An Estimative Model of Automated Valuation Method in Italy

Marina Ciuna, Francesca Salvo and Marco Simonotti

Abstract The Automated Valuation Method (AVM) is a computer software program that analyzes data using an automated process. It is related to the process of appraising an universe of real estate properties, using common data and standard appraisal methodologies. Generally, the AVM is based on quantitative models (statistical, mathematical, econometric, etc.), related to the valuation of the properties gathered in homogeneous groups (by use and location) for which are collected samples of market data. The real estate data are collected regularly and systematically. Within the AVM, the proposed valuation scheme is an uniequational model to value properties in terms of widespread availability of sample data, allowing the use of statistical models, and in the opposite conditions of the absence of data of comparable properties. Under these conditions the 'appraisal model' has a unique shape, when its coefficients are calculated with a mathematical-statistical model and when they are determined by an estimative process. The main part of the appraisal model is unique and the universal in the valuation, for which the mathematical-statistical and estimative procedures are the underlying part. Of course, the accuracy of the valuation increases with the number of available data, other conditions being equal, and the valuations, carried out in the absence of data (but in the presence of other market information), require extra-statistical appraisal procedures involving a complete knowledge of the real estate market (Ciuna and Simonotti 2011). However such knowledge is also required in the AVM performed by quantitative models with regard to the data sampling and the verify of the results (Kauko and d'Amato 2008a, b). The appraisal model is based on uniequational 'appraisal functions', on indices measured in the market and on tests. In first approximation, the linear form is preferred for simplicity, for the modularity (the majority of models are linear or linearized or attributable to additive forms), for

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the understanding of the calculations, the intermediate elements (e.g. marginal prices of the real estate characteristics) and the results of the valuation.

Keywords AVM · Valuation · Market segment · Appraisal function

1 Introduction

The AVM (Automated Valuation Method) regards the process of valuation of a set of properties, and takes place with models and methods based on the direct detection of punctual data of individual contracts and other market indications, mainly concerning the parameters of the market segments, and the statistical and economical indicators. The appraisal procedures may be statistical, statistical-estimative and estimative, in accordance with the purposes of the valuation, the availability of data, the type of property and the means and the time put forward by the valuer (d'Amato and Kauko 2008). The purpose of this work is to present an appraisal model of AVM characterized by power to operate even in cases in which there is in the conditions with few data, only one property datum or no data, according to a unique pattern represented by the appraisal function. The valuation model proposed is based on the appraisal function that in the basic form establishes a relationship of cause and effect between the market price, the characteristics of the property and the parameters of a given segment in a specific context of the real estate market. The peculiarity of the model is the possibility to construct functions with a minimum amount of information available, starting from a sample of market prices to a single datum or no data, in the latter case doing the valuation with the other market information, in a structured way. The appraisal model is in fact able to use any kind of real estate information, perfecting with the knowledge of the market and considering the result according to the purpose of the valuation.

2 Basis of the Valuation

The bases of the valuation are the economic variables taken as the foundation of real estate valuation. The main foundations of the valuations are the market prices and the rents as true references to the real estate market. The respective bases of the valuation are then the market value and the market rent. The appraisal model poses as the basis of valuation the market value, the market rent and the net income referred to the market rent. These bases of valuation are defined according to the current valuation standard the market value is "the estimated amount for which an asset or liability should exchange on the valuation date between a willing buyer and a willing seller in an arm's-length transaction after proper marketing and where the parties had each acted knowledgeably, prudently, and without compulsion." (IVS 2011, IVS Framework); the market rent is "the estimated amount for which a property would be leased on the valuation date between a willing lessor and a willing lessee on appropriate lease terms in an arm's length transaction, after proper

marketing and where the parties had each acted knowledgeably, prudently and without compulsion" (IVS 2011, IVS Framework); the net market income is the amount obtained by subtracting to the market rent, as defined, the operating expenses incurred by real estate. Generally the AVMs are applied to appraise the market value, the market rent and the net income of the residential properties, for which there are a number of sample data. Although AVMs development requires skilled analysis and attention to quality assurance, AVMs are characterized by the use and application of statistical and mathematical techniques. This distinguishes them from traditional appraisal methods in which an appraiser physically inspects properties and relies more on experience and judgment to analyze real estate data and develop an estimate of market value. Provided that the analysis is sound and consistent with accepted appraisal theory, an advantage to AVMs is the objectivity and efficiency of the resulting value estimates (Salvo et al. 2015). Of course, sound judgment is required in model development and an appraiser should review the values produced by the model (IAAO 2003). However, the valuation on large scale may be related to properties with destinations other than residential. These properties are intended for special uses or special property (agricultural, industrial and commercial real estate, secondary real estate, etc.) that have a limited market and which often show specific structure, shape and size. The market prices and rents of instrumental properties include the effects of the a typicalness of these properties, the system of non-competitive market, the difficulty of building renovation and production conversion, as well as the effects induced by the dynamics of the productive sector. The appraisal model aims to provide a process for uniform valuation of residential and special properties (Salvo et al. 2015).

3 Market Segment and Area

The real estate market is segmented for the diversity, the atypical nature and complexity of the properties. The real estate market is divided into segments, which represent the basic unit, not further divisible, of the economic-estimative analysis. The market segment is defined with respect to a set of parameters for which two or more housing units fall in the same market segment if they have similar economic-estimative parameters, whether if they are similar housing units for valuation. A segment of the real estate market is classified by: the type of contract (rent, purchase, etc.); the destination; the location; the type of property; the building type; the characters of supply and demand; the shape of the market and the price level. The market segments are defined not only by the parameters but by a series of indicators, ratios and market indices. A market segment may include a single property, a group of real estate, a block or a neighborhood. The segments are defined by the boundaries within which the properties are subject in the same way to the economic forces that largely determine the market price and rent. The properties of the same real estate market segment may have a spatial discontinuity even within restricted geographical areas (Ciuna et al. 2014).

In the present study we consider the contract of sale for the price and the lease for the rents, as they are typically the most popular, there is a ratio between them with appraisal meaning (the gross rent multiplier, the capitalization rate, etc.) and they can be easily extended to other contracts and rights (lease, leasehold, etc.) or to other situations (comparable market analysis). The market segment is typically a small unit of analysis (parts of the building, blocks, neighborhoods, etc.) but with the definition of the parameters it is possible to group similar or comparable segments in a larger unit (also spatially), taking into account the different parameters. For example, the market data of real estate from two spatially neighboring segments can be combined into a single sample introducing between the characteristics of collected properties, the different localization (e.g. central and semi-central). Likewise, the market data of real estate segments for the different building types can be grouped in the same sample data, taking into account the type (e.g. for apartments: the multi-storey buildings and chalet). Based on the study of the segmentation process, the model proposes as a basic unit of application of the AVM in the market area. The market area delimits, by means of a continuous line, a set of market segments, for the purposes of the construction and application of the functions of the model (Renigier-Biłozor and Biłozor 2016a, b). The market area is defined according to four main parameters: the type of contract, the destination, the type of building and the type of property (Simonotti 1998). The boundaries of the market area varies according to the parameters taken into consideration. In practice, the market areas, individual by the main coordinates, are layered in a sectional for the purpose of simplifying the valuation procedure, and their mapping follows perimeter lines that correspond physically to the boundary lines between buildings and areas. The definition of the market area can be done by aggregation of similar properties with inductive procedure, taking into account the fact that more widen is the perimeter of the market area, greater is the variability of the properties. This implies a priori knowledge of the market. The market area may include: segments in which there are market prices and rents; segments in which there are only market prices and no rents; segments in which there are only the rents and no market prices; segments in which there are not neither the market prices nor the rents. The model aims to settle the principles of international standards, because it is based on the detection of market data on a uniform appraisal methodology and on the ability to operate quality controls.

4 Appraisal Function

In the appraisal model the general form of the function used to appraise the market value, referred to the market area, the characteristics and parameters of the real estate segment, can be proposed as follows in the following Formula (1):

$$V = L_0 + \sum_{f=1}^{n} p_f \cdot x_f + \sum_{f=1}^{n} q_g \cdot X_g \quad (1)$$

where:

V market value of the property;
L_0 constant term;
p_f marginal price of the generic real estate characteristic f (con f = 1, 2, …, n);
q_g marginal price of the generic market segment parameter g (con g = 1, 2, …, m);
x_f generic real estate characteristic;
X_g generic market segment parameter.

The marginal price of the real estate characteristic represents the variation in the market value varying the characteristic. The marginal price of the parameter segment expresses the variation in the market value varying the parameter. The function used to appraise the market value is presented in the deterministic form. The general form of the function used to appraise the market rent refers to the market area, to the segment parameters and the real estate characteristics can be proposed as follows in the following Formula (2):

$$R = l_0 + \sum_{f=1}^{n} r_f \cdot x_f + \sum_{g=1}^{m} v_g \cdot X_g \quad (2)$$

where:

R annual market rent of the property;
l_0 constant term;
r_f marginal income of the generic real estate characteristic;
v_g marginal income of the generic market segment parameter;
x_f generic real estate characteristic;
X_g generic market segment parameter.

The marginal income of the characteristic expresses the variation in the real estate market rent varying the characteristic. The marginal income of the segment parameter expresses the variation in the real estate market rent varying the parameter. The function used to appraise the market rent is presented in the deterministic form. The equivalent statistic appraisal function of the market value and the market rent is the following Formula (3):

$$y_j = b_0 + \sum_{f=1}^{n} b_f \cdot x_{jf} + \sum_{g=1}^{m} B_g \cdot X_{jg} + e_j \quad (3)$$

where:

y_j total market price or annual market rent of the generic real estate j (with j = 1, 2, …, m);
b_0 constant term (euro);

b_f coefficient of the generic real estate characteristic;
B_g coefficient of the generic market segment parameter;
x_{jf} generic real estate characteristic;
X_{jg} generic market segment parameter;
e_j stochastic error.

The statistical appraisal function of the market value and the market rent is presented as a multiple linear regression equation according to the most commonly used symbols. The regression equation considers the price or the market rent collected as explained variable and the parameters of the real estate segment and the characteristics as explanatory variables. The statistical appraisal function of the value and market rent is submitted in stochastic form.

There is complete identity between the appraisal Formulas (1) and (2) and the statistical Formula (3) in the constant component (L_0, l_0 and b_0) and in marginal prices and income of real estate characteristics (p_f, r_f and b_f) and in marginal prices and incomes of the parameters (q_g, v_g and B_g).

The general formula of net market income can be proposed as follows in the Formula (4):

$$R_N = R - c_E \tag{4}$$

where:

R_N annual net income of the property;
R annual total market rent of the property;
c_E annual operating cost of the property.

The net income is appraised by subtracting to the market rent, obtained by the estimation function, the operating expenses incurred by the property owner.

5 Construction of the Appraisal Function

The peculiarity of the appraisal model consists in the possibility of constructing the prediction function with the statistical models and estimation procedures (the market comparison method, the method of the depreciated reconstruction cost and direct capitalization method) according to the valuation standards (d'Amato 2008; d'Amato and Siniak 2008).

To this purpose, the model considers four specific situations: in the first situation there is a sample of market prices or a sample of market rents sufficiently numerous for the construction of a statistical model; in the second situation there is a market price or a market rent of a real transaction; in the third situation there is a sample of market prices or a sample of market rents of comparable properties, in itself few to be treated statistically, but perfectly suitable for use in the appraisal process; in the fourth situation, finally, there are not any real estate data (market prices and market

rents), but we know the functions of market areas similar and close to that for which we want to estimate the function.

6 Function of the Market Value

The construction of the appraisal function of the market value follows four situations regarding the availability of market data. In the first situation, collected a statistical sample of prices related to the market area, known the characteristics of the contracted real estate property and the parameters of the market segment, the appraisal function of the market value can be presented as multiple linear regression equation according to the general Formula (3). The uniequational model can also be calculated in a mathematical-statistical way different by the regression analysis, provided that the locational factor and the marginal prices of the characteristics and of the segment parameters are specified. The model in this form then provides directly the constant term and the marginal prices of the characteristics and parameters of the real estate segment. In the first situation for the individual property being appraised the market value V_0 is equal to Formula (5):

$$V_0 = b_0 + \sum_{f=1}^{n} b_f \cdot x_{0f} + \sum_{g=1}^{n} B_g \cdot X_{0g} \tag{5}$$

where:

x_{0f} generic real estate characteristic of the property being assessed;
X_{0g} generic segment's parameter of the property being assessed.

In principle, the appraisal function is able to estimate individually by the interpolation all properties of the market area. In the second situation, detected in a given area of the market, the market price P_j, note the real estate characteristics and parameters of the market segment of the contracted property, for the individual property being appraised according to the general Formula (1) the market value V_0 is equal to Formula (6):

$$V_0 = L_0 + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \tag{6}$$

The constant term of the appraisal function of the market value is mainly related to the localization of the property and the effect of other characteristics and segment parameters different from those reported in the function. The constant term of the appraisal function of the market value can be calculated by setting an appraisal comparison equation for which the difference in the price between the two properties is a function of the differences presented by their characteristics and their parameters of segment. The functional relationship between the known market price

P_j of the generic comparable property j and its characteristics and parameters of real estate segment according to the Formula (1) is the following in the Formula (7):

$$P_j = L_0 + \sum_{f=1}^{n} p_f \cdot x_{jf} + \sum_{g=1}^{m} q_g \cdot X_{jg} \tag{7}$$

The appraisal comparison equation of the market value refers to the comparison between the property being appraised and the generic comparable property respectively according to the Formulas (6) and (7), as follows in the Formula (8):

$$V_0 - P_j = \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} - \sum_{f=1}^{n} p_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg} \tag{8}$$

Consequently the market value V_0 of the individual property being appraised is equal to the Formula (9):

$$V_0 = (P_j - \sum_{f=1}^{n} p_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg}) + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \tag{9}$$

where the constant term is setted equal to Formula (10):

$$L_0 = P_j - \sum_{f=1}^{n} p_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg} \tag{10}$$

The appraisal model in this form requires an exogenous valuation of the marginal prices of the characteristics and real estate segment parameters. In principle, the apraisal function is able to estimate individually, by interpolation, all properties of the market area. In the third situation, detected an appraisal sample (not very large) of price P_j (j = 1, 2, …, k) referred to the market area, known the parameters of the market segment and the characteristics of the traded properties, the appraisal function can be achieved by setting k equations according to the Formula (9) in the following way (Simonotti 1985), Formula (11):

$$\begin{cases} V_0 = P_1 - \sum_{f=1}^{n} p_f \cdot x_{1f} - \sum_{g=1}^{m} q_g \cdot X_{1g} + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \\ V_0 = P_2 - \sum_{f=1}^{n} p_f \cdot x_{2f} - \sum_{g=1}^{m} q_g \cdot X_{2g} + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \\ \cdots = \cdots \\ V_0 = P_k - \sum_{f=1}^{n} p_f \cdot x_{kf} - \sum_{g=1}^{m} q_g \cdot X_{kg} + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \end{cases} \tag{11}$$

In the third situation, for the property being appraised, the market value V_0 is then equal to Formula (12):

$$V_0 = \frac{1}{k} \cdot (\sum_{j=1}^{k} P_j - \sum_{f=1}^{n} p_f \cdot \sum_{j=1}^{k} x_{jf} - \sum_{g=1}^{m} P_g \cdot \sum_{j=1}^{k} X_{jg}) + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \tag{12}$$

where the divisor is the number of sampled data. In the third situation the constant term L_0, according to the Formula (12) is equal to Formula (13):

$$L_0 = \frac{1}{k} \cdot (\sum_{j=1}^{k} P_j - \sum_{f=1}^{n} p_f \cdot \sum_{j=1}^{k} x_{jf} - \sum_{g=1}^{m} \Pi_g \cdot \sum_{j=1}^{k} X_{jg}) \tag{13}$$

The appraisal model, in this form, requires an exogenous valuation of the marginal prices of the characteristics and of the parameters of the real estate segment. In order to consider the possibility of resolution of the equations system of the Formula (9), the system can be presented in the following Formula (14):

$$\begin{cases} V_0 + \sum\limits_{if=1}^{n} p_f \cdot (x_{1f} - x_{0f}) + \sum\limits_{g=1}^{m} q_g \cdot (X_{1g} - X_{0g}) = P_1 \\ V_0 + \sum\limits_{f=1}^{n} p_f \cdot (x_{2f} - x_{0f}) + \sum\limits_{g=1}^{m} q_g \cdot (X_{2g} - X_{0g}) = P_2 \\ \cdots = \cdots \\ V_0 + \sum\limits_{f=1}^{n} p_f \cdot (x_{kf} - x_{0f}) + \sum\limits_{g=1}^{m} q_g \cdot (X_{kg} - X_{0g}) = P_k \end{cases} \tag{14}$$

The unknowns of the equations system are the market value of the property being appraised and the marginal prices of the characteristics and parameters of the real estate segment. The known terms of the system are represented by the recorded market prices. To the valuation of the market value are valid the mathematical conditions of resolution of the equations system, including, eventually, appropriate solution conditions mathematically approximate. In principle, the appraisal function is able to estimate individually by the interpolation all properties of the market area. In the fourth situation it is assumed that there are not any data of market prices, characteristics and parameters of the real estate market segment, but we know the appraisal functions of market areas next to the one for which we want to build the function. In this circumstance the appraisal model allows the interpolation of the appraisal functions of the next two areas, or of their neighbors segments, by varying marginal prices (if with the same parameters of the segment) or by introducing the parameter or parameters that are different. The appraisal function can be achieved by setting the equations of the market areas A and B selected in accordance with the Formula (1) in the following Formula (15):

$$\begin{cases} V_A = L_{0A} + \sum_{f=1}^{n} p_{fA} \cdot x_f + \sum_{g=1}^{m} q_{gA} \cdot X_g \\ V_B = L_{0B} + \sum_{f=1}^{n} p_{fB} \cdot x_f + \sum_{g=1}^{m} q_{gB} \cdot X_g \end{cases} \tag{15}$$

where the locational factor L_{0A} of the market area A according to the Formula (7) is equal to Formula (16):

$$L_{0A} = P_{jA} - \sum_{f=1}^{n} p_f \cdot x_{jfA} - \sum_{g=1}^{m} q_g \cdot X_{jgA} \tag{16}$$

and the locational factor L_{0B} according to the Formula (7) of the market area B is equal to Formula (17):

$$L_{0B} = P_{jB} - \sum_{f=1}^{n} p_f \cdot x_{jfB} - \sum_{g=1}^{m} \Pi_g \cdot X_{jgB} \tag{17}$$

In the fourth situation, the market value V_0 of the individual property being appraised is equal to Formula (18):

$$V_0 = \left(\frac{L_{0A} + L_{0B}}{2}\right) + \frac{p_{fA} + p_{fB}}{2} \cdot \sum_{f=1}^{n} x_{0F} + \frac{q_{gA} + q_{gB}}{2} \cdot \sum_{g=1}^{m} X_{0g} \tag{18}$$

The appraisal model in this form requires an exogenous valuation of the marginal prices of real estate properties. The inclusion in the appraisal function of the parameter of segment solves the case of the segments without data in the valuation of the marginal prices of the parameters. In principle, the interpolated appraisal function is able to estimate individually, by extrapolation, all properties of the market area.

7 Function of the Market Rent

The construction of the appraisal function of the market rent follows four situations regarding the availability of market data. In the first situation, collected a statistical sample of the rents related to the market area, known the real estate characteristics and the parameters of segment, of the contracted properties, the appraisal function of the market rent can be presented as multiple linear regression equation according to the general Formula (3). The uniequational model can also be calculated with mathematical-statistical analysis different from the regression, provided that in the form there is specified the locational factor and the marginal income of the real estate characteristics and of the parameters of the segment. The model in this form

then provides directly the constant term and the marginal income of real estate characteristics and parameters of the segment. In principle, the appraisal function is able to estimate individually, by interpolation, all properties of the market area. In the second situation, detected in a given market area, a market rent R_j, known the characteristics and parameters of the real estate segment of the contracted property, for the individual property being appraised, according to the general Formula (3), the market rent R_0 is equal to Formula (19):

$$R_0 = l_0 + \sum_{f=1}^{n} r_f \cdot x_{0f} + \sum_{g=1}^{m} v_g \cdot X_{0g} \tag{19}$$

The constant term of the appraisal function of the market rent is mainly due to the location of the property and the effect of other characteristics and segment parameters different from those specified in the function. The comparison equation of the market rent refers to the comparison between the generic comparable property and the property to be appraised, according to the Formula (2) and the Formulas (7), (8) and (9), is the following Formula (20):

$$R_0 = (R_j - \sum_{f=1}^{n} r_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg}) + \sum_{f=1}^{n} r_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g}) \tag{20}$$

where the constant term is equal to Formula (21):

$$l_0 = R_j - \sum_{f=1}^{n} r_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg} \tag{21}$$

The appraisal model, in this form requires the exogenous valuation of marginal income of the real estate characteristics and of the parameters of the segment (paragraph 6). In principle, the apprasial function is able to estimate individually, by interpolation, all properties of the market area. In the third situation, detected an appraisal sample (not numerous) of market rents R_j (j = 1, 2, …, k) refers to the market area, known the characteristics of the traded properties and the parameters of the market segment, the appraisal function can be achieved by setting k equations according to the Formula (11) in the following way, Formula (22):

$$\begin{cases} R_0 = R_1 - \sum_{f=1}^{n} r_f \cdot x_{1f} - \sum_{g=1}^{m} v_g \cdot X_{1g} + \sum_{f=1}^{n} r_f \cdot x_{0f} + \sum_{g=1}^{m} v_g \cdot X_{0g} \\ R_0 = R_2 - \sum_{f=1}^{n} r_f \cdot x_{2f} - \sum_{g=1}^{m} v_g \cdot X_{2g} + \sum_{f=1}^{n} r_f \cdot x_{0f} + \sum_{g=1}^{m} v_g \cdot X_{0g} \\ \cdots = \cdots \\ R_0 = R_k - \sum_{f=1}^{n} r_f \cdot x_{kf} - \sum_{g=1}^{m} v_g \cdot X_{kg} + \sum_{f=1}^{n} r_f \cdot x_{0f} + \sum_{g=1}^{m} v_g \cdot X_{0g} \end{cases} \tag{22}$$

In the third situation, for the individual property being appraised, the market rent R_0 is then equal to Formula (23):

$$R_0 = \frac{1}{k} \cdot \left(\sum_{j=1}^{k} R_j - \sum_{f=1}^{n} r_f \cdot \sum_{j=1}^{k} x_{jf} - \sum_{g=1}^{m} q_g \cdot \sum_{j=1}^{k} X_{jg}\right) + \sum_{f=1}^{n} r_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \tag{23}$$

In the third situation the constant term l_0, according to the Formula (23), is equal to Formula (24):

$$I_0 = \frac{1}{k} \cdot \left(\sum_{j=1}^{k} R_j - \sum_{f=1}^{n} r_f \cdot \sum_{j=1}^{k} x_{jf} - \sum_{g=1}^{m} q_g \cdot \sum_{j=1}^{k} X_{jg}\right) \tag{24}$$

The appraisal model in this form requires an exogenous valuation of the marginal income of the real estate characteristics and of the parameters of the segment. In order to consider the possibility of resolution of the equations system of the Formula (14), the system can be presented in the following form, Formula (25):

$$\begin{cases} R_0 + \sum\limits_{f=1}^{n} r_f \cdot (x_{1f} - x_{0f}) + \sum\limits_{g=1}^{m} q_g \cdot (X_{1g} - X_{0g}) = R_1 \\ R_0 + \sum\limits_{f=1}^{n} r_f \cdot (x_{2f} - x_{0f}) + \sum\limits_{g=1}^{m} q_g \cdot (X_{2g} - X_{0g}) = R_2 \\ \cdots = \cdots \\ R_0 + \sum\limits_{f=1}^{n} r_f \cdot (x_{kf} - x_{0f}) + \sum\limits_{g=1}^{m} q_g \cdot (X_{kg} - X_{0g}) = R_k \end{cases} \tag{25}$$

The unknowns of the equations system are the market rent of the property being appraised and the marginal income of real estate characteristics and of the parameters of the segment. The known terms of the system are the collected market rents. For the valuation of the market rent are valid the mathematical conditions of resolution of the equations system, including, where appropriate, solution conditions mathematically approximate. In principle, the appraisal function is able to value individually, by interpolation, all properties of the market area. In the fourth situation it is assumed that there are not any information on market rents, property characteristics and parameters of the market segment, but we know the appraisal functions of market areas next to the one for which we want to build function. In this circumstance the appraisal model allows the interpolation of the appraisal functions of the next two areas, or neighbors segments, by varying the marginal income (if with the same parameters of the segment) or by introducing the parameter or parameters of difference. The appraisal function can be achieved by setting the equations of the market areas A and B selected in accordance with the Formula (1) in the following way, Formula (26):

$$\begin{cases} R_A = l_{oA} + \sum_{f=1}^{n} r_{fA} \cdot x_f + \sum_{g=1}^{m} q_{gA} \cdot X_g \\ R_B = l_{oB} + \sum_{f=1}^{n} r_{fB} \cdot x_f + \sum_{g=1}^{m} q_{gB} \cdot X_g \end{cases} \tag{26}$$

where the locational factor l_{0A} of the market area A according to the Formula (21) is equal to Formula (27):

$$l_{oA} = R_{jA} + \sum_{f=1}^{n} r_f \cdot x_{jfA} + \sum_{g=1}^{m} q_g \cdot X_{jgA} \tag{27}$$

and the locational factor l_{0B} of the market area B according to the Formula (19) is equal to Formula (28):

$$l_{oB} = R_{jB} + \sum_{f=1}^{n} r_f \cdot x_{jfB} + \sum_{g=1}^{m} q_g \cdot X_{jgB} \tag{28}$$

In the fourth situation, the to market rent R_0 of the individual property being appraised is equal to Formula (29):

$$R_0 = \left(\frac{l_{0A} + l_{0B}}{2}\right) + \frac{r_{fA} + r_{fB}}{2} \cdot \sum_{f=1}^{n} x_{0f} + \frac{q_{gA} + q_{gB}}{2} \cdot \sum_{g=1}^{m} X_{0g} \tag{29}$$

The appraisal model in this form requires an exogenous valuation of marginal income of the real estate characteristics. The inclusion in the appraisal function of parameter of segment solves the case of segments with the absence of data in the valuation of the marginal income of the parameters (Renigier-Biłozor et al. 2014a). In principle, the appraisal function is able to estimate individually, by interpolation all properties of the market area (Ciuna 2014a, b).

8 Appraisal Procedures

The procedures to the valuation of the market value and market rent refer to current valuation standard, which indicate the market comparison method, the depreciated reconstruction cost method and the income capitalization method.

9 Market Comparison Method

The market comparison method is an appraisal procedure of the market value or the market rent of the property, by comparing the property being appraised and a set of similar comparable properties, recently traded and with price or rent known (Salvo et al. in print). The market comparison method applies: in the situation where we have the market price or the market rent of a real transaction (depending on the situation) (Salvo and De Ruggiero 2011, 2013); and in the situation where we have a sample of market prices or a sample of market rents of comparable properties, in itself not very numerous, but suitable for use in the valuation process (the third situation) (Kaklauskas et al. 2012a, b). The framework of the coefficients of the appraisal function concerns: the constant factor, the marginal rents and prices of the characteristics and parameters (Borst et al. 2008).

10 Constant Term

The extra-statistical valuation of the constant term concerns the appraisal procedures. The constant term expresses the characteristics for which two samples of real estate data, taken from two different market areas, while presenting the same characteristics and the same parameters of the segment, show two different price levels. For the market area, the appraisal function assigns to the constant term an abstract meaning, as it is not referred to the market segment, as in traditional valuations, but to the market area. The same is for the marginal price of real estate characteristics. The calculation of the constant term in the construction of the appraisal functions of the market value and the market rent is divided: in the situation where we have the market price or market rent for a single transaction (depending on the situation); and in the situation of the appraisal sample of prices or market rents of comparable properties (third position).

11 Analysis of the Marginal Prices of the Characteristics

The marginal price of a real estate characteristic represents the variation of the total price of the property varying the characteristic. The marginal prices can be expressed in terms of value and in percentage terms. In the appraisal analysis the marginal prices are accounting prices, i.e. prices that perform instrumental tasks and are estimated a priori according to the purposes of the valuation. The analysis of the marginal prices of the main characteristics of the properties can be carried out for all of the real estate characteristics. However, for the purposes of the MA, the interest is reduced to the real estate characteristics for which the valuation of the marginal prices and income applies directly available market information. They are generally recurring ratios between economic-estimative sizes of the real estate market and

applied by the operators. These commercial ratios are naturally available and are recorded directly from the market. The marginal price of the main area of the property is calculated by multiplying the average price for the position ratio σ_P, which locates on the cartesian floor the relative position of the curve of marginal price and that known of the average price. If the curve of the average price is above the marginal price, then the position ratio is less than unity (Simonotti 2001). The position ratio σ_P between the marginal price p_i and the average unit price $\bar{p}_i$ of the same surface characteristic s_i, it indicates the following Formula (30):

$$\sigma_p = \frac{p_i}{\bar{p}_i} \tag{30}$$

Therefore, the marginal price of the surface characteristic is calculated as the product of the unit price directly calculable on the market data and the position ratio σ_P, as follows in the Formula (31):

$$p_i = \bar{p}_i \cdot \sigma_P \tag{31}$$

To calculate the average price of the main surface in the presence of secondary and accessory surfaces is necessary to detect the market ratio of these surfaces with the main surface. The surface ratio π_{Pf} expresses the ratio between the marginal price p_f of the generic secondary surface (with f = 2, 3, …, h) and the marginal price of the main surface p_1, as follows in the Formula (32):

$$\pi_{Pf} = \frac{p_f}{p_1} \tag{32}$$

In the ratios of the secondary area is generally supposed that these are worth less than the main surface that have the greater importance in the property ($\pi_{Pf} < 1$). However, sometimes the secondary surfaces are more relevant and more useful of the main surface because they can be the subject of significant real estate valuations ($\pi_{Pf} > 1$). The commercial ratios are explicitly indicated by the market. Knowing the total price of the property P_j, being s_{j1} the main surface, s_{jf} the generic secondary surface, the unit price $\bar{p}_{j1}$ of the main surface is equal to Formula (33):

$$\bar{p}_{j1} = \frac{P_j}{s_{j1} + \sum_{f=2}^{h} \pi_{Pf} \cdot s_{jf}} \tag{33}$$

then the marginal price p_{j1} of the main surface, according to the Formula (31), is equal to Formula (34):

$$p_{j1} = \frac{P_j}{s_{j1} + \sum_{f=2}^{h} \pi_{Pf} \cdot s_{jf}} \cdot \sigma_p \tag{34}$$

In the ratio is calculated the average price, in the denominator appears a fictitious surface called commercial area. The marginal price p_{jf} of the secondary areas according to the Formula (32) is equal to Formula (35):

$$p_f = \pi_{Pf} \cdot p_{j1} \tag{35}$$

i.e. the product of the marginal price of the main surface and the market ratio of the secondary surface in consideration. The marginal price of the outer surface of the property is obtained by multiplying the average price for the appropriate position ratio. The unit price of the built area is based on the survey: (a) the market prices of the land built when these soils have an independent market; (b) the market prices of the building areas, considering the potential transformation of the land in building area and subtracting the cost of demolition; (c) the market prices of the buildings constructed by including the impact of developed land on the market value of the property. The marginal price p_T of the outer surface, based on the survey of market prices of built lands (a), calculated their average price $\bar{p}_T$ and appraised the position ratio σ_T, is equal to Formula (36):

$$p_T = \bar{p}_T \cdot \sigma_T \tag{36}$$

The marginal price p_T of the outer surface, based on the survey of market prices of building lands (b), calculated their average price $\bar{p}_E$, the cost of demolition unit c_E and appraised the position ratio σ_E, is equal to Formula (37):

$$p_T = (\bar{p}_E - c_E) \cdot \sigma_e \tag{37}$$

The marginal price p_T of the outer surface, based on the survey of market prices of the properties (c), is based on the impact of built-up land, which expresses the ratio between the market value of the built land and the market value of the property (including the building and the land). Calculated the average unit price $\bar{p}_I$ of property, measured the impact of built land λ and appraised the position ratio σ_I, the marginal price p_T of the outer surface is equal to Formula (38):

$$p_T = \bar{p}_I \cdot \lambda \cdot \sigma_I \tag{38}$$

For example, considering only the surface's characteristics indicated, other things being equal property characteristics and parameters of the market segment, the appraisal function of the market value V_0 according to the Formula (5) is as follows in the Formula (39):

$$V_0 = \left[P_j - p_{j1} \cdot (s_{j1} + \pi_{P2} \cdot s_{j2} + \lambda \cdot s_{j3})\right] + p_{j1} \cdot (s_{01} + \pi_{P2} \cdot s_{02} + \lambda \cdot s_{j3}) \tag{39}$$

where:

P_j market price of the generic property;
s_{j1} main area of the generic property;
s_{j2} secondary area of the generic property;
s_{j3} outer surface area of the generic property;
s_{01} main area of the property being appraised;
s_{02} secondary area of the property being appraised;
s_{03} outer surface area of the property being appraised;
p_{j1} marginal price of the main surface according to the Formula (34);
π_{P2} market ratio of secondary surface according to the Formula (32);
λ incidence of the built area according to the Formula (38).

The possibility of inserting in the function other real estate characteristics over the surface's characteristics may cover characteristics such as the date, the level of the floor, the technological installations, the maintenance status, the number of toilettes and all those characteristics for which the market expresses a ratio or other indications of the market, and can apply the comparison appraisal procedures, such as the paired data analysis (PDA) (Ciuna and Simonotti 2014).

12 Analysis of the Marginal Incomes of the Characteristics

The marginal income of a real estate characteristic is the change in the total market rent of the property varying the characteristic. The marginal income can be expressed in terms of value and in percentage terms.

In the appraisal analysis the marginal income are prices that perform instrumental tasks and are estimated a priori, according to the purposes of the valuation. The analysis of the marginal income of the main characteristics of the properties can be carried out for all of the real estate. However, for the purposes of the MA interest is reduced to the real estate characteristics for which the valuation of marginal income applies directly available market information. The marginal income of the main area of the property is calculated by multiplying the average income for the position ratio σ_R, which locates on the Cartesian floor the relative position of the curve of marginal income with respect to that known of the average income. To calculate the average income of the surface in presence of primary and secondary accessory surfaces is necessary to detect the market ratio of these surfaces with the main surface. The market ratio π_{Rf} expresses the ratio between the marginal income r_f of the generic secondary surface (with f = 2, 3, …, h) and the marginal income of the main surface r_1, as follows in the Formula (40):

$$\pi_{Rf} = \frac{r_f}{r_1} \tag{40}$$

Knowing the total market rent R_j of the generic property, and the main surface s_{j1}, the generic secondary surface s_{jf}, then the marginal income r_{j1} the main surface is equal to Formula (41):

$$r_{j1} = \frac{R_j}{s_{j1} + \sum_{i=2}^{h} \pi_{Rf} \cdot s_{jf}} \cdot \sigma_R \tag{41}$$

In the fraction is calculated the average rent, in the denominator appears the commercial area. The marginal income r_{jf} of the secondary areas of the property, in accordance with the general Formula (36), is equal to Formula (42):

$$r_{jf} = \pi_{Rf} \cdot r_{j1} \tag{42}$$

i.e. the product of the marginal income of the main surface and the market ratio of the secondary surface considered. The marginal income of the outer surface of the property we can get from the marginal price of outer surface according to the Formulas (36), (37) and (38) multiplying by the capitalization rate of the soil i_T as follows in the Formula (43):

$$r_T = p_T \cdot i_T \tag{43}$$

If we have the market incomes of properties, the marginal income of the outer surface, can be calculated, according to the incidence of the built land. Known the average income $\bar{r}_I$ of the real estate properties, the impact of the built land λ and the position ratio σ_I, the marginal income r_T of the outer surface is equal to Formula (44):

$$r_T = \bar{r}_I \cdot i_T \cdot \lambda \cdot \sigma_I \tag{44}$$

As an example, considering only the surfaces characteristics indicated, being coeteris paribus the other characteristics and parameters of real estate market segment, the appraisal function of the market rent R_0, according to the Formula (39), is the following Formula (45):

$$R_0 = \left[R_j - r_{j1} \cdot (s_{j1} + \pi_{R2} \cdot s_{j2} + \lambda \cdot s_{j3})\right] + r_{j1} \cdot (s_{01} + \pi_{R2} \cdot s_{02} + \lambda \cdot s_{03}) \tag{45}$$

The possibility of inserting in the function other real estate characteristics over the surfaces characteristics may cover characteristics such as the date, the level of the floor, the technological installations, the maintenance status, the number of

toilettes and all those characteristics for which the market expresses a market ratio or other indications of the market, and can apply the appraisal comparison procedures, such as the PDA.

13 Analysis of the Marginal Prices and Incomes of the Parameters

In the market comparison method, the appraisal functions are based on the collection of the market prices and rents (type of contract) in a definite market area (location), second the use of the property (destination). Consequently the parameters of the market segment insured into the appraisal function are essentially: the type of property and the type of building. The type of property indicates whether it is a contract relating to land and buildings; if the property is in a market of used property, the renovated or restored, new or almost new; whether it is in a condominium unit or exclusive ownership. The type of building refers to the character of the building or construction, or indicate if it is reinforced concrete structure, masonry, metal structure, or mixed; if it is multi-storey building, house, warehouse, shed, building complexes or other. Also other parameters of the market area can be taken into account (d'Amato 2010). The appraisal comparison is conducted between market segments that make up the market area and is carried out with the set parameters, including the parameters for which market segments differ in the area and excluding the parameters coeteris paribus. The marginal prices and incomes of the segment parameters are generally expressed in percentage and are taken positive or negative in relation to their effect on the market price. The valuation of the adjustment of a parameter answers the question: in what percentage the market price or income level of comparable segment differs from the level of the price or income of the segment for which we want to build a function, being other parameters coeteris paribus? To answer this question one must consider that quantitative market information is available about the parameters, the market ratios and the average levels indicative of prices and incomes. In practice, this information may take various forms and different employment opportunities and are assessed with the information and knowledge of the market operators. They are in fact indicative measures sometimes referred to wider contexts of the market area, which are not considered in the valuation for their absolute amount but in a relative sense compared to their mutual ratios. The marginal prices and incomes of the segment's parameters are generally estimated with the PDA, by comparing two or more segments that have equal amounts for all parameters except for the one that we should estimate the marginal price or income. It is a practical procedure that can be applied in situations of data availability and of substantially equal conditions. Specifically, the marginal price and the income of the segment's parameter could be based on the ratio between the supply prices or the quotations of properties represented by insertions (Active Listings) (Conditionally Sold Listings) and possibly

by other atypical lists (Expired/Suspended/Terminated Listings), while not recognizing any interest in the valuation, because they don't meet the definition of market value and rent, but admitting a meaning to their ratio that is the adjustment percentage (Salvo et al. 2013a, b). So for example, the marginal price of the parameter type of property can be based on the ratio between the average supply prices $\bar{p}_N$ for new buildings and the supply prices of used buildings $\bar{p}_U$, being the other parameters coeteris paribus, as follows in the Formula (46):

$$\Pi_{NU} = \frac{\bar{p}_N - \bar{p}_U}{\bar{p}_U} \tag{46}$$

The PDA can be used in an extended form to calculate the marginal price of a parameter of the segment. As an example, in the comparison between two market segments 1 and 2, in which were recorded market prices P_1 and P_2 of the two properties, which differ in the parameter segment g, considering only the surface characteristics, being coeteris paribus other real estate characteristics and parameters of market segment (excluding the parameter), the marginal price of the parameter of segment q_g according to the Formula (39), is equal to Formula (47):

$$q_g = \frac{P_1 - P_2 + p_1 \cdot [s_{21} - s_{11} + \pi_2 \cdot (s_{22} - s_{12}) + \lambda \cdot (s_{23} - s_{13})]}{X_{1g} - X_{2g}} \tag{47}$$

where:

s_{11} main surface of the first property;
s_{12} secondary surface of the first property;
s_{13} outside surface of the first property;
s_{21} main surface of the second property;
s_{22} secondary surface of the second property;
s_{23} outside surface of the second property;
X_{1g} parameter of the segment of the first property;
X_{2g} parameter of the segment of the second property;
p_1 marginal price of the main surface;
π_2 market ratio of secondary surface;
λ impact of the built area.

The PDA then work adjustments of the parameters of the market segments comparable to that reference. The appraisal comparison between the market segments is carried out on the parameters specified in the definition of the market areas. The marginal prices and incomes relate to the specific knowledge of the market area, of the segments and then of the market as a whole.

14 Cost Method

The application of the cost method falls into the fourth situation in which we do not have any data property (market price and market rent).

The method of the depreciated reconstruction costs, or cost method, consider a property built into its component parts: the built land and buildings. The cost method is based on a comparison of the property to be appraised and similar properties, considering the characteristics of the area and differences in age, in the state and utility buildings (Salvo et al. 2015; Ciuna 2010, 2011).

The cost method is composed of three elements:

- the valuation of the market value of the built land;
- the valuation of the reconstruction cost of buildings, which insists on the ground;
- the valuation of the depreciation.

The main procedures for appraising the market value of a built land are based on the survey of: (a) the market price of the land built; (b) the market price of the building areas; (c) the market price of the constructed real estate properties (defined as land and buildings).

The appraisal function of market value, based on observed market prices of land built (a), adds to the general function of the Formula (1) referred to the built land, the cost of rebuilding of the property net of depreciation. The appraisal function of the market value V_0 of the property to be appraised according to the Formula (5) is equal to Formula (48):

$$V_0 = V_{T0} + c_R \cdot (1 - d) \cdot z = L_0 + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} + c_R \cdot (1 - d) \cdot z \tag{48}$$

where:

V_{T0} market value of the built land;
c_R unit cost of rebuilding of the building;
z consistency of the building (area, volume, etc.);
d depreciation percentage presented by the building.

The appraisal function of the market value of the built land based on the collection of market prices of building land (b) considers the potential transformation of the land into building area by deducting the cost of demolition. The appraisal function of the market value of the land built subtracts to the general function of the Formula (5) refers to the building, the cost of demolition and adds the rebuilding cost of the property net of depreciation. The appraisal function of market value V_0 of the property to be appraised according to the Formula (5) is equal to Formula (49):

$$V_0 = V_{A0} - c_D \cdot z + c_R \cdot (1 - d) \cdot z \tag{49}$$

where:

V_{A0} market value of the built land;
c_D unit cost of demolition of the building.

The appraisal function for the market value of the built land based on the collection of market prices of the properties (c) includes the impact of the built land on the market value of the property (land and building). The appraisal function of the market value of the built land considering the function of the general Formula (1) refers to the property, multiplying it by the ratio of built land and adding the cost of rebuilding of the property net of depreciation. The appraisal function of the market value V_0 of the property to be appraised according to the Formula (5) is equal to Formula (50):

$$V_0 = V_{I0} \cdot \lambda + c_R \cdot (1 - d) \cdot z \tag{50}$$

where:

V_{I0} market value of the property;
λ percentage of built land on the market value of the property.

In the cost method the appraisal function for the market rent is based on the appraisal function of the market value, considering the capitalization rate of the built land and the capitalization rate of the building. The appraisal function of the market rent in the cost method follows the cases of the market value of the built land, based on the survey of market prices of built land, building areas and constructed buildings (Salvo et al. 2014). In synthetic terms according to the previous cases, in the cost method the appraisal of the market rent R_0, based on the survey of market prices of land built (a), according to the Formula (48) is as follows in the Formula (51):

$$R_0 = V_{T0} \cdot i_T + c_R \cdot (1 - d) \cdot z \cdot i_F \tag{51}$$

where:

i_T annual capitalization rate of built land;
i_F annual capitalization rate of the building.

In the cost method, the appraisal function of the market rent R_0, based on the survey of market prices of building land (b) according to the Formula (49) is the following Formula (52): on the survey of prices in the real estate mar

$$R_0 = (V_{A0} - c_D \cdot z) \cdot i_T + c_R \cdot (1 - d) \cdot z \cdot i_F \tag{52}$$

In the cost method, the appraisal function of the market rent R_0, based on the detection of market prices of the properties (c) according to the Formula (50) is the following Formula (53):

$$V_0 = V_{I0} \cdot \lambda \cdot i_T + c_R \cdot (1 - d) \cdot z \cdot i_F \tag{53}$$

In the cost method, the appraisal function of the market value and the market rent is based on the survey of prices in the real estate market (Ciuna et al. 2015a, b).

15 Income Capitalization Method

The income capitalization method is applied in a situation where there are not prices of comparable properties, but there are one or more market rents. The income capitalization method (or approach based on the expected results) considers the ability of a property to generate an income. The method is based on a comparison of the property to be appraised and market rents for similar properties known and on the conversion of income into capital value using a capitalization rate. The direct capitalization method consists of two elements: the valuation of market rent of the property through the appraisal function; the search of the capitalization rate. In the direct capitalization method, the general form of the function to estimate the market value according to the Formula (1) considers the capitalization rate of the property. The function of the estimated market value V_0 of the property to be appraised according to the Formula (19) is equal to Formula (54):

$$V_0 = \frac{R_0}{i} = \frac{l_0 + \sum_{f=1}^{n} r_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g}}{i} \tag{54}$$

where:

i annual capitalization rate of the property.

The capitalization rate is the ratio between the income and the market price of a property. His research is based on data collection in the real estate market. According to the valuation standards, the search of the capitalization rate should reflect only the data and information of the real estate market (Biłozor and Renigier-Biłozor 2016). The direct capitalization method can be used in the inverse formulation designed to calculate the market rent once we know the market value and the capitalization rate. In this circumstance the appraisal function of the market rent in accordance with the Formula (1) considers the capitalization rate of the property. The appraisal function of market rent R_0 of the property to be appraised according to the Formula (5) is equal to Formula (55):

$$R_0 = V_0 \cdot i = (L_0 + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g}) \cdot i \tag{55}$$

In the direct capitalization method, the appraisal functions of the market value and the market rent are based on the survey of prices and rents of the real estate market (Renigier-Biłozor et al. 2014b; d'Amato 2015).

16 Appraisal Test

In general, the accuracy of the valuation of the market value and rent is related to the uniformity of conditions related to the market segment and the degree of similarity of the characteristics of comparable properties. Under these conditions the segment parameters (location, type, etc.) and for some real estate properties (e.g., access, age, etc.) are coeteris paribus. Those characteristics and parameters therefore are excluded from the estimative analysis; while for the characteristics that have different modality (e.g. the surface, the outer area, etc.) and which are introduced in the analysis, the accuracy of the valuation increases with decreasing variability. In other words, the valuation is much more accurate than most similar are the properties comparable to the property being appraised. The accuracy of the appraised value and the market rent in the market area, which include multiple market segments, it is linked to the characteristics of the comparable properties collected and the parameters for which the segments differ. For the estimative analysis, the parameters of the market segment can be considered macro-characteristics, the choice and the measurements of which influence the precision of the valuation. One the real estate market area should therefore include market segments, different for the fewest number of parameters. The appraisal procedures must be tested to ensure that they have achieved the required standard for their use. This is done through the ratio study and the diagnostic statistics, in which the estimated values are compared to observed prices (prices and market rents). It is a subsequent verification of the discrepancy between the predictions and the data collected (d'Amato and Kauko 2012). The appraisal error can be measured by various indices and percentages. The measures of the performance valuations take into consideration: the appraisal level, referred to the difference between the values (or incomes) and the appraised market prices (or rents) of a defined group of properties; and the valuation consistency, which regards the equity within a group of properties and between groups of real estate properties. The uniformity between the groups can be analyzed in terms of horizontal equity and vertical equity. The measures of performance are tested with the minimum and maximum indices and coefficients of verification (IAAO 1999; Kauko and d'Amato 2011).

17 Conclusions

The appraisal model of indicates the basis for the valuation in the market value and the market rent of residential properties and special destination. The appraisal model proposes the use of the universal "appraisal functions" of the market value and rent related to the market area, consisting of one or more market segments. The shape of the appraisal function is linear and report the real estate characteristics and the parameters of real estate segment. The peculiarity of the appraisal model is the ability to build the prediction function with the statistical models and with appraisal procedures depending on the availability of market data (d'Amato 2004). For a sufficiently large number of data samples for the construction of a statistical model, the appraisal function is an equation of multiple linear regression. The uniequational statistical model can also be calculated in a mathematical-statistical manner different by the analysis of regression, provided it contains the marginal prices and incomes of real estate properties and parameters of the segment. For samples of market data, few in number, which can not be treated statistically, for only one datum or in the absence of data, the appraisal function is determined with the appraisal procedures (market comparison method, cost method and direct capitalization method) (Simonotti et al. 2015). The appraisal model in fact allows to determine the appraisal function for the areas of real estate market without data operating on the marginal prices of the parameters of the segment, on the appraisal functions of the market areas, upstream, on prices and rents of the next market areas. The appraisal model aims to provide a uniform procedure for value through the modular appraisal functions, which form a system of interrelationships between the market areas, between the data and the market information, including the estimative and statistical procedures and between statistical and estimative verification tests. The appraisal model is in accordance with the international valuation standards. The appraisal model, based on the appraisal functions, presents: uniformity of application regarding the estimation of market values and incomes for all properties taking place solely and evenly with the appraisal functions; immediate understanding by the operators for which the linear prediction functions, which show the locational factor and prices and incomes, the property's characteristics and parameters of the segment, allowing comparisons and tests; modularity regarding the possibility of calculating the appraisal function with statistical procedures and appraisal methods. The appraisal model based on the appraisal functions is offered as a simple and economical model for the estimation of the market value and rent of the property.

References

Biłozor, A., & Renigier-Biłozor, M. (2016), The procedure of assessing usefulness of the land in the process of optimal investment location for multi-family housing function. In *World Multidisciplinary Civil Engineering-Architecture-Urban Planning Symposium*. WMCAUS 2016 Praga.

Borst, R. A., Des Rosiers, F., Renigier, M., Kauko, T., & d'Amato, M. (2008). Technical comparison of the methods including formal testing accuracy and other modelling performance using own data sets and multiple regression analysis. In T. Kauko, M. d'Amato (Eds.), *Mass appraisal an international perspective for property valuers*. Wiley Blackwell.

Ciuna, M. (2010). L'Allocation Method per la stima delle aree edificabili. *AESTIMUM*, *57*, 171–178.

Ciuna, M. (2011). The valuation error in the compound values. *AESTIMUM* [S.l.], 569–583, Aug. 2013. ISSN:1724-2118.

Ciuna, M., & Simonotti, M. (2011). Linee guida per la rilevazione dei dati del mercato immobiliare. *Geocentro*, nn.15 e 16.

Ciuna, M., & Simonotti, M. (2014). Real estate surfaces appraisal. *AESTIMUM, 64*, Giugno, pp. 1–13.

Ciuna, M., Salvo, F., & Simonotti, M. (2014a). The expertise in the real estate appraisal in Italy. In *Proceedings of the 5th European Conference of Civil Engineering (ECCIE '14). Mathematics and Computers in Science and Engineering Series* pp. 120–129, North Atlantic University Union, ISSN:2227-4588, Firenze, 22/24 November 2014.

Ciuna, M., Salvo, F., & De Ruggiero, M. (2014). Property prices index numbers and derived indices. *Property Management, 32*(2), 139–153. doi:(Permanent URL):10.1108/PM-03-2013-0021.

Ciuna, M., Salvo, F., & Simonotti, M. (2014b). Multilevel methodology approach for the construction of real estate monthly index numbers. *Journal of Real Estate Literature, 22*(2), 281–302.

Ciuna, M., Salvo, F., & Simonotti, M. (2015a). Compensation appraisal processes for the realization of hydraulic works in an agricultural area. In *Proceedings of XLIV INCONTRO DI STUDI Ce.S.E.T. Il danno. Elementi giuridici, urbanistici e economico-estimativi* (pp. 69–82). Bologna, Italy. November 27–28, 2014. ISBN:978-88-99459-21-5.

Ciuna, M., Salvo, F., & Simonotti, M. (2015b). Parametric measurement of partial damage in building. In *Proceedings of XLIV INCONTRO DI STUDI Ce.S.E.T. Il danno. Elementi giuridici, urbanistici e economico-estimativi* (pp. 171–188). Bologna, Italy. November 27–28, 2014. ISBN:978-88-99459-21-5.

d'Amato, M. (2004). A comparison between RST and MRA for mass appraisal purposes. A case in Bari. *International Journal of Strategic Property Management, 8*, 205–217.

d'Amato, M. (2008). Rough set theory as property valuation methodology: The whole story, Chap. 11. In T. Kauko, M. d'Amato (Eds.), *Mass Appraisal an International Perspective for Property Valuers* (pp. 220–258). Wiley Blackwell.

d'Amato, M. (2010). A location value response surface model for mass appraising: An "iterative" location adjustment factor in Bari, Italy. *International Journal of Strategic Property Management, 14*, 231–244.

d'Amato, M. (2015). Income approach and property market cycle. *International Journal of Strategic Property Management, 29*(3), 207–219.

d'Amato, M., & Kauko, T. (2008). Property market classification and mass appraisal methodology, Chap. 13. In T. Kauko, M. d'Amato. (Eds.), *Mass appraisal an international perspective for property valuers* (pp. 280–303). Wiley Blackwell.

d'Amato, M., & Kauko, T. (2012). Sustainability and risk premium estimation in property valuation and assessment of worth. *Building Research and Information, 40*(2), 174–185 (March–April 2012).

d'Amato, M., & Siniak, N. (2008). Using fuzzy numbers in mass appraisal: The case of belorussian property market. In T. Kauko, M. d'Amato (Eds.), *Mass appraisal an international perspective for property valuers* (Chap. 5, pp. 91–107). Wiley Blackwell.

IAAO. (1999). *Standard on ratio studies*.

IAAO. (2003). *Standard on automated valuation models (AVM$_S$)*.

IVSC. (2011). *International valuation standards*. London: IVSC.

Kaklauskas, A., Zavadskas, E. K., Kazokaitis, P., Bivainis, J., Galiniene, B., d'Amato, M., et al. (2012a). Crisis management model and recommended system for construction and real estate. In N. T. Nguyen, B. Trawi´nski, R. Katarzyniak, & G.-S. Jo (Eds.), *Advanced methods for*

computational collective intelligence in studies in computational intelligence series edited by Janusz Kacprzyk (pp. 333–343). Berlin: Springer Verlag.

Kaklauskas, A., Daniūnas, A., Dilanthi, A., Vilius, U., Irene, L., Gudauskas, R. et al. (2012b). Life cycle process model of a market-oriented and student centered higher education. *International Journal of Strategic Property Management, 16*(4), 414–430.

Kauko, T., & d'Amato, M. (2008a). Introduction: Suitability issues in mass appraisal methodology. In T. Kauko, M. d'Amato. (Eds.), *Mass appraisal an international perspective for property valuers* (pp. 1–24). Wiley Blackwell.

Kauko, T., & d'Amato, M. (2008b). Preface. In T. Kauko, M. d'Amato, (Eds.), *Mass appraisal an international perspective for property valuers* (p. 1), Wiley Blackwell.

Kauko, T., & d'Amato, M. (2011). *Neighbourhood effect, international encyclopedia of housing and home*. Elsevier Publisher.

Renigier-Biłozor, M., & Biłozor, A. (2016a). Proximity and propinquity of residential market area —Polish and Italian case study. In *16th International Multidisciplinary Scientific Geoconferences SGEM*. Bułgaria. (web of science).

Renigier-Biłozor, M., & Biłozor, A. (2016b). The use of geoinformation in the process of shaping a safe space. In: *16th International Multidisciplinary Scientific Geoconferences SGEM*. Bułgaria.

Renigier-Biłozor, M., Wiśniewski, R., Biłozor, A., & Kaklauskas, A. (2014a). Rating methodology for real estate markets—Poland case study. *Pub. International Journal of Strategic Property Management. 18*(2), 198–212. ISNN:1648-715X.

Renigier-Biłozor, M., Dawidowicz, A., & Radzewicz, A. (2014b). An algorithm for the purposes of determining the real estate markets efficiency in Land administration system. *Public Survey Review, 46*(336), 189–204.

Salvo, F., De Ruggiero, M. (2011). Misure di similarità negli Adjustment Grid Methods. *AESTIMUM, 58*, 47–58. ISSN:1592-6117.

Salvo, F., De Ruggiero, M. (2013). Market comparison approach between tradition and innovation. A simplifying approach. *AESTIMUM, 62*, 585–594, ISSN:1592-6117.

Salvo, F., Ciuna, M., & d'Amato, M. (2013). Appraising building area's index numbers using repeat values model. A case study in Paternò (CT). In *Dynamics of land values and agricultural policies*. (p. 63–71), Bologna: Medimond International Proceedings, Editografica, ISBN:978-88-7587-690-6, Palermo, 22–23/11/2012.

Salvo, F., Ciuna, M., & d'Amato, M. (2013). The appraisal smoothing in the real estate indices. In: (a cura di): *Maria Crescimanno, Leonardo Casini and Antonino Galati, Dynamics of land values agricultural policies* (pp. 99–111), Bologna: Medimond Monduzzi Editore International Proceeding Division, ISBN:978-88-7587-690-6, Palermo, 22–23/11/2012.

Salvo, F., Ietto, F., & Cantasano, N. (2014). The quality of life conditioning with reference to the local environmental management: A pattern in Bivona country (Calabria, Southern Italy). *Ocean and Coastal Management*, ISSN:0964–5691, doi:10.1016/j.ocecoaman.2014.10.014.

Salvo, F., Francini, M., & Palermo, A. (2015). The "Urban Damage" into the description of the plan's alternative equalization addresses for mitigate the effects. In (a cura di): Alessandra Castellini Lucia Devenuto, *Proceedings of XLIV INCONTRO DI STUDI Ce.S.E.T. Il danno. Elementi giuridici, urbanistici e economico-estimativi* (pp. 189–206). Mantova: Universitas Studiorum S.r.l., Bologna, 27/28 November 2014.

Salvo, F., Ciuna, M., Simonotti, M. (2015). Compensation appraisal processes for completion of hydraulic works in an agricultural area. In (a cura di): *Alessandra Castellini Lucia Devenuto, Proceedings of XLIV INCONTRO DI STUDI Ce.S.E.T. Il danno. Elementi giuridici, urbanistici e economico-estimativi* (pp. 69–82). Mantova: Universitas Studiorum S.r.l., Bologna, 27/28 November 2014.

Salvo, F., De Ruggiero, M., Zupi, M. (2015). The valorization of public real estate. A first outcome of the experiences in progress and a methodological proposal. In *XLIII INCONTRO DI STUDI Ce.S.E.T. Sviluppo economico e nuovi rapporti tra agricoltura, territorio e ambiente. AESTIMUM* (Vol. 67, pp. 135–146). FIRENZE: Firenze University Press, ISSN:1592-6117, Verona, 21/23 november 2013.

Simonotti, M. (1985). La comparazione e il sistema generale di stima. *Rivista di economia agraria, 4*, 543–561.

Simonotti, M. (1998). *La segmentazione del mercato immobiliare per la stima degli immobili urbani*. Roma: Atti del XXVIII Incontro di studio CeSET.

Simonotti, M. (2001). I rapporti estimativi e le funzioni di stima. Estimo e territorio, n. 9.

Simonotti, M., Salvo, F., Ciuna, M. (2015). Appraisal value and assessed value in Italy. *International Journal of Economics and Statistics, 3*, 24–31. ISSN:2309-0685.

Salvo, F., Simonotti, M., Ciuna, M., & De Ruggiero, M. (in print). Measurements of rationality for a scientific approach to the Market Oriented Methods. *Journal of Real Estate Literature*. ISSN:0927-7544.