Chapter 18
Cost Model for Assessing Losses to Avionics Suppliers During Warranty Period

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Abstract Reduction of warranty maintenance costs is a critical issue to the manufacturers of avionic products. A method to reduce expected warranty costs is the determination of all components of financial losses to avionic product suppliers during the warranty period with further minimisation of these losses. This study interlinks the warranty, reliability and maintenance indicators of avionic products. Mathematical models are proposed for analysing and assessing financial costs to avionic system suppliers during the warranty period. The developed mathematical models consider the warranty period, reliability indicators with respect to permanent and intermittent failures, redundancy, number of spare parts, cost of restoration and transportation and penalties for exceeding the duration of warranty repair or replacement. Numerical examples illustrating the proposed models are provided.

18.1 Introduction

Efficient operation of an aircraft is largely determined by the proper regulation of the relationship between an aircraft supplier (manufacturer) and aircraft operator (airline). The most acute problem faced in this relationship is during the warranty period, where the main cost for aircraft systems' repair is borne by the supplier, i.e. the supplier will remedy the defect in a system free of charge and in a reasonable time, by either repairing or replacing the defective system. In the process of meeting

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the terms of a contract for the supply of aircrafts, it is necessary to choose a warranty maintenance strategy and to evaluate all the penalties that may be presented to the supplier in the event of non-compliance with the warranty. The regularity of aircraft flights and the costs that may be incurred by both sides will depend largely on the chosen strategy of warranty maintenance. Thus, the most important task that must be performed during the process of purchasing and commissioning new aircraft is the justification of the warranty maintenance and repair strategy and assessment of the supplier costs during the warranty period. This problem is topical because, for example, in 2015, aerospace product warranty claims paid by U.S.-based companies amounted to $1.6 billion [1]. The warranty policies are classified as free replacement warranty (FRW) and pro-rata warranty (PRW). Under the FRW, the manufacturer takes the responsibility to repair (or replace) any failed product free of charge during the warranty period. The PRW policy implies that the buyer shares the repair cost with the manufacturer during the warranty period. In this study, we consider only FRW policies. Modelling of FRW policies has received a lot of attention in the literature. The Product Warranty Handbook edited by Blischke and Murthy [2] is a collection of research papers devoted to warranty cost models. A review of the literature on warranty models and analysis methods was conducted by Thomas and Rao [3], wherein they analysed different FRW cost models. Chukova and Hayakawa [4] employed an alternating renewal process to model the operating and repair times and evaluated the warranty costs over the warranty period for FRW. Jun and Hoang [5] considered the discounted warranty cost model for repairable series systems under FRW policy. The proposed approach incorporates the system structure information, the value of time and the impact of repair actions. Huang et al. [6] evaluated the problem of estimating the expected warranty costs in case of intermittent and heterogeneous usage intensity. FRW policies for repairable and non-repairable products were analysed. Wang [7] evaluated four new warranty cost models including imperfect repairs. Warranty of a k-out-of-n system with imperfect maintenance was also analysed. Park [8] studied warranty cost models based on the quasi-renewal processes, exponential distribution and assumption that a repair service is imperfect. Blischke et al. [9] presented an integrated compilation of existing literature on warranty data collection. Cost models for one- and two-dimensional warranties were considered. The most important warranty cost models were analysed by Diaz et al. [10]. A method for minimizing additional costs of warranty service through optimal service strategies and efficient management logistics was shown. Shafiee and Chukova [11] conducted a systematic review of mathematical models, where the warranty is interlinked with the maintenance strategy. The research directions in warranty and maintenance were outlined, and a list of new and challenging topics were identified.

It should be noted that the considered models do not take into account the features of avionic systems' operation and maintenance.

18.2 Objectives of the Study

Avionics represent a significant component of aircraft costs: up to 40% in civil aircraft and more than 50% in military aircraft [12]. The complexity in avionics is continuously growing. Most avionic systems have a block-modular structure. Each of the systems of the flight control and navigation complex is a redundant system consisting of two or three line replaceable units (LRUs). In turn, the LRU includes a set of easily removable shop replaceable units (SRU), which are typical replacement elements for the LRU. Each LRU has a built-in test equipment (BITE), which monitors the operability of the LRU. In accordance with the architecture of avionic systems, there are three possible levels of maintenance. The upper level is the field or flight line maintenance, where the LRU can be removed and replaced if it was rejected by the BITE during flight. At the intermediate-level (I-Level) of maintenance, the failed SRUs are replaced to restore the operability of dismantled LRUs. The third level of maintenance consists of the restoration of SRUs by replacement of the failed chips at the depot or by the supplier. In some cases, to reduce the cost of maintenance, the intermediate level is eliminated [13]. This means that LRUs removed on the flight line are sent directly to the depot or supplier instead of heading first to the intermediate level. In this case, the major element of cost savings comes from the reduction of maintenance personnel at the intermediate (base) level [14]. Different strategies of warranty maintenance may differ primarily in the warehouse spare part system, presence of automatic test equipment (ATE) and the depth of the recovery of failed systems at the buyer's end. Obviously, the mathematical models that describe the total supplier cost should take into account the chosen warranty maintenance strategy. A high level of flight safety is ensured by the redundancy of avionic systems. Therefore, when evaluating the operating costs due to warranty claims, we should take into account the type of redundancy for each avionic system. Thus, the purpose of this study is to develop mathematical models for evaluating the financial costs to avionic system suppliers during the warranty period, taking into account the above-mentioned features of avionics architecture, maintenance and operation.

18.3 Analysis of Costs Components of Avionics Suppliers

An analysis of the warranty obligations of the avionics suppliers allows us to identify the following components of the financial costs: cost of repair or replacement of failed LRUs; penalties for delays in repairing or replacement of failed LRUs; capital expenditures related to the chosen variant of the warranty maintenance and operating costs; costs associated with the buy-back of the excess spare parts from the airline at the end of the warranty period and transportation costs.

The warranty period T_W can be represented in flight hours T or in calendar duration T_0 (years). In some cases, the length of the warranty period is specified separately in calendar duration and in the number of flight hours. In such cases, the warranty period is equal to the value, whichever is reached first.

Suppliers of avionics products guarantee the restoration or replacement of the failed items at their own expense. The repair of the defective product can be carried out by the supplier, as well as at certified repair centres. In any case, the supplier pays for the restoration. The penalty for any delay in delivery of repaired or replaced avionics product is associated with aircraft downtime due to the failure of a product under warranty from supplier. The amount of the penalty for each day (hour) of downtime can either be specified in the contract between the supplier and the buyer, or can be equal to the rent paid towards borrowing the missing items from other airlines. Capital expenditure related to the chosen variant of warranty maintenance and operating costs are associated with the presence of the supplier representatives at the buyer, as well as with the possibility of repairing the defective product at the buyer end. The possibility of buying the excessive spare parts should be stipulated in the contract between the supplier and the buyer. To reduce this loss component, the supplier should use accurate data on the reliability characteristics of the avionics products and operating conditions while calculating the optimal number of spare parts. Transportation costs of the supplier are primarily associated with the delivery of repaired items and spare parts to the buyer and shipping the failed items to the repair facilities. An analysis of the sales contracts for avionics products shows that the transportation costs can be shared between the buyer and the supplier. The contract usually specifies the party responsible for bearing transportation costs.

Thus, the financial costs to the supplier during the warranty period are defined by the following formula:

$$C_S(T_W) = R(T_W) + P(T_W) + C_E(T_W) + C_{SP}(T_W) + C_{TC}(T_W) \tag{18.1}$$

where $R(T_W)$ is the cost of repair or replacement of the failed LRUs during the warranty period, $P(T_W)$ is the penalty for exceeding the duration of the warranty repair or replacement within the warranty period, $C_E(T_W)$ is the capital expenditure related to the chosen variant of warranty maintenance and corresponding operating costs, $C_{SP}(T_W)$ is the cost of purchasing the excess spare parts for the LRUs from the buyer at the end of the warranty period and $C_{TC}(T_W)$ is the total transport cost borne by the supplier during the warranty period.

To identify the components of supplier losses we need to model the process of operation and maintenance of avionic systems.

18.4 Maintenance Model During the Warranty Period

The avionics LRU failures can be classified into permanent and intermittent failures. The condition of each LRU is continuously tested by its BITE and in case of a permanent failure, the LRU is switched off. If an intermittent failure occurs during flight, the LRU is usually not switched off but the on-board computer records the information on such events. The following procedure is set for recovery operations during the warranty period. If a permanent or intermittent failure occurred during a flight, the LRU is dismantled after landing and directed to the supplier for repair. We assume that the LRU becomes as good as new after repair.

18.4.1 Probabilities of LRU Repair

Consider an LRU that should operate for a finite time interval T, which is the warranty period. Assume that a random variable Ξ ($\Xi \geq 0$) denotes the time to a permanent failure, with a failure density function $\omega(\xi)$. Let us introduce the following notations for possible restoration of the LRU at time $j\tau$: $A(j\tau)$ is the event consisting of restoration of the LRU at time $j\tau$ after the j-th flight; $H_{IF}(j\tau)$ and $H_{PF}(j\tau)$ are the events corresponding to the restoration of the LRU after intermittent and permanent failure, respectively; $P_R(j\tau)$, $P_{IF}(j\tau)$ and $P_{PF}(j\tau)$ are, respectively, the probabilities of the events $A(j\tau)$, $H_{IF}(j\tau)$ and $H_{PF}(j\tau)$, where τ is the mean time between aircraft landings at the base airport. To determine $P_R(j\tau)$, $P_{IF}(j\tau)$ and $P_{PF}(j\tau)$, we introduce the probability distribution function (PDF) of random variables Ξ, Θ_1, …, Θ_j, which we denote as $\Omega(\xi, \theta_1, \ldots, \theta_j)$, where Θ is the time to intermittent failure with PDF $\psi(\theta)$ and

$$\Theta_j = \Theta - (j - 1)\tau \tag{18.2}$$

is the remainder of the operating time to intermittent failure after j–1 flights (j = 1, 2, …). Using the multiplication theorem of PDFs, we get [15]

$$\Omega\left(\xi, \theta_1, \ldots, \theta_j\right) = \omega(\xi)\Omega_0\left(\theta_1, \ldots, \theta_j|\xi\right) \tag{18.3}$$

where, $\Omega_0(\theta_1, \ldots, \theta_j|\xi)$ is the conditional PDF of random variables Θ_1, …, Θ_j under the condition that $\Xi = \xi$.

Further, we use two conditional probabilities associated with intermittent failures. The conditional probability of an intermittent failure occurring during the j-th (j = 1, 2, …) flight, under the condition that $\Xi = \xi > j\tau$, is formulated as follows:

$$P_{IF|\xi}\left[\overline{\tau, (j-1)\tau}; j\tau|\xi\right] = P\left[\bigcap_{i=1}^{j-1}\Theta_i > \tau \bigcap \Theta_j < \tau|\xi\right] \tag{18.4}$$

The conditional probability of an intermittent failure not occurring during the j-th flight is stated as follows:

$$P_{\overline{IF}|\xi}\left[\overline{\tau,(j-1)\tau};j\tau|\xi\right]=P\left(\bigcap_{i=1}^{j}\Theta_i>\tau|\xi\right) \tag{18.5}$$

Since a random variable Θ_i (i = 1, …, j) is defined in the finite time interval (0, T − (i - 1)τ], we need to introduce the conditional PDF $\Omega_1\{\theta_1, \ldots, \theta_j \mid \xi \cap [0 < \Theta_i \leq T - (i-1)\tau,\ i = 1, \ldots, j]\}$, which is expressed through the conditional PDF $\Omega_0(\theta_1, \ldots, \theta_j|\xi)$ as follows:

$$\Omega_1\left\{\theta_1,\ldots,\theta_j|\xi\bigcap\left[0<\Theta_i\leq T-(i-1)\tau,i=\overline{1,j}\right]\right\}=$$
$$\Omega_0\left(\theta_1,\ldots,\theta_j|\xi\right)\Big/\int\limits_0^T\int\limits_0^{T-\tau}\ldots\int\limits_0^{T-(j-1)\tau}\Omega_0\left(u_1,\ldots,u_j\,|\xi\right)du_1\ldots du_j \tag{18.6}$$

By integrating (18.6) over the corresponding limits, we determine the probabilities (18.4) and (18.5):

$$P_{IF|\xi}\left[\overline{\tau,(j-1)\tau};j\tau|\xi\right]=\frac{\int\limits_\tau^T\int\limits_\tau^{T-\tau}\ldots\int\limits_\tau^{T-(j-2)\tau}\int\limits_0^{\tau}\Omega_0\left(u_1,\ldots,u_j|\xi\right)du_1\ldots du_j}{\int\limits_0^T\int\limits_0^{T-\tau}\ldots\int\limits_0^{T-(j-1)\tau}\Omega_0\left(u_1,\ldots,u_j|\xi\right)du_1\ldots du_j} \tag{18.7}$$

$$P_{\overline{IF}|\xi}\left[\overline{\tau,(j-1)\tau};j\tau|\xi\right]=\frac{\int\limits_\tau^T\int\limits_\tau^{T-\tau}\ldots\int\limits_\tau^{T-(j-1)\tau}\Omega_0\left(u_1,\ldots,u_j|\xi\right)du_1\ldots du_j}{\int\limits_0^T\int\limits_0^{T-\tau}\ldots\int\limits_0^{T-(j-1)\tau}\Omega_0\left(u_1,\ldots,u_j|\xi\right)du_1\ldots du_j} \tag{18.8}$$

The probabilities of the LRU restoration due to the occurrence of intermittent or permanent failure are formulated as follows:

$$P_{IF}(j\tau)=P[H_{IF}(j\tau)]=P\left\{\bigcup_{\nu=0}^{j-1}\left\{A(\nu\tau)\bigcap\{B_1[(j-\nu)\tau]\backslash B_2[(j-\nu)\tau]\}\right\}\right\} \tag{18.9}$$

$$P_{PF}(j\tau)=P[H_{PF}(j\tau)]=1-P\left\{\bigcup_{\nu=0}^{j-1}\left\{A(\nu\tau)\bigcap B_1[(j-\nu)\tau]\right\}\right\} \tag{18.10}$$

where

$$B_1[(j-\nu)\tau] = [\Xi > (j-\nu)\tau] \bigcap \left(\bigcap_{i=\nu+1}^{j-1} \Theta_i > \tau \right) \tag{18.11}$$

is the joint occurrence of the following events: the LRU begins to work at time t_ν; it does not fail up to time $j\tau$ and no intermittent failure occurs during the flights $\nu + 1$, …, $j - 1$;

$$B_2[(j-\nu)\tau] = [\Xi > (j-\nu)\tau] \bigcap \left(\bigcap_{i=\nu+1}^{j} \Theta_i > \tau \right) \tag{18.12}$$

is the event different from $B_1[(j - \nu)\tau]$ only in the fact that in the j-th flight also there was no intermittent failure;

\ is the symbol denoting the difference between the two events.

The LRU will be restored at time $j\tau$ if either of the events $H_{IF}(j\tau)$ or $H_{PF}(j\tau)$ occurs. Therefore,

$$A(j\tau) = H_{IF}(j\tau) + H_{PF}(j\tau) \tag{18.13}$$

Assuming that $H_{IF}(j\tau)$ and $H_{PF}(j\tau)$ are mutually exclusive events and applying this condition to (18.13), the addition theorem of probability gives

$$P_R(j\tau) = P[A(j\tau)] = P_{IF}(j\tau) + P_{PF}(j\tau) \tag{18.14}$$

If the LRU was restored at time $\nu\tau$, random variable Ξ is defined in the interval $(0, T - \nu\tau]$ with conditional PDF

$$\omega(\xi | 0 < \Xi \le T - \nu\tau) = \omega(\xi) \Big/ \int_0^{T-\nu\tau} \omega(x)dx \tag{18.15}$$

Taking into account (18.15) and applying to (18.9)–(18.12), the addition and multiplication theorems of probability, we obtain

$$P_{IF}(j\tau) = \sum_{\nu=0}^{j-1} \frac{P_R(\nu\tau)}{\int_0^{T-\nu\tau} \omega(x)dx} \int_{(j-\nu)\tau}^{T} P_{IF|\xi}\left(\overline{\tau, (j-\nu-1)\tau}; (j-\nu)\tau | \vartheta\right) \omega(\vartheta)d\vartheta \tag{18.16}$$

$$P_{PF}(j\tau) = 1 - \sum_{\nu=0}^{j-1} \frac{P_R(\nu\tau)}{\int_0^{T-\nu\tau} \omega(x)dx} \int_{(j-\nu)\tau}^{T} P_{\overline{IF}|\xi}\left(\overline{\tau, (j-\nu-2)\tau}; (j-\nu-1)\tau|\vartheta\right)\omega(\vartheta)d\vartheta \tag{18.17}$$

where $P_R(0) = 1$, $P_{IF}(0) = 0$ and $P_{PF}(0) = 1$.

As is well known, the exponential distribution provides an appropriate distribution of permanent failures of complex systems [16]. It has been reported in several publications that the exponential distribution is also an appropriate distribution for avionics products [17, 18]. This is because LRUs in modern avionics consist of a large number of electronic chips. A flight director system may consist of 460 digital integrated circuits (ICs), 97 linear ICs, 34 memories, 25 ASICs and 7 processors [18]. For these components, external failure mechanisms (electrical overstress, electrostatic discharge and so on) and intrinsic failure mechanisms (dielectric breakdown, electromigration and hot carrier injection) can cause the components to fail. Different failure modes contribute to a constant LRU failure rate. This is possible only with the exponential distribution of failure over time

$$\omega(t) = \lambda e^{-\lambda t} \tag{18.18}$$

where λ is the permanent failure rate of LRU.

Assume that intermittent failures are also subject to the exponential law with PDF

$$\psi(t) = \theta e^{-\theta t} \tag{18.19}$$

As is well known, the exponential distribution has the memoryless property. Therefore, conditional probabilities (18.7) and (18.8) are converted to the following form:

$$P_{IF|\xi}\left[\overline{\tau, (j-1)\tau}; j\tau|\xi\right] = \left(1 - e^{-\theta\tau}\right)\prod_{i=1}^{j-1}\left\{e^{-\theta\tau} - e^{-\theta[T-(i-1)\tau]}\right\} \bigg/ \prod_{i=1}^{j}\left\{1 - e^{-\theta[T-(i-1)\tau]}\right\} \tag{18.20}$$

$$P_{\overline{IF}|\xi}\left[\overline{\tau, (j-1)\tau}; j\tau|\xi\right] = \prod_{i=1}^{j}\left\{e^{-\theta\tau} - e^{-\theta[T-(i-1)\tau]}\right\} \bigg/ \prod_{i=1}^{j}\left\{1 - e^{-\theta[T-(i-1)\tau]}\right\} \tag{18.21}$$

Substituting (18.18), (18.20) and (18.21) into (18.16) and (18.17) we obtain

$$P_{IF}(j\tau) = \left(1 - e^{-\theta\tau}\right)\sum_{\nu=0}^{j-1} P_R(\nu\tau)\frac{\left[e^{-\lambda(j-\nu)\tau} - e^{-\lambda T}\right]\prod_{i=\nu+1}^{j-1}\left\{e^{-\theta\tau} - e^{-\theta[T-(i-1)\tau]}\right\}}{\left[1 - e^{-\lambda(T-\nu\tau)}\right]\prod_{i=\nu+1}^{j}\left\{1 - e^{-\theta[T-(i-1)\tau]}\right\}} \tag{18.22}$$

$$P_{PF}(j\tau) = 1 - \sum_{\nu=0}^{j-1} P_R(\nu\tau)\frac{\left[e^{-\lambda(j-\nu)\tau} - e^{-\lambda T}\right]\prod_{i=\nu+1}^{j-1}\left\{e^{-\theta\tau} - e^{-\theta[T-(i-1)\tau]}\right\}}{\left[1 - e^{-\lambda(T-\nu\tau)}\right]\prod_{i=\nu+1}^{j-1}\left\{1 - e^{-\theta[T-(i-1)\tau]}\right\}} \tag{18.23}$$

It should be noted that beginning from the fourth or fifth flight probabilities (18.22) and (18.23) usually reach the steady-state values

$$P^*_{IF}(\tau) = \left(1 - e^{-\theta\tau}\right)\left(e^{-\lambda\tau} - e^{-\lambda T}\right)/\left[\left(1 - e^{-\lambda T}\right)\left(1 - e^{-\theta T}\right)\right] \tag{18.24}$$

$$P^*_{PF}(\tau) = 1 - \left(e^{-\lambda\tau} - e^{-\lambda T}\right)/\left(1 - e^{-\lambda T}\right) \tag{18.25}$$

18.4.2 Expected Repair Costs

Expected repair costs during the warranty period are determined as follows:

$$R(T) = mN\left\{C_{IF}\sum_{j=1}^{[T/\tau]} P_{IF}(j\tau) + C_{PF}\sum_{j=1}^{[T/\tau]} P_{PF}(j\tau)\right\} \tag{18.26}$$

where m is the number of identical LRUs in a redundant avionic system, N is the number of aircraft under warranty of supplier, C_{IF} and C_{PF} are, respectively, the mean cost of repairing LRU with intermittent and permanent failures by the supplier and $[T/\tau]$ is the integer number of ratio T/τ.

Equation (18.26) is simplified to

$$R(T) = \frac{mNT}{\tau}\left[C_{IF}P^*_{IF}(\tau) + C_{PF}P^*_{PF}(\tau)\right] \tag{18.27}$$

if the steady-state values of probabilities $P_{IF}(j\tau)$ and $P_{PF}(j\tau)$ are used.

Example 1 In modern wide-body aircraft there are usually three instrument landing systems (ILS), i.e. $m = 3$. Assume that $N = 4$, $T = 10{,}000$ flight hours, $\tau = 5$ h, $\lambda = 10^{-4}$ h^{-1}, C_{IF} = ₤1,000 and C_{PF} = ₤2,000. Using (18.24) and (18.27), $P^*_{IF}(\tau)$ and $R(T)$ are calculated as a function of θ.

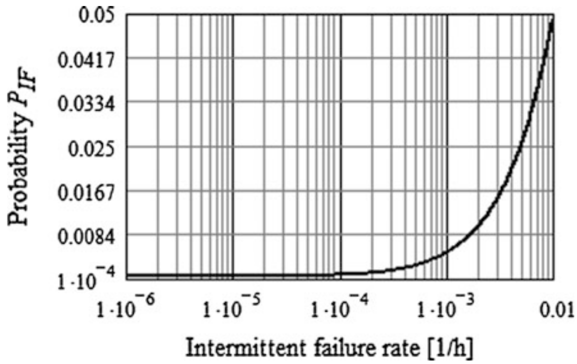

Fig. 18.1 Dependence of probability $P^*_{IF}(\tau)$ on intermittent failure rate

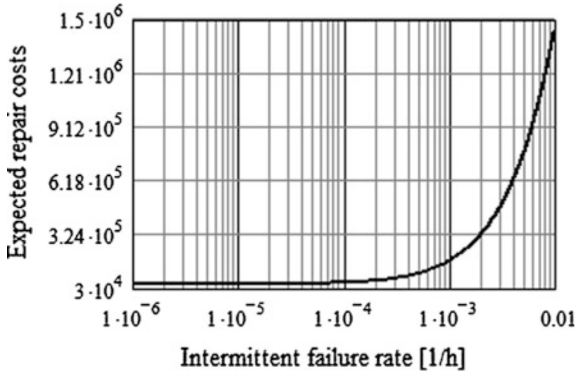

Fig. 18.2 Dependence of expected repair costs on intermittent failure rate

The dependency of $P^*_{IF}(\tau)$ and $R(T)$ as a function of θ are shown in Figs. 18.1 and 18.2, respectively. As can be seen from Fig. 18.1, the probability $P^*_{IF}(\tau)$ is 7.9×10^{-4} when $\theta < 9 \times 10^{-5}$ h^{-1}, and it begins to rise sharply when $\theta > 10^{-4}$ h^{-1}. From Fig. 18.2, it follows that the expected repair cost is £60,700 when $\theta < 8 \times 10^{-5}$ h^{-1}. However, when $\theta > 10^{-4}$ h^{-1}, the shape of $R(T)$ is similar to the shape of $P^*_{IF}(\tau)$.

Thus, the expected costs of warranty repairs may greatly depend on the intermittent failure rate.

18.4.3 Expected Penalty Costs

Optimal number of spare LRUs largely depends on the warranty repair time of defective LRUs. When the duration of the warranty repair time is exceeded, one of the following situations may take place.

Situation 1. There is at least one spare LRU in the warehouse. All aircraft under warranty by the supplier fly on schedule.

Situation 2. There are no spare LRUs in the warehouse. All aircraft under warranty of the supplier fly on schedule.
Situation 3. There are no spare LRUs in the warehouse. At least one aircraft is standing idle on the ground, while awaiting emergency delivery of spare LRUs.

In the first or second situation, the aircraft owner does not bear financial losses. When the third situation arises, the aircraft owner incurs losses due to violation of the schedule, flight cancellation, compensation to passengers, etc. If the supplier makes an expedited delivery under existing contract t_{ed}, it does not lead to financial losses. However, if the spare LRU is delivered after a time $t_{ed} + \Delta t_{delay}$, the supplier bears financial losses for each unit of time interval (t_{ed}, $t_{ed} + \Delta t_{delay}$). These financial losses may be equal either to the penalty for each unit of downtime ($C_0 \Delta t_{delay}$) or to the rental cost of LRU paid to another airline ($C_{rent} \Delta t_{delay}$). C_0 is the penalty per unit of time for exceeding the duration of the warranty repair time and C_{rent} is the rent per unit of time for the use of LRU, leased from another airline.

It should be noted that the frequency of occurrence of the third situation is dependent on whether the supplier complies with the warranty repair duration t_{WR}. The greater the delay of the LRU recovery or replacement in the first and second situations, the higher the probability of occurrence of the third situation.

Let us denote the optimal number of spare LRUs in the warehouse of the airline by n_{opt}, calculated based on the values of t_{WR} and t_{ed}. Then, when exceeding the duration of the warranty repair or replacement of the LRU by the amount Δt_{delay}, the average time of aircraft delay is determined by the formula

$$\Delta T_{ad} = \Delta t_{LRU}\left(n_{opt}, t_{WR} + \Delta t_{delay}, t_{ed} + \Delta t_{delay}\right) + t_D + t_M - t_S \tag{18.28}$$

where Δt_{LRU} is the average time of delay in satisfying the demand for spare LRUs at the base airport, t_D and t_M are the average durations of dismantling and mounting the LRU on the aircraft board and t_S is the average duration of aircraft stop at the base airport while performing a typical route.

It is obvious that the following inequality holds:

$$\Delta t_{LRU}\left(n_{opt}, t_{WR} + \Delta t_{delay}, t_{ed} + \Delta t_{delay}\right) > \Delta t_{LRU}\left(n_{opt}, t_{WR}, t_{ed}\right) \tag{18.29}$$

Consequently, the probability of finding the avionic system in the state of waiting for a spare LRU at the aircraft stop is increased.

Equations for the calculation of $\Delta t_{LRU}(n_{opt}, t_{WR}, t_{ed})$ are given in [19].

The total average penalties for exceeding the duration of the warranty repair or replacement within the warranty period are determined as follows:

$$P(T) = \frac{mNT}{\tau} P_{delay} C_p \Delta T_{ad} \tag{18.30}$$

where P_{delay} is the probability that LRU failed in flight is awaiting a replacement during the aircraft stop at the base airport and C_p is the penalty (C_0) or the rent (C_{rent}) per unit of time.

The probability P_{delay} is given by

$$P_{delay} = \frac{\sigma\left[\Delta t_{LRU}\left(n_{opt}, t_{WR}, t_{ed}\right) + t_D + t_M - t_S\right]}{M\left(S_{op}\right) + M\left(S_{inop}\right) + \sigma\left[\Delta t_{LRU}\left(n_{opt}, t_{WR}, t_{ed}\right) + t_D + t_M - t_S\right]} \tag{18.31}$$

where $M(S_{op})$ and $M(S_{inop})$ are, respectively, the expected mean time spent by the LRU in the operable (S_{op}) and inoperable state (S_{inop}) and σ is the indicator function

$$\sigma = \begin{cases} 0 \text{ if } t_S \geq (\Delta t_{LRU} + t_D + t_M) \\ 1 \text{ if } t_S < (\Delta t_{LRU} + t_D + t_M) \end{cases} \tag{18.32}$$

Generalized equations for $M(S_{op})$ and $M(S_{inop})$ are given in [19]. In the case of exponential distribution of time to permanent and intermittent failure, $M(S_{op})$ and $M(S_{inop})$, are determined as follows [19]:

$$M\left(S_{op}\right) = \frac{\tau}{1 - e^{-\theta\tau}}\left[1 - e^{-(\lambda+\theta)T}\right] + \left[\left(1 - e^{-\lambda\tau}\right)\times \left(\frac{1}{\lambda} - \frac{\tau}{1 - e^{-\theta\tau}}\right) - \tau e^{-\lambda\tau}\right]\frac{1 - e^{-(\lambda+\theta)T}}{1 - e^{-(\lambda+\theta)\tau}} + \tau e^{-(\lambda+\theta)T} \tag{18.33}$$

$$M\left(S_{inop}\right) = \left(\tau - \frac{1 - e^{-\lambda\tau}}{\lambda}\right)\left[\frac{1 - e^{-(\lambda+\theta)T}}{1 - e^{-(\lambda+\theta)\tau}}\right] \tag{18.34}$$

As can be seen from (18.30)–(18.32), if σ = 0, then $P_{delay} = 0$ and $P(T) = 0$.

Example 2 Calculation of the expected penalty costs during warranty period for an avionic system if $T = 10{,}000$ h, $N = 5$, $m = 2$, $\tau = 5$ h, $\lambda = 10^{-4}$ h^{-1}, $\theta = 2 \times 10^{-4}$ h^{-1}, $t_{WR} = 120$ h, $t_{ed} = 16$ h, $C_0 = £20{,}000$ and $t_M = t_D = 0.25$ h.

The dependence of $P(T)$ as a function of the number of spare LRUs is illustrated in Table 18.1. Several conclusions can be made from an analysis of Table 18.1. Firstly, it is obvious that the greater number of spare LRUs, the lower the expected penalty costs. Secondly, an increase of warranty repair duration ($2t_{WR}$) or expediting delivery time ($2t_{ed}$) results in a significant increase in the expected penalty costs. As can be seen from Table 18.1, the worst case is when both t_{WR} and t_{ed} are doubled.

Table 18.1 Calculated values of $P(T)$ for different values of t_{WR} and t_{ed}

Number of spare LRUs (n)	Expected penalty costs (£)			
	(t_{WR}, t_{ed})	($2t_{WR}$, t_{ed})	(t_{WR}, $2t_{ed}$)	($2t_{WR}$, $2t_{ed}$)
1	2.08×10^5	3.22×10^5	4.96×10^5	7.3×10^5
2	0	3.02×10^4	6.1×10^3	1.28×10^5
3	0	0	0	0

18.4.4 Capital Expenditures

Capital expenditures depend on whether the supplier is ready to install additional equipment for testing the dismantled LRUs. For example, the following two variants of warranty maintenance are evident. In the first variant, the restoration of all dismantled LRUs is carried out at the supplier factory. Here the capital expenditures are equal to zero. This variant can be cost-effective in the case of a small number of aircraft that have supplier warranty. The use of this variant when a large number of aircraft have supplier warranty requires an increase in the number of spare LRUs. In case of a shortage of spare LRUs, the expected penalty costs will increase according to (18.30). The second variant assumes that automatic test equipment (ATE) is used in the airline for re-testing the dismantled LRUs. The purpose of ATE is to avoid the shipment of falsely dismantled LRUs to the supplier for repairing.

Experience of modern aircraft operation confirms a rather high percentage of unscheduled removals of LRUs, which causes a significant increase in the number of spare LRUs needed to provide the required level of flight regularity. Furthermore, unscheduled LRU removals result in increased repair costs during the warranty period.

If the aircraft buyer does not purchase ATE, but it is profitable for the supplier to install ATE in the airline for rechecking dismantled LRUs, the capital expenditure recalculated to the beginning of operation of the first delivered aircraft is

$$C_{ATE,i}(1+\delta)^{i-1}\Big/(1+\varepsilon)^{i-1},\ i=1,\ \ldots,\ T_0 \tag{18.35}$$

where T_0 is the duration of warranty expressed in calendar years, $C_{ATE,i}$ is the cost of ATE in the i-th year (i = 1, …, T_0), δ denotes the increase of ATE cost due to inflation in labour costs and so on and ε is the depreciation in monetary value, since the expected cost of ATE would be affected by inflation, increased labour and other costs.

The current supplier costs in the j-th year of the interval (0, T_0) associated with the renting of area to accommodate ATE in the airline, payment to staff, payment for electricity, and other necessary expenses is designated as C_j. Then, the capital expenditure for the organization of the warranty maintenance and operating costs during the interval (0, T_0) is equal to:

$$C_E(T_0) = C_{ATE,i}(1+\delta)^{i-1}\Big/(1+\varepsilon)^{i-1} + \sum_{j=i}^{T_0} C_j(1+\delta)^{j-1}\Big/(1+\varepsilon)^{j-1} \tag{18.36}$$

If i = 1, i.e. the supplier installs ATE in the airline at the beginning of the warranty period, then (18.36) is converted to the form:

$$C_E(T_0) = C_{ATE,1} + \sum_{j=1}^{T_0} C_j (1+\delta)^{j-1} \Big/ (1+\varepsilon)^{j-1} \tag{18.37}$$

18.4.5 Cost of Purchasing an Excess of Spare LRUs

The supplier calculates the required number of spare LRUs in the airline's warehouse during the warranty period. The main parameters that determine the required number of spare LRUs are as follows: T—warranty period, N—number of aircraft that are under warranty by the supplier, λ—LRU permanent failure rate, θ—LRU intermittent failure rate, t_{WR}—warranty repair duration and t_{ed}—expedited delivery time of a spare LRU.

Let N^*, λ^*, θ^*, t^*_{WR} and t^*_{ed} be the values of the initial data used by the supplier for the calculation of the optimal number of spare LRUs, n^*. Since parameter N cannot be different from the specified N^*, the excess of spare LRUs may appear only as the difference between the values of λ^*, θ^*, t^*_{WR} and t^*_{ed} and the actual values of these parameters. For example, if the actual values of parameters λ and θ are less than λ^* and θ^*, there will be an excess of spare LRUs. Further, if the supplier repairs and replaces the failed LRUs within a time shorter than t^*_{WR} and t^*_{ed}, this will lead to an excess of spare LRUs.

Let n be the total number of spare LRUs purchased by the airline during the warranty period to ensure flight regularity. Then, the excess of spare LRUs to be re-purchased at the end of the warranty period is determined by

$$\Delta n = n^* - n \tag{18.38}$$

The expected costs for the purchase of the excess spare LRUs are determined by

$$C_{SP}(T_0) = \Delta n C_{LRU}(T_0) \Big/ (1+\varepsilon)^{T_0 - 1} \tag{18.39}$$

where $C_{LRU}(T_0)$ is the cost of LRU at the end of the warranty period.

Example 3 Calculation of expected costs for re-purchasing the excess of spare LRUs at the end of warranty period for an avionic system if $T_0 = 3$ years, $T = 10{,}000$ h, $N = 10$, $m = 3$, $C_{LRU} = £20{,}000$, $\tau = 5$ h, $\lambda^* = 10^{-4}$ h^{-1}, $\theta^* = 2 \times 10^{-4}$ h^{-1}, $t^*_{WR} = 160$ h, $t^*_{ed} = 24$ h, $t_M = t_D = 0.25$ h and $\varepsilon = 0.1$.

For these initial data, the optimal number of spare LRUs n^* is 4. Now assume that some parameters had different values during the warranty period, for example, $\theta = 1 \times 10^{-4}$ h^{-1}, $t_{WR} = 120$ h and $t_{ed} = 16$ h. In this case, $n = 3$ and $\Delta n = 1$. Using (18.39), we calculate that $C_{SP}(T_0) = £16{,}530$.

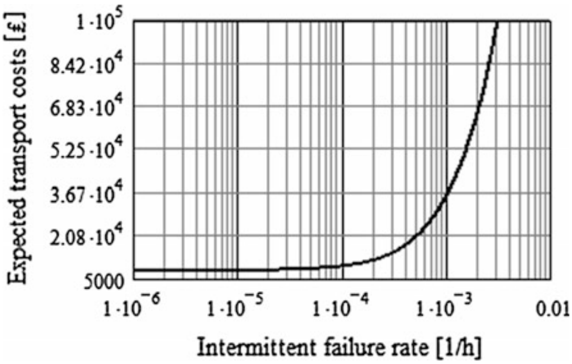

Fig. 18.3 Dependence of expected transport costs on intermittent failure rate

18.4.6 Expected Transport Costs for the Supplier

Avionics suppliers or buyers or both bear all transportation costs for shipping failed and restored LRUs. If transportation costs are fully borne by the buyer, the supplier expenses are zero. If the supplier pays for the transportation, fully or partially, the transport costs during the warranty period are determined analogous to (18.27)

$$C_{TC}(T) = \frac{mNTC_{tc}}{\tau}\left[P^*_{IF}(\tau) + P^*_{PF}(\tau)\right] \tag{18.40}$$

where C_{tc} is the transportation cost of shipping a failed LRU to the supplier and shipping the restored LRU back to the buyer.

Example 4 Calculation of expected transport costs for an avionic system, assuming T = 10,000 h, N = 10, m = 3, C_{tc} = £100, τ = 5 h and $\lambda = 10^{-4}$ h^{-1}.

The dependence of expected transport costs as a function of intermittent failure rate is shown in Fig. 18.3. As can be seen from Fig. 18.3, $C_{TC}(T)$ begins to rise significantly when $\theta > 10^{-4}$ h^{-1}.

If the supplier installed ATE at the buyer end, then only LRUs with permanent failures are directed to the supplier for repairs. In this case, (18.40) is simplified to

$$C_{TC}(T) = mNTC_{tc}P^*_{PF}(\tau)/\tau \tag{18.41}$$

18.5 Conclusions

In this chapter, we have analysed the components of the financial costs to avionic system suppliers during the warranty period. Mathematical models have been developed for the evaluation of the supplier's costs, taking into account the warranty duration, number of aircraft, reliability indicators with respect to permanent and intermittent failures, number of LRUs in a redundant avionic system, number of spare LRUs, cost of restoration and transportation and penalties for exceeding the duration of the warranty repair or replacement. These results enable a reasonable determination on the number of spare LRUs, minimize the amount of penalty costs in the case of breach of warranty by the supplier and minimize some other supplier costs. Practical use of the obtained results will significantly reduce the maintenance costs of avionics during the warranty period.

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