Newtonian and Non-Newtonian Pulsatile Blood Flow in Arteries with Model Aneurysms

S. Tabakova, P. Raynov, N. Nikolov and St. Radev

Abstract The cardiovascular diseases depend directly on the blood flow dynamics. The mathematical modeling and numerical simulations are expected to play an important role to predict the genesis of the atherosclerosis and the formation and rupture of the aneurysms. In the present work the numerical solutions for the oscillatory flow velocity due to the Newtonian and the non-Newtonian (Carreau) model are constructed for a straight long tube and for a tube (artery) with a model aneurysm. The numerical solutions are obtained by the finite-difference method (FDM) for the straight tube and by the software ANSYS/FLUENT for both geometries. The numerical results obtained by the ANSYS/FLUENT for a straight long tube are validated by the analytical and numerical solutions using the FDM for the Newtonian and Carreau models for different Womersley numbers, correspondent to different tube radii. The obtained peak wall shear stresses from the oscillatory flow in the straight long tube are lower than those in the tube with the model aneurysm, which can be used as an indicator for further clinical examinations.

S. Tabakova (✉) · N. Nikolov · St. Radev
Institute of Mechanics, BAS, Acad. G. Bontchev str., bl. 4, 1113 Sofia, Bulgaria
e-mail: stabakova@gmail.com

N. Nikolov
e-mail: n.nikolov@imbm.bas.bg

St. Radev
e-mail: stradev@imbm.bas.bg

S. Tabakova
Technical University - Sofia, Branch Plovdiv, 25 Tzanko Djustabanov str.,
4000 Plovdiv, Bulgaria

P. Raynov
University of Food Technologies, 26 Maritsa Blvd., 4000 Plovdiv, Bulgaria
e-mail: plamsky@mail.bg

1 Introduction

In most cases the in vivo measurement techniques are unable to prevent the cardio-
vascular diseases evolution, which depends directly on the blood flow dynamics. One
of the most dangerous diseases is that of the formation and rupture of different artery
aneurysms. The study of the blood flow in tubes can be treated as a flow model in
different types of arteries.

The blood is a suspension of particles and plasma, which has a non-Newtonian
character as a fluid. It is a typical representative of the shear thinning fluids with
an apparent viscosity dependent on the shear rate, i.e. its viscosity continuously
decreases or increases with the shear rate increase or decrease reaching two different
upper and lower plateaus independent of the further change of the shear rate. Several
non-Newtonian models are used to express the blood rheology: the Carreau model
[1–5], the Carreau-Yasuda model [1, 6–9], the Casson model [1, 3, 8], the Power law
model [1, 3, 4] and others. Some of these models, such as that of Carreau, give a non-
linear dependence of the shear stress on the shear rate. Since the shear rate changes
significantly in the arteries with non-constant cross section, the viscosity could not be
taken as a constant. This means that there are no analytical solutions for the blood flow
in arteries. The proper knowledge of the viscosity leads to a proper knowledge of the
Wall Shear Stresses (WSS), which are of a major importance for the prediction of an
aneurysm rupture. The problem becomes more complicated if the pulsatile character
of the blood flow is considered. It occurs that the blood flow can be approximated with
the Newtonian fluid flow in the larger arteries, e.g. in the aorta, while in the narrow
arteries the non-Newtonian character of the blood flow is essential. The analysis of
non-Newtonian flows in infinitely long tubes is very important when studying the
blood flow in different types of arteries. The well known analytical solution proposed
by Womersley [10] is often applied to approximate the pulsative velocity of blood
flow in arteries, when the fluid is regarded as Newtonian. However, if non-Newtonian
models are applied for the blood viscosity, the flow solution can be obtained only
numerically. For example, the Lattice Boltzmann Method is applied in [8] for the 2D
oscillatory Newtonian and non-Newtonian flows in straight and curved tubes. The
viscosity is given by the models of Casson and Carreau-Yasuda. The authors show
that the difference between the velocity and the Wall Shear Stresses (WSS) calculated
by the Newtonian viscosity model and by the non-Newtonian models increases with
the decrease of the Womersley number, which expresses the relation between the
oscillatory inertia and viscous forces.

In this paper we investigate numerically the non-Newtonian oscillatory flow of
blood in a long straight tube and in a tube with a model aneurysm, using the Car-
reau viscosity model. The numerical simulations are performed by the software
ANASYS/FLUENT for different Womersley numbers correspondent to different
tube radii. The obtained solution for the velocity in the straight tube will be compared
with the analytical solution for the Newtonian fluid model and with the numerically
obtained solution (by the FDM) for the Carreau viscosity model given in [11].

2 Problem Statement

The blood is assumed incompressible with constant density ρ and apparent viscosity μ_{app} (constant for the Newtonian blood model and defined by a non-linear function of the shear rate for the non-Newtonian blood model).

Two different axisymmetrical geometries are considered in cylindrical coordinates (x, r, φ), where x is the axial coordinate: a straight circular tube with radius R and a circular tube with entry radius R and a model aneurysm given by the Gaussian shape function [12], as shown in Fig. 1:

$$r(x) = R + H \exp(-\frac{x^2}{2W^2}), \tag{1}$$

where H and W are the aneurysm height and width.

The equations of motion and continuity in a vector form are:

$$\rho(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}) = -\nabla p + \nabla \cdot \mathbf{T}, \tag{2}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{3}$$

where $\mathbf{v} = (u, v, w)$ is the velocity vector, p is the pressure, $\mathbf{T} = f(\dot{S})$ is the viscous stress tensor, with $\dot{S}$—the shear rate tensor.

For a very long (infinite) tube, we obtain $v = w = 0$ and $u = u(r, t)$ from the Eq. (3). In this case the shear stress tensor has only one non-zero term $\tau = \mu_{app}(\dot{\gamma})\dot{\gamma}$, where $\dot{\gamma} = \dfrac{\partial u}{\partial r}$. The system (2) transforms into a single equation for the axial velocity u:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r}(\mu_{app} r \frac{\partial u}{\partial r}) \tag{4}$$

Fig. 1 The Gaussian model of an aneurysm at $R = 1$, $H = R$ and $W = R$
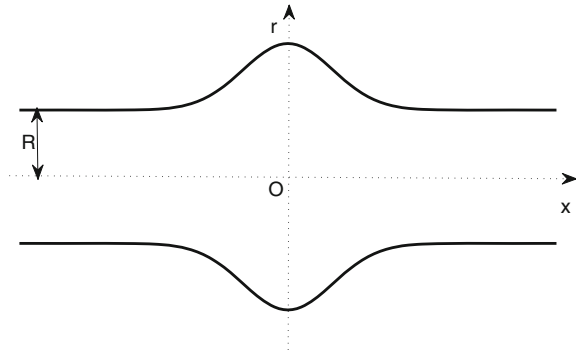
The boundary conditions for the velocity u are the no-slip condition $u = 0$ at $r = R$ and the symmetry condition $\dfrac{\partial u}{\partial r} = 0$ at $r = 0$. For the pressure gradient we consider the case of an oscillatory function in time $-\dfrac{\partial p}{\partial x} = A\cos(nt)$, where A is the pulse amplitude and n is the angular frequency.

The Carreau model of blood when treated as a non-Newtonian fluid is chosen with apparent viscosity μ_{app} that is usually given [2] by the following expression - further denoted by μ_c:

$$\mu_c = \mu_\infty + (\mu_0 - \mu_\infty)[1 + \lambda^2\dot{\gamma}^2]^{(n_c-1)/2}, \tag{5}$$

where λ and n_c are empirically determined. For human blood [2]: $\mu_0 = 0.056\,\mathrm{Pa\,s}$, $\mu_\infty = 0.00345\,\mathrm{Pa\,s}$, $\lambda = 3.313\,\mathrm{s}$ and $n_c = 0.3568$.

3   Analysis of the Results

3.1   Long Straight Tube

The Eq. (4) is dimensionlized using the following characteristic scales: R as a characteristic length ($r = RY$), $1/n$ as a characteristic time ($t = T/n$), μ_∞ as a characteristic viscosity ($\mu_c = \bar{\mu}_c\mu_\infty$):

$$\frac{1}{Y}\frac{\partial}{\partial Y}\left(\bar{\mu}_c Y\frac{\partial u}{\partial Y}\right) - \alpha^2\frac{\partial u}{\partial T} + \frac{R^2 A}{\mu_\infty}\cos(T) = 0, \tag{6}$$

where $0 \le Y \le 1$, $\dfrac{\partial u(0, T)}{\partial Y} = 0$, $u(1, T) = 0$ and $\alpha = R\sqrt{\dfrac{\rho n}{\mu_\infty}}$ is the Womersley number.

The analytical solution of Eq. (6) is the so called Womersley solution [10] for the Newtonian viscosity model:

$$u_n = Real\left[\frac{iA}{n\rho}\left(\frac{J_0(i^{3/2}\alpha Y)}{J_0(i^{3/2}\alpha)} - 1\right)exp(iT)\right], \tag{7}$$

where J_0 is the Bessel function of 'zero-th' order.

The presented here results are for a human carotid artery with a radius $R = 0.0031\,\mathrm{m}$ at blood density $\rho = 1000\,\mathrm{kg/m^3}$, pulse frequency of oscillations $n = 2.4\pi$ (correspondent to 72 heart beats per minute) and a pressure gradient amplitude $A = 6000\,\mathrm{Pa/m}$ (45 mm mercury column per meter). In this case $\alpha = 4.58$ and the maximum Reynolds number achieved during the blood oscillatory flow is around 530. The cross-section mean velocity based on the solution (7) is:
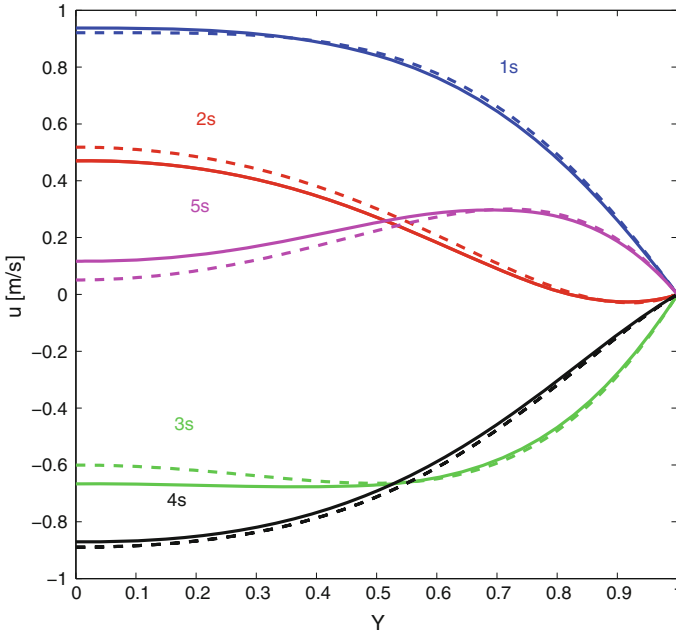
Fig. 2 Comparison between the axial velocities u of the Newtonian model (*dotted lines*) and of the Carreau model (*solid lines*) at different instants of time t: 1, 2, 3, 4, 5 s. (the different colors correspond to different times)

$$\bar{u}_n = 0.5868 \sin(T + 0.3605) \tag{8}$$

For the Carreau viscosity model Eq. (6) is solved numerically by the FDM of Crank-Nicholson with time and space steps $O(10^{-3})$ giving relative error $O(10^{-6})$. The numerical solution has been approximated to obtain its mean velocity:

$$\bar{u}_c = 0.5799 \sin(T + 0.3867) \tag{9}$$

Newtonian and Carreau velocities are presented in Fig. 2 for different times t. It is well seen the fast change of the velocity profile in time. However, the Carreau velocities are quite similar to the Newtonian ones, except in the symmetry axis region, i.e., near to $Y = 0$, which is due to the big difference in viscosities at the small velocity gradient there. As a whole, the difference between the two solutions increases with the decrease of the Womersley number, i.e., with the decrease of the tube radius (artery), found in our previous paper [11]. This has been observed also in the 2D case [8, 13, 14].

The WSS can be obtained from the velocity solution by the following formula:

$$WSS = \mu_{app} \frac{\partial u}{\partial r} \Big|_{r=R} \tag{10}$$

Table 1 Absolute values of the Wall Shear Stress (WSS)

	Newtonian (Pa)	Carreau (Pa)
1 s	3.25	3.5
2 s	0.88	0.95
3 s	3.75	4
4 s	1.5	1.7
5 s	2.85	3

The absolute values of the WSS for the considered example are given in Table 1 for the times $t = 1, 2, 3, 4, 5$ s.

The obtained peaks of WSS for the human carotid artery are in the experimental limits [15]: 2.5–4.3 Pa. The WSS of the Carreau model are slightly higher from those of the Newtonian model and are in a small phase shift, as found in [11].

The full system of equations (2) and (3) have been solved numerically by the software ANSYS/FLUENT for a straight long tube (1000 times longer than its radius) using a mesh of 40000 elements and 84042 nodes. The obtained results for the axial velocity u have been verified by the Womersley solution Eq. (7) (for the Newtonian fluid) and by the numerical solution found by the FDM (for the Carreau viscosity model). The relative error for both cases is less than 4 %.

3.2 Artery with Model Aneurysm

The numerical calculations of the Eqs. (2) and (3) in the case of the model aneurysm given by the shape formula (1) has been performed at $H = W = R = 0.0031$ m. The aneurysm is situated in the middle of the tube, which is long enough to achieve the straight tube flow (discussed in the previous subsection) in the regions before and after the aneurysm. Here the length is taken to be 0.62 m, such that the axial coordinate is -0.31 m $\leq x \leq 0.31$ m. The used mesh for the calculations with the ANSYS/FLUENT in this domain consists by 156000 cells and 160040 nodes. The boundary condition at the inlet of the tube for the velocity is to be equal to the mean axial velocity u_n from Eq. (8) for the Newtonian model and by u_c from Eq. (9) for the Carreau model. The other boundary conditions are a constant pressure at the outlet and the usual no-slip condition on the wall. The results show that besides the axial velocity u, the velocity vector has also a radial non-zero component v in the aneurysm region. The appearance of v is connected to the toroidal vortices in the aneurysm part. This can be seen from Fig. 3, where the velocity vectors are colored according to the axial velocity magnitude at time $t = 5$ s in the Newtonian viscosity case.

It is interesting to show the axial velocity distribution along the symmetry axis $r = 0$ for different times. In the aneurysm region its character is uneven depending
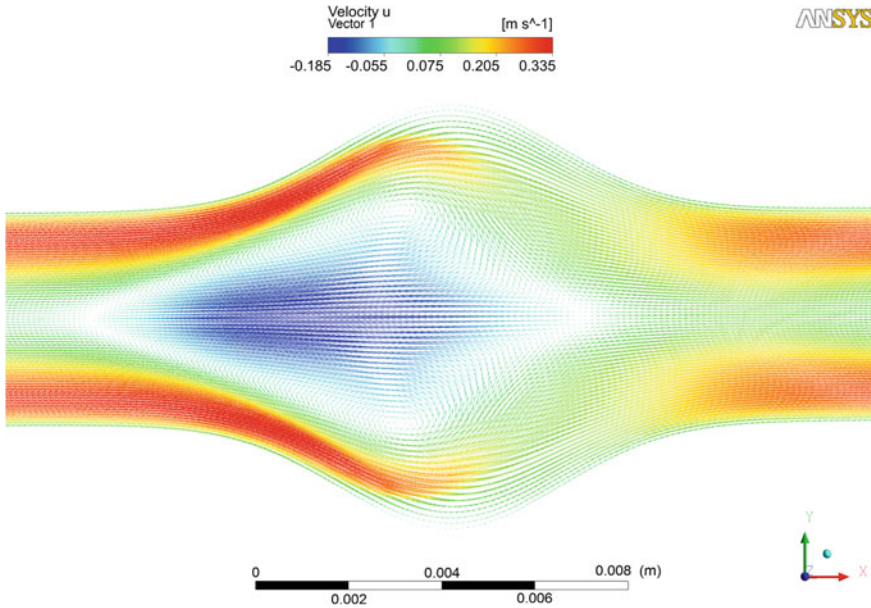
Fig. 3 The velocity vectors of the Newtonian model at time $t = 5\,\mathrm{s}$ colored by the magnitude of the axial velocity u

on the time instant, which is shown in Fig. 4. As it is seen the Newtonian and Carreau velocity profiles are similar in the straight tube region (before and after the aneurysm), which is in a good comparison with the corresponding plots in Fig. 2.

The WSS in the aneurysm region are calculated by the full shear rate, as the radial velocity as well as its gradient on the wall are non-zeros:

$$WSS = \mu_{app}(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r}) \mid_{r=R} \qquad (11)$$

The obtained absolute values of the WSS ($|WSS|$) in the tube with the aneurysm are quite different from those in the straight tube region, which is plotted in Fig. 5 for different times. The comparison between the results in Fig. 5 and those in Table 1 shows that some of the peak $|WSS|$ for the aneurysm are more than two times higher than those of the straight tube. In general the Carreau $|WSS|$ are slightly higher than the Newtonian ones, but in the aneurysm region they are almost equal in some places along the aneurysm width, while in others are quite different. This fact can be used as an indicator for further clinical studies.
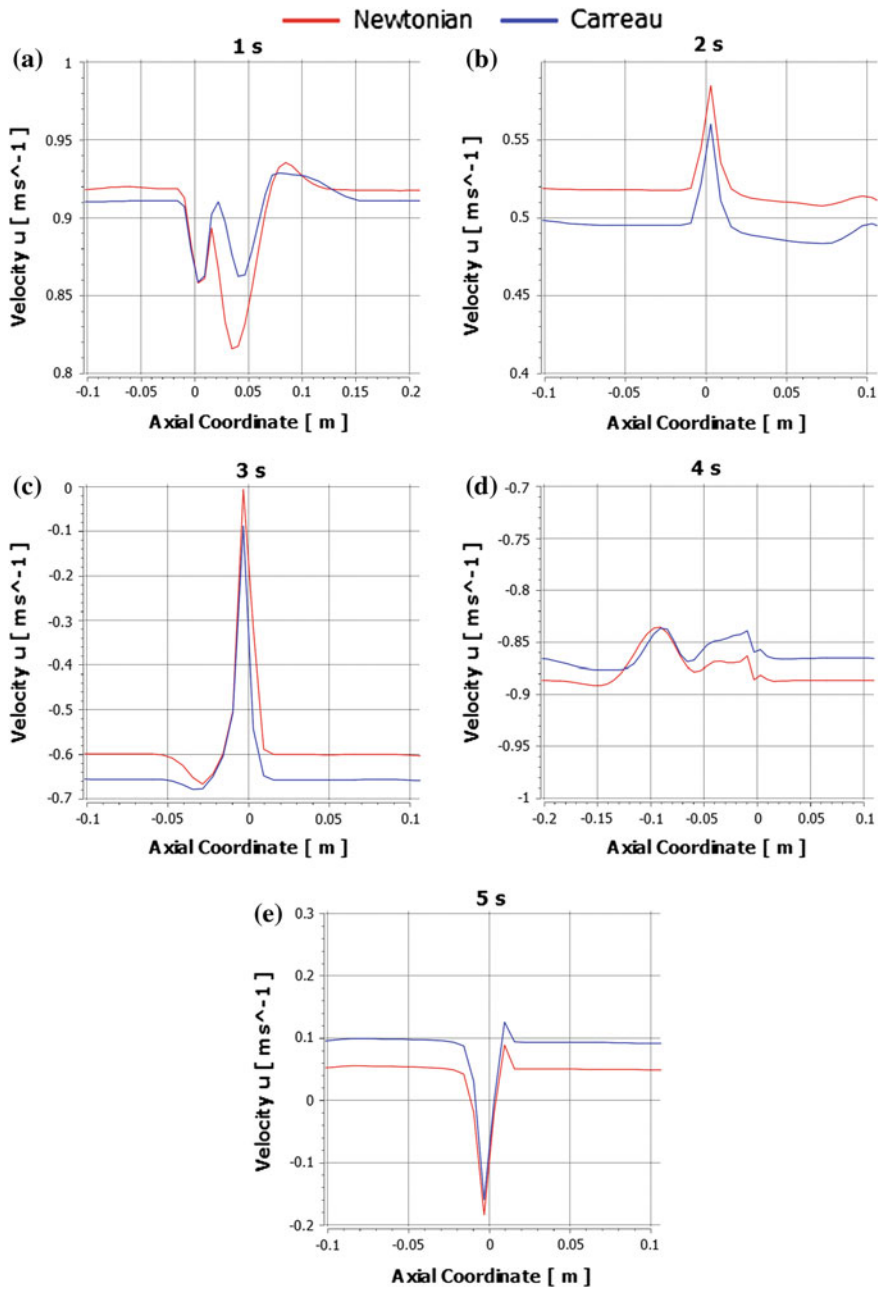
Fig. 4 Comparison between the axial velocities $u(x, 0, 0)$ on the centerline of the Newtonian model (*red lines*) and of the Carreau model (*blue lines*) at different instants of time t: (a) 1 s, (b) 2 s, (c) 3 s, (d) 4 s and (e) 5 s
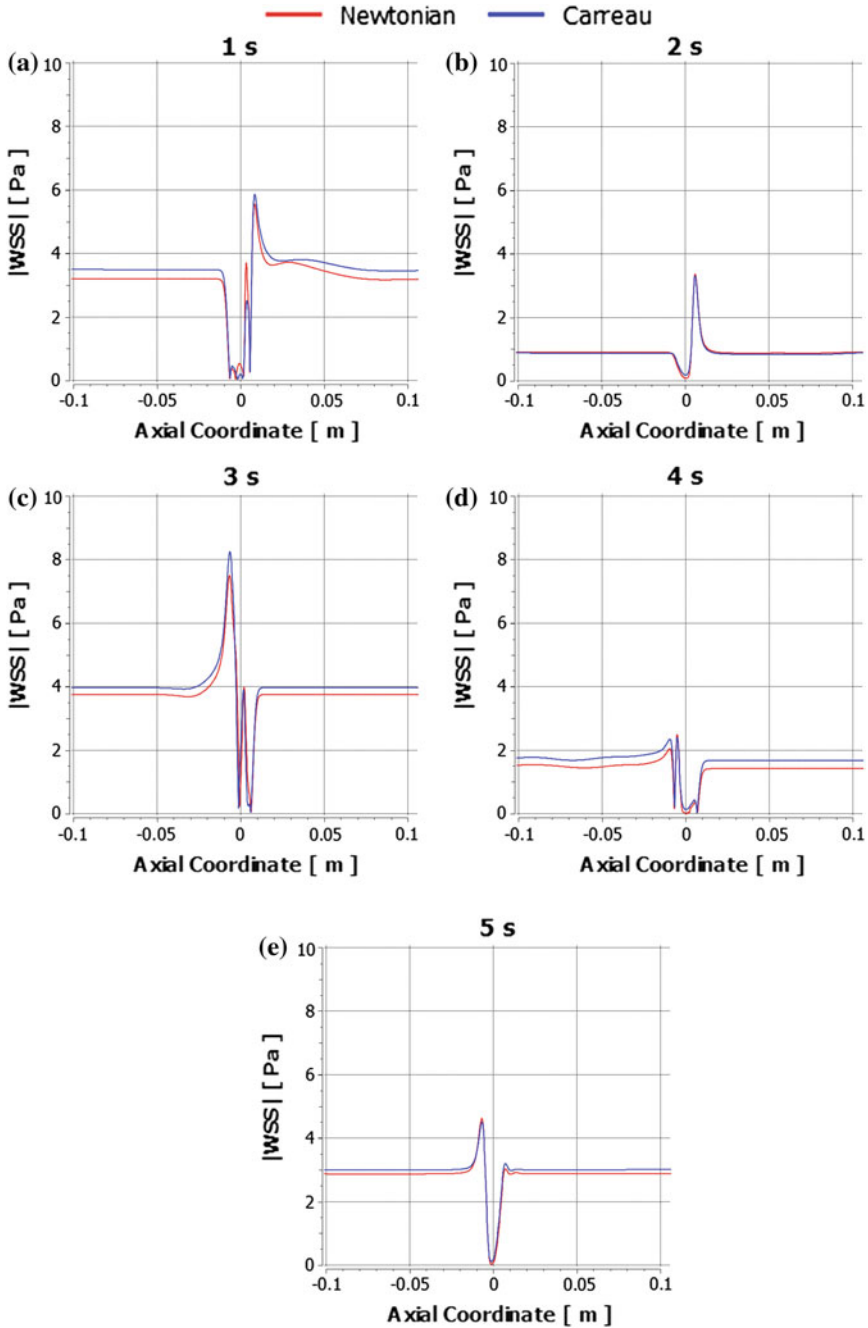
Fig. 5 Comparison between the absolute values of the WSS of the Newtonian model (*red lines*) and of the Carreau model (*blue lines*) at different instants of time *t*: (a) 1 s, (b) 2 s, (c) 3 s, (d) 4 s and (e) 5 s

4   Conclusion

The numerical solutions for the oscillatory flow velocity due to the Newtonian and Carreau model are constructed numerically for a straight long tube and for a tube (artery) with a model aneurysm. The numerical solutions are obtained by a finite-difference method (FDM) for the straight tube and using the software ANSYS/FLUENT for both cases. The numerical results obtained by the ANSYS/FLUENT for the velocity and WSS in a straight long tube are validated with the analytical and numerical solutions by FDM for Newtonian and Carreau models. The obtained peak WSS from the oscillatory flow in a tube with model aneurysm are higher than those in a straight long tube.

The obtained results for the Carreau model flow characteristics can be used for future studies with experimentally registered oscillatory pressure gradient. In order to predict the real WSS it is necessary to use the geometry of a patient based artery with aneurysm and to take into account the fluid structure interaction of the blood flow in a deformable artery, whose characteristics are based on experimental results of the wall artery structure.

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