Chapter 24
Gender Differences Concerning Pupils' Beliefs on Teaching Methods and Mathematical Worldviews at Lower Secondary Schools

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Abstract This article documents the development of a questionnaire concerning pupils' beliefs on teaching methods and mathematical worldviews. A representative poll leaded to some remarkable gender differences that are reported here as a first application of this questionnaire. These differences can be seen in more instructivist and less apply-oriented attitudes of the female group and more constructivist, process- and applied-oriented and less instructivist attitudes of the male group. Additionally, the constructivist and instructivist scales correlate positively only in the male group.

24.1 Pupil's Beliefs on Teaching Methods and Worldviews

The Swiss Conference of Cantonal Ministers of Education is planning a nation-wide assessment of basic competencies in mathematics at the end of compulsory education in grade 9 (cf. EDK 2015). This assessment is intended to take place in 2016. The School of Teachers Education Northwestern Switzerland is the leading house for constructing the mathematical tasks of the performance test and for a part of the context questionnaire. This questionnaire will contain a socio-demographic part and a "mathematical" part focused on pupils' attitudes, affects, emotions, and self-efficacy concerning the teaching and learning of mathematics. These kinds of pupils' beliefs are described e.g. by McLeod (1992) in summary. Studies like Schoenfeld (1989) indicate that they have an explorative power for pupils' mathematical performance. Hence, it became a common standard to accompany a mathematical performance test by a context questionnaire containing the topics mentioned above.

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The Swiss questionnaire will contain some scales that were already used in studies like TIMSS and PISA and that are mainly focused on self-efficacy and the motivational and emotional aspects of pupils' beliefs, affects, and attitudes (cf. e.g. OECD 2013, pp. 184–186 and 194). My intention was to broaden this questionnaire by two new topics: (a) pupils' preferences in teaching methods and (b) pupils' beliefs on mathematical worldviews. To create suitable scales, I carried out a first pretest in fall 2014 with 256 participants (cf. Girnat 2015). In spring 2015, I conducted a second larger and representative pretest with 956 participants to overhaul the scales and to include more covariates like gender, the type of school, and school marks. In this article, I give an overview on the scales and the findings of the second pretest related to gender differences. The focus is set on gender differences, since these differences are in general very typical for many aspects of mathematics education (cf. Gallagher and Kaufman 2005, Pajares and Graham 1999, for self-efficacy, motivation constructs, and mathematics performance; and cf. Stipek and Gralinski 1991, for achievement-related beliefs and emotional responses).

24.2 Setting Up the Scales

The basic idea for creating scales to measure pupils' preferences in teaching method is the antagonism of instructivist and constructivist teaching methods. According to Duit (1995), constructivist learning theories are based on the assumption that learning is a learner's active construction of knowledge related to his prior experiences and convictions. Insofar, constructivist learning environments are characterised by properties that are supposed to enforce these construction processes like pupil-centeredness, autonomy, inclusion of the pupils' prior knowledge, social negotiations of meanings, and possibilities to explore and discover insights by self-directed activities. Instructivist environments, on contrary, are marked by teacher-centeredness and a mostly passive understanding of the pupils' learning process focused on understanding and re-enacting teachers' explanations or examples and getting routine by solving series of similar tasks.

The first pretest in fall 2014 was designed to examine the factorial structure of 22 items that were intended to express typical aspects of instructivist and constructivist teaching methods. The participants were prompted to rate the items on a six step Likert scale of agreement/disagreement. The question was how useful they regarded the teaching method expressed by the items to learn mathematics. This is a difference to common scales on teaching methods where the question is what teaching methods are used in the classroom, and not how pupils value these methods (cf. e.g. OECD 2013, p. 194). After collecting the data, an exploratory factor analysis was carried out (cf. Tabachnick and Fidell 2001), following Horn's parallel analysis to determine the number of factors to extract (cf. Horn and Engstrom 1979). I used the psych package (Revelle 2015) with R (R Core Team 2014). The explorative factor analysis leaded to a five factor solution (cf. Girnat 2015): As expected, the instructivist items form a single scale (called instrlearn in the following), but the constructivist items were arranged into four different ones: learning by exploration and discovery (disclearn), using real-world situations to understand mathematics (realref), social learning and learning in groups (soclearn), and pupils' autonomous choice of tasks and topics (autlearn). The last factor autlearn was dropped, since there were no significant relations to the pupils' properties, like gender or marks, observable. Afterwards, some new items were created to gain almost the same amount of items for each scale in the second pretest. The following list contains all the items used in the second pretest. The items of the first pretest are marked by an *.

disclearn_1) I love to puzzle out solutions to tasks. I also love to solve tasks by trial and error.
disclearn_2*) I like tasks and problems that encourage me to discover different mathematical insights by myself.
disclearn_3) It's exciting when we discover how to solve a task on our own before the teacher has explained it to us.
disclearn_4*) In mathematics you can discover a lot on your own.
disclearn_5) In mathematics, you can come up with creative solutions without theoretical background knowledge.
disclearn_6*) In mathematics, you can fiddle and puzzle out a lot on your own. This is the best way to come to a solution.
disclearn_7) If you are working on a mathematical problem, you often come up with new insights spontaneously and automatically.
instrlearn_1) It is important that our teacher provides us with consistent rules, methods and notations, and that everyone then follows these precisely.
instrlearn_2*) I think it's useful to solve a lot of tasks in order to understand a method correctly.
instrlearn_3*) I learn mathematics well, if the teacher first demonstrates a new method and we then repeat this method with several tasks.
instrlearn_4*) I want to see rules and examples that show me how to solve my tasks.
instrlearn_5*) Doing exercises should be based on training the exact same method again and again until we can all handle the task.
instrlearn_6*) It's best if the teacher first demonstrates the solution of a task and we repeat his method step by step to solve the task afterwards.
instrlearn_7*) The teacher should present mathematical topics and methods to us. He shouldn't encourage us to discover them on our own.
soclearn_1*) I learn mathematics well, if we collaborate in groups to solve a problem and develop our own solution.
soclearn_2*) I prefer it if we as pupils explain to each other how to solve a task rather than the teacher explaining to us how to do it.
soclearn_3) I like to work in pairs or bigger teams.
soclearn_4*) I often understand a mathematical topic first, if I discuss it with classmates or colleagues.
soclearn_5) I understand mathematics better if we collaborate in groups as opposed to being shown by the teacher on the blackboard.
soclearn_6) I learn a lot when I work on a task together with other classmates.
realref_1*) When we introduce a new mathematical theme, I like to start with a real situation from everyday life and then explore the mathematical theme in this context.
realref_2*) I find it interesting to solve everyday life problems using mathematics.

realref_3*) Mathematical task don't need to be related to everyday life. I don't need such illustrations.
realref_4*) Tasks should always be related to everyday life. They shouldn't only relate to pure mathematics.
realref_5*) A mathematical theme only makes sense to me, if I can see how it helps to solve real-life problems from everyday contexts.
realref_6) Mathematical tasks should always be related to reality.

The second part of scales concerning mathematical worldviews is based on the ideas of Grigutsch et al. (1998). They introduced four basic dimensions to describe a teacher's mathematical worldview: The formalism aspect (concerning rigour, logic, deduction, formalism, and technical terms as typical characteristics of mathematics); the apply aspect (stressing the practical use of mathematics in everyday life, in the professional world, and for society); the process aspect (underlining the creativity of doing mathematics); and the scheme aspect (related to a standpoint that regards mathematics as a bound of rules, algorithms, and prescriptions to be followed). The items of the second part of my questionnaire are developed according to Grigutsch et al., but there were three differences: (1) the process aspect was integrated into the scale disclearn (as items 4, 5, 6, 7), since this aspect seems to be more related to the learning of mathematics than to its nature; (2) the "formalism aspect" was renamed into "system aspect", since formalism seems just to be one of its parts; (3) most of the items were linguistically simplified and they were adapted to the pupils' horizon of mathematical experiences. This seemed to be necessary, since the original items were created to investigate teachers' mathematical worldviews. The following list contains all the items used in the second pretest (the items already used in the first pretest are again marked with an *). The participants were asked to rate these items on a six step Likert scale of agreement/disagreement. The question was how strong they agreed that the content expressed by an item was a characteristic property of mathematics.

applyasp_1*) Mathematical knowledge is important for everyday life.
applyasp_2*) Mathematics is necessary for many occupations.
applyasp_3*) In mathematics education, we often deal with topics that have no practical use.
applyasp_4*) Many mathematical themes are of practical use.
applyasp_5*) Mathematics is important to our society.
applyasp_6*) Without mathematics, you won't get far.
schemasp_1*) You have to follow the teacher's examples and sample solutions exactly to manage your tasks successfully.
schemasp_2*) Ideally, the solution of a task looks the same in every pupil's exercise book.
schemasp_3*) To solve mathematical tasks successfully depends on having learnt the right methods off by heart. Otherwise you'll get lost.
schemasp_4) In mathematics education, it is most important to learn predefined ways of solving problems off by heart and to apply them correctly.
schemasp_5*) It's impossible to invent mathematics on your own. You depend on having mathematics shown and explained to you.
schemasp_6) You'll only be able to learn mathematics if someone explains you its methods and you imitate them.
systasp_1) It's necessary to understand mathematical methods. It's not enough just to apply them.

systasp_2*) A solution has to be written down in a formally correct notation to be correct.
systasp_3*) All parts of mathematics are systematically linked to each other.
systasp_4*) You have to be able to think logically and to justify theorems if you do mathematics.
systasp_5*) In mathematics it's important to use technical terms and conventional notations.
systasp_6) In order to understand new themes it is important to understand previous ones.
systasp_7) In mathematics, it is indispensible to know exactly what symbols and technical terms stand for.

24.3 Rechecking the Scales

The second pretest was used to recheck the scales. This was carried out in two steps: At first, an exploratory factor analysis was used to see if the result of the first pretest could be reproduced. Aside from two problematic items (realref_1 and applyasp_3), the number of factors and the assignment of the items to the factors could be reproduced. Secondly, for each factor or latent variable, a confirmatory factor analysis was carried out (cf. Brown 2006), using the lavaan package (Rosseel 2012) with R. In several cases, there was evidence that the one factor solution was not the best possibility and that it might advisable to split the single factor in two ones. Table 24.1 contains the fit indices for the measurement models. Due to the ordinal nature of the questionnaire's data, the diagonally weighted least squares method with a correction for the means and variances (WLSMV) was used to estimate the parameters and to set up the test statistics (cf. Beaujean 2014, p. 98 for the WLSMV method, and pp. 153–166 for the fit indices). In some cases, a two factor solution seems to be the better alternative. If so, both the single and the two factor solution are reported.

As Table 24.1 shows, all of the two factor solutions have got substantially better fit indices than the single factor solutions. However, statistical properties should never be the only reasons to prefer one model above the other. The choice of a model has also to be based on content to be valid. In the five cases of "split" factors, the two factor solutions also seem to contribute an enhancement with regard to the content: (1) The two factors of disclearn separate the state and trait aspect of learning by discovery (implying that it was no good idea to combine the attitudes to teaching methods with mathematical worldviews); (2) the items of instrlearn_a single this aspect of instructivism out that is related to repetitive exercises, whereas instrlearn_b addresses the instructions of the teachers; (3) soclearn_a is related to social arrangements in general, while soclearn_b stresses the communicative learning effect of social situations; (4) the difference between schemasp_a and _b is that schemasp_b specifically addresses the technique "learning by heart", whereas schemasp_a is more general; (5) systasp_a expresses the logical and systematic aspect of mathematics, whereas systasp_b is related to formal correctness. Concerning these analyses, the two factor solutions are to prefer.

Table 24.1 Fit indices of the confirmatory factor analysis of the measuring models

Scale	χ^2	df	p value χ^2	CFI	RMSEA	RMSEA <0.05	SRMR	Cor
disclearn	57.352	14	0.000	0.992	0.057	0.198	0.044	—
disclearn_a (1, 2, 3) disclearn_b (4, 5, 6, 7)	12.427	21	0.493	1.000	0.000	1.000	0.020	0.835
instrlearn	46.911	14	0.000	0.986	0.050	0.471	0.046	—
instrlearn_a (2, 3, 5) instrlearn_b (1, 4, 6, 7)	20.108	21	0.093	0.997	0.024	0.988	0.030	0.793
soclearn	41.192	9	0.000	0.986	0.062	0.134	0.046	–
soclearn_a (1,3) soclearn_b (2,4,5)	7.591	10	0.108	0.997	0.031	0.794	0.023	0.731
realref (2, 4, 5, 6)	1.102	5	0.576	0.995	0.000	0.929	0.010	–
applyasp	9.980	9	0.352	1.000	0.011	0.993	0.022	–
schemasp	27.617	9	0.001	0.990	0.047	0.559	0.037	–
schemasp_a (1, 2, 5, 6) schemasp_b (3, 4)	11.154	8	0.193	0.998	0.021	0.972	0.024	0.809
systasp	49.450	14	0.000	0.987	0.052	0.380	0.052	–
systasp_a (1, 3, 4, 6) systasp_b (2, 5, 7)	19.408	13	0.111	0.998	0.023	0.990	0.033	0.805

24.4 Gender Difference I: The Means

To investigate group difference between the means of latent variables, the first step consists in checking the strong or scalar invariance of the measurement models, i.e. that the loadings and intercepts can be treated as equal in all groups (cf. Beaujean 2014, pp. 61–69). For this task, the R package semTools (semTools Contributors 2015) was used. In every case, the strong invariance was given.

Table 24.2 contains the mean differences. To calculate the differences, all latent variables were standardised and the female group was set to be the reference group. Therefore, the female group always has zero as its mean, and the mean of the male group directly expresses the difference to the mean of the female group. Since the latent variables are standardised, the differences can be interpreted as effect sizes using the thumb rule that 0.2 indicates a small, 0.5 a medium, and 0.8 a strong effect

Table 24.2 Mean differences (using the female group as the reference group)

Scale	Mean difference male	Standard error of difference	*p* value mean difference	Correlation female	Correlation male
disclearn_a	0.685***	0.095	0.000	0.795	0.873
disclearn_b	0.296***	0.081	0.000		
instrlearn_a	−0.417***	0.081	0.000	0.750	0.851
instrlearn_b	−0.200**	0.075	0.007		
soclearn_a	0.179*	0.077	0.020	0.665	0.789
soclearn_b	0.115	0.076	0.131		
realref	0.200**	0.072	0.006	–	–
applyasp	0.285***	0.076	0.000	–	–
schemasp_a	0.172*	0.076	0.023	0.817	0.790
schemasp_b	0.162*	0.076	0.033		
systasp_a	−0.107	0.073	0.144	0.766	0.847
systasp_b	0.012	0.074	0.872		

The mean differences are marked with the usual asterisks to indicate the significance levels (* stands for $p \leq 0.05$, ** for $p \leq 0.01$, and *** for $p \leq 0.001$).

(cf. Cohen 1988). In case of the "split" factors, the table also contains correlation between the two factors separated for each group.

Nine of the twelve mean differences are significant; remarkably, the differences concerning the system aspect are not. Among the significant differences, the strongest effects are observable in the field of the constructivism/instructivism dichotomy: The male group prefer the explorative activities (disclearn_a) much more than the female group. The difference between disclearn_a and disclearn_b is interesting: disclearn_b stands for the conviction that mathematics "in general" is a field of creativity and discovery. In this case, the gender difference is small. Disclearn_a expresses the willingness to explore and discover mathematical insights on your own, i.e. including the motivational background and the cognitive, emotional or motivational obstacles like anxiety or low self-efficacy. In this case, the gender difference is the highest one observed in this study. This difference seems to be similar to the state/trait distinction in psychology, also regarded as relevant for mathematical beliefs (cf. Stipek and Gralinski 1991). Furthermore, the female group estimates instructivist teaching method higher than the male group. The more relevant difference can be located in the scale that expresses "learning by repetitive exercises" (instrlearn_a), but also the female group prefers the teachers' instructive and explaining activities (instrlearn_b). The effects on real-world connections (realref and applyasp), scheme aspect and learning in groups (soclearn_a) are smaller, but still significant. With one exception (schemasp), the correlation between the "split" factors are lower within the female group. This indicates that the female perceptions of teaching methods and mathematical worldviews is more "fine-grained" than the male ones, i.e. female pupils discriminate these beliefs more precisely.

24.5 Gender Difference II: Correlations

Table 24.3 contains the covariances between the latent variables of the scales. The covariances are separately reported for the female and male group, with the first value being the covariance of the female group. Since all the latent variables are standardised, the covariances can be understood as correlations. Bold entries indicate significance at least on 0.05 level. The most interesting results are located in the first two columns: These columns contains the correlations between the two "discovery scales" and the other ones. It is remarkable that "learning by repetition" (instrlearn_a) correlates positively with both "disclearn" scales, i.e. this type of instructivist learning is not seen as opposed to constructivism, but as an addition (more by the male pupils than by the female ones). This is different in case of instrlearn_b, the teacher-centered explanations: The female group perceive this teaching method as opposed (−0.177) or neutral (0.044) to constructivist discovery, whereas the male group understands it as slightly (0.167) or remarkably (0.407) supportive. Furthermore, learning by discussing in groups (soclearn_b) correlates to constructivist discovery within the male group (0.306), but not within the female one (0.087), similar in case of the two scheme aspects with slightly positive correlations in the male group, but not in the female one. These results are remarkable, since in literature (cf. Duit 1995, see above) constructivist and instructivist teaching methods are normally seen as being opposed to each other. The correlations reported here, however, indicates that especially the male pupils regard these methods as additions to each other, and not as antipodes. The female pupils see them partly as neutral and partly also as additions.

Overall, the general predominance of positive correlations does not indicate that it might be possible to divide the scales into two parts as the theoretical literature of teaching methods typically suggests: a more constructivist part (disclearn_a/b, soclearn_a/b, realref, and applyasp) and a more instructivist one (instrlearn_a/b, schemasp_a/b, and systasp_a/b). This result may advice to examine the pupils' perceptions of teaching methods more intensively and to compare the results to theoretical expectations.

24.6 Reflection and Further Research

As stated above, the scales presented in this paper are intended to be used in a context questionnaire accompanying a performance test. Primarily this context allows examining the potential of these scales. Two questions are of great interest: (1) How are the relations between these scales and the results of the performance test? (2) How are the relations to the other context scales (mostly related to emotional and motivational issues)? Possibly the scales presented here can identify causes of emotions and motivation or can operate as mediators to raise or decrease these affects.

Table 24.3 Covariances between the standardised latent variables, separated by gender

Scale	dl_a	dl_b	il_a	il_b	sl_a	sl_b	rr	aa	sc_a	sc_b	sy_a
disclearn_b	**0.775**										
	0.914										
instrlearn_a	**0.273**	**0.374**									
	0.501	**0.550**									
instrlearn_b	**−0.177**	0.044	**0.634**								
	0.167	**0.407**	**0.774**								
soclearn_a	**0.210**	**0.291**	**0.323**	**0.279**							
	0.399	**0.497**	**0.396**	**0.311**							
soclearn_b	0.087	0.122	**0.300**	**0.271**	**0.638**						
	0.306	**0.365**	**0.362**	**0.325**	**0.779**						
realref	**0.154**	**0.245**	**0.464**	**0.417**	**0.503**	**0.441**					
	0.459	**0.463**	**0.496**	**0.449**	**0.570**	**0.587**					
applyasp	**0.484**	**0.468**	**0.442**	**0.212**	**0.334**	**0.231**	**0.543**				
	0.659	**0.670**	**0.562**	**0.403**	**0.441**	**0.392**	**0.722**				
schemasp_a	−0.073	0.017	**0.437**	**0.709**	**0.475**	**0.373**	**0.576**	**0.371**			
	0.225	**0.303**	**0.631**	**0.726**	**0.443**	**0.574**	**0.636**	**0.637**			
schemasp_b	−0.035	0.099	**0.415**	**0.639**	**0.355**	**0.403**	**0.528**	**0.340**	**0.801**		
	0.279	**0.345**	**0.582**	**0.591**	**0.530**	**0.534**	**0.657**	**0.590**	**0.795**		
systasp_a	**0.335**	**0.411**	**0.630**	**0.336**	**0.396**	**0.306**	**0.649**	**0.662**	**0.490**	**0.438**	
	0.585	**0.656**	**0.747**	**0.518**	**0.530**	**0.433**	**0.614**	**0.873**	**0.614**	**0.636**	
systasp_b	**0.279**	**0.340**	**0.562**	**0.444**	**0.382**	**0.337**	**0.645**	**0.617**	**0.601**	**0.586**	**0.765**
	0.541	**0.586**	**0.614**	**0.480**	**0.455**	**0.403**	**0.678**	**0.803**	**0.713**	**0.703**	**0.854**

But even not concerning further investigations, the most remarkable findings of the study presented here are the indications that there are some strong and significant gender differences in this field of beliefs: (1) The most important mean differences can be located in the perceptions of instructivist and constructivist teaching methods. These differences indicate instructivist and less apply-oriented attitudes of the female pupils and more constructivist, process- and applied-oriented and less instructivist attitudes of the male group. (2) The gender differences in the correlation matrix indicate that the perception of "the whole situation" established by teaching methods and mathematical worldviews is in some aspects quite different. Within the male group, the correlations are positive without exception, i.e. that male pupils perceive different teaching methods and mathematical worldviews more as complements than as opposites. Concerning the female group, the situation is gradually different, insofar the two "discovery scales" of a constructivist view have zero or negative correlations with some scales that expresses more instructivist points of view.

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