A Three-Dimensional No-Equilibrium Chaotic System: Analysis, Synchronization and Its Fractional Order Form

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Abstract Recently, a new classification of nonlinear dynamics has been introduced by Leonov and Kuznetsov, in which two kinds of attractors are concentrated, i.e. self-excited and hidden ones. Self-excited attractor has a basin of attraction excited from unstable equilibria. So, from that point of view, most known systems, like Lorenz's system, Rössler's system, Chen's system, or Sprott's system, belong to chaotic systems with self-excited attractors. In contrast, a few unusual systems such as those with a line equilibrium, with stable equilibria, or without equilibrium, are classified into chaotic systems with hidden attractor. Studying chaotic system with hidden attractors has become an attractive research direction because hidden attractors play an important role in theoretical problems and engineering applications. This chapter presents a three-dimensional autonomous system without any equilibrium point which can generate hidden chaotic attractor. The fundamental dynamics properties of such no-equilibrium system are discovered by using phase portraits,

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Lyapunov exponents, bifurcation diagram, and Kaplan–Yorke dimension. Chaos synchronization of proposed systems is achieved and confirmed by numerical simulation. In addition, an electronic circuit is implemented to evaluate the theoretical model. Finally, fractional-order form of the system with no equilibrium is also investigated.

Keywords Chaos · Hidden attractor · No-Equilibrium · Lyapunov exponents · Bifurcation · Synchronization · Circuit · SPICE

1 Introduction

In 1963, Lorenz found a chaotic system when studying a model for atmospheric convection [50]. The most well-known feature of a chaotic system is the sensitivity on initial conditions, named "butterfly effect". This means that a small variation on initial conditions of a system will generate a totally different chaotic trajectory. After the invention of Lorenz, there has been significant interest in chaos theory, chaotic systems, and chaos-based applications [5–8, 18, 19, 67, 72, 104]. Especially, various new chaotic systems have been discovered such as Rössler's system [61], Arneodo's system [4], Chen's system [18], Lü's system [51], Vaidyanathan's systems [79, 85, 87, 88, 90], time-delay systems [11], nonlinear finance system [75], four-scroll chaotic attractor [2] and so on [58, 82]. Complex behaviors of chaotic system were used in different applications. True random bits were generated by using a double-scroll chaotic attractor [103]. Volos et al. controlled autonomous mobile robots via chaotic path planning [96]. Han et al. implemented a fingerprint images encryption scheme based on chaotic attractors [27]. Hoang and Nakagawa proposed applications of time delay systems in secure communication due to their complex dynamics [31]. Application of synchronization of Chua's circuits with multi-scroll attractors in communications was introduced in [24]. In addition, Akgul et al. presented engineering applications of a new four-scroll chaotic attractor [2].

When studying chaotic systems, their equilibrium points play important role [69, 98]. As have been known, most reported chaotic systems have a countable number of equilibrium points [68]. Therefore, chaos in these systems can be proved by using Shilnikov criteria where at least one unstable equilibrium for emergence of chaos is required [66]. However, a few chaotic systems without equilibrium have been proposed recently [34]. We cannot apply the Shinikov method for verifying chaos in such systems because they have neither homoclinic nor heteroclinic orbits. Chaotic systems without equilibrium are categorized as systems with "hidden attractor" and have been received significant attention [44, 46].

In this chapter, a novel system is introduced and its chaotic attractors are displayed. The special is that such new system does not have equilibrium points. This chapter is organized as follows. The related works are summarized in the next section. The model of the new system is proposed in Sect. 3. Dynamics and properties of the new system are investigated in Sect. 4. The adaptive anti-synchronization scheme is studied in Sect. 5. Section 6 presents a circuital implementation of the theoretical model. Fractional-order form of the new no-equilibrium system is proposed in Sect. 7. Finally, conclusions are drawn in the last section.

2 Related Work

The terminology "hidden attractor" has been proposed recently when Leonov and Kuznetsov introduced types of attractors: self-excited attractors and hidden attractors [42, 44, 46, 47]. A self-excited attractor has a basin of attraction that is excited from unstable equilibria. In contrast, hidden attractor cannot be found by using a numerical method in which a trajectory started from a point on the unstable manifold in the neighbourhood of an unstable equilibrium [33]. "Hidden attractor" plays a vital role in nonlinear theory and practical problems [41, 46, 54, 59, 65]. Therefore, various noticeable results relating to this topic has been reported in recent years. The presence of hidden attractors has witnessed in a smooth Chua's system [48], in mathematical model of drilling system [45], in nonlinear control systems [43], or in a multilevel DC/DC converter [107]. Hidden attractors appear in a 4-D Rikitake dynamo system [94], in 5-D hyperchaotic Rikitake dynamo system [92], in a 5D Sprott B system [52] or in a chaotic system with an exponential nonlinear term [56]. Other works on hidden attractors were introduced in [16, 35, 64, 69] and references therein.

Interesting that chaotic systems without equilibrium belong to a class of nonlinear systems with "hidden attractor" [34]. A few three-dimensional chaotic systems without equilibrium points have been discovered. Wei applied a tiny perturbation into the Sprott D system to create a new system with no equilibia [99]. Wang and Chen proposed a no-equilibrium system when constructing a chaotic system with any number of equilibria [98]. Especially, Jafari et al. found catalog of chaotic flows with no equilibria [34].

Moreover, four-dimensional chaotic systems without equilibrium points have been investigated recently. Based on a memristive device, a novel four-dimensional system has been proposed [57]. The peculiarity of the memristive system is that it does not display any equilibria and exhibits periodic, chaotic, and also hyperchaotic dynamics. Vaidyanathan has presented analysis, control and synchronization of a ten-term novel 4-D highly hyperchaotic system with three quadratic nonlinearities [81]. The author have been shown that it is a novel hyperchaotic system does not have any equilibrium point. Dynamics, synchronization and SPICE implementation of a memristive system with hidden hyperchaotic attractor have been reported in [55]. Investigation of new systems without equilibrium is still an attractive topic and should receive further attention.

3 Model of the No-Equilibrium System

Jafari et al. introduced a list of simple chaotic flows without equilibrium (denoted NE_1-NE_{14}) [34]. Interestingly, the system NE_8 can display coexisting hidden attractors [64]. The system NE_8 is described as

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x - yz \\ \dot{z} = xy + 0.5x^2 - a, \end{cases} \tag{1}$$

where x, y, z are state variables and a is a positive parameter. The Lyapunov exponents of system NE_8 in (1) are $\lambda_1 = 0.0314$, $\lambda_2 = 0$, $\lambda_3 = -10.2108$ and the Kaplan–Yorke dimension is $D_{KY} = 2.0031$ (for $a = 1.3$) [34].

Based on system NE_8 in (1), in this work we study a new system in the following form

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x - yz \\ \dot{z} = xy + ax^2 + by^2 - c, \end{cases} \tag{2}$$

where a, b, c are three positive parameters and $c \neq 0$. A detailed study of dynamics and properties of no-equilibrium system in (2) is presented in the next section.

4 Dynamics and Properties of the No-Equilibrium System

The equilibrium points of the system in (2) are found by solving $\dot{x} = 0$, $\dot{y} = 0$, and $\dot{z} = 0$, that is

$$y = 0, \tag{3}$$

$$-x - yz = 0, \tag{4}$$

$$xy + ax^2 + by^2 - c = 0, \tag{5}$$

From (3), (4), we have $x = y = 0$. Therefore Eq. (5) is inconsistent. In the other words, there is no real equilibrium in the system (2).

We consider the system (2) for the selected parameters $a = 0.5$, $b = 0.1$, $c = 1.3$ and the initial conditions are

$$(x(0), y(0), z(0)) = (0, 0.1, 0). \tag{6}$$

Lyapunov exponents, which measure the exponential rates of the divergence and convergence of nearby trajectories in the phase space of the chaotic system [68, 72], are calculated by using the algorithm in [102]. As a result, the Lyapunov exponents of the system (2) are

$$\lambda_1 = 0.0453, \lambda_2 = 0, \lambda_3 = -3.2903. \tag{7}$$

The non-equilibrium chaotic system is dissipative because the sum of the Lyapunov exponents is negative. It is worth noting that this non-equilibrium system can be classified as a nonlinear system with hidden strange attractor because its basin of attractor does not contain neighbourhoods of equilibria [44, 46]. The 2-D and 3-D projections of the chaotic attractors without equilibrium in this case are presented in Figs. 1, 2, 3 and 4.

It has been known that the Kaplan–Yorke dimension, which presents the complexity of attractor [23], is defined by

$$D_{KY} = j + \frac{1}{\left|\lambda_{j+1}\right|} \sum_{i=1}^{j} \lambda_i, \tag{8}$$

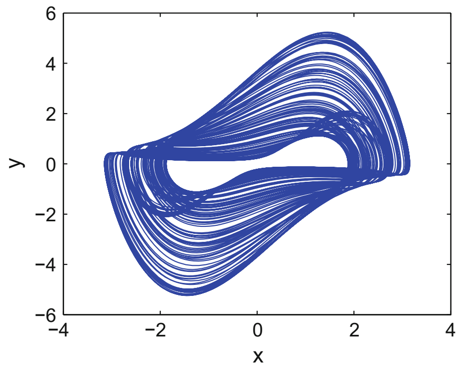


Fig. 1 2-D projection of the chaotic system without equilibrium in (2) in the (x, y)-plane

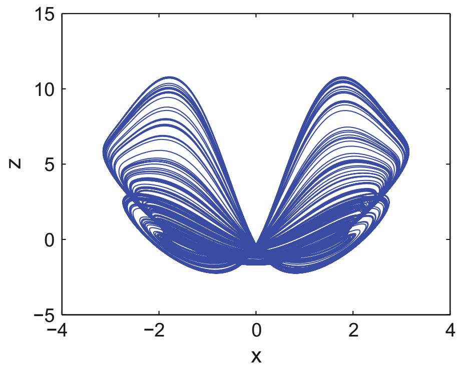


Fig. 2 2-D projection of the chaotic system without equilibrium in (2) in the (x, z)-plane

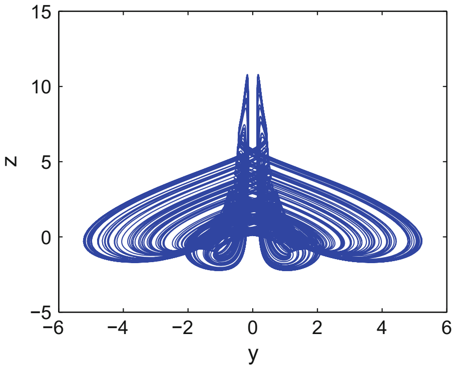


Fig. 3 2-D projection of the chaotic system without equilibrium in (2) in the (y, z)-plane

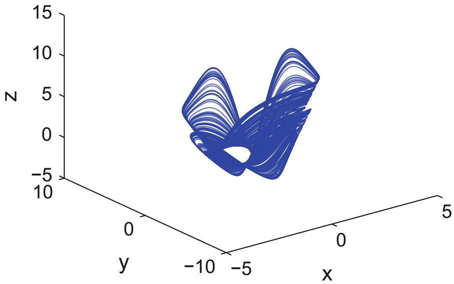


Fig. 4 3-D projection of the chaotic system without equilibrium in (2) in the (x, y, z)-space

where j is the largest integer satisfying $\sum_{i=1}^{j} \lambda_i \geq 0$ and $\sum_{i=1}^{j+1} \lambda_i < 0$. Thus, the calculated fractional dimension of no-equilibrium system in (2) when $a = 0.5$, $b = 0.1$, $c = 1.3$ is

$$D_{KY} = 2 + \frac{\lambda_1 + \lambda_2}{|\lambda_3|} = 2.0138. \tag{9}$$

Equation (9) indicates a strange attractor.

It is easy to see that the system in (2) has rotational symmetry with respect to the z-axis as evidenced by their invariance under the transformation from (x, y, z) to $(-x, -y, z)$. Therefore, any projection of the attractor has symmetry around the origin. In the other words there is a symmetric coexisting attractor as shown in Fig. 2. In addition, the attractor of the system is displayed in Fig. 4. The bifurcation diagrams of system in (2) illustrated in Fig. 5 indicate the presence of muti-stability. For example, there are coexisting attractors when $b = 0.15$ as shown in Figs. 6, 7, 8, and 9.

Fig. 5 Bifurcation diagram of the chaotic system without equilibrium (2) when varying the value of the parameter b for $a = 0.5$, $c = 1.3$, and the initial conditions $(x(0), y(0), z(0)) = (0, 0.1, 0)$ (*blue*), and $(x(0), y(0), z(0)) = (0, -0.1, 0)$ (*red*)

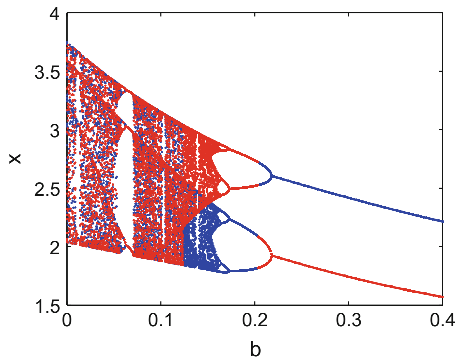


Fig. 6 Coexisting attractors of the chaotic system without equilibrium in (2) in the (x, y)-plane

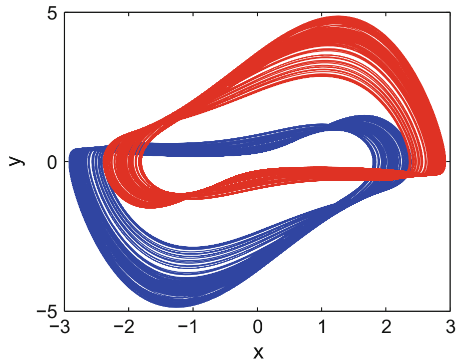
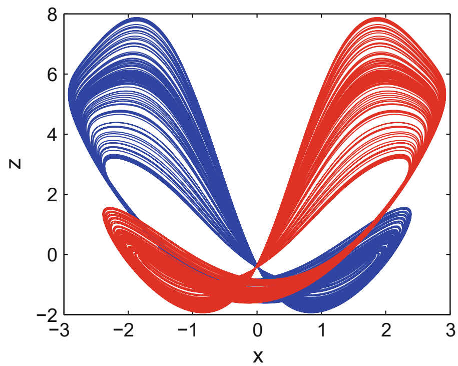


Fig. 7 Coexisting attractors of the chaotic system without equilibrium in (2) in the (x, z)-plane for the initial conditions $(x(0), y(0), z(0)) = (0, 0.1, 0)$ (*blue*), and $(x(0), y(0), z(0)) = (0, -0.1, 0)$ (*red*)



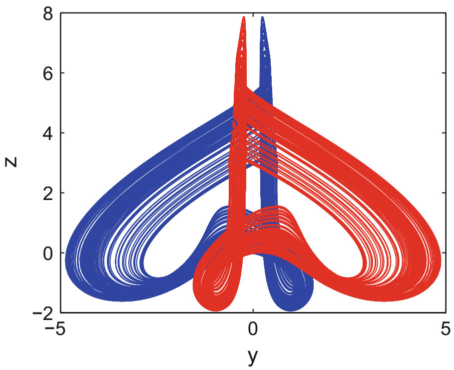


Fig. 8 Coexisting attractors of the chaotic system without equilibrium in (2) in the (y, z)-plane for the initial conditions $(x(0), y(0), z(0)) = (0, 0.1, 0)$ (*blue*), and $(x(0), y(0), z(0)) = (0, -0.1, 0)$ (*red*)

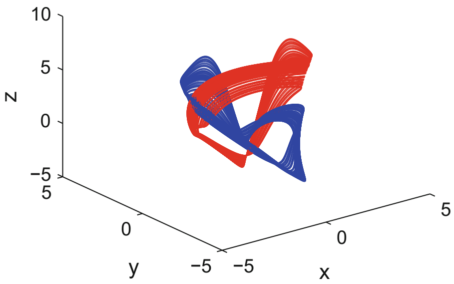


Fig. 9 Coexisting attractors of the chaotic system without equilibrium in (2) in the (x, y, z)-space for the initial conditions $(x(0), y(0), z(0)) = (0, 0.1, 0)$ (*blue*), and $(x(0), y(0), z(0)) = (0, -0.1, 0)$ (*red*)

5 Adaptive Anti-synchronization of the No-Equilibrium System

The most vital practical feature relating to chaotic systems is the possibility of synchronization of two coupled chaotic systems [12, 21, 38, 53]. Synchronization of nonlinear systems has been discovered extensively in literature [17, 24, 39, 59, 70, 80, 83, 84, 91, 106]. Some important obtained results can be listed as follows: synchronized states in a ring of mutually coupled self-sustained nonlinear electrical oscillators [101], ragged synchronizability of coupled oscillators [71], various synchronization phenomena in bidirectionally coupled double-scroll circuits [95], observer for synchronization of chaotic systems with application to secure data transmission was studied in [1], or shape synchronization control [32]. Moreover, various kinds of synchronizations have been reported, for example lag synchronization [60], frequency synchronization [3], projective-anticipating synchronization [30], anti-synchronization [78], adaptive synchronization [86, 93], hybrid chaos synchronization [39], generalized projective synchronization [89], fuzzy control-based function synchronization [15] or fast synchronization [37] etc. It is interesting that fuzzy adaptive synchronization of uncertain fractional-order chaotic systems has been introduced in [14].

In this Section, we study the adaptive anti-synchronization of identical no-equilibrium systems with three unknown parameters. Here the no-equilibrium system in (2) is considered as the master system as

$$\begin{cases} \dot{x}_1 = y_1 \\ \dot{y}_1 = -x_1 - y_1 z_1 \\ \dot{z}_1 = x_1 y_1 + a{x_1}^2 + b{y_1}^2 - c, \end{cases} \tag{10}$$

in which x_1, y_1, z_1 are state variables. The slave system is considered as the controlled no-equilibrium system and its dynamics is described as

$$\begin{cases} \dot{x}_2 = y_2 + u_x \\ \dot{y}_2 = -x_2 - y_2 z_2 + u_y \\ \dot{z}_2 = x_2 y_2 + a{x_2}^2 + b{y_2}^2 - c + u_z, \end{cases} \tag{11}$$

where x_2, y_2, z_2 are the states of the slave system. Here the adaptive controls are u_x, u_y, and u_z. These controls will be designed for the anti-synchronization of the master and slave systems. $A(t)$, $B(t)$ and $C(t)$ are used in order to estimate unknown parameters a, b and c.

The anti-synchronization error between no-equilibrium systems (10) and (11) is given by the following relation

$$\begin{cases} e_x = x_1 + x_2 \\ e_y = y_1 + y_2 \\ e_z = z_1 + z_2. \end{cases} \tag{12}$$

As a result, the anti-synchronization error dynamics is described by

$$\begin{cases} \dot{e}_x = e_y + u_x \\ \dot{e}_y = -e_x - \left(y_1 z_1 + y_2 z_2\right) + u_y \\ \dot{e}_z = \left(x_1 y_1 + x_2 y_2\right) + a\left({x_1}^2 + {x_2}^2\right) + b\left({y_1}^2 + {y_2}^2\right) - 2c + u_z. \end{cases} \tag{13}$$

Our aim is to construct the appropriate controllers u_x, u_y, u_z to stabilize system (13). Therefore, we propose the following controllers for system (13):

$$\begin{cases} u_x = -e_y - k_x e_x \\ u_y = e_x + \left(y_1 z_1 + y_2 z_2\right) - k_y e_y \\ u_z = -\left(x_1 y_1 + x_2 y_2\right) - A(t)\left({x_1}^2 + {x_2}^2\right) \\ \qquad -B(t)\left({y_1}^2 + {y_2}^2\right) + 2C(t) - k_z e_z. \end{cases} \tag{14}$$

in which k_x, k_y, k_z are positive gain constants for each controllers and the estimate values for unknown system parameters are $A(t)$, $B(t)$, and $C(t)$. The update laws for the unknown parameters are determined as

$$\begin{cases} \dot{A} = e_z\left(x_1{}^2 + x_2{}^2\right) \\ \dot{B} = e_z\left(y_1{}^2 + y_2{}^2\right) \\ \dot{C} = -2e_z. \end{cases} \tag{15}$$

Then, the main result of this section will be introduced and proved.

Theorem 15.1 *If the adaptive controller (14) and the updating laws of parameter (15) are chosen, the anti-sychronization between the master system (10) and the slave system (11) is achieved.*

Proof It is noting that the parameter estimation errors $e_a(t)$, $e_b(t)$ and $e_c(t)$ are given as

$$\begin{cases} e_a(t) = a - A(t) \\ e_b(t) = b - B(t) \\ e_c(t) = c - C(t). \end{cases} \tag{16}$$

Differentiating (16) with respect to t, we have

$$\begin{cases} \dot{e}_a(t) = -\dot{A}(t) \\ \dot{e}_b(t) = -\dot{B}(t) \\ \dot{e}_c(t) = -\dot{C}(t). \end{cases} \tag{17}$$

Substituting adaptive control law (14) into (13), the closed-loop error dynamics is defined as

$$\begin{cases} \dot{e}_x = -k_x e_x \\ \dot{e}_y = -k_y e_y \\ \dot{e}_z = (a - A(t))\left(x_1{}^2 + x_2{}^2\right) \\ \qquad + (b - B(t))\left(y_1{}^2 + y_2{}^2\right) - 2(c - C(t)) - k_y e_y \end{cases} \tag{18}$$

Then substituting (16) into (18), we have

$$\begin{cases} \dot{e}_x = -k_x e_x \\ \dot{e}_y = -k_y e_y \\ \dot{e}_z = e_a(t)\left(x_1{}^2 + x_2{}^2\right) + e_b(t)\left(y_1{}^2 + y_2{}^2\right) - 2e_c(t) - k_z e_z. \end{cases} \tag{19}$$

We consider the Lyapunov function given as

$$V(t) = V\left(e_x, e_y, e_z, e_a, e_b, e_c\right)$$
$$= \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2 + e_c^2\right). \tag{20}$$

The Lyapunov function (20) is clearly definite positive.

Taking time derivative of (20) along the trajectories of (12) and (16) we have

$$\dot{V}(t) = e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z + e_a\dot{e}_a + e_b\dot{e}_b + e_c\dot{e}_c. \tag{21}$$

From (17), (19), and (21) we get

$$\dot{V}(t) = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 + e_a\left[e_z\left({x_1}^2 + {x_2}^2\right) - \dot{A}\right]$$
$$+e_b\left[e_z\left({y_1}^2 + {y_2}^2\right) - \dot{B}\right] - e_c\left(2e_z + \dot{C}\right). \tag{22}$$

Then by applying the parameter update law (15), Eq. (22) become

$$\dot{V}(t) = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2. \tag{23}$$

Obviously, derivative of the Lyapunov function is negative semi-define. According to Barbalat's Lemma in Lyapunov stability theory [40, 63], it follows that $e_x(t) \to 0$, $e_y(t) \to 0$, and $e_z(t) \to 0$ exponentially when $t \to 0$, i.e. anti-synchronization between master and slave system is achieved. This completes the proof. □

We illustrate the proposed anti-synchronization scheme with a numerical example. The parameters of the no-equilibrium systems are selected as $a = 0.5$, $b = 0.1$, $c = 1.3$ and the positive gain constant as $k = 4$. The initial conditions of the master system in (10) and the slave system in (11) have been chosen as $x_1(0) = 0.0$, $y_1(0) = 0.1$, $z_1(0) = 0$, and $x_2(0) = 0.5$, $y_2(0) = 1$, $z_2(0) = 0.8$, respectively. We assumed that the initial values of the parameter estimates are $A(0) = 1$, $B(0) = 0.4$, and $C(0) = 1.5$.

We see that when adaptive control law in (14) and the update law for the parameter estimates in (15) are applied, the anti-synchronization of the master in (10) and slave

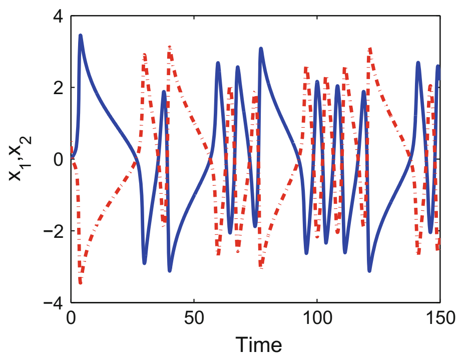


Fig. 10 Anti-synchronization of the states $x_1(t)$ and $x_2(t)$

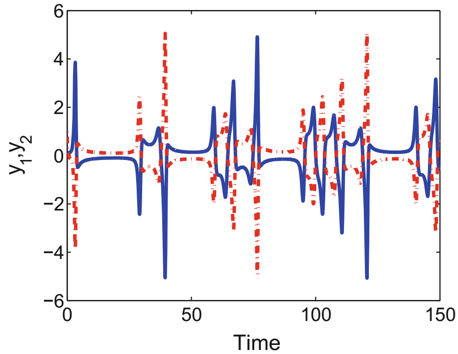


Fig. 11 Anti-synchronization of the states $y_1(t)$ and $y_2(t)$

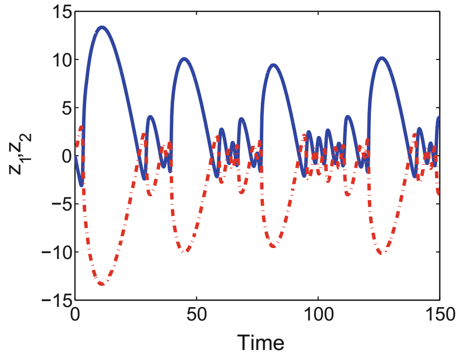


Fig. 12 Anti-synchronization of the states $z_1(t)$ and $z_2(t)$

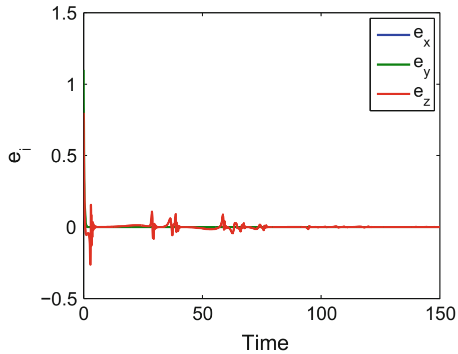


Fig. 13 Time series of the anti-synchronization errors e_x, e_y, and e_z

system in (11) occurs as illustrated in Figs. 10, 11 and 12. Time series of master states are denoted as blue solid lines while corresponding slave states are plotted as red dash-dot lines in such figures. Moreover, the time-history of the anti-synchronization errors e_x, e_y, and e_z is reported in Fig. 13. The anti-synchronization errors converge to the zero, which indicates that the chaos anti-synchronization between the no-equilibrium systems is realized.

6 Circuit Implementation of the No-Equilibrium System

Electronic circuits have been used for emulating theoretical chaotic models [13, 17, 22, 74]. In addition, circuit implementation of chaotic models plays an important role from the point of application view. Circuital realization of chaotic systems has been applied in various engineering fields such as secure communication, signal processing, random bit generator, or path planning for autonomous mobile robot etc. [10, 24, 62, 96, 97, 103].

Therefore, in this section, we introduce an electronic circuit which emulates the theoretical model in (2). By using the operational amplifiers approach [22], the circuit is designed and presented in Fig. 14. The state variables x, y, z of no-equilibrium system in (2) are the voltages across the capacitor C_1, C_2, and C_3, respectively. As seen in Fig. 14, theoretical model in (2) is realized by using only common electronic components such as resistors, capacitors, operational amplifiers and analog multipliers. By applying Kirchhoff's laws to the electronic circuit in Fig. 14, its corresponding circuital equations are derived in the following form

$$\begin{cases} \frac{dv_{C_1}}{dt} = \frac{1}{R_1C_1}v_{C_2} \\ \frac{dv_{C_2}}{dt} = -\frac{1}{R_2C_2}v_{C_1} - \frac{1}{10R_3C_2}v_{C_2}v_{C_3} \\ \frac{dv_{C_3}}{dt} = \frac{1}{10R_4C_3}v_{C_1}v_{C_2} + \frac{1}{10R_5C_3}v_{C_1}^2 + \frac{1}{10R_6C_3}v_{C_2}^2 - \frac{1}{R_7C_3}V_c, \end{cases} \tag{24}$$

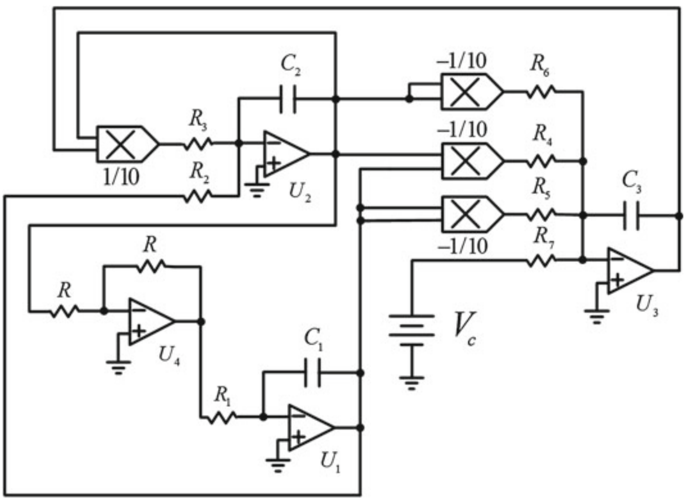


Fig. 14 Schematic of the designed circuit which modelling system without equilibrium in (2)

in which v_{C_1}, v_{C_2}, and v_{C_3} are the voltages across the capacitors C_1, C_2, and C_3, respectively.

In this work, the power supply to all active devices are $\pm 15V_{DC}$ and we use the operational amplifiers TL084. The values of components in Fig. 14 are chosen as follows: $R_1 = R_2 = R_6 = R_7 = R = 10k\Omega, R_3 = R_4 = 1k\Omega, R_5 = 2k\Omega, V_c = 1.3V_{DC}$, and $C_1 = C_2 = C_3 = 10nF$. For the chosen set of components, the values of parameters in system (2) are: $a = 0.5$, $b = 0.1$, and $c = 1.3$.

The designed circuit has implemented in SPICE. The obtained results are reported in Figs. 15 and 16 which display the attractors of the circuit in different phase planes (v_{C_1}, v_{C_2}), (v_{C_1}, v_{C_3}), and (v_{C_2}, v_{C_3}) respectively (Fig. 17). It is easy to see that there is a good agreement between the theoretical attractors (Figs. 1–2) and the circuital ones (Figs. 15 and 16). It can be concluded that the circuit simulations are consistent with the numerical simulations. Moreover, the designed circuit, which is built by using off-the-shelf electronic components, can be applied in practical applications.

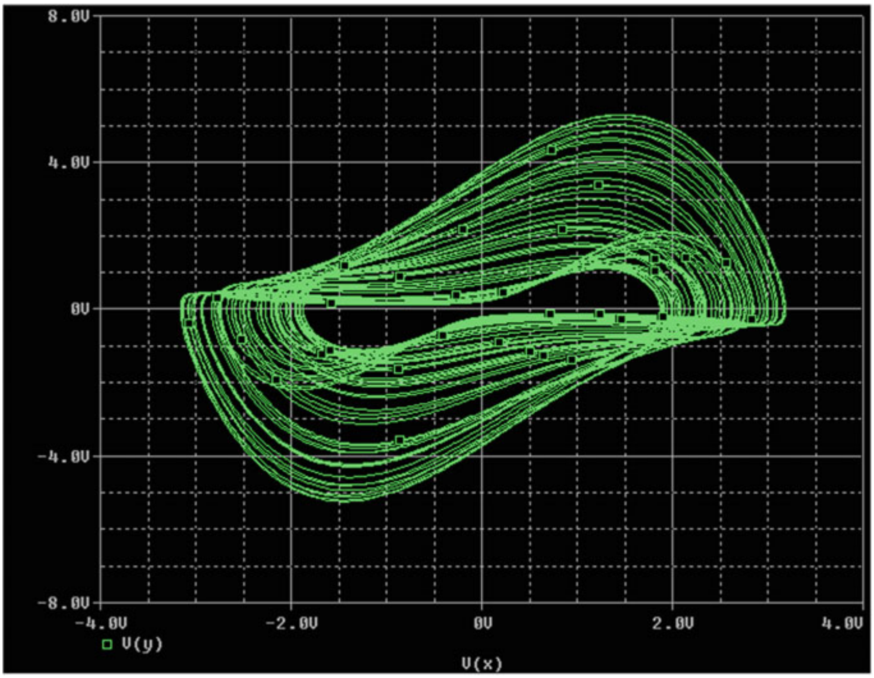


Fig. 15 Obtained SPICE attractor of the designed circuit in the (v_{C_1}, v_{C_2}) phase plane

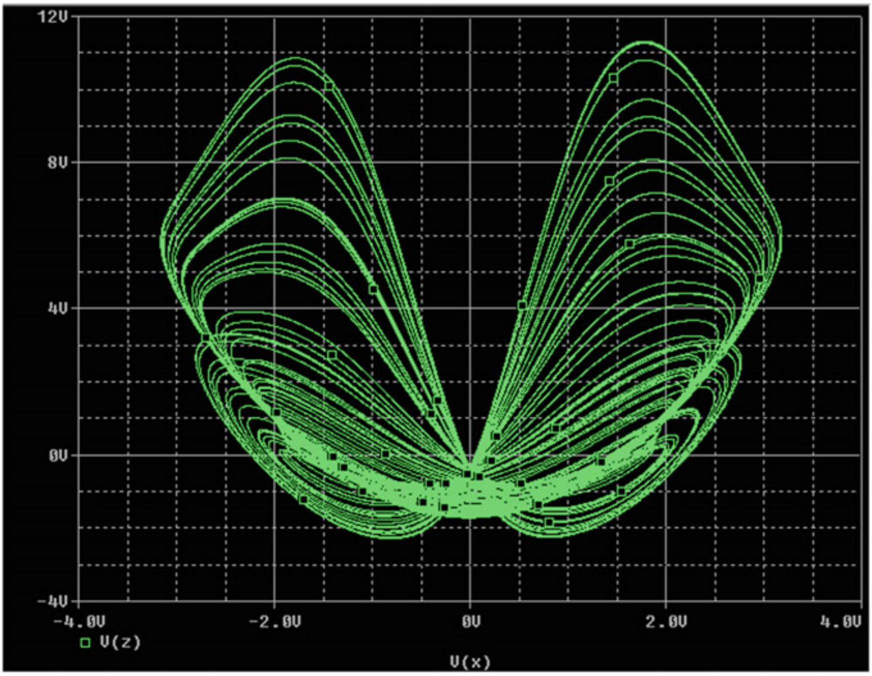


Fig. 16 Obtained SPICE attractor of the designed circuit in the (v_{C_1}, v_{C_3}) phase plane

7 Fractional Order Form of the No-Equilibrium System

As have been known that, practical models such as heat conduction, electrode-electrolyte polarization, electronic capacitors, dielectric polarization, viso-elastic systems are more adequately described by the fractional-order different equations [9, 29, 36, 73, 77, 100]. Existence of chaos in fractional-order systems are investigated [26, 28, 49, 105]. In this section, we consider the fractional-order from of the no-equilibrium system which is described as

$$\begin{cases} \frac{d^q x(t)}{dt^q} = y \\ \frac{d^q y(t)}{dt^q} = -x - yz \\ \frac{d^q z(t)}{dt^q} = xy + ax^2 + by^2 - c, \end{cases} \tag{25}$$

where a, b, c are three positive parameters and $c \neq 0$ for the commensurate order $0 < q \leq 1$. Fractional-order system (25) has been studied by applying Adams–Bashforth-Mounlton numerical algorithm [20, 25, 76]. It is interesting that chaos exists in fractional-order system (25). Figures 18, 19, and 20 display chaotic attractors generated from fractional-order system (25) for the commensurate order $q = 0.99$, the parameters $a = 0.5$, $b = 0.1$, $c = 1.3$ and the initial conditions

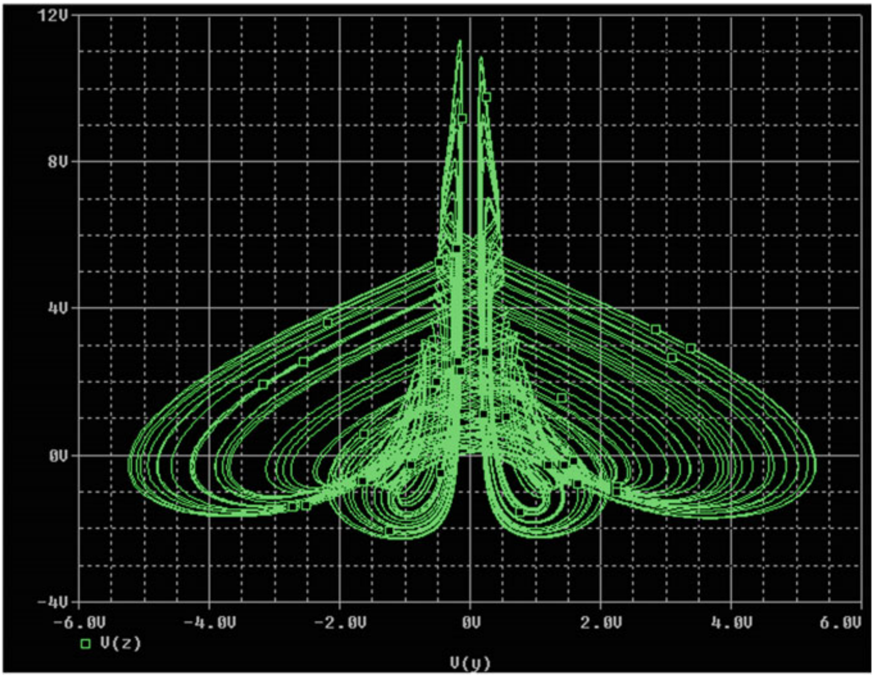


Fig. 17 Obtained SPICE attractor of the designed circuit in the (v_{C_2}, v_{C_3}) phase plane

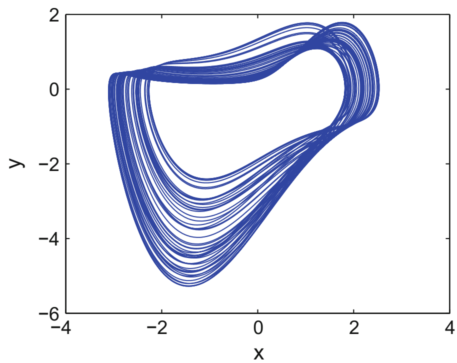


Fig. 18 2-D projection of the fractional-order system without equilibrium (2) in the (x, y)-plane

$$(x(0), y(0), z(0)) = (0, 0.1, 0). \tag{26}$$

This research would enable future engineering applications by considering the advantages of the system without equilibrium and the fractional order theory.

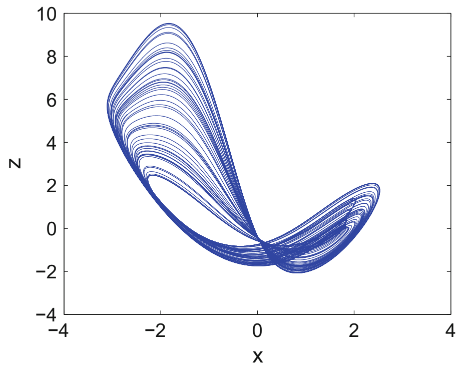


Fig. 19 2-D projection of the fractional-order system without equilibrium (2) in the (x, z)-plane

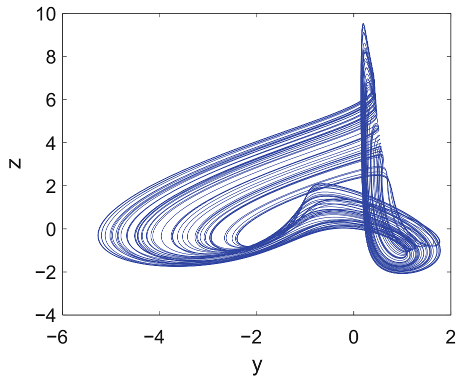


Fig. 20 2-D projection of the fractional-order system without equilibrium (2) in the (y, z)-plane

8 Conclusion

This work introduces a new autonomous chaotic system with special features. There is no any equilibrium points in the proposed system, therefore it is classified as a system with hidden attractor. There is a coexistence of different attractors in the system when changing the values of initial conditions. We have discovered the dynamical properties of such system without equilibrium by using phase portraits, bifurcation diagram, Lyapunov exponents and Kaplan–Yorke dimension. The possibility of synchronization of no-equilibrium systems is studied through an anti-synchronization scheme. The proposed no-equilibrium system are suitable for chaos-based engineering applications because of its complex behavior as well as its feasibility, which has been confirmed by designing an electronic circuit. Fractional order of the proposed system has been given and the result showed that the attractor has no equilibrium.

Potential applications of the proposed system should be investigated. Further studies about fractional-order chaotic systems without equilibrium will be presented in our future works.

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