Modeling of Data Communication Networks using Dynamic Complex Networks and its Performance Studies

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Abstract To study the underlying organizing principles of various complex systems, designing an efficient graph-based model for data representation, is a fundamental aspect. As the topological structure of the network changes over time, it is a challenging task to design a communication system having ability to respond to randomly changing traffic. We are interested to find out the suitable and fair traffic flow rates to each system for getting optimal system utility using dynamic complex network model. In this context, we design and simulate a growth model of the data communication network based on the dynamics of in-flowing links which is motivated by the concept that newly added node will connect to the most influential nodes already present in the system. The connectivity distribution of the evolved communication networks follows power law form, free from network scale. We analyze Kelly's optimization framework for a rate allocation problem in communication networks at different time instants, and optimal rates are obtained with the consideration of arbitrary communication delays.

Key words: Complex Networks, Dynamic Networks model, Communication Processes, System Utility

1 Introduction

Systems such as social, telecommunication, computer, biological, citation, etc. can be modeled as a graph considering distinct elements represented by nodes and there is a connection (links) between them. The graph has nontrivial topological properties, connections between elements are neither purely regular nor purely random. These systems are very large, can be modeled in the form of a network,

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H. Cherifi et al. (eds.), *Complex Networks & Their Applications V*,
Studies in Computational Intelligence 693,
DOI 10.1007/978-3-319-50901-3_3

helps us to understand the behavior of the system, called complex networks. Complex networks are currently being studied across many fields of science systems in nature. In complex networks [2, 11, 15, 16], links often exhibit various features: they can be directed, have different weights assigned to it, be active only at certain times. The demographic features of random graphs using the probabilistic approach in network structure analysis was developed by Erdos and Renyi (ER), they investigated random network model [6].

Watts and Strogatz (WS) have proposed a model, which generates complex network having small world properties [22]

The more complex network model, Scale-free model was proposed by Barabasi-Albert ([1]). The model is defined in two steps:

- Expansion: Starting with a small number (n_0) of nodes, at each instant of time a new node appears with $a(\leq n_0)$ links which are connected to the existing nodes in the system.
- Preferential connection: The Π probability that a newly added node will be attached to node i only when the value of influential parameter (k_i) of that node is maximal.

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

After time t, the network will contain total $n = t + n_0$ nodes and at links. Network evolves into a scale invariant case and hence the scaling exponent is independent of a total number of links a.
Limitations of BA model are as follows:

- Both invariant, expansion and preferential connections are compulsory.
- It is assumed that new connection is established only when new nodes are added to the system. But, in real life, connections are made continually.
- In some systems, re-association or rewiring of the existing links can happen, and they are also following preferential connection, but if reattachment dominates over expansion, then this will destroy the behavior, i.e., the power-law scaling in the system.

To make the network dynamic, an important ingredient of the dynamics is a preferential connection of links (outflowing/inflowing). Tadic [20] has focused on outflowing links and shown that both the outflowing and inflowing links follow a heavy-tailed distribution with distinct exponents. Momentary alteration of the outflowing links inside the networks effect on both the outflowing and inflowing links. After establishing a correlation between the outflowing and inflowing links, it is shown that the local structure of the network is qualitatively different compared to the case without an update. The expansion, as well as update, are taking place at unique time scale, a new node $n(=t)$ appears in the network (expansion), and a number $X(t)$ of new links are scattered. There is an increasing interest in investigating not only the process dynamics on networks [18, 19] but also the dynamics of networks [7]. There is a need to extend the basic network concept to include time relations between

nodes arose, leading to many models for Time-Varying Graphs (TVGs) [5, 10, 21]. Although the nodes are placed in the space randomly, network structure depends on the distribution of links.
The structure of connections has an immediate impact on the accessibility of particular node, and it is the backbone for the stability of the network. If the number of connected components increases, then there must be at least one path between each pair of the node. Social networks are one of the examples of dynamic network where, people are represented by nodes and if two people are connected then, there will be a connection between them. Contacts are not static, it is temporal and depends on the state(active/inactive) of nodes. Some activity parameter is used to generate temporal links and an adaptive network is formed by incorporating memory effect to know about past connections. In [3], reciprocal action of individual activity and network structure are shown. State of the node determines the dynamic activity of human interaction and states are also decided by the connection between nodes.
Another example is communication networks, which can respond to randomly changing traffic flow rates by reassigning traffic routes and by reallocating resources. As expansion and updates, both are happening at unique time scale, so the design and control of such kind of network is a challenging task. Topology is changing at each time-stamp. Due to change in topology, the performance of the network is also affected [12]. The exponent is independent of a total number of links a.
Modern communication networks are faced with multiple challenges at different layers and modeling their rate control behavior [9, 13, 14, 17] with volatile and dynamic connectivity setting is a prominent issue. Real life network settings are extremely volatile, and still communication takes place albeit with degraded quality and possible setback in performance. There is a new kind of thinking to understand the underlying reasons for volatile spatiotemporal behavior and how one can re-engineer them for optimal performance for this change.

Rather than closing our eyes to these kinds of hard technical difficulties, a framework is proposed to model arbitrarily changing directed networks in both space and time with the help of proposed mathematical models in [1, 20]. It is shown that the degree distribution of the networks follows the power law and hence scale free in nature. We analyze Kellys optimization framework for a rate allocation problem in communication networks at different time instants, and optimal rates are obtained considering user's willing to pay and network cost.
Section 2 states about mathematical modeling of the network, Section 3 provides a real life mobile communication network examples with arbitrary link changes by maintaining certain set of rules and followed by algorithmic steps, Section 4 presents a numerical example illustrating the algorithm and Section 5 describes the conclusion and explains the future directions of this work.

2 Mathematical model and related work

In this section we give a brief description about rate allocation problem. We contemplate a network with a set E of links and a set of R users. Let C_e be the capacity of the link, where, $e \in E$. For each user $k \in R$, a route r_k has been assigned for a particular time instant $t_i \in T$, where $t_i \mid 1 \leq i \leq \tau$ contains a nonempty subset of E. A zero-one matrix A of the size $E \times R \times t$ is defined where, $A_{k,e,t} = 1$, if e is in the route of user k at time t, otherwise zero. When the user k is assigned a rate $x_{k,t}$ then utility of user k at rate $x_{k,t}$ is given as $U_{k,t}(x_{k,t})$ is increasing, strictly concave function of $x_{k,t}$ over the range $x_{k,t} \geq 0$. Aggregate utility is calculated by summing up all utilities of user k at rate $x_{k,t}$ and is denoted as $\sum_{k\in R,t\in T} U_{k,t}(x_{k,t})$. Rate allocation problem can be formulated as the following optimization problem.

$$\begin{aligned} & SYSTEM(U_t, A_t, C_t) \\ & maximize \sum_{k\in R,t\in T} U_{k,t}(x_{k,t}) \\ & A_n^T x_t \leq C_t \text{ and } x_t \geq 0 \end{aligned} \tag{1}$$

where, $n = (1, 2, ..., \tau)$, τ is the total number of time instants. A_n is the matrix formed in the time interval t_{n-1} to t_n. The constraint shown above tells us that the flow through a link can not exceed the capacity of particular link [8]. For handling large scale of the system, it is inconvenient to allocate each user an optimal rate. Hence, Kelly has divided this problem into two simpler problems named as user's optimal problem and network's optimal problem [9]. Let each user k is demanded a price per unit flow as λ_k. A user chooses an amount to pay at per unit time is $P_k(t)$ according to the incurred cost with the user. Hence, user receives a flow, $x_k(t) = P_k(t)/\lambda_k$ then user's optimal price will be

$$\begin{aligned} & User_k(U_k(t), \lambda_k(t)), \\ & \text{maximize } U_k(x_k(t)) - p_k(t), \\ & p_k > 0 \end{aligned} \tag{2}$$

On the other hand, network wants to maximize weighted log function of $p_k(t)$. Therefore, network utility function can be written as

$$\begin{aligned} & NETWORK(A_t, C_t, p_t), \\ & maximize \sum_{k\in R,t\in T} P_k(t) log(x_k(t)), \\ & A_n^T x_t \leq C_t \text{ and } x_t \geq 0. \end{aligned} \tag{3}$$

The values of λ_k, P_k and x_k are considered variable with time. Each user in the network, $k \in R$ initially computes the price per unit flow by using the Eqn. (4) and it is willing to pay, $P_k(t)$. It adjusts its rate based on the feedback provided by the links in the network. Each user attempts to make equilibrium by its willingness to pay the total price for the complete duration. Finally, one can always find out unique stable value of the price per unit flow $\lambda_k^\star$, rate $x_k^\star$ and willingness to pay and $P_k^\star$ and corresponding convergence vectors will be $\lambda^\star = \lambda_k^\star, k \in R$, $P^\star = P_k^\star, k \in R$ and $x^\star = x_k^\star, k \in R$.

For each user, k is given price per unit flow as λ_k and the amount for which user is willing to pay, $P_k(t)$ at time t. Hence, the rate assigned to user k is $x_k(t) = P_k(t)/\lambda_k$. Utility of each user k at a particular time instant is assumed by strictly concave function of users rate at that time instant. Suppose that each user adopts a rate based flow control. At each time instant each link $e \in E$ charges a price per unit flow of $\mu_e(t) = g_e(\sum_{k:e \in E} x_k(t))$ where $g_e(\bullet)$ is an increasing function of the total flow through it and $g_e(y)$ is

$$g_e(y) = c_e.(y/C_e)^{\omega}$$

where, c_e is constant and assumed one, C_e is the capacity of resource $e \in E$. The defined price function arises when resources are modeled as $M/M/1$ queue. $M/M/1$ queue is a queue having some length with the single server. Processes are arriving with certain rate and then service is provided to that process by the server. Suppose processes are arriving at rate λ and μ is the service rate. Hence, $\rho = \lambda/\mu$, where ρ is the average proportion of time when the server is occupied or busy. C_e is the service rate and packet will receive a mark when there is already ω packets in the queue. Now consider the following system of differential equation

$$\frac{dx_k(t)}{dt} = \sigma_k(P_k(t) - x_k(t) \sum_{e \in E} \mu_e(t)) \tag{4}$$

Each user firstly computes it's willingness to pay as $P_k(t)$ then it adjusts its rate based on the feedback provided by the links in the network and trying to balance its willing to pay and total price. Eqn. (4) consists of two components: a steady increase in the rate proportional to $P_k(t)$ and steady decrease in the rate proportional to the feedback provided by the network.

3 Proposed work

Like the Internet, communication networks use a specific set of rules to connect the components and directed links are used to access data. In the communication network, degree distribution of both out-flowing and in-flowing links follow a heavy tail distribution with separate exponent values. In the proposed model, we have given preference for in-flowing link because the newly created link is attached to the node which has highest in-flowing link probability. Set of rules which are used in the formation of dynamic networks, yield that the distributions of both out-flowing and in-flowing links are interdependent. Another important feature of the model is that the connection between pairs of nodes is not fixed in time, but it may change on the time scale of the network's expansion(updates of links).
Here, a communication network is formed with scale-free property by modifying the BA model [4] and model [20]. The modified directed network is formed by maintaining the following rules.

1. Directed nature of linking.
2. Expansion and update are done at unique time scale. At each time unit t, a new node $n(=t)$ is added to the network (expansion) and total number $X(t)$ of new connections are established and allocated to the nodes. Newly created links are divided into two groups: added link and updated link. Distribution of the links is done using following rules specified below.

- Enter the value of fraction β, γ, such that $\beta < 1$ and $0.5 < \gamma \leq 1$.
- A fraction $f_\beta(t) = \beta X(t)$ of new links are out-flowing links from the new appeared node $n = t$ and added with the nodes existing in the network at $(t-1)$ based on priority, here β is a fraction with $\beta < 1$.
- Another remaining fraction $f_{(1-\beta)}(t) = (1-\beta)X(t)$ are the updated (removed and rewired) links within existing nodes excluding the newly added nodes.

  Updated links may have two types:
  - A fraction $f_{up}(t) = \gamma f_1(t)$links are rewired with the value of fraction γ, $0.5 < \gamma \leq 1$. It helps to maintain the growing nature of the network.
  - Fraction $f_{dlt}(t) = (1-\gamma)f_1(t)$ are removed from the network.
- The parameter δ is the ratio of updated and added links in the model and is given by $\delta = \frac{f_1(t)}{f_0(t)} = \frac{1-\beta}{\beta}$, which is independent of the added number of links $X(t)$ and known as **correlation parameter**.

3. We can define two functions preferential update and preferential attachment.

While talking about communication network, the concept of preferential linking driven by the demand of the node for the flowing data into the network. In addition to this, preference for the update is given to only a few nodes, rather than updating all nodes at each time instant. Moreover, some of the nodes want to update out-flowing links more frequently than others. Apart from the newly appeared node, larger update probability is given to most active nodes at time t, i.e., an out-flowing link from the node $k \leq n$ appears according to preferential attachment. Removal of links are done randomly but the rearrangement of links done based on preferential attachment.

Algorithmic steps are given for expansion and updation of network.

Attributes of links contain *linkid*, named, delay and capacity. We have to send packets from multiple sources to multiple destinations based on shortest path. Shortest path is measured in terms of hop count. Multiple users can send data from specific source (*S*) to destination (*D*) based on shortest path and these S-D sets are generated according to user's choice. If number of users increases, then the congestion level will increase according to the selection of paths.
Initially, shortest path for user is found and after that optimal data rate of the user is calculated by using these steps:

Algorithm 1 Network Evolution

Input: A small number (m_0) for seed network , $m(\leq m_0)$ for distribution, β, γ and $timer$.
Output: Evaluated network.
while $T \leq timer$ **do**
  Add a node at each time instant.
  for m: 1 to $f_\beta(t)$ **do**
    Select a node of higher probability to attach with.
  end for
  for n : 1 to $f_{up}(t)$ **do**
    Select an arbitrary source and link it to the node having higher inflowing link probability.
  end for
  for p : 1 to $f_{dlt}(t)$ **do**
    Randomly select v a link to remove.
  end for
end while

Algorithm 2 Finding shortest path and optimal rate for each user

for $i := 1$ to $numPair$ **do**
  Find shortest path between source and destination
  for $j := 1$ to $numofNode$ **do**
    Calculate frequency of occurring of active node during path formation
    $rate(j) = \frac{capacity}{frequency(j)}$;
  end for
end for
for $r = 1$ to $numofPair$ **do**
  Update feedback for each element of S-D pair
  ratePath(r) = minRate(elementofPath);
  $A(r) = rand(1, 10)$;
  $Wpay(r) = ratePath(r) * (\frac{a}{ratePath(r)+b})$;
  $meu1(r) = meu$;
end for
Use the value of ratePath, A, Wpay and Meu1 to find out the rate of convergence of each user.

Evolution of the network is done at a unique time instant. Here we have taken initial size of the network of $(100+m)$ nodes i.e., $t_0 = 100+m$ units and $\delta t = 100$, hence $t_{i+1} = t_i + \delta t$ and the series will look like $T = (t_0, t_1, t_2, ...t_\tau)$ and the value is, $T = (100+m, 200+m, 300+m, ..., 100\tau+m)$. Each user firstly computes and shows a willingness to pay as $P_k(t)$ then it adjusts its rate based on the feedback provided by the links in the network and trying to balance it is willing to pay the total price. Eqn. (4) consists of two components: a steady increase in the rate proportional to $P_k(t)$ and steady decrease in the rate proportional to the feedback provided by the network. Initial values of willingness to pay for the user, feedback of the network and the rate of the resources are provided to the solver for finding out the optimal rate of each user. At each time instant user increases its willingness to pay but due to congestion in the network rate and becomes stable after some time.

4 Simulation and results

In most of the real world networks, the degree of the majority of nodes has low value, but there exist few hub nodes, having a high degree. Some social networks are found to have degree distributions that approximately follow a heavy-tailed distribution: $P(k) \sim k^{-\alpha}$, where $2 < \alpha \leq 3$, known as scale-free networks. In a scale-free network, numerous nodes with few links coexist with a few hub nodes, having connected with thousands or even millions of links. To make all the values for large k visible use of a log-log plot is needed. We can either use logarithmic axes, with powers of 10 or we can plot $logp_k$ in function of $logk$. Here, logarithmic axes, with powers of 10 is taken for plotting the probability distribution of node degrees over the whole network and the degree distribution shows power law behavior. The value of β can be obtained from δ as $\beta = \frac{1}{(1+\delta)}$. There are four possible cases of the value of the δ, depending on updated and newly added link in the network.

In Fig. 1, it is shown that evolved network follows power law degree distribution when network has different values of nodes along with correlation parameter δ.

1. $\delta = 0 (\beta = 1)$ i.e, only expansion is happening no update (rearrangement and removal). The degree distribution of the network having $N = 10000$ nodes and scaling exponent $\alpha = 2.664$, is shown in Fig. 1(d).
2. $\delta < 1 (\beta > 0.5)$, more number of new links are getting added than updated. The degree distribution of the networks having N = 10000 nodes and the values of β = 0.6 (expansion), γ = 0.5(rearrangement) and $\alpha = 2.455$, shown in Fig. 1 (b).
3. $\delta > 1 (\beta < 0.5)$, more number of links are updated than added. Degree distribution of the networks having $N = 10000, \beta = 0.25, \gamma = 0.7$ and $\alpha = 2.065$ is shown in Fig. 1(c).
4. $\delta = 1$, when both the value of updated and added links are same, degree distribution of the networks with $N = 10000, \beta = 0.5, \gamma = 0.5$ and $\alpha = 2.486$ is shown in Fig. 1(a).

From the graph shown in Fig. 1, it is analyzed that, by increasing the parameter β in the range $(0,1)$, corresponds to decrease of the correlation parameter δ in the interval $(\infty,0)$, the slope of the distributions increases.

The network is formed using the algorithm 1. Evolved network is formed by putting the values of parameters as: size of the seed network $m_0 = 5$, Number of links which is distributed at each time instant $m(\leq m_0)$, β, γ and *timer*. User's routes for sending packets are varying according to time. At each time instant, a new node appears with m links and expansion as well as re-arrangements are done. As the network becomes larger and larger, many paths are available for sending packets for each user between desired source and destination. All routes are equally weighted hence, users can select any of these routes for sending packets.

Each user can send data along one of the shortest paths to the destination with a

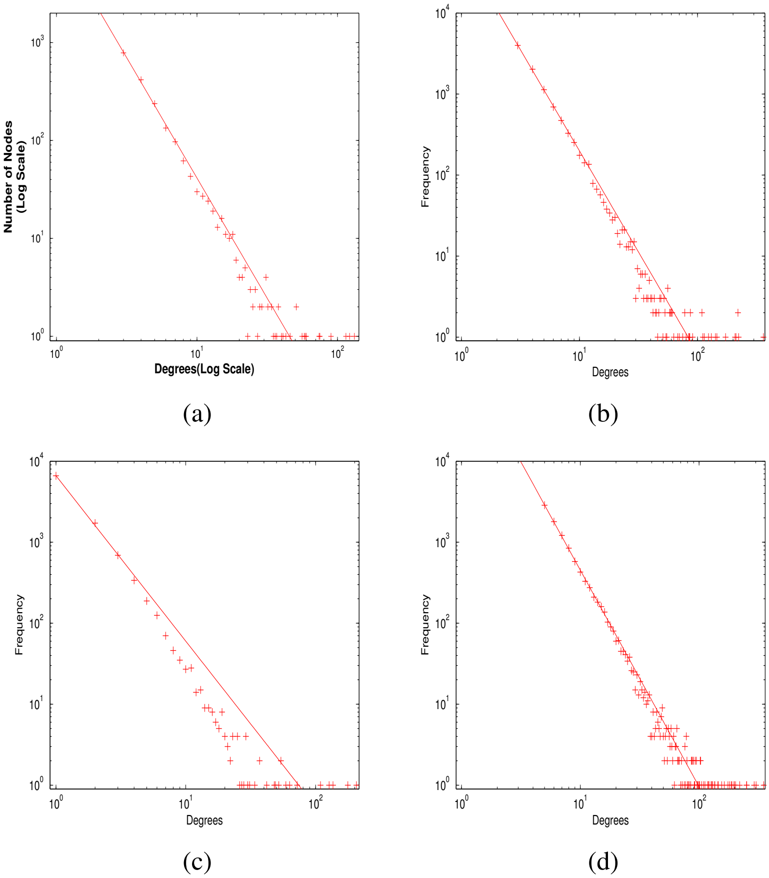

Fig. 1: Degree distribution of the network when number of nodes are and average ratio of updated and newly added links are **(a)** $N = 10000$, $\delta = 1$, **(b)** $N = 10000$, $\delta = 0.67$, **(c)** $N = 10000$, $\delta = 3$ and **(d)** $N = 10000$, $\delta = 0$

maximum flow rate of individual links. Multiple users need to share the resources hence, data sending rate got reduced, and it can no more send data with a maximum rate. User's rate depends on two parameters; it's own willingness to pay and network's feedback. Using rate control theorem given in (4), an optimal data sending rate of each user is obtained. In Fig. 2, User1's and User2's data sending rates are shown at different time instants. Instead of, increased network size, optimal rates are not increasing. User rates depend on the demand of particular resources coming in the shortest route. If demand is high, then data sending rate will be less.

Multiple users want to establish connections between a distinct pair of nodes and hence, a shortest possible communication path is chosen. There may exist a multiple number of shortest routes having the same number of hop count, but betweenness

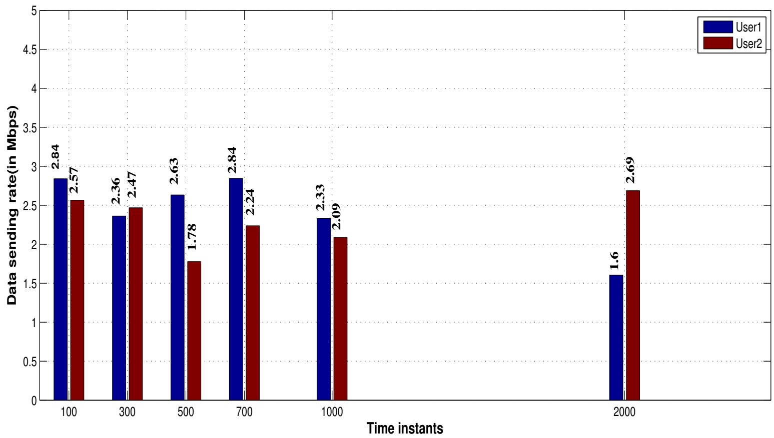

Fig. 2: Conservation of data sending rates of User1 and User2 at different time instants

centrality of all shortest paths would not be same. Hence, data flow rate of the paths having high betweenness value will be less. Optimal rates are also dependent on betweenness. User's optimal rates along with their betweenness values are shown in Table 1. User's optimal rates are also shown in figure 3.

Table 1: User's optimal rate through the shortest routes having different betweenness values(maximum and minimum), when number of nodes $N = 100$

	Source	Destination	Betweenness (Minimum)	Betweenness (Maximum)	Optimal Rates (Minimum Betweenness)	Optimal Rates (Maximum Betweenness)
User1	7	38	0.1931	0.1966	6.316335	7.737758
User2	77	96	0.0906	0.5877	3.383048	4.446214
User3	21	37	0.0616	0.3066	6.489473	9.356067
User4	68	79	0.0856	0.3872	4.409398	5.202014
User5	13	36	0.0751	0.4118	7.057041	8.913657
User6	20	47	0.0185	0.1467	5.134327	6.963006
User7	36	62	0.0608	0.1006	5.519844	6.575628
User8	24	65	0.085	0.3955	3.780126	7.232818
User9	40	6	0.0762	0.2017	6.978428	12.434885
User10	18	75	0.1483	0.3413	5.832557	7.118301
User11	24	85	0.0818	0.3847	4.081002	8.247708
User12	39	2	0.2144	0.3314	9.000121	11.311826

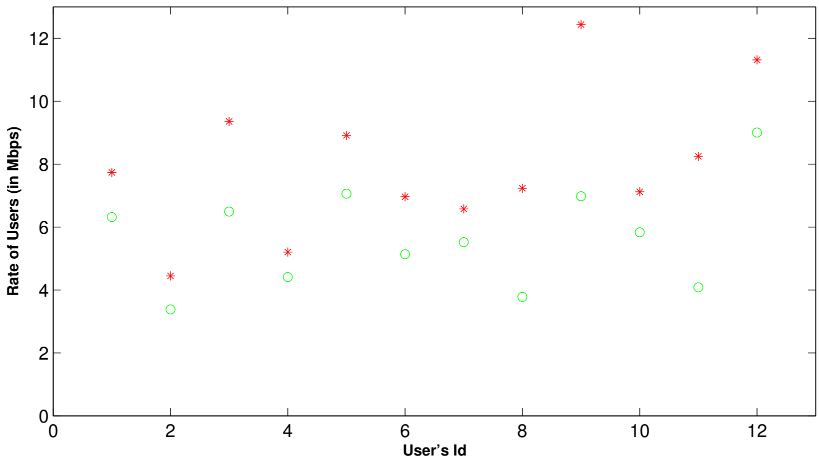

Fig. 3: User's optimal rate through the shortest routes having different betweenness values(maximum and minimum), when number of nodes $N = 100$

5 Conclusions and Future directions

In this paper, a model is proposed to represent complex dynamic systems in the form of complex networks and their representation is also given by using mathematical expression. The proposed model is simple, flexible and efficient for the representation and modeling of dynamically changing networks. At each time instance, a new node appears with few links, either for expansion or update based on the value of fractions β and γ. Expansion and update (removal and rewiring) of links are done based on the preferential basis (most influential nodes). Network changes at each time instant and it grows according to the value of time. Various experiments are performed for finding out the topological structure of the evolved network and the rate control behavior is also studied. At each time slot, user's route changes and hence data sending rates also change accordingly. Rate control theorem proposed by Kelly [9], formulated for static network, is used for obtaining optimal user data sending rates to maximize the system utility.
In this paper User's willingness to pay is taken as constant value and it is proportional to the initial capacity(maximum) of that User. It can vary dynamically according to the rate assigned to the User. We have not considered the role of delays while solving System utility. User's routes are selected by considering shortedness, betweenness centrality and initial capacity of users are taken according to their in-degree. It can be extended by considering different objective functions by using parameters such as reputation, influence etc.

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