

Towards a Robust Fractional Order PID Stabilizer for Electric Power Systems

Magdy A.S. Aboelela and Hisham M. Soliman

Abstract This chapter deals with the design and application of a robust Fractional Order PID (FOPID) power system stabilizer tuned by Genetic Algorithm (GA). The system's robustness is assured through the application of *Kharitonov's* theorem to overcome the effect of system parameter's changes within upper and lower bounds. The FOPID stabilizer has been simplified during the optimization using the Oustaloup's approximation for fractional calculus and the "nupid" toolbox of Matlab during simulation. The objective is to keep robust stabilization with maximum attained degree of stability against system's uncertainty. This optimization will be achieved with the proper choice of the FOPID stabilizer's coefficients (k_p , k_i , k_d , λ , and δ) as discussed later in this chapter. The optimization has been done using the GA which limits the boundaries of the tuned parameters within the allowable domain. The calculations have been applied to a single machine infinite bus (SMIB) power system using Matlab and Simulink. The results show superior behavior of the proposed stabilizer over the traditional PID.

Keywords Power system • Power system stabilizer (PSS) • Genetic algorithm • Robust control • Single machine infinite bus (SMIB) • Kharitonov's theorem • Matlab/simulink

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1 Introduction

Low or negative damping in a power system can lead to spontaneous appearance of large power oscillations. Several methods for increasing the damping in a power system are available such as static voltage condenser (SVC), high voltage direct current (HVDC) and power system stabilizer (PSS). Operating conditions of a power system are continually changing due to load patterns, electric generation variations, disturbances, transmission topology and line switching [18].

To enhance system damping; the generators are equipped with power system stabilizers that provide supplementary feedback stabilizing signals in the excitation systems [23]. The control strategy should be capable of manipulating the PSS effectively. The PSS should provide robust stability over a wide range of operating conditions, easy to implement, improves transient stability, low developing time and least cost [18]. Various topologies and many control methods have been proposed for PSS design, such as adaptive controller [33], robust controller [3, 8, 9], extended integral controller [41], state feedback controller [19], fuzzy logic controller [4] and variable structure controller [11]. In Kothari et al. [15] an adaptive fuzzy PSS that behaves like a PID controller that provides faster stabilization of the frequency error signal with less dependency on expert knowledge is proposed. In Malik et al. [20], an indirect adaptive PSS is designed using two input signals, the speed deviation and the power deviation to a neural network controller.

The robust PSS has the ability to maintain stability and achieves desired performance while being insensitive to the perturbations. Among the various robustness techniques, H_∞ optimal control [5] and the structured singular value (SSV or μ) technique [31] have received considerable attention. But, the application of μ technique for controller design is complicated due to the computational requirements of μ design. Besides the high order of the resulting controller, also introduces difficulties with regard to implementation [14, 34].

The H_∞ optimal controller design is relatively simpler than the μ synthesis in terms of the computational burden [5, 35, 36].

Since power systems are highly non-linear, conventional fixed-parameter PSSs cannot cope with wide changes of the operating conditions. There are two main approaches to stabilize a power system over a wide range of operating conditions; namely adaptive control [1, 10, 37] and robust control [3, 19, 35]. However, adaptive controllers have generally poor performance during the learning phase; unless they are properly initialized. Successful operation of adaptive controllers requires the measurements to satisfy strict persistent excitation conditions; otherwise the adjustment of the controller's parameters fail [2, 5, 13].

This chapter is organized as follow: In Sect. 2, we present a brief introduction to fractional calculus and its approximation. Section 3 presents the GA. Section 4 illustrates the system under investigation. Section 5 presents the problem formulation and the problem solution is discussed in Sect. 6. The design procedure of

FOPID PSS is introduced in Sect. 7 with different loading and working conditions. Section 8 and some references are given in Sect. 9. The chapter has three Appendices A, B, and C.

2 Fractional Order PID Controller ($PI^\lambda D^\delta$) Design

The PSS proposed in this chapter belongs to the class of robust controllers. It relies on the Kharitonov's theorem and GA optimization. The use of the Kharitonov's theorem enables us to consider a finite number of plants to be stabilized. The resulting controller will be able to stabilize the original system at any operating point within the design range. We propose to tune the controller's parameters using the genetic algorithm optimization technique [11, 12, 16].

Proportional-Integral-Derivative (PID) controllers are widely being used in industries for process control applications. The merit of using PID controllers lie in its simplicity of design and good performance including low percentage overshoot and small settling time for slow industrial processes. The performance of PID controllers can be further improved by appropriate settings of fractional-I and fractional-D actions [24, 25, 28, 29].

In a fractional PID controller, the I- and D-actions being fractional have wider scope of design. Naturally, besides setting the proportional, derivative and integral constants K_p, T_d and T_i respectively, we have two more parameters: the power of s in integral and derivative actions- λ and δ respectively. Finding [$k_p, k_i, k_d, \lambda, \text{ and } \delta$] as an optimal solution to a given process thus calls for optimization on the five-dimensional space. Classical optimization techniques cannot be used here because of the roughness of the objective function surface. We, therefore, use a derivative-free optimization, guided by the collective behavior of social swarm and determine optimal settings of $k_p, k_i, k_d, \lambda, \text{ and } \delta$ [1].

The performance of the optimal fractional PID controller is better than its integer counterpart. Thus the proposed design will find extensive applications in real industrial processes. Traces of work on fractional PID are available in the current literature [1, 7, 22, 24–29, 32] on control engineering. A frequency domain approach based on the expected crossover frequency and phase margin is mentioned in Vinagre et al. [39]. A method based on pole distribution of the characteristic equation in the complex plane was proposed in Petras [24]. A state-space design method based on feedback poles placement can be viewed in Dorcak et al. [7]. The fractional controller can also be designed by cascading a proper fractional unit to an integer-order controller [26].

Moreover, researchers reported that controllers making use of fractional order derivatives and integrals could achieve performance and robustness results superior to those obtained with conventional (integer order) controllers. The Fractional-order PID controller (FOPID) controller is the expansion of the conventional PID controller based on fractional calculus [1].

The differential equation of the $PI^\lambda D^\delta$ controller is described in time domain by

$$u(t) = k_p e(t) + k_i D_t^{-\lambda} e(t) + k_d D_t^\delta e(t) \quad (1)$$

The continuous transfer function of the $PI^\lambda D^\delta$ controller is obtained through Laplace transform as

$$G_c(s) = k_p + k_i s^{-\lambda} + k_d s^\delta \quad (2)$$

It is obvious that, the FOPID controller does not only need the design three parameters k_p , k_i and k_d , but also the design of two orders λ, δ of integral and derivative controllers. The orders λ, δ are not necessarily integer, but any real numbers [25].

3 Genetic Algorithm Operation

To illustrate the working process of genetic algorithm, the steps to realize a basic GA are listed below [11, 12, 16]:

- Step 1: Represent the problem variable domain as a chromosome of fixed length; choose the size of the chromosome population N , the crossover probability P_c and the mutation probability P_m .
- Step 2: Define a fitness function to measure the performance of an individual chromosome in the problem domain. The fitness function establishes the basis for selecting chromosomes that will be mated during reproduction.
- Step 3: Randomly generate an initial population of size N : x_1, x_2, \dots, x_N .
- Step 4: Calculate the fitness of each individual chromosome: $f(x_1), f(x_2), \dots, f(x_N)$.
- Step 5: Select a pair of chromosomes for mating from the current population. Parent chromosomes are selected with a probability related to their fitness. High fit chromosomes have a higher probability of being selected for mating than less fit chromosomes.
- Step 6: Create a pair of offspring chromosomes by applying the genetic operators.
- Step 7: Place the created offspring chromosomes in the new population.
- Step 8: Repeat Step 5 until the size of the new population equals that of initial population, N .
- Step 9: Replace the initial (parent) chromosome population with the new (offspring) population.
- Step 10: Go to Step 4, and repeat the process until the termination criterion is satisfied.

A GA is an iterative process. Each iteration is called a generation. A typical number of generations for a simple GA can range from 50 to over 500. A common practice is to terminate a GA after a specified number of generations and then examine the best chromosomes in the population. If no satisfactory solution is found, then the GA is restarted [21, 31].

The GA moves from generation to generation until a stopping criterion is met. The stopping criterion could be maximum number of generations, population convergence criteria, lack of improvement in the best solution over a specified number of generations or target value for the objective function.

Evaluation functions or objective functions of many forms can be used in a GA so that the function can map the population into a partially ordered set. The computational flowchart of the GA optimization process employed in the present study is given in Fig. 1.

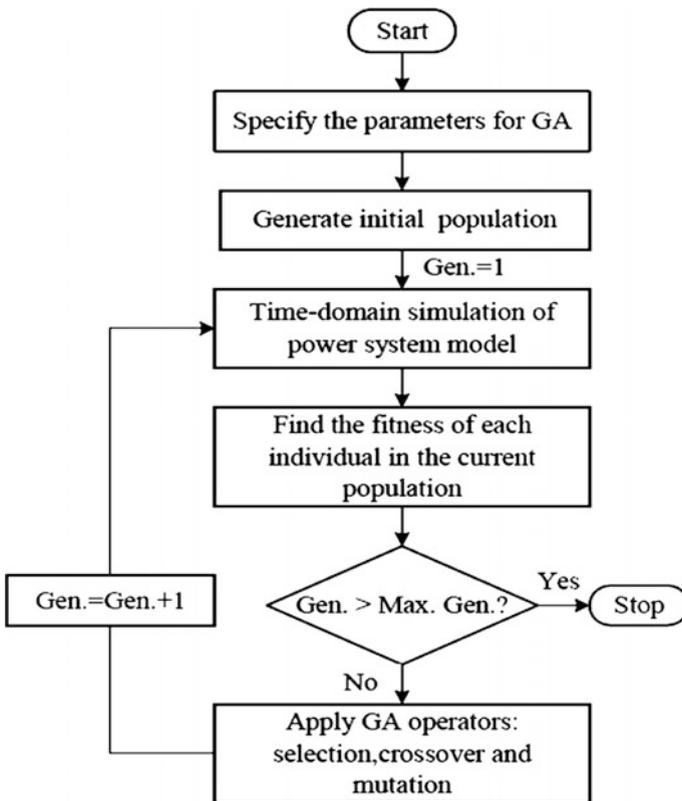


Fig. 1 The computational flowchart of the GA

4 System Investigated

A single machine-infinite bus (SMIB) system is considered for the present investigations. A machine connected to a large system through a transmission line may be reduced to a SMIB system, by using Thevenin’s equivalent of the transmission network external to the machine. Because of the relative size of the system to which the machine is supplying power, the dynamics associated with machine will cause virtually no change in the voltage and frequency of the Thevenin’s voltage (infinite bus voltage). The Thevenin’s equivalent impedance shall henceforth be referred to as equivalent impedance (i.e. $Re + jXe$) [6].

Figure 2 shows the system under study which consists of a single machine connected to an infinite bus through a tie-line. The machine is equipped with a static exciter. The non-linear equations of the system are

$$\begin{aligned}
 \dot{\omega} &= \frac{T_m - T_e}{M} \\
 \dot{\delta} &= \omega_0 \omega \\
 \dot{E}'_q &= \frac{1}{T'_{do}} \left(E_{fd} - \frac{x_d + x_e}{x_d' + x_e} E'_q + \frac{x_d + x'_d}{x_d' + x_e} V \cos \delta \right) \\
 \dot{E}_{fd} &= \frac{1}{T_E} (k_E E_{ref} - k_E V_t - E_{fd})
 \end{aligned}
 \tag{3}$$

The synchronous machine is described as the fourth order model. The two-axis synchronous machine representation with a field circuit in the direct axis but without damper windings is considered for the analysis. The equations describing the steady state operation of a synchronous generator connected to an infinite bus through an external reactance can be linearized about any particular operating point as follows:

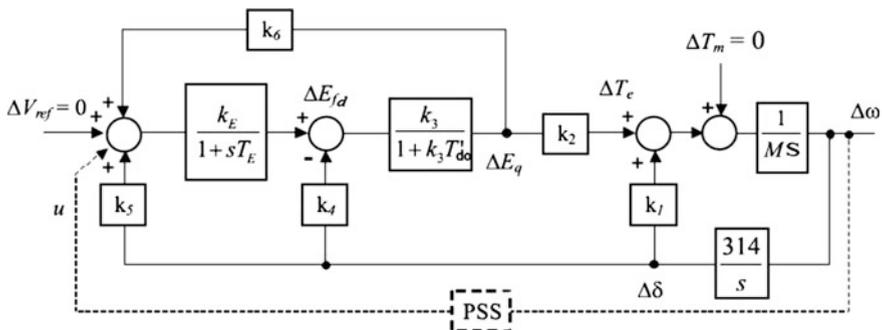


Fig. 2 The block diagram for closed loop SMIB System

$$\Delta T_m - \Delta P = M \frac{d^2 \Delta \delta}{dt^2} \quad (4)$$

$$\Delta P = K_1 \Delta \delta + K_2 \Delta E'_q \quad (5)$$

$$\Delta E'_q = \frac{K_3}{1 + sT'_{d0}K_3} \Delta E_{fd} - \frac{K_3 K_4}{1 + sT'_{d0}K_3} \Delta \delta \quad (6)$$

$$\Delta V_t = K_5 \Delta \delta + K_6 \Delta E'_q \quad (7)$$

The synchronous machine is described by Heffron-Philips model as described in Fig. 2. The K-constants are given in Appendix A. The data definitions are given in Appendix B. The system data are illustrated in Appendix C.

The interaction between the speed and voltage control equations of the machine is expressed in terms of six constants k_1 – k_6 . These constants with the exception of k_3 , which is only a function of the ratio of impedance, are dependent upon the actual real and reactive power loading as well as the excitation levels in the machine [6].

The system equation can be expressed in the following state variable form:

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (8)$$

$$\begin{aligned} X(t) &= [\Delta \delta \quad \Delta \omega \quad \Delta E'_q \quad \Delta E_{fd}]^T, \\ A &= \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ \frac{-k_1}{M} & 0 & -\frac{k_2}{M} & 0 \\ -\frac{k_4}{T'_{d0}} & 0 & -\frac{1}{T} & -\frac{1}{T'_{d0}} \\ -\frac{k_5 k_E}{T_E} & 0 & -\frac{k_6 k_E}{T_E} & -\frac{1}{T_E} \end{bmatrix}, \\ B &= \begin{bmatrix} 0 & 0 & 0 & \frac{k_E}{T_E} \end{bmatrix}', C = [0 \quad 1 \quad 0 \quad 0]. \end{aligned} \quad (9)$$

5 Problem Formulation

The system can be represented by the block diagram proposed by deMello and Concordia [40] which can be cast as shown in Fig. 2. The parameters of the model are load dependent, thus, they have to be calculated at each operating point. Analytical expressions for the parameters k_1 – k_6 , as derived in Soliman et al. [35], Soliman and Sakr [36], are listed in Appendix A. The parameters, k_1 – k_6 , are functions of the loading condition (P and Q). By varying P and/or Q to cover a wide range of system loading, the parameters K_1 – K_6 are computed.

The use of the high-gain voltage regulators usually destabilizes the system. This effect is usually complemented compensated by the inclusion of a stabilizing signal generated by the PSS to provide the required damping. In most cases, the speed deviation signal $\Delta\omega$ is used as an input to the PSS.

To design the PSS, it is convenient to represent the system in the transfer function form as shown in Fig. 3. An analytical expression for the transfer function is derived based on the obtained parameters by using Mason’s rule. The resulting transfer function is

$$\frac{\Delta\omega}{U}(s) = \frac{bs}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0} \tag{10}$$

The transfer-function coefficients expressed in terms of the k -parameters are:

$$\begin{aligned} a_4 &= MTT_E \\ a_3 &= M(T + T_E) \\ a_2 &= M = 314k_1TT_E + k_Ek_3k_6M \\ a_1 &= 314k_1(T + T_E) - 314k_2k_3k_4T_E \\ a_0 &= 314(k_1 - k_2k_3k_4 + k_Ek_1k_3k_6) \\ b &= k_Ek_2k_3 \end{aligned} \tag{11}$$

The coefficients of the transfer function are load-dependent. So, the PSS has to be adjusted at different loads. To scan the whole range of operation, the load dependency may require the analysis of a large number of points with a new model generated at each operating condition.

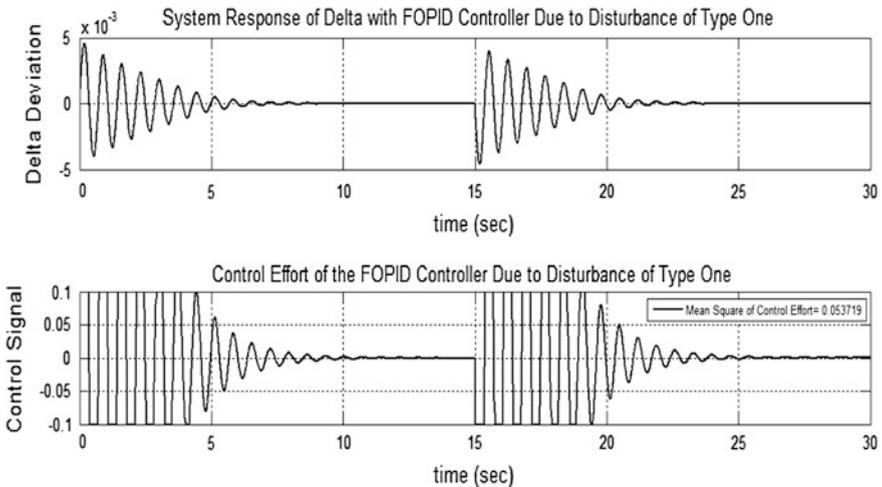


Fig. 3 System response to 0.2 pu torque disturbance at ($P = 0.8, Q = 0.3$)

A proposed technique, based on the Kharitonov’s theorem and GA, is used to design a fixed parameters robust FOPID controller to stabilize the non-linear system over the specified range of operating conditions $[P_{\min}, P_{\max}]$ and $[Q_{\min}, Q_{\max}]$. In this technique, the problem is transformed to simultaneous stabilization of a finite number of extreme plants. We will show in the next section that we need to stabilize exactly eight characteristic polynomials.

5.1 Mathematical Tools and Problem Solution

5.1.1 Kharitonov’s Theorem

The Kharitonov’s theorem studies the robust stability of an interval polynomial family [40]. A polynomial

$$p = a_n s^n + a_{n-1} s^{n-1} + \dots + a_0 \tag{12}$$

is said to be an interval polynomial if each coefficient a_i is independent of the others and varies within an interval having lower and upper bounds that is,

$$a_i = [a_i^-, a_i^+] \tag{13}$$

The Kharitonov’s theorem states that “An interval polynomial

$$p = \sum_{i=0}^n [a_i^-, a_i^+] s^i \tag{14}$$

is robustly stable if and only if the following four Kharitonov’s polynomials

$$\begin{aligned} p_1 &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + \dots \\ p_2 &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + \dots \\ p_3 &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + \dots \\ p_4 &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + \dots \end{aligned} \tag{15}$$

are stable”.

Assuming that the coefficient function a_i depends continuously on the vector $= [P \ Q]^T$ (machine loading P and Q), we define the bounds

$$\begin{aligned} a_i^{*-} &= \min_r(a_i) \\ a_i^{*+} &= \max_r(a_i) \end{aligned} \tag{16}$$

and simply construct the polynomial described by

$$p^*(s) = \sum_{i=0}^n [a_i^{*-}, a_i^{*+}]s^i \tag{17}$$

Then the robust stability of polynomial (17) implies the robust stability of (12), El-Metwally et al. [10].

5.1.2 Oustaloup’s Recursive Filter to Approximate FOPID

Some continuous filters have been summarized in [36]. Among the filters, the well-established Oustaloup recursive filter has a very good fitting to the fractional-order differentiators. Assume that the expected fitting range is (ω_b, ω_h) . The filter can be written as

$$G_f(s) = K \prod_{K=-N}^N \frac{s + \omega'_k}{s + \omega_k} \tag{18}$$

where the poles, zeros, and gain of the filter can be evaluated such that

$$\begin{aligned} \omega'_k &= \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+N+\frac{1}{2}(1-\gamma)}{2N+1}} \\ \omega_k &= \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+N+\frac{1}{2}(1+\gamma)}{2N+1}} \end{aligned} \tag{19}$$

and

$$K = \omega_h^\gamma$$

Thus, the any signal $y(t)$ signal can be filtered through this filter and the output of the filter can be regarded as an approximation for the derivative term of the FOPID with $\gamma = \delta$ or the integral counterpart with $\gamma = -\lambda$. The resulted transfer function of the FOPID is the sum of the proportional term k_p plus the filter approximation of the integral term $(k_i s^{-\lambda})$ plus the derivative term $(k_D s^\delta)$. The result will be the approximated transfer function of the FOPID controller $G_c(s)$ as given by Eq. (2).

In general $G_c(s)$ can be assumed to be in the form:

$$G_c(s) = \frac{N(s)}{D(s)} \tag{20}$$

As shown in Fig. 3, the closed loop characteristic equation can be written as

$$1 + G_c(s)G_p(s) = 0 \tag{21}$$

where $G_p(s) = \frac{A\omega}{U}(s)$ is the plant transfer function [10].

5.1.3 The 16 Kharitonov’s Polynomials

Given the plant family with Kharitonov’s polynomials N_1, \dots, N_4 and D_1, \dots, D_4 for the numerator and denominator, respectively, we define the 16 Kharitonov’s plants as El-Metwally et al. [10].

$$G_c^i(s) = \frac{N_{i_1}(s)}{D_{i_2}(s)}, i_1 = 1, 2, \dots, 4 \text{ and } i_2 = 1, 2, \dots, 4 \tag{22}$$

where $i = 1, 2, \dots, 16$. If the controller can stabilize all the 16 closed loop polynomials given as

$$1 + G_c^i(s)G_p(s) = 0 \tag{23}$$

Then the closed loop system (23) is robustly stable, where $i_1 = 1, 2, \dots, 4$ and $i_2 = 1, 2, \dots, 4$, El-Metwally et al. [10].

Applying the above mathematical tools to the single machine–infinite bus system (Fig. 1), we have the vector r which is composed of two independent components.

$$r = [P \quad Q]^T \tag{24}$$

In the system under study, the numerator of the transfer function is a first order polynomial (bs). Thus, the coefficient b has two extreme values b^+ and b^- ; that is, the 16 plants corresponding to (23) are reduced to 8 plants only.

6 Problem Solution

To stabilize the system over the required ranges of P and Q , the following eight polynomials must be stable.

We will use the genetic algorithm to find the values of k_p, k_i, k_d, λ , and δ that correspond to the following optimization problem

$$\min_{k_p, k_i, k_d, \lambda, \text{ and } \delta} (\max (\lambda_e)) \tag{25}$$

Subject to

$$\begin{aligned} k_p^{min} &\leq k_p \leq k_p^{max} \\ k_i^{min} &\leq k_i \leq k_i^{max} \\ k_D^{min} &\leq k_D \leq k_D^{max} \\ \lambda^{min} &\leq \lambda \leq \lambda^{max} \\ \gamma^{min} &\leq \gamma \leq \gamma^{max} \end{aligned} \tag{26}$$

where λ_e is a vector containing the real parts of the roots of the eight equations resulting from (25). This means that the parameters k , z and p must stabilize the eight polynomials in Eq. (25). On the other hand, the swarm optimization algorithm attempts to push the closed loop poles to the left as far as possible by minimizing the maximum real part of the roots resulting from (25). The problem can be tackled using a different approach. If we divide the range of P and Q into small steps, the resulting grid will represent the possible operating points.

For each point on the grid, a model can be derived. Applying the genetic algorithm optimization technique to stabilize such systems is possible. However, there is no guarantee that stability is preserved for intermediate points inside the grid. The proposed technique eliminates this shortcoming via the Kharitonov's theorem.

7 PSS Design for Different Machine Loadability

The design objective, in this chapter, is to implement the machine loadability, of the system under study, over the range $Q \in [-0.4, 0.4]$ and $P \in [0.2, 1.2]$. The design procedure can be summarized as follows:

- Develop the linearized model as shown in Fig. 2. The machine parameters and the k -parameter calculations are given in the Appendices A and C.
- Based on the analytical expressions for a_0, a_1, \dots, a_4 and b in (11), calculate the maximum and minimum values of the aforementioned parameters using any standard optimization technique. Note that a_3 and a_4 do not depend on the values of P and Q .
- Using (29) and replacing a_i by a_i^* , construct the four Kharitonov's polynomials as in (15). Compute the roots of the 8 extreme polynomials and take the largest real part of the roots as the objective function to be minimized.
- Use the GA to find a solution for the optimization problem (26) such that the roots of (25) lie in the left hand side of the s -plane away from the imaginary axis as much as possible. Thus the shortest settling time of oscillations is achieved

The above procedure is applied to the system under study as follows: Consider the system transfer function (10). The extreme values of its coefficients are calculated as

$$\begin{aligned} a_i^{*-} &= \min_{P,Q}(a_i) \text{ and } a_i^{*+} = \max_{P,Q}(a_i^*) \\ b_i^{*-} &= \min_{P,Q}(b_i) \text{ and } b_i^{*+} = \max_{P,Q}(b_i^*) \end{aligned} \tag{27}$$

The results of the above calculations are

$$a_4^* = 1, a_3^* = 22, a_2^* \in [64 \ 106], a_1^* \in [388 \ 1002],$$

$$a_0^*[392 \ 2624] \text{ and } b^* \in [2.7 \ 12.4]$$

Then, the four Kharitonov’s polynomials are:

$$p_1 = 392 + 388s + 106s^2 + 22s^3 + s^4$$

$$p_2 = 2624 + 1003s + 64s^2 + 22s^3 + s^4$$

$$p_3 = 2624 + 388s + 64s^2 + 22s^3 + s^4$$

$$p_4 = 392 + 1003s + 106s^2 + 22s^3 + s^4$$
(28)

7.1 Design of a Robust PSS Using GA

The plant transfer function (10) is analyzed using eight extreme plants given by

$$G_p(s) = \frac{\Delta\omega}{U}(s) = \frac{b^- s}{p_i}, i = 1, 2, \dots, 4$$

$$G_p(s) = \frac{\Delta\omega}{U}(s) = \frac{b^+ s}{p_i}, i = 1, 2, \dots, 4$$
(29)

To reach the optimization goal, proper adjustment of the GA parameters are needed. This requires the determination of population size (N = 100 is sufficient), the bit size for each binary parameter (16 is reasonable size), and the upper and lower bounds of the optimization of FOPID PSS (for $k_p, k_i,$ and $k_d,$ [0 100] is an acceptable range but for λ and δ [0 1.5] is found to be a proper choice in our case [38].

The results obtained using the GA on FOPID PSS design procedure mentioned in this chapter are delineated in Table 1. The same procedure can be successfully applied to the case of PID PSS considering the limits of λ and δ of the FOPID PSS as [1 1]. Results of this case are also shown in Table 1.

The proposed PSS is tested over three operating condition.

Table 1 GA estimated parameters for PID and FOPID PSS

Controller	k_p	k_i	k_d	δ	λ
PID (minimum = -1.3961)	45.36	45.452	62.2	N/A	N/A
FOPID (minimum = -1.3849)	48.50	93.666	79.8	0.61	1.3

7.2 The Normal Loading Test

The first operating point is $P = 0.8$ pu and $Q = 0.3$ pu represents the normal loading conditions. The system was exposed to a 0.20 p.u step increase in the input torque reference at 0.5 s. The disturbance was removed at 15 s, i.e. the signal duration is 14.5 s, and the system returned to the original operating point by the end of disturbance. The regulated system without a stabilizer is stable at this point [10]. However, the mechanical disturbance pushes the system close to the stability bound. Figure 3 shows the machine speed deviation and the machine power angle (δ). It is clear that if the power system stabilizer is not employed, the rotor angle oscillation will have a very slow damping behavior. On the other hand, the proposed FOPID stabilizer successfully suppresses and damps the oscillations in almost three seconds. The controller signal is shown in Fig. 3. It is clear that the controller is utilizing the full control range limited by the maximum standard power system stabilizer signal ± 0.1 pu.

The Simulink models for the FOPID PSS applications are illustrated in Figs. 4 and 5. The FOPID PSS block is represented by “NIPID” block of “ninteger” blockset of Matlab [14, 38].

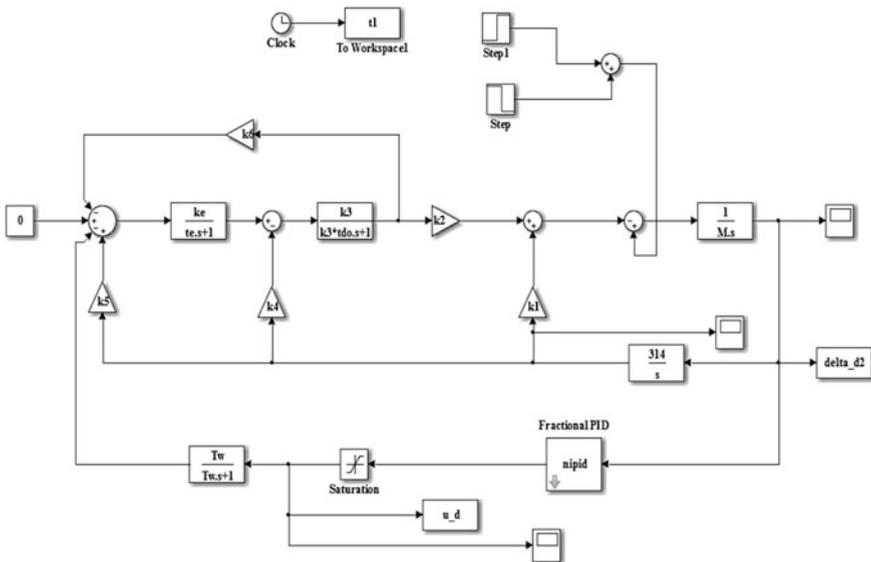


Fig. 4 Matlab/Simulink Model with FOPID PSS and Torque Disturbance Signal

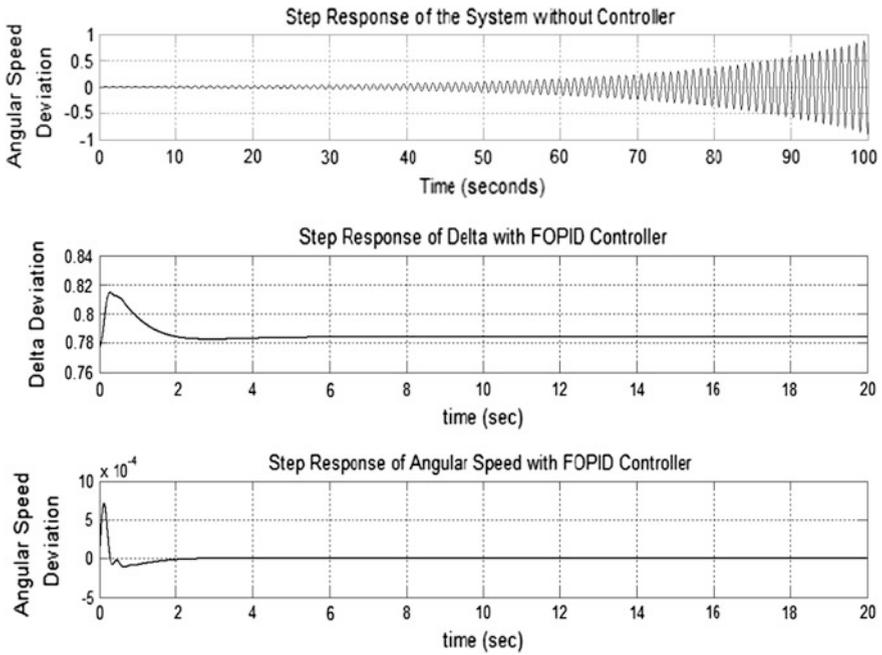


Fig. 6 $\Delta\delta$ and $\Delta\omega$ after adding FOPID PSS type in normal operation at ($P = 1.2, Q = 0.2$)

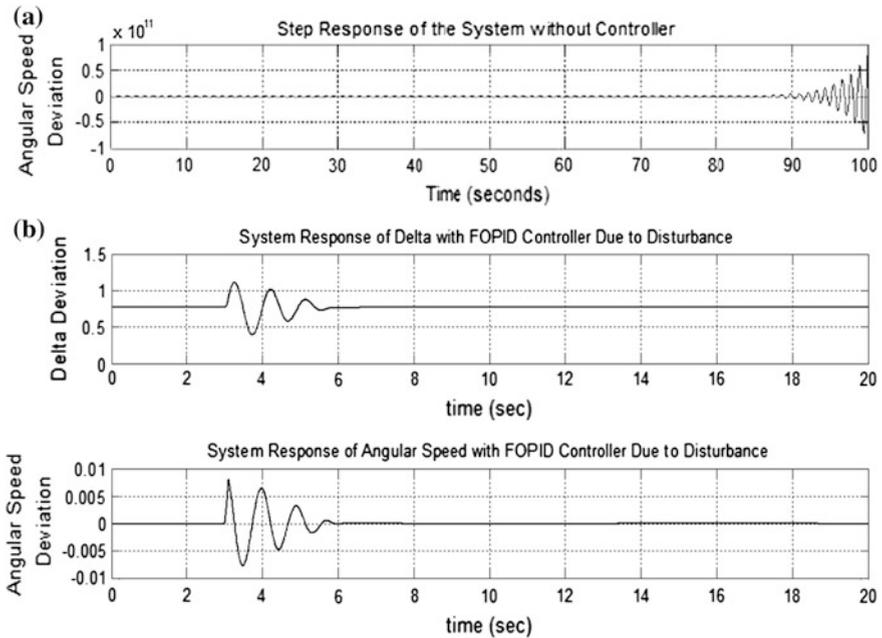


Fig. 7 a $\Delta\delta$ without Controller ($P = 1, Q = -0.4$). b $\Delta\delta$ and $\Delta\omega$ due to a three line to ground fault at 3 s staying for 100 ms after adding FOPID PSS type ($P = 1, Q = -0.4$)

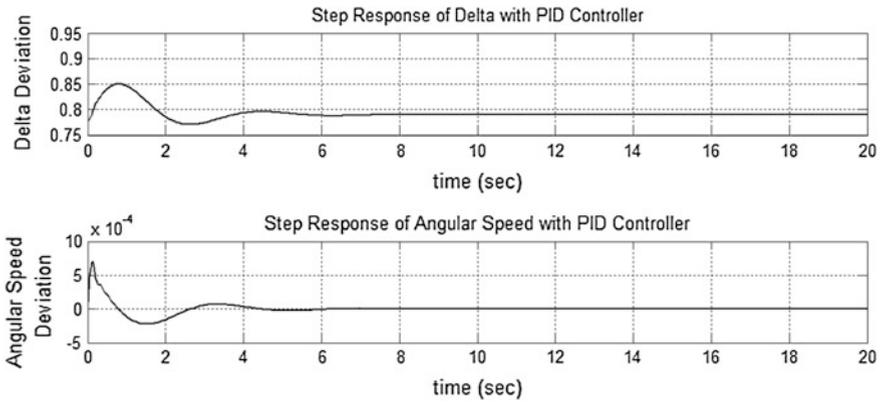


Fig. 8 $\Delta\delta$ and $\Delta\omega$ after adding PID PSS type in normal operation ($P = 0.8, Q = 0.3$)

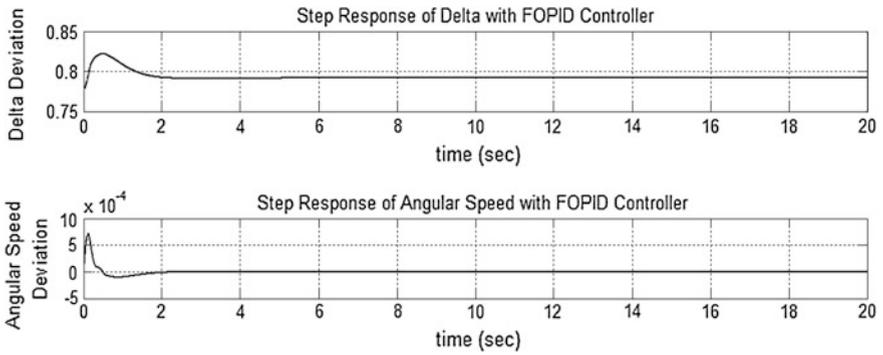


Fig. 9 $\Delta\delta$ and $\Delta\omega$ after adding FOPID PSS type in normal operation ($P = 0.8, Q = 0.3$)

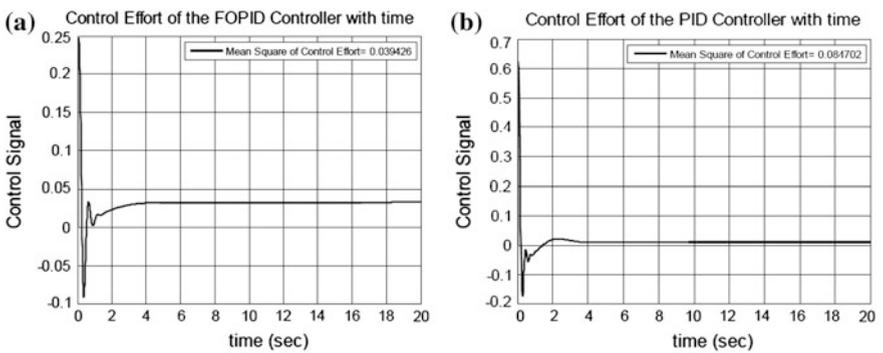


Fig. 10 **a** Control Effort of the FOPID PSS in normal operation ($P = 0.8, Q = 0.3$) **b** Control Effort of the PID PSS in normal operation ($P = 0.8, Q = 0.3$)

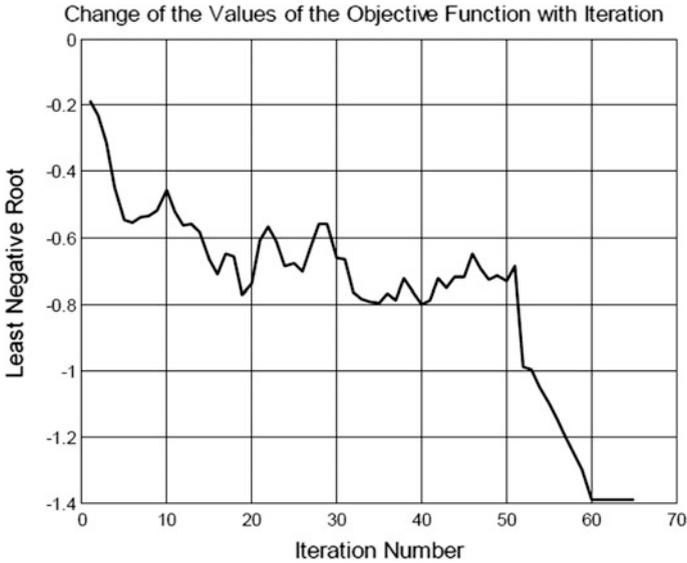


Fig. 11 The Objective Function vs. Iterations in case of FOPID PSS

Moreover, the minimum negative eigenvalue of the stabilized SMIB system using the PID and FOPID PSS is almost the same as shown in Table 1. The change of this value for the case of FOPID PSS with iteration is delineated if Fig. 11.

It is apparent that the presented tuning algorithm for the fractional order PID controllers has been found robust at different loading conditions of a single machine connected to an infinite bus (SMIB) power system. The convergence rate of the presented algorithm is noticeable which encourage the application of the fractional order PID (FOPID) controllers on some other industrial applications.

8 Conclusion

The design of a robust FOPID PSS using the Kharitonov's theorem has been proposed. The k -parameters of the model are parameterized in terms of the operating point (P , Q). Accordingly, the coefficients 'bounds of the transfer function relating the stabilizing control signal to the speed deviation have been calculated over the whole range of operating points. The design is based on simultaneous stabilization of eight extreme plants to achieve a satisfactory dynamic performance. The calculations are based on the GA optimization algorithm. Simulation results based on a non-linear model of the power system confirm the ability of the proposed compensator to stabilize the system over a wide range of operating points as illustrated with various examples.

The performance of the conventional PID PSS, designed with the same procedure, as compared with the FOPID PSS shows less oscillation damping of both the changes in angle δ and the angular speed ω .

For future work, authors recommend the extension of the method to the case of multi machines power systems. Also some other evolutionary techniques such as bat inspiration, gravitational techniques and imperialist colony may be tried to determine the best tuning of the fractional order PID controllers.

Appendix A: Derivation of k-Constants

All the variables with subscript 0 are values of variables evaluated at their pre-disturbance steady-state operating point from the known values of P_0 , Q_0 and V_{i0} .

$$i_{q0} = \frac{P_0 V_{i0}}{\sqrt{(P_0 x_q)^2 + (V_{i0}^2 + Q_0 x_q)^2}} \quad (\text{A1})$$

$$v_{d0} = i_{q0} x_q \quad (\text{A2})$$

$$v_{q0} = \sqrt{V_{i0}^2 - v_{d0}^2} \quad (\text{A3})$$

$$i_{d0} = \frac{Q_0 + x_q i_{q0}^2}{v_{q0}} \quad (\text{A4})$$

$$E_{q0} = v_{q0} + i_{d0} x_q \quad (\text{A5})$$

$$E_0 = \sqrt{(v_{d0} + x_e i_{q0})^2 + (v_{q0} - x_e i_{d0})^2} \quad (\text{A6})$$

$$\delta_0 = \tan^{-1} \frac{(v_{d0} + x_e i_{q0})}{(v_{q0} - x_e i_{d0})} \quad (\text{A7})$$

$$K_1 = \frac{x_q - x'_d}{x_e + x'_d} i_{q0} E_0 \sin \delta_0 + \frac{E_{q0} E_0 \cos \delta_0}{x_e + x_q} \quad (\text{A8})$$

$$K_2 = \frac{E_0 \sin \delta_0}{x_e + x'_d} \quad (\text{A9})$$

$$K_3 = \frac{x'_d + x_e}{x_d + x_e} \quad (\text{A10})$$

$$K_4 = \frac{x_q - x'_d}{x_e + x'_d} E_0 \sin \delta_0 \quad (\text{A11})$$

$$K_5 = \frac{x_q}{x_e + x_q} \frac{v_{d0}}{V_{t0}} E_0 \cos \delta_0 - \frac{x'_d}{x_e + x'_d} \frac{v_{q0}}{V_{t0}} E_0 \sin \delta_0 \quad (\text{A12})$$

$$K_6 = \frac{x_e}{x_e + x'_d} \frac{v_{q0}}{V_{t0}} \quad (\text{A13})$$

Appendix B

Nomenclature

All quantities are per unit on machine base.

D	Damping Torque Coefficient
M	Inertia constant
ω	Angular speed
δ	Rotor angle
I_d, I_q	Direct and quadrature components of armature current
x_d and x_q	Synchronous reactance in d and q axis
x'_d and x'_q	Direct axis and Quadrature axis transient reactance
E_{fd}	Equivalent excitation voltage
K_E	Exciter gain
T_E	Exciter time constant
T_m and T_e	Mechanical and Electrical torque
T'_{do}	Field open circuit time constant
V_d and V_q	Direct and quadrature components of terminal voltage
K_1	Change in T_e for a change in δ with constant flux linkages in the d axis
K_2	Change in T_e for a change in d axis flux linkages with constant δ
K_3	Impedance factor
K_4	Demagnetizing effect of a change in rotor angle
K_5	Change in V_t with change in rotor angle for constant E'_q
K_6	Change in V_t with change in E'_q constant rotor angle

Appendix C

The system data are as follows:

Machine (p.u):

$$\begin{aligned}x_d &= 1.6 & x'_d &= 0.32 \\x_q &= 1.55 & T'_{d0} &= 6 \text{ s} \\D &= 0.0 & M &= 10 \text{ s}\end{aligned}\tag{C1}$$

Transmission line (p.u):

$$r_e = 0.0 \quad x_e = 0.4\tag{C2}$$

Exciter:

$$K_E = 25.00 \quad T_E = 0.05 \text{ s}\tag{C3}$$

Nominal Operating point:

$$\begin{aligned}V_{i0} &= 1.0 & P_0 &= 0.8 \\Q_0 &= 0.3 & \delta_0 &= 45^\circ \\& & \omega_0 &= 314\end{aligned}\tag{C4}$$

Others

$$\begin{aligned}k_3 &= 1/2.78 \\v &= 1.0 \\T_w &= 5\end{aligned}\tag{C5}$$

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