Optimal Fractional Order Proportional—Integral—Differential Controller for Inverted Pendulum with Reduced Order Linear Quadratic Regulator

M.E. Mousa, M.A. Ebrahim and M.A. Moustafa Hassan

Abstract The objective of this chapter is to present an optimal Fractional Order Proportional—Integral-Differential (FOPID) controller based upon Reduced Linear Quadratic Regulator (RLQR) using Particle Swarm Optimization (PSO) algorithm and compared with PID controller. The controllers are applied to Inverted Pendulum (IP) system which is one of the most exciting problems in dynamics and control theory. The FOPID or PID controller with a feed-forward gain is responsible for stabilizing the cart position and the RLQR controller is responsible for swinging up the pendulum angle. FOPID controller is the recent advances improvement controller of a conventional classical PID controller. Fractional-order calculus deals with non-integer order systems. It is the same as the traditional calculus but with a much wider applicability. Fractional Calculus is used widely in the last two decades and applied in different fields in the control area. FOPID controller achieves great success because of its effectiveness on the dynamic of the systems. Designing FOPID controller is more flexible than the standard PID controller because they have five parameters with two parameters over the standard PID controller. The Linear Quadratic Regulator (LQR) is an important approach in the optimal control theory. The optimal LQR needs tedious tuning effort in the context of good results. Moreover, LQR has many coefficients matrices which are designer

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dependent. These difficulties are talked by introducing RLQR. RLQR has an advantage which allows for the optimization technique to tune fewer parameters than classical LQR controller. Moreover, all coefficients matrices that are designer dependent are reformulated to be included into the optimization process. Tuning the controllers' gains is one of the most crucial challenges that face FOPID application. Thanks to the Metaheuristic Optimization Techniques (MOTs) which solves this dilemma. PSO technique is one of the most widely used MOTs. PSO is used for the optimal tuning of the FOPID controller and RLQR parameters. The control problem is formulated to attain the combined FOPID controllers' gains with a feed forward gain and RLQR into a multi-dimensions control problem. The objective function is designed to be multi-objective by considering the minimum settling time, rise time, undershoot and overshoot for both the cart position and the pendulum angle. It is evident from the simulation results, the effectiveness of the proposed design approach. The obtained results are very promising. The design procedures are presented step by step. The robustness of the proposed controllers is tested for internal and external large and fast disturbances.

Keywords Fractional order Proportional-Integral-Derivative · Inverted pendulum · Linear quadratic regulator · Particle swarm optimization technique · Proportional-Integral-Derivative · Reduced linear quadratic regulator · Robustness verification

1 Introduction

The Inverted Pendulum (IP) System is a classical benchmark for the control designers. The IP system is a physical system consists of pendulum carried by a cart and swinging around the fixed pivot [1]. The concept of the IP system is used in many modern technological applications like the landing of aircraft, space satellites, launching and guidance of the missile operations, spacecraft, statistics applications, and biomechanics [2]. Also, IP system is used on a large scale in many areas and applications including medical, transportation, robotics, aerospace, and military [3]. The IP system is considered the heart of many industrial applications. Some of these industrial applications are control of our ankle joint during quiet standing up, Segway's, quad rotor helicopters and walking robots [4]. IP system is the subject of an interesting from the standpoint of control because of their intrinsic nonlinearity [5]. It is used to illustrate the ideas in the nonlinear control and control of the chaotic system. The evaluation of various control theories is based on the inverted pendulum system.

IP system is a physical system consists of a bar which is usually made of aluminum and swinging around the fixed pivot. This fixed pivot will be installed on the vehicle which moves in the horizontal direction only. The center of gravity of the normal pendulum is under the axis of rotation and therefore, his condition is stable when it is directed to the bottom while the center of gravity of the inverted pendulum is over its axis of rotation. In the inverted pendulum problem, the pendulum tried to be in a vertical position to be heading up. The swinging up of the pendulum makes the situation of the system abnormal. So, the permanent controller should be applied to the system to keep the pendulum vertically upright. The nonlinearity and inherently instability of the system adding complexity to the problem especially when the proposed controllers will apply to the nonlinear system without any linearization [6]. Fast swinging up of the pendulum angle and stabilizing the cart at a certain position is required. The proposed controllers should be robust against the various system disturbances. Different control techniques are applied on the inverted pendulum to show the performance and effectiveness of the techniques [7].

In the IP problem, a lot of control techniques are proposed to make the pendulum balance in inverted to be heading up. Since this situation is abnormal, the status of "unbalanced" Basically, a permanent effort is needed to keep it this way, at any moment this effort is stopped, the system will collapse but return again to put the natural stability beyond. Normally be inaugurated with a turnover point centered on a moving vehicle accidentally. If the pendulum starts from a vertical position without applying any control strategy, it will begin to fall off and the cart will move in the opposite direction which means that any change in pendulum moving will effect on the cart and vice versa. The desired objectives of inverted pendulum control are:

(a) Maintaining the pendulum vertically upright.
(b) Stabilizing the cart.

The major objectives of the chapter are:

1 Modeling the IP system and presenting the linearized model at the certain operating point.
2 Design Reduced Linear Quadratic Regulator (RLQR) with minimum tuning parameters.
3 Design HybridPID controller in conjunction with feed forward and RLQR.
4 Design Hybrid Fractional Order PID controller with RLQR.
5 Develop multi-objective function which guarantees overall system stability in terms of minimum overshoot, settling time and steady state error for the both outputs of the IPsystem.
6 Propose Particle Swarm Optimization (PSO) technique for tuning the IP control system parameters.
7 Verify the robustness of the proposed controllers on changing the IP system parameters.
8 Validate the effectiveness of the proposed controllers using various types of disturbances.

The rest of the chapter is organized as follows: Sect. 2 illustrates a comprehensive literature survey of all related works. Section 3 formulates the dynamic model of a simple IP system. In Sect. 4, different classical and metaheuristic optimization techniques are used for optimality regions classification and verification. Additionally, the selection criteria of the most robust controller is reported. Simulation results are considered in Sect. 5. The conclusions and the perspectives are drawn in Sect. 6. Finally, the future work is illustrated in Sect. 7.

2 Related Work

There are many types of control techniques that are applied on the IP which has two outputs: position and angle. The presented methods for IP control are classified into seven groups:

- Classical methods such as PID controllers [8].
- Adaptation methods [9].
- Artificial methods such as fuzzy logic control [10], neural network [11], Genetic Algorithm [12] and PSO [13].
- Hybrid control [14].
- Sliding mode control [15].
- Time optimal control [16].
- Predictive control [17].

Some of these techniques are applied for tuning the angle controller gains while the position controller gains are constant. These strategies try to find the best gains to achieve the desired angle response. After that, the same procedures are carried out for tuning the position controller gains while maintaining the angle controller gains constant. The old control strategies deal with the IP as a single input single output system (SISO). In recent years, there are many control strategies deal with the IP as Single Input Multi Output System (SIMO). Some of control techniques are applied to the Inverted Pendulum system as follow:

Fuzzy Logic Controller

Fuzzy logic controller is used based on the single input rule modules. The input terms of the fuzzy controller are: the angle, angular velocity of the pendulum, the position and velocity of the cart and the output term is driving force. The authors in [18] represented a nonlinear plant with a Takagi-Sugeno fuzzy model. Each control rule is derived by using "parallel distributed compensation" in the controller design. To solve linear matrix inequality problems, Convex programming techniques are used as the control design problems can be reduced to LMI.

Lyapunov Approach

Lyapunov approach is used in PID adaptive control for self-tuning method for a class of nonlinear control systems [19]. There are three PID control gains parameters are adjustable and updated online with a stable adaptation mechanism. By introducing a supervisory control and a modified adaptation law with projection, the stability of closed-loop nonlinear PID control system is analyzed. Finally; a tracking control of an inverted pendulum system is used to demonstrate the control performance. Properties of simple strategies for swinging up an inverted pendulum are discussed in [20]. It turns out that the inverted pendulum swing behavior depends mainly on, the ratio of the maximum acceleration of the pivot to the acceleration of gravity. There are great ideas to minimum time solutions by make a comparison of energy-based strategies with minimum time strategy.

Energy Control Methods

In [21] generalized energy control methods are used to swing-up and stabilization of a cart–pendulum system with some restriction such, cart track length and control force. By using energy control principles, the pendulum is swung up to the upright unstable equilibrium configuration with Starting from a pendant position. In order to prevent the cart from going outside the limited length, an "energy well" must be built within the cart track. When getting adequate amount of energy by the pendulum and maintained it, it goes into a "cruise" mode. Finally, the stabilizing controller is activated around a linear zone about the upright configuration when the pendulum is closed to the upright configuration. This way has worked well both in simulation and a practical setup and derived the conditions for stability by using the multiple Lyapunov functions approach. The feedback of an inverted pendulum is not linear although inverted pendulum is one of the typical examples of nonlinear control systems. A new method to design back stepping-like controller is proposed by Saeki in [22]. By combining Saeki's method with the energy function method, produces a swing-up controller. Firstly, to prevent the effect of the pendulum, the control input is given of the cart. Secondly, design the input that guarantees the convergence of the acceleration of the cart to the desired value. Thirdly, an energy function was used to design the swing-up control law. The energy function-based controller is used to swing up the pendulum and the potential function-based controllers used to stabilize the inverted pendulum.

Sliding Mode Control

In [15], the authors developed a Second-order sliding mode control synthesis for under-actuated mechanical systems and operates it under uncertainty conditions. The output is specified in such a way that the corresponding zero dynamics is locally asymptotically stable in order to locally stabilize an under-actuated system around an unstable equilibrium. And then, provide the desired stability property of the closed-loop system by applying a quasi-homogeneous second-order sliding mode controller, driving the system to the zero dynamics manifold in finite time [23]. It does not rely on the generation of first-order sliding modes, although the present synthesis exhibits an infinite number of switches on a finite time interval, while providing robustness features similar to those possessed by their standard sliding mode counterparts. The performance issues of the proposed method are illustrated in numerical and experimental studies of a cart–pendulum system.

Optimal Control

Optimal control with time invariant nonlinear controller is presented in [24] for the inverted pendulum, which is defined for all pendulum angles. The external field is calculated by solving the Euler–Lagrange equations backward in time. The time-optimal feedback control that brings a pendulum to the upper unstable equilibrium position is obtained in [25]. The technique is based on the maximum principle and analytical investigations and numerical computations.

The nonlinear model predictive control is applied in [26] to an inverted pendulum apparatus. A standard sequential quadratic programming approach is used to solve non-convex constrained optimization problem involves 61-variables with 241-constraints.

In this chapter, a new objective function with new artificial intelligent based technique for tuning the controllers' gains of SIMO inverted pendulum system is proposed. The difficulty of the proposed strategy comes from that any change in angle will effect on the position and vice versa. The tuning process of FOPID controller and RLQR is not aneasy task as there are five parameters for the FOPID and two weighting matrices for RLQR. These gains directly affect the angle and cart response so it is a complicated problem.

3 Mathematical Modeling

Modeling and control of the IP are the prerequisites of autonomous walking. The primary approach to derive the model is the Euler-Lagrange approach. The IP is one of the most difficult systems in control theory due to the non-linearity [27]. It is inherently unstable system with single input and multi-outputs so applying classical control methods did not lead to good results. If there is a stick on hand and the objective is to make it always in a vertical position, it is needed to move the hand to keep the stick in a vertical position. On the other hand, a force is applied to keep the stick in a vertical position. Similarly in the control of inverted pendulum, a force is applied to make the pendulum always upright vertical without any deviation about zero. The IP system has two degrees of freedom of motions as shown in Fig. 1. The first degree of freedom is the motion of the cart along the x-axis and the second

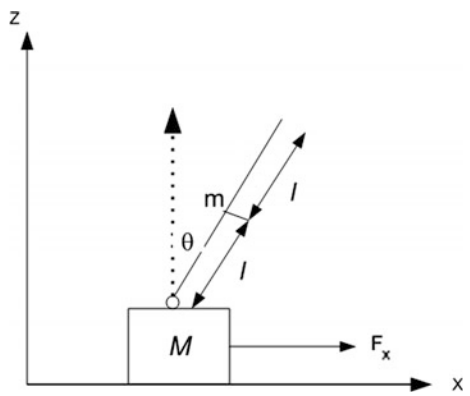


Fig. 1 Inverted pendulum system modeling

Table 1 Inverted pendulum system parameters

Symbol	Parameter	Value	Unit
M	Mass of the cart	0.455	Kg
m	Mass of the pendulum	0.21	Kg
l	The distance from the pivot to the mass center of the pendulum	0.61/2	m
g	The acceleration of gravity	9.8	m/s^2

degree of freedom is the rotation of the pendulum aboutthex-zplane. The mathematical model can be defined as, a set of mathematical equations representing some of the phenomena in a way that gives insight into the origins and the consequences of the behavior of the system. The more accurate the mathematical model is, the more complex the equations will be. The mathematical model should be easy to understand. So accuracy and the simplicity are the two main parameters that should take into consideration while modeling. It can be seen that the equations describing the system are non-linear. Taylor series expansion is used in order to obtain a linear model to convert the non-linear equations to linear ones; finally, produce a linear model that will be helpful in linear control design. The system has two equilibrium points: one point is stable such as the pendant position and the other point is the unstable equilibrium point such as the inverted position. For our purpose, the second is required to make linearization to the model about it. So, a very small deviation from the vertical is assumed.

The parameters of the IP system are illustrated in Table 1.

The IP System is nonlinear and inherently unstable system. The modeling equations of the IP system are very important which allow the controller to stabilize the cart position and swinging up the pendulum angle. The dynamic differential equation of the system is derived according to the Euler-Lagrange equations. The IP system is a highly coupling system as it not allowable to derive each output equation individually. The Euler-Lagrange formula can derive the system as a multi-outputs system and get the state-space representation of the system states.

3.1 Applying Lagrangian to Inverted Pendulum System

The following steps should be followed to put the IP system in Lagrangian formula [28]:

(a) Obtain the kinetic and potential energies.
(b) Substitute in the Lagrangian formula $L = K - P$
(c) Find $\partial L/\partial q$
(d) Find $\partial L/\partial \dot{q}$ then find $\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}})$.
(e) Solve the Euler-Lagrange equation with the generalized force $\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) - \frac{\partial L}{\partial q} = Q_q$

where:

Q_q The generalized forces
q The generalized coordinates

The nonlinear differential equations of the motion are as follow:

$$\ddot{X} = \frac{F_x - mg\ \cos\theta\ \sin\theta + ml\dot{\theta}^2\ \sin\theta}{M + m\sin^2\theta} \tag{1}$$

$$\ddot{\theta} = \frac{(M+m)g\ \sin\theta - ml\dot{\theta}^2\ \sin\theta\ \cos\theta - F_x\cos\theta}{Ml + ml\sin^2\theta} \tag{2}$$

Here, the states of the system are defined as the following to represent the state space on the inverted pendulum:

$$X_1 = X/X_2 = \dot{X}/X_3 = \theta/X_4 = \theta,$$

where:

X The position of the cart that move along the x-axis
$\dot{X}$ The velocity of the cart that moves along x-axis
θ The angle position from the vertical position
θ The velocity of the pendulum that swings along Z-axis

When the system is linearized, the state space representation could easily be obtained as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -4.5231 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 46.9609 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 2.1978 \\ 0 \\ -7.2059 \end{bmatrix} F \tag{3}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_X \tag{4}$$

4 The Proposed Control Techniques:

The proposed control techniques for the IP system in this chapter are:

- *Linear Quadratic Regulator.*
- *Proportional Integral Derivative Controller.*
- *Fractional Order Proportional Integral Derivative Controller.*
- *Particle Swarm Optimization based PI/RLQR and FOPID/RLQR.*

(a) ***Linear Quadratic Regulator***

Given a linear time-invariant state-space model of the system:

$$\dot{x} = Ax + B \tag{5}$$

$$y = Cx + Du \tag{6}$$

The LQR is used to minimize the following cost function [29]:

$$J = \frac{1}{2}\int_{0}^{\infty} [x^T Qx + u^T Ru]dt \tag{7}$$

where:

Q and R are weighting matrices which are selected by the designer.

This selection process depends on the experience of the designer which in turn is a tedious effort in multi-dimensions problems. In LQR, the following Riccati equation should be solved [29]:

$$PA + A^T P - PBR^T B^T P + Q = 0 \tag{8}$$

The matrix P is obtained by solving the above equation so the controller gain can be calculated according to Eq. (9) as explained in [14]:

$$K = R^{-1} B^T P \tag{9}$$

The Q and R matrices are the main design parameters which greatly affect on the controller gain. In this chapter; PSO technique is used to get Q and R matrices according to specific constraints.

(b) ***Proportional Integral Derivative Controller***

The proportional-integral-derivative (PID) controller is used in most control systems. It consists of three gains: proportional gain (K_p), integral gain (K_i) and derivative gain (K_d). Each of the PID controller gains has an action on the error. The error is the difference between a setpoint designed by the user and some measured process variables. The continuous form of a PID controller, with input e and output U, is presented in Eq. (10).

$$U_{PID} = K_P e(t) + K_i \int_{0}^{t} e(t)dt + K_d \frac{d}{dt} e(t) \tag{10}$$

(c) ***Fractional Order PID Controller with Feed Forward Gain (FOPID)***

Fractional Order Proportional-Integral-Derivative (FOPID) controller is the recent advances improvement controller of a conventional classical PID controller [30]. The earliest studies concerning fractional calculus presented in the 19th century made by some researchers such as Liouville (1832), Holmgren (1864), and Riemann (1953) as introduced in [31], and others made some contributions in this field in the past. Fractional-order calculus deals with non-integer order systems. It is the same as the traditional calculus but with a much wider applicability. Fractional Calculus is used widely in the last two decades and applied in different fields in the control area.

Fractional order Proportional-Integral-Derivative controller achieves great success because of its effectiveness on the dynamic of the systems. Designing FOPID Controller is more flexible than the standard PID Controller because they have five parameters with two parameters over the standard PID controller. The operator ${}_aD_t^q$ is commonly used in fractional calculus which is defined as the differentiation integration operator and discussed as presented in Eq. (11):

$$ {}_aD_i^q = \left\{ \begin{array}{cc} \frac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int\limits_a^t (d\tau)^{-q} & q < 0 \end{array} \right\} \tag{11} $$

where:

q	Fractional order (can be complex)
a and t	The limits of operation

There are different definitions for fractional derivatives. The widely used definitions are as following:

(a) Grunwald–Letnikov definition.
(b) Riemann–Liouville definition.
(c) Caputo definition.

These definitions will be discussed below:

(a) Grunwald–Letnikov definition

The Grunwald–Letnikov definition is given by Eq. (12):

$$ {}_aD_t^q f(t) = \frac{d^q f(t)}{d(t-a)^q} = \lim_{N\to\infty} \left[\frac{t-a}{N}\right] \sum_{j=0}^{N-1} (-1)^j \binom{q}{j} f(t - j\left[\frac{t-a}{N}\right]) \tag{12} $$

(b) Riemann–Liouville definition

The Riemann–Liouville definition is the easiest definition and defined as presented in Eq. (13):

$$ {}_aD_t^a f(t) = \frac{d^q f(t)}{d(t-a)^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t (t-\tau)^{n-q-1} f(\tau) d\tau \tag{13} $$

where:

n The first integer $(n - 1 \leq q < n)$

Γ The Gamma function $(\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt)$

(c) Caputo definition

The Caputo definition is given by Eq. (14):

$$ {}_aD_t^q f(t) = \left\{ \begin{array}{cc} \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^{(m)}}{(t-\tau)^{q+1-m}} d\tau & m-1 < q < m \\ \frac{d^m}{dt^m} f(t) & q = m \end{array} \right\} \tag{14} $$

where:

m: The first integer larger than q

Fractional differential equation simulation is not easy as compared with the ordinary differential ones. Approximation and numerical methods are used for solving fractional order differential equations Fractional order control calculus presented by Tustin for the position control of massive objects a half century ago. Provided some of the other researches were presented by Manabe around (1960). However, the fractional-order control was not included in the control engineering because of the major limitations of the possibilities and a lack of adequate amount of mathematical knowledge and computational power at this. The researchers have concluded in the past decades that the (fractional order differential equations) could model diverse systems fuller than integer-order ones and provide an excellent instrument for describing dynamic processes. In fractional order controllers, in addition to parameters of the classical proportional-integral-derivative constants, there are two extra parameters (λ and μ) as discussed in [32]. The parameters λ and μ are the order of s in integral and derivative respectively so a specific algorithm is required to make tuning for the parameters of the FOPID Controller. This will improve the system performance in terms of flexibility and durability better than the classical PID controller.

The differential equation of the FOPID controller is described as:

$$ U(t) = K_P e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t) \tag{15} $$

After the introduction of this definition, it became easy to see that the classical types of PID controller such as integral order PID, PI, or PD become special cases of the most general fractional order PID controller. In other words, the FOPID controller expands the integer-order PID controller from point to plane, as shown in

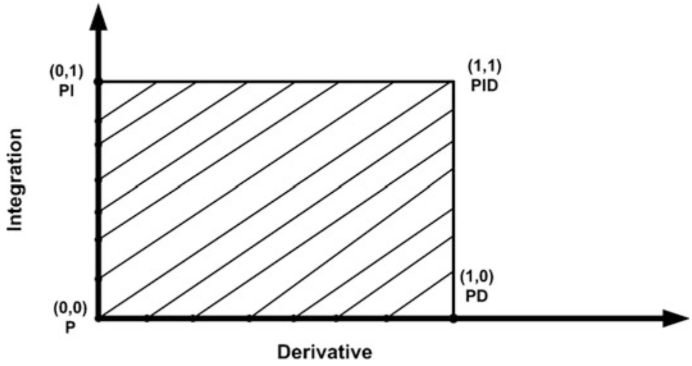


Fig. 2 Schematic view of PID for all probabilities

Fig. 2. Taking Laplace Transform of Eq. (15), the controller expression in s-domain is obtained as:

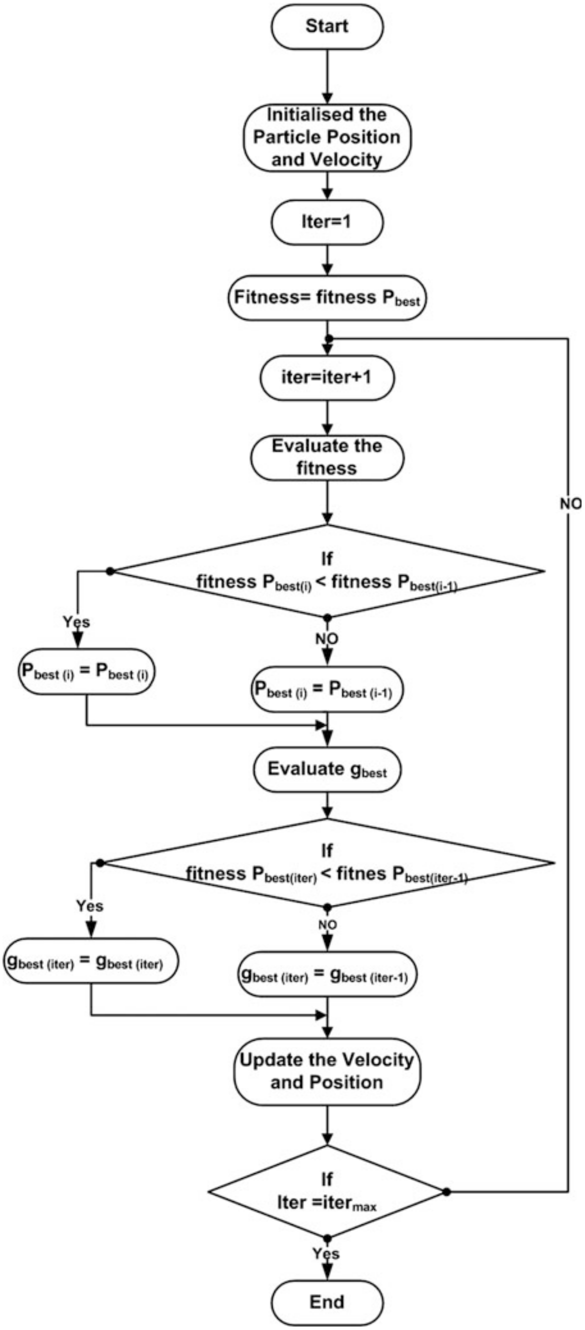
$$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{16}$$

(d) ***Particle Swarm Optimization***

Particle Swarm Optimization (PSO) is a stochastic optimization technique developed by Eberhart and Kennedy in 1995 as given in [33]. The PSO algorithm is inspired by social behavior of bird flocking, animal hoarding, or fish schooling. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles as explained in [34]. PSO has been successfully applied in many areas [35, 36], [37] and [38, 39].

PSO simulates the behavior of bird flocking. When a group of birds flying in the sky searching for the food. The food is located at the specific place through the searching area but not all the birds know where the food is. Each bird estimates a position of the food and the bird which have the least distance from the food position, will follow the group. By iterations, the birds can reach to the food easily. PSO started with random values for the particles and searching for the optimal solution that achieves the minimum values of the objective functions. During each iteration, the best value of the objective functions obtained in each iteration is called local best (*pbest*). The best value of the local best values obtained through the iterations is called global best (*gbest*) as explained in [40]. After finding the local best and global best values in each iteration, the particles update its velocity and position according to Eqs. (17)–(18) respectively as introduced in [33].

Fig. 3 Flowchart of the PSO



$$V_{k+1}^i = wv_k^i + c_1 r_1 (pbest^i - x_k^i) + c_2 r_2 (gbest - x_k^i) \quad (17)$$

$$x_{k+1}^i = x_k^i + v_{k+1}^i \quad (18)$$

where:

v_k^i	Velocity of i^{th} particle at k^{th} iteration
x_k^i	Current position of the i^{th} particle,
r_1 and r_2	random numbers generated uniformly between 0 and 1
c_1	Self-confidence (cognitive) factor
c_2	swarm confidence (social) factor
w	is the inertia weight factor

The 1st term in Eq. (18) represents the effect of the inertia of the particle, the 2nd term represents the particle memory influence, and the 3rd term represents the swarm (society) influence. The flow chart of the procedure is illustrated in Fig. 3. The velocities of the particles on each dimension may be clamped to a maximum velocity V_{max}, which the parameter is specified by the user. If the sum of the accelerations causes the velocity on that dimension to exceed V_{max}, then the velocity is limited to V_{max}.

4.1 Control Strategy:

Various control strategies are applied to the IP. PID controller is one of the most popular ones among them. Some researches concentrate on swinging up the angle in vertical upright without considering the dynamics of the cart. Employing two PID controllers to stabilize the cart position and swinging up the pendulum angle was presented in [41]. The tuning of two PID controllers is atedious effort by using conventional methods. Recently, the artificial intelligent computational techniques were used to tune the PID parameters. Linear Quadratic Regulator (LQR) is suggested as a replacement for one of the two PID controllers [42] for swinging up the pendulum angle. LQR design is depending on solving the Riccati equation. Riccati equation is based on two designing matrices Q and R which have seventeen parameters.

They must be positive definite and positive semi-definite respectively. Tuning of seventeen parameters of LQR, feedforward gain, and in addition to the three parameters of FOPID or PID controller is time-consuming and more complex. The reduction of tuning parameters is one of the most important topics in the computational evolutionary field [43]. A reduced order LQR with FOPID and with PID controller was presented in this chapter. In this proposed technique the tuning parameters were reduced to be thirteen instead of twenty-one in the case of using PID and reduced from twenty-three to fifteen in the case of using FOPID. It simplifies the optimization problem and has a great effect on the computation time.

Designing the gains of LQR is mainly depending on the choice of the Q and R matrices which selected by the designer. This may take a long time to obtain the best values of the two matrices parameters. Therefore, trial and error method is time-consuming. The process of selecting the matrices becomes more difficult when the system has a large dimension of system state space matrices. In this chapter, an evolutionary optimization based RLQR controller, FOPID controller and compensating gain (K_f) design for an inverted pendulum system is introduced and compared to another one with PID controller. The weighting matrices Q and R are positive semi-definite and positive-definite respectively. This means that the term $x^T Qx$ in Eq. (7) is always positive or zero at each time t for all functions x(t). Furthermore, the second term in Eq. (7) is always positive at each time t for all values of u(t). Therefore J is always positive at each time. To ensure that the weighting matrices Q and R are positive semi-definite and positive-definite respectively and to reduce the dimension of Q and R matrices as explained in [43], it is assumed that:

$$Q = W^T * W \tag{19}$$

$$R = V^T * V \tag{20}$$

where:

W amatrixof m * n dimension
V matrix of k * l dimension

In this chapter, It is assumed that: m = 2, n = 4, k = 1, l = 1.

So the modified Riccati equation is given by Eq. (21):

$$PA + A^T P - PB(V^T * V)^T B^T P + (W^T * W) = 0 \tag{21}$$

The proposed optimization technique is used to tune the W and V matrices to guarantee that Q and R will be positive semi-definite and positive-definite respectively. After that, the modified Riccati equation is solved to find the reduced Linear Quadratic Regulator gains according to the following equation:

$$K = (V^T * V)^{-1} B^T P \tag{22}$$

Hint: The modified Riccati equation can be solved in Matlab by using the command lqr (A, B, Q, R)
where:
A and B: System State-space matrices.
Q and R: weighting matrices of RLQR gains.
K: RLQR gain ($K = [K_1 K_2 K_3 K_4]$)

The four gains (K_1, K_2, K_3, K_4) of the RLQR will be calculated. The states that affect the pendulum angle are cart velocity, pendulum position, and the pendulum velocity so the feedback from the angle output having these states. Hence, there is no necessity to use the controller gain K_1 which controls the cart position state.

4.2 Problem Formulation

The optimization problem has 15 variables $(K_P, K_i, K_d, \lambda, \mu, K_F, Wmatrix$ and $Vmatrix)$ in the case of using FOPID controller. Also, it has 13 variables $(K_P, K_i, K_d, K_F, Wmatrix$ and $Vmatrix)$ in the case of using PID controller. PSO run to find the best values for all variables that achieve the minimum Overshoot, Steady state error and Settling Time. The difficulty of the algorithm is to achieve the minimum Overshoot, Steady state error and settling time for both the cart position and the pendulum angle at the same time as presented in Eq. (23). In this chapter, the algorithm runs according to a Multi-objective function that has the constraints which give an acceptable response to the two outputs.

The global _best_ Fitness is determined according to:

$$Global - best - fitness = min(e_{ss},\ o.s,\ T_s) \tag{23}$$

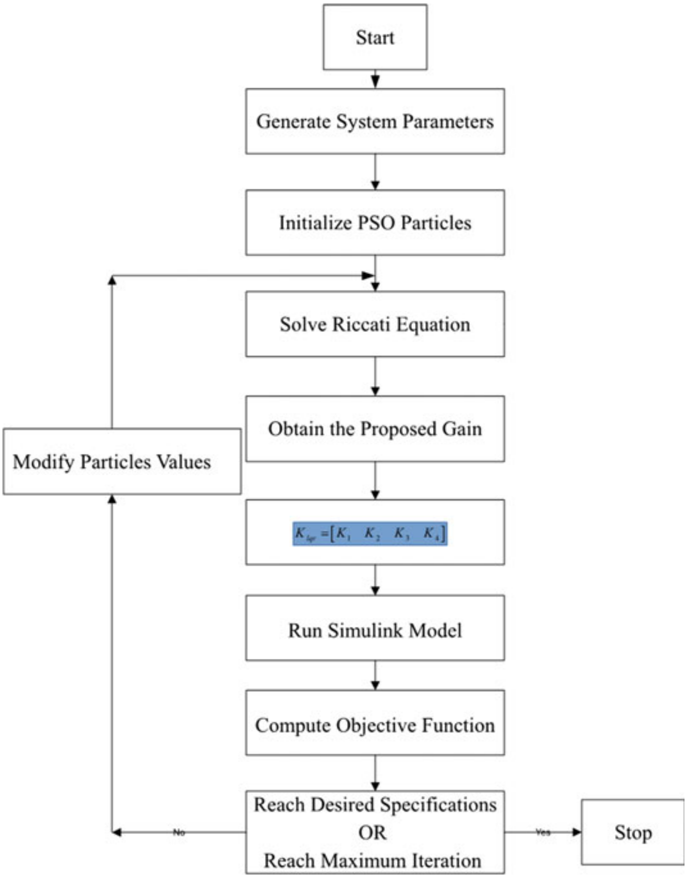


Fig. 4 The flow chart of the proposed control strategy

The steps of the proposed algorithm are illustrated in Fig. 4 as follow:

Step1: Generating the inverted pendulum system parameters.
Step2: Initialize values of PSO particles.
Step3: Solve Riccati equation.
Step4: Obtain the proposed RLQR gain.
Step5: Consider last three values only of the RLQR gain.
Step6: Run the Simulink model.
Step7: Computing the objective function of the algorithm.
Step8: Check achieving minimum value for steady state error, settling time and overshoot.
Step9: Stop the algorithm when achieving minimization for the objective function or exceeding the maximum iteration number.

5 Simulation and Results:

The IP is among the most difficult systems to control in the field of control engineering. It consists of two control loops as presented in Fig. 5. For the purpose of effective comparison, the system is equipped with FOPID with RLQR and PID with RLQR. The first one is FOPID with RLQR (FOPID/RLQR) controller. The (FOPID/RLQR) gains are responsible for stabilizing the cart and swinging up the pendulum to be in a vertical position. The second one is PID with RLQR (PID/RLQR) controller. In the first control loop there is a feedback signal from the cart position output to the summing point with the input signal (unit step) then a feed forward controller is applied. The feedback in the second control loop is extremely different as the factors that effect on swinging up the angle are the speed of the cart, the pendulum angle and the angular velocity of the pendulum. A feed-forward estimator is used to estimate the speed of the cart and the angular velocity of the pendulum as presented in the model as illustrated in Fig. 5. The estimations for the speed and angular velocity are collected and then introduced into the inverted pendulum. A feed forward amplifier (K_f) affects the dynamic response of the inverted pendulum.

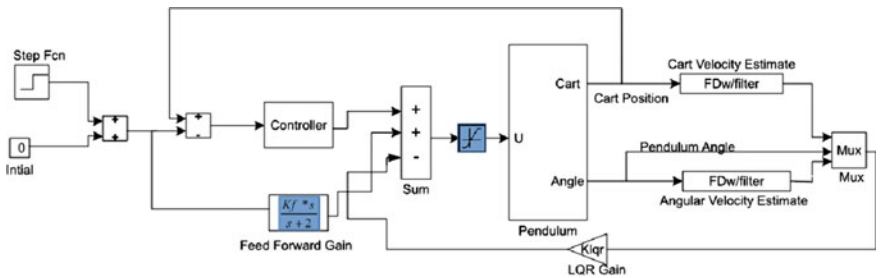


Fig. 5 Block diagram of inverted pendulum system

Table 2 The obtained gains from PSO

	K_P	K_i	K_d	λ	μ	K_f
FOPID/RLQR	10	19.4	−2.1	−1	1.6	55.7
PID/RLQR	10	0.009	−29.2320	1	1	−50

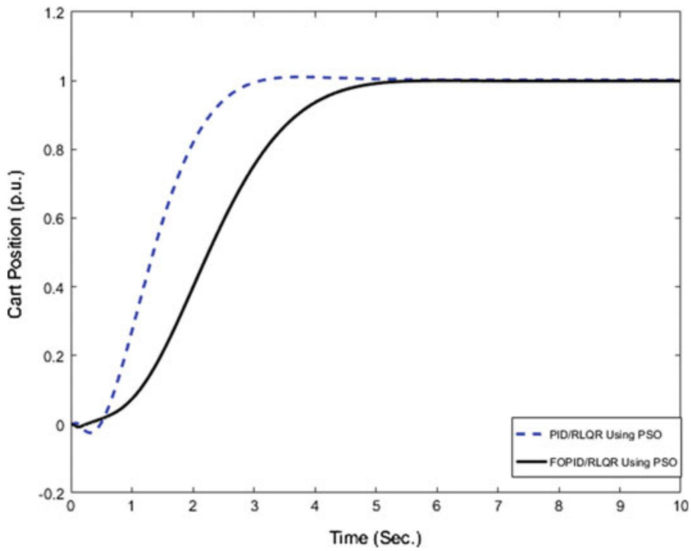


Fig. 6 System response of cart position with different controllers

The obtained gains from PSO are given in Table 2. The results are presented in Figs. 6 and 7.

The RLQR gains for both FOPID/RLQR and PID/RLQR are K_{lqr} = [−50.03 −353.79 −85.9] and K_{lqr}=[−62.03 −251.45 −72.69] respectively.

The simulation results illustrate that both controllers had succeeded in stabilizing the cart position and swinging up the pendulum angle effectively. Although, FOPID/RLQR controller stabilized the angle of the inverted pendulum with less over shoot and under shoot than PID/RLQR. Moreover, PID/RLQR controller stabilized the inverted pendulum position with less settling time than FOPID/RLQR. Tables 3 and 4 presented the output specifications of the cart position and the pendulum angle respectively.

To ensure that the proposed controllers are robust, three robustness tests are performed to measure the effectiveness of the system. The cases are as follows:

(a) Set points with different amplitudes.
(b) Increasing the step input with different ranges.
(c) System parametersperturbation.

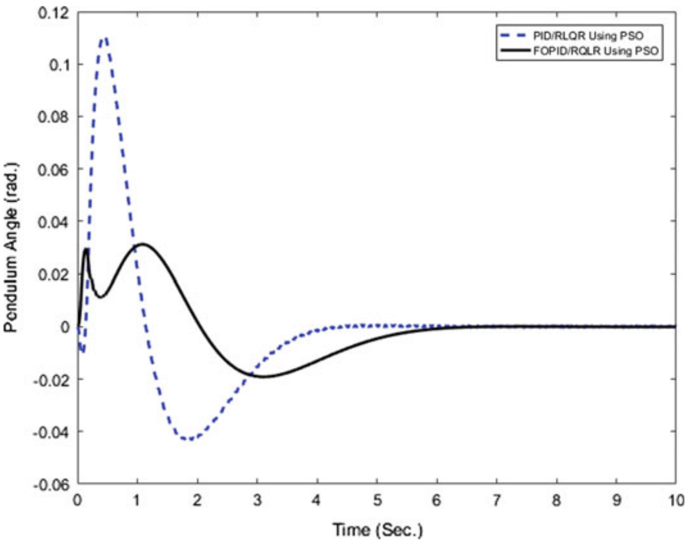


Fig. 7 System response of pendulum angle with different controllers

Table 3 The output specifications of the cart position

Time response specifications	PID/RLQR using PSO	FOPID/RLQR using PSO
Rise time (s)	1.5638	2.58
Settling time (s)	2.7873	4.6234
Overshoot (%)	1.2%	0.00072206%
Undershoot (%)	2.4988%	0.8306%

Table 4 The output specifications of the pendulum angle

Time response specifications	PID/RLQR using PSO	FOPID/RLQR using PSO
Maximum value (rad.)	0.1109	0.0312
Minimum value (rad.)	−0.0432	−0.0191
Settling time (s)	3.5758	5.6728

(a) **Set points with different amplitudes**

A series of set points as shown in Fig. 8 are applied to the IP system to validate the effectiveness of the proposed controllers. It is evident from the results the effectiveness of the controllers in stabilizing the cart position and swinging up the pendulum angle. Both controllers are succeeded in stabilizing the cart position and swinging up the pendulum angle as shown in Figs. 9 and 10.

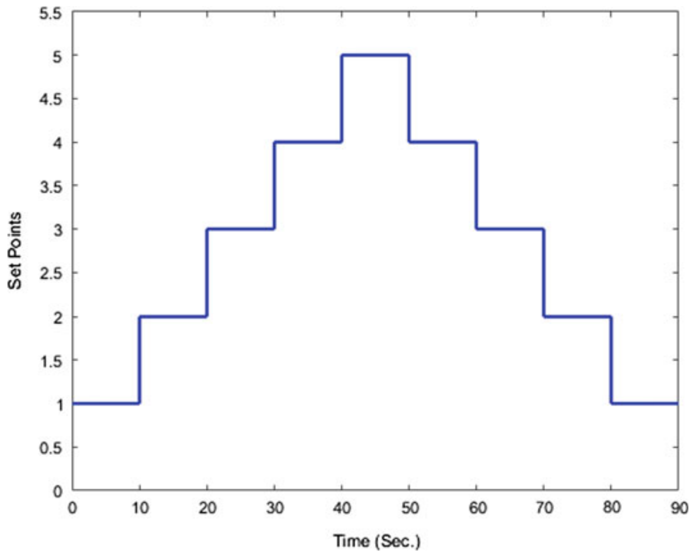


Fig. 8 Series of set points with different amplitudes

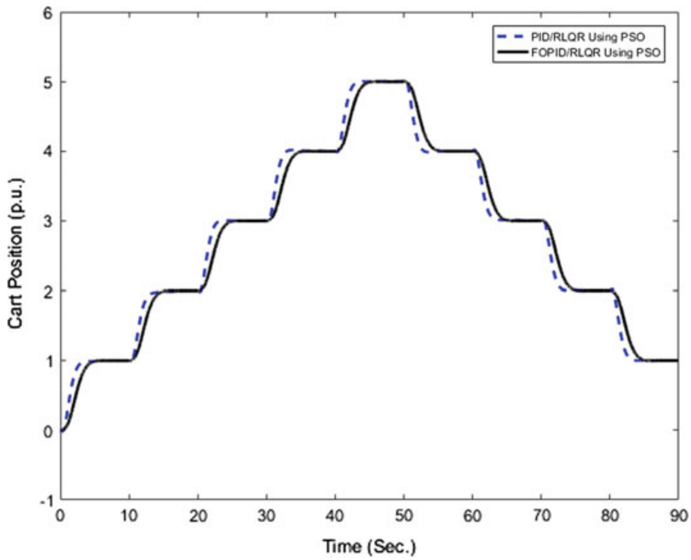


Fig. 9 System cart position response of different controllers with series of set points

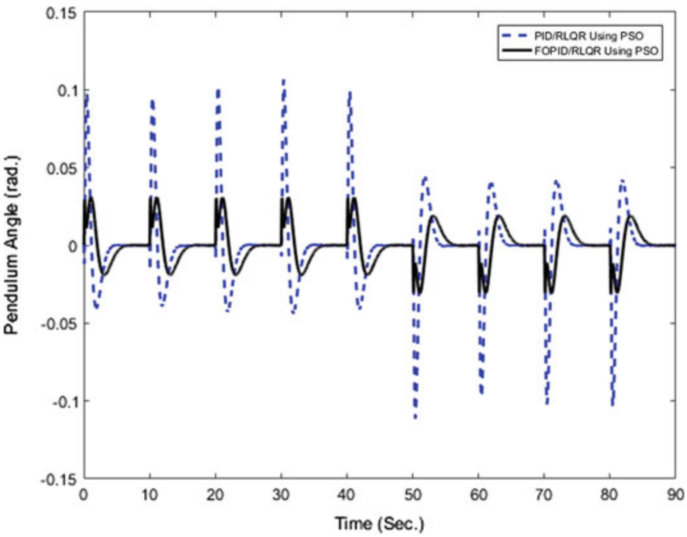


Fig. 10 System pendulum angle response of different controllers with series of set points

Robustness Verification

(a) **Increasing the step input with different ranges.**

To measure the effectiveness of the system, the step input is increased with different ranges. The controller which cannot withstand the increasing of the step input will be not a robust controller design. Firstly, step input is increased with 10% then it will be increased by 20%. Figures 11 and 12 illustrate dynamic responses of the cart position and the pendulum angle respectively when the step input increased with 10%. It is noted that the two proposed controllers succeeded in keeping balance to the inverted pendulum system when the step input increased with 10%.

The step input increased with 20% from its value. Although PID/RLQR controller using PSO can stabilize the inverted pendulum system with 10% increasing of the step input, it cannot keep the system in balance with 20% increase. FOPID/RLQR controllers using PSO only the controller that succeeded in the robustness test related to the increasing of the step input. Figure 13 presented the response of the cart position. Figures 14 and 15 illustrated the pendulum angle response in case of PID/RLQR and FOPID/RLQR using PSO.

(b) **System parameters perturbation.**

This test is one of the most important tests in checking the robustness of the inverted pendulum system. In this test, inverted pendulum parameters are increased with 10% from their values. Each controller will be applied to the nonlinear system with the new parameters. If the FOPID/RLQR controller is succeeded in controlling the same system with the new parameters, the controller will be very robust. Table 5 Illustrates

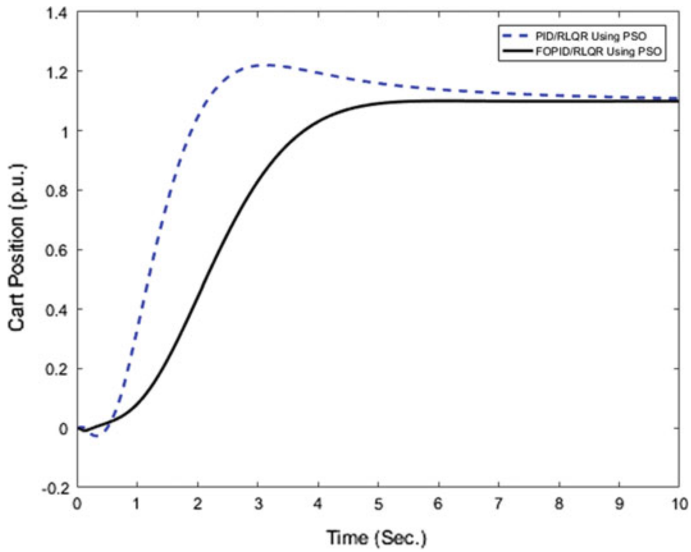


Fig. 11 Dynamic response of cart position with 10% increase of step input

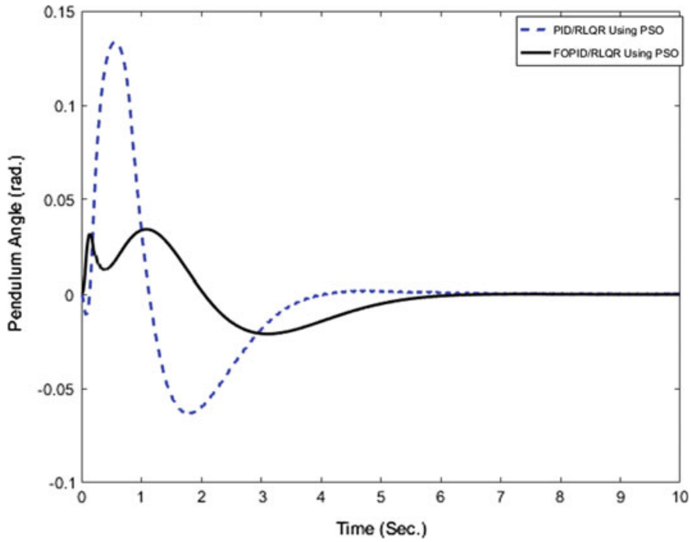


Fig. 12 Dynamic response of pendulum angel with 10% increase of step input

the IP system parameters with 10% increasing. It is noted that the FOPID/RLQR using PSO can stabilize the cart position and swing up the pendulum angle while PID/RLQR using PSO failed in balancing the system (Figs. 16, 17 and 18)

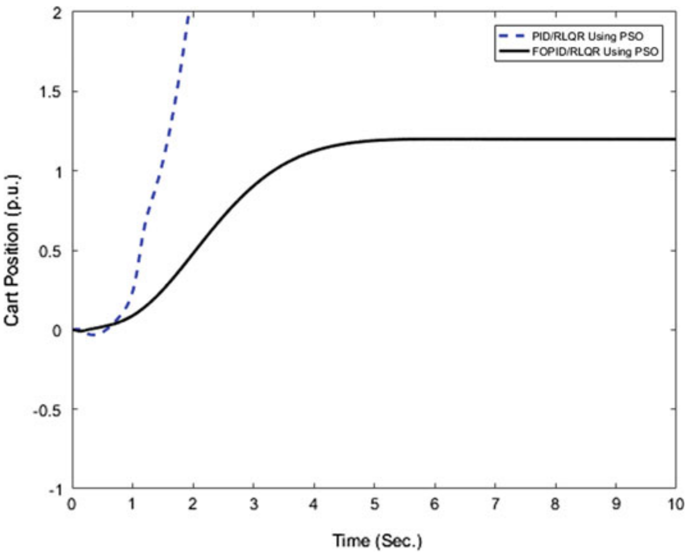


Fig. 13 Dynamic response of cart position with 20% increase of step input

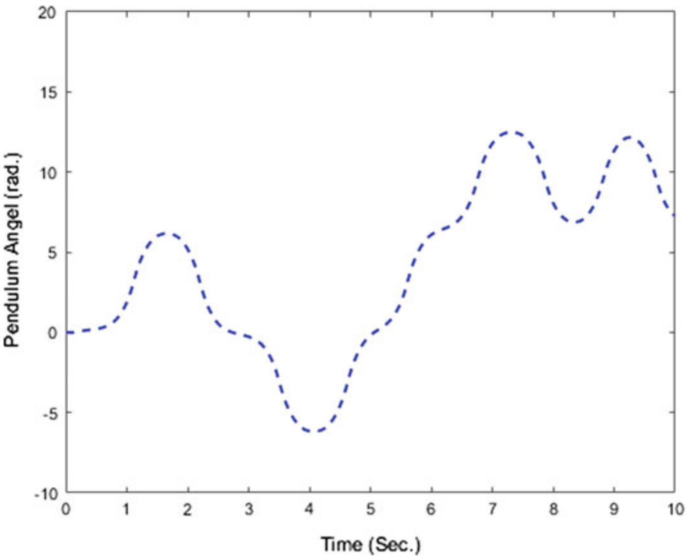


Fig. 14 Dynamic response of pendulum angel with 20% increase of step input using PID/RLQR controller

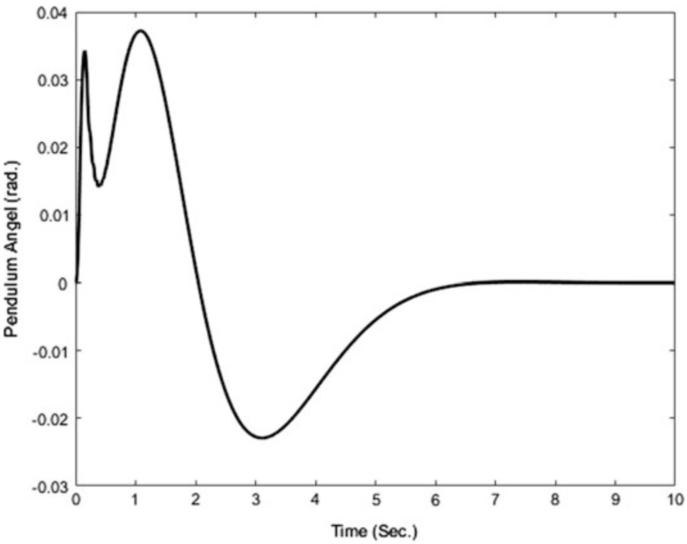


Fig. 15 Dynamic response of pendulum angel with 20% increase of step input using FOPID/RLQR controller

Table 5 Parameters of the inverted pendulum system with 10% increasing

System parameters	Mass of the cart (M)	Mass of the pendulum (m)	Distance from the pivot to the mass center of the pendulum (l)
Parameters values	0.455	0.21	0.61/2
Increasing with 10%	0.5005	0.2310	0.3355

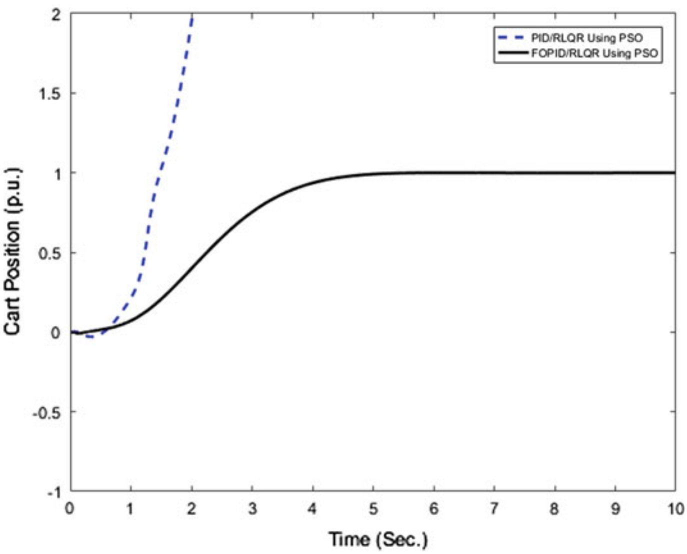


Fig. 16 Dynamic response of cart position (10% Increase of System Parameters)

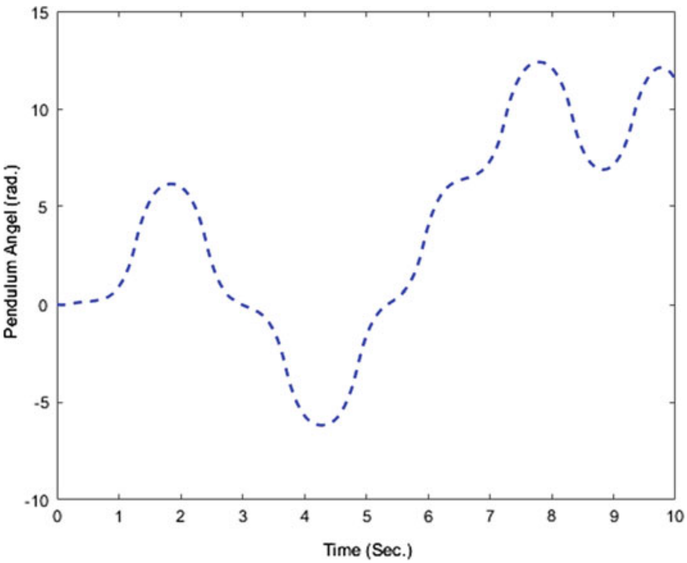


Fig. 17 Dynamic response of pendulum angle using PID/RLQR controller (10% increase of system parameters)

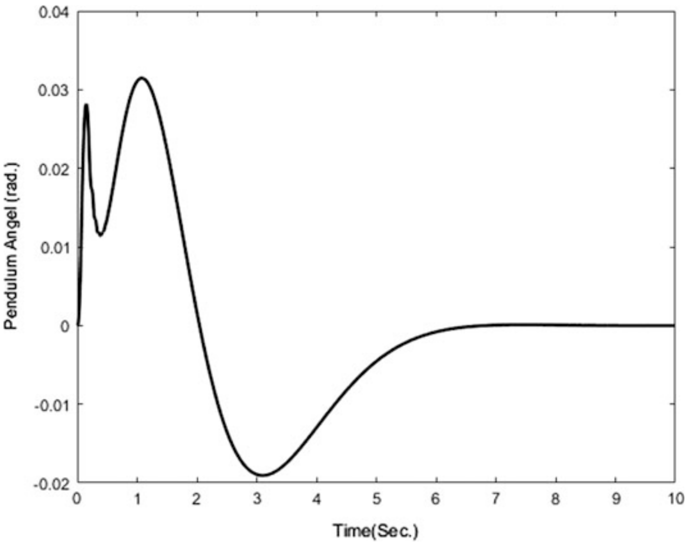


Fig. 18 Dynamic response of pendulum angle using FOPID/RLQR Controller (10% increase of system parameters)

6 Conclusion

In this chapter, a new effective control method integrating both PSO-based Fractional Order Proportional Integral Derivative (FOPID) and Reduced Linear Quadratic Regulator (RLQR) was introduced. This chapter demonstrates that PSO can solve searching and tune the controller parameters more efficiently than conventional ones. The inherited instabilities in the inverted pendulum were treated effectively. Modeling of the inverted pendulum was performed using MATLAB. The simulation was conducted in order to cover the full range of operating conditions and severe disturbances. The application of the proposed control method showed its ability to stabilize the inverted pendulum. The obtained results are very promising.

7 Future Work

In control system engineering there are a lot of techniques for balancing the inverted pendulum system. PSO is one of the evolutionary computational techniques which proposed in this chapter. Balancing the inverted pendulum system with other techniques is left as a future work. Adaptive control and using other computational techniques can be used for this purpose.

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