapore, South Korea, and Taiwan. We conducted our literature review via the Education Resource and Information Center (ERIC), allegedly the world's largest digital library for education literature, as well as via Google Scholar. We obtained peer-reviewed research articles primarily using the search terms of "textbook" and "mathematics," and refined our searches further by adding several groups of terms, including "textbook research," "textbook content," and "textbook analysis." Next, we systematically examined past issues of peer-reviewed research journals in mathematics education to identify the relevant literature, including the following:

ZDM—The International Journal of Math Education
ESM—Educational Studies in Math Education
CI—Cognition and Instruction
SSM—School Science and Mathematics
JCS—Journal of Curriculum Studies
IJME—International Journal of Science and Mathematics Education
JRME—Journal for Research in Mathematics Education

These journals were selected based on two criteria. First, their scope of publication covers a great range of mathematics education research, and secondly, they are all highly ranked. Nevertheless, research articles that were published in other journals also received attention, though we mainly identified these articles through ERIC searches, not directly from the journals. Our search was also limited to peer-reviewed research articles published between 1988 and the first half of 2015, because the reform movement began with the formation of the National Council of Teachers of Mathematics (NCTM) in 1988.

It should be mentioned that our collection of the relevant literature is by no means complete, which is a limitation of our study. While the ERIC database includes a variety of sources, the main body of the literature we have identified consists of peer-reviewed journal articles based on original empirical studies. Thus, we did not pay attention to sources such as books, doctoral dissertations, and papers presented at conferences. We must point out that although we tried to make the survey as comprehensive as possible within our criteria, it is possible that some important peer-reviewed research work in this area was missed in the selection process. This is due to a variety of reasons, including the scope and focus of the study and the fact that not all research is accessible via ERIC or published in journals. These challenges are common among survey studies like this one.

Coding Framework

After we selected the literature, we first constructed a database by classifying all the articles using the six criteria established for the study, including topics, research questions, grade level, education systems analyzed, analytical framework used, and major findings reported. This analysis helped us provide an overview of general tendencies in textbook analysis studies. Next, we further analyzed each article based on the following framework for content analysis and problem analysis (see Table 1.1).

Content analysis refers to comparing learning goals, lists of topics (content coverage), topic placement, textbook size, allocation of content, allocation of time, repetition of content, development of concepts and procedures, the use of technology, and the use of worked examples. Problem analysis means classifying the textbook exercises and problems/tasks by various kinds of schemes, such as the characteristics of mathematical features, contextual features, cognitive demand, cognitive expectations, depth of knowledge required for solving problems, and the relevance of non-textual elements. For the analytical foci in problem analysis, including cognitive demand, cognitive expectations, and cognitive depth of knowledge, we referred to Stein, Grover, and Henningsen (1996), Son and Senk (2010), and Webb (1999), respectively.

Charalambous et al. (2010) called the former type of textbook analysis horizontal analysis, especially focusing on the overall structures of textbooks (i.e., what mathematics is taught at what grade level), and the latter type as vertical analysis, focusing on the treatment of a particular mathematical topic. Li, Chen, and An (2009) called the former type macroanalysis and the latter type microanalysis. In our study, content analysis exceeds the horizontal analysis or macroanalysis by including textbook size, allocation of content, allocation of time, repetition of content, development of concepts and procedures, the use of technology, and the use of worked examples. The findings reported in each article were categorized based on the sub-dimensions shown in Table 1.1 to describe how mathematics textbooks in different education systems structure learning opportunities for their

Table 1.1 A framework for classifying the literature on textbook analysis research

Analytical foci	Subcomponent
Content analysis (macro, horizontal)	• Content coverage (topic placement)
	• Size/length of book
	• Introduction and development of concepts and procedures
	• Repetition of content
	• Others (e.g., the use of technology and worked examples)
Problem analysis (micro, vertical)	• Mathematical features (number of steps required: single vs. multiple)
	• Contextual features (purely mathematical or illustrative)
	• Response type (numerical answer only or explanation required)
	• Cognitive demand (degree to which students are required to engage cognitively: high or low)
	• Cognitive expectations (kind of knowledge/process required in solving problems: conceptual knowledge, procedural knowledge, representations, mathematical reasoning, and problem-solving)
	• Depth of knowledge (the complexity of mental processing that occurs to answer a question or perform a task: level 1, 2, 3, or 4)
	• Relevance of non-textual elements (e.g., photos, pictorial illustrations, mathematical representations, and pictures)

students, and whether there is any unique signature that represents the textbooks in each education system. Each sub-dimension in both content and problem analysis will be discussed in detail in the findings section.

Results

Overall Tendencies in Textbook Analysis

Appendix gives a list of the peer-reviewed research articles analyzed in our survey. In total, we identified 31 articles that addressed international textbook comparisons between the USA and the five high-achieving Asian education systems. Figure 1.1 illustrates the frequency of the education systems surveyed in comparisons of mathematics textbooks. All but one article included the USA in the comparisons. The majority of textbook analysis research focused on China, followed by studies comparing Japan and South Korea to the USA. In total, 17 textbook analysis studies comparing China to other education systems were identified in our data source. Singapore and Taiwan were analyzed relatively less frequently. Only five textbook analysis studies were conducted based on Singapore, and four studies involved Taiwan.

Table 1.2 presents a frequency count of the type of analysis (content, problem, or both), grade level, and topic. Of the 31 articles surveyed, six strictly analyzed the content of the textbooks. An article was counted as strictly content analysis if the

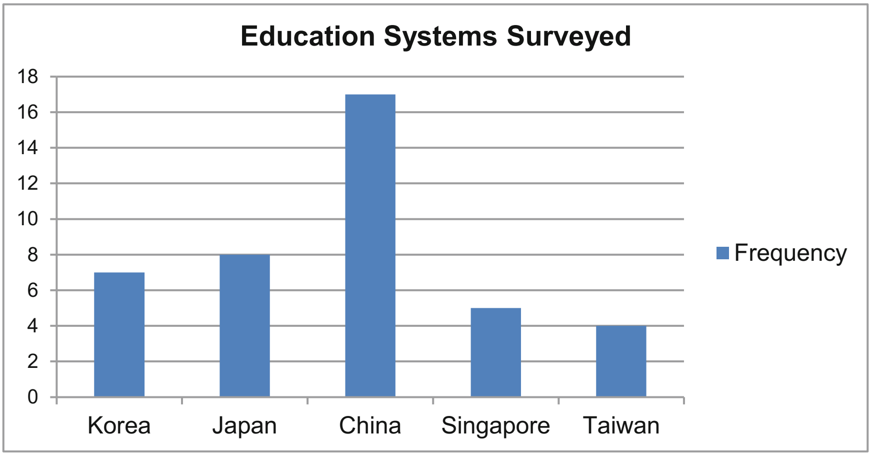

Fig. 1.1 Frequency of textbook analysis research by education system

Table 1.2 Frequency of research articles with respect to analysis, grade level, and topics

Themes	Frequency
Analysis foci	
• Content	6
• Problem	11
• Both	14
Grade level	
• 1	8
• 2	6
• 3	7
• 4	7
• 5	8
• 6	10
• 7	11
• 8	10
• 9	4
• 10	5
• 11	2
• 12	1
Topic	
• Whole numbers	5
• Fractions	8
• Integers	3
• Algebra	4
• Probability	2
• Geometry	2
• Others (e.g., average, percent)	7

researchers looked at the overall structure of the book, which includes factors such as the grade placement of topics, size of the books and number of pages, methods of introducing topics and developing concepts and procedures, the use of technology, and the number of worked examples. Eleven of the articles strictly analyzed the problems and tasks within the student textbooks or supplemental materials (e.g., student workbooks, review materials, and assessments). These articles addressed factors such as tasks that require an answer only vs. those that also require explanation, frequency of tasks involving real-world applications, and cognitive demand and expectations required in solving problems. The 14 remaining articles analyzed both content and problem tasks.

With respect to grade level, the most frequently analyzed grade levels were the elementary grades 1–5, followed by the middle grades, especially grades 6, 7, and 8. High school levels (grades 9–12) were the least analyzed. Note that occasionally topics do not appear at the same grade level in the US and Asian education systems. This represents a particular challenge for those engaged in textbook analysis if grade bands are used, such as elementary, middle, and high school grades. Thus, we presented the frequency of the topic analyzed by grade. Because research articles that analyzed more than one grade level were counted once for each grade level covered, the total frequency counts for grade level is higher than the 31 total articles surveyed.

A wide range of topics was discovered in our survey. The most widely analyzed topic was fractions. This includes the introduction of fractions as well as operations with fractions. A total of eight articles had some fractional component to them (Alajmi, 2012; Charalambous et al., 2010; Li et al., 2009; Son, 2012; Son & Senk, 2010; Sun, 2011; Sun & Kulm, 2010; Yang, Reys, & Wu, 2010). Son and Senk (2010), for example, examined how fraction multiplication and division were introduced, and what types of problems were used to facilitate the development of procedural and conceptual understanding in textbooks in the USA and South Korea. Five surveyed articles analyzed tasks and the placement of either whole number operations (Fuson et al., 1988; Watanabe, 2003) or introductions to whole number operations (Boonlerts & Inprasitha, 2013; Kang, 2014; Xin, Liu, & Zheng, 2011). For example, Watanabe (2003) compared and contrasted the number of lessons, problem situations, and types of representations used in the initial treatment of multiplication of whole numbers in the USA and Japan. Four studies focused on algebraic thinking, including the distributive property, the equals sign, and algebraic problems (Ding & Li, 2010; Hong & Choi, 2014; Li, 2007; Li, 2007). For example, Ding and Li (2010) compared elementary textbooks in the USA and China when analyzing instances of the distributive property. Other topics include probability, average, the use of justifications, and explanations of work.

After establishing the type of analysis conducted, grade level, and topics, each article's framework and findings were explored in greater detail. The following sections detail the common themes discovered in our survey, broken down into content analysis and problem analysis.

Table 1.3 Frequency of textbook analysis research by sub-dimensions of content analysis

Content analysis	Frequency
Topic placement	10
Size/length of book	5
Repetition of content	2
Development of concepts and procedures	10
Others (e.g., the use of technology)	6

Content Analysis: Common Themes

Content analysis includes comparing content coverage, topic placement, textbook size, allocation of time, methods of introducing topics and developing concepts and procedures, repetition of content, the use of technology, and the characteristics of worked examples. A total of 20 articles fell into the category of content analysis. Of these, six strictly utilized content analysis and the remaining studies analyzed both content and problems. Table 1.3 presents the frequency of research articles addressing each of the subcomponents in content analysis. The most prevalent themes for content analysis were *topic placement* and *development of concepts and procedures*.

Topic Placement

Topic placement refers to the grade or chapter of the textbook where the content appeared. Nine surveyed articles reported on topic placement (Boonlerts & Inprasitha, 2013; Cai, Lo, & Watanabe, 2002; Choi & Park, 2013; Fuson et al., 1988; Hong & Choi, 2014; Kang, 2014; Li et al., 2009; Son, 2012; Son & Senk, 2010; Yang et al., 2010). Of the nine articles focusing on topic placement, eight reported that mathematical ideas tended to occur earlier in Asian education systems than in the USA (Cai et al., 2002; Choi & Park, 2013; Fuson et al., 1988; Hong & Choi, 2014; Li et al., 2009; Son & Senk, 2010). For instance, in their analysis of fifth- and sixth-grade textbooks Son and Senk (2010) found that multiplication of fractions appears earlier in Korean texts than in a standards-based US textbook (*Everyday Mathematics*). Some multiplication and division of fractions topics that appear in Korean textbooks did not appear at all in *Everyday Mathematics*. Similar results were found in Li et al.'s (2009) analysis of fraction division. They found that US textbooks vary the introduction of fraction division anywhere between grades six and eight. However, all the covered Chinese and Japanese textbooks introduce fraction division in sixth grade. Yang et al. (2010) also found similar results for computation of fractions in Singapore, Taiwan, and the USA. In their study, US students in the sixth grade were only expected to compare fractions, while in Taiwan sixth-grade students were expected to subtract proper fractions. Fifth-grade students in Singapore were required to subtract mixed numbers. In their analysis of computing averages, Cai et al. (2002) similarly found that China, Japan, and Taiwan introduce the topic as early as fourth grade, but averages do not appear until fifth grade in the USA.

Continuing the theme of the earlier introduction of content in Asian education systems, in their analysis of the addition and subtraction of whole numbers, Fuson et al. (1988) found uniformity of grade level placement in Japan and China, but not in the USA. Topics seem to appear earlier and disappear earlier in Asian texts than in US texts. Similarly, Hong and Choi (2014) found that topics relating to quadratic equations in grades 9, 10, and 11 are introduced earlier in Korean textbooks than in standards-based US textbooks, and some topics appearing in Korean textbooks did not appear at all in the standards-based US textbooks.

One study reported that geometry topics occur during the same grades in both Korea and the USA. However, this study looked at the intended curriculum of both countries, rather than at actual textbooks (Choi & Park, 2013). The final study that analyzed topic placement for the introduction and development of multiplication found that Singaporean textbooks begin teaching multiplication in first grade, while Japanese and Thai textbooks begin in grade 2 (Boonlerts & Inprasitha, 2013). However, this study did not compare its findings with US textbooks.

Introduction and Development of Mathematical Concepts and Procedures

Introduction and development of mathematical concepts and procedures was another prevalent theme in our survey. We found ten articles that analyzed this concept, reporting on methods of topic introduction, development of concepts and procedures, and how topics related to other content areas (Cai et al., 2002; Cheng & Wang, 2012; Han, Rosli, Capraro, & Capraro, 2011; Kim, 2012; Li, Ding, Capraro, & Capraro, 2008; Li et al., 2009; Son & Senk, 2010; Sun, 2011; Sun & Kulm, 2010; Watanabe, 2003). Of the ten, three articles analyzed how fractions and fraction operations are introduced and developed in US and Chinese series (Son & Senk, 2010; Sun, 2011; Sun & Kulm, 2010). Sun and Kulm (2010) analyzed one Chinese textbook series and one standards-based US textbook series (*Connected Mathematics*) with respect to fraction concepts. They found that the US textbooks focused on measurement and part-whole sub-constructs, while Chinese texts emphasized fractions as division and fractions as part-whole. According to Sun and Kulm, 55.6% of the content in the standards-based US textbook series involved the use of fraction strips as a measurement tool and 30.5% of the content related to the part-whole sub-construct in a real-world context. In contrast, 66.7% of the content in the Chinese textbook series focused on fractions as division, while 27.8% of the content was devoted to the part-whole sub-construct with an emphasis on equal sharing.

Sun (2011) also found variations in developing fraction division between four US and three Chinese series. According to Sun (2011), the Chinese textbook solidifies new concepts through abbreviated problem sets with conceptual connections, while the US series first utilizes repetition for retention of the fraction division procedure and then develops fluency. For example, Chinese texts treated fraction division as the inverse computation of fraction multiplication, with further explanation on how these two computations are conceptually related. In contrast, US textbooks used the idea of the reciprocal and the "flip-and-multiply" algorithm. Although the US textbooks also used verbal explanations and illustrations, they did not explain why the procedure works.

Continuing the exploration of developing mathematical concepts and procedures in US and Chinese texts, Cheng and Wang (2012) analyzed one standards-based (*Investigations*) and one traditional US series (*Mathematics*), along with two Chinese textbooks, for the development of number sense. They found that textbooks in both education systems stress the counting property of number sense. However, US traditional and reformed textbooks pay much more attention to number sense properties, such as various ways of counting, number meaning and representation, place value and base-ten concepts, and different number composition, compared to the Chinese traditional and reformed textbooks.

Further analysis of fractional concepts was performed by Li et al. (2009) on US, Chinese, and Japanese textbook series. The concept of fraction division was analyzed in three standards-based (*Mathematics in Context, Connected Mathematics*, and *MathScape*) and one traditional US textbook (*Glencoe*), alongside three Chinese and three Japanese series. Like Sun (2011), Li et al. (2009) found that all the books from Japan and China treated fraction division as an inverse computation of fraction multiplication, with further explanation of how these two computations are conceptually related. For example, Japanese textbooks introduced "$\frac{5}{8} \div \frac{1}{3}$" through a context-based problem and then introduced at least two ways of solving the problem: one with the anticipated computation of fraction division (i.e., the inverse computation of fraction multiplication, $\frac{5}{8} \div \frac{1}{3} = \frac{5}{8} \times \frac{3}{1} = \frac{15}{8}$) and the other with an alternative solution using multiplication (i.e., $\frac{5}{8} \div \frac{1}{3} = \left(\frac{5}{8} \div 1\right) \times 3 = \frac{5}{8 \times 1} \times 3 = \frac{5 \times 3}{8 \times 1}$). However, US textbooks emphasized how the division computation can be explained in a way similar to the division of whole numbers. For example, US textbooks tended to explain the meaning of division using "partitive" or "measurement" interpretations, with whole number division word problems. According to Li et al. (2009), Chinese texts tended to provide verbal explanations, numerical computations, and pictorial representations in connection with fraction division computation. In contrast, in the US textbooks verbal explanations and numerical expressions were only used to complement the process of fraction division. Japanese texts used some explanations, but were the first to use line segment representations to show relationships.

Analyzing the concept of averaging in US, Chinese, Japanese, and Taiwanese textbook series, Cai et al. (2002) found that the US textbook series focused on averages as measures of central tendency where Asian series focused on the meaning of average. Additionally, being able to solve complex problems is an explicit standard for China and Taiwan. All series used the process of evening out to find an average, but the Asian series used it as a model to mediate learning of the concept while the US series used it as representative of a data set. Watanabe (2003) also analyzed US and Japanese texts for representations of the multiplication of whole numbers. He found that all series use array situations and equal sets. Also, both the Japanese and the US textbook series provided in-depth discussions of the rationale for specific instructional decisions. However, the Japanese series did not emphasize the multiplier/multiplicand distinction and focused on only one property instead of multiple properties/strategies.

The introduction and development of mathematical concepts and procedures also varied between US and Korean textbook series. One standards-based US textbook series (*Everyday Mathematics*) and three Korean textbooks were analyzed by Son and Senk (2010). Looking at fraction multiplication and division, Son and Senk found that while the US textbook series introduced fraction multiplication as "part of a fractional part," the Korean textbooks introduced it as repeated addition and expanded this meaning from "part of whole units" to "part of a fractional part." Han et al. (2011), who analyzed two Korean, two Malaysian, and four US traditional textbooks for probability, also reported variations in terms of definitions of probability, and noted that only one textbook included experimental probability.

Textbook Size and Allocation of Content

Textbook size and *allocation of content* was the third most-researched theme in our survey. Textbook size refers to the total number of pages in the textbook, the overall dimensions of the textbook, and the number of pages per chapter. Allocation of content refers to the number of pages spent on a particular mathematical concept. Five articles fell under this theme (Alajmi, 2012; Choi & Park, 2013; Li, 2007; Saminy & Liu, 1997; Yan & Lianghuo, 2006). All five studies reported very similar findings. When comparing US and Korean textbooks, Choi and Park (2013) reported that US textbooks had more textbook pages and more overall chapters than Korean textbooks. The US textbooks also tended to be physically larger and have a significantly greater number of pages than Japanese texts (Alajmi, 2012; Saminy & Liu, 1997). However, while Saminy and Liu (1997) reported that Japan had a larger number of chapters in each textbook than the USA, Alajmi (2012) reported later that the US series had a greater number of chapters than Japanese textbooks. Finally, Li (2007) reported that US textbooks are longer than those in Hong Kong, China, and Singapore.

Allocation of time related to specific content areas tended to be greater in the US than in Asian education systems. Choi and Park (2013) found that more pages of the Korean textbooks were allocated to geometry, but the number of chapters devoted to geometry was similar in the USA and Korea. In terms of fractions, US textbooks had a greater number of chapters on fractions and a greater percentage of pages with fractions, as well as a greater number of fraction lessons than the textbooks in Japan (Alajmi, 2012). Similarly, Yan and Lianghuo (2006) found that the overall number of problems in the US textbook they analyzed was greater than the overall number of problems in the Chinese texts. However, they reported that the number of problems in a single section was nearly the same for textbooks in both countries. Yan and Lianghuo (2006) did not explain how each section can have similar amounts of problems when the overall number is higher in US textbooks. They also found that the ratio of exercise problems to text problems was higher in the USA than in China. Exercise problems are tasks which students are to complete on their own, whereas text problems are designed to include teacher intervention during the lesson. Other dimensions of content analysis, including repetition of content, the use of technology, and the use of worked examples, were not prevalent in our survey.

Summary and Implications of Content Analysis

Our survey of textbook analysis studies emphasizing content analysis suggests some variations and commonality in terms of learning opportunities presented in the US and Asian mathematics textbooks. First, in both the early grades and the later grades, mathematical topics tend to appear earlier in Asian education systems than they do in the USA. Textbooks in the USA tend to be physically larger as well as to contain more pages than Asian textbooks. Individual textbooks vary in size, but it is commonly reported that US textbooks spend more time on specific content areas, as well as on revisiting previously taught material. With regard to textbook exercises, the USA tends to have a larger ratio of exercise (practice) problems to text problems (in-class activities) than Asian countries. Furthermore, there are some commonalities but variations in how certain mathematical ideas are introduced and developed among the surveyed education systems.

Problem Analysis: Common Themes

Problem analysis entails classifying textbook exercises and problems by various kinds of schemes, as shown in Table 1.4. In problem analysis, problems can be defined as tasks appearing in textbooks, teacher manuals, and supplemental materials which are done during the instructional lesson or as independent practice by the students. Although a mathematical task can be a set of problems with a particular goal, we used *problems* and *tasks* interchangeably. Thus, problem analysis includes mathematical task analysis. Our survey of the problem analysis articles was based on the dimensions of problem requirements set forth by Li (2000). We categorized findings based on the following dimensions: *mathematical features* (the number of steps required in solving problems), *contextual features* (whether problems are presented in purely mathematical contexts, real-world contexts, or with illustrations), *response types* (type of answers required), *cognitive demand* (Stein et al., 1996), *cognitive expectation* (Son & Senk, 2010), *depth of knowledge* (Webb, 1999), and *the relevance of non-textual elements* like photos, pictorial illustrations, visual representations, and pictures (Kim, 2012). Table 1.4 gives a frequency count of peer-reviewed articles addressing these components. A total of 25 articles fell into the problem analysis group. Of these, 11 were strictly problem analysis and the remaining studies analyzed both content and problems.

Table 1.4 Frequency of the sub-dimensions of problem analysis

Problem analysis	Frequency
Mathematical features	
Single/multiple computational steps	4
Contextual features	
Purely mathematical/real world or illustrative	12
Response type	
Answer only/numerical expression required/explanation required	6
Cognitive demand	
High (doing mathematics/procedures with connections) Low (memorization; procedures without connections)	1
Cognitive expectation	
Conceptual knowledge, procedural knowledge, problem-solving, representation, and mathematical reasoning	10
Depth of knowledge:	1
Level 1; Level 2; Level 3; Level 4	
Relevance of non-textual elements	5

In the USA, there are two major textbook formats with differing pedagogical approaches, referred to here as traditional (commercial) and reform-oriented (standards-based) curricular materials. Standards-based materials are those that adopt the recommendations of the NCTM (1989, 2000) with the support from National Science Foundation, i.e., to include a classroom pedagogy that fosters the understanding of discrete concepts through communication and problem-solving (Senk & Thompson, 2003). Traditional textbooks tend to utilize direct instructional methods and reinforce concepts through individual practice (Senk & Thompson, 2003). While many traditional textbooks cite the content recommendations of the NCTM, ideological and political disputes have allowed them to retain their traditional pedagogy (Schoenfeld, 2004). In the USA, the choice of a mathematics textbook often occurs at the school level, and school districts have had a choice between traditional and reform curriculum materials (Reys, Reys, Lapan, & Holliday, 2003). Both types can be described as highly utilized across the USA. However, standards-based textbooks have been reported to have a higher level of conceptual questions than traditional textbooks (Son & Senk, 2010). This is important to recognize when interpreting the results of problem analysis. Table 1.4 shows that the most prevalent themes for problem analysis are *contextual features* and the depth and breadth of *cognitive requirements (cognitive expectation, cognitive demand,* and *depth of knowledge)*.

Mathematical Features

Mathematical features involve the number of steps required to answer a problem: tasks are classified as a single computational procedure or multiple computational procedures (Li, 2000). Four surveyed articles reported on mathematical features of the problems, with mixed results (Li, 2000; Li et al., 2009; Son & Senk, 2010; Yan & Lianghuo, 2006). When comparing the multiplication of fractions in US standards-based and Korean textbook series, Son and Senk (2010) found that 17% of the tasks in the Korean series required a student to use multiple steps to solve a problem, while only 2% of the US textbooks required the use of multiple steps. When analyzing five US and four Chinese textbooks, Li (2000) found that in both US and Chinese textbook series, the majority of the problems (80% in both countries) required a single step only. Unlike Li (2000), Yan and Lianghuo (2006) did find differences between US standards-based and Chinese texts, without looking at a specific content area. They found that in the US textbook over 63% of the tasks required only a single computational step, while 52% of the tasks in the Chinese textbook required only a single computational step. Supporting these results, Li et al. (2009) found that in both standards-based and traditional US textbooks, fraction division problems mainly required a single computation step, while Chinese and Japanese texts included many more multistep problems.

Contextual Features

The most prevalent theme for problem analysis was *contextual features*. This term refers to the setting of the task, which involves whether a problem that is presented with illustrations including representations and/or real-life contexts or presented purely mathematically (Li, 2000). Twelve of the articles we surveyed were counted as addressing this aspect (Alajmi, 2012; Choi & Park, 2013; Han et al., 2011; Hong & Choi, 2014; Kang, 2014; Li, 2000, 2007; Son, 2012; Son & Senk, 2010; Sun & Kulm, 2010; Yan & Lianghuo, 2006; Yang et al., 2010). Kang (2014) analyzed one traditional (*Harcourt*) and one standards-based US textbook series (*Investigations*), along with one Korean textbook series. In his analysis of addition and subtraction in first grade, Kang found that the traditional US textbook series lacked attention to real-life contexts, with a whopping 91% of the tasks being purely mathematical. The Korean textbook contained 68% purely mathematical questions, with the US standards-based textbook having the fewest purely mathematical questions at 63%. For fourth-grade materials, Kang reported similar findings. Han et al. (2011) analyzed US and Korean textbooks for probability. Four US traditional textbook series (*Glencoe, Saxon, McDougal Littell,* and *Prentice Hall*) and two Korean textbooks were found to focus on routine, closed-ended, non-contextual problems. Choi and Park (2013) found similar results when looking at geometry tasks. They found that while the standards-based US textbook (*Connected Mathematics 2*) typically introduced geometry using real-life applications, only a small portion of real-life tasks were included in Korean texts.

When comparing US and Chinese textbooks, the results of our survey indicate that both US and Chinese texts contain a majority of purely mathematical tasks. Li (2000) found that textbooks in both countries (5 in the USA and 4 in China) had a majority of tasks that were purely mathematical and only required a single computational step for integer addition and subtraction. Similarly, Yan and Lianghuo (2006) found that a standards-based US textbook developed by the University of Chicago School Mathematics Project and a Chinese textbook published by the People's Education Press contained a majority of purely mathematical tasks. However, the US textbook contained more application problems than the Chinese series. Sun and Kulm (2010) found similar results when analyzing the learning of fractions. Sun and Kulm noted that the US standards-based textbook also used more real-world representations (51%) than the Chinese textbook (11%).

In comparing US, Chinese, and Singaporean textbooks, Li (2007) found that all the textbooks were dominated by purely mathematical contexts. However, the five US textbooks analyzed had an average of 15% real-world problems with only 6% of the Asian tasks being real-world examples. Li reports that 62% of the US and 90% of the Asian series required the use of routine procedures. When comparing US, Taiwanese, and Singaporean textbooks for the development of fractions, Yang et al. (2010) found that the standards-based US series had more real-world problems than the Taiwanese and Singaporean series. Over 95% of the tasks in the US series were coded as real-world, while only 48% and 55% were coded as real-world in Taiwanese and Singaporean textbooks, respectively.

Response Types

Response types, the third most-researched theme in problem analysis, examined tasks that require students either to provide an answer only or to explain or justify their reasoning (Li, 2000). Six surveyed articles reported on the type of student responses (Hong & Choi, 2014; Li, 2007, 2014; Son & Senk, 2010; Sun & Kulm, 2010; Xin et al., 2011). Findings were mixed when comparing response types in US and Chinese texts (Li, 2007, 2014; Sun & Kulm, 2010; Xin et al., 2011). When looking at algebraic problems, Li (2007) found that out of five US curriculum series, one Chinese textbook, and one textbook from Singapore, US texts put more emphasis on explanation. Approximately 6.9% of the problems in US textbooks required an explanation, followed by China at 2.9% and finally Singapore at 0.1%. Later, Li (2014) found that out of five US textbooks and four Chinese textbooks, none of the Chinese textbook problems required an explanation when analyzing problems that immediately followed the introduction of integer addition and subtraction. However, 19% of the problems in US textbooks required an explanation. Unlike Li (2007, 2014), when analyzing the development of fraction concepts Sun and Kulm (2010) found that the Chinese series used more questions that required an explanation or justification than the US standards-based textbook series did. Furthermore, Xin et al. (2011) analyzed four traditional and two standards-based US series, along with one series from China. They found that the US traditional books had the fewest problems that required an explanation, followed by the Chinese books, whereas the standards-based series had the most problems that required an explanation.

Comparing multiplication and division of fractions in US and Korean textbooks, Son and Senk (2010) found that Korean texts had a greater number of problems that required an explanation than the US standards-based textbook (*Everyday Mathematics*) had. However, different tendencies were reported by Hong and Choi (2014). In their analysis of quadratic equations in three US standards-based textbooks and two Korean textbooks, Hong and Choi found more than 30% of the problems in the US series required explanations while only 15.7% of problems in the Korean books required an explanation. This tendency suggests that based on the content area, grade level, and the type of textbook analyzed in each country, the findings of textbook analysis of response types can be quite varied.

Cognitive Demand

Table 1.5 presents similarities and differences among cognitive demand, cognitive expectation, and depth of knowledge. As mentioned earlier, according to Stein et al. (1996), cognitive demand refers to the kind and level of thinking required when students are working on mathematical problems and tasks. According to Stein, Grover, and Henningsen, different mathematical problems place differing cognitive demands on students and can be categorized into two kinds—(1) low cognitive demand tasks (i.e., " procedures without connections" and "memorization") and (2) high cognitive demand tasks (i.e., "doing mathematics" and "procedures with connections"). In a similar vein, Webb (1999, 2002) developed the Depth-of-Knowledge framework that measures (more) specific cognitive demands of mathematical problems. The framework, which features three cognitive complexity levels (i.e., low, moderate, and high) and four levels of knowledge depth (Level 1: Recall/Reproduce; Level 2: Basic application of skill/concept; Level 3: Strategic

Table 1.5 Framework used for analysis of mathematical problems in textbooks

Focus questions	Aspects investigated
1. What is expected in terms of the depth or level of cognitive demand?	• Cognitive demand – High level – Low level
2. What is expected in terms of the breadth of cognitive complexity?	• Cognitive expectation – Conceptual knowledge – Procedural knowledge – Mathematical reasoning – Representation – Problem-solving
3. What is expected in terms of the depth of cognitive complexity?	• Depth of knowledge (DOK) – Level 1: Recall/reproduce – Level 2: Skill/concept – Level 3: Strategic thinking – Level 4: Extended thinking

thinking; and Level 4: Extended thinking), references the complexity of mental processing that must occur to answer a question or perform a task. While cognitive demand and depth of knowledge explain the depth of cognitive complexity of the mathematical problems presented in textbooks, cognitive expectation, as operationalized by Son and Senk (2010), can capture the problems' breadth of cognitive complexity by examining their cognitive expectations, i.e., knowledge and process required for students to solve mathematical problems and tasks.

Only one article was classified as addressing cognitive demand based on Stein, Grover, and Henningsen's framework (Hong & Choi, 2014). In their analysis of quadratic equations in three US standards-based textbooks and two Korean textbooks, Hong and Choi (2014) found that US standards-based textbooks included a higher percentage of problems with higher level cognitive demands than Korean textbooks. However, the majority of problems in both US standards-based and Korean textbook series require lower level cognitive demand: around 87% of the problems in the US standards-based text, and around 94% of the problems in the Korean textbooks.

Cognitive Expectations

Cognitive expectations, the second most-researched theme in problem analysis, refer to the type of mathematical knowledge or processes students should acquire and use when solving mathematical problems, and includes conceptual knowledge, procedural knowledge, problem-solving, representation, and mathematical reasoning. This measure is a composite of the work of Li (2002), Kilpatrick et al. (2001), and NCTM (2000). Referring to NCTM (2000), Son and Senk combined Li's (2002) four components of problems' cognitive expectations with Kilpatrick et al.'s (2001) five interrelated components of mathematical proficiency. Son and Senk then operationalized the cognitive expectation of tasks as the kind of knowledge and processes required when students are working on mathematical problems. Ten articles were counted as addressing cognitive expectation (Ding & Li, 2010; Kang, 2014; Li, 2000, 2007; Li et al., 2008; Son & Senk, 2010; Xin, 2007; Xin et al., 2011; Yan & Lianghuo, 2006; Yang et al., 2010). Note that although these articles did not directly refer to either Son and Senk (2010) or Li (2000), to some extent they looked at the *kind* of knowledge and processes required when students work on mathematical problems and tasks, by categorizing mathematical problems as either concept-based tasks or procedural tasks.

Our survey of articles on cognitive expectations yielded varied results based on the textbooks analyzed and the countries compared. Overall, the articles that compared Korean and US textbooks reported that US standards-based textbooks had a heavier focus on conceptual problems than Korean texts (Kang, 2014; Son & Senk, 2010). While Kang (2014) compared a traditional US textbook, US standards-based textbooks, and Korean textbooks, Son and Senk (2010) compared standards-based textbooks to Korean textbooks. Kang (2014) found that the Korean textbooks fell in between the traditional and standards-based books when it came to a focus on conceptual understanding. The standards-based textbook had the highest number of problems involving conceptual understanding while the traditional US textbook had the least. In a similar vein, Son and Senk reported that while the US series introduced conceptual understanding before algorithms, Korean texts developed conceptual understanding and procedural fluency simultaneously.

The majority of articles which analyzed the cognitive expectation of problems compared US and Chinese textbooks. However, their results differ from the Korean comparisons. Three articles found that US and Chinese texts are similar in their distribution of procedural and conceptual problems, regardless of the type of US book analyzed—Li (2000), Yan and Lianghuo (2006), and Xin (2007). Li (2000) found that the majority of the problems in both US and Chinese textbooks required a single computational procedure. Li analyzed five US textbooks and four Chinese texts, none of which were described as standards-based or traditional. Yan and Lianghuo (2006) found similar results without focusing on a single concept for analysis. Yan and Lianghuo looked at one standards-based US series and one Chinese textbook and found that both books contained a majority of routine, procedure-based problems. However, the Chinese texts had more problems that required multiple steps (problems that cannot be solved using one direct operation). Xin (2007) analyzed word problem-solving tasks, and again found that US traditional textbooks and Chinese textbooks had a similar distribution of word problems.

Alternatively, three other surveyed articles reported Chinese textbooks having higher numbers of conceptually based problems than US textbooks (Ding & Li, 2010; Li et al., 2009; Xin et al., 2011). When analyzing problems in multiplication and division, Xin et al. (2011) found that one Chinese series required a higher level of conceptual understanding than the four standards-based and two traditional US textbook series. When analyzing the distributive property, Ding and Li (2010) found that the Chinese textbook they analyzed aimed at conceptual understanding and utilized a heavy amount of word problems. The two US textbook series analyzed were not defined as being standards-based or traditional, but the authors found that the distributive property was mostly used for computation, rather than for conceptual understanding.

A comparison of US, Chinese, and Singaporean texts revealed that Singaporean and Chinese texts expected more from students when they were analyzing traditional algebraic problems than did the five US texts (Li, 2007). However, only 62% of the US textbooks and 90% of the Asian textbooks were based on routine procedures, indicating that US texts have a higher number of conceptually based problems. Finally, Yang et al. (2010) found that US standards-based textbooks emphasize conceptual knowledge while Singaporean and Taiwanese texts were more focused on procedural knowledge.

Depth of Knowledge

Only one article was found to use Webb's depth of knowledge framework. Son (2012) analyzed one standards-based US textbook (*Everyday Mathematics*) and two Korean textbook series on the topic of fraction addition and subtraction. In particular, to characterize the reform efforts in South Korea, she compared the quality of the mathematical problems in the reformed version of the Korean textbooks to the previous version. When comparing the two Korean series, the reformed version of the Korean textbooks provided better opportunities for students to learn fraction addition and subtraction than its previous version, in terms of the depth of cognitive complexity. However, the selected standards-based US series provided a more balanced level of depth of knowledge than the revised version of the Korean reform textbooks, by providing more opportunities for students to use strategic thinking and extended thinking.

Relevance of Non-textual Elements to Concepts or Problems

Existing research has emphasized the importance of both visual representations and pictorial representations in teaching and learning, because they provide students with concrete and concise images of related concepts (NCTM, 2000). *Non-textual elements* refer to the context in which a mathematical concept or problem is visually presented, whether in the form of drawings, illustrations, pictures, or mathematical representations such as mathematical diagrams. While contextual features only capture the presence of *non-textual elements* in textbooks, *relevance of non-textual elements* provides additional information on how non-textual elements are related to mathematical problems or concepts.

Four of the articles we surveyed focused on the relevance of *non-textual elements* (Kim, 2012; Li, 2007; Li et al., 2009; Saminy & Liu, 1997). Saminy and Liu (1997) found that an American textbook used 60 picture sets within 13 pages for the concept of subtraction, while a Japanese book used only 28 picture sets within 8 pages. Thus, the American text in their study had 4–5 pictures per page while the Japanese one had only 3–4. In addition, Saminy and Liu reported that in the American text, some pictures might be visually entertaining to students, but they did not always relate to the concept the textbooks intended to teach. This is because one important criterion in the section of pictures in the American textbook was the book's themes, e.g., "Tumble through the Jungle" or "Fun through the Seasons." For example, in the unit on subtraction, two pictures of forests (one in spring and one in winter) were presented to teach the subtraction facts of eight, but they were not relevant to the concept of subtraction. In contrast, in the Japanese text, mathematical concepts are the primary organizing criterion for the selection of pictures. Similarly, Li (2007), who analyzed algebra content in mathematics textbooks from four education systems, reported that the US algebra textbooks had illustrations and figures on nearly every page while the textbooks from Hong Kong, China, and Singapore were basically black-and-white with only a few illustrations. Li et al. (2009) focused on the role and relevance of pictorial representations in illustrating mathematical concepts in three countries' textbooks. They found that in US and Chinese textbooks, pictorial representations played a major role in illustrating the solution to a fraction division problem, and followed up with explanations and numerical representations. In contrast, Japanese textbooks mainly relied on numerical expressions to explain the fraction division algorithm.

Similar findings were observed by Kim (2012), who reported a significant difference across topics and textbooks in regard to students' learning opportunities though non-textual elements. Two standards-based (*Connected Mathematics, MathThematics*) and one traditional US textbook series (*Holt Middle School Mathematics*), along with three Korean textbook series, were analyzed. In her analysis of non-textual elements in mathematics textbooks, Kim developed a conceptual framework which includes the following four aspects: *accuracy* (i.e., how non-textual elements represent concepts and ideas correctly in mathematical ways), *connectivity* (i.e., how closely non-textual elements are related to the mathematical content), *contextuality* (i.e., whether mathematical problems are presented in realistic contexts), and *conciseness* (i.e., how a non-textual element is concise and neat in presenting a concept or problem without any unnecessary or distracting factors). Looking at angle, slope, and prime factorization, Kim found that overall, non-textual elements were accurate, well connected, and concise in both countries. However, mathematical representations tended to be used more often than pictorial representations in both US and Korean textbook series, and the pictorial representations were relatively weaker in terms of accuracy, connectivity, and conciseness compared with mathematical representations.

Summary and Implications of Problem Analysis

Many articles reported on the contextual features (pure math vs. illustrative) of textbook tasks. The majority of tasks in both US and Asian textbooks are purely mathematical. Results were mixed when comparing response types in US and Asian textbooks. Some studies showed that US textbooks had a greater number of tasks that required students to explain or justify their reasoning, compared to Chinese texts. However, other studies reported contradictory findings. The same contradictions are found when comparing US and Korean texts. These conflicting results can be attributed to the types of US textbooks, the content area, and the grade levels (e.g., elementary vs. secondary) analyzed. Similarly, with regard to cognitive requirements (i.e., cognitive demand, cognitive expectation, and depth of knowledge), results are mixed. Some studies suggested that the US and Chinese textbooks have similarly cognitively demanding tasks, while others report that Chinese books had more tasks that require high cognitive demand. Considering the type of US textbooks analyzed did not resolve this conflict. However, there were clearer results when comparing Korea to other Asian countries. The US textbooks tended to have a greater number of cognitively demanding tasks than textbooks in Korea (at the secondary level), Taiwan, and Singapore, regardless of the type of US textbook series analyzed. Non-textual elements (e.g., drawings, pictures) tended to be used more frequently in US texts than in Asian countries. However, they were not always relevant to the mathematical content in US textbooks.

Summary and Limitations

Our survey of 31 textbook analysis studies has shed some light on the relationship of textbook characteristics among the USA and five high-achieving Asian education systems. It appears that most studies utilizing textbook analysis have consistently revealed, to a greater or lesser degree, the inadequacy of textbooks in presenting mathematics content, topics, and problem-solving. Remarkable differences were found in textbooks from different series and particularly from different education systems. These results indicate both challenges and a need for researchers, curriculum developers, policymakers, and schoolteachers to conduct further research and action. More specifically, by breaking down our analysis into the structure of the content and problems in those textbooks, we were able to determine common themes throughout the articles we surveyed. Additionally, we were able to determine the most widely analyzed education systems, grades, and topics. We found that textbooks in China were compared most frequently with US series. Taiwanese textbooks were least often compared to their US counterparts. The majority of articles focused on elementary grades (1–5), with high school grades (9–12) being the least analyzed. Conceptual components of fraction development and operations with fractions were the most analyzed topic in our survey.

Based on comparative content analysis of textbooks in American and Asian education systems, several conclusions can be drawn. In both the early grades and the later grades, mathematical topics appear earlier in Asian education systems than they do in the USA. Additionally, textbooks in the USA tend to review material more frequently than in Asian education systems. While this was not a prevalent finding among the majority of articles, it is important to consider for textbook publishers and curriculum developers. Textbooks in the USA also tend to be physically larger and contain more pages than Asian textbooks. Results on the number and length of chapters vary, but the US books tend to have more chapters and more pages per chapter on specific content areas. With regard to textbook exercises, the USA tends to have a greater number of problem tasks as well as exercise-to-text problems than Asian countries. Non-textual elements are more colorful and are also used more frequently in US texts than in Asian series (Li, 2007; Mayer, Sims, & Tajika, 1995). However, non-textual elements are not always relevant, or well connected to the math content (Carter et al., 1997).

Topic introduction and development tend to be different in US textbooks than has been reported for Asian textbooks. Based on these findings, we attempted to identify textbook signatures that might represent the features of each education system's textbooks (see Table 1.6). Researchers might examine the hypothesis that different textbooks offer different learning opportunities to students and consequently contribute to differences in student achievement. Specifically, based on the results of this study, we hypothesize the following characteristics of textbooks that may partly account for the superior achievement of Asian students to US students on international assessments.

Table 1.6 Textbook signatures in the US and five high-achieving Asian education systems

Textbook	Content analysis	Problem analysis
US Traditional	1. Larger than Asian series 2. More pages than Asian series 3. Later presentation of topics than Asian series 4. Greater number of exercises than Asian series 5. Greater number of illustrations than Asian series and illustrations that relate less closely to concepts	1. Greater number of tasks requiring a single step than Asian series 2. Fewer number of real-world tasks than Asian series and US standards-based series 3. Heavier focus on conceptually based problems than Korean textbooks, but fewer than standards-based US textbooks. For US vs. Chinese books, equal focus on concepts, though some studies say China has more
US Standards-Based	1. Larger than Asian series 2. More pages than Asian series 3. Later presentation of topics than Asian series 4. Greater number of exercises than Asian series 5. Greater number of non-textual elements than Asian series and non-textual elements that relate less closely to concepts	1. Greater number of tasks requiring a single step than Asian series 2. Greater number of real-world tasks than Asian series and US traditional series 3. Heavier focus on conceptually based problems than Korean series. For US vs. Chinese books, equal focus on concepts, though some studies say China has more. 4. More hands-on activities than Chinese books 5. Connects concepts to other mathematical content 6. More use of pictorial representations
Korea	1. Topics occur earlier than US standards and traditional 2. Texts have less pages and less chapters than US standards and traditional 3. More pages devoted to geometry than US standards-based	1. Required greater use of multiple steps in dealing with fractions than US standards-based 2. Amount of purely mathematical tasks fell in between US standards-based and US traditional 3. Conflicting findings regarding the requirement of an explanation when comparing US texts 4. Fewer conceptually based problems than US standards and traditional books 5. Less non-textual elements than the USA, but were more connected to content

(continued)

Table 1.6 (continued)

Textbook	Content analysis	Problem analysis
China	1. Topics occur earlier than US standards-based and traditional series 2. Textbooks are shorter than US standards-based and traditional series 3. Smaller number of overall problems than US standards-based series 4. Fewer exercise-to-text problems than US standards-based series 5. Fewer illustrations than US series	1. Greater number of tasks that require multiple steps than US standards-based and traditional series 2. Fewer number of real-world problems than US standards-based series 3. Conflicting findings regarding the requirement for an explanation when compared to US texts 4. Similar focus on procedural and conceptual tasks when compared to US traditional and standards-based series, but conflicting results find that China has more conceptual tasks
Japan	1. Topics occur earlier than US standards-based and traditional series 2. Texts are smaller, with fewer pages than US standards-based and traditional series 3. Fewer pages, chapters, and lessons regarding fractions than US traditional series 4. Fewer illustrations than US traditional series; non-textual elements do not always relate to concepts	1. Greater number of tasks that require multiple steps than US standards-based and traditional series 2. Fewer number of real-world tasks than US traditional texts
Taiwan	1. Topics occur earlier than US standards-based and traditional series	1. Fewer real-world problems than US standards-based texts 2. More focused on procedural questions than US series
Singapore	1. Textbooks are shorter than US standards-based and traditional 2. Fewer illustrations than the USA	1. Fewer real-world problems than US standards-based texts 2. Fewer problems that require an explanation vs. US or Chinese series 3. More focused on procedural questions than US series

First, decreasing the number of superfluous and irrelevant illustrations would reduce the number of distractions students encounter. Textbook publishers, especially those in the USA, might increase the amount of time dedicated to the depth of knowledge needed for specific content areas. Second, changes to the actual problem tasks students are exposed to can be updated as well. Tasks' levels of cognitive demand and their context (real world vs. pure math) can play a significant role in student achievement on tests comparing US and Asian education systems. However, since results on response types varied dramatically based on education systems and type of textbooks analyzed, we cannot determine that response type is a contributing factor to the assessment gaps.

Finally, when analyzing different representations of mathematical concepts, it appears that US textbooks seem to use multiple representations, ones not necessarily directly connected to the mathematical concepts, and they revisit concepts in order to solidify mathematical fluency. Asian textbooks tend to be succinct in their representations of mathematical concepts. Given the results of varied representations, it may be beneficial to consider limiting the amount of variation before moving on to alternate representations.

Implications and Future Directions

No matter how large a gap might exist between the intended and potentially implemented curriculum, teachers who provide additional instruction for their students can narrow this gap (Cai & Wang, 2010; Cai et al., 2014). In our collection of textbook analysis studies, many studies only looked at the intended curriculum or potentially implemented curriculum, and not the implemented curriculum. Only seven studies indicated this aspect as one of the limitations of their respective studies. As mentioned previously, we do not support the direct connection between textbook analysis and student achievement because there are many other factors to be considered. Textbook analysis might be able to predict, but can never conclude with confidence, the actual classroom use of texts. We thus highlight the important role of teachers and suggest that teachers need to be aware of what is intended and what is presented in textbooks for teaching mathematics, and then work toward helping students make sense of mathematics.

This chapter summarized the findings from prior research on textbook analysis with respect to content and problem analysis, and raised questions and issues for mathematics education researchers to further advance the research on mathematics textbook analysis. First, we think that it is necessary for researchers to establish a more solid fundamental conceptualization and theoretical underpinning for analytical frameworks. Unfortunately, our survey of common themes for textbook analysis faced an added layer of difficulty due to the lack of a common analytical framework. Many researchers used their own frameworks to suit the needs of the particular content they were examining. It was thus difficult to compare their findings directly, so we identified common themes based on findings, rather than on a given analytical framework.

To address these problems, future researchers might use a framework that encompasses wider educational and social contexts. Few textbook analyses focus on gender, ethnicity, and equity issues in exploring students' learning opportunities presented in textbooks. Moreover, only two studies looked at the language used in the books—Herbel-Eisenmann (2007) and O'Keeffe and O'Donoghue (2015). Only looking at the tasks themselves is not enough: the author's voice can have an impact on how students see those tasks. Furthermore, we observed some methodological issues in our collection of textbook analysis studies. Patton (2002) stated that validity and reliability are two factors which any researcher should be concerned about while designing a study, analyzing results, and judging the quality of the study. Given that different researchers use different frameworks, it is important to establish clear research questions and valid analysis methods, including coding systems. However, clearly defined research questions were lacking in several studies (e.g., Ding & Li, 2010). Eight studies did not clearly articulate their framework with examples of tasks using the coding method (e.g., Sun & Kulm, 2010). Of the 31 articles reviewed, only 16 studies clearly stated some type of inter-rater reliability along with percentages of agreement of the inter-rater reliability.

Indeed, as Fan, Zhu, and Miao (2013) pointed out, there is a strong need for more confirmatory research about the relationship between textbooks and students' learning outcomes. As reported earlier, the results are mixed when comparing learning opportunities presented in US and Asian textbooks and even within the same education system, so the research evidence for a positive correlation between textbooks and students' learning outcomes is weak and inconclusive (Fan et al., 2013). This is because the results are often based on a comparison of selected textbooks, investigating the differences between textbooks in different countries and comparing grade levels in these countries. In these studies, the issues of whether the selected textbooks are a good representation of all the available textbooks, and whether the selected framework and methodology are solid, were often ignored or taken for granted. Similar to our perspective and findings, Cai and Cirillo (2014) also call for more careful attentions to the variety of the methods used in textbook analysis studies in order to move the field forward in understanding the potential role of the curriculum on students' learning. Future researchers should therefore consider the following questions:

- How many textbooks should we analyze?
- Which textbook(s) should we analyze?
- What text in the textbook(s) series should we analyze?
- How much of that text should we analyze?
- How should we analyze it?
- What research questions should guide our analysis?
- What framework should we use?
- How can a researcher persuade his or her audiences that the research findings of textbook analysis are relevant and worthwhile?

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Appendix: Surveyed Textbook Analysis Articles

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