Research in Mathematics Education
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Chiara Andrà
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Peter Liljedahl *Editors*

Teaching and Learning in Maths Classrooms

Emerging Themes in Affect-related Research: Teachers' Beliefs, Students' Engagement and Social Interaction

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Foreword

The acronym MAVI stands for **MA**thematical **VI**ews and speaks to the focus of the conference in a broad and inclusive sense, that is: affective issues in Mathematics Education.

The conference is an unavoidable appointment for any researcher interested in the role of beliefs, motivation, attitudes, emotions, will and values in mathematics teaching and learning processes. Theoretical and methodological issues are brought forth and/or refined by a group of researchers who have the sole intent of enjoying the discussion of (new) ideas, welcoming anybody who has a different perspective and getting the best to improve his/her own research.

In 2015, the 21st edition of this annual international conference took place in Milan and attracted new researchers, "besides the ones belonging to the group since many years." Germany and Finland are the birthplaces for the conference, in that Guenter Toerner and Erkki Pehkonen from respective countries have launched the first edition of it. Since then, researchers from both Germany and Finland have attended the various editions of MAVI, together with colleagues from Austria, Italy, Sweden, Israel, Spain, Estonia, Denmark, Australia and Canada. In 2015, the MAVI conference was enriched by the presence of researchers from Japan and Nigeria.

We all have different backgrounds, different research interests and different academic statuses. Special attention is paid to young researchers, who represent the majority of the contributors. The spirit of the conference is, in fact, not only inclusive: it is dedicated to Ph.D. students and young researchers, who are welcome to come and present the status of their research in order to get insightful feedback from their colleagues. Extended time is dedicated to the discussion of each presentation, so that the balance between the time for frontal presentation and discussion is in favour of the latter. No keynote speakers, no plenaries, no parallel sessions: the entire group participates in the whole conference, and no distinction is made among participants on the basis of their experience, academic status or age.

Those who intend to participate have to submit a contribution, which goes through a peer-review process of different phases: in phase 1, before the conference starts, two reviewers read the paper and submit their advice; in phase 2, each author reviews his/her paper, prepares for the conference presentation and during the conference receives questions, feedback, suggestions and comments during a long discussion dedicated to his/her work; in phase 3, after the conference, the paper is revised again, on the basis of what the author has learned from the discussion.

The result is a high-quality collection of cutting-edge research reports. Year after year, new research themes emerge, others are extended and deepened, and foundational constructs are debated and enriched with new perspectives. This is what the reader will find in the next pages.

MAVI21 Conference Organizers

Chiara Andrà
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Contents

Contributors

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Chapter 1
Introduction

Esther Levenson

This book is essentially made up of the 25 papers presented at the 21st MAVI conference in Milan. On the one hand, it may appear to the reader as a mere collection of papers. On the other hand, several of the papers have a common research theme, although the focus may be on different elements. Some of the studies are directly related to previous studies presented at MAVI, written by long-time members of the MAVI community. Other studies, although not directly related to previously presented MAVI papers, are indirectly related and when taking a look at the bigger picture, add to our understanding of the research presented. This introduction is written and organized in order to help the reader get the most out of this book by describing the common threads that run along the papers while placing them in the larger picture of MAVI conferences.

The first section is dedicated to classroom practices and beliefs regarding those practices. Three papers take a look at prospective or practicing teachers' views of different practices such as decision-making (Gonzales), the roles of explanations in the classroom (Levenson and Barkai), and the use of play in mathematics classrooms (Lake). A fourth paper, Tirosh et al., investigates preschool teachers' self-efficacy beliefs for solving patterning tasks. This paper may be seen as a direct continuation of previous studies reported in MAVI (e.g., Tirosh et al. 2011, 2014) regarding teachers' self-efficacy beliefs for various mathematical tasks carried out in preschool, showing the relationships between teachers' knowledge and self-efficacy beliefs. One paper (Ahtee, Näveri and Pehkonen) reports solely on students' views and focuses on the way they perceive their teacher's activities during a mathematics lesson. The methodology used in this paper, having students draw a picture of a mathematics lesson, was also used by Pehkonen et al. (2011), and presented in the 17th MAVI conference. Taking into consideration that classroom practices

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are experienced by both teachers and students, Palmer and Karlsson look at both teachers' and students' perspectives, in this case, focusing on problem-solving. Previously, problem solving beliefs were discussed only from a teachers' point of view (e.g., Näveri et al. 2011; Pehkonen 1999). In this book, however, Palmer and Karlsson report that students and teachers have different images of problem solving that may influence the way teachers teach problem solving and the way learners learn problem solving.

Of major interest to MAVI participants, and a long-debated issue, is the relationship between teachers' professed beliefs and classroom practice (e.g., Ribeiro and Carrillo 2011), and teacher change (Philippou and Christou 1996). This is the focus of the second section in this book. Investigating teachers' professed beliefs is a challenge. Andrà employs the use of teachers' auto-biographical narratives, claiming that metaphors may open a window on the structure of teachers' beliefs, while Beswick asked teachers to respond to an open written questionnaire describing their best and worst mathematics students. Semi-structured interviews were used to explored teachers' conceptions of arithmetic as a specific mathematical discipline (Eichler, Bräunling and Männer) and to investigate the impact of the physical environment on students' learning (Fahlstrom).

Three papers in the second section deal directly with teacher change—Brunetto and Kontorovich, Heyd-Metsuyanim, and Liljedahl. Teacher change is notoriously difficult, even when the teachers themselves are interested in changing their practice. At times, this difficulty is caused by teachers' emotions and their identification with students' emotions. For example, even when teachers agree that classroom norms should be developed such that students feel comfortable making mistakes, teachers tend to emotionally identify with their students and to avoid cognitively demanding and discussion-based instruction (Heyd-Metsuyanim). Emotions and change were also linked in the previous MAVI conference where Liljedahl (2014) related how prospective teachers' emotions are linked to the hierarchy of their motives. In this volume, Liljedahl discusses how teachers' active participation in task design and task piloting can promote changes in their mathematics practice.

The third section of this book centers on the undercurrents of teaching and learning mathematics, what goes on just beneath the surface, but rises in various situations, causing tensions and inconsistencies. Two papers take into consideration parents, one paper focusing on teachers' conflicting views of parent involvement (Rouleau and Peter Liljedahl) and one focusing on parents' own conflicting views of their involvement (Albersmann and Bosse). Conflicting views and tensions are not necessarily detrimental. Kontorovich and Zazkis show how presenting learners with tasks that give rise to conflicting views, may stimulate learning. Inconsistencies are sometimes caused by the tensions felt between affective and social concerns. These tensions may influence patterns of participation (Tuohilampi), attitudes towards the place of mathematics in science education (Aderonke, Oyebola, and Akinloye), as well as how one identifies themselves (Branchetti and Morselli) and others (Hess-Green and Heyd-Metzuyanim) as mathematics learners. While in this section, Branchetti and Morselli, and Hess-Green and Heyd-Metzuyanim, discuss learners' identities, in past MAVI conferences, several studies investigated teachers' identities of themselves as mathematics teachers (e.g., Lutovac and Kaasila 2012; Palmer 2013).

The last section of this book takes a look at emerging themes in affect-related research. Some of the papers relate to the development of new research tools (Goldin, Girnat) while others describe extending research to new directions by re-analyzing existing data (Pieronkiewicz, Sumpter). At the 20th MAVI conference, Törner noted that as early as the 1940s, researchers investigated the influence of attitudes on assessment. In this section, instead of investigating affective elements which influence assessment, two papers discuss attitudes towards assessment. Cusi, Morselli, and Sabena investigate the role of technologically enhanced formative assessment methods, while Signorini investigates teachers' emotions and beliefs towards standardized mathematics assessment, comparing differences between school levels and discussing their educational relevance.

As can be seen from this introduction, many of the papers presented in this book continue traditional MAVI themes while others build on those themes towards new directions. Although the book was divided into sections according to themes, we invite the reader to search for commonalities between papers in different sections, and to explore additional themes and avenues of affect research in mathematics education.

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Part I
Classroom Practices: Explanation, Problem-Solving, Patterning, Decision-Making, Drawings and Games

Chapter 2
Prospective Primary Teachers' Beliefs Regarding the Roles of Explanations in the Classroom

Esther Levenson and Ruthi Barkai

Abstract This study classifies and discusses the views of 23 prospective primary teachers in Israel regarding the roles of explanations in the mathematics classroom, explanations given by teachers and those given by students. Results indicated that prospective teachers perceive explanations as playing various roles although greater emphasis is placed on building content knowledge than on developing a mathematical disposition. Results also hinted that perspectives of explanations may reflect on teachers' beliefs regarding mathematics and their beliefs regarding the teaching and learning of mathematics.

2.1 Introduction

Explanations are central to mathematics education. They are given during various instructional activities such as concept handling, carrying out procedures, and conjecturing. Mathematics educators promote the giving of explanations in the classroom as a means for encouraging communication and enhancing mathematical reasoning (NCTM 2000). How prospective teachers (PTs) view the roles of explanations, both explanations given by teachers as well as explanations given by students, may eventually affect how they use explanations in the classroom. For teacher educators, who are interested in developing not only PTs' mathematics knowledge, but also their pedagogical content knowledge, it is important to recognize that knowledge is often intertwined with mathematical and pedagogical beliefs (Kinach 2002). Thus, the first aim of this study is to investigate PTs' views regarding the roles of teachers' explanations and the roles of students' explanations in the mathematics classroom. We differentiate between teachers' and students' explanations because the teacher and students sometimes play different roles in the classroom, which in turn may affect the roles of explanations given by each. Taking into consideration that different beliefs are often inter-related (Beswick 2005), we

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also consider the possibility that beliefs related to the roles of explanations in the classroom may reflect on other views, such as what it means to do mathematics as well as what it means to teach and learn mathematics. Thus, a second aim of this study is to examine how perspectives of explanations may reflect on epistemological and pedagogical beliefs regarding mathematics and mathematics education.

2.2 Theoretical Background

At the heart of this study are explanations. This section begins by offering possible ways of defining the term "to explain" and continues by reviewing various roles explanations may play in the mathematics classroom. It then briefly describes some research related to teachers' beliefs and possible connections to explanations.

Yackel (2001) used the term "explain" in the following way: "Students and the teacher give mathematical explanations to clarify aspects of their mathematical thinking that they think might not be readily apparent to others" (p. 13). In other words, to explain can mean to clarify. However, to explain may also mean to describe the steps in a procedure. Yackel (2001) describes an incidence of a child asked to figure the sum of 16+8+14. The child "explained" how the sum was reached, by describing how he took one from the 16 and added it to the 14 and then added the easier sum of 15+15+8. If the child in this story had been asked why she used this method, she might "explain" that it is a practical and efficient procedure for adding these three numbers. Thus "to explain" may mean to rationalize. However, if the answer to the question of why is based on mathematical properties, it might be called a justification (Cioe et al. 2015). Thus, in the mathematics classroom, a request to explain might also mean to justify.

In analyzing the roles explanations may have in the learning process, we take a socio-cultural perspective on mathematical development (Vygotsky 1978). According to this view, learning takes place through social interactions where an individual's mathematical activity is influenced by participating in cultural practices such as explaining one's ideas. As such, explanations are tools for semiotic mediation as the teacher mediates some mathematical content through a didactical intervention (Bartolini Bussi and Mariotti 2008). Mathematical concepts also interact with spontaneous concepts which emerge from the child's reflections on everyday experience. Thus, the social interaction must take place within the zone of proximal development (ZPD) of the individual (Vygotsky 1978) and the teacher's role as a mediator is to recognize this zone for each student and offer appropriate assistance. Assistance may take the form of a teacher's explanation but may also be a prompt from the teacher for the student to explain aloud his or her thinking. In this case, the student's explanation to the teacher may serve the purpose of revealing that student's ZPD. Students' explanations may have additional roles. Given by a knowledgeable or more advanced student to another student, students' explanations may also serve as a semiotic mediator between the content and the receiver of the explanation. A student's explanation may also assist in the internalization process defined by Vygotsky (1978) as "the internal reconstruction of an external operation" (p. 56).

Learning may also be seen partly as a process of acculturation into mathematical practices (Yackel 2001). As such, the teacher's explanations may serve to demonstrate what counts as an acceptable mathematical explanation. Students' explanations may be viewed as a way for participating in the mathematics classroom community. This view is in line with theories which perceive learning as increasing participation in a community of practice which includes experts (e.g., teachers) and novices (e.g., students) (Lave and Wenger 1991). Explanations which take place in communities of practice, which focus on the connections between mathematical topics, may also have a role in developing a mathematical disposition and meta-mathematical views. Developing a mathematical disposition is related to developing a sense for what it means to do mathematics and thus being able to do mathematics for yourself. Frade (2005), referring to Ernest (1998), claimed that meta-mathematical views include views of proof and definition and the scope and structure of mathematics as a whole. Zazkis and Leikin (2010) referred to meta-mathematical issues as "cross-cutting themes that may appear within any mathematical content" (p. 274) and so explaining how one solution is the same or different from another solution, may promote meta-mathematical habits of mind.

While few studies focused on teachers' and PTs' views of the roles of explanations, several studies investigated their more general beliefs regarding mathematics and teaching mathematics. One commonly held belief among primary teachers is that mathematics consists of a set of rules, skills, and procedures (e.g., Stipek et al. 2001) and thus learning mathematics means memorizing those rules and becoming proficient at carrying out procedures. Other commonly held beliefs among prospective teachers include the belief that teaching means transmitting knowledge and information (Stuart and Thurlow 2000) or that teaching mainly consists of offering clear explanations (Richardson 1996). Among practicing teachers, Beswick (2005) found that teachers, who believed that a major part of mathematics is computation, also believed that the teacher is responsible for explaining mathematical content clearly. Taking into consideration that how one interprets the roles of explanations can reflect on one's beliefs related to the nature and teaching and learning of mathematics, this study examines primary PTs' views of the roles of explanations in the classroom and then discusses how these views may reflect on other beliefs.

2.3 Method

Participants in this study were 23 prospective primary school teachers, from diverse backgrounds, in their third year of a 4-year program studying at a teachers' college in Israel. In their second and third year they had participated in field work which included going to a primary school once a week and observing mathematics classes. In their second year, they also taught mathematics to small groups of children and in their third year, they received experience teaching whole classes under the guidance of a teacher mentor who was affiliated with the educational college.

One questionnaire was handed out. PTs were told that explanations in the classroom may play various roles. They were then asked to answer two open questions: (1) In your opinion, what are the roles of explanations given by the teacher during mathematics lessons? (2) In your opinion, what are the roles of explanations given by students during mathematics lessons? All questionnaires were answered in the presence of the researchers.

Each question was analyzed separately. A first analysis was carried out in order to identify statements which referred to the content of an explanation (i.e., what is meant by the term "to explain"). For example, one teacher wrote that a role of teachers' explanations is to "provide strategies for working out problems." Those statements were then categorized according to the different meanings of "to explain" reviewed in the background section. This led to three categories of definitions for the term "to explain": to clarify or simplify, to describe or tell the steps in a procedure, and to justify or convince.

A second analysis was carried out to identify statements related to the roles of explanations in teaching and learning. An initial analysis was carried out according to the roles found in the literature and a search for related keywords such as: mediation, internalization, participation, and discourse. A further analysis was carried out searching for additional common keywords not categorized under the initial process. An example of such a keyword was "new", as in "new material" or "new knowledge". Another example which arose was the term "self-confidence". Finally, in line with mathematics educators (Zazkis and Leikin 2010) who separated mathematical content knowledge from meta-mathematical knowledge and disposition, we differentiated between roles related to each of these types of knowledge.

2.4 Results

2.4.1 What Does It Mean "to Explain"?

When asked to write down the roles of teachers' explanations, 18 PTs wrote statements that were similar to definitions for the term "to explain" (see Table 2.1—the number in parentheses represents responses to the question of the roles of students' explanations).

When asked to write down the roles of students' explanations, three PTs, which happened to be three of the above mentioned 18 PTs, wrote statements categorized as definitions (see the numbers in the parentheses in Table 2.1). As seen from Table 2.1, greater emphasis was given to explanations that describe solution methods and clarifications, than on explanations which act as justifications.

Table 2.1 Categories related to **definitions**

Category	Illustrative examples: The role of a teacher's (students') explanation is to…	Freq. PTs
Describe how/tell the steps in a procedure	… provide tools for solving problems; … provide strategies for working out problems	7(3)
Clarify	… clarify a point that the teacher feels students did not understand; … to sharpen students' understanding of difficult points; … help students understand the given data of a problem.	11
To justify or convince	… to explain why a solution is correct; … to validate the use of a certain solution method; … to convince that the formula/method/operation is valid.	3(1)

Table 2.2 Roles of **teachers'** explanations: building content knowledge

Role	Illustrative examples: The role of **a teacher's explanation** is to…	Freq. PTs
Introducing new content	… teach new material ; … to introduce a new topic	6
Mediating mathematical content to the student	… answer students? questions; … give the student an opportunity to understand and to connect to the material	15
Promoting the process of internalization	… help the student internalize the material and understand it so the student will not just memorize and recite it.	2

2.4.2 Roles of Explanations

PTs' responses to the question of the roles of teachers' explanations are presented first, followed by their responses to the roles of students' explanations. In each section, we first describe roles related to promoting mathematical knowledge and then roles related to promoting meta-mathematical knowledge and developing a mathematical disposition.

2.4.2.1 Roles of Explanations Given by Teachers

Seventeen PTs wrote in some form or another that the role of a teacher's explanations is to promote students' mathematical knowledge (see Table 2.2).

We categorized teachers' statements by adapting Vygotsky's approach to mathematics education (e.g., Goos 2004). To begin with, scientific (or in our case, mathematical) concepts enter the classroom via the teacher. This is related to the first category, *Introducing new content*. These new concepts come in contact with learners' everyday concepts and often need mediating by an adult. This is related to the second category, *Mediating mathematical content to the student*. Finally, the everyday and scientific concepts merge and become an integral part of the student's own reasoning. This is related to the third category, *Promoting*

Table 2.3 Roles of **teachers'** explanations: developing a mathematical disposition

Role	Illustrative examples: The role of **a teacher's explanation** is to...	Freq. PTs
Widening students? perspectives of mathematics	... to connect to other topics learned in the past; ... to promote different ways of thinking and creativity; ... to arouse curiosity	5
Promoting discourse and participation	... to connect between different students? explanations and promote mathematical discourse	2

Table 2.4 Roles of **students'** explanations: building content knowledge

Role	Illustrative examples: The role of **a student's explanation** is to...	Freq. PTs
Students mediate content to other students	... help other students understand if they don't understand the teacher's explanations; ... explain to friends using language that children will understand	9
Internalization	... encourage deeper thought processes and promote that student's learning	5

the process of internalization. The frequency of teachers who related to each role, along with illustrative examples, is presented in Table 2.2. Six PTs wrote statements related to developing meta-mathematical knowledge and a mathematical disposition (see Table 2.3). Statements which described how teacher's explanation may be used to encourage students to see a wider and more general view of mathematics (beyond the specific content being learned) were recognized in the role, *Widening students' perspectives of mathematics*. Statements related to explanations as a way of participating in the mathematics classroom community were acknowledged in the category *Promoting discourse and participation*. In this category we also included statements related to promoting self-confidence as studies have shown that students' with a higher self-confidence are more likely to participate in classroom discourse (e.g., Silver and Smith 1996).

2.4.2.2 Roles of Explanations Given by Students

Regarding how students' explanations may help to build content knowledge, several studies have adapted the notion of the ZPD to collaborative learning where each student in the group has some piece of knowledge but requires the others' contributions to make progress (Goos 2004). This is related to the role—*Students mediate content to other students*. On the other hand, a student's explanation, whether given to the teacher, another student, or to him or herself, may promote understanding among the individual student giving the explanation and assist in the internalization process. Thus, *Internalization* is the second role related to building content knowledge (Table 2.4).

Table 2.5 Roles of **students'** explanations: developing a mathematical disposition

Category	Illustrative examples: The role of **a student's explanation** is to...	Freq. PTs
Widen students' perspectives of mathematics	... offer opportunities for seeing different points of views	1
Promoting discourse and participation	... encourage other students in the class to participate in the classroom discussion; ... develop a more "open" discourse, encouraging other students to participate; ... develop students' ability to express themselves mathematically; ... to raise students' self-confidence	12

Table 2.6 Frequency of statements related to teachers' and students' explanations

The role of...	Teachers' explanations	Students' explanations	Total
Building content knowledge	26	15	41
Developing a mathematical disposition	8	16	24
Total	34	31	65

Students' explanations, like teachers' explanations may also promote meta-mathematical knowledge and a mathematical disposition. Findings related to this role are summarized in Table 2.5. When comparing the results to the roles of teachers' explanations (see Table 2.3) it is interesting to note that more PTs viewed *Promoting discourse and participation* as a role of students' explanations than as a role of teachers' explanations. In addition to the above roles, when analyzing PTs' statements related to students' explanations, 19 PTs wrote 24 statements related to a role we called *Shedding light on students' conceptions*, that is, how students' explanations can be used by teachers to assess what students know, what they misconceive, and where mistakes are located. For example, one teacher wrote that a student's explanation allows the teacher to "check the level of understanding of the student and to see if there are any difficulties". While this role may not be directed related to building content knowledge, it views students' explanations as a way for teachers to recognize the boundaries of a student's ZPD, enabling the teacher to find appropriate ways to mediate mathematical content by seeing in the students' ideas links to the language and concepts of mathematics (Goos 2004)

In summarizing the results, we take into account statements categorized as definitions as well as those categorized as roles. The first two categories of definitions—*telling the steps in a procedure* and *clarifying*—were included in the role of building content knowledge. The third definition—*to justify* or *convince*—was related to developing meta-mathematical knowledge and a mathematical disposition. Taking into consideration that some teachers wrote more than one statement categorized under the same role, Table 2.6 summarizes the number of statements written for each major role (not including the role of *Shedding light on students' conceptions*).

In general, more statements related explanations in the classroom to building content knowledge than to developing meta-mathematical knowledge and disposition. In addition, more statements were written by PTs connecting teachers' explanations, rather than students' explanations, to building students' content knowledge. However, when developing meta-mathematical knowledge and disposition, more statements were written in connection to students' explanations than to teachers' explanations. The possible implications of these results are discussed in the next section.

2.5 Discussion

When looking at the results of this study, we first note that the PTs held various views regarding the roles of explanations in the classroom, including views that took into consideration both promoting mathematical knowledge as well as developing a mathematical disposition. In general, PTs attributed these roles to both teachers' as well as students' explanations. However, when taking a closer look, we notice some imbalances. These imbalances may reflect upon other beliefs.

Teachers' beliefs are often divided into their beliefs about mathematics and their beliefs about teaching mathematics. Regarding beliefs about what it means to do mathematics, we look at statements that were categorized as definitions and note that more teachers referred to explaining as describing procedures rather than as validating. This is in line with Stipeck et al. (2001) and may reflect a belief that mathematical activity mostly includes applying strategies as opposed to justifying and reasoning. In addition, more PTs' statements related explanations to building content knowledge than to developing meta-mathematical knowledge. This could reflect a belief that mathematics is made up of a series of isolated topics, not necessarily connected. The emphasis placed on content, rather than on the global picture of mathematics may also reflect a pedagogical belief that more time needs to be spent teaching content, even allowing that mathematics is a rich domain made up of inter-related topics and shared processes.

Pedagogical beliefs may also be reflected in the roles PTs assigned to teachers' explanations versus those they assigned to students' explanations. For example, PTs wrote that teachers' explanations can be used to introduce new content. While this makes sense and is in line with theories which view the teacher as the introducer of mathematical content (Bartolini Bussi and Mariotti 2008), other studies (e.g., Nunokawa 2010) claimed that students' explanations may serve to generate new objects of thought by directing new explorations. Another example is that several PTs wrote that teachers' explanations can widen students' perspectives of mathematics, but only one PT claimed that students' explanations could play this role. Yet, several studies have shown that students are capable of connecting concepts from different mathematical areas (e.g., Levav-Waynberg and Leikin 2012). That PTs seemed to stress the roles of teachers' explanations over students' explanations is in line with previous studies which found that PTs equate teaching with explaining. In other words, it is the teachers' job, and not the students' job, to explain. It also may reflect so-called traditional beliefs about learning mathematics versus more inquiry-oriented, constructivist beliefs. According to Stipek et al. (2001), teachers who hold traditional beliefs tend to exercise more control over students' mathematical activities while teachers who hold more inquiry-oriented beliefs allow for more student autonomy. However, it was not always the case that PTs stressed teachers' explanations over students' explanations.

As opposed to the above findings, few PTs wrote that teachers' explanations may encourage student participation and discourse, but several PTs assigned this role to students' explanations. In other words, while they viewed students' explanations as way for participating in class, they did not view teachers' explanations as a way of participating. This may indicate that PTs do not necessarily believe in learning as participating in a community or that the teacher is an expert participant. Perhaps, they want to encourage students' participation in class, but do not necessarily view the teacher as a legitimate and integral participant in the learning community.

Finally, we note limitations of this study. Two open questions were used to investigate teachers' perspectives of the roles of explanations. On the one hand, open questions allow participants to freely express their thoughts without instilling limitations. On the other hand, the roles of explanations may depend on the context, such as the age of the students, the specific task, and the circumstances surrounding the explanation (Morselli and Levenson 2014). The timing of an explanation may also affect its role. For example, if teachers' explanations are given without taking advantage of students' previous work, then that teacher may not share a Vygotskian view of learning. In addition, we do not learn how strongly the teachers hold their beliefs 'do they think these are important or essential roles of explanation, or merely possible or useful roles. Likewise, the emotional dimension of beliefs did not emerge. Despite these limitations, results of this study may be shared with PTs to explicitly discuss with them various roles explanations may play in the classroom. In addition, researchers investigating beliefs often construct detailed questionnaires to investigate different aspects of teachers' beliefs (e.g., Beswick 2005). Results of this study may assist in building a questionnaire that would allow for a more detailed and focused investigation of beliefs related to the roles of explanations and their relationship to beliefs regarding mathematics and its teaching.

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Chapter 3
Defining, Drawing, and Continuing Repeating Patterns: Preschool Teachers' Self-efficacy and Knowledge

Dina Tirosh, Pessia Tsamir, Esther Levenson, Ruthi Barkai, and Michal Tabach

Abstract Patterning activities, specifically those related to repeating patterns, may encourage young children's appreciation for underlying structures. This paper investigates preschool teachers' knowledge and self-efficacy for defining, drawing, and continuing repeating patterns. Results indicated that teachers were able to draw and continue various repeating patterns but had difficulties defining repeating patterns. In general, teachers had a high self-efficacy for all tasks. However, teachers' had a significantly lower self-efficacy for defining repeating patterns than for drawing and continuing repeating patterns.

3.1 Introduction

In Israel, the preschool curriculum encourages teachers to engage children with pattern activities with the aims of having children identify, draw, and continue repeating patterns as well as use mathematical language to describe these patterns (Israel National Mathematics Preschool Curriculum [INMPC] 2008). In order for teachers to carry out such activities they should be knowledgeable of the subject matter as well as believe that they have the knowledge required to teach that subject. Taking into consideration that self-efficacy is both domain and task specific (Tirosh et al. 2014) this paper investigates preschool teachers' knowledge for defining, drawing, and continuing repeating patterns, as well as their self-efficacy for accomplishing these tasks.

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3.2 Theoretical Background

Mathematics has been described as "a living subject which seeks to understand patterns that permeate both the world around us and the mind within us" (Schoenfeld 1992, p. 334). Several national curricula have recognized the potential of pattern activities in promoting early algebraic thinking among young children. For example, the National Council of Teachers of Mathematics (2000) Algebra Standard for Pre-K-2 states that "algebraic concepts can evolve and … develop … through work with classifications, patterns, and relations?" (p. 91). Exploring patterns during the elementary years may enhance the meaning of algebra during the secondary years. Algebraic thinking relates to finding and using generalizations. "Every pattern is a type of generalization in that it involves a relationship that is 'everywhere the same'" (Papic et al. 2011, p. 240). Thus, working with patterns can promote algebraic thinking. At the preschool level, educators have specifically noted that exploring repeating patterns may promote children's appreciation of underlying structures (Starkey et al. 2004).

Repeating patterns are patterns with a cyclical repetition of an identifiable 'unit of repeat' (Zazkis and Liljedahl 2006). For example, the pattern ABBABBABB … has a minimal unit of repeat of length three. According to the Israel National Mathematics Preschool Curriculum (2008), "patterning activities provide the basis for high-order thinking, requiring the child to generalize, to proceed from a given 'unit', to a pattern in which the unit is repeated in a precise way" (p. 23).

Young children naturally engage in pattern activities such as building block towers with an ABAB pattern (Seo and Ginsburg 2004). However, while most children by the end of kindergarten will be able to copy a repeating color pattern, few will be able to extend or explain it (Clarke and Clarke 2004). Being able to copy a pattern may not necessarily indicate that the child recognizes the structure of the pattern. Papic et al. (2011) found that some preschool children may be able to draw an ABABAB pattern from memory by recalling the pattern as single alternating colors of red, blue, red, blue, basically recalling that after red came blue and after blue came red. However, when shown a more complicated pattern such as ABBC, they could not replicate the pattern. Rittle-Johnson et al. (2013) found that when young children were asked to duplicate or extend an ABB pattern, some children could not produce more than one unit of repeat correctly while others reverted to producing an ABAB pattern.

In the above studies, children were observed without adult intervention. However, when given proper assistance, young children are capable of recognizing the unit of repeat in a repeating pattern and come to comprehend the underlying structure of the pattern (Papic et al. 2011). In other words, for children to achieve the benefit of engaging in pattern activities, adult guidance is essential. Yet, few studies investigated teachers' knowledge specifically related to the teaching of patterns. Those that did, found that teachers provide limited worthwhile patterning opportunities for children and when children engaged spontaneously in patterning, teachers sometimes failed to capitalize on the child's interest, missing out on opportunities to extend children's interest and knowledge in patterning (Fox 2005). There is clearly a need for systematically studying preschool teachers' knowledge for teaching patterns, as well as a need for providing professional development for preschool teachers related to patterning.

Another factor related to teachers' classroom actions is teachers' self-efficacy. For example, Bates et al. (2011) found that teachers who reported higher mathematics self-efficacy were more confident in their ability to teach mathematics than teachers with a lower mathematics self-efficacy. Hackett and Betz (1989) defined mathematics self-efficacy as, "a situational or problem-specific assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular [mathematics] task or problem" (p. 262). In a past study, we investigated preschool teachers' self-efficacy related to defining and identifying triangles and circles (Tirosh et al. 2014). In that study, it was found that teachers had a high self-efficacy for identifying triangles and circles as well as a high score for correctly identifying both figures. On the other hand, teachers had a significantly lower self-efficacy for identifying circles than for identifying triangles, whereas their correct identifications of circles was quite high. In other words, their actual knowledge of identifying circles was higher than their perceived self-efficacy. Even within the same domain, teachers' self-efficacy may vary according to the task (Tsamir et al. 2015).

The current study investigates preschool teachers' knowledge and self-efficacy regarding three different patterning tasks – defining repeating patterns, drawing repeating patterns, and continuing repeating patterns. Specifically, we ask: (1) Are preschool teachers able to define, draw, and continue a repeating pattern? (2) What are preschool teachers' self-efficacy beliefs regarding their ability to define, draw, and continue repeating patterns? (3) What are the differences and relationships between teachers' knowledge and self-efficacy for defining, drawing, and continuing repeating patterns?

3.3 Method

For the past several years, we have been providing professional development for preschool teachers aimed at promoting their knowledge and self-efficacy for teaching mathematics in preschool (e.g., Tirosh et al. 2014). This paper reports on a subgroup of preschool teachers who participated in a section of the program devoted to patterning concepts. The mandatory Israel National Mathematics Preschool Curriculum (2008) is still fairly new and preschool teachers are just becoming familiar with the standards. Informal interviews with some preschool teachers revealed that most of the patterning activities taking place in the kindergartens consisted of children drawing boarders or frames for pictures, albeit boarders which were made up of repeating patterns. Few activities explicitly aimed to develop

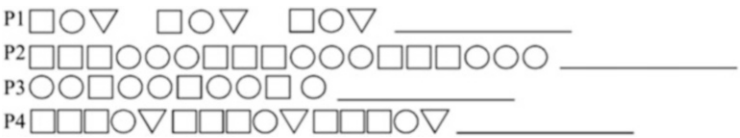

Fig. 3.1 Continue the pattern

Table 3.1 Mean self-efficacy per patterning task ($N = 27$)

	Mean	SD
Defining	3.33	0.55
Drawing	3.81	0.40
Continuing	3.85	0.36

children's appreciation for pattern structure or for the unit of repeat in a repeating pattern.

The main data for this study was gathered from a group of 27 teachers, teaching 4–6 year old children in municipal preschools. All had a first degree in education. Before the program began, teachers were asked to fill out a two-part questionnaire which began with self-efficacy statements and continued with knowledge questions. The self-efficacy statements were as follows: I am able to say what a repeating pattern is; I am able to draw an example of a repeating pattern; I am able to continue a repeating pattern. A four-point Likert scale was used to rate participants' agreements with self-efficacy statements: 1—I do not agree that I am capable; 2—I somewhat agree that I am capable; 3—I agree that I am capable; 4—I strongly agree that I am capable. Following those statements, were two questions: (1) What is a repeating pattern? (2) Draw an example of a repeating pattern. Teachers' responses were analysed by one author (as described in the results section) and validated by a second author. Lastly, teachers were shown four patterns (see Fig. 3.1) and were requested to draw a continuation of each pattern.

3.4 Results

3.4.1 Self-efficacy

In general, the teachers ($N = 27$) had a relatively high self-efficacy for defining, drawing, and continuing repeating patterns (see Table 3.1) with the lowest self-efficacy being for defining repeating patterns.

In order to further analyze the results, paired-samples t-tests were carried out (Table 3.2). Results indicated that teachers' self-efficacy for defining repeating patterns was significantly less than both the drawing and continuing tasks. However, when it came to self-efficacy regarding the tasks of drawing patterns versus continuing patterns, no significant difference was found.

Table 3.2 Comparing self-efficacies for different tasks ($N = 27$)

Tasks	Mean difference	t-Value	df	p-value
Defining versus drawing	–0.48	–4.32	26	0.000
Defining versus continuing	–0.52	–4.65	26	0.000
Drawing versus continuing	–0.04	–0.57	26	0.574

3.4.2 Knowledge: Defining Repeating Patterns

The preschool curriculum does not offer a specific definition for repeating patterns, thus we did not expect teachers to offer identical definitions. Instead, we analyze teachers' definitions by referring to the definition used in the beginning of this paper: Repeating patterns are patterns with a cyclical repetition of an identifiable 'unit of repeat' (Zazkis and Liljedahl 2006). This definition may be broken up into three, not necessarily discrete, attributes: (1) there is a specific core unit made up of a string of elements, (2) the string of elements are not randomly laid out but have a fixed structure and thus the string is identifiable as a unit, and (3) the unit is repeated (i.e., the unit appears more than once). Responses were analyzed in terms of if and how each of the attributes was presented. For example T2 wrote that a repeating pattern is "a number of items (at least 2) which repeat themselves in a fixed way - a series which repeats itself". T2's definition relates to the content of the unit (a number of items), that the unit has a fixed structure, and that it is repeated. In other words, T2's statement related to all three attributes. Likewise, T13 related to all three attributes writing that a repeating pattern is "a fixed sequence of items which repeats in a fixed order". Some teachers wrote that a repeating pattern is a form, or a sequence which repeats itself. Terms such as 'form' and 'sequence' were considered to combine the first two attributes into one succinct word. In the case of the Hebrew word for 'form', the word is used in a similar manner as in a 'cake form', implicitly indicating structure.

Out of 27 participants, only two teachers left this question blank. One teacher (T6) wrote only "it repeats itself" without referring to what repeats itself and without referring in any way to structure. A different teacher (T123) (the numbering of the teachers reflects that this was a sub-group of a larger group) wrote that a repeated pattern is "a sequence of shapes/colors that are in the same place and direction". This teacher referred to the content of the pattern and its structure, but did not mention the issue of repetition. Five teachers included in their statements terms that had to do with the content and repetition, but not with structure. For example, T128 wrote, "shapes that repeat themselves". This is questionable because there is no indication that the shapes must appear in a consistent or set way each time. Such a definition could lead to the following string, which is not a repeating pattern: □ △ △ △ □ □ .... Similarly, T140 wrote, "one or more shapes that repeat a few times (more than 2)".

Five teachers wrote that a repeated pattern "is a pattern which repeats itself". It is difficult to tell from such a statement if the teachers are aware of the attributes of a repeating pattern. It could be that they are defining this type of pattern by placing it into the general category of patterns and then identifying its special attribute of repetition. However, it could also be that they are aware that there is a specific unit of repeat which repeats itself, and they call this unit of repeat a pattern. Finally, one teacher (T11) did not refer to the repetition of a whole unit, but rather that every element appears at regular intervals. However, her statement did refer to all three attributes.

To summarize this section, nine teachers' responses (including non-responses) were considered insufficient for describing repeating patterns. Five statements were questionable. The rest, 13 teachers, wrote statements that described repeating patterns by referring to a structured content which repeats itself.

3.4.3 Knowledge: Drawing Repeating Patterns

All of the teachers' drawings (teachers drew patterns consisting of shapes familiar to children – squares, circles, hearts, etc.) were of repeating patterns; one teacher drew two patterns. Tables 3.3 and 3.4 respectively summarize the different structures of the patterns teachers drew and the number of cycles in each pattern.

As can be seen, nearly three-quarters of the drawn patterns had a unit of repeat whose length was three. Most teachers drew a pattern with three complete cycles, although no instructions were given regarding the length of the pattern. That is, teachers could have just drawn the next element in the pattern. The amount of elements to be drawn was left to the teachers' discretions. Interestingly, three teachers drew patterns that did not end in a complete cycle. One of those teachers ended the pattern with three dots (…) perhaps indicating that she had stopped in the middle. However, an additional five teachers also drew three dots at the end of their pattern. Thus, the teacher who ended her pattern mid-cycle might simply have been indicating, along with the other five teachers, that a repeating pattern may theoretically go on without ending.

Table 3.3 Frequency (%) of structures and cycles in teachers' drawn patterns

Unit structure	AB	ABC	ABB	AAB	ABCD	ABAC
Frequency	6(21)	14(50)	3(11)	3(11)	1(4)	1(4)

Table 3.4 Frequency (%) of cycles in teachers' drawn patterns

Number of cycles	2	3	4	2 1/2	2 2/3
Frequency	4(14)	18(64)	3(11)	2(8)	1(4)

3.5 Knowledge: Continuing Repeating Patterns

Recall that four repeating patterns were presented to the teachers and they were requested to continue each pattern (see Fig. 3.1 above). Twenty-five (out of 27) teachers responded to this part; two of those teachers responded only to the first question. All of the continuations that teachers drew for the first two patterns and for the last pattern were correct ways of continuing the pattern. One teacher drew an incorrect continuation for the third pattern. Figure 3.2 summarizes the different ways each of the patterns (P1, P2, ...) was continued (C1, C2, ...) and the frequency of teachers who drew each continuation (listed in parentheses). For each pattern, the first continuation presented is the minimal set of items that completes the pattern with a complete unit of repeat. Starred (⋆) continuations indicate continuations which are correct but result in a pattern ending mid-cycle. Note that the last continuation of the third pattern (P3C5) is correct, if we consider the possibility that the unit of repeat is all 10 elements presented in that pattern.

Regarding continuations which ended the pattern mid-cycle, note that Pattern 4 had a relatively long unit of repeat consisting of five elements, and yet most teachers still chose to draw all five elements. Moreover, for Pattern 2, which had a unit of repeat of length six, 13 teachers drew a complete unit of repeat. Approximately 10% of the continuations ended the pattern mid-cycle. Finally, we note that for three out of the four patterns, most teachers added just enough elements to end the pattern with a complete unit. The exception was for Pattern 3, the only pattern presented on the questionnaire that did not end with a complete unit. To end that pattern with a complete unit, it was necessary to add less than a complete unit. Only two teachers did this.

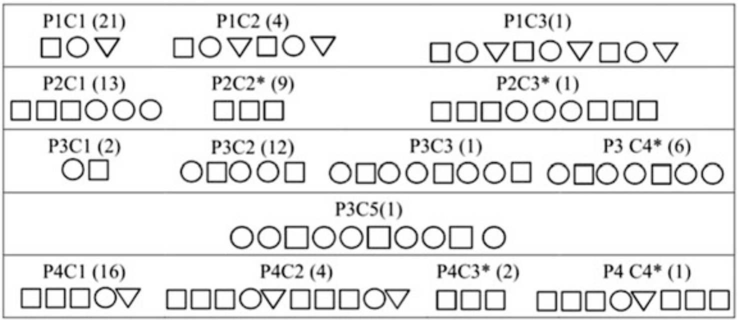

Fig. 3.2 Frequency of teachers' correct continuations of the patterns

Table 3.5 Comparing knowledge and self-efficacy for defining

Knowledge Self-efficacy	1	2	3	4	Total
0	–	–	6	3	9
1	–	–	3	2	5
2	–	1	7	5	13
Total	0	1	16	10	27

3.5.1 Comparing Knowledge to Self-efficacy

In general, teachers' high self-efficacy for drawing and continuing repeating patterns matched their level of knowledge. Regarding the tasks of defining repeating patterns, the picture is more complex. On the one hand, teachers had a lower self-efficacy for defining than they had for drawing and continuing, which coincided with the difficulties they had in actually writing definitions for repeating patterns. On the other hand, their self-efficacy was still relatively high. This might lead us, at first, to believe that teachers had a higher self-efficacy than was perhaps justified. However, this is not the entire picture. Scoring the teachers' definitions on a scale of 0–2 (0 being an insufficient definition, 1 being a questionable definition, and 2 being a sufficient definition), Table 3.5 offers a look at the cross-tabulated knowledge and self-efficacy scores.

These findings suggest inconsistencies in both directions. On the one hand, nine teachers gave insufficient definitions and yet believed highly or very highly in their ability to define repeating patterns. On the other hand, out of 13 teachers who wrote a sufficient definition, only 5 strongly believed they were able to define repeating patterns.

3.6 Discussion and Implications

When planning professional development for teachers, it is important to take into consideration the mathematical knowledge teachers bring to their learning along with their self-efficacy in that specific domain. Regarding drawing repeating patterns, although most teachers drew a basic ABC structured pattern, several drew more complex structured patterns and a few drew an even more basic ABAB structure. Building on this knowledge, teacher educators can discuss with teachers how children might approach these more complex patterns and how to build tasks based on various patterns that may promote children's knowledge of patterning. Teachers' continuations of given patterns indicated for the most part a strong tendency to end patterns with a complete unit of repeat. However, repeating patterns, such as repeating decimals, do not always present themselves by ending in a complete unit. This study suggests that the issue of ending or not ending a pattern in a complete cycle might be an aspect of pattern knowledge in need of more attention.

Regarding the task of defining repeating patterns, it might be argued that children do not need to learn a formal definition for repeating patterns. However, it is still important for teachers to know the attributes of repeating patterns a well as to accurately use words to describe mathematical concepts. One study, for example, found that the amount of preschool teachers' mathematics-related talk was found to be significantly associated with the growth of children's conventional mathematical knowledge over the school year (Klibanoff et al. 2006).

For the most part, teachers in this study had a positive self-efficacy for drawing and continuing repeating patterns, but a less definitive belief in their ability to define. It might simply be, as shown in other studies, that knowledge is not necessarily correlated with self-efficacy (Tsamir et al. 2015). On the other hand, one of the sources for self-efficacy beliefs is performance attainments (Bandura and Schunk 1981); success raises self-efficacy while failure lowers it. Mixed self-efficacy beliefs could be the result of teachers' past experiences with mathematical definitions, and this affected their response to this question. Just as studies have shown that mathematics self-efficacy predicts children's choices of the types of problems they prefer to engage with (Bandura and Schunk 1981), it might be that teachers with a low self-efficacy for defining patterns may avoid giving verbal descriptions of repeating patterns. In general, for teachers with an already high self-efficacy, professional development may increase teachers' knowledge so that it is in line with their self-efficacy beliefs. For teachers with a low self-efficacy, professional development may help increase their self-efficacy, showing teachers just how knowledgeable they really are.

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Chapter 4
Primary School Students' Images of Problem Solving in Mathematics

Hanna Palmér and Lena Karlsson

Abstract This paper focuses on primary school students' images of problem solving in mathematics. The teachers of these students have been participating in a professional development programme on problem solving in mathematics involving them reading literature and conducting problem-solving lessons in their classes. One semester after the completion of this professional development programme, interviews were carried out with both teachers and students. These interviews show that the students have very different images of problem solving, both in relation to each other and in relation to the teachers. These different images may influence what these students think about problem solving and what they learn about and by problem solving, and may also influence the potential for their teachers to teach problem solving.

4.1 Introduction

The empirical material in this paper is from a study exploring the potential in combining entrepreneurship and problem solving in mathematics in Swedish primary schools. According to the Swedish syllabus, entrepreneurship is to pervade all teaching in primary school (Swedish National Agency for Education 2011) and it seems to be commonly taken for granted that entrepreneurship is something that is only positive. In this study, instead of taking that rather unconsidered stance, we try to investigate what happens with mathematics in general and problem solving in particular when entrepreneurship makes an entrance in mathematics lessons.

The study was conducted at eight schools, but in this paper we will only focus on two of these, since they are "special" in terms of problem solving. Before becoming involved in the study, the teachers from these two schools had participated in a national professional development programme named *Boost for Mathematics*. Within this programme they had focused especially on problem solving. In this paper, we will present the images the students from these two schools expressed

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about problem solving at the beginning of the study. The word *images* refer to what the notion of problem solving seems to imply for these students. In the paper these images will be discussed in relation to the professional development programme as well as in relation to the study on entrepreneurship.

4.2 Problem Solving

According to the Swedish curriculum for primary schools, "[m]athematics is by its nature a creative, reflective, problem-solving activity" (p. 62). Mathematics teaching should help children to develop their skills to formulate and solve mathematical problems, as well as to evaluate the strategies and methods to be used (Swedish National Agency for Education 2011). In the curriculum, problem solving is described both as a purpose (ability to formulate and solve problems) and as a strategy (way to acquire mathematical knowledge). This emphasis in the curriculum can be understood as a reaction to a national inspection of mathematics teaching conducted in 2009, which showed that mathematics teaching in Sweden was dominated by individual counting, with limited possibilities for students to develop their ability to solve problems (Swedish Schools Inspectorate 2009). Furthermore, research shows that students who work with challenging problem-solving tasks in school develop their understanding of mathematical concepts in a better way than other students (Hiebert and Grouws 2007).

This emphasis on problem solving in the curriculum is not new, however, and similar formulations can be seen in policy documents in other countries (Cai 2010; English and Sriraman 2010). Even though it is emphasised in policy documents in several countries, there is "no general agreement on what the teaching of mathematics through problem solving should really look like" (Cai 2010, p. 255), but there are common traits. A problem-solving task should be a challenge for a student; s/he should not know in advance how to proceed to solve the task. Instead, the student should develop new (for the student) strategies, methods and/or models when solving the task. This implies that what is a problem-solving task for one student may not be a problem-solving task for another student.

4.3 Boost for Mathematics

All teachers of mathematics in Sweden from grade 1 up to secondary school will, by the end of 2016, have taken part in a professional development programme named *Boost for Mathematics* (Swedish National Agency for Education 2015). The programme was initiated by the government in 2012 with the aim of improving mathematics teaching and thereby students' learning. The programme has been developed by researchers and is organised around teacher collaboration, where teachers work in groups with experienced tutors. The teachers can choose content by working with different modules, for example, geometry, problem solving and number sense. These modules have specialised content for lower primary, upper primary, lower secondary and upper secondary school. Each module is divided into eight parts and each part is divided into four steps. In step A, the teachers read a text and often view a number of films; in step B, they have group discussions with the tutor and they make a plan for a lesson; in step C, every teacher teaches the lesson in their own class; and finally, in step D, the group of teachers and the tutor meet again and follow up on the lesson. Even though the modules are specialised for lower primary and upper primary, the main message in the two problem-solving modules is common. It is emphasised that a problem-solving task should have the potential to be solved in several different ways. Examples of problem-solving strategies mentioned are drawings; searching for patterns; working backwards; making lists, charts and/or tables; and simplifying the task. It is also emphasised that the teacher should adapt the problem-solving tasks to the individual student's knowledge and experience. One way to do this is to add facts or take some facts away. Other ways are to change the context, the question, the level of abstraction or the numbers in the task. Then all students in a class can work with the "same" problem-solving task, but with different numbers or with different sets of questions to be answered.

4.4 Why Are Students' Images of Importance?

Mathematics teaching in Swedish primary schools varies considerably, thereby creating differences in students' experiences of mathematics and how it is taught (Swedish Schools Inspectorate 2009). The different social and cultural contexts within which children learn mathematics influences what they learn, what they think mathematics is, and how they think about mathematics learning (Perry and Dockett 2008). What is expected of children in mathematics classrooms is seldom formulated in documents; instead, sociomathematical norms influence the learning opportunities for both students and teachers (Yackel and Cobb 1996). Sociomathematical norms influence students' images of mathematics, which in turn influence both how students behave and how they perform. Sociomathematical norms and student images can also vary between different mathematical areas (Mason 2003). Thus students' images of problem solving may influence both how they behave and how they perform when they do problem solving (or whatever they define as problem solving). When the teaching of mathematics changes in some way, for example, with the introduction of entrepreneurship in problem-solving lessons, it often implies renegotiation and modification of sociomathematical norms. This means that the sociomathematical norms already established in a class influence how changes in mathematics teaching can, or cannot, be successfully implemented by teachers in the classroom (Wester 2015). Thus, if we want to investigate what happens with mathematics in general and problem solving in particular when entrepreneurship makes an entrance in mathematics lessons, it is important to know the images that the students have of mathematics teaching in general and of problem solving in particular. These images influence both how students behave and how they perform as well as how the changes can be implemented.

4.5 The Study

The primary schools involved in the study were selected based on the teachers' interest in being involved. In Sweden, as in other countries around the world (Tatto et al. 2009), most primary school teachers are educated as generalists, teaching several subjects, one of which is mathematics. As mentioned, the teachers in the two schools selected for this paper had taken part in the professional development programme *Boost for Mathematics*. They had worked with two of the modules—number sense and problem solving. Thus, the students of these teachers had been involved in at least eight problem-solving lessons during the previous semester when their teachers worked with the problem-solving module. The ten participating teachers from the two schools were interviewed at the beginning of the study. In the interviews they were asked to describe their mathematics teaching, and after that they were asked about problem solving. Quite often, however, the teachers themselves started to talk about problem solving as they described their mathematics teaching, and in that way the two questions merged.

The students were also interviewed at the beginning of the study; they comprised 195 students from grades 1 to 5 (7–11 years old). The guardians were given written information about the study and had approved their children's participation. To equalise the power imbalance between the researcher and the children, the students were interviewed in pairs (Alderson and Morrow 2011). Just like the teachers, the students were asked to describe their mathematics lessons in general and problem solving in particular. The specific question was "Do you know what problem solving in mathematics is?" If the students answered "yes", they were asked to "tell and/or give examples of what it is".

Notes were taken during the interviews (with teachers and students), and the interviews were also recorded to make it possible to listen to the original wording if needed. When analysing the interviews with the teachers, a typology was made of how they talked about problem solving. Typologies are objectifications and differ from actual types (e.g. the teachers) in that they are analytic constructions used to characterise key patterns from several actual types. None of the teachers in the study personifies the typology to be presented; instead, the typology is based on common features of interest regarding the studied phenomena (Hammersley and Atkinson 2007). When analysing the interviews with the students, the empirical material was given codes grounded in the information; that is, no pre-constructed codes were used, but rather the empirical material was labelled, line by line, with as many codes as possible (Kelle 2007). Based on the question "How do the students describe what problem solving is?", segments in the empirical material were initially inductively labelled with codes such as *don't know, social interpretation, task on paper* and *tricky task*. After that, these codes were deductively connected, based on similarities, into three main categories: *non-mathematical explanations, explanations connected to mathematics but without emphasis on any special features in the tasks* and *explanations connected to mathematics with emphasis on special features in the tasks*.

4.6 Results

In this section the typology based on the interviews with the teachers will be presented first, followed by a presentation of the interviews with the students.

4.6.1 What Images of Problem Solving Do the Teachers Have?

In this section the ten teachers' descriptions of how they work with problem solving will be presented as a typology. The teachers talked very similarly about problem solving in the interviews, maybe as a result of the *Boost for Mathematics* programme.

> In my mathematics, teaching problem solving is one part, and training of skills is another. Those two are the most important parts. Problem solving implies tasks that can be solved by using different strategies; there are many different ways to find an answer. Sometimes there is not just one single answer to the tasks; it is not just right or wrong. Often the students first work on their own for some time, after that they work in pairs and finally we discuss the task together in the class. We call this way of working O-C-A (own – couple – all). I would estimate that I work with problem solving quite often, at least once a week. Sometimes we use tasks from the textbook and sometimes we use tasks from other sources.

4.6.2 What Images of Problem Solving Do the Students Have?

This section will focus on the students' answers in the interviews. As shown above, the question was labelled "problem solving in mathematics". Thus, it was a leading question and the design of the interviews gave the students clues regarding problem solving being connected to mathematics. The students from each grade will be presented together even if they belong to different classes, since there were no visible trends in the answers within each class; instead, the answers were scattered. The only trend found in the answers was between grades; more students in lower grades did not know what problem solving was, and older students more frequently connected problem solving to mathematics, but with great variety in these connections. Of course, differences between students' answers may not only have to do with different images but also with age and abilities to communicate verbally. However, in the interviews the younger children spoke more than the older children. The results will be presented below in three tables based on the three categories mentioned above: *non-mathematical explanations, explanations connected to mathematics but with no emphasis on any special features in the tasks* and *explanations connected to mathematics with emphasis on special features in the tasks*. Each table will be illustrated with some empirical examples. (It should be emphasised that, as examples, these are just illustrative and do not represent the entirety of each category.)"

Table 4.1 Number of students giving non-mathematical explanations

	Don't know	Social interpretation
Grade 1 (50 students)	26	16
Grade 2 (22 students)	5	
Grade 3 (35 students)	3	1
Grade 4 (46 students)	2	1
Grade 5 (42 students)		
Total	36	18

Table 4.2 Number of students giving explanations connected to mathematics but with no emphasis on any special features in the tasks

	Task in textbook	Task on paper	Task written with words	O-C-A
Grade 1 (50 students)		3		
Grade 2 (22 students)		2	6	
Grade 3 (35 students)		2	5	
Grade 4 (46 students)	2	7		2
Grade 5 (42 students)		6	4	4
Total	2	20	15	6

The younger students in particular did not connect problem solving to mathematics, but instead gave answers either indicating *not knowing* what problem solving was or a *social interpretation* of problem solving (Table 4.1).

Thus, the design of the interview, implying that problem solving is connected to mathematics, does not seem to have influenced these students' answers. A social interpretation implies that the students connected problem solving to solving problems in "real life", for example:

> You help someone. If someone has lost something or if the ball gets stuck in a tree. Not saying mean things. (Student, grade 1)

> I know what it is! Rob the bank, look for clues and solve the case. (Student, grade 1)

Of course, many problems in real life have to do with mathematics, but there were no such examples given by these students. In grade 1, all but eight students gave non-mathematical explanations of problem solving. Few of the younger students gave answers connected to mathematics, but several of the older ones did, but with no emphasis on any special features in the tasks (Table 4.2). This implies that the students do connect problem solving to mathematics lessons, but to the designs of the lessons and not of the tasks.

The students answering task in textbook referred to the heading in the textbook indicating that some tasks are problem-solving tasks.

> They are at the end of each chapter. (Student, grade 4)

Also, answers connecting problem solving to task on paper or task written with words indicate not connecting problem solving to any special features in the tasks, but instead focusing on where it can be found or how it is presented.

Table 4.3 Number of students giving explanations with emphasis on special features in the tasks

	Problem solving explained with emphasis on special features in the tasks	
	Problem solving task	Task to be solved with different strategies
Grade 1 (50 students)	5	
Grade 2 (22 students)	9	
Grade 3 (35 students)	21	3
Grade 4 (46 students)	28	4
Grade 5 (42 students)	26	2
Total	89	9

There is a task written with words where you have to figure out what to count. (Student, grade 5)

You solve it on a paper. For example, twenty crowns. Some people have to split them. How many will they get each? (Student, grade 4)

It is a task on a paper. (Student, grade 4)

Answers categorised as *O-C-A* (own–couple–group) indicate that the students connected problem solving to the structure of the lesson. O-C-A was expressed as something that did not need to be explained further, but upon being asked, the students described how they first worked alone, after that in couples and finally as a whole group. They did not, however, mention anything about what they worked on.

O-C-A. The answers are not that important. (Student, grade 5)

Explanations categorised as *explanations connected to mathematics with emphasis on special features in the tasks* were mainly given by the older the students, but also by a small number of students in lower grades.

Table 4.3: Number of students giving explanations with emphasis on special features in the tasks.

Some of these students focused on the design of the task while others focused on the solutions to the tasks (*task to be solved with different strategies*). Regarding the design of the tasks, the students emphasised that a problem-solving task was tricky, puzzling and/or dodgy and therefore demanding to solve.

It is almost like a question. But more tricky. Because of that it takes a little time to solve it. (Student, grade 4)

You have to solve a problem that is puzzling. (Student, grade 2)

When you get a question and are supposed to use different strategies. (Student, grade 4)

You are to solve the problem. There are different strategies to do this. Pictures. They [the strategies] are on a paper. (Student, grade 5)

4.7 Discussion

In this final section we will discuss similarities and differences between teachers' and students' answers as well as how the students' answers might influence the implementation of entrepreneurship in the mathematics lessons in the future. In the interviews, the teachers mainly talked about problem solving as a purpose (ability to formulate and solve problems). The teachers explained that problem solving is an important but separate part of mathematics teaching. However, the students had several different images of what this "separate part" implies. The majority of the younger students made no connections between problem solving and mathematics. This was the case even though the questions in the interview actually gave them some clues regarding problem solving being connected to mathematics. When the teachers talked about problem solving, they talked about both the special features in problem-solving tasks and about the structure of the lessons. The students, however, did not talk about both; they either talked about the structure of the lesson or about the special features of the tasks. Regarding the structure of the lesson, the teachers emphasised O-C-A, while the students mentioned this but mainly emphasised being given tasks on paper or tasks written in words. Several of the students knew that problem solving implies a task with special features, but few gave answers that indicated awareness that the task could be solved in different ways, which was emphasised by the teachers.

The different images held by the students will influence their expectations when they are told that they are to work with entrepreneurship and problem solving in the new project. For example, for some students this will be associated with real-life situations while for others it will be associated with tasks that can be solved in different ways. Such differences will probably influence both how the students behave and how they perform as well as how the changes can be implemented.

Finally, some words about the *Boost for Mathematics* programme. As mentioned, sociomathematical norms may influence whether changes in mathematics teaching become successful or not, and may also influence the learning opportunities for both students and teachers. Further, sociomathematical norms can vary between different mathematical areas. In the *Boost for Mathematics* programme nothing is written about how teachers should, or should not, *talk with* the students about problem solving. The students are to *do* problem solving, but only the teachers are to *talk about* problem solving. Thus, it seems to be up to the students to interpret what problem solving is, as well as figure out why they are doing it. It is questionable whether this really should be up to the students; as seen in this paper, the students' images differ and these differences may influence what they think about problem solving, what they learn about and by problem solving, and will also influence the potential for their teachers to teach problem solving. And, as mentioned, these differences will probably influence what happens with mathematics in general, and problem solving in particular, when entrepreneurship makes an entrance in mathematics lessons.

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Chapter 5
Secondary School Mathematics Teachers' Conceptions on Data-Based Decision-Making: Insights from Four Japanese Cases

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Abstract The role of informed decision-making in today's society is very important at personal, professional, societal, and even educational levels. Thus, mathematics teachers are required, now more than ever, to appropriately engage students with tasks having the potential to promote decision-making skills. This article reports on the features and competence demands of tasks that are thought to promote decision-making skills by a purposeful sample of twenty-three Japanese secondary school mathematics teachers, focusing on a case study of two pairs of teachers who selected the same task. A qualitative analysis on the collected data revealed some commonalities between the answers, as well as correspondences and discrepancies between what the participants and statistics educators think a task with the potential to promote decision-making is.

5.1 Introduction

Statistics has become very important at all levels of citizenry in today's society, in which large amounts of data are available to almost everyone. Then, to be part of modern society in a competent and critical way, citizens need to be able to interpret such data in a broad sense, and understand the variability and heterogeneity which cause uncertainty in interpreting information, in facing risks, and in making decisions. In the particular case of the latter, many statistics educators, curriculum developers and international agencies agree on the increasing importance for students to gain competence in using, handling and interpreting data to inform decision-making at personal, professional, and societal levels (Garfield and Ben-Zvi 2008).

The last reform to the Japanese mathematics course of study echoes these ideas. At secondary school level, the latest mathematics course of study emphasises—in

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the domains “Practical Use of Data” at junior high school, and “Analysis of Data” at senior high school—nurturing the attitude and ability to purposely process daily-life data, capture its trends and features, and make decisions based on such analysis (MEXT 2008, 2009). Thus, due to the significant place held by fostering decision-making skills in Japanese secondary school mathematics, teachers must be able to properly design instruction aimed to develop students’ decision-making skills.

Despite all these facts, what decision-making is and how to promote the skills related to it are not defined in either the mathematics courses of study, or in the teaching guides. Thus, teachers are left to determine by themselves how decision-making could be promoted, which raises particular concern, due to the reported need for appropriate training in statistics education in the case of future and in-service mathematics teachers in Japan (Isoda and González 2012; González 2014).

The aforementioned facts point to the importance of doing research on how Japanese mathematics teachers conceptualize the promotion of decision-making in their students. To shed light on this issue, the present study addresses the following research questions: (1) What kind of tasks do Japanese secondary school mathematics teachers regard as having the potential to promote decision-making, in particular when teaching statistical contents? (2) When teaching statistical contents, what knowledge and skills do Japanese secondary school mathematics teachers believe to be associated with the promotion of decision-making? This is a similar approach to the one adopted by Levenson (2011), who asked similar questions to explore beliefs and values regarding creative mathematical tasks, as well as to identify the properties of such tasks.

5.2 Theoretical Background

5.2.1 Decision: What Is It?

A decision is defined as “the broader process within which a choice among specific options will be made” (Brown 2005, p. 1). Through this process, the decision-maker is ultimately able to determine what action to take (Brown 2005, pp. 1, 236–237). Making a decision demands from the decision-maker to engage in the many phases of the decision-making process (Arvai et al. 2004; Edelson et al. 2006):

Definition: here decision-makers define the specific decision that has to be made, as well as a broad set of end objectives in the context of the impending decision.
Planning: during this phase, the identification, design, and choice of an optimal way to achieve ends objectives is determined. The choices must be a set of appealing and purposeful alternatives from the objectives previously defined.
Data: in this phase, recalling and seeking of information is carried out, as well as collecting statistical data relevant to the achievement of end-objectives.
Evaluation: during this phase, decision-makers must assess the implications of different choices for the decision.

Weighing impact: in this phase, decision-makers weigh the impacts of the different options on stakeholders based on their own values. Thus, this stage gives decision-makers the opportunity to see how different values can lead to different decisions.

Making and justifying a decision: during this phase, decision-makers select the course of action that better addresses their objectives, in the light of the decision-makers' constraints, considerations, assumptions, value systems, stakeholder impact, etc., and provide an informed justification for such a decision.

5.2.2 Decisions: What Types Are There?

In the present study, decisions will be classified in four types: three of them (i.e., personal, professional and civic) were identified by Brown (2005, pp. 5–7), while the last type (i.e., object-related) is proposed by the author.

Personal decisions: those that decision-makers make on their own behalf.

Professional decisions: those that professionals and specialist decision aiders make on behalf of others in a work capacity (e.g., as in medical practice).

Civic decisions: these are decisions made on a public issue, such as when decision-makers, as citizens, take a private position on someone else's (e.g., government's) choice, for which they have no direct responsibility.

Object-related decisions: those made about parameters or particular features of statistical objects (i.e., decisions regarding language situations, concepts, propositions, procedures and arguments) involved in a given statistical problem.

5.2.3 Decision-Making, Values and Value-Focused Thinking

Traditional decisions methods mostly emphasize the exploration of alternatives. This approach is called "alternative-focused thinking" (AFT). According to Keeney (1988, 1992), AFT may not work in a decision-making situation because (1) alternatives can be misleading the decision, and (2) the attention is limited to available alternatives, which may not reflect what the decision maker really wants: what he/she values.

The importance of the decision-maker's values in any decision process is stressed by several authors (e.g., Keeney 1988, 1992; Edelson et al. 2006; Edwards and Chelst 2007). Values are defined by Keeney (1992, pp. 6–7) as follows:

> Values are principles used for evaluation. We use them to evaluate the actual or potential consequences of action or inaction, of proposed alternatives and of decisions. They range from ethical principles that must be upheld to guidelines for preferences among choices.

In other words, values are abstractions that help organize and guide preferences, and can ultimately be understood as what the decision-maker wants to achieve via the decision (von Winterfeldt and Edwards 1986, p. 38).

Keeney (1988, 1992) proposes “value-focused thinking” (VFT) in any decision context, as opposed to AFT. VFT would provide, among other things, (1) the identification of creative alternatives to better achieve what is desired; (2) a wider range of alternatives in comparison to AFT; and (3) articulated values, by stating decision-maker’s own values in the form of alternatives and objectives (Keeney 1988).

5.3 Methodology

5.3.1 Data-Collection Instrument and Participants

In order to address the research questions of this empirical study, an assignment-like survey was designed, asking respondents the following open questions:

1. From a textbook, teacher’s guide, student workbook, internet, academic journal, or other type of source, choose a task or activity that, in your opinion, would promote decision-making skills in your students in secondary school mathematics when you teach contents in the mathematical domains “Practical Use of Data” or “Analysis of Data”. You may also develop a task or activity by yourself.
2. Attach a copied or printed version of the chosen task or activity, and report its source.
3. Briefly explain why, in your opinion, the chosen task or activity has the potential to promote decision-making skills.

A purposeful sample of twenty-three Japanese secondary school mathematics teachers was surveyed. The participants were teachers who participated in a national academic meeting on mathematics education in Japan. The data collection went from mid-July to October 2014. Thirteen of the respondents were working at junior high school, while the rest were working at senior high school. The participants were between 24 and 63 years old, having between 1 and 41 years of teaching experience (with eleven of them with at least 10). In this paper, it is reported a preliminary analysis of the data gathered from four teachers belonging to this sample.

5.3.2 Data Analysis

During the initial phase, all the questionnaire answers were translated from Japanese into English by the author of this paper. Then, they were read repeatedly in order to gain an overall impression. After this general analysis, the main analysis was undertaken, aiming to identify what was common to all participants. A "bottom up" approach to coding was initially used to analyze the tasks' features and the participant's reasons for choosing such tasks, in order to ensure that the themes or categories extracted were grounded in the data. The author reviewed all the given answers to the questionnaire and identified answers that occurred frequently. Such answers appearing to contain similar content were initially given the same code by the author. A process of reduction and clustering of categories followed, resulting in summary groupings of themes sharing common meaning.

5.4 Findings

5.4.1 Tasks' Features and Reasons for Choice

From the qualitative analysis performed on the tasks collected from all the 23 teachers, several features were identified. Furthermore, from the grounded analysis of the reasons given by teachers about why their chosen task has the potential to promote decision-making, six category clusters of competence aspects that participants seem to associate with decision-making were identified. Those results are shown in Table 5.1. For a detailed discussion of the categories in Table 5.1, regarding the first twelve participants of this study, the interested reader should refer to González (2015).

The data analysis revealed that two pairs of surveyed teachers chose the same tasks. Although this was a surprising discovery at first, the research possibilities offered by this fact appeared interesting, in particular the opportunity of studying how different teachers conceptualize and deal with decision-making while considering the same task.

Two of these teachers—hereafter Teacher 1 (T1) and Teacher 2 (T2)—selected the "Ski Jump" task, quite famous in Japan since it appeared in the 2012 National Assessment of Academic Ability and Learning Situation for Grade 9 Mathematics. Both T1 and T2 are junior high school teachers, T1 is 63 years old (y.o.) with 41 years of experience (YoE), and T2 is 26 y.o. with 2 YoE. The other two teachers—hereafter Teacher 3 (T3) and Teacher 4 (T4)—also selected an identical task, the "Let's share Pocarius" task, from a grant-in-aid research report for the Japan Society for the Promotion of Science (JSPS). JSPS research reports are resources commonly consulted by researchers, pre- and in-service teachers in Japan. Both T3 and T4 are senior high school teachers, T3 is 24 y.o. with 2 YoE, and T4 is 62 y.o. with 36 YoE. Both tasks—referred hereafter as Task 1 and Task 2, respectively—are depicted in Fig. 5.1, and the reasons given by these four teachers about why either Task 1 or Task 2 has the potential to promote decision-making are shown in Table 5.2. Then, after seeing the answers given by T1, T2, T3 and T4 through the lenses of the features outlined in Table 5.1, the results summarized in Tables 5.3 and 5.4 were obtained. From the information summarized in Table 5.3, we can

Table 5.1 Features of a task with potential to promote decision-making, and competence aspects associated with decision-making, according to the collected data

Tasks' features
Number of choices offered
The task explicitly requests students to think of several possible solutions/to solve the problem in different ways
The task invites students to engage in open inquiry and investigation
The task required to connect different statistical concepts
The task is a multi-step one, comprised of several mini-tasks
The task explicitly demands from students to communicate and/or justify their procedures
Different types of statistical representations
The task includes the use of manipulatives
The task is set in a real-life context
The task can be solved in several ways
Type of decision requested (i.e., personal, professional, civic, object-related)
Environment in which the task is supposed to take place (i.e., indoors, outdoors)
Decision-making competence aspects
Decision-making involves opportunity to build students' own decision criteria
Decision-making involves personal or societal values
Decision-making demands from students to make use of their own mathematical and statistical literacy skills
Decision-making involves engagement with different steps of the open-ended approach
Decision-making involves engagement with a familiar real problem
Decision-making requires inter-personal processes such as discussion, communication, argumentation, negotiation, and collaboration

see more similarities than differences between Task 1 and Task 2. For example, both tasks aim to promote decision-making by engaging students in statistical investigations, connecting different statistical concepts, being set in a real-life context, and having more than one solving way. This is aligned with the findings of many previous studies on instruction of decision-making at school level (e.g., Arvai et al. 2004; Edelson et al. 2006; Edwards and Chelst 2007; Garfield and Ben-Zvi 2008; Pfannkuch and Ben-Zvi 2011).

One of the few differences between these tasks is concerning the type of decision requested. Task 1 requires an object-related decision, whereas Task 2 requires a civic one. In this regard, specialists say that, although decision-making skills are not specific to any particular type of decision, developing such skills has been done mainly in the context of personal and civic decisions, to which students can most easily relate (cf. Brown 2005, p. 155; Edelson et al. 2006). Also, by just requesting object-related decisions, completing the task seems to depend more on students' content knowledge.

The other big difference between these tasks is the number of predetermined alternatives. Task 1 offers two, which limits the detection of the alternatives to the given ones (e.g., a likely answer from students could be "either Harada or Funaki,

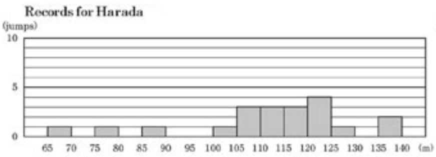
Task 1: The "Ski Jump" task

Misaki researched the jump distances of the two members of the Japanese team, Masahiko Harada and Kazuyoshi Funaki. The two histograms shown below summarize the jump distances of these two athletes from several international competitions held during the 1998 season, prior to the Nagano Olympics. From these histograms, we can realize/understand that, for example, these two athletes made 3 jumps longer than or equal to 105 m but shorter than 110 m.

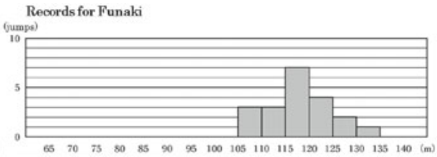
Based on the data shown in the histograms, Misaki wanted to think about whom of these two athletes would have jumped farther if both would make one more jump.

If you are going to choose one athlete who might make the longer jump based on your comparison of the two histograms and the characteristics you identified, which athlete would you pick? From (a) and (b) below, select one athlete. Then, explain why you chose that athlete by comparing the characteristics of the histograms of these athletes. You may justify a choice of either athlete.

(a) Harada



(b) Funaki



Task 2: The "Let's share Pocarius" task

Mr. Tōru's school received 600 packs of the powdered sports drink Pocarius. He distributed them between the clubs having activities during summer. The number of members and activity days of the summer clubs are shown in the table below:

Club	N° of members	N° of activity days	Club	N° of members	N° of activity days
Basketball	20	14	Badminton	15	8
Soccer	50	12	Chorus	25	24
Tennis	30	18	Science	10	24

If it were you, how many packs would you give to each club? Please explain your reasoning.

Fig. 5.1 Two tasks chosen by multiple participants in this study

since both could make jumps of similar length"), which cannot be chosen since it is not an alternative. On the contrary, Task 2 offers no options from the start, and hence students need, as in VFT, to articulate their own values by deciding what they want to achieve, and then trying to figure out how to get it (Keeney 1992, pp. 4–6).

From identifying what competence aspects teachers relate with decision-making, it would be possible to determine teachers' conceptions of decision-making (i.e., how teachers consciously understand decision-making and its promotion through the teaching of statistical contents). Some similarities were found between how the two teachers who chose Task 1 conceive decision-making. For example, both teachers expressed that decision-making involves an opportunity for students to build their own rules for decision, which is one of the main features of the decision-making process (cf. Brown 2005; Edelson et al. 2006; Garfield and Ben-Zvi 2008, p. 277). Also, both teachers associated decision-making with skills related to mathematical and statistical literacy, by using expressions such as "mathematical grounds" and "following the PPDAC cycle". Moreover, both teachers indicated that decision-making involves engagement with different steps of the open-ended approach, which is also true, since "dealing with the openness" is a main feature of the decision-making process (cf. Edelson et al. 2006; Edwards and Chelst 2007). Finally, none of the teachers explicitly mentioned about decision-making involving a familiar real problem. Students relate more easily to such problems, particularly to those in which they are asked for personal or civic decisions (e.g., Brown 2005, p. 155; Edelson et al. 2006).

Table 5.2 Reasons given by the participants for having chosen either Task 1 or Task 2

	Reasons for selection
T1	Because I think that, when students are selecting only one option among many, they are making decisions based on certain criteria (their own values and mathematical grounds).
T2	The learning issue is "to predict the next jumping record", and for that students will estimate a likely record in the future from records registered in the past. During that stage of "Analysis", students will decide about "which record is likely to happen?" In addition, this whole learning activity is organized following the PPDAC cycle. During the stage "Plan", students are to decide about "what is the required data for the solution?"; and for the stage "Conclusion" they said "extreme values being low could be because the weather was bad. On that moment, it is possible that other athletes' records were much worst". Then, based on the problem context about the analysis results, the decision is determined. So, by means of the aforementioned learning issue and learning activity, I think decision-making skills while using statistics are being fostered.
T3	In this diversified society, I think children have to make choices to progress by themselves in this society. In other words, it is necessary to enhance children?s decision making skills. And, regarding to this, the thing with which mathematics education can contribute to is, in my opinion, to develop their own criteria to decide from a given information, and to foster the ability to build consensus by persuading others Therefore, I chose this problem because problems like this, making children consider real problems familiar to them, and I think it is a good problem to encourage discussion and communication. Moreover, another reason for choosing it was that it does not matter the school level.
T4	In an atmosphere of social fairness, everybody will say his/her solution, will discuss and finally, to some extent, will end up achieving a common understanding answer.

Table 5.3 Features of Task 1 and Task 2

Features	Task 1	Task 2
Number of choices offered	2	0
The task explicitly requests students to think of several possible solutions/to solve the problem in different ways	Y	N
The task invites students to engage in open inquiry and investigation	Y	Y
The task required to connect different statistical concepts	Y	Y
The task is a multi-step one, comprised of several mini-tasks	Y	N
The task explicitly asks students to communicate/justify their procedures	Y	Y
Different types of statistical representations	1	1
The task includes the use of manipulatives	N	N
The task is set in a real-life context	Y	Y
The task can be solved in several ways	Y	Y
Type of decision requested by the task (Pe = personal; Pr = professional; C = civic; O = object-related)	O	C
Environment in which the task is supposed to take place (I = indoor, O = outdoor)	I	I

Table 5.4 Competence aspects associated with decision-making, according to the reasons for task choice provided by teachers T1, T2, T3 and T4

Competence aspects	T1	T2	T3	T4
Decision-making involves opportunity to build students' own decision criteria	✓	✓	✓	
Decision-making involves personal or societal values	✓			✓
Decision-making demands from students to make use of their own mathematical and statistical literacy skills	✓	✓	✓	
Decision-making involves engagement with different steps of the open-ended approach	✓	✓		
Decision-making involves engagement with a familiar real problem			✓	
Decision-making requires inter-personal processes such as discussion, communication, argumentation, negotiation, and collaboration		✓	✓	✓

Regarding the differences between teachers who chose Task 1, only T1 explicitly mentioned the enactment of personal values as an important aspect of decision-making, while only T2 explicitly mentioned the importance of inter-personal processes.

In the case of the conceptions of decision-making held by the two teachers who chose Task 2, T3 and T4, a pair of similarities was found. Both teachers did not acknowledge explicitly that decision-making involves engagement with different steps of the open-ended approach. Also, both teachers explicitly pointed out the importance of social and inter-personal processes in the decision-making process. In fact, the role that communication and personal interaction play in the creation of alternatives during the decision-making process is fundamental (Keeney 1988, p. 468), and many researchers (e.g., Arvai et al. 2004; Edwards and Chelst 2007) are of the idea that the decision-making process is much stronger with discussion and feedback, and hence recommend that to effectively engage in decision-making in the classroom.

Four differences were found between the conceptions of decision-making held by T3 and T4. There were about decision-making being an opportunity for students to create their own decision criteria (acknowledged just by T3); decision-making involving values (expressed just by T4); decision-making requiring the enactment of mathematical and statistical literacy skills (acknowledged just by T3); and decision-making involving a familiar real-life problem (expressed just by T3).

5.5 Conclusions

Regarding the research question "What kind of tasks do Japanese secondary school mathematics teachers regard as having the potential to promote decision-making, in particular when teaching statistical contents?", twelve particular features of such tasks emerged from a grounded analysis applied to the collected data. Such features seem to be of a structural nature (e.g., being a multi-step task), a cognitive nature (e.g., requiring linking different statistical concepts to be solved), an affective nature (e.g., requiring the articulation of students' values when no alternatives are given), or a technical nature (e.g., being set in a real-life context).

From analyzing teachers' reasons for task choice, it was possible to answer the research question "When teaching statistical contents, what knowledge and skills do Japanese secondary school mathematics teachers believe to be associated with the promotion of decision-making?" Also, from such reasons, although two of them were somehow short, it was possible to sketch participants' conceptions of decision-making. Six competence aspects were found, all of them aligned with those identified in the specialized literature. Such aspects can be of a cognitive nature (e.g., demanding from students to use their mathematical and statistical literacy skills), an affective nature (e.g., involving values), or a social nature (e.g., requiring engagement in inter-personal processes). Awareness and development of these conceptions during pre- and in-service teacher training would enable teachers to appropriately foster decision-making skills, as intended by mathematics curriculum developers in many countries.

Regarding the four teachers who selected the same tasks, similarities and differences in their conceptions of decision-making were found. For example, neither T2 nor T3 explicitly expressed that decision-making involves the consideration of values. However, all four teachers seem to believe that the decision-making process is related to personal, societal, and disciplinal values. For example, it seems that T3, implicitly, is fostering decision-making skills through demanding from his students to articulate their own values by making them explicit in the form of alternatives (Keeney 1988).

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Chapter 6
Teachers' Activities During a Mathematics Lesson as Seen in Third Graders' Drawings

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Abstract Third-graders from 19 classrooms ($N = 316$) were asked to draw a picture on a mathematics lesson. Based on these drawings we have produced a list which can be used to analyze how young pupils describe in their drawings their teacher's activities during mathematics lessons. This list contains items: Teacher is giving information on mathematics, Teacher is giving instructions, Teacher is asking questions, Teacher is giving feedback, and Teacher is reflecting. In addition, the list contains information on whether the pupils have drawn the teacher at all, whether the teacher is quiet or talking, and what the teacher's location is in the classroom. In order to show the functioning of the list we give some results of the analysis.

6.1 Introduction

Classroom is a social environment, where pupils spend around 20–30 h per week during the 6-year primary education in Finland. Thus, the classroom environment is significant in shaping pupils' perceptions. Neumann (2013) examines the three learning relationships between pupils and teachers, namely those in which the teacher is at the forefront, those in which the pupils are at the forefront, and those in which both the teacher and the pupil share the forefront. Based on this he divides pupil-centered learning into three contours: learning contexts that center in pupils, that center on pupils, and that center with pupils. Learning context that center in pupils means that learning happens within pupils with more or less no assistance from the teacher. In the second contour, the teacher plans the lesson and it is the pupils' responsibility to learn the material. Learning contexts that center with pupils bring the teacher into partnership with the pupils and learning happens as the teacher and pupils are collaborating.

In the literature of teaching and learning, it is often referred to teacher-centeredness and student-centeredness. According to Thomas et al. (2001) main characteristics of teacher-centeredness are that (1) the teacher is at the center of

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instruction and learning, and (2) the classroom environment is organized to facilitate the teacher as the knowledge conduit. Respectively, the main characteristics of student-centeredness are that the students are at the center of learning and the teacher only guides or facilitates all activities.

Usually, communication and activities in mathematics teaching has been studied by direct observing (e.g. Smith and Glynn 1990), by interviews (e.g. Chaviaris and Kafoussi 2010) and/or by questionnaires (Kosko and Wilkins 2010). Instead, free hand drawing has seldom been used to study pupils' images of teaching. However, on one hand third-graders' responses in answering questionnaires are not always reliable due to their young age, and on the other hand, interviews and classroom observations are time demanding methods. Therefore, in this paper we concentrate to find out an alternative method to see how young pupils describe in their drawings their teacher's activities.

The data of our article consists of third-graders' drawings that were collected in the autumn of 2010 in Finland. In the earlier articles we have looked at what kind of teaching methods and communication (Tikkanen et al. 2011), and emotional atmosphere (Laine et al. 2013) can be found in the pupils' drawings. We have also looked at the differences of teaching methods and communication found in third-graders' mathematics lessons in Finland and USA (Hart et al. 2014).

The aim of this research is to produce a method how to analyse teachers' activities during a mathematics lesson from pupils' drawings. This kind of subjectively perceived classroom of Finnish primary pupils in mathematics has rarely been studied. A reason for the small amount of research may be the lack of suitable measurement instruments for young children.

6.2 Theoretical Framework

In cognitive science, a mental image is defined as a mental representation or a mental referent. Kosslyn (1988) proposed that a mental image is a mental representation of an object, an event or a situation whose features are spatially and temporally organised. The formation of such a representation can be based on direct experience with its referent. In his theory of imagery Kosslyn (1996) has explained that the images may have depictive, picture-like qualities that could not be regarded as purely propositional, language-like mental representations. His theory suggests that the images are short-term memory representations generated from long-term memory representations that may have a depictive or propositional form. The depictive form may be used less, as propositional knowledge increases and deduction becomes easier. If pupils have few propositional linguistic representations of the image's referent they tend to use their visual image.

De Beni and Pazzaglia (1995, see also De Beni et al. 2007) list different kind of mental images such as common vs. bizarre images, memory vs. imagination images, single vs. interactive images, general vs. specific and autobiographic vs. episodic-autobiographic images. General image represents a concept without any reference to a particular example or to specific characteristics of the item. A general image of a 'table' could be described as a surface with four legs. A specific image represents a single well-defined example of the concept without reference to a specific episode. For example specific images of a 'ball' might be a tennis ball, a football or a basketball. An episodic-autobiographical image represents the occurrence of a single episode in the subject's life connected to the concept. All of these images are content defined. The studies of Pitta (1998) demonstrate that pupils in comprehensive school have among others general, specific and episodic autobiographical images both of mathematical contents and of mathematics learning and teaching. Our data consists of drawings about mathematics lessons. The drawings produced by pupils are thus mainly episodic autobiographical images. An episodic autobiographical image of learning mathematics might be for example "Last week I succeeded to solve hard problems in mathematics with my classmates".

Drawings help pupils to overcome the difficulties in disclosing their thoughts, feelings and opinions to an adult researcher (Zambo and Zambo 2006). Drawings are useful, because they require little or no language mediation. According to Weber and Mitchell (1996) pupils' classroom drawings form rich data to study children's conceptions on teaching. Pupils' drawings have made an alternative and complementary contribution among conventional research methods by conveying their images about mathematics, mathematics teaching, their teacher, their peers and classrooms in mathematics lessons. According to Losh et al. (2007) primary pupils conceptualise and clearly distinguish the professional of teachers among scientists and veterinarians via drawings.

Picker and Berry (2000) compared lower secondary pupils' images of mathematicians in five countries using drawings with a questionnaire. About 20% of the drawings portrayed a school teacher, and classrooms which basically looked the same from country to country with only small differences. Drawings of the teacher showed that s/he neither always mastered the teaching group nor the topics to be learned. S/he seemed to be cleverer and better than her/his pupils, but s/he lacked common sense, style and calculation skills.

Bulut (2007) used drawings with writings in order to clarify fifth-graders' views of changes in mathematics teaching. The data revealed that mathematics teaching in Turkey had become more student-centred. In order to understand what pupils value in their mathematics learning process, Seah (2007) conducted a study in Australia comparing 118 primary pupils' drawings about their individual impressions of effective mathematics lessons. The data revealed that mathematics lessons featured a co-valuing of fun, teacher and pupil experience and the teacher's explicit explanation or instruction on board work by both pupils and their teachers. In the framework of motivation theory, Dahlgren and Sumpter (2010) compared second- and fifth-graders' conceptions on mathematics and mathematics teaching via drawings with the written questionnaire in Sweden. All pupils presented mathematics teaching as an individual activity with a focus on the textbook. Most of the second-graders had positive attitude toward mathematics whereas a larger proportion of the fifth-graders had a negative one.

6.3 The Purpose of the Study

A pupil's drawing gives a "snapshot" of a teacher's activities experienced by her/his pupil. Our study aims to find out how third-graders describe what the teacher is doing during a mathematics lesson, to what kind of issues they pay attention in their drawings. Therefore, we first need to generate a method to analyse young pupils' drawings in order to answer the following question:

How can we identify a teacher's activities as seen in young pupils' drawings?

In order to show the functioning of the method we have applied it in the case of mathematics lesson in third grade.

6.4 Methodology

6.4.1 Participants and Data Gathering

Drawings were collected and analyzed from a total of 316 third-graders (about 8–9-year-olds) from the classes taught by nineteen different teachers in nine primary schools in Great-Helsinki (Finland). The pupils did the drawing task during their mathematics lessons in the beginning of their third school year (autumn 2010). The task for the pupils was, as follows:

"Draw your teaching group, the teacher and the pupils, in a mathematics lesson. Use speaking and thinking bubbles to describe discussion and thinking. And mark "me" in your drawing."

The drawings collected by the teachers were obtained from 165 boys, 150 girls and one pupil who did not indicate the gender. The contents of the speaking and thinking bubbles enabled us to investigate the communication in classroom.

6.4.2 Data Analysis

The starting point for the classification of the pupils' drawings is the analysis method developed by Tikkanen in her dissertation (Tikkanen 2008). This was further developed by Pehkonen et al. (2011). According to this method, a drawing as a data source can be divided into content categories. A content category means the phenomena on which data are gathered, for example, a teacher's location in the classroom or pupils' comments on mathematics.

6.4.3 *Different Trials*

In the first stage, our idea was to develop a method to reveal the range of teacher-centeredness vs. student-centeredness used in the classrooms as seen in the pupils' drawings. For this, we used Tikkanen's classification scheme for pupils' drawings (Tikkanen 2008), and also the list combined by Markic and Eilks (2010). The central idea was to find from the drawings items connected to the description of the teacher's activity, also the teacher's location with respect to pupils was marked. Each feature was scored in a dichotomous fashion with an indication of "present" or "not present" in each drawing. After several trials we came to the conclusion that with such a simple list with yes or no markings we cannot say much about the teacher-centered or student-centered teaching/learning situation in a drawing. This is understandable because both teacher-centered and student-centered teaching/learning are actually rather complicated ideas and they contain a wide range of meanings (Neumann 2013).

Our second trial was to use a specialist consulting group: We used 24 mathematics teacher educators and researchers in a conference on mathematics education in autumn 2012. They were asked to mark in per cents how much a certain sentence in a pupil's drawing like "Teacher is questioning" indicates teacher-centred or student-centred teaching. Their answers varied from 25% to 75% with the mean 52%. Due to the huge variations in respondents' answers, we had to abandon also this approach.

Finally, we listed from all the drawings in one classroom all possible teachers' activities during the mathematics lesson as seen in the pupils' drawings. This list was then completed by going through all the third-graders' drawings from 19 classes.

6.4.4 *An Example of the Drawings*

In the drawing shown in Fig. 6.1, the smiling teacher is sitting behind her desk (TA21). Code TA21 refers to the first subcategory in the content category TA2 (teacher's position). She has written a task on the blackboard (TA1). She is thinking positively about her pupils (TA61).

The drawing instruction given to the pupils was quite open, thus there is a large variability among the drawings. Here we have chosen one drawing that will illustrate the coding. It does not represent any specific prototype. It contains speech bubbles whereas some of the drawings were very simple ones e.g. pupils were substituted with desks. More drawings are presented e.g. in the publication of Pehkonen et al. (2016).

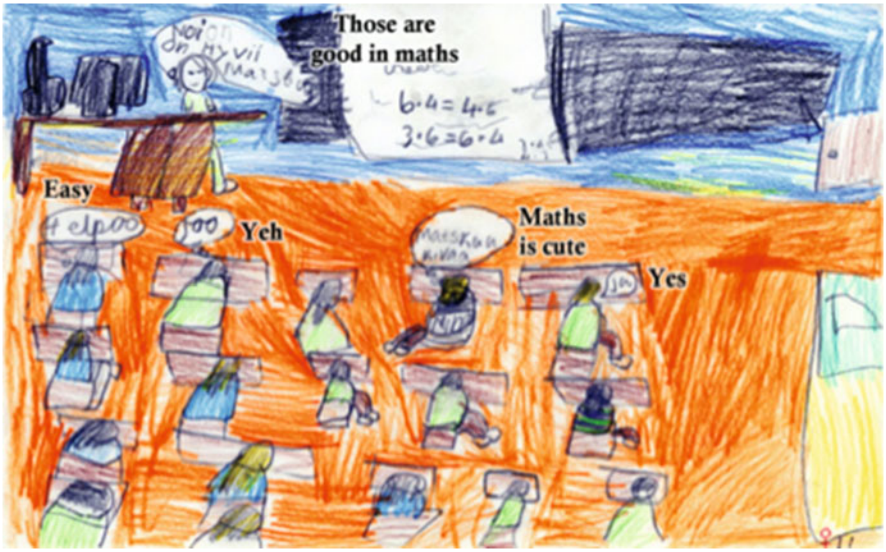

Fig. 6.1 An example of a girl's drawing

6.5 Results

In Table 6.1 we have divided all the possible teachers' activities which we found in the third-graders' drawings into five categories: Teacher is giving information on mathematics, Teacher is giving instructions, Teacher is asking questions, Teacher is giving feedback, and Teacher is reflecting. In addition, these categories are divided to sub-categories. The list also contains information on whether the pupils have drawn the teacher at all, whether the teacher is quiet or talking, and what the teacher's location is in the classroom.

To show the working of this method we have used it to calculate 19 teachers' activities from 316 Finnish third-graders' drawings including the differences between the girls' (150) and boys' (165) drawings. According to the pupils' drawings the teacher is usually sitting behind her desk during the mathematics lesson (49%). The boys drew the teacher slightly more often (57%) sitting behind her desk than the girls (almost significant). In 60% of the drawings the third-graders had drawn the teacher teaching mathematics e.g. having written a task on the board or helping the pupils in their problems. In this case, there were significantly more girls' drawings (70%) than boys' drawings (52%). Nearly half (45%) of the pupils also drew the teacher giving instructions with half (22%) of these related to mathematics and 15% related to keeping up order. Here again there were

Table 6.1 The coding of teachers' activities during a mathematics lesson as seen by the third-graders in their drawings

Code	Title	Comments
TA0	**No teacher**	No teacher can be identified
TA1	**Teacher is quiet**	Teacher is neither speaking nor thinking
TA2	**Teacher's location**	
TA21	Behind or close to the desk	When difficult to decide between TA21 and TA22, choose TA22 if there is a task on the board.
TA22	Near the blackboard	
TA23	Among the pupils	
TA3	**Teacher informs on mathematics**	Teacher shows/talks about the task/s on the board
TA31	There is a task on the board	
TA32	Teacher is teaching	
TA33	Teacher is helping	
TA4	**Teacher gives instructions**	
TA41	... related to studying mathematics	Teacher gives permission to answer
TA42	... not related to studying	"Open the door, please".
TA43	Teacher keeps order	"Be quiet".
TA5	**Teacher is questioning**	
TA51	Teacher asks about a task.	There is a test in the class
TA52	Teachers asks something else	"Who will do the calculation?"
TA6	**Feedback**	
TA61	Teacher gives positive feedback related to mathematics	"Good", "That is correct".
TA62	Teacher criticizes	"Wrong"
TA7	**Reflecting**	
TA71	Teacher reflects on mathematics	
TA72	Teacher reflects on something else	"I wonder how pupils are doing".

Extra comment. When the pupil has drawn a cartoon, the categorizing is made from the first picture which contains both the teacher and at least one pupil.

significantly more girls' drawings (55%) than boys' drawings (35%). However, one has to notice that the differences between girls and boys is mainly due to the fact that girls are writing much more on the speech bubbles than boys. Only in a quarter of the drawings the teacher is asking questions, and again in the girls' drawings slightly more often than in the boys' drawings. The pupils had written very little about the teachers giving feedback (12%), and then it was almost always positive: only two pupils, one girl and one boy had drawn a negative feedback.

6.6 Discussion and Conclusions

The main aim of this article is to analyse teachers' activities from young pupils' drawings. By going through 316 third-graders' drawings from nineteen classes we listed all possible teachers' activities during the mathematics as seen in the pupils' drawings (see Table 6.1). However, one has to notice that there are so many different ways for a teacher to act that our list does not include all the possibilities. This means that the coding has to be checked in each new case.

Teacher is a central element in the mathematics lessons. Only about 10% of the pupils did not draw the teacher at all, and only in 15% of the drawings the teacher was drawn without saying or thinking anything. In 50% of the drawings the teacher was sitting or standing near the desk and in 60% of the drawings she was giving information about mathematics. Half of the instructions the teacher was giving concerned mathematics and a third about the order in the classroom. In this case, there is also a difference between the girls and the boys. A third of the girls referred to mathematics but only a sixth of the boys. Nearly all the feedback that the teachers were giving is positive. It would be interesting to know why only a quarter of the third-graders include the teacher as asking questions in their drawings.

Pupils' drawings reveal important information that is difficult to be obtained from young children using more conventional methods (cf. Tikkanen et al. 2011; Weber and Mitchell 1996). Especially by connecting words and images the drawers reflect their feelings and attitudes towards their teacher. These drawings are snapshots from certain even though quite usual situations during mathematics lessons. Thus they give the researchers and school authorities a possibility to have a look what is happening in classrooms. The variation between the teachers was quite large. For example, the majority of the pupils in one classroom drew their teacher standing quietly near her desk whereas in another classroom the teacher was mostly either near the board or among the pupils giving information or helping the pupils.

Altogether, pupils' drawings seem to be a versatile way to collect information about teachers' activities in mathematics lessons. There are, however, several factors that may influence the drawings pupils will produce. Some pupils may have difficulties to draw complicated pictures and, therefore, they might draw only such objects or situations that are easy for them to draw. On the other hand, many of the pupils who were not able to draw persons just used stick figures or wrote the names of their classmates on the desk. Also the fact how the task is given may have an effect on how pupils will concentrate on working and how they understand what they are expected to do.

However, one has to be careful in making definite conclusions from the drawings. Different pupils may derive different meanings from the very same happening. However, here in most of the classrooms we have quite a lot of pupils (from 16 to 19). Furthermore, one of the researchers (LN) followed once a month a mathematics lesson from nine of these teachers. According to her the general overview obtained from the drawings is in accordance with her observations.

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Chapter 7
Serious Frivolity: Exploring Play in UK Secondary Mathematics Classrooms

Elizabeth Lake

Abstract As part of a PhD researching teacher's positive emotions in secondary mathematics education, I am exploring the role of play as both instigator and product of emotions. Play has recognised social, creative, cognitive and especially emotional benefits for young children. However, as students mature, do they still respond to play? Teaching mathematics in secondary school is not normally associated with play, but as illustrated below, can appear in many forms. I will argue that, in the hands of a skilled practitioner, playfulness can benefit teacher, students, and their mathematics learning. This paper explores the unique nature of play in everyday classrooms using a sample of observations and interviews with experienced UK teachers. I conclude that play appears in many subtle forms, playing various roles in a mathematics classroom.

7.1 Defining Play

> The opposite of play… [ ]… is not a present reality or work, it is vacillation, or worse, it is depression. (Sutton-Smith 2001)

It is hard to imagine life without some degree of play. Play for me is associated with humour, with laughter and with positive social relationships. Humans are designed to be playful and engagement in play may enrich and engender happiness. In this paper, I explore the role that play takes in learning mathematics. Bibby (2011) claims there is a lack of effective use of play in secondary, so I will assess whether this lack is the case within a sample of UK mathematics classrooms. I will review what we mean by the activity of play in learning, exemplify different dimensions of play as informed by teachers and I will discuss some implications of the disposition of teacher playfulness within a secondary mathematics context.

Definitions and usage for the term play range from 'engaging in activity for enjoyment and recreation rather than a serious or practical purpose', to 'amusing oneself by engaging in imaginative pretence', or 'engaging in an activity without

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proper seriousness or understanding'. Playing can be participating in a sporting match. Alternatively, one can play a piece or card in a game with rules. One can be in a play, play along in a storyline, play a role, or play someone for a fool. We play instruments or a song, whilst an angler plays a fish. However, play is mostly associated with children learning to be adults as well as with relaxation or merrymaking. Play may be deemed frivolous, not serious or even real, as in 'I was only playing'.

Yet it seems that humans need play. From a list of 24 psychological needs (Murray 1938), play is located among affection needs. Play is defined as distinct from the other affective needs of affiliation, nurturance, rejection or succorance, as 'having fun with others'. Thus locating play as a social as well as an individual need. In a seminal text on play, 'Homo Ludens', [Man the player] Huizinga (1949) defines play as

> a free activity standing quite consciously outside 'ordinary' life as being 'not serious' but at the same time absorbing the player intensely and utterly. It is an activity connected with no material interest, and no profit can be gained by it. It proceeds within its own proper boundaries of time and space according to fixed rules and in an orderly manner. It promotes the formation of social groupings that tend to surround themselves with secrecy and to stress the difference from the common world by disguise or other means. (p. 13)

I seek to define play in context through seeking some essential characteristics to look for, to determine whether an episode in a mathematics classroom really is play. These five characteristics will frame discussion of episodes selected from classroom observations. Firstly, that the teacher chooses to and directs the play. The focus of my wider research centres on teachers, and this has guided episode selection. Secondly, means or activity is valued more than outcome, so play is process orientated despite a teacher seeking positive outcomes. Thirdly, that structure and rules emanate from the mind of the teacher as the instigator. Fourthly, there needs to be some imaginative remove from real or serious life, and finally, play needs an alert and active frame of mind (Reifel 1999) considered here as playfulness. In this paper, I am not considering the whole lesson, which in itself may be viewed as a kind of play; rather the focus is on short episodes from within a lesson.

Distancing play from real life in an educationally purposed context may be problematic. This position also neglects the emphasis within mathematics education on problem solving, and real life examples. Radford (1998) draws on a Vygotskian view of play; that activity in play is neither independent of context nor independent of particular motives. Radford suggests that both motives underpinning the actions in play and plots come from cultural reality, so any dialogue and actions displayed in play will be coherent with an individual's version of cultural reality. This coherence implies, in terms of mathematics teaching, that we can still define activity that has a motive of mathematical learning as play, as this is the contextual reality for a teacher.

7.2 The Multiple Roles and Value of Play

Play can support the learning of mathematics through playing by oneself, socially or mathematically. Storytelling of the form 'Once upon a time…' is a domain of childhood, yet playing through storytelling for adults is still a pleasant experience, albeit in forms such as fantasy games, novels and films. Internal monologues and connections formed by engaging in fantasy play are supportive of creative thinking and innovation. Predicting the future, as in imaginative play, is a useful skill that is essential for mathematical problem solving. Play is preparation, by acting, imitating or practicing for perfection. Brown (2008) suggests humans are designed to play because it is advantageous for adaptability. Further, there is a strong association between play and creativity in brain processes. For example, rats lose strategies and risk-taking if prevented from engaging in play (Diamond et al. 1964). We know play for young children supports brain development, making connections, brain crafting, firing up and generating passion and drive (Brown and Vaughan 2009). Play can also be therapy (in the Freudian sense), which allows re-ordering of people, events and circumstances into patterns, a very mathematically relevant skill. Play in a form appealing to an adult leads to arousal or stimulus seeking and avoids boredom (Mandler 1984).

Children also learn limits of social interaction through play. Even if play is not just preparation for adulthood, this role is still a dimension of play, one pertinent to adolescents. Engaging in play supports the formation of rituals (rules of ball play) and establishment of norms (rules of the game) (Vygotsky 1978). Only the form of acceptable play as students mature alters, in line with cultural and social norms. For example, work is deemed a serious business. Play as a social endeavour benefits the balancing of emotions. For example, play can take a role in reduction of conflict, or in encouraging positive emotions in others. In any context, people must adapt and behave in accordance with conscious, shared mental conceptions of what is appropriate, and play can support this aligning. People that play together (such as sports teams or groups with a common purpose) ideally build trust, trust that comes from play signals which may be vocal or facial, appearing through body or gesture. Play has a communicative role, reducing barriers to communication and reducing personal distance. Belonging and building emotional bonds are one of the main roles of playing together for all ages (Reifel 1999).

Unsurprisingly, there is far more research about mathematical play in relation to young children rather than teenagers, drawing on a long tradition of play based learning in the UK. Perry and Dockett (2007) write of play specifically in relation to mathematics in early childhood. They suggest that many early mathematical understandings that create meaning will have been formed through play. They emphasise the role that play has in creating a situation supportive of innovation, risk taking and problem solving. Relocating activity in play rather than in 'real life' is useful when learning mathematics. The separation from reality that play allows means that a safe place is created where risks can be taken, supporting a reduction of potential shame or embarrassment, similarly for developing curiosity and for exploration into uncertainties. Such a fictional mode of thinking, and keeping that mode distinct from the literal, is innate to the human mind (Abraham and Yves von Cramon 2009).

Play is complementary to learning mathematics in early learning contexts as play has an integrative role within learning, through consolidation and making connections across experiences, or forming new representations. But mathematics and play seem to disconnect as students pass into adolescence. Perry and Dockett (2007) remind us that the role of the teacher is pivotal in making play connections and I would suggest remains pivotal in secondary mathematics. If we want students to enjoy and play whilst learning mathematics, then there needs to be a supportive environment, space and time for encouragement of play. We will see in what follows if the teachers studied for this research provide such an environment. However, a contradiction exists in that at the same time as we know play benefits learning, it is possible to teach mathematics without socially experienced or observable play.

7.3 How Do Teachers Play in Their Lessons, and How Do They Speak of Play?

I have collected data from interviewing and observing eight experienced UK secondary mathematics teachers. This data includes a post-observation discussion using video recall, discussion centred on episodes selected on the basis of expression of positive emotions by the teacher. Episodes of play are frequent in just a few selected minutes of observation. The examples below illustrate play as it appeared within selected episodes that exhibit the five play characteristics of director, process, choice, remove and frame of mind. The range of examples illustrate imaginary play including nursery behaviours, storytelling, banter, sharing of humour, teasing, puzzles as play, physical play as in modelling, discovery play and in general playfulness in tone, voice and manner. After presenting the examples, I will explore how these forms act to benefit learning in mathematics.

In a lesson about the nth term for sequences (generating $4n - 2$ from terms), one teacher, Adam, enticed his students into imaginary play. The students enjoy becoming a flock of sheep with a shepherd, “You’ve got to imagine Mark’s got like a funny hat thing, and you’re the sheep” [Pointing to class who begin bleating]. A scenario is used to reinforce the concept of n as natural numbers within sequences. An associative physical form of play emerges as the student positioned as ‘shepherd’ stands to count his ‘sheep’. In the same episode, Adam uses repetition of the phonic form of n, used in early years, to further emphasise the point,“nuh… nuh… nuh… we use nuh. En [n] for number.” Further, he encourages the student use of a nursery rhyme (“12345 once I caught a fish alive”). These students are 14–15 years old, not primary school children, yet they willingly engage in play, and as one tells the teacher later, “I ‘get’ algebra now”. The rewarded teacher implied he would count sheep again.

Another teacher, Gus, continually engaged his year 7 students in fast paced banter, playing through apparently irrelevant flights of fancy. The imaginative element illustrated below emerged from a serious mathematical point about communicating mathematically, one which gave time for students to finish the preceding task. A slip of tongue over pronouncing the name David in a responsive quip from one of the students led to an enjoyed extended deviation and storytelling, relating both the teacher's short grey hair and imminent retirement to Ginola's famous flowing locks.

Teacher:	Nice one Jeff. No, seriously, did you know David Ginola used to played football for Arsenal?
ALL [Singing]	Arsenal are useless... [Inaudible]...
Teacher:	...and when he finished...when he finished his footballing career, he got a model...as a hair model...he used to advertise shampoo [more laughter]
Jeff:	Funny. You could do shampoo when you finish your teaching. [The students knew Gus was retiring] You could do that.
Teacher:	Yeah, can you imagine this saying...? 'Grecian zero, for men who want to be grey' [more laughter] [Gus has very short grey hair]

As the laughter ended, the whole class spontaneously re-engaged in the mathematics. This pattern happened often in the observed lesson; periods of concentration interspersed with banter, anecdotes and lots of playful laughter. In post observation discussion Gus explains his view of this patterning. He believes that students who enjoy themselves will learn more, reducing anxiety about mathematics.

Edward's form of playing illustrates sharing of humour as selected as humorous by the teacher. He plays a video of a polar bear catching a seal as part of a lesson introducing proportion through animal hunting data. Afterwards he commented

> It is one of my favourite clips, and I've played that... this is the second time that I've used it, without fail, every class has laughed at the bit where the polar bear pops up its head behind the seal. (Edward)

Adam in a different lesson is playing a spontaneous humorous con on his students, telling the class that he will ask the exam board if their chosen names for common sequences can be added to the curriculum. This complex example of playing has mathematical purpose and gives the students ownership of the mathematical terminology. It also shows the teacher imagining future pleasure from extending the joke.

> I thought... if they can actually kind of come up with the sequence themselves, rather than me telling them, 'these are square numbers and these are triangle numbers,' then they'll get a bit more ownership of it and remember it more. —He adds— So I'll tell them, I've got them tomorrow, I'll tell them 'I've been in touch with the exam board and they've said they'll take that, your definition' [Laughs]. (Adam)

In the data there are more traditional examples of play, defined as common strategies for teaching that may use games, such as engaging in puzzles. For example, Debbie uses a jigsaw type of puzzle, a Tarsia, as a means of engaging her students in mathematics through problem solving. Debbie comments on playing with her hands as a means of explaining a written problem,

> So I was trying to explain that when it [Aeroplane] hits the ground… []… it's fallen from the sky, it's got to 180 metres when it has hit the ground, but it has gone down a further 45, how far has it gone down?… []… So I was trying to do it with hands, so they sort of got the… that's the sky, that's the ground, it's fallen into the sea further, so… I think she did then go on to answer the question correctly, and she could explain to me why, which was good." (Debbie)

She tells of playfulness from an early age, "I can remember when I was very small registering my teddy bears, having them all lined up and taking a register." I observed Debbie using her whole body whilst teaching and effectively using mannerisms, noises and gestures usually associated with teaching younger children. For example, at one point, her students join in singing the theme to Balamory, a nursery TV programme.

In interview, Carol talks of play and mathematics in relation to her own mathematical learning. For example, as a child, she entered a magic square competition. This play, in the form of puzzle solving, was encouraged, "He [Dad] recognised that I had an interest, and he sort of perhaps pushed that and promoted that a little bit." Her classroom is bright, with lots of student display work, a teacher form of play, usually more associated with primary and not often high profile in secondary mathematics classrooms. Carol chose for observation an experiential learning lesson, where students played with coins and dice to discover relative frequency.

In summary, as well as play as described above, there are examples of physical body play, playfulness from the teacher, as when Debbie moves her arms to model an aeroplane flight path, or Adam's student sheep. 'The sheep' example also illustrates how imaginative pretend play can appear in a mathematics lesson. Gus, a keen football fan, telling of David Ginola's hair, also takes students on a flight of fantasy. A teacher sharing amusing videos shares pleasure in a playful way, as does choosing exploratory play to teach. There is also a place for neoteny, such as when Adam repetitively says 'nuh' for explaining n in sequences, uses nursery counting songs, or when Debbie and her class sing a children's TV theme song. These examples illustrate a wide range of play in everyday mathematics classrooms in just a few short episodes. Next I will discuss these episodes in relation to the five characteristics of play, before discussing the implications in terms of deviation from the rules, and any emotional benefits.

7.4 The Five Characteristics of Play

The five play characteristics, director, process, choice, remove and frame of mind assume play is about motivation and that play exists on a continuum from pure play to none. The examples selected show the teacher choosing to play and directing the play since this is my specific area of interest. Play as used here is predominantly process rather than product orientated, with examples taken from classroom activity. The students may not be aware of the learning product intended by the teacher. A further defining characteristic in this context is that the structure and rules of play emanate freely from the mind of the teacher. Although the teacher chooses to play, the rules often need to be negotiated, and may, as in the case of Adam, become the norms of the class. In play organised by a teacher, students are not usually able to change rules, which is why Adam's proposed intervention is potentially powerful in terms of ownership of mathematics. Adam, in 'the sheep' story, demonstrates what Vygotsky called 'socio-dramatic play'. This form is among the most complex forms of play, in terms of rules and the playful acting out of roles or scenes. The fundamental rule here is that you must abide by shared understandings of the role that you are playing (Vygotsky 1978). Examples of imaginative remove from 'real' or 'serious' life appear particularly in the observations of Gus or Adam. Play of all sorts has 'time in' (period of fiction) and 'time out' (temporary return to reality), though this distinction is more obvious for some forms of play. Gus has a clear distinction in his teaching between 'time in' and 'time out'. In this case, time out is engaging in mathematics. Yet during 'time in' he does not say, 'I am just playing', since to acknowledge that play is play removes the magic spell. Gus seamlessly switches between 'time in' and 'time out', and at times, the distinction is blurred. The fifth characteristic relates to willingness, that a teacher needs a propensity for play. It seems that in all these examples, enjoyment of play forms when participants have an alert and active frame of mind. Therefore, an engaging pace is important for effective play in a mathematics classroom. This dimension is apparent in the selected episodes, as is a disposition to engage in play on the part of the teacher.

7.5 Implications

Perry and Dockett (2007), although chiefly discussing primary mathematical play, consider the ideal relationship between teacher and students that is play supportive. They suggest that students should not be unguided, as an active teacher role can deepen the sophistication of play. They suggest a teacher of mathematics should be a provocateur, who challenges, generates situations through questions, or surprises for example. Yet modelling playfulness in conjunction with learning mathematics only works if the teacher is so disposed. I would suggest that deviation is associated with play. A teacher can decide to deviate from norms of their classroom, to be creative, accept the associated risk and vulnerability and use deviative forms of play as a teaching strategy. Goffman (1997) suggests that there may be less potential for conflict between teacher and students where some expected roles have been abandoned. In this case, and others, teachers have more impact as not all teachers are playful. On an individual level, play draws and fascinates the player precisely because it is structured by rules that the player herself or himself has invented or accepted.

Whilst collecting the data, I recorded the teacher's galvanic skin response (GSR) using a sensor, with an interest in experienced internal emotions whilst teaching. The sample is small, and the inconclusive results require further investigation, but there were some dips in the associated graphs (reduced stress or excitement) associated with episodes of intense laughter for Adam and Gus. Engaging in play might even be calming for a teacher as Gus suggests in interview. "There are bits which are stressful, but when the class and I are working together well, then it's great. We've got to do… all we do is just work together and it's relaxing." There seems to be a link between positive emotions and reduction in stress, but because play is not a response to external demands or immediate strong biological needs, the person at play is relatively free from the strong drives and emotions experienced as pressure or stress. Yet there is a contradiction. If play relaxes, managing play in classroom context requires teacher intensity and energy. The pace during playful episodes is generally faster, more dynamic, and this may add to a need for intense management. The teacher also has to manage behaviour carefully whilst engaging in extended scenarios, especially as play may appear as an abandonment of expected rules. It is easy to see how the students might lose the mathematical purpose whilst experiencing 'time in'.

Mandler's (1984) suggestions for the purposes of emotions may correlate to some of the purposes of play. If so, then play, as for emotions, can help to deal with mismatches between actual and intended actions and to address discrepancies or uncertainties as well as having an adaptive role. Emotions also serve to process first encounters that may be strange or unusual events, these encounters too may be a source of arousal. Therefore, teacher initiated play in a mathematics classroom can act to bring positive emotions to the fore. We know students talk of boredom, but experienced teachers experience boredom too. Play can work against this.

Play, as expressing positive emotions, acts to reduce emotional distances. In any power relationship distances are inevitable, in this case between teacher as authority and student as child. Play, used appropriately, may act to break down such barriers and play a balancing role for both students and teachers. If we consider the teacher as a mediator between mathematics and students, then play may act to facilitate this role, especially for experienced teachers. So far, I have portrayed play as positive. However, not all the observed teachers engaged in identifiable forms of play. For example, Helen used a game to teach, but this was not considered play, by myself or Helen, as the primary purpose was to support exam success. This raises two questions: What might act to prevent play in the forms described above? And, given barriers and risks to play, why do some teachers still engage in play?

7.6 Secondary Mathematics Lessons with No Observable Play

Not all the teachers observed and interviewed spoke about or used play. There seem to be limitations in the forms of play observed, or at least differences in teacher views of play. A more traditional view might be to see play simply as games or as a teaching tool. For example, Helen, a strategic outcome orientated teacher, separates play and learning. Playing games and activities (she says) can be allowed only after exams because they are high risk in terms of paying attention to exam success, even to the point of time wasting.

> …we are coming up towards a test, —you kind of want to make every second really focussed and really count, and really relevant and really going to help them with that test rather than perhaps being a bit more exploratory and a bit more outside the curriculum, outside the box. (Helen)

This may be a common position even when speaking to others of the rhetorically acceptable advancement of play within mathematics teaching, a discrepancy because they do not really believe in the value of play. One reason for not engaging in play is that it can be chaotic (Perry and Dockett 2007) and hence entail risk. Further, playfulness takes intensity and effort to sustain. So there has to be an effective reward for engagement in play, especially social play in a work heavy context such as teaching.

As discussed above, play is dependent on willingness to potentially ‘look silly’, as well as on what is important to a person, one’s own values in relation to the teaching of mathematics. Engaging in play involves revealing self, so there is a degree of vulnerability involved in playing in the context of a mathematics classroom. Teachers are also risking damaging their relationship with a group of students if they ‘pitch’ it wrong. Teachers also risk criticism of neoteny, as European culture is one where childishness is often a criticism. Yet Brown (2008) suggests that humans are adaptable because they are among the most neotenous species on Earth. There are also potential cultural or social barriers to play. For example, that play is a waste of time, or associated with guilt for not ‘working’, or a common view that play is only a rehearsal for adulthood and has no place as one gets older and dignity prevails.

7.7 Benefits for Teachers Who Engage in Play

Yet if engaging in play works as for some teachers, the rewards are significant. Play can balance and enhance relationships through shared satisfaction and enjoyment. Adam or Gus especially use play, where play serves to break and divide a lesson and keep the attention of students. They seek novelty, and have found an effective unique teaching style, one that generates and supports positive learning.

One reason for engaging in play might be that when routine sets in, a self-aware teacher is likely to seek novelty and deviation, different ways of playing to entertain both themselves and students. It may also be the case that prior experiences have shown them that it is an effective strategy for them. Play experienced as leading to pleasure will trigger seeking of similar emotions and hence becomes an enticement to engage in more similarly rewarding activities. Therefore playful teachers are willing to risk (risk is part of play too), but that they have found that they can use play to model connective learning of mathematics through playfulness and humour. Therefore, they expect enjoyment. The expression of positive emotions evoked by an expectation of enjoyment is likely to make the risk successful, as the students see this modelling and expectation, and respond positively.

7.8 Conclusions and Implications

The evidence points towards play being good for adolescents, mentally and physically, and, as I have argued, mathematically. Although I am converted to the value of play, a question remains as to whether the effective use of playing by teachers in secondary mathematics is a disposition, or whether it can be learnt. I would suggest that balancing play with managing classroom conduct is an art rather than a skill. The implications from the discussion above suggest play requires willingness, but also confidence, self-discipline and energy. A teacher needs to be willing to use their "divinely superfluous neurons" (Brown and Vaughan 2009). Certainly play is not passive. What may be of importance is the modelling activity implied by engaging in play, as for effective teaching of mathematics.

Playfulness models a valuable form of engagement in mathematics, one supportive of experiment and creativity that is means rather than outcome focussed. It is also a form of teaching that generates positive emotions, but also positive emotions promote playfulness. If harnessed appropriately to the teaching and learning of mathematics then we have a very powerful tool to add to the teacher's repertoire, one which retains teacher interest, sustains student engagement and is socially supportive. So yes, there a place for serious frivolity in the secondary mathematics classroom.

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Part II
Teachers' Beliefs, Changing Beliefs and the Role of the Environment

Chapter 8
In-Service Math Teachers' Autobiographical Narratives: The Role of Metaphors

Chiara Andrà

Abstract This paper examines the narratives of 50 in-service secondary school math teachers regarding lessons designed for preventing gambling abuse in their classes. Findings concerning teachers' beliefs, Bruner's constructivist approach and Lakoff and Johnson's embodied mind paradigm are the basis for the investigation. The results reveal that teachers systematically organise their thoughts in a metaphorical, embodied way, and some common patterns in teachers' narrative emerge, e.g. the uses of metaphors of "light", of "journey" and orientational metaphors.

8.1 Introduction: To Tell Is to Be

The current state of research in the field of affect acknowledges the key role of narratives as means to access both students' and teachers' beliefs, emotions, motivation, attitudes, values, and so on. Oral interviews, written questionnaires of various sort, and stories, have been largely employed to access the individuals' inner worlds, particularly in relation with mathematics, its teaching and learning (see e.g., Pepin and Roesken-Winter 2015). Researches on teachers' beliefs and attitudes have been particularly enriched when teachers' narrative were available, and analyses of narratives have complemented direct classroom observations.

In Psychology, Bruner (2004) underlines the power of autobiographical accounts, maintaining that "we seem to have no other way of describing 'lived time' save in the form of narrative" (p. 692). According to this view, teachers telling their experiences in first person are actually reporting the most faithful account of what happened to themselves, their emotions, their beliefs, when living these experiences. Accounts of this sort are not through-the-clear-crystal recital of something univocally given, but can be taken as rather cognitive achievements, as reflections and elaborations of what teachers actually lived.

The research focus is on teachers' beliefs about mathematics and its teaching, in a particular *context*: a professional development course for in-service secondary

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school math teachers interested in teaching probability to prevent gambling abuse. In this case, the concept of "context" is multifaceted, since it ties back to several meanings:

- the *context of gambling abuse*, which is emotionally and motivationally charged, not only for gamblers but also for teachers that aim at preventing their students to become pathological gamblers;
- the *context of professional development course*, and the tensions the teachers may experience with respect to their role, their knowledge, and what is proposed in the course (see also Liljedhal et al. 2015);
- the *context of the classroom*, where teachers teach after having attended the course.

The relationship between the teachers and any of these contexts needs to be taken into account, and examined. To this end, I introduce the theoretical references that frame my investigation.

8.2 Theoretical Framework, or the Systematicity of Teachers' Accounts of Their Lived Experiences

In the field of affect in Math Education it is well acknowledged that beliefs, even if they represent a key concept, are still not clearly defined (Pepin and Roesken-Winter 2015). Skott (2015), however, states that there are four characteristics of beliefs that are accepted by almost all researchers: *conviction*—beliefs can be thought of as knowledge that are true at least in the eyes of the beholder; *commitment*—beliefs are value-laden and relate to motivation; *stability*—beliefs are considered stable, in comparison with emotions that are more unstable and can change quickly. Beliefs are expected to change only after substantial, new experiences; *impact*—beliefs are conceived of as influencing both the individual's perception and his/her practice.

The latter feature is particularly relevant for research on teachers' beliefs: teachers hold beliefs about mathematics, math teaching, and about themselves, and these beliefs deeply influence their classroom practice. With this respect, it has been argued by Fives and Buehl (2012) that between beliefs and behaviour there is not that much congruity, as if beliefs are organised in clusters: the effect of this clusterization is that beliefs one gets access to in interviews are different from the ones that emerge in classroom practice. Skott (2015) maintains that these observations are fully compatible with the claim that beliefs explain behaviour, and suggests to see the context as a constrain on the opportunities for 'belief enactment', acknowledging a central role to social interaction.

Following this view of "context", I would like to exploit an idea suggested by Bruner (2004), concerning the power of autobiographical narratives: he claims that human beings construct themselves autobiographically, thus autobiographical accounts of teachers' experiences can provide us with a trustworthy insight on teachers' beliefs. This idea has been already exploited by Di Martino and Zan (2011) to study students' attitudes towards mathematics in terms of what they liked, their perceived competence in math, and possible links between "I like" and "I can". Coppola et al. (2013) applied a similar lens to analyse prospective primary school teachers' attitudes towards math and its teaching. The aforementioned studies have provided experimental evidence that teachers' narratives may open a window on affective elements that shape teachers' practices. In the present work, with this in mind, I exploit a different idea suggested by Bruner, namely to apply a narratological lens, which allows to take the role of the context into account, given the participationist nature of my research.

Narratology acknowledges that in a story there are five basic elements: the characters, the setting, the actions, the plot, and the moral (see, e.g., Bal 2009). When the story is autobiographical, Bruner (2004) observes that the narrator and the central figure in the narrative is the same, and this can create dilemmas such as "defacement", namely the act of turning around oneself to create one's own image, and instability. However, Bruner underlines that it is this instability that makes autobiographical stories highly susceptible to cultural, interpersonal and linguistic influences. Consequently, Bruner suggests that in autobiographical stories two elements play a key role: the *character* and the *setting* (or, the context). Particularly, it is the relationship established between the two that counts. To characterise such a relationship, he defines a path from the *figure*, which fulfils a function in the plot but does not own it, to the *individual*, which transcends society and is responsible of his future, in a sense creating a space where his rights are respected. To sum up, the narrator can see himself as dominated by the setting, with scarce possibilities to intervene, or he can tell about himself as the owner of his own destiny. In between these two extremes, there can be a variety of nuances, which are worth to be investigated when the narrator is a math teacher, who tells about her relationship with the context of the classroom and the math activity.

It is, thus, necessary to define a way to take into account all the possible nuances in between the "passive" *figure* and the "super-active" *individual*, and I would like to introduce a lens of analysis that has been already used in the field of affect, namely the use of metaphors (see, e.g., Portaankorva-koivisto 2013; Oksanen and Hannula 2013). Metaphors have a broad, general meaning in my research, following Lakoff and Johnson's understanding of them. With Lakoff and Johnson (2003), I see metaphors as pervasive of any thought and any action: since communication is based on the same conceptual system that we use in thinking and acting, *language* is an important source of evidence for what *thinking* is like. Most of our conceptual system is metaphorical in nature: for example, our conventional way of talking about arguments presuppose a metaphor we are hardly conscious of: the metaphor 'argument is war'. This metaphor is not merely in the words we use, it is in our very concept of 'argument'. We talk about argument in a certain way ("your claim is *indefensible*", "he *attacked* every *weak point* of my argument", for example), because we conceive of it that way, and we act according to the way we conceive of it. Lakoff and Johnson identify some basic metaphorical concepts that *structure* our everyday activities: 'argument is war', 'time is money', 'ideas are objects, linguistic expressions are containers (or the conduit metaphor), and communication is sending'. They also observe that the structuring involved by these metaphorical concepts is partial, otherwise one concept would actually be the other, and not merely understood in terms of the other.

Another interesting set of metaphors for the aim of my research are the *orientational* ones: up-down, in-out, front-back, central-peripheral. For example, 'conscious is up, unconscious is down'; 'control is up, being subject to control is down'; 'rational is up, emotional is down'. Such spatial orientations arise from the fact that we have bodies which function as they do in our physical environment: for example, the metaphor 'happy is up' is grounded on the erect posture that is linked with a positive emotional state. In some cases, spatialisation is such an essential part of a concept that it is hardly impossible to imagine any alternative metaphor. Given that metaphors are inseparable from their experiential basis, and that metaphors structure our way of thinking, acting and telling, I search for metaphorical expressions in teachers' autobiographical accounts of their experiences, since with Bruner I see a two-way relationship between life and narrative: one mirrors the other, and viceversa.

8.3 Methodology

In October 2014 a group of 50 in-service secondary school math teachers participated to a professional development course aimed at providing teachers with didactical tools and knowledge to teach a series of math lessons in their classes to prevent gambling abuse, within the Italian research project BetOnMath. Before the professional development course started, teachers were asked to write the reasons why they were willing to attend this course on gambling abuse prevention (task 1), and after it teachers were asked to write a report (task 2). Tasks 1 and 2 were designed to allow emotions, motivation, and beliefs about teaching math to emerge.

Task 1 has been sent via email to all the participants, and it reads: "If you have to imagine to teach probability to prevent gambling abuse, how would you see it?"

Task 2 is made of several steps. The first step resembles a lesson image: after the professional development course, the teachers have been invited to examine the courseware (slides, materials for groupwork activities in classroom, simulators of betting games), to prepare their lessons and to answer to the following questions: which topics have been already taught? on which parts do you expect to face more difficulties? on which parts do you feel more comfortable? on which ones do you expect to spend more time? are the slides a good support? which is your expected role during group activities? which words do students use to talk about gambling?

The subsequent steps of task 2 ask the teachers to report on: previous students' experiences and knowledge that emerged after each lesson; teachers' difficulties faced in each lesson; the actual role played by the slides; the time dedicated to group activities, and the actual role played by the teacher; possible changes in the students' ways of talking about gambling. Since the course has been divided into three lessons, teachers were asked to report on all these issues three times: once after every lesson.

Data have been coded using a process of *analytic induction* (Patton 2002), which starts with a set of a priori codes and relies on the use of a constant comparative method. In the case of these data, the codes came from the framework of Bruner's approach to autobiographical accounts (paying specific attention to the relation between the narrator and the context), as well as from Lakoff's and Johnson's classification of metaphors into orientational ones (in-out, center-periphery, and so on) and structuring ones (e.g., 'argument is war'). Analytic induction, through its constant comparative method, also allows for the emergence of new themes (Patton 2002) and this happened with the research presented here.

8.4 Data Analysis I: Before the Course Starts

Task 1 was sent via email to teachers, who replied quickly and seemed willing to participate. They were asked to tell how they see the course on gambling prevention, before any specific information about it had been shared. Data have been collected and four different kinds of metaphors emerged:

- **The metaphor 'argument is war'**: after having pointed out that to teach probability is a *struggle*, a teacher added "I see it like David against Goliath, because David is young but smart, and even if Goliath is bigger and stronger than David, the latter *wins*. It is also true that we, teachers, are like David: we have few and poor means, but we will definitely *win*". The arguments provided by the teachers in future math lessons are seen as weapons to defeat the arguments that students bring against a rational approach to gambling.
- **The metaphor of light**: a teacher said "I see it like *illuminating the road* with knowledge: dangers become visible". Another teacher: "I am giving them a *torch* and a *map* to get out from the labyrinth, because knowledge provides tools to interpret reality and lights to deal with bewilderment". And another teacher: "If one knows *how a machine works*, it is possible to make it work better and to avoid mistakes that can damage it".
- **The metaphor of protection**: one teacher said "I am providing my students with an *helmet* to drive their motorcar", and another one wrote "gambling is like a *jump* and teaching is like giving a *parachute*, because gambling is inebriating and can kill you if you are ignorant".
- **The metaphor of journey**: one teacher said "it's like becoming aware of the *road* you are traveling".

The metaphor of war resembles the structuring metaphor 'argument is war' identified by Lakoff and Johnson: in teachers' words, we can infer that they are willing to *defeat* gamblers' arguments with new knowledge. As regards the relationship between the narrator and the context, this kind of metaphors sets the teachers rather on the side of the *individual*, who creates his own destiny: the teacher declares her certainty to defeat “the giant”.

The metaphor of light resembles the up-down orientational metaphor: the road can be seen with the eyes, located ‘up’ in the body, a map shows the track *from above*; the man operating on the machine is ‘up’ (controlling it), while the machine is ‘down’ (being controlled by him). As regards the narrator-setting relationship, also in this kind of metaphors the individual seems to emerge, but in his *potentiality*: to make dangers visible allows one to avoid them, but it does not assure that he won’t stumble; to provide a map allows one not to get lost, without assuring that it will; and to provide instructions about the functioning of a machine does not guarantee that one would make no mistakes. Hence, a *potential individual* seems to be associated with this kind of metaphors.

The metaphor of protection seems to be related with the in-out orientational metaphor: if the students are *inside* the protection given by the teacher (the helmet, the parachute), they can have inebriating experiences with less risk. Also in this case, there is a *potential individual* that emerges by focusing on the relationship between the character and the setting.

Finally, the metaphor of journey has been already observed by other researchers (e.g., Portaankorva-koivisto 2013). A different relation between the narrator and the setting emerges: the former travels a road which seems to be already determined, and the only possible action is to become aware of it. Hence, the narrator seems to have less agency on the context.

8.5 Data Analysis II: Course Image and Report After Each Lesson

8.5.1 Step One of Task 2: Lesson Image

Teachers’ writings about the first phase of task 2, namely accounts of their expectations with respect to the course to be taught to their students, are rich in spatial metaphors. I now report the most frequent ones, connecting them with the statements reported in Lakoff and Johnson’s book. Moreover, I classify the metaphors according to the different kinds of relationship between the narrator and the context, following Bruner’s suggestion. Table 8.1 reports the first kind.

The metaphors listed in Table 8.1 yield us thinking that the agency of the teacher is limited, and a predominant role is played by the setting. In such a setting, the students act and react, they apply knowledge to a new real situation. Moreover, the students’ actions and reactions influence the teacher’s work: only if the groupwork takes off, in fact, the teacher can synthesise the results. In some cases, the teacher seems not to have a role in shaping the group activities, and she limits herself wishing that the solution emerges, or observing that shy students are not set apart,

Table 8.1 Metaphorical expressions used by teachers in the first step of task 2, pointing to a teacher who is not fully dominating the context

	The character is not fully dominant
Up-down	"If the groupwork will *take off*, my role will be simply to synthesise"; "I wonder to what extent my students succeed in *lowering* their knowledge into a specific real situation"; "I wish that the solution will *emerge* from the classroom"; "I am afraid that slides will make my lesson *flat*".
Front-back	"I am afraid not to be able to *follow* all the suggestions provided, and to allow my students to grasp all the details".
Center-periphery	"Groupwork activities allow all the students not to be *set apart*".
In-out	"Groupwork activities allow shy students to *get out in the open*"; "Slides help to *dig deeper* into concepts".
Conduit	"the complexity of concepts to be *transmitted*".
Time-is money	"My colleagues do not want to *waste* their time in teaching gambling prevention"; "I have *spent too much time* with the installation of the simulator".

Table 8.2 Metaphors pointing to a teacher who has limited agency

	Teachers have an agency on the setting
Up-down	"it will be difficult to *grab* all my students' ideas"; "I'll try not to give them *direct solutions*, so that they will learn to correct themselves without my control"; "I'll try to understand if they have *reached* some *awareness* about gambling risks".
Front-back	"Slides are a valid *starting point*".
Center-periphery	"I start, *faithful* to the slides"; "I will try to reach the *bull's eye*".
In-out	"My role will be *maieutic*".
Ideas-are-objects	"It will be difficult *to transform* the intuitive notions in suitable mathematical laws".
Conduit	"It's hard to *pass the message*".
War	"I will do my best to make the formalisation of concepts to be a *conquer* of my students".

as if she cannot intervene. This holds also for slides, about whose use the teacher is afraid not to follow faithfully, for example.

Metaphors in Table 8.2 depict a different scenario, where teachers have an agency on the setting. In these statements, the teachers play an active role, they relate with the setting which can constrain their actions, but the setting itself offers possibilities and is full of potentialities. The use of the verb "to try" and expressions like "it will be difficult" speak to the possibility to succeed, even recognising the difficulty of the teacher's task.

Statements in Table 8.3, finally, recognise full agency to the teacher. In these metaphors, it seems that the setting does not have an agency on teachers' actions. We can notice, however, that the huge majority of these statements regards the content to be taught, rather than classroom dynamics: the teachers feels to have

Table 8.3 Metaphors pointing to a teacher who has full agency on the context

	Full agency of the teacher
Up-down	"I will start with *concrete examples* and gradually reach complexity"; "Slides are a good *support* for the lesson"; "I will let the students reason *autonomously*"; "I expect that the students will ask for my *support*, but I will try to provide them with only little suggestions, to let them work *alone*"; "I will guide them towards the solution of the problems and towards a *higher awareness* about gambling".
Front-back	"I have intentionally *left behind* probability"; "I have deliberately chosen to *start from zero*".
Center-periphery	"I have not yet *introduced* probability".
War	"slides are *dead images*, and the first step towards classroom discussion consists in *repressing* any disturbing form"; "[in the past] probability content had been *sacrified* to dedicate more time to study calculus"; "I will guarantee that the 'good students' will not *oppress* the ones that are shy or slow".

full agency in terms of the choices to be made about the math content of the lesson. Conversely, looking at the first group of statements, the teachers might feel not to have a significant role in shaping the students' work, nor to change a courseware (the slides) that is completely new for them. We can also notice that the issue of 'control' emerges: it seems that the teacher feels more able to control the content rather than classroom dynamics. This is in line with our understanding of metaphors, in particular with the up-down orientational one: to control is 'up', to be controlled is 'down'.

As pertains Tables 8.1, 8.2, and 8.3, we can further observe that the up-down orientational metaphor takes different forms, for example: real world and examples are 'down', ideas are 'up'; as regards the slides, support is down, the teaching is up; as regards the classroom atmosphere, happy is up, sad is down; as regards students' autonomous work, to control is up, to be under control is down; and conscious is up, unconscious is down. In front-back orientational metaphors, we can see that teachers are oriented to the future ('in front') if slides are a starting point, and the teacher who *follows* is 'back', while the lesson is in 'front' of her.

8.5.2 Subsequent Steps of Phase 2: Lessons Report

Some metaphors, among the ones listed in the previous section, show up also in teachers' accounts of their lessons. Autonomy-is-up, support-is-down, time-is-money, ideas-are-up, conscious-is-up, emerge also in teachers' subsequent steps of task 2: these metaphors, as in the previously reported writings, regard the role of the slides, the role of the teacher, and awareness about gambling and its mathematical modeling. For sake of space, I am not reporting them here.

Some new metaphorical expressions have been used in the after-the-lesson reports:

- "The biggest part of the lessons has been *centred* on slides, I would have had to leave more space to the discussion", "they have *lost the target*, lingering on useless details", "the class has been able to reach *the very heart* of the problem": these are new centre-periphery orientational metaphors;
- "I have noticed a *decrease* of interest when we started theorising": interest is up, boredom is down; "I had to feed the birds", which is connected to the up-down orientational metaphor (birds fly, birds are up as ideas are up); "I have seen students' *awareness emerging from experience*", "for the students it has been a true *discovery*", "students now know that betting games are insidious": consciousness and reflection are up, experience without reflection is down;
- "Some students have worked in a *superficial* way", "I will assign some exercises *in order to sediment* the contents": these are in-out orientational metaphors;
- "I am impressed that even my low-achieving class had been able to *go so much ahead*": a front-back metaphor.

These metaphors widen the range of possibilities teachers resort to when they talk about their lived experiences in classroom.

8.6 Discussion

Data reported in this paper show that teachers tend to believe to have less agency on group dynamics and the courseware, and to have full control of the math content to be taught. As a consequence, regarding the former, teachers express their wonder, their wish and their being afraid of it, while the latter is related to actions teachers do. Hence, teachers' will emerge clearly in the former setting, while the latter seems more like a report of choices that teachers usually make when teaching. Some emotions emerge (surprise, worry), and emotions are mostly related to teachers who believe not to fully dominate the setting: emotions turn out to be passive reactions to the situations. One teacher is surprised to see how far her student can reach, many are afraid not to faithfully follow the instructions. On the other hand, statements pertaining the "fully dominant" teacher do not show emotions.

Furthermore, this study shows that our thinking and communicating is inherently metaphorical: task 1 has a different nature of task 2, since the former asks to describe an expectation while the latter asks to report on lived experience, and this reflects the kind of language the teachers use. In task 1, teachers' metaphors are more explicit, as if they deliberately choose to use similarities (like David and Goliath). In task 2, metaphors are more hidden, but they are at the very heart of teachers' reports. At the same time, metaphors are embodied: they are not mere stylistic arrangements, but they constitute the very nature of what teachers say, even when teachers themselves are not aware of using them. We can, in fact, actually perceive the lesson concretely centred on slides, namely with slides in the center of the class as well as in the center of the teachers' mind, and the students' thinking at the border; we can see awareness emerging, physically, or the content embodily sedimenting inside students.

To conclude, metaphors can open a window on the structure of teachers' beliefs, since metaphors structure our thinking and our communicating. Metaphors should not be taught of as pure mental constructs, but as grounded in our physical body and in our social experience of the world.

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Chapter 9
A Contribution to the Relation Between Teachers' Professed and Enacted Beliefs

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Abstract In this paper we define principles of an approach to investigate the relationship between mathematics teachers' professed and enacted beliefs. Based on these principles we analysed the belief systems of two primary teachers and, further, observed these teachers in each six lessons. Our results show that particularly the teachers' main beliefs explain mostly these teachers' classroom practices while minor episodes of the classroom practices could be explained by peripheral beliefs.

9.1 Background

Wilson and Cooney (2002, p. 128) described a main motivation for investigating mathematics teachers' beliefs as these beliefs represent a "significant determiner of what gets taught [and] how it gets taught". However, the research referring to the relationship between a teacher's beliefs and his classroom practice shows at best ambivalent results. Thus, some researchers reported a consistency between teachers' professed beliefs and enacted beliefs—i.e. beliefs that could explain a teacher's classroom practice—while other researchers noted an inconsistency (Thompson 1992; Philipp 2007; Buehl and Beck 2014).

Inconsistencies between the teachers' professed and enacted beliefs seem to be more apparent in reports of mathematics education research than consistencies. This could be the case since "these correlative relationships are to be expected and, as such, are seen as 'uninteresting'" (Liljedahl 2009). Further, unexpected inconsistencies are in some sense more interesting since inconsistencies have to be explained. Research yields mainly five explanations of inconsistencies between professed and enacted beliefs. A first explanation of inconsistencies refers to the social context including administration, students or parents. For example, Sztajn (2003) showed that two teachers, who seem to hold very similar beliefs concerning

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mathematics and mathematics teaching and learning show different classroom practices in two classes with different social contexts. Skott (2009) claims that the context in a broad social approach could explain relationships between professed and enacted beliefs that could only be noted as inconsistencies when the social context is neglected.

Further, inconsistencies are explained by the mathematics-related context including grades, textbooks or the mathematical discipline: For example, the research of Hoyles (1992) imply that inconsistencies between professed and enacted beliefs could be explained by differences of the context, in which professed beliefs and enacted beliefs are investigated. By contrast, acknowledging the discipline-specificity of mathematics related beliefs could explain the consistency between professed and enacted beliefs (Peterson et al. 1989; Eichler 2011; Schoenfeld 2011).

Particularly Skott (2001) explained inconsistencies with the focus of the investigated beliefs in a specific situation. For example, he explained the inconsistency of a teacher's professed and enacted beliefs with the possible overlap of beliefs towards mathematics and beliefs that refer more globally to the teaching and learning of mathematics. A similar interpretation is given by Leatham (2006) to explain the inconsistency reported by Raymond (1997).

Another reason for observed inconsistencies could be based on the difference between central and peripheral beliefs Green (1971). For example, Cooney (1999) explained inconsistencies between teachers' professed and enacted beliefs also in his own research (Cooney 1985) with a lack of differentiation of central and peripheral beliefs (see also Putnam and Borko 2000). Finally, Leatham (2006, p. 91) mentioned the difference between "what teachers state and what researchers think those statements mean". Thus, inconsistencies between teachers' professed and enacted beliefs could be explained by a misunderstanding of the researcher on the one side and, on the other side, by a misunderstanding of the investigated teacher (Philipp 2007).

The reported studies are based on a qualitative design focusing on a difference between teachers professed and enacted beliefs. In contrast, studies with a quantitative design—following the aim of rejecting hypotheses referring to the independence of professed and enacted beliefs—emphasise consistencies between teachers professed and enacted beliefs. Related studies showed consistencies between

- the teachers' professed beliefs and the sort of tasks that these teachers use in their classroom practices (Peterson et al. 1989; Staub and Stern 2002). These studies imply a consistency of professed and enacted beliefs referring to a teaching orientation (transmission view versus constructivist orientation).
- the teachers' professed beliefs referring to efficient guidance in the classroom, the students' cognitive activation and the students' learning support on the one side, and, on the other side, the teachers' enacted beliefs in respect to the mentioned three aspects measured by the students' evaluation of their teachers' classroom practice (Dubberke et al. 2008).

- the teachers' professed beliefs represented by completed questionnaires and the teachers' enacted beliefs represented by the coding of videotaped classroom practices (Stipek et al. 2001).

Based on the brief literature review the aim of this paper is to make a contribution to the research of the relationships between mathematics teachers' professed and enacted beliefs. We firstly outline our theoretical framework including particularly the construct of teachers' beliefs and the synthesis of the aforementioned literature review. Afterwards we define principles for analysing the relationships between teachers professed and enacted beliefs and we report parts of our research results.

9.2 Theoretical Framework

Following Pajares (1992) and Philipp (2007), the term beliefs represents an individual's personal conviction referring to a subject that represent a disposition of the ways of receiving information and acting in a specific situation (c.f. Eichler and Erens 2015). Further, following Green (1971) and Thompson (1992), we focus on the internal organisation of beliefs, i.e. an individual's belief system. Belief systems are characterised by three aspects. Firstly, a distinction could be made between central beliefs, i.e. strongly held beliefs, and peripheral beliefs, i.e. beliefs of a lesser importance (for the differentiation of central and peripheral beliefs in our research c.f. Bräunling and Eichler 2015). Further, a belief system is characterised by at most quasi-logical relations among different beliefs. That means that from the researcher's perspective different beliefs of a person could be contradictory. Finally, primary and derivative or rather primary and subordinated beliefs could be distinguished (c.f. Bräunling and Eichler 2015).

We use the definition of the so called "world views" regarding mathematics (Grigutsch et al. 1998) to describe our teachers' beliefs (c.f. Eichler and Erens 2014). Grigutsch et al. (1998) describe four different world views, i.e.

- a process oriented view: mathematics is presented as a heuristic and a creative activity that allows solving problems using different and individual ways.
- an application-oriented view: the utility of mathematics for real world problems in emphasised.
- a formalist view: mathematics is characterised by a strongly logical and formal approach.
- a schema view (partly described as toolbox aspect): mathematics is presented as a set of calculation rules and procedures to apply for routine tasks.

In addition, we used a distinction of a teacher's learning orientation. For this we distinguish a transmission view or a constructivist orientation (Staub and Stern 2002). In addition to this distinction we further defined a co-constructivist orientation. Following this orientation teachers hold beliefs that students must construct their own knowledge (constructivist orientation). However, these teachers also hold beliefs that the teacher is responsible to guide and to structure the students' individual acquisition and construction of knowledge (c.f. Strohmer et al. 2012).

Regarding this framework and regarding the existing studies focusing on the relation between teachers' professed and enacted beliefs, our research is based on the four principles. Firstly, following Cooney (1999), it is crucial to analyse professed beliefs or teaching goals regarding the aspect of centrality since it seems to be a plausible assumption that teachers tend to enact mainly central beliefs and at most secondarily also peripheral beliefs. By contrast, the existence of peripheral beliefs could be crucial to explain classroom practices that are not in line with a teacher's central beliefs.

Further, a system of central and peripheral beliefs includes the assumption that a teachers' classroom practice could show different beliefs dependent on a specific situation of a lesson. For this reason, following also a conclusion of Skott (2001), it seems to be mandatory to observe a teachers' classroom practice for a substantial time period. By contrast, the observation of 2 h (e.g. Skott 2001) could be insufficient to evaluate a grade of consistency between a teacher's professed and enacted beliefs. This conclusion is also in line with a quantitative design. For example Stipek et al. (2001, p. 221) reported the highest correlation of 0.75 between the teachers' professed beliefs of "math as operation" and the teachers' classroom practice emphasising performance. This correlation implies that a teacher's central belief must not necessarily be observed in every lesson.

Also the social context must be regarded in every phase of analysing the teachers' beliefs. That means that the social context has to be included in the investigation of the teachers' (professed) belief system. This means that a teacher has to reflect on his beliefs with respect to his school, his students etc. For example, in an own study (Eichler 2011) for one teacher an important characteristic of mathematics is its axiomatic basis. However, he believes that in the social context, namely in his school with his students, it is senseless to focus on axioms since he believes that his students would not be able to grasp the meaning of axioms.

Finally, the mathematics-related context is crucial to analyse the relevance of the teachers' professed beliefs for their classroom practices. For this reason the investigation of the teachers' professed beliefs and the teachers' enacted beliefs must be based on the same grade of the students, the same textbook and, particularly, the same mathematical discipline (c.f. Eichler and Erens 2015). For example, if a teacher talks about his central beliefs referring to geometry having in mind a certain group of students, he potentially enact different beliefs in a arithmetic lesson with another group of students. The aforementioned considerations have a an impact on the method of our study.

9.3 Method

Our sample consists of two experienced arithmetic teachers of primary schools that were chosen out of sample of 20 arithmetic teachers in a larger study (Bräunling and Eichler 2015). The method of investigating the teachers' (professed) beliefs is the same as in the larger sample (ibid.). In this larger sample, we collected data with semi-structured interviews focusing particularly on arithmetic as specific mathematical discipline (consideration 4 concerning the mathematics-related context). Thus, the interview includes clusters of questions referring to arithmetic content, beliefs towards teaching arithmetic, beliefs towards teaching mathematics, the nature of mathematics, students' learning or materials used for the classroom practice, e.g. textbooks. For every statement the teachers' were asked to give concrete examples of their classroom practice (consideration 3 concerning the social context). In addition, the interviews incorporate prompts to evaluate given arithmetic tasks or fictitious statements of teachers or students that represent one of the views mentioned above, e.g. an application oriented view. Further, we used a questionnaire adapted form an existing scale referring to teachers' views (Grigutsch et al. 1998) to triangulate the findings procured from the interviews. The adaption consists of the transformation of the focus of the items from mathematics to arithmetic.

For analysing the interview data, we used a qualitative coding method (Kuckartz 2012). The codes gained by interpretation of each episode of the verbatim transcribed interviews indicate beliefs towards arithmetic teaching. In addition to inductive codes we used deductive codes derived from the (adapted) world views of Grigutsch et al. (1998) as well as derived from the different teaching orientation (see above). The inductive codes were mainly used to differentiate beliefs behind the world views or the overarching teaching orientations. We weighted the codes with 1 or 2. If a teacher mentions a belief without a precision we weighted the code with 1. If a teacher explains a belief more deeply giving for instance a concrete example or task of his classroom practice, we weighted the code with 2. We analysed the sum of the weighted codes as triangulation to the qualitative interpretations. In a further triangulation we compared the results of the sum of weighted codes with the results of the questionnaire. These three steps were used to distinguish central and peripheral beliefs (c.f. Bräunling and Eichler 2015). For example, if the sum of the weighted codes concerning a world view as well as the related sum of ratings in the questionnaire is high, we understand this as evidence for a central belief. However, we used in addition our interpretation of the transcripts to evaluate the relation of different central beliefs or to analyse beliefs that define or explain the central beliefs. Similarly, we defined peripheral beliefs by a low sum of weighted codes.

To investigate the teachers' enacted beliefs we used the method of an observation (Bennewitz 2012). Both teachers were observed in six lessons. The classroom practice was documented by a protocol including notes about different phases (e.g. group work, discussion in the whole class etc.), the tasks discussed in the class, participating students and the information on the blackboard and all materials given to the students. In addition, the teacher's voice was audiotaped for every lesson.

The partly transcribed audiotapes (for example, we did not transcribed episodes aiming to plan activities without a mathematical focus) were connected with the other documents of the classroom practice and analysed also with a coding method that is close to the method used for the interview transcripts. For example, we used the same deductive codes (see above) to analyse the transcripts, or to analyse tasks posed for students. Further, we analysed the time the teachers spent for specific teaching phases based on the beliefs concerning a teaching orientation.

9.4 Results Referring to the Teachers' Professed Beliefs

We firstly use the standardised sum of the weighted codes to characterise both teachers, Mrs. G and Mr. I, referring to their professed beliefs (Fig. 9.1).

Although both teachers are similar, their belief systems also include differences. For example, both teachers express that formalism is an at most peripheral belief (low sum of weighted codes; see Fig. 9.1). In contrast for both teachers it is a central belief to emphasise a process orientation (high sum of weighted codes, see Fig. 9.1) that is briefly illustrated in the following quotes:

Mrs. G: Problem solving is crucial for me. I like students' discussions concerning different ways of arithmetical computations. Posing a problem and to discuss possible solutions [?]. It is important to discuss these things, not to solve problems only for one self.

Mr. I: At the end the ability to solve problems in everyday life situations is the main goal of arithmetic teaching.

As well as the different episodes in the interviews, the teachers' reactions to the prompts, and the ratings of the four overarching teaching goals in the questionnaire give strong evidence that for both teachers, the process orientation is the central belief towards arithmetic teaching. However the two teachers' differ considerably referring to beliefs representing application orientation and a schema view. For Mrs. G the application orientation is as central as the process orientation. In fact,

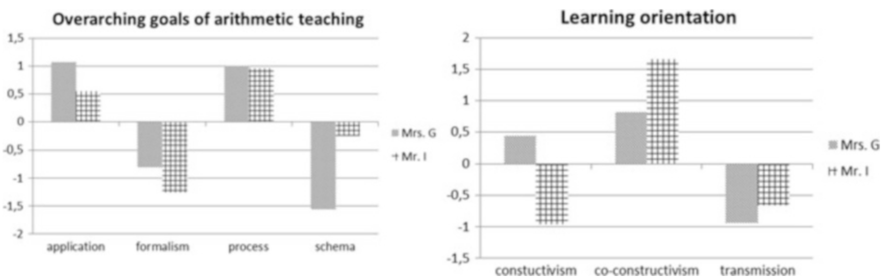

Fig. 9.1 Sum of weighted codes

in many statements of Mrs. G, application orientation and process orientation are intertwined:

Mrs. G: Yes, to apply mathematics is very very important. However, not only to apply mathematics is crucial, but also to discuss how to solve a real world problem.

By contrast, for Mr. I application orientation is a central but a subordinated belief that is only partly crucial in his teaching:

Mr. I: Reading tables or graphs are important, because they have a big relevance in everyday life. [?] When you examine written computation algorithms real life is not important. In fact, if you are in a shop you have to be able to make rough calculations.

Further, Mr. I holds schema oriented beliefs. Thus, he believes that for several arithmetic computations drill is an important prerequisite for other goals like process orientation:

Mr. I: For me it is important to automatise a lot of things in arithmetic. That is a main goal of primary schools. [?] However, it is possible to combine the drill with interesting things when tasks show unexpected results.

Referring to the teaching orientation both teachers emphasise a moderate constructivism emphasising students' explorations that are guided by the teacher. However, Mr. I also values also a more direct guidance dependent of a specific teaching subject and Mrs. G refuses every teaching approach representing a transmission view:

Mr. I: Firstly the students solve problems alone, and afterwards we discuss different solutions. [?] However, we have different types of students. For some of these students the frowned teacher centred instruction is just the best way.

Mrs. G: It is a no-go if a teacher tells me what I have to do. This is a no-go. If I do this in this way, I place the students under disability.

9.5 Results Referring to the Teachers' Enacted Beliefs

With reference to the classroom practice, we firstly regard Mrs. G. Her classroom practice is characterised by student-centred phases. Over 50% of the total time of observation the students work alone or in small groups only slightly moderated by Mrs. G, who partly commented her classroom practice towards the observer:

Mrs. G: In these phases, I have a look for those, who are not yet able for cooperative learning. This has a value in my teaching.

In the student-centred phases, a main characteristic of her classroom practice is to initiate students' individualised work that is often focused on real world problems:

Mrs. G: In this group you have to look for areas. Look also towards the flooring. Talk about that in your group. Afterwards you have to make a report for the whole class. I request the other group to compute the perimeter for each room and to consider for which material we need the perimeter.

At the end of the student-centred phases, Mrs. G usually guided a discussion of the results (about 8% of the total time). A characteristic of these phases is that Mrs. G did not stop the discussion when a student gives a solution. In fact, she always asked the students for an explanation as the example of a task concerning producing simple and complex subtraction tasks illustrates:

The task was to provide a number x that yields an easy or rather a complex subtraction task for $220 - x$.

Mrs. G: [A student gives a solution by $x = 30$] Ok, 30. However, why this is a complex task? You are right, but what is more complex in this task? [The student: Because, there is a 30, but here is only 20. For this reason, I have to remove 1 from the hundreds] Ok. If we have to go down into the next hundred it is more difficult.

These few episodes of the classroom practice represent a lot of further evidence that Mrs. G prefers extended phases of student-centred work followed by guided discussions with the whole class (co-constructivism). She mainly follows a problem solving approach whereat most of the problems are based on a real world situation. Finally, she often let the students' explore mathematics in a process sometimes based on self-developed tasks (process orientation and application orientation). Thus, the classroom practice of Mrs. G is mostly consistent with her professed beliefs. Only in one lesson, Mrs. G partly showed a classroom practice that could be understood as enacting beliefs representing a transmission view. However, Mrs. G justified the related episodes with a lack of time. Accordingly, she changed her teaching style to a more constructivist orientation after a short time period.

Mr. I also showed a classroom practice that is consistent to his professed beliefs. It is important for our research that also the slight differences in the belief systems of both teachers result in different classroom practices. For example, the classroom practice of Mr. I gave evidence that he has a stronger tendency for teacher-centred phases than Mrs. G. In contrast to Mrs. G, in the classroom practice of Mr. I student-centred phases have a (smaller) amount of about 30% of the total time, whereas his classroom practice include more phases that are teacher-centred as the following:

The task is: How many clothes-pegs a person needs to hang up a certain number of towels.

Mr. I: You need three clothes-pegs for two towels. Correct. If you hang up the next towel, how many pegs do you need? A student: 4.

Mr. I: Yes. How many pegs do you need to hang up four towels?

A student: 5.

Mr. I: I see! Did something struck you yet? To hang up three towels you needed four, to hang up four towels you need five. If I want to hang up the next towel, I need another peg.

A student: 6.

Mr. I: Aha. What strikes you? ?

However, the classroom practice of Mr. I had also extensive episodes, in which the students work alone or in small groups only slightly moderated by Mr. I.

Similarly to Mrs. G, the classroom practice of Mr. I included many episodes in which he enacted process oriented goals. However, according to the differences of the professed goals of Mrs. G and Mr. I, Mr. I also emphasised a schema orientation:

Mr. I: You have computed 9 plus 1 equals 10. You must write 0 and you have a carry of 1. You have to repeat this a couple of times. Then it will work.

9.6 Discussion and Conclusion

This report is based on a synthesis of the research focusing on the relationship between teachers' professed and enacted beliefs. A consequence of this synthesis was to define principles for analysing the mentioned relationship in terms of a consistency of professed and enacted beliefs. These principles are to firstly identify central and peripheral beliefs that could explain central and peripheral aspects of a teacher's classroom practice. Further principles are to focus on the teaching of a specific subject in a specific (social) context. Although we reported only two cases in this paper, our results show that the mentioned principles yield an empirical consistency that is—from a theoretical perspective—a characteristic of teachers' beliefs.

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Chapter 10
Raising Attainment: What Might We Learn from Teachers' Beliefs About Their Best and Worst Mathematics Students?

Kim Beswick

Abstract Teachers' beliefs about the capacities of students to learn mathematics have been linked to the environments that they establish in their mathematics classes, the pedagogy they employ, and what they see as appropriate goals for mathematics teaching. Eighteen teachers of secondary mathematics were asked to describe the best and the worst students of mathematics that they were currently teaching, and to describe how they planned to ensure that each student made progress in his/her mathematics learning in the coming year. The findings highlight the potential of efforts to teach mathematical thinking as defined by the proficiency strands of the Australian Curriculum: Mathematics, as well as particular work habits, to enhance the attainment of students perceived as less capable.

10.1 Introduction

The Australian government, like many others, is concerned to improve the country's ranking based on international tests of its students' mathematical achievement. It has been recognised that raising the overall attainment of Australian students in mathematics requires improving the performance of students at all levels (Jensen et al. 2012). The development and progressive implementation across all states and territories since 2010 of a national curriculum is regarded as an important contributor to improving the quality of education across the country (Australian Curriculum Assessment and Reporting Authority [ACARA], 2014). In addition to specifying content, the curriculum mandates the teaching of mathematical proficiencies—understanding, fluency, problem solving, and reasoning. Teaching the proficiencies has the potential to assist low attaining students but we know that teachers' judgements about the attainment and potential for attainment of their students inform their choices about the mathematics content that students are offered and the pedagogies that teachers employ (Beswick 2007/2008; Watson 2001a). The

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study reported here attempted to answer two research questions related to the extent to which Australian secondary mathematics teachers might use the proficiencies in the Australian Curriculum to raise the attainment of all students. They were:

1. What do teachers believe distinguishes poor mathematics students from those who are capable students of mathematics?
2. How might these judgements impact their pedagogical choices?

10.2 Mathematics Learning for Low Attaining Students

In Australian secondary schools lower attaining students in mathematics are typically grouped together from at least Year 9. Beswick (2005) found that students in mixed or lower ability classes were more likely to be taught by relatively inexperienced teachers, who were less mathematically qualified, and with beliefs less likely to be aligned with a Problem Solving orientation (Ernest 1989) than were their peers in high ability classes. Students in lower ability groups perceived their classroom environments to be less aligned with constructivist principles than did their peers in other classes. Similarly, Straehler-Pohl et al. (2014) reported that lower ability groups tend to have the least experienced and least well qualified teachers and work in classrooms in which academic expectations are low, while Gervasoni and Lindenskov (2011) noted the inextricable link between underperformance and poor quality learning and teaching environments.

Beswick (2007) identified teachers' beliefs about their students' capacities to learn mathematics as key to the establishment and maintenance of reform oriented classroom environments. She reported evidence that teachers' beliefs about such things as the appropriateness of conceptual understanding as a goal for mathematics teaching, the role of concrete materials for supporting conceptual development compared with answer getting, and the relative importance of basic calculation skills differed according to whether or not students were regarded as having difficulty learning mathematics (Beswick 2007/2008). In each case the differences pointed to students having greatest difficulty with the subject experiencing curricula and pedagogies focussed on low level skills rather than on the development of understanding.

Informal and ongoing assessments are an integral part of teachers' work and are necessarily based on inferences from what students do and the mathematics that students' articulate or write (Watson 2001a). Watson (2001a) explained how unconscious bias can influence teachers' judgements as they make those inferences. She also reported evidence of the capacity of low attainers to think in mathematically sophisticated ways (Watson 2001b).

10.3 Mathematical Proficiency

The term "mathematical proficiency" was adopted by Kilpatrick et al. (2001) to describe the ultimate goal of school mathematics. They noted that consensus on the goal of school mathematics has changed over time but a broad shift from computational facility alone to their own view that encompassed aspects of mathematical thinking and the ability to apply mathematics to problems is evident. Consistent with this, Wu and Zhang (2006) pointed to an increasing emphasis on problem solving in the curricula of countries in both the east and the west.

Kilpatrick et al. (2001) conceptualised mathematical proficiency as comprising five interdependent strands—conceptual understanding, procedural fluency, strategic competence (concerning the formulation, representation and solution of problems), adaptive reasoning, and productive disposition. Mathematics curricula around the world highlight similar constructs that should be developed in students as they learn school mathematics. For example, in the US, the Common Core State Standards (CCSS) include seven process Standards for Mathematical Practice. These draw upon earlier standards developed by the National Council for Teachers of Mathematics (NCTM 2000) and the proficiencies identified by Kilpatrick et al. (2001). They relate to perseverance in problem solving, abstract and quantitative reasoning, argumentation and critique, mathematical modelling, strategic use of tools, precision, and attention to structure (National Governors Association Center for Best Practices 2010). In Singapore, problem solving is positioned at the centre of the mathematics curriculum with skills, processes (reasoning, applications and modelling, and thinking skills), concepts, metacognition, and attitudes regarded as contributing to problem solving capacity (Kaur 2014). In Australia, the four mathematical proficiencies—fluency, understanding, reasoning, and problem solving—correspond closely with the first four of Kilpatrick et al.'s (2001) proficiencies. The definitions provided in the Australian curriculum are shown in Fig. 10.1.

10.4 The Study

The study was conducted in the context of a 2014 professional learning (PL) program developed collaboratively by Department of Education and university staff in one Australian state. The program ran over four spaced days and was aimed at encouraging classroom teachers to adopt evidenced based approaches to planning, teaching and assessing mathematics using the Australian Curriculum. Schools were encouraged to facilitate the participation of three or more teachers from their school. In recognition of the key role of school leadership in providing instructional leadership, a single day was provided for leaders from each of the schools in which the teachers worked.

Understanding: Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

Fluency: Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.

Problem Solving: Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

Reasoning: Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.

Fig. 10.1 Proficiency strands from the Australian Curriculum: Mathematics

10.4.1 Participants

The 18 teachers were participants in the PL program and drawn from 6 secondary schools in the same region. They comprised 11 males and 7 females. Their teaching experience ranged from 6 weeks to 33 years with 11 reporting fewer than 10 years of experience. Six reported having studied mathematics (not mathematics education) at university with two of these citing the highest level they had studied as each of 1st, 2nd and 3rd year. Eight had studied mathematics to Year 11 or 12 (the final year of secondary school) while two reported their highest level of mathematics study as Year 10. Six nominated mathematics as their preferred teaching area with other subjects mentioned being Science (7 teachers), Physical Education (3), Geography (1), English/Drama (1), and Early Childhood (1). One did not nominate a preferred teaching area. In summary, the group was diverse but predominantly out-of-field to a greater or lesser extent.

Think about the students to whom you teach mathematics.

a) Which of the students is best at mathematics? Without identifying the student describe what makes this student good at mathematics? How can you tell that this student is good at mathematics?
b) Describe how you plan to ensure that this student makes progress in mathematics this year.
c) Which of the students is worts at mathematics? Without indentifying the student describe what makes this student poor at mathematics? How can you tell that this student is poor at mathematics?
d) Describe how you plan to ensure that this student makes progress in mathematics this year.

Fig. 10.2 Relevant questions from the teacher questionnaire

10.4.2 Instrument

The items that provide the focus of this paper were part of a comprehensive pen and paper profile instrument based on similar instruments whose use has been reported elsewhere (e.g., Beswick et al. 2012) and completed by participants at the start of the first PL day. The relevant questions are shown in Fig. 10.2.

10.4.3 Data Analysis

The teachers' responses to each of the questions shown in Fig. 10.1 were listed, similar responses grouped, and categories identified. In addition, elements of the responses that related to the proficiency strands of the Australian Curriculum: Mathematics were identified. These are italicised in Tables 10.1 and 10.2.

10.5 Results

The characteristics of 'poor' and 'good' students that comprised teachers' responses to the relevant questions shown in Fig. 10.2 are shown in Table 10.1 along with the number of times that each was mentioned.

In Table 10.1 language related to the each of the four proficiencies was used to describe the skills and knowledge of 'good' students. In contrast with this, descriptions of 'poor' students focussed on their lack of proficiency and included that they lacked understanding, the ability to explain, and were unable to transfer their skills to unfamiliar problems. By far the most commonly mentioned (7 times) distinguishing characteristic of 'poor' students was, however, their poor basic computational skills.

Table 10.1 Characteristics of poor and good mathematics students

	Poor mathematics students …	Good mathematics students …
Skills and knowledge	have poor *basic computational skills* (7)	are *fluent* with tables (1)
	have poor *understanding* (3)	*understand concepts* (1)
	lack prior knowledge (3)	have broad background knowledge (3)
	have difficulty grasping concepts (2)	can pick up new methods and *explanations* quickly and grasp concepts intuitively (3)
	have no mental strategies (1)	
	are unable to *explain* (1)	are able to *reason and explain* their strategies (2)
	are unable to talk about questions (1)	can respond to questions in class (1)
	can't *transfer* skills to unfamiliar problems (1)	are able to *transfer knowledge* to complex/*unfamiliar contexts* (2)
	require each problem to be broken down into small steps (1)	can *problem solve* (2)
	are very slow to complete tasks (1)	
		can produce *multiple solutions* and *understand* more than one method (4)
		can *justify/prove* their answers (2)
		think outside the box (1)
		are successful (1)
		make few mistakes in written work (1)
Affect	have negative attitudes/dislike mathematics (4)	have a positive attitude/love maths (2)
	won't try difficult work (3)	
	lack effort (1)	
	are unwilling to complete tasks (1)	want to do maths (1)
	think they can't learn (1)	are self-confident (1)
	lack a desire to learn (1)	are keen to learn, self-motivated (3)
	fear exploring (1)	are willing to explore/takes risks and unafraid of being wrong (3)
	fear algebra (1)	are unafraid of algebra (1)
		stay motivated (2)
		enjoy success (1)
		see the relevance (1)

(continued)

Table 10.1 (continued)

	Poor mathematics students ...	Good mathematics students ...
Work habits	miss lessons (3)	
	are inattentive (1)	
	won't ask for help (1)	can help others (1)
	only like/do repetitive work with a calculator (1)	attempt the hardest problems (1)
	are disruptive (1)	
	have poor concentration (1)	
		are willing to listen (1)
		methodically break down problems, identifies relevant information (2)
		show working via step-by-step written work (2)
		take initiative to figure things out, test self (2)
		are goal setting (2)
		actively check work (1)
		are organised (1)
		make lots of effort in class (1)
		complete all set work (1)
		learn from mistakes (1)
		like to discuss things and think about meaning (1)
Other	have out of school influences that reinforce the lack of importance of maths (1)	

In terms of affect, 'good' students were characterised as confident, motivated, fearless, and positive about mathematics, whereas 'poor' students were described as unwilling, unmotivated, fearful, and negative. Consistent with these differing affective profiles, 'good' students were seen as making an effort, organised, willing to listen and learn, and showing initiative in their learning. 'Poor' students were regarded as identifiable by negative work habits including missing classes and being disruptive. In addition, one teacher mentioned that 'poor' students have non-school influences that reinforce the lack of importance of mathematics.

Table 10.2 shows the teachers' suggestions regarding how they would ensure that their worst and best mathematics students would continue to make progress, along with the number of times that each was mentioned. Consistent with the identification of poor computational skills as characteristic of 'poor' mathematics students there were six references to the need to help these students to build these skills. Most responses, however, related to the kinds of tasks considered appropriate.

Table 10.2 Ways of supporting poor and good mathematics students to make progress

	Poor mathematics students need…	Good mathematics students need…
Affect	to develop confidence (3)	
	praise (1)	
	encouragement (1)	encouragement to work hard, push themselves and continue to learn (1)
	to develop motivation to learn (1)	
	to be taught why mathematics is important and useful (1)	
	success resulting from effort and hopefully leading to better attendance (1)	
Tasks	(some chance of) success, work that's not too hard (5)	challenging tasks (6)
		open-ended tasks (4)
		problem solving (3)
	tasks that are relevant to their interests (3)	
	tasks at their level (2)	tasks that continue to extend and stretch their knowledge and *understanding* (1)
	applications, real-world relevant tasks (3)	applications (2)
	lots of practical/hands-on (2)	
	engaging tasks to develop confidence (1)	
	easily accessible tasks (1)	
	inclusive tasks (1)	
	the correct steps, correct learning (1)	questions/tasks that provide correct techniques which will enable them to extend themselves (1)
	questions that allow them to show their knowledge without complex formulae (1)	
	tasks designed to move them on from where they currently are (1)	
	work that requires *explanations*, to be asked for verbal *explanations* of how they obtained their answers (1)	lots of opportunity to talk (1) opportunities to *explain* and help others (1)
		logic problems (1)
		lots of *reasoning/understanding problems* with chance to check work (1)

(continued)

Table 10.2 (continued)

	Poor mathematics students need...	Good mathematics students need...
		further development of *reasoning* skills (1)
		extended activities/work to continue when finish early (1)
		lots of brand new questions (1)
Skills	to continue building number knowledge (1)	
	back to basics (1)	
	to spend time on strategies to improve multiplication *fact recall* (1)	
	to build on basic skills/mental strategies to assist them (1)	
	work on developing memory (1)	
	repetition of essential skills (1)	opportunities to learn new skills rather than revisiting skills already learned (1)
		to progress skills to the next logical step (1)
		help *explaining understanding* to other class members (1)
Structures	one-on-one help (1)	individual questioning to ensure *understanding* of content (1)
	smaller groups with students at the same level (1)	a streamed (same ability) group so working with others who are 'good' at maths (1)
	a modified program, keeping them involved in whole class activities but enabling them to access the task (1)	access to the whole curriculum including more demanding coursework (1)
	opportunity to participate and interact with students of different abilities (1)	
Other		regular feedback (1)
		formative assessment (1)
		summative assessment (1)

For ‘poor’ students the over-riding concern was that they experience success, with relevance, being at the appropriate level and hands-on the next most commonly suggested characteristics of tasks for these students. In contrast, appropriate tasks for ‘good’ students were described as challenging, open-ended, and involving problem-solving. In addition, three of the proficiencies (understanding, reasoning and problem solving) were included in responses for ‘good’ students (8 mentions in all) compared with just one reference to tasks requiring explanations for ‘poor’ students. In terms of addressing the negative affective characteristics of ‘poor’ students the teachers’ responses were more likely to identify needs than to suggest ways of addressing them—providing praise, encouragement, and success were each mentioned once. Organisational structures for teaching were mentioned by four teachers in relation to ‘poor’ students and by three teachers in relation to ‘good’ students. In each case the suggestions included individual help. Homogeneous ability grouping was suggested just once and for ‘good’ students.

10.6 Discussion and Conclusion

Poor computational skills seems to be a hallmark of poor mathematics students but interestingly there was only one reference to strong computational skills for ‘good’ students of mathematics. Instead ‘good’ students were more likely to be identified by their ability demonstrate aspects of mathematical proficiency. The finding in relation to ‘poor’ students is consistent with Beswick’s (2007/2008) finding that teachers believed basic computation to be more important for students who had difficulty learning mathematics, than for other students. These data suggest that teachers may also believe that mathematical proficiency is the province only of ‘good’ mathematics students rather than something that should and can be taught to all students as mandated by the Australian curriculum.

The teachers offered little by way of suggestions for addressing the shortcomings they identified in ‘poor’ students’ work habits and affective responses. The desire to provide weaker students with tasks that are relevant and that offer the opportunity for success in mathematics is undoubtedly well-meaning but it is unclear what was meant by success. It could be assumed that ‘good’ students are successful and hence helping other students to succeed would involve helping them to exhibit the characteristics of ‘good’ students, including being able to demonstrate mathematical proficiency. It seems more likely, given the emphasis in these teachers’ responses on basic computational skills, that for ‘poor’ students, success was seen in terms of improving those skills.

It would be interesting to ask teachers about an average ability student as well as their best and worst mathematics students. This would provide some insight into the extent to which the characterisations of ‘good’ and ‘poor’ mathematics students and their teachers’ plans for their learning represent the ends of a continuum or continua. In Table 10.1, for example, many of the characteristics that have been aligned could be answered in relation to an average student as somewhere between (e.g., poor students have poor understanding whereas good students understand concepts—an average student might understand some concepts). Other characteristics and particularly many of the teaching strategies in Table 10.2 appear unconnected so a midpoint is hard to imagine.

Teachers will be unlikely to take seriously the requirement to teach the mathematical proficiencies if they believe that they are appropriate for only some students or that only some students are capable of learning them. It also seems that teachers may need help with ways in which the proficiencies can be developed in students and especially in those who struggle to learn mathematics. Indeed it is possible that uncertainty about how to teach the proficiencies underpins or at least reinforces a belief that they cannot be taught. Similarly, assistance with ways in which to help all students to adopt work habits resembling those of 'good' students of mathematics may also be helpful.

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Chapter 11
Numeracy Task Design: A Case of Changing Mathematics Teaching Practice

Peter Liljedahl

Abstract Over the last 15 years, numeracy has become more and more prominent in curriculum initiatives around the world. Yet, the notion of numeracy is still not well defined, and as such, often not well understood by the teachers who are charged with the responsibility of helping our students to develop these skills. In this article I explore the work of a team of mathematics teachers brought together for the purpose of developing a set of numeracy tasks for use within district wide numeracy assessments. Results indicate that these teachers' experiences designing these tasks, and pilot testing them in their own classrooms, propelled them to make massive changes in their own mathematics teaching practice.

11.1 The Numeracy Movement

Around the world it has long been recognized that students are completing their compulsory education without the mathematical skills to cope with the demands that life and work require of them. This recognition has launched, simultaneously around the world, what is commonly called the *Numeracy Movement* (Hillyard 2012), which recognizes that mathematics alone is not helping students to achieve their, and society's, goals. What is needed is not more mathematics.

> . . . efforts to intensify attention to the traditional mathematics curriculum do not necessarily lead to increased competency with quantitative data and numbers. While perhaps surprising to many in the public, this conclusion follows from a simple recognition—that is, unlike mathematics, numeracy does not so much lead upwards in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life's diverse contexts and situations. (Orrill 2001, p. xviii)

What is needed is more contextualization and an increased ability to deal with this contextualization. As such, to be numerate means to be willing and able to use mathematical knowledge across a wide variety of contextual (even real) situations.

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> Mathematical literacy[1] is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. (OECD 2003, p. 14)

But this definition, like the hundreds of definition existing in curriculum documents, white papers, and research reports are not actionable. Instead, they are idealizations: idealizations of what numeracy can be, and what numeracy can achieve. They are more a call to action than a blueprint for action. Yet action is expected.

This call for action, driven largely by the OECD's focus on mathematical literacy (as manifest in their PISA and PIAAC assessments) has resulted in a massive restructuring of curriculum all over the world (Biedermann 2014). For example, although lacking a national curriculum, Western Canada has declared numeracy as one of the cross-curricular key areas of learning from Kindergarten to grade 9, and as one of the aims of mathematics education within the 10–12 curriculum. And like other places in the world, the only real support that teachers receive in realizing these curriculum goals is a definition of numeracy.

> Numeracy can be defined as the combination of mathematical knowledge, problem solving, and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (Mathematics K to 12: Mathematics Curriculum Documents 2008, p. 11)

Regardless, since 2002, the British Columbia Ministry of Education has required each school district in British Columbia to provide an annual report on the numeracy performances of students within their district. These reports must include an explanation of how numeracy is being measured within the district, what populations are being measured, what improvements have been seen over time, and what the district plans are to improve numeracy performance in the coming year.

Much is required of teachers in such environments. They are being expected to implement the ideals of numeracy, not as a distinct topic in mathematics, but in more subtle and dispositional ways, both within their mathematics curriculum and across all curricula. And often with little more support than a definition.

Considering this high expectation on mathematics teacher change, I have become curious about the professional growth of teachers within this context. In particular, what are the experiences of mathematics teachers as they struggle to make sense of these emergent, and ill-supported, demands? What is the nature of any of the resultant changes they may undergo in their thinking about mathematics and teaching of mathematics and what are the mechanisms of these changes? Answers to these questions will go a long way towards informing the mathematics education

[1] Although it can be argued that there is a distinction between the terms *numeracy* and *mathematical literacy*, the fact is that preferential use of the terms seems to be geographic, with some countries choosing to use the former while other opt for the latter (Hoogland 2003). As such, for the purposes of this article, the terms will be used interchangeably.

community at large about teacher change in general, and the implementation of numeracy initiative in particular.[2]

11.2 Teacher Change

Teacher education literature is full of examples of teachers' changing their practice. Usually, these examples are found in research examining specific professional development models such as action research (Jasper and Taube 2004), lesson study (Stigler Hiebert 1999), communities of practice (Wenger 1998), or more generally, collegial discourse about teaching (Lord 1994). Such research has very effectively delineated different mechanisms by which teachers change while participating in a variety of professional development setting. Conclusions show that with time and continued intervention, support, and collaboration teachers can make significant and robust changes to their practice.

As a mathematics inservice teacher educator working in a variety of professional development settings I have witnessed teacher change of the form exemplified in the aforementioned research. But I have also witnessed change of a different kind—rapid and profound change in practice—examples of which are not often found in the literature.

11.3 Rapid and Profound Change

In prior research (Liljedahl 2010), I identified and articulated this phenomenon of mathematics teachers' *rapid and profound change*. This research showed that this phenomenon can be nuanced into five distinct and non-hierarchical mechanisms of change which I have come to call: (1) *conceptual change*; (2) *accommodating outliers*; (3) *reification*; (4) *leading belief change*; and (5) *push-pull rhythm of change*. These five mechanisms of change stand in stark contrast to the more pedestrian mechanisms of change articulated above. Three of these mechanisms are articulated below.

Accommodating outliers is born from the work of Piaget (1968) and explains instances in which teachers are able to keep various aspects of their practice and experiences disjoint from each other. Sometimes when, in professional learning settings, they are asked to consider these experiences in unison, a process of *accommodation* occurs, from which emerges a new view on what it means to teach and learn mathematics.

Reification is borrowed from the work of Wenger (1998) and explains the observed phenomenon wherein a teacher makes rapid changes to their practice after participating in a process in which they "give form to [their] experiences by producing objects that congeal this experience into *thingness*" (Wenger 1998, p. 58).

[2]For an extended version of this paper please see Liljedahl (2015).

Leading Belief Change acknowledges that teachers practice is not distinct from their beliefs about teaching and learning mathematics (Chapman 2002) and that these beliefs do not exist in isolation from each other. Instead, they cluster together along lines of relevance to form robust systems of beliefs (Chapman 2002; Green 1971). These belief clusters can, however, exist distinct from each other, or in tension with each other. And sometimes, when changes to one of these clusters is made, it causes a significant and rapid reorganization of beliefs in a different cluster, resulting in complementary significant changes in teaching practice.

11.4 Methodology

The data for the research presented here comes from a team that was assembled to craft the numeracy tasks that would be used to gather the aforementioned numeracy performance data required by the British Columbia Ministry of Education. This team met every 3 weeks from September until February, for a total of six meetings. Each meeting was 3 h long and took place during school time. The terms of reference for the team were scant with the only requirement being that at the end of the six meeting the two numeracy tasks were to be ready for district wide use. The only resources afforded the team to help them in this endeavour were the release time to meet, the aforementioned definition in use in Western Canada, and a facilitator.

My initial role within this project was as this facilitator. In this capacity I set the agenda for the six meetings. In the first meeting I put three questions to the team. The first was simply what they thought numeracy was. The second question asked them to think of a student that they had taught that they believed to be exceptionally numerate and to then articulate for the group what *qualities* that students possessed that made them numerate. My third question was to now think about what a numeracy task should look like. In the second meeting we began to design a number of preliminary numeracy tasks for the teachers to take away and test with their own students. In the third through fifth meetings we debriefed the teachers' experiences in implementing the tasks and refined (or redesigned) the tasks based on their feedback. The last meeting was used to finalize the wording, formatting the two tasks, and to write a script to help teachers across the district implement the tasks within their own classrooms.

11.4.1 Participants

The design team was comprised of four grade 5 and six grade 8 teachers. Of these, six were female and four were male, four had taught for over 15 years and two had taught for less than four. Only one of the teachers had a university education in mathematics. In short, the design team was a diverse collection of elementary and middle school teachers representative of the gender, experiential, and educational makeup of the school district.

11.4.2 Method

This research is ethnographic in nature in that, despite being the facilitator, I was an observer within the design team community. However, having the role of facilitator made the collection of data difficult as I found myself too embroiled in the task design activities, impromptu conversations, commitments, and actions, to adopt the removed stance of observer. Instead, I adopted a stance of noticing (Liljedahl 2010; Sherin et al. 2011). This stance allowed me to work within the design team setting to achieve our explicit goals, while at the same time staying tuned to the experiences of the teachers involved. If something of interest occurred, I was able to subtly shift from facilitator to observer to researcher, and to begin to probe more deeply the comments, conversations, and experiences of the teachers through questioning, individual interviews, and classroom visits. As such, data consists primarily of the field notes taken during and immediately after each meeting, audio recordings of interviews with individual participants, as well as field notes from classroom visits.

11.4.3 Analysis of Data

These data were coded using a process of analytic induction (Patton 2002). This process, like grounded theory (Charmaz 2006), relies on the use of a constant comparative method. Unlike grounded theory, however, in analytic induction this process starts with a set of a priori codes. In the case of these data, these codes came from the framework of rapid and profound change (Liljedahl 2010) as well as from the numeracy movement context in which this design team was situated. Analytic induction, through its constant comparative method, also allows for the emergence of new themes (Patton 2002) and this happened with the research presented here.

11.5 Results and Discussion

Initial observations showed that the teachers in the design team were making significant changes in their teaching. Analysis showed that the mechanism of this change was more or less the same for all the teachers. Rather than discuss these changes across all of the members of the design team, two cases—that of Frank and Victoria—have been chosen to represent the changes the rest of the design team experienced.

11.5.1 Frank

Frank is a middle school teacher with 12 years of teaching experience who classifies himself as a social studies and language arts specialist. Frank came to the numeracy design team because he has become increasingly unhappy with his students performance in mathematics and his inability to "put his finger on what is wrong".

Frank had no initial ideas of what numeracy was—or, at least, none that he would offer. From the beginning he positioned himself within the group as the person who has never attended a mathematics workshops or been part of a design team and that he was "just keen to learn". When the discussions about the qualities of numerate persons began Frank eventually began to talk about students who "just get it". He was very clear that he was not talking about gifted students, but students who just had a very good sense of what was going on when they were working on problems. Towards the end of this discussion Frank became very animate, almost upset, at the realization that these skills are so important to life and yet our K-12 curriculum does nothing to foster these within students.

> What are we doing in math if we are not working on these things? I see students every year who have all the facts, but if I ask them what 2 + 3 is they can't answer without writing something down. What are we doing to kids when they can sit and multiply out three digit numbers but they can't think clearly about simple everyday concepts?

During the pilot testing process, Frank became concerned with two things he was observing in his students. First, he was bothered by the poor performance of the majority of his students. At the same time, he was worried by the lack of challenge for some of his top students. The design team had been working hard to ensure that the task allowed every student to start.

> All my students were able to start. This is not the problem. The problem is that my weak students were too challenged and my really strong students are not being challenged enough. Somehow, we need to make the task harder without making it harder.

The task design process reified Frank's thinking into the activity of problem solving, which Frank realized his students had been missing in their experiences. He began asking me for problems and resources of problems which he then began using almost exclusively within his teaching. At the same time he began problematizing everyday occurrences in his classroom.

> I realize that I make lots of numeracy-like decisions every day within my teaching, and that if I start getting my students to make some of these then they will start to really experience numeracy at its best. So, for example, last week it was time to start basketball in PE. Normally, I make teams at the beginning of a unit and then those teams stay together for the whole unit. Well, rather than making the teams myself I put the students in groups and told them to come up with a proposal for who should be on what team so that the teams are fair. It turned into a whole week project.

11.5.2 Analysis of the Case of Frank

The changes in Frank's teaching began with the first meeting when, despite not having any particular thoughts about numeracy, he *accommodated the outliers* of some of his past students. This resulted in his focus on students who "just get it". The pilot testing after the second meeting shifted this focus towards challenging his students, which was further *reified* through the task design process into a belief that problem solving was important. This belief was the *leading belief change* catalysing Frank to change his beliefs about the teaching and learning of mathematics. As a result, Frank began to use problem solving in his day to day teaching of mathematics, as well as other subjects.

11.5.3 Victoria

Victoria is a former high school teacher with 8 years teaching experience. At the time of the research, she had recently started teaching grade 8 at a middle school where her teaching partners had gladly given her all of the mathematics and science courses to teach. Victoria is the only member of the design team who has a degree in mathematics and has been trained as a mathematics teacher. Victoria has come to the design team with very strong traditional views about what numeracy is and what a numeracy assessment should look like.

> Isn't numeracy just basic number facts? So, a numeracy assessment should just be a test of basic facts. We really need this in this district. That is why I am here.

Interestingly, when I asked the group to think of qualities of a numerate student Victoria' answer did not mention basic facts.

> I taught a boy last year who was so good. He could solve things in more than one way. He could explain his thinking. And he was always trying to make connections to other things we had learned.

This was a significant departure from her initial stance. As much as Victoria valued fluency of basic facts, she also seemed to value the more diverse skills of "making connections" and "solving things in more than one way". Like Frank, this initial focus became a steadfast focus for Victoria as the project evolved.

During the task design work of the design team the diverse skills that Victoria valued were *reified* into a desire to have students produce multiple solutions. This view was refined further after pilot testing the numeracy tasks with her own students—something she mistakenly thought would go well.

> I guess my students are used to more structured problems. My problems tend to be linked more closely to specific things I am teaching.

In this moment, Victoria realized that the connectedness she was looking to impart in her students was not possible through her current teaching methods.

> When I said that in our first meeting I was thinking about the way I teach. I really do value multiple solution methods and I want my students to see that there is often more than one way to do things. So, I always teach them how to do things in more than one way. And then what I want to see is that my students can do these multiple ways that I have shown them. This numeracy task requires something completely different form the students. This isn't

> about me having shown multiple ways. This is about students being able to identify how to solve it for themselves. They just don't have any experience doing this.

The numeracy tasks we were developing, with their openness and ambiguity, were asking for different skills from the students. After this, Victoria began to notice that one of the things that her students lacked was an ability to deal with the freedom that the tasks offered. This realization shifted Victoria's thinking to a new belief of teaching in which students needed to "identify how to solve [problems] for themselves". This new belief about what students need to be able to do, led in turn, to new beliefs about what it means to teach and learn mathematics. In particular, that she needs to stop being so directed in her teaching and offer, instead, opportunities for students to "figure it out on their own". Over the course of the design team this led to a wholesale reformulation of Victoria's teaching style.

> Instead of teaching first and then giving them questions second I give them the questions first. I just do my lesson backwards.

11.5.4 Analysis of the Case of Victoria

Like Frank, Victoria went through a process of *accommodating outliers—reification—leading belief change*. During the first meeting the *accommodation of outliers* led to Victoria placing importance on solving "things in more than one way". The task design process *reified* this idea into a focus on multiple solutions and the pilot testing created a *leading belief change* that restructured her beliefs around what it means to teach and learn mathematics.

11.6 Emergent Themes

Emerging out of, and cutting across, these aforementioned cases are seven themes. Whereas the aforementioned framework *rapid and profound change* (Liljedahl 2010) explains the mechanism of change that the teachers underwent as participants of the numeracy design team, these seven themes are the fuel that drives this mechanism. In what follows I briefly present three of these themes.

11.6.1 Past Students

As seen in the two cases of Frank and Victoria, the activity of considering the qualities of a past 'numerate' student was a powerful trigger for each of them to *accommodate an outlier*. Through this process they brought into the main qualities that they may not see in their students on a daily basis. In each case, these qualities were identified as something worth assessing through the eventual numeracy tasks being developed. Interestingly, for each member of the design team the particular quality that they championed in the first meeting shifted subtly along the way from being something that they wanted to measure in their students to being something that they wanted to foster within their students—both through the numeracy task and in mathematics more generally.

In making this shift two things are happening. First, the teachers are disaggregating the nuanced qualities of 'good students'. Because of their association with successful students these qualities are automatically seen as important. Second, they are seeing these qualities as something within their domain of influence—as something that they can foster within their classrooms and through their teaching.

11.6.2 Task Design

The definition of numeracy that the teachers had to work with exists in the abstract plane, somewhere between the intuitive understanding of what it means to be numerate and a will for it to be something other than mathematics. It lacks concreteness. That concreteness comes through the process of task design. That is, it is not until the participants began to actually articulate these ideas in a *reified* form of a task that the embodied qualities of the definition could be seen clearly. And like the process of thinking of a numerate past student, the process of task design was the impetus behind profound changes. In each case, it was at this stage of the process that the ideas emerging from the *accommodated outliers* changed, and took on a form that was more articulate.

Frank moved from the intangible articulation of students who "just get it" to the more actionable idea of challenging students. And Victoria transitioned from wanting students to do "things in more than one way" to looking for more than one solution. Without the task design to *reify* the definition, and the participants' initial intuitive notions, the idea of numeracy would have remained vague and abstract, much the way the local curriculum documents represents it.

11.6.3 Poor Student Performance

The unexpected poor student performance on the pilot testing also acted like a catalyst of change. On the heels of surprising and poor student performance each of the teachers subtly shifted their beliefs about numeracy—and what it meant to be numerate—to mathematics more broadly. The realization that their students were wholly incapable of what they deemed to be an important quality in numeracy meant, to each participant, that these qualities were absent in their teaching practice in general. This realization created the impetus to expand their new beliefs from the context of the numeracy task in particular to their mathematics teaching practice in general. And in the process, these new beliefs caused a perturbation of their existing beliefs about mathematics teaching and learning.

Frank shifted his belief that a numeracy task should challenge every student to the importance of problem solving in mathematics, and beyond. Victoria, realizing that her students could only mimic what she gave them, turned her lessons upside down and started each day with a problem for them to solve.

11.7 Conclusion

It is clear from the results presented above that the teachers in this project made significant—even rapid and profound—changes in their mathematics teaching practice. The mechanism of this change was through a chained sequence of *accommodating outliers*, *reification*, and *leading belief change*. As such, this research extends my earlier work on rapid and profound change (Liljedahl 2010) which looked at 42 cases wherein only one mechanism of change was at play for each teacher.

These mechanisms of change were fuelled by three distinct experiences within the design team: a consideration of a past numerate student, numeracy task design, and the unexpected poor results of their own student during pilot testing. Unlike the previous research (Liljedahl 2010) on rapid and profound teacher change, wherein the catalysing experiences were treated descriptively, the potentially prescriptive nature of the catalysing experiences in the research presented here offers a means by which change may be occasioned.

However, these catalysing experiences were effective only in that they occurred within a context that was largely unfamiliar to the teachers. The lack of pragmatic clarity as to what numeracy is, coupled with a lack of resources around this important construct, afforded the emergence of a more intuitive and grounded entry into numeracy.

Taken together, the idealized and ill-supported definitions of numeracy present in the local context, combined with the expectation that the teachers design a comprehensive tool for assessment, created within the numeracy design team a perfect storm highly conducive to teacher change. However, this storm would not have been possible had there not been, at the outset of the project, a tension between numeracy and mathematics wherein numeracy stood, not beside (or inside) of mathematics as is often the case in our definitions, but in opposition to it—as something new, as something different. And in so doing, numeracy offered these participants a different context in which to think about, and experiment with, the teaching of numeracy (né mathematics).

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Chapter 12
Math Lessons: From Flipped to Amalgamated, from Teacher- to Learner-Centered

Domenico Brunetto and Igor Kontorovich

Abstract We introduce a collaboration framework for teachers and teacher educators, who are interested in designing learner-centered mathematics lessons that amalgamate instructional videos. The framework is spiral, when each of its rounds consists of four phases: understanding the teaching context, developing a plan of an amalgamated lesson, carrying out the lesson and looking back. The implementation of the framework is expected to foster teachers' technological pedagogical content knowledge. To exemplify the framework in action we present a case of an experienced high-school teacher. The case highlights the complexity of designing learner-centered lessons even for a knowledgeable teacher with predispositions towards integration of technology in the classroom.

12.1 Introduction

The goal of this paper is to present and illustrate a collaborative framework for teachers and teacher educators, who are interested in designing learner-centered mathematics lessons that integrate instructional videos. Briefly, by learner-centered lessons we refer to an environment in which students are given the freedom to construct their knowledge through personal experience with the content, for instance in problem-solving or collaboration with classmates. In this context, the role of a teacher is to facilitate such experiences rather than provide instructions (e.g., Rogers 1983). Indeed, teachers, who are also the adopters of this innovation, can be seen as the communication channel through which the innovation can be spread to students.

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The motivation for our framework stems from two bodies of knowledge: on Massive Open Online Courses (MOOCs) and on developing teachers' technological pedagogical content knowledge (TPACK).

12.1.1 MOOCS and Flipped Classrooms

The first MOOC was initiated in the Manitoba University in 2008, but Stanford's MOOC on artificial intelligence from 2011 was the one that drew public attention to the new educational model. The MOOCs' potential to deliver high-quality education to masses of people regardless of their ethnicity, gender, geographical location, health and economic status has been inspiring for many researchers and practitioners. As a result, in a short period of time conspicuous platforms for hosting MOOCs have been launched in Europe and the US (see Daza et al. 2013, for an elaborated review).

However, Konnikova (2014) and others indicate that the enthusiasm for MOOCs has waned in the past years. From their perspective, while MOOCs succeeded in providing many high-quality resources at low cost or even for free, the typical participants of the courses come with solid backgrounds, live in developed countries, are self-determined and enrol out of curiosity or to advance in a job. Another phenomenon is the low completion rate of MOOCs: Brahimi and Sarirete (2015) report that the usual completion rate is less than 10% and Konnikova presents examples of courses where one third of the enrolled students finish. In this way, it can be argued that MOOCs have not reached (yet?) the audiences and the scales for which they were initially intended.

In the context of high education, the idea of exclusively online courses has been transformed into a *blended education* (e.g., Kim et al. 2014). While a variety of approaches to this notion exist, the Sloan Consortium defines it as an "instruction that has between 30 and 80% of the course content delivered online" (Bart 2014, p. 2). The popularity of such flipped (or "inverted" or "hybrid') classrooms in the context of higher education can be explained with the challenges of academic studies. Indeed, a typical academic syllabus is dense, the classes are teacher-centered and contain masses of students from various backgrounds. Accordingly, the initial premise of blended education was that the online part of the course will enable teachers to allocate time and efforts for learner-centered activities in the classroom (e.g., Daza et al. 2013).

Following upon Konnikova (2014) we acknowledge the quality of resources (e.g., instructional videos) used in some mathematical MOOCs and argue that amalgamating them in school lessons can be helpful in designing a learner-centered environment. Clearly, such amalgamation requires advanced knowledge and skills of a lecture or teacher.

12.1.2 TPACK and Its Development

The concept of TPACK is usually discussed in the school context and it accounts for teachers' ability to utilize the potential of technological instances for the benefits of students' learning experience (e.g., Stoilescu 2015). In some countries environments that integrate technology became a part of a learning standard and teachers' TPACK development is a part of teacher preparation programs (e.g., Niess 2005).

In their elaborated review, Koehler et al. (2013) overview the challenges that accompany introducing technology into the classroom and argue that the core components of a "good teaching with technology" are content, pedagogy and technology, as well as, the relationships between and among them. Their TPACK framework has been used for advancing and evaluating pre-service mathematics and science teachers (e.g., Niess 2005).

Developing teachers' TPACK is inseparable from integrating technology in the classrooms. When reviewing the critique for the TPACK framework, Stoilescu (2015) mentions that it does not provide clear paths for such integration and that it has been rarely used in the case of in-service expert teachers. The framework introduced in this paper addresses these critiques by sketching a possible path for integrating MOOC videos in mathematics lessons. The framework in work is illustrated with the case of an experienced high-school teacher.

12.2 Theoretical Foundation of the Framework

Our framework is a combination of Schoenfeld's (2011) model of teaching-in-context with a modification of Pólya's (1945) method for mathematical problem solving.

Schoenfeld (2011) provided a detailed theoretical account of how and why teachers make specific decisions and actions in their teaching. Schoenfeld considers teaching as a dynamic and context-dependent act that is derived from the interplay of teacher's *goals, resources and orientations*. A goal, whether explicit or tacit and unarticulated, is something that a teacher wants to accomplish. Goals can be classified in terms of grain sizes. For instance, a short-term goal can be associated with a single lesson, while a long-term goal can refer to what a teacher wants her students to know as a result of a school experience. Resources include all kinds of "goods" that are available for a teacher. For example, the technology in the classroom; students' knowledge; teachers' knowledge, interpersonal skills and relations with students. Orientations are used for indicating teachers' affective instances, such as beliefs, values, predispositions and tastes. For example, orientations shape teacher's perceptions regarding what the students of a particular class can and cannot do, and whether it is useful to use technology in their classroom. Similarly to goals, some orientations that govern teacher's decisions can be latent.

Pólya's (1945) *How To Solve It* is considered as one of the milestones in mathematics education. The book reports a new approach to solve a problem exploiting four phases: (1) understanding the problem, which is aimed at comprehending the givens of the starting situation and characteristics of the desired situation; (2) devise a plan, which is concerned with closing the gap between the situations; (3) carry out the plan, where the plan is executed; and (4) look back, in which the quality of the solution is examined and a preparation is made for solving future problems.

In Pólya's view, the four-phase method can be used for teaching problem solving. We adapt Pólya's (1945) method for solving our "problem", which is designing learner-centered mathematics lessons, when MOOC videos serve as a tool for the "solution".

12.3 Proposed Framework

Our framework is built on three premises, which are informed by the complexity of the innovative-decision process (Rogers 1983) and the specificity of this complexity in the context of in-service teachers (Stoilescu 2015).

First, according to Rogers (1983), attitudes play a significant role in the decision-making regarding accepting or rejecting an innovation. Our framework appeals to teachers who are predisposed towards integration of technology in their classrooms and need support in carrying it out. Applying Rogers' classification, these teachers can be associated with early adopters and early majority who participate in the innovation diffusion willingly, but are not the gatekeepers who bring the innovations in education from outside of the system.

Second, teacher knowledge is a context-dependent structure (e.g., Shulman 1987), and accordingly, our framework is informed by the pedagogical context of the concrete teachers.

Third, we perceive the framework implementation as a collaborative practice of mathematics educators and teachers, the expertises of whom complement each other (see Ethell and McMeniman 2000, for teachers as expert practitioners). Accordingly, we build on the premise of teachers' active participation in all the phases of the framework and leadership in some of them.

The framework consists of four phases that are intended for spiral execution:

Understanding the teaching context The outcome of this phase is an analysis of teachers' resources, orientations and long-terms goals (Schoenfeld 2011). A particular focus of attention is directed towards the aspects that can contribute and hinder the integration of instructional videos in learner-centered lessons. Accordingly, at this phase the discussions of teacher educators and a teacher can be concerned with the technological infrastructure of the school, teacher's perceptions of students and their abilities, and ideas regarding how technology can assist a teacher in the lesson. In the discussion the teacher is exposed to various MOOC videos and possible scenarios of integrating them in the classroom.

Devising a plan for amalgamated lessons At this phase decisions should be made regarding the classes in which the chosen videos will be integrated, the mathematical topics of the lessons that will be carried out and the (short-term) goals of these lessons. Accordingly, the discussions between teacher educators and the teacher are focused on designing students' engagement with the videos and on the activities that precede and proceed the engagement. For instance, mathematical problems and tasks can be created for reaching the lessons' goals.

Carrying out the lesson This phase is concerned with collecting data on the actual teaching of the planned lessons. The data can contain video-recordings of the lessons, notes of the delivering teacher, participating students and observing educators.

Looking back The data collected at the previous stage are used for stimulating the reflection of the teacher and educators. The reflection is driven by two sets of questions. The first set is concerned with an experience of carrying out the planned lessons, and it contains such question as "What were the strength and shortcomings of the lessons? What should be preserved for the next time the lesson is carried out, what should be modified, and how?" The second set of questions reanalyses teachers' resources, orientations and long-term goals with the focus on the evolved technological, pedagogical and content knowledge. This set is targeted at consequal development of learner-centered lessons and engagement with the next cycle of framework implementation.

12.4 The Case of Veronica

Veronica is one of two teachers who have been recruited within a pilot study aimed at analysing how MOOC videos can been used into classroom practice. In the second half of the school year 2014–2015, the teachers met us several times, during the first meeting, a Pre-Calculus MOOC, realised by Polimi Open Knowledge, has been presented to them. Polytechnic of Milan has launched its own series of MOOCs, the Pre-Calculus one has mathematical content and it is specifically dedicated to students that aim at enrolling to the first-year university courses. The mathematical content reflects (and recap) the mathematical curriculum of the last years in high school.

In this paper we report on Veronica's experience in the first cycle of framework implementation.

12.4.1 Understanding the Teaching Context

In terms of resources, Veronica has a masters degree in mathematics and 30 years of teaching experience. She teaches in a technologically highly-equipped school and has experience in supervising technology-related projects. Veronica told us that in her teaching she often uses various technological devices, such as laptops and an interactive whiteboard, as well as software, such as GeoGebra and Wikispaces. When we asked her to explain how she uses them, it turned out that they help her in representing mathematical ideas to the class, spare time and ease on her communication with the students. Note that these implementations do not necessarily indicate a learner-centered environment in terms of Rogers (1983).

When we asked Veronica to explain her interest in integrating MOOC videos in her lessons, her response addressed her personal long-term goals and orientations:

> It is innovative in terms of didactic methodology, and it is an opportunity to use videos produced by experts to develop the teaching around the students, I am very willing to take part in this because it allows me to sharpen my teaching style, to improve myself, and to be involved in new challenges which are useful for studying new ways for reducing the gap between students and math.

As a part of a wider project concerned with the MOOC platform developed in the Politecnico of Milano, we exposed Veronica to a number of short video clips from the Pre-calculus course. Her reaction indicated some of her orientations: "It looks like a good way to foster the collaborative learning, which is also a keyword in the last ministerial directive". However, after we told her about possible scenarios for "flipping" the classroom, she said that not all of her classes fit for such learning due to students' lack of experience in working autonomously. She concluded that in any case, she will need to design new practices for integrating the videos.

Regarding her long-term goals related to students' learning, Veronica explained:

> I want my students to approach technologies and multimedia with critical thinking, they should be able to use software and applets properly, they should view this kind of videos, which are very dense, getting the main idea and being able to discern the details.

Based on the above resources, orientations and goals, we account Veronica for being a resourceful teacher, who is predisposed towards and experienced in integrating technology in her teaching.

12.4.2 Devising a Plan for Amalgamated Lessons

Veronica chose to carry out amalgamated lessons in two of her 11th grade classes. She described one of the classes as "strong" (Class A in the continuation of the paper) and said that its students "[. . .] are willing and motivated to work in groups, accept challenge, and deal with math problems". She addressed the second class (Class B in the continuation of the paper) as the one that needs much more guidance and described the students as “Not so open to innovations, they hardly accept something which is different from a traditional lesson”.

When we asked Veronica, why she had chosen such different classes, her answer was that she wanted to exploit her collaboration with us, the researchers, for learning how to integrate technology in different classes. We were inspired by this response and associated it with Veronica’s motivation for developing her own TPACK.

Veronica decided to develop two lessons (one for each class), the common (short-term) goal of which was to recall the properties of monomial functions (e.g.,) and their inverses? root functions (); students learned these topics in lower grades. She chose to use the same 6 min video clip for both lessons. In the clip, a lecture addressed the graphs and definitions of monomial and root functions for natural, integer, rational and real ’s. The root functions were presented as inverses of monomial functions, and the notions of oddness, evenness, and symmetry were briefly explained.

Veronica planned to ask the students of Class A to watch the video clip before the lesson and to answer a list of questions that addressed the definitions, properties, graphs and relations among monomial and root functions. In the lesson, she wanted to divide the class into small groups and engage them in solving a challenging problem. When considering candidates, we proposed Veronica the paper-folding idea, which is concerned with the geometrical progression that emerges when a piece of paper is consequently folded in halves. Veronica liked this idea because “it comes from the real life and is very mathematical. Moreover, it shows a non-trivial connection between monomial functions and geometric sequences”. She developed the idea into a problem presented in Fig. 12.1.

The students of Class B were planned to watch the video clip twice during the lesson. After the first time, the students create a table with concepts that are familiar and unfamiliar for them. Then, Veronica replays particular parts of the video clip to help the students with indicating the definitions of the key concepts. Afterwards, the students are divided into small groups and discuss the concepts of oddness, evenness and inverse functions. In group discussions the students are requested to sketch examples of such functions with GeoGebra and summarize the group work in writing.

Note that opposed to Class A that was planned to work autonomously, for Class B Veronica prepared elaborated guidelines. Thus, the developed plans are in-line with her orientations regarding the perceived capabilities of both classes.

> Consider a thin piece of paper. At the beginning fold it in half, then fold the folded paper in half, then again, and so on.
>
> a. How can we describe this situation mathematically?
> b. What will be the thickness of the folded paper after 100 folds, if the thickness of the original piece of paper is 0.1 mm?

Fig. 12.1 The folding-paper problem designed by Veronica

12.4.3 Carrying Out the Lessons

Veronica started the lesson in Class A with asking whether the students watched the videos and if they have any questions about the concepts. There were no questions, and the class turned to the group work on the Paper-folding problem. All the groups solved the problem correctly.

In one of the groups, the students attended the (b) part of the problem and asked Veronica how she came up with the number of 0.1 mm. Veronica redirected the question to the class and asked them to consider possible ways for determining the thickness of a piece of paper. The groups approached the question differently. One of the groups tried to measure the thickness with standard rulers. When a student in another group noted these unsuccessful attempts, she recalled that “there exists an instrument for measuring thin things, but I don’t remember how it’s called”. Then her group engaged in looking for thickness gauge in the Internet and exploring how it works. A student in another group connected between Veronica’s question and the given problem, and suggested to fold the paper again and again until the thickness becomes measurable with a standard ruler. His group liked the idea and engaged in developing a formula for the thickness of a piece of paper as a function of the measured thickness of the folded paper and the number of folds. Another student noted that this solution is not always realizable because the number of times that a piece of paper can be folded is quite limited (see Gallivan, n.d. for an exploration of this idea and an empirical proof for 12 folds).

In Class B after watching the video clip for the first time, Veronica invited the students to create a table of familiar and new concepts, as it was planned. The students asked to watch the videos again because they did not pay attention at the first time. After the second time, they still did not engage in filling the table and Veronica decided to change the planned lesson: she created a table for the whole class on the whiteboard and replayed particular parts of the video, around ten seconds each, with the concepts and properties that she considered important (e.g., image and domain, graphs). After each part, students elicited the concepts and Veronica explained them by extending the explanations of the lecturer in the video clip. This routine continued until the end of the lesson and it did not leave time for the planned group work.

12.4.4 Looking Back

Overall, Veronica reflected that devising amalgamated lessons is a time and effort-consuming endeavour for a teacher. However, she appreciated our support and especially the opportunities that we provided to yield what is important for her. In her words:

> I was worried a little bit that you are going to be invasive. So it really helped that I could work autonomously and do the best I can. Your advices and comments helped to make it better.

In regard to the lessons, she said that the students in both classes surprised her:

> One class accepted the activity as a challenge, the other one accepted it with resignation because they are used to learn in frontal lectures. Actually, I expected something like that, but the class [A] was more mature than I supposed. In class [B] many negative aspects stood out, the main one of which is that they did not complete the activity, were passive and not interested. They were even less autonomous than I imagined and definitely don't have enough critical thinking. Even though I wanted to guide them how to learn math differently.

We, in our turn, directed Veronica's attention to a paradoxical situation: On the one hand, she believed in the capabilities of Class A to work autonomously (see understanding the teaching context phase), so she trusted the students to prepare for the lesson, designed a problem that she perceived as challenging (see developing an amalgamated lesson phase), and even when she deviated from the developed plan, the lesson remained learner-centered (see carrying out the lesson phase). On the other hand, Veronica did not believe in the capabilities of the students of Class B to work autonomously, so she prepared elaborated guidelines aimed at coaching them to work autonomously. Even though the students were planned to work in groups, the guidelines were intended to govern their work step by step. Furthermore, her deviation from the plan turned the lesson into a traditional and teacher-centered, which is something that she so attempted to avoid.

Veronica thanked us for the reflection and said that she feels more aware of her own teaching. She added that in light of her experience she would like to make some changes and create more amalgamated lessons with us.

12.5 Concluding Remarks

In this paper we presented a collaboration framework, in which teachers and teacher educators design learner-centered mathematics lessons that integrate instructional videos. While the literature is rich with frameworks for pre-service teachers (e.g., Niess 2005), our framework appeals to in-service teachers who are predisposed towards integration of technology in their classrooms. In addition, the framework takes into account the teaching context of a teacher and considers her as an active collaborator. In other words, we attempted the framework to be learner-centered, when teacher is "in the shoes" of a learner.

Some might argue that we "push at the open door", in the sense that it is not especially challenging to carry out someone's predispositions. However, idiomatically speaking, our framework is concerned with a path behind that door rather than opening it. Moreover, according to Rogers (1983) people with compatible predispositions towards innovations (early adopters and early majority) consist around 66% of the population.

We illustrated the framework in work with the case of an experienced and knowledgeable teacher. As a result of framework implementation, the teacher crystallized her pedagogical resources, orientations and goals, was exposed to mathematical resources that were new for her, considered various scenarios for integrating instructional videos in her classrooms, designed mathematical tasks and reflected on her practice. Encouraged by teachers' self-report, we propose that these activities contributed to her TPACK.

The framework equipped us, the teacher's educators, with the lens that provided explanations for the emergence of some positive and negative outcomes of her teaching. Specifically, the framework enabled to indicate that while being skillful and willing to design learner-centered lessons, the teacher carried out such a lesson in the class that she perceived as capable of coping with one. In the class where she was skeptical about students' ability to handle such a lesson, she employed a teacher-centered approach and supposed that it will prepare the class for activities of a learner-centered type. We hypothesize that this "perception-action" pair creates a self-reinforcing circle in which "capable" classes seem to the teacher even more capable and "incapable" seem even more incapable. In the next round of framework implementation we plan to exploit teacher's own experience in a learner-centered framework for breaking the ill-fated circle for her students.

In the presented case, the technological component of each lesson was assimilated by the teacher to support the chosen learner-centered or teacher-centered approach. Accordingly, we, the educators who worked with this teacher, learned that integration of technology in the classroom does not necessarily promote learner-centered environment. Reinforced by this finding we are now working with additional teachers on designing amalgamated lessons, the goals of which go beyond recalling the previously learned materials.

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Chapter 13
Emotional Expressions as a Window to Processes of Change in a Mathematics Classroom's Culture

Einat Heyd-Metzuyanim

Abstract In this paper the lens of Symbolic Interactionism is used to examine the changing norms of a 6th grade classroom in which two teachers attempt to implement the ideas of "reform" cognitively demanding and discourse rich instruction. The findings point to tensions between the declared norms of "it's OK to be wrong" and the teachers' unreflective emotional alignment with students' embarrassments and frustrations that adhere to the old norms. The paper concludes with the importance of attending to emotions in situations of change to further teachers' success in implementing cognitively demanding and discussion-based instruction.

13.1 Introduction

In the last few decades, increasing efforts have been made to train mathematics teachers to shift their practice from teacher-centred, lecture-and-drill type of instruction to student-centred, dialogic and problem-solving based instruction (Boston and Smith 2011; Mercer and Littleton 2007). Almost all of these efforts have been focused on the cognitive aspects of instruction. Though many of those promoting "reform" type of instruction have stressed that such instruction necessitates changes in social or socio-mathematical norms (Yackel and Cobb 1996) and in the positioning of students vis-à-vis the teacher and the mathematics (Boaler and Greeno 2000), little research has examined how change from traditional to reform types of instruction actually takes place in the classroom. Even less research has focused on one of the most important aspects of such change: emotions and feelings, both of the students and of the teacher. The present study seeks to examine this process of change in a classroom of two teachers who participated in a professional development program and were attempting to change their instruction. Our goal is

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to examine the process of change in classroom culture through the lens of feelings and emotional expressions exhibited by the students and reacted to by the teachers.

13.2 Theoretical Background

As early as 1989, Paul Cobb and his team (Cobb et al. 1989) directed their analytical gaze to students emotional acts in the classroom. In a study of an "experiment class", where one member of the research team taught a 2nd grade math classroom for a whole year using constructivist, problem-based instruction, they documented the lack of embarrassment, shame and other negative emotional acts that was quite different than those they had witnessed in traditional mathematics classrooms. They claimed these positive emotional outcomes to be a result of the teacher's insistence on norms that: a. emphasized perseverance, thinking and effort and disregarded quick answers and b. explicit countering of any expressions of embarrassment due to mistakes mainly at the beginning of the year.

Even though Cobb and his colleagues' work on the importance of socio-mathematical norms has received widespread attention, their initial focus on emotions has had surprisingly little follow-up, neither in Cobb's group's works, nor in other studies of classroom instruction. Studies on emotional experiences of students have usually focused on problem-solving activities and looked at individual students (DeBellis and Goldin 2006; Op't Eynde et al. 2006), but not necessarily at interactional processes. A notable exception can be seen in the work of Evans et al. (2006) who linked between the discursive positioning available for students and their emotional expressions in small-group problem solving situations.

In line with growing evidence that cognitively demanding and discussion-based instruction (what I simplify here as "reform" instruction) results in better learning outcomes and positive mathematical identities (Boaler and Staples 2008; Schoenfeld 2014), substantial efforts have been made in the last two or three decades to train teachers in these reform practices. Yet these attempts have been challenging. Despite curriculum changes, professional development efforts and substantial financial investment, mathematics classroom practices are often still surprisingly similar to those practiced decades ago (McCloskey 2014). The reasons offered for these challenges have ranged from teachers' lack of appropriate pedagogical content knowledge (Ball et al. 2001) to inconsistencies in policy and structural settings outsides the classroom (Penuel et al. 2011). Almost no attention has been given to the complex process of instantiating change in classroom culture and to the emotions that this change elicits both in students and in teachers. In what follows, I turn to symbolic interactionism's take on emotions as a theoretical lens for examining such processes of change.

13.2.1 Emotions Within Symbolic Interactionism

Symbolic interactionism (Blumer 1969; Fields et al. 2006), a stream in sociology, claims that emotions are shaped by both culture and the human capacity to react to and make sense of our feelings. This activity of shaping and reacting to one's own and others' feelings has been termed "emotion work" (Fields et al. 2006).

A special emphasis is put within symbolic interactionism on those emotions that are often called the "social" or "self-conscious" emotions (Lewis 2008)—shame, pride, and embarrassment. With regard to the last Field states:

> Loss of face is an emotional experience – we feel embarrassed, guilty, or ashamed when we make a bad impression on others or fail to uphold our end of the social pact. Working together to save face keeps social life moving and maintains social institutions and patterns of interactions.(Fields et al. 2006, p. 157)

Methods for analysing these "face saving" acts have been developed within sociolinguistics (Brown and Levinson 1987; Goffman 1956). These methods tend to minute details of talk, mostly implicit norms of conversation, that enable participants in conversation to save each other from embarrassment.

The lens of symbolic interactionism and socio-linguistics will be used in this study to examine through student emotional expressions and teachers' emotion work, the underlying norms of the classroom and the tension between old norms of traditional teacher-centred classroom cultures and the norms of discussion-based, dialogic and student-centred classroom cultures.

13.2.2 The Context of the Study: The 5 Practices and Accountable Talk

The present study comes on a background of a year-long professional development (PD) program for middle-school mathematics led by the co-PIs of this study, Margaret Smith and Victoria Bill of the Institute For Learning (IFL), University of Pittsburgh. The PD was centered mainly on the "5 Practices for orchestrating productive mathematical discussions" (Smith and Stein 2011). In a nutshell, these are practices for selecting high-cognitively demanding tasks, anticipating the different solutions paths that would be attempted by students (both correct and incorrect), monitoring students' progress, sequencing the different solutions strategies to be presented on the board and helping students link between the different solutions to form a more robust understanding of the mathematical concept at hand. If implemented well, cognitively demanding instruction is supposed to produce a change in classroom norms, that is, change in the meta-discursive rules that govern what is supposed to be said and who is supposed to say it (Sfard 2008). Instead of the teacher "telling" and the students "following", such norms dictate that students' should be the ones authoring the mathematical narratives in the classroom, while the teachers' role is to facilitate discussion and to lead it towards important mathematical ideas (Smith and Stein 2011).

Observing the participating teachers' efforts to change classroom norms, we saw them often tripping over difficulties that they were not fully aware of and that had to do with implicit messages and minute emotional interactions in the classroom. To examine this closely, I chose to focus in the present case study on these emotional interactions in one classroom. My question was: what can emotional expressions in the classroom, and the implicit messages they convey, tell us about teachers' attempts to change the norms in their classroom?

13.3 Method

The study included seven middle-school classrooms in an Urban district in Eastern US. The teachers were part of a larger group participating in the "5 practices" PD described above. The PD was initiated by the district and supported by a state grant.

In this paper, I focus on one 6th grade classroom which was taught by two teachers. Ms. Andrews, a general education teacher with 6 years of experience out of which 3 years were in teaching mathematics; and Ms. Jacobs—a special education teacher that had been working together with Ms. Andrews for 6 years. Both teachers co-taught the classroom at all times and Ms. Jacobs tended specifically to the special ed. students only in separate learning periods. Both Ms. Andrews and Ms. Jacobs attended the PD and both were very diligent in applying what they had learned to their classroom, their classroom showing marked changes in the discourse structure throughout the year. Both teachers reported changes in their practice at the end of the year and were very enthusiastic about the PD. These features, in addition to rich data that had been obtained from their pre- and post-lesson interviews, led us to focus on the classroom for the present case study.

Ms. Andrews and Ms. Jacobs' class included around 25 students (some left or joined during the year). Eleven of them were identified as special education students receiving an IEP (Individualized Education Program). The school was a low-performing school in an urban district and almost all of the students in the class had not passed the previous year's standardized State Exams in mathematics.

The classroom was video-recorded 4 times during the year. Each session included a pre- and a post-conference with the two teachers and a video recording of a 45 min lesson, as well as introductory and summary interviews that were held separately with each teacher. The classroom lessons were audio and video recorded with audio recorders on each desk and two video cameras.

All the lesson and interview recordings were fully transcribed. From these transcriptions, we first searched for occurrences of talk about emotion or points where the mathematical talk (mathematizing) turned to talk about the students (subjectifying) (Heyd-Metzuyanim and Sfard 2012). We did not, however, include in these behavior management talk, such as when the teacher stopped to reprimand a student for being off-task or the like. After this initial scan, we turned back to the video recordings, searching for emotional expressions of students during classroom discussion.

In addition, we scanned for all the instances in the interviews where the teachers referred to students' emotions or to feelings. In a few instances, the teacher and interviewer's reflections in the post-conference about a student's feelings led us back to the video recordings to watch more closely the classroom situation that was talked about in the interview and that would not have captured our attention since the emotional expressions perceptible to the camera were so mild.

13.4 Findings

The scope of this paper does not allow me to present all the critical emotional points that were identified in the lessons. Instead, I bring one episode that exemplifies the emotional tensions viewed in this classroom.

13.4.1 "Roger I'm Not Trying to Pick on You"

This episode is taken from the second lesson, in which the students were given the following task:

Jonny, Jeremy and Dorothy were playing another round of the card game. Their scores this time were as follows: Jonny: -1, Jeremy: -2 and Dorothy: -4. Who won the game?

The students worked on the problem in groups while the teachers circulated between them. Then, the teachers called several students to present their solutions. First was called Robert, who claimed Dorothy (who had -4 points) had won.

231 Roger:	I think the winner is Dorothy, because 4's higher than 1 and 2.
232 T.A:	Okay… (Roger mumbles) Anything else?
236 Roger:	Even though 1 is closer to the 0, it's still bigger than … it's still … 4 is bigger than 1, even though it's farther away from the zero.
237 T.A:	Okay. Alright, so let's look at our sentence stem, or if you have a comment of your own. … So, Roger you can call on someone that has their name up, or hand up.
238 Roger:	Dawson.
239 Dawson:	I'm wondering why you - why - I'm more - I'm curious why you thought that Dorothy won.
240 Roger:	Because 4's bigger than 1.

T.A. asks for other opinions. T.J jumps in and explains there was a debate between Roger and other students in his group and that Roger convinced his friend

244 Dawson:	Where - one - wouldn't it be Jonny, 'cause 1 is closer to the 0?
245 Std A:	Yeah.

246 Std B:	But how is it -
247 Roger:	But 4 is bigger than 1

At this point Roger was thanked and sent to his seat and Leonard was called to the board. Leonard put his sheet on the overhead and said:

> I put ‘Johnny won because whatever his score was, he only lost one point which was negative 1’.

Andrew agreed with him: “Because, the other day we talked about – if you want to owe Ms. Jacobs – whether you wanted to owe Ms. Jacobs 4 dollars or 1 dollar? And if you owe 4 dollars, you’re losing 4 dollars from yourself”.

After having clarified Leonard and Andrew’s claims, Ms. Andrews turned back to Roger:

300 T.A:	Roger, what do you think about that?
301 Roger:	(mumbles) [I guess you were right, and] [Indistinct]
302 T.A:	What?
303 T.J:	Roger, I want to point out – I’m not trying to pick on you, there were a lot of students who came up with the same thing. I just chose you to come up. But, do you still think Anabella won, or are we starting to sway you a little bit, Roger?
304 Roger:	’cause…
305 T.A:	You’re not sure?
306 T.J:	Alright, so who else? [Laron]?

The most noticeable moment in this episode, in terms of emotion work, is in line 303, where Ms. Jacobs abruptly turned from the mathematical discussion and from queries that had to do solely with Robert’s “thinking” to a statement about what she was (or was not) doing to him while asking him these questions. Analyzing this moment through symbolic interactionism means looking at the meanings used by each of the participants as a basis for their emotion work. With regard to these meanings, one can see that Ms. Jacobs intrusion may have not been necessary at all. Roger did not show any observable signs of embarrassment or distress besides, perhaps, a lowered tone of voice and some mumbling. Ms. Jacobs’ remark was, thus, a result of her interpretation of this moment, as potentially embarrassing for Roger. This interpretation was so strong for her that she thought it worth halting the mathematical discussion and diverging everyone’s attention from the mathematics to the social positioning of Roger.

The symbolic-interactionist lens further directs our analytical gaze towards the implicit assumptions and messages underlying Ms. Jacob’s face-saving action. First, Ms. Jacobs’ choice to frame the explanation as “I’m not picking on you” means she was expecting Roger to interpret the situation as one in which she was picking on him. This means that Ms. Jacobs was aware that for many students (such as Roger), a situation where they were asked to defend their argument in front of the class would not be interpreted as a neutral situation, having to do solely with cognitive argumentation of what is right or wrong mathematically, but as a socially inappropriate situation where they were condemned or reprimanded for being "wrong". Such an interpretation of the situation is very much aligned with traditional classroom settings in which students are expected to produce correct answers to teachers' questions. Interestingly, Ms. Jacobs did not tackle this interpretation directly. She did not say something like "Roger, you are entitled to have your own opinion, as long as you justify it" or a similar statement that would make the desired norms of classroom discussion explicit. Rather, she justified her claim that she was "not picking" on Roger by saying "there were a lot of students who came up with the same thing. I just chose you to come up". This justification was more aligned with the old rules of the game since its implicit message was perhaps you are wrong but many others may be wrong too. Such a justification alleviates somewhat the embarrassment of being wrong by distributing it between other students, but it does not explicitly undermine the interpretation that being wrong is not a cause for embarrassment. On the contrary, it somewhat strengthens this interpretation by aligning with the assumption that there is, in fact, something to be embarrassed about.

13.4.2 Teachers' Emotional Struggle with "Not Telling the Answer"

The duality between the teachers' explicit attempts to instill the norm that it was "OK to be wrong" and their alignment with interpretations that contradicted this norm was evident in another major emotional struggle of the teachers—the one we termed "the struggle not to tell". This could be clearly seen in both Ms. Andrews' and Ms. Jacobs' reflections after the first lesson.

> Well I think the hardest thing for me is just that I, I want them to know the answer so I want to tell them and I'm trying to not tell them but sometimes I find myself saying things that I [offer] and [I'm like] "Oh, I really shouldn't have given them that hint."

Ms. Jacobs expressed a similar sentiment:

> Yeah me too, like I really want them to . . . just tell them how to do the ratio table because they were struggling with it. (2nd lesson pre-conference, December 2014)

Again, underlying this struggle between the students that request help from the teachers and the teachers' resistance on the ground that "they shouldn't be telling" were some interpretations that conflicted with the goals for learning that the PD was promoting. For instance, Ms. Andrews explained she wants the students "to know the answer" and "to know the information", goals that are aligned with the value of "being correct" and with repeating knowledge that has been passed on by an external authority rather than thinking for oneself and coming up with one's own ideas. Ms. Jacobs explained her wanting to "just tell them" with "because they were struggling with it", hinting that she was interpreting the situation as frustrating and anxiety provoking for the students. Again, such frustration is justified on the ground of norms that value "being right" over independent thinking.

The above emotional and unreflective interpretations of the teachers are especially significant because of the teachers' insistence, in multiple occasions during the lessons, on stating that "it is OK to be wrong". This insistence, they reported at the end of the year, had mixed results.

> And even though we've tried to establish a good place to have an open conversation, they still don't want to be wrong. I think it's the age, too. I think they just don't want to be wrong, so it's just easier to say "I don't know", because then I can't be wrong, versus if I tell you where it (the solution) came from and someone disagrees with me, then – I'm wrong and that's embarrassing. (Ms. Andrews, Final Interview, May 2015)

Ms. Jacobs, on the other hand, did see improvement in this respect:

> So I think our initial concern was that there wouldn't be discussion, because basically no one would be able to come up with the right answers, or that they would be? the kids would be embarrassed, but – I think that's a lot of how you set up the [atmosphere] in your class. Of "it's okay to be wrong" and we stressed that a lot in the beginning of the year. And they just became more comfortable with it. (Ms. Jacobs, Final Interview, May 2015)

13.5 Discussion

The case of Ms. Andrews and Ms. Jacobs' classroom proves that a focus on emotions can point to details and complexities in the process of change that may have otherwise gone unnoticed. Overall, these teachers made significant efforts and their classroom culture did change, as evidenced by their own accounts and by measures of Accountable Talk moves that increased considerably throughout the 4 lessons observed during the year. Yet the reluctance of some students to "be wrong" and the teachers' own interpretations with regard to moments of error and confusion show that the process was far from being straightforward and was in a state of negotiation between old and new norms. The old norms being *mathematics learning is a business where you ought to be right* vs. new norms being *mathematics learning is a business where you ought to think for yourself, convince others and justify your arguments*. Out loud, the teachers were encouraging the latter, new norms. Emotionally, however, they were often aligning themselves with the old norms, either by succumbing to the students' emotional reactions or by doing emotional work that was aligned with these old norms.

None of these tensions, however, seemed to be in the teachers' awareness. This unawareness is in line with former findings showing a teacher's emotional reactions and her succumbing to students' embarrassment may be a totally automatic and unreflective process (Heyd-Metzuyanim 2015). This points to the necessity of unearthing these emotional processes and talking about them explicitly with teachers.

Teachers should learn that the process of change in their classroom culture has an inherent tension built into it with regards to emotions. Emotions rise from change of reality from expectations and change in norms necessarily envelopes such a change. They should therefore be prepared to deal with emotions of embarrassment and frustration that are aligned with "the old rules of the game" and learn how to directly tackle them and how to control their own feelings of empathy to students' frustrations.

The focus on emotions also problematizes the views of teachers as agents of change in the classroom. Although many view teachers as having full agency on instantiating and changing classroom norms, the analysis of emotional interactions in the classroom shows that this view may be somewhat naïve. Though teachers can indeed learn new practices and exercise them in the classroom, the process by which classroom culture changes is beyond the sole control of the teacher. Involved in it are all the prior experiences the students bring with them to the classroom, which result in their emotional reactions to the change that the teachers are trying to instantiate. This may be the reason that the 2nd grade teacher in Cobb and colleagues' study (Cobb et al. 1989) was able to succeed by stating explicitly rules of what should be considered as embarrassing and what is not while the teachers in our own study refrained from such explicit statements about feeling rules.

While illuminating some points for thought, the present study also has a few limitations, mainly regarding the methods of inquiry. We did not have access to students' accounts of their own subjective experience, neither were we able to elicit many accounts of emotional experiences from the teachers. The first constraint would probably be amenable with different Institutional Review Board constraints. The second, however, is much more complex. It was difficult to talk with teachers about emotions, especially negative ones such as embarrassment and shame. Teachers usually preferred talking about the cognitive and mathematical actions of students. Talking about emotions such as embarrassment and shame in their classroom seemed to be threatening their perceptions of themselves as good teachers and unnecessarily intruding or overly "psychological". It is therefore necessary to devise further tools and interview protocols that would bypass these defence mechanisms.

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Chapter 14
Mathematics Teachers' Conceptions of the Classroom Environment

Magnus Fahlström

Abstract This study explores mathematics teachers' conceptions of how the physical environment in classrooms affects their students' chances for learning. Semi structured interviews were performed with a few Swedish teachers with experience from tackling different physical settings when teaching mathematics. When analysing the interview transcripts preliminary findings are that: teachers appreciate flexibility and control over the physical settings in the classroom; inadequate acoustics are extra problematic in mathematical activities involving verbal interactions between students in small groups; mathematics task solving in peace and quiet is a common part of mathematics lessons and it easily gets disturbed by external noise.

14.1 Introduction

When learning happens it takes place in an environment. The teachers in charge of the learning situation for the students in the classroom face different settings in the physical environment. The research body on the physical environment's effect on humans is substantial. When basic single factors have been examined, most of the results are conclusive. Looking at research on the effect of physical school environment on learning a common start are basic fundamental aspects of the physical environment. These fundamental aspects of the physical environment are: temperature and air quality, lighting, noise and acoustics (Earthman 2004; Schneider 2002; Weinstein 1979). Moreover, the teachers' influence and control over the physical settings are crucial for their perception of the educational value of the classroom environment (Uline et al. 2009). The importance of the physical learning environment has been known for a long time, for instance, Maria Montessori emphasizing that inadequate stimulus from the physical world will stress the senses in a non fruitful way (Montessori 1914). Still little is known about what is subject specific in the physical context, the objective of this study is to explore mathematics teachers' conceptions of the physical classroom environment. The research question posed is:

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What factors can be identified and what are their effects, when studying mathematics teachers' conceptions of the physical classroom environment's impact on their students' learning in mathematics? Learning here is considered to be the intended outcome of the activities the teachers engage their students in. Conception is defined as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Philipp 2007, p. 259).

14.2 Background

In this section some research on teachers' conceptions of physical school conditions will be presented together with a brief review of the research on the physical indoor settings in schools and their effect on students' learning outcome.

14.2.1 Teachers' Conceptions

In a study on teacher attitudes about classroom conditions the researchers studied two groups of teachers. One group working in buildings rated as in good condition and one group working in buildings rated as in poor condition. They found significant difference between the two groups of teachers. The attitudes of teachers in buildings rated good were more positive than the teachers in buildings rated as poor. On the specific question of support or hindrance to teaching almost half of the teachers in buildings rated in poor condition agreed to the statement that the classroom hindered their efforts in teaching. Nearly 80% of the teachers in good buildings disagreed to the same statement, but more than 70% of them agreed to the statement that the classroom helps their students learn (Earthman and Lemasters 2009). The size of schools is also a factor; smaller schools are linked to more positive teacher attitudes towards teaching in general (Schneider 2002). Another research team performed a case study of one rural and one urban school with high rated school facilities with a large part of the student population coming from low socioeconomic background. The researchers report that the positive perceptions of the school found among students and teachers in high rated school buildings appeared to support students' academic learning there. Further findings are that teachers value the flexibility to arrange the space in their classroom in various ways depending on activity (Uline et al. 2009).

14.2.2 Temperature and Air Quality

The concept of human comfort zone for temperatures is thoroughly researched and used in many studies. Research reveals a strong correlation between student achievement and comfortable temperature as well as student absence and poor test results are correlated to problems with indoor air quality (Earthman 2004).

14.2.3 Noise

There is quite a substantial amount of research on the effect of noise on humans and a great part of that is on children learning in noisy environments. Early studies focus on chronic or acute external noise for example from road traffic, trains or aircrafts (Ising and Kruppa 2004). One study looked at how sudden bursts of noise lead to a reduction of effective teaching time (Weinstein 1979). The most common distraction in schools comes from irrelevant speech. The reason for the distraction is the mental resources diverted from the primary task to decode the irrelevant speech (Knez and Hygge 2002).

14.2.4 Lighting

Many studies have shown a positive correlation between proper lighting and higher student achievement (Earthman 2004). There is a great deal of research on different kinds of lighting, from daylight to a variety of artificial lighting strategies (Benya 2001). As a result of more use of computers in schools glare- and flicker-free lighting is needed (Barnitt 2003).

14.2.5 Combined Effects

There is not as much research in the combined effect of physical factors than single factors (Higgins et al. 2005). There is often contradicting recommendations, for example air quality supporting systems often make noise and sometimes the artificial lighting introduces a buzzing noise (Earthman 2004). When studying noise, temperature, and indoor lighting, both synergetic and antagonistic effects were found between the factors (Hygge and Knez 2001). The issue of relying on subjective or objective indicators when it comes to assessing the effects of the indoor environment was examined by a team of researchers. They found that subjective indicators are better predictors of overall indoor comfort (Fransson et al. 2007).

14.3 Method

There are few studies in the area of mathematics teachers' conceptions of factors in the physical setting in school that potentially have an impact on students' opportunity to learn mathematics as intended. Here a qualitative approach by semi structured interviews was chosen. In order to conduct the interviews an interview guide was designed and developed. The guide was tested in a couple of pilot interviews. From those tests it was concluded that some questions had to be more specific. After fine-tuning those questions the procedure for the interviews was finalized. Since there is no quantitative claim in this study there was no random sample selection. Purposive sampling was used, trying to cover the school years in the public Swedish school system. The interviews were carried out with two teachers teaching school years 4–6, two teachers teaching years 7–9, and three teachers from upper secondary school. In all, seven teachers were able to participate during the data collecting period of this study. Their teaching experience spanning from 1 year to 18 years, adding up to a total sum of 80 years. The interviews took place in neutral places chosen at convenience for the interviewees, lasting 30–60 min. In the previously developed guide the procedure for the interview is defined. The semi structured part of the interview starts with the interviewer reading the exact wording of the research objective to the interviewee in order to get a uniform base for all of the interviews. In the next step the interviewees are asked to tell about teaching situations in general, which they perceived as disturbing or distracting for themselves or their students. The purpose of this initial open general question is to get some topics to pursue for the interviewer with questions like: "how do you mean when. . . ?" The respondents are then asked to tell about specific situations in relation to disturbance or distraction, a question aiming to produce narratives related to the objective of the study. Question words like: *how*, *when*, *who*, and *by what* are an aid when trying to get as rich data as possible. To balance this, in the last phase of the interview, they are asked to tell about successful teaching situations where there was no disturbance or distraction. Besides balancing, the reason for this is that other factors and effects may arise in conceptions of non disturbing/distracting situations compared to the first phase concerning disturbance/distraction. During these two main phases, follow up questions are posed to clarify and specify what is specific for mathematics teaching and not. There was no direct question about the teachers' perception of the status of the classrooms they taught in, but the interview transcripts indicate that it ranges from poor to good status. The interview transcripts were analysed by narrative analysis in the assumption that the interviews had produced narratives. Narratives can be defined in a number of ways. Andrews et al. (2008) have compiled an overview of narrative research theories and branches. When responding, the teachers often tell of experiences from several situations through semi fictional or pseudo events. To handle these elicited pseudo events, narratives are considered to be stories of experience rather than events Andrews et al. (2008). The actual practical analysis work started with several read through of each transcript. Keywords related to basic physical factors and disturbance/distraction was highlighted as in directed content analysis (Hsieh and Shannon 2005). In the second stage of the analysis the text around these keywords were examined for small picture narratives, i.e. situations or thematic categories. During this process, text passages that consisted of stated or perceived causalities where coded as important text chunks and labelled with a category. In the final stage of the analysis the thematic categories from all transcripts were grouped in a large scheme. Those groups were categorized in higher level themes as in conventional content analysis (Hsieh and Shannon 2005). These categories constitute the headlines for the subsections of the results section.

14.4 Results

In the following some of the preliminary results from the analysis of the interview transcripts will be presented under headlines corresponding to the higher level themes that were the results of the analysis. The excerpts from the interview transcripts are translated from Swedish to English. In the translation the content of the excerpts was the main goal rather than the exact phrasing. In the transcripts R*n* represents respondent number *n*.

14.4.1 Mathematical Activities

When students interact with each other verbally in smaller groups the acoustics and the layout of the classroom are important. As respondent number one puts it:

R1: Well, when they work with problem solving themselves, maybe in groups of two or three so that everyone gets engaged.
Interviewer: Ok [confirming].
R1: In these situations it can happen that someone gets disturbed by the sound from other groups and looses focus if the noise level in the classroom gets too elevated.
Interviewer: Ok [confirming].
R1: In those cases I wish for a cosier classroom with special corners so that some groups could be screened of a bit for those in need of that.

Or as respondent number three says:

R3: If we take mathematics as a subject of discussion—we want to discuss mathematics in groups—in some groups it works great—you have focus on the task—you investigate—you compare—and in a different group—it simply does not work.
Interviewer: Ok [confirming].
R3: Yes. Because I think that a classroom that provides a calmness—a calm impression—makes students who loose focus easy and like—for some students a certain classroom can be perceived as messy.
Interviewer: In what way is it messy?
R3: Well, it can be different types of chairs, desks and a lot of things on the walls that gives a messy feeling.

Individual work in mathematics text books is easier disturbed by external noise since the students work in total silence according to respondent number four:

R4: So specific for math class is that a greater part of the time students work alone with individual tasks. That is also why I feel that you get more disturbed by noise because it is peace and quiet most part of the lesson.

When instructing and lecturing in mathematics at the white board, proper lighting is needed for the students to se every detail. Respondent number four tells of the following experience:

R4: I taught mathematics in a temporary building for 1 year. There I know that the lighting was no good. It was a white board on the wall, but it was not illuminated with directed light, which led to that the students sitting a few rows back could not fully se what I had written.

On the matter of white board respondent number seven walks up to the whiteboard during the interview and pulls down the projection screen beside the white board and praise the positioning of it in relation to the white board:

R7: I use to show example tasks in mathematics on the screen and solve them on the white board next to it. That is good for the students because they can se the task here in front of the classroom and the students don't have to look down in the text books.

Summary Mathematical Activities Poor acoustics is a factor with disturbing noise as the effect whether the mathematical activity is the primary sound source or the activity is performed in silence and the sound source is external. Tidiness of the classroom is a factor with a calming effect if it is good or disturbing effect if it is messy. Poor lighting is a factor with impaired vision as the effect and good layout of the front of the classroom is a factor with an aiding effect for the students' visual input.

14.4.2 Internal Factors

The size of the classroom has several impacts. To have the opportunity to keep extra workplaces for different purposes is something that is desired:

R5: Yes, the ideal would be to have a seating like this [slightly skewed rows] and then also to have some places for group work in the room so that you can move and choose.

Just not to have a crowded classroom is a common wish:

R3: It is obvious that the students are affected by sitting cramped and getting everyone very up close.

A crowded classroom makes it difficult for the teacher to walk up to and reach each student and the comfort for the students is impaired:

R7: To be able to move around and get close to the students without any obstacles in order to help them is definitely something on the checklist.

Poor air quality is not unusual with crowded classrooms:

R3: Sometimes you walk in to a classroom where there have been many people and you feel that there is no oxygen left.

One secondary problem from the ventilation system is sound that travels in the ventilation canal between classrooms:

R2: Our air ventilation system does not make much noise, but it is inefficiently sound insulated. If they watch a movie or something in the adjacent classroom then we can hear it in here.

Other indoor noise that travels can be in older multi-level buildings. Especially noise from chairs and desks being moved in the above classroom is a distractor:

R4: You have class and students work silently and then you get disturbed by chairs and tables being moved in the classroom above.

The acoustic insulation within classrooms and the soundproofing between them are important. The sound of students from other classes on a break can be a problem that gets worse with inadequate soundproofing:

R2: Something I believe that one can be disturbed by for example are, sounds from the corridor outside and the locker halls.

The doors to the classroom are an important factor for reducing noise between adjacent areas in the school building:

Interviewer: So the sound got through?
R1: Yes, through the door that was between the classrooms.

The sound absorption inside the classroom is crucial when students are supposed to interact verbally during class:

R1: If you have poor classrooms it does not take much noise really before it feels disturbing for some individuals.
Interviewer: Poor classrooms, in what way?
R1: Yes, worn down and lack of sound insulation perhaps.

Poor lighting is rarely mentioned, but when mentioned it concerns illumination of the whiteboard:

R7: There is no directed lighting above the whiteboard. I have told that to the maintenance but they still have not mounted it.

The good amount of daylight that comes inside the classroom is honoured by a teacher that has it:

R5: But I have it very well suited with daylight here in my classroom. And I find that very positive.

The furniture is mentioned several times. First and foremost the teachers want uniform, durable and ergonomic chairs and desks:

R1: And then they have to be able to sit in an ergonomic adequate way.
R3: And then it shall look a bit inviting when you enter a classroom, no litter and not different types of chairs and desks.

One respondent who works in a newly furnished classroom honours the extra height of the students' chairs and tables they have got. The fact that the teacher need not bend that much when instructing single students thus saving the back is the main benefit:

R7: These tables and chairs that we just invested in here, they are brand new. They are good in the sense that they are quite high. The students sit higher and I don't have to bend as much.

Summary Internal Factors Acoustics is present as a factor here as well with effects analogous to the former theme, but contrastingly related to activity outside the classroom by others instead of the internal mathematical activity inside. The size of the classroom is a factor. If the size is generous the effect is flexibility and control over seating, activity, and movement in the classroom. The opposite effects come from a crowded classroom together with the possible effect of poor air quality. Lighting and tidiness are also factors here with daylight added as a factor that has an overall positive effect. Ergonomic furniture is a factor with the effect of minimizing discomfort.

14.4.3 External Factors

The school buildings are often placed so that road traffic and train traffic will not be a problem to the school:

R4: For example car traffic or train traffic or something, in my experience, we do not have any problems with that here.

Reported issues from external sources are noise from other kids playing outside the classroom, visual distraction from activities outside classroom windows, and sunlight that makes it to hot inside the classroom:

R7: Someone has placed a playground outside the window here.
R4: What are they up to now then? Are they making a new sidewalk over there?
R4: In the summertime we have asked for sun protective film on the windows.

Privacy control and sun blocking window tints are often used to solve the issue of visual distraction and sun heating:

R2: Yes, we have privacy window tints on all windows facing outwards now and it has made big difference.

The noise that comes from outside is very much controlled by the quality of the windows:

R4: Where we have mathematics, we have good windows and doors.
Interviewer: For soundproofing you mean?
R4: Yes, that is what I mean.

A combined problem in this circumstance is that when it is to hot, windows are opened and then the noise gets in together with the desired cool air:

R7: And then when you air the classroom, there is sound from children playing outside the window.

Summary External Factors Many of the factors in this theme have the effect to cause the need for factors found in the previous themes. For instance the location of the school determines the amount of external soundproofing needed. The location of the school is also related to the factor visual distraction through windows, not mentioned earlier, with an effect to create a need for privacy window tints. Daylight is mentioned as positive earlier and it comes from the sun. The sun is a factor with the possible effect of blinding light and causing an uncomfortable temperature raise. A following effect of this is the need for sun protective film on the windows.

14.5 Discussion

The research question asked for factors and their effects in mathematics teachers' conceptions of the physical classroom environment. All the basic factors in the physical environment are present in the result and those are inline with Earthman (2004), for instance. Their effects are disturbing or distracting in some way alternatively imposing the need for counter measures. All factors are related to some activity by someone or something outside the classroom or inside the classroom. Factors related to the mathematical activities present in this study vary depending on the type of the mathematical activity. It is testified that when the teacher is instructing at the front of the classroom proper lighting is important together with good whiteboard and screen positioning in order for the students to see what is written and displayed. Mathematical activity and communication are very symbolic at their nature and therefore extra sensitive for poor vision. During individual work some student might need individual support from the teacher. In that case the classroom size and seating arrangement affects whether the teacher can reach the student without obstacles or not, according to present findings. Several external sources of distraction or disturbance emerged in the results when the mathematical activity is silent individual task solving. The fact that working (i.e. thinking) in total silence is extra sensitive for disturbance is partly related to previous research, where cognitive (i.e. mathematical) tasks were easier disturbed in silence (Knez and Hygge 2002). The internal acoustic insulation in the classroom is an important factor when the primary sound source is the verbal interaction inside the classroom.

For instance, if the activity is mathematical problem solving in groups, inadequate sound insulation might impair the intended learning outcome of that activity. There is a distinction here though: It is not necessarily bad if the groups can hear fragments of mathematical communication from other groups. What is reported in the results is that when the noise level gets too elevated, the aim of the activity is at risk. The results also reveals a wish for the ability to adjust the classroom for all these different activities and needs, for instance rearrange the seating and create some partitioning of the classroom. Additional factors, also with the possible effect of promoting the ability to achieve this flexibility to switch between these mathematical activity types are: comfortable, uniform, and adequate furniture together with non crowded classrooms where the students have enough space around themselves. These results are quite rich of information of factors and effects found in seven mathematics teachers' conceptions of the physical classroom environment. In order to do a more general identification of factors and effects a larger sample is needed. Nevertheless, the dense content of these seven teachers' conceptions related to the physical classroom environment indicates the importance of studies of this kind as a complement to other research about physical environment. Finally, these different types of mathematical activities that emerged to plot the factors and their effects in the teachers' conceptions also signal diverse beliefs of how mathematics education is conducted.

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Part III
Understanding the Undercurrents: Tensions, Inconsistencies and the Social Turn

Chapter 15
Teacher Tensions: The Case of Naomi

Annette Rouleau and Peter Liljedahl

Abstract Tensions are endemic to the teaching profession. Viewed as dichotomous forces, tensions shape the experiences of mathematics teachers, affecting both their practice and professional growth. In this article, we identify and examine some of the tensions experienced by Naomi in her practice of teaching mathematics. While previous research presents the image of teachers as dilemma managers who accept and cope with continuing tensions, our research suggests that a desire to resolve these tensions may impact teaching practice and professional growth needs.

15.1 Introduction and Theoretical Background

Teachers are often faced with dilemmas. Lampert (1985) suggests that these dilemmas arise because the state of affairs in the classroom is not what the teacher wants it to be. Conflicts can surface when mathematics teachers encounter a disparity between what they want to do and what they are asked to do, or between what they want to do and what they know how to do. These competing influences create what Adler (2001) and Berry (2007) refer to as teacher tension and encompasses the inner turmoil teachers experience when faced with contradictory alternatives for which there are no clear answers. Considering that tensions are endemic to the teaching profession, mathematics education would benefit from (1) identifying these tensions, (2) understanding how teachers cope with these dichotomous forces, and (3) examining how this impacts their teaching practice and professional growth requirements. This paper is part of a larger qualitative study regarding the tensions inherent in the teaching of mathematics, in which we intend to focus on all three areas. Extending the work we began in Liljedahl et al. (2015), the goal of this particular paper is to explore a framework for identifying and examining these tensions, in order for us to better understand the wants and needs of mathematics teachers.

Whether categorized as personal, practical, or contextual, identifying tensions within the practice of mathematics is beneficial in providing a language for

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discourse (Adler 1998; Ball 1993; Barbosa and de Oliveira 2008). Indeed, Adler (1998) refers to the language of tensions as "a powerful explanatory and analytic tool, and a source of praxis for mathematics teachers." (p. 26). Naming and exploring the tensions can present a view of teacher thinking that is broader than "decision making" (Lampert 1986). To highlight this, Lampert (1985) shares an illustrative example of the tension she experienced upon choosing where to sit her students during mathematics lessons. No matter which arrangement she chose, it would be to the detriment of some of her students. Outwardly appearing as a simple "decision", the thought process entailed in managing her tension demonstrates the complexity involved.

For Lampert (1985) then, tensions came to be seen as problems to be managed rather than solved. In her study, Lampert (1985) examined how teachers cope with conflicts. She introduced the notion of teachers as dilemma managers who accept conflict as endemic, and useful in shaping both identity and practice. In doing so Lampert (1985) echoed John Dewey (1922) who tells us that:

> Conflict is the gadfly of thought. It stirs us to observation and memory. It instigates to invention. It shocks us out of sheep-like passivity, and sets us noting and contriving. Not that it always effects this result; but that conflict is a 'sine qua non' of reflection and ingenuity (Dewey 1922, p. 301).

This suggests that the tensions experienced by teachers can be both useful and utilized. Once acknowledged and identified, they can become a source of reflection and praxis.

Berry (2007) agreed and a framework to identify, understand and utilize the tensions inherent in teaching emerged from her work. As a former teacher who moved into the role of a teacher educator, she completed a self-study of her practice in order to improve her understanding of the process of learning to teach teachers. Building upon the work of Adler (2001) and Lampert (1985), she utilized the notion of tension as a framework for both doing and understanding her research. The result was twelve tensions expressed as dichotomous pairs that "capture the sense of conflicting purpose and ambiguity held within each" (Berry 2007, p. 120). Noting that these tensions do not exist in isolation, she used their interconnectedness as a lens to examine her practice:

1. Telling and growth
   - between informing and creating opportunities to reflect and self-direct
   - between acknowledging prospective teachers' needs and concerns and challenging them to grow.

2. Confidence and uncertainty
   - between making explicit the complexities and messiness of teaching and helping prospective teachers feel confident to progress
   - between exposing vulnerability as a teacher educator and maintaining prospective teachers' confidence in the teacher educator as a leader.
3. Action and intent
   - between working towards a particular ideal and jeopardising that ideal by the approach chosen to attain it.
4. Safety and challenge
   - between a constructive learning experience and an uncomfortable learning experience.
5. Valuing and reconstructing experience
   - between helping students recognise the "authority of their experience" and helping them to see that there is more to teaching than simply acquiring experience.
6. Planning and being responsive
   - Between planning for learning and responding to learning opportunities as they arise in practice (Berry 2007, p. 32–33).

Although initially applied to teacher education, it is possible the tensions that emerged from Berry's (2007) framework can be used both as a way to identify the competing conflicts experienced by mathematics teachers, and as a way to describe them. As such, our research question is aimed at identifying similar tension pairs within teachers' practice of teaching mathematics. While our eventual goal is to explore how these dichotomous forces impact teaching practice and professional growth needs, in this study our purpose is only in the applicability of using Berry's framework to emerge sets of tensions from the practice of a mathematics teacher. In what follows the methodology is addressed, one particular case is analyzed, and the results and conclusions are discussed.

15.2 Methodology

These questions are part of an ongoing research project in which we will examine the tensions of teachers from Kindergarten to University. In the end we will have data from 25 participants (5 primary, 5 intermediate, 5 junior high school, 5 senior high school, and 5 University teachers). In our previous work we tested our theories with fictional, aggregated data. The results of this test showed viability of the Berry framework as a basis for understanding and articulating the tensions teachers experience (Liljedahl et al. 2015). In this paper we extend this work by applying the framework to real data. In what follows we present a brief analysis of one of our first participants, a teacher named Naomi.

Naomi has been teaching for 11 years: 1 year teaching grade five on a remote First Nation reserve, 3 years teaching grade four in an urban K-8 school followed by 7 years teaching grade six in the same school. Her undergraduate degree required her to complete one introductory mathematics course and one mathematics for education course. A self-described "mathematics-phobe,[1]" she has had no further mathematics education other than occasional mandated professional development.

Data was collected over a 1 year period during which Naomi was a participant in a District Learning Team led by one of the authors. Notes were kept of conversations with Naomi that occurred naturally during breaks in the sessions. The opportunity was taken during these casual interactions to probe more deeply into questions Naomi had asked or about perspectives she had shared. Field notes were also taken during two classroom observations and during the lesson debrief. More formally, Naomi was engaged in two semi-structured follow up interviews that were designed to illuminate tensions present in her practice of mathematics. These interviews ranged from 20 to 45 min in length and were transcribed in their entirety. The data collected were then scrutinized using Berry's (2007) framework as an a priori frame for identifying and coding tensions. Dichotomous tensions in Naomi's practice emerged from the data and were identified and categorized accordingly.

15.3 Analysis

In the following analysis, Berry's (2007) framework will be used to analyze data from the perspective of Naomi in her practice of teaching sixth grade children. For the purposes of brevity, only three of the tensions will be discussed here.

15.3.1 Telling and Growth

For Naomi, the teaching of mathematics has been influenced by the constructivist notion of learning. She understands this to mean that the primary role of teaching is not to lecture, explain, or otherwise transmit mathematical knowledge, but to create opportunities for the student to construct their own knowledge. Naomi wants to avoid *"telling"* and instead focus on growth of understanding through experience. She explains that her teaching style is *"like night and day"* in comparison with her own mathematical education, which she described as rote memory, drill and kill, and algorithms. Instead, she uses *"hands-on, collaborative activities where students talk*

[1] A "mathematics-phobe" describes a person who has an aversion to mathematics.

and work through things together to come up with many different ways to do things." Naomi values when students come to her with a new mathematical understanding that she has not taught. Yet a tension arises when she is faced with students who are not accustomed to being taught in the *"new style of teaching"*. She shares that she doesn't know how to deal with that, and sometimes reverts back to *"the old school"* despite believing that it's really not in the best interests of the child. Naomi says she attempts to help the students see the benefits of her teaching style but that there are days when she hands out mathematics worksheets. Her statement, *"and you don't want to do that, you want to be egging these kids on to try and do as much as they can"* reveals the tension Naomi feels. It was apparent that this tension is a driving force in Naomi's professional growth and development when she followed with *"I'm always looking for better ways to teach math."* This is a tension that is neither managed nor resolved, instead it has become the impetus for change.

15.3.2 Confidence and Uncertainty

Teachers who feel weak in mathematics have the dilemma of whether or not to share that weakness with their students. Berry (2007) suggests there's a tension between exposing vulnerability and maintaining the respect of students, which Naomi discovered when she first began teaching. Her unexpected answer to the question, "What are you best at in teaching math?" was *"Um, honestly, showing the kids my weaknesses in math."* Naomi goes on to reveal that mathematics is her weakest subject and that she struggled with it throughout her life. When she began teaching, she hid this from her students because she thought they expected her to know everything. Naomi was able to accomplish this deception by a lot of traditional, direct teaching from the text and passing over questions that she couldn't answer. This eventually felt uncomfortable for her and she began telling her students when she didn't know something, stating *"But then I decided, maybe it would be okay if they knew."* Using the word *"maybe"* in her rationale indicates that perhaps Naomi is not completely at ease with her decision but she concludes with, *"and I think that the kids actually love it even more because the teacher really is struggling, and they get to teach me."* Naomi has made a decision that is acceptable to her and that she can live with. Her tension is not resolved but it is managed.

15.3.3 Action and Intent

Berry (2007) tells us that tension can arise when what we intend to do is in conflict with our approach in working towards that intention. In other words, what we do can inadvertently undermine our goals. Naomi provides an example of this tension when she describes her reliance on summative assessments in mathematics stating, *"I would say that with math, my hardest struggle would be not to rely solely on summative assessments."* She notes that her teaching style requires the use of formative assessment but she frequently forgets to document the students' learning and reverts back to her traditional summative assessment adding, *"I think because I get carried away in classes and I'm not actually documenting the formative assessments as I'm going through."* This causes tension for Naomi because her view of summative assessment is that it doesn't provide an accurate reflection of what the kids are doing *"especially the kids that have text anxiety or they're having a bad day."* Her intent appears to be to assess the students in a way that best matches the way the content was learned but her actions in relying on summative assessment interferes with that aim. And, despite acknowledging that the results from the summative assessment occasionally surprise her, Naomi manages this tension by combining the information from the summative assessment with what she has observed in class. *"It (summative assessment) doesn't give a true reflection of what the kids are doing. You're looking at summative assessment marks but also trying to reflect, bring in some of your observations."* Naomi shared that her decision to volunteer to be part of the District Learning Team was in part because of her desire to learn more about assessment in mathematics. Again, here is a tension in which seeking professional growth is seen as part of the outcome in resolution.

15.4 Discussion

In examining the data, it is possible to see limitations in applying Berry's (2007) framework to a teacher/student situation. Further tensions are evident that do not appear to fit within any of the categories provided by Berry (2007). Notably are the tensions the participant experiences with parents and colleagues. Naomi willingly spends time working with parents to help them understand *"new math"* but acknowledges that *"it takes a lot of my time and effort."* With colleagues, Naomi voices a tension between a latent desire to conform to *"what everyone else is doing because that's just easier"* and doing what she believes to be pedagogically sound. Teachers feel a great deal of pressure to conform to the norms and standards of their school, their mentors, and their grade partners. While this is especially true for beginning teachers, even experienced teachers feel tension when abiding by the norms conflicts with personal pedagogical beliefs.

Another tension emerged in the area of standardized testing. Naomi expressed conflict over preparing her students to write a standardized math test. She felt that the test was as much a *"judgement"* of her ability to teach mathematics as it was a method of assessing student learning. A study by Walls (2008) suggests that standardized testing challenges as well as reinforces teachers' perceptions of themselves as mathematics teachers. This was certainly true for Naomi who experienced this dichotomy.

It would appear that in order to encompass Naomi's tensions surrounding parents, colleagues, and standardized testing, that new categories of tension pairs would be of possible benefit. We expect that as our research progresses and broadens that we will also be capturing other tensions that are specific to the practice of teaching mathematics. This will require expanding or perhaps altering Berry's (2007) original framework to allow for their incorporation.

It is also apparent that there is overlap between the tension pairs and that several could fit under other categories. For example, Naomi's first tension regarding constructivist learning could potentially be recast as either safety and challenge or confidence and uncertainty. Safety and challenge could describe her capitulation to students uncomfortable with her teaching style. When challenged, she falls back to the safety and familiarity of worksheets, despite wanting her students to learn mathematics differently from her own learning experience. Alternatively, confidence and uncertainty could also be used capture the tension she feels. Her willingness to acquiesce to her students' highlights her own lack of confidence in teaching and learning mathematics. Examining the tensions with several different lenses could provide a richer perspective into the ways teachers experience and cope with tensions.

It may also be beneficial to consider further categorizing the tensions according to whether they are conflicts of pedagogy, conflicts of subject matter or possibly conflicts from external, systemic influences. What information might be revealed that could help in understanding how teachers experience tension? As well, on several occasions Naomi mentioned the difference between her current practice and when she first began teaching. It would be beneficial to compare the tensions felt by a new teacher with those of an experienced teacher and then examine any differences in how they manage those tensions.

15.5 Conclusion

Naomi appears to fit Lampert's (1985) image of a teacher as a dilemma manager who accepts and copes with continuing tension. She initially manages the tensions that surface in her practice while never fully resolving her competing conflicts. Our results show that where Naomi may differ is in living with the consequences of her decisions, as some of her managed tensions continue to resurface. She acknowledges that she wished she *"knew how to deal with that"*. This results in new tensions between what she wants to do and what she knows how to do. Dewey (1922) refers to this as harmonizing conflict and suggests that new tensions are bound to result from the settling of previous tensions, albeit in a "new form or on a different plane" (p. 285). The new tensions Naomi experiences may potentially fuel a desire for change in her teaching practice. In our future research we hope to discover if this is applicable to other teachers' experiences of teaching mathematics and what impact that might have on the design and delivery of professional development. By further investigating teacher tension, we believe the field will be better informed to improve teacher education and professional development efforts related to this phenomena. It is beneficial to mathematics education to have a fuller understanding of these tensions that drive teachers' needs, and shape both the individual and their practice.

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Chapter 16
Towards Inconsistencies of Parents' Beliefs About Teaching and Learning Mathematics

Natascha Albersmann and Marc Bosse

Abstract Due to our observations and theoretical considerations, we assume that there are parents who believe in the concept of constructivist learning but who reject this notion when supporting their own children in learning mathematics. In this study, both qualitative and quantitative data was gathered in order to identify factors contributing to possible inconsistencies. We found out that belief-inconsistent parents (a) assess their mathematical competence as relatively low, (b) aim at preventing pressure and frustration in mathematical learning situations with their children, and (c) meet resistance from their children but (d) overall experience rather less resulting conflicts.

16.1 Introduction and Motivation

Parental influences on their children's mathematical developments are especially notable in direct supportive situations, like homework situations, in which parents and their children get in contact with mathematics in an active way. Parents' involvement is particularly beneficial for children when it is, for example, autonomy supportive, focuses on the process of learning, and is accompanied by positive affect. However, it has negative repercussions for children if the involvement is controlling, performance focused, and accompanied by negative affect (Pomerantz et al. 2007; Wild et al. 2006). One factor influencing parents' supportive behavior is their beliefs about the teaching and learning of mathematics. In the context of a doctoral research project on the conscious integration of parents' into their children's mathematical education, we were able to observe that there are parents who acknowledge the relevance of learning processes following a constructivist

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setting. One could further expect that these parents are well-equipped in order to support their children in a beneficial way.

During the 19th MAVI conference in Germany 2013, the community discussed the question to what extent beliefs are stable or modifiable. We have learned that there are different theoretical positions, some seeing beliefs as immutable worldviews, others regarding them as complex systems with changeable and volatile parts. A theoretical approach about the role of context was presented during a plenary talk at MAVI 20 in Sweden (J. Skott, personal communication, October 1, 2014). Based on these considerations and the experiences of parent-child interactions in the context of the doctoral research project, we raise the question of whether parents' beliefs about teaching and learning mathematics are consistent with their beliefs about supporting their children. Furthermore, we are interested in identifying characteristics that may have an influence on possible inconsistencies.

In the following, we will firstly focus on theoretical considerations of beliefs related to mathematical teaching and learning as well as supportive mathematical learning situations. In addition to this, we will outline the aspect of contextuality of beliefs. Secondly, we will describe our methodological approach and give a more detailed description of the process of data analysis. Based on the discussion of our results, which will be presented as the third part, we will finally draw some conclusions giving an outlook on potential measures in order to enhance parents' supportive strategies.

16.2 Theoretical Framework

16.2.1 Beliefs About Teaching and Learning Mathematics

Empirical research has shown that two perspectives of beliefs about teaching and learning mathematics can be separated (Leuchter et al. 2006). On the one hand, people with a *transmission view* believe that mathematical knowledge is transferred from a teacher to a learner via a planned and directed knowledge-transmission-process. In this manner, mathematical learning takes place in teacher-centered schooling situations, e.g. lessons based on lectures. On the other hand, people with a *constructivist view* believe that learning mathematics is an active and self-regulated process of discovering and constructing knowledge. In this sense, it is the teacher's role to provide appropriate learning opportunities and material as a supporter, advisor, and mentor.

Depending on which of the both views a teacher shares, her or his teaching changes (Peterson et al. 1989; Staub and Stern 2002). The existing research primarily focuses on *teachers'* beliefs. In our opinion, it is reasonable to use the two-perspective model for describing parents' beliefs, too, since there is almost no research on parents acting as their own children's mathematics "teachers".

16.2.2 Beliefs About Supporting Children in Learning Mathematics

In order to describe parents' beliefs about supporting their children, we draw on a differentiation by Renshaw and Gardner (1990, see also Wild et al. 2006; Wittler 2008). They examined the relationship between parents' interpretations of a specific task and their support strategies and distinguished between. While Renshaw and Gardner (1990) use the subsuming term *orientations*, we draw on the concept of beliefs, since orientations can be interpreted as an umbrella category (Schoenfeld 2011) addressing the same construct as the term *beliefs* in context of this paper

Depending on parents' views on learning mathematics they either orientate themselves towards the *process* or the *product* of the learning situation. Process-oriented parents focus on the quality of a learning process and therefore they believe in the importance of a deeper engagement with the content and its understanding. These parents enable their children to discover mathematics for themselves and to construct their own knowledge. By doing this, parents favor an autonomy supportive behavior. When parents follow a product-orientation, they focus on the results of a learning situation rather than on the process. They acknowledge more controlling strategies, like direct instructions paired with a constant evaluation of their children's learning outcomes. Therefore, the process-oriented view can conceptually be related to a constructivist understanding of teaching and learning mathematics, while the product-oriented view shows conceptual parallels to the transmission view.

16.2.3 Contextuality of Beliefs

Following Törner (2002), every belief refers to a belief object. However, such a belief object cannot be treated like a separate and virtual entity. Instead it may be semantically, functionally, procedurally, situationally etc. connected to other belief objects (Rolka 2006). This configuration of belief objects induces the organization or clustering of object-corresponding beliefs, a systematization which is often called *belief system* (Törner 2002).

As another starting point for constructing our theoretical framework, we want to refer to a principle, which is considered in socio-cultural approaches (Skott 2014; Wenger 1998): *contextuality*. In these theories, it is assumed that practice, community membership and identity are influenced by contextual factors. We want to enrich our theoretical framework by adding the notion of contextuality; however, we will not use a socio-cultural framework for our analysis.

Radford (2008) explains that it is legitimate and reasonable to integrate ideas of other theories as long as there are neighboring principles in both theories. Prediger et al. (2008) suggest that the local integration of theoretical principles is one way of theory networking. As outlined above, belief objects are not separated from each other but are connected via contextual frames. Such a configuration of belief objects is virtually a context that is also considered in socio-cultural approaches. For example, the situation of supporting one's own children constitutes a specific configuration of belief objects (= a context) and a corresponding belief system. A situation of teaching and learning mathematics in which one's own children are not involved also provides a specific configuration of belief objects (= another context). Therefore, there are two belief systems that do not necessarily have to be the same. While the first context definitely comprises the belief object of "supporting one's own children" the second one does not do so. Other belief objects, though, might exist in both contexts. From a psychological viewpoint, mental structures and processes define which objects are considered as a part of a specific context. From a socio-cultural viewpoint, the configuration of belief objects is influenced by norms, values, traditions etc.

In summary, theoretical and empirical considerations lead to the assumption that beliefs may be inconsistent due to their respective context. We assume that someone's belief about the learning of mathematics does not necessarily have to be like her or his belief about her/his own children's learning of mathematics. In the following, we want to pick up on this assumption when analyzing parents' beliefs about teaching and learning mathematics in general and within the context of supporting their own children.

16.3 Methodology

16.3.1 Sampling

The here presented data originates from a study conducted in the context of a family math project called math-experience-days. The project was carried during the school year 2014/2015; the participants were parents and their children attending the 5th grade (10–11 years old) of a German higher-level secondary school, a so-called gymnasium. About 110 parents had the opportunity to participate in the project, but only 37 actually did. All of these parents have fundamental reasons for participation, like personal interest in mathematics or positive collaborative mathematical experiences with their children. They also livein rather high socio-economic conditions. Combining these aspects, the sample is highly selective. In the following, we will indicate a specific parent from the study by using the letter P for parent, combined with a respective number *X* (P*X*).

16.3.2 Data Collection and Research Instruments

Before having participated in the family math project, the parents were asked to answer a questionnaire with both closed as well as open items. In order to reduce the effects of social desirability, the study was conducted anonymously.

In the first step of our analysis, we concentrate on quantitative data addressing both parents' beliefs about supportive behavior in the context of mathematical learning situations with their children and parental beliefs about the teaching and learning of mathematics in general (see Table 16.1).

Parental beliefs with regard to supportive behavior were measured with a set of 14 items (No. 1, 2, 3, 4, and 5) with a 4-point Likert scale ranging from *totally true* (1) to *not true at all* (4). The items were adopted from the national questionnaire supplementary to PISA 2003 (Ramm et al. 2006), originating from the PALMA-project (Pekrun et al. 2006, 2002) and the longitudinal study *Fostering Self-Determined Forms of Learning Motivation at Home and in School* (Wild et al. 2006), both part of the DFG-Priority Program *BIQUA—Quality of Schools*. These items can either be assigned to the construct of process-orientation (No. 1 and 2) or product-orientation (No. 3, 4, and 5). Regarding beliefs about the teaching and learning of mathematics, the questionnaire comprises 14 items (No. 6 and 7) with a 6-point Likert scale from the TEDS-M study (Laschke and Blömeke 2013). The Likert scale ranges from *I totally agree* (1) to *I do not at all agree* (6).

In the second analytical step, additional quantitative as well as qualitative data is taken into account (see Table 16.2). The relevant quantitative data was measured with a 4-point Likert scale ranging from *totally true* (1) to *not true at all* (4). The item sets No. 8, 9 and 11 stem from the national parent questionnaire supplementary to PISA 2003 (Ramm et al. 2006) and item set No. 12 is originally from the PALMA-project (Pekrun et al. 2002). Additionally, two items characterizing homework conflicts from a study by Wittler (2008) are integrated.

Because of the small number of items used to describe some constructs, all reliability scores are acceptable (see Tables 16.1 and 16.2). However, the variable of performance-oriented punishment is an exception and, hence, will not be considered

Table 16.1 Data considered in the first analytical step

No.	Variable	# items	α	Ref.
1	Learning orientation	3	0.54	[a]
2	Autonomy support	4	0.646	[a]
3	Performance-oriented pressure	3	0.738	[a]
4	Performance-oriented punishment	2	0.106	[a]
5	Performance-oriented reinforcement	2	0.53	[a]
6	Transmission view	8	0.701	[b]
7	Constructivist view	6	0.618	[b]

[a]Ramm et al. (2006)
[b]Laschke and Blömeke (2013)

Table 16.2 Data considered in the second analytical step

No.	Variable	Data type/# items	α	Ref.
8	Intrinsic valuation	5	0.804	[a]
9	Extrinsic valuation	3	0.589	
10	General role in support	Qualitative		
11	Self-assessment of mathematical competence	2	0.776	[a]
12	General learning support	6	0.881	[b]
13	Child's mathematical performance	Qualitative		
14	Expectancies for child's mathematical performance	Qualitative		
15	Principles of support	Qualitative		
16	Role in a supportive situation	Qualitative		
17	Affect in a supportive situation	Qualitative		
18	Homework conflicts	2	0.823	[c]

[a]Ramm et al. (2006)
[b]Pekrun et al. (2002)
[c]Wittler (2008)

in the analysis. Moreover, qualitative data is included in the comparative analysis (No. 10, 13, 14, 15, 16 and 17). Because there are content-related overlaps between the qualitative and the quantitative items, the gathered qualitative data serves as an additional validation and exemplification of parents' responses.

16.3.3 Data Analysis (Step 1)

The collected data was stored in SPSS and prepared for further analysis. In order to make the data gathered by two *different* Likert scales comparable, the 4-point Likert scale was transformed into a 6-point Likert scale by using the formula $x_6 = \frac{5}{3}x_4 - \frac{2}{3}$. Afterwards, mean values were calculated for every variable so that a decimal from 1 to 6 could be assigned to every parent for every variable. In order to get a first overview of the relationship between the different variables, Pearson's correlations matrix was calculated and the correlations were tested for significance.

16.3.4 Substantial Interim Result Forcing Us to Adapt the Data Analysis

In contrast to our hypothesis of different belief structures due to contextuality, Pearson's correlation shows that *constructivist view* (No. 7) and *process orientation in supportive behavior* (No. 1 and No. 2) correlate significantly ($r = 0.411$, $p < 0.05$). While *constructivist view* also significantly correlates with *autonomy support* (No. 2) ($r = 0.340$, $p < 0.05$), there has not been found a correlation between *constructivist view* and *learning orientation* (No. 1). For further investigations of parents' beliefs about teaching and learning mathematics in general and beliefs in the context of supporting their own children, we decided to pick *autonomy support* as the relevant variable for representing supportive behavior due to the positive correlation. We want to draw our attention to reasons that may lead to differences in the two belief domains even though *constructivist view* and *autonomy support* correlate.

16.3.5 Data Analysis (Step 2)

In order to identify those parents who hold a clear constructivist view on mathematics teaching and learning but who reject autonomy support in situations of supportive behavior, differences between the mean values of the variables *autonomy support* (No. 2) and *constructivist view* (No. 7) were calculated. After that, those parents with the highest difference were considered and it was examined whether they actually reject autonomy support and agree with a constructivist view. Additionally, qualitative data (No. 15 and 16) was checked for further validations of a missing autonomy supportive view or the approval of it. Those belief-inconsistent parents were selected for further investigations.

In the next step, we searched for common features among the belief-inconsistent parents concerning the variable values of both quantitative (No. 8, 9, 11, 12 and 18) and qualitative (No. 10, 13, 14, 15, 16 and 17) variables. In terms of quantitative variables, commonality was determined when the variable mean values were the same. In terms of qualitative variables, commonality was determined when the same thematic codes developed from the texts or text fragments appeared.

Finally, a belief-consistent group was defined by choosing the 50% of parents with the lowest difference D_{PX} and $D_{PX} > 0$.[1] Following, it was checked whether the characteristics that are possibly constitutive for the group of belief-inconsistent parents appeared when checking the same relevant dimensions for the belief-consistent group or not. If the belief-consistent group can be distinguished from the belief-incongruent parents regarding the discovered variable values, we can propose the hypothesis that these variable values contribute to a shift of beliefs in the context of supporting one's own children.

[1]Cases in which the value of the autonomy support variable is higher than the value of the constructivist view variable (which results in a negative difference) are not in the focus of this paper.

16.4 Results and Discussion

According to our methodology, five parents (P04, P07, P19, P13, P34) with the highest difference ($D_{P04} = 1.83$; $D_{P07} = 1.58$ $D_{P19} = 1.25$; $D_{P13} = 1.17$ and $D_{P34} = 1.17$) between the averages of *autonomy support* (No. 2) and *constructivist view* (No. 7) were selected.[2] Shifting our attention from the differences to the mean averages, it became apparent that parent P34 does not generally negate autonomy support ($\acute{x} = 2.67$) but supports the constructivist view so much ($\acute{x} = 1.50$), that the difference is high. Hence, this parent has been excluded from further analysis. As the qualitative data concerning variables No. 15 and 16 contradicts the interpretation of lacking autonomy support, P19 has also been excluded from the analysis. Accordingly, the parents P04, P07 and P13 were identified as belief-inconsistent parents. Thus, there are indeed parents who believe in the relevance of constructivism for mathematical learning but who do not believe in autonomy supportive strategies in the context of their children's mathematical learning. The question arises whether there are constitutive characteristics contributing to such a shift in the belief systems.

In the context of characterizing parents with inconsistent beliefs, the variables No. 11, 15, 17, and 18 were identified. Three of these variables (No. 15, 17, and 18) are related to the context of mathematical learning situations with their children. In the following, we will refer to them as context-dependent factors. Variable No. 11 does neither relate to the context of teaching and learning mathematics in general nor to the mathematical learning and support of one's own children as the *self-assessment of mathematical competence* is rather a personal factor. All three belief-inconsistent parents assess their mathematical competence as relative low which is reflected in a mean value of 4.33. However, as a result of the PISA study in 2003 it was claimed that parents show autonomy supportive behavior, even though they assess their mathematical competence as low (Ehmke and Siegle 2005). This result indicates that variable No. 11 has less influence on parents' autonomy support. Therefore, we will disregard parents' self-assessment of mathematical competence in our concluding remarks.

When analyzing the context-dependent variables No. 15, 17, and 18, some additional commonalities were extracted. With regard to the principles of support (No. 15), all three belief-inconsistent parents seek to prevent frustration and pressure when supporting their children. P07 for example states: "My principles still are (I'm working on it): Keep patient, don't exert pressure [...]". P13 tries to reduce pressure by providing sufficient time for learning mathematics and P14 tries to reduce frustrations by communicating some task-simplifying tricks.

Despite these intentions, parents stated that they experienced strong resistance from the part of their children during actual mathematical learning situations at home (No. 17). As the data shows, these resistances appear in context of parental

[2] The sixth-highest value is $D_{P36} = 0.83$, the seventh-highest value is $D_{P11} = 0.58$. Therefore, it is reasonable to concentrate on the top 5.

directive instructions: "Collaborative homework situations are always difficult because my child does not like to follow instructions." (P04) However, with regard to homework conflicts in general, the belief-inconsistent parents report rather less real conflict situations, which is reflected in a mean value of 3.5 (No. 18).

An explanation for these results could be that experiencing resistance from the part of their children may lead parents to behave rather directly instructive, which contradicts autonomy support. This assumption is strengthened by P07, referring to affect in supportive situations (No. 17): "The situation regularly blows up. My child begins to cry when not understanding, hence, I try even more to impart the topic, I get more impatient, more irritated—a vicious cycle" (P07). As a result of our data analysis, we want to highlight this named vicious cycle as a model of belief-inconsistent parents' supporting situations. The moment these parents draw on direct instructions and their children show resistances while not following the parents' directives and not understanding the mathematics the directives are supposed to lead to, the parents are likely to exert even more directive instructions in order to accomplish the task and to reduce frustration and pressure. In other words, this strategy is a vicious cycle which belief-inconsistent parents are yet unable to escape from.

However, the belief-inconsistent parents estimate the overall amount of conflict in homework situations rather as moderate than severe. When analyzing the qualitative data, it furthermore becomes apparent that all belief-inconsistent parents do not give up but remain in the mathematical learning situations with their children in order to overcome the resistance and accomplish the mathematical problem.

Finally, a belief-consistent group was defined according to our methodology. The aim was to test the factors that are possibly constitutive for the belief-inconsistent group by checking whether they appear with the belief-consistent parents. By doing this, P22, P33, P37, P01, P21, as well as P03 (each $D_{PX} = 0$), P08 ($D_{P08} = 0.08$), P26 ($D_{P26} = 0.17$), P31 ($D_{P31} = 0.17$), and P02 ($D_{P02} = 0.25$) are explored as representatives for the belief-consistent group. Except for P02, all parents of this group dissent in at least two of the four variables, which are relevant for characterizing parents with inconsistent beliefs. P02 who dissents in only one variable can be disregarded since she/he is the closest to the belief-inconsistent parents in terms of the difference between the variables of autonomy support and constructivist view. Thus, the belief-consistent group can be distinguished from the belief-inconsistent parents regarding the discovered four variables.

16.5 Conclusions

Belief-inconsistent parents believe in the general concept of constructivist learning but reject the notion of autonomy when supporting their children in learning mathematics. We draw the conclusion that these belief-inconsistent parents (a) assess their mathematical competence as relative low, (b) aim at preventing pressure and frustration in mathematical learning situations with their children, and (c) experience resistance from the part of their children but (d) rather less resulting conflicts. Moreover, it became apparent that the belief-inconsistent parents seem to be willing to support their children although they experience resistance and moderate conflicts. One can conclude that a fundamental motivation for supporting their children is given and can be used as a prerequisite for further thoughts on parental programs.

Focusing on the enhancement of parents' mathematical competence does not seem to be a fruitful starting point for such programs, because the variable of parental self-assessment of their mathematical competence lacks relevance for autonomy support (Ehmke and Siegle 2005). In the wake of this paper, alternative parental programs might actually focus on strategies to avoid resistance in mathematical learning situations through autonomy supportive behavior. Enabling parents to actively experience the potential of autonomy support and its effectiveness for their children's learning process could be one aspect of such programs. As a consequence, it is of fundamental importance that parents as well as their children are involved in such programs.

In order to break the vicious cycle, parents need to experience the dynamic underlying their directives and children's resistances. We recommend designing programs which help parents in order to recognize the relevance of constructivist learning approaches and hence, the importance of autonomy supportive strategies especially in the context of their own children's mathematical learning.

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Chapter 17
Evoking the Feeling of Uncertainty for Enhancing Conceptual Knowledge

Igor' Kontorovich and Rina Zazkis

Abstract This paper is focused on mathematical conventions, which account for the decisions of the mathematics community regarding definitions, names and symbols of concepts. We argue that tasks that request learners to create and discuss not necessarily historically valid, but convincing explanations of mathematical conventions, provide them with opportunities to enhance conceptual knowledge. Specifically, the tasks are designed to evoke the feeling of uncertainty that can be resolved through active engagement with mathematical concepts. We analyze the tasks using different theoretical lens and exemplify two responses of teachers who engaged with one of the tasks. We conclude by suggesting avenues for using the tasks in research and practice.

17.1 Mathematical Conventions, Explanations and Tasks

In this paper we introduce tasks, in which learners are asked to create explanations for mathematical conventions and discuss them with their peers. The task is expected to provoke a feeling of uncertainty among learners, as there are no structured criteria of a persuading explanation for a convention. In this paper we analyse how the emerged feeling of uncertainty may lead learners to active engagement with mathematical concepts. We first turn to the tasks themselves and invite readers to consider their personal explanations.

Please, address the following questions:

1. Why are angles in the Cartesian plane measured counter-clockwise and not clockwise?
2. Why does 0! equal 1?
3. Why are the axes in the Cartesian plane orthogonal?

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4. Why should a in the function $f(x) = a^x$ always be positive, if $(-2)^x$, for example, is defined for natural exponents?
5. Why are odd and even functions called so?
6. Why is an absolute value denoted by the symbol $|\cdot|$?
7. Why are reciprocals and inverse functions denoted by the same symbol of $^{-1}$?

The reasoning for establishing a particular mathematical convention cannot be *proved mathematically*, i.e. presented as "[…] a sequence of transformations of formal sentences, carried out according to the rules of the predicate calculus" (Hersh 1993, p. 391). However, mathematical conventions can be *explained*. We adopt Balacheff's (1988 in Hanna 1990) view on what an explanation entails: "We call an *explanation* the discourse of an individual who aims to establish for somebody else the validity of a statement" (p. 2).

On the one hand, literature on evolutions of mathematical concepts provides historical explanations for establishing some mathematical conventions (e.g. Cajori 1993; Wilder 2013). The validity of these explanations is warranted by the authority of the literature (see Harel and Sowder 2008, for an analogous idea of proof by authority). On the other hand, explanations that appeal to a subjective sense of reasonableness of a learner can be created.

Following upon Harel and Sowder (2008), we consider creating explanations for mathematical conventions as a process consisting of *ascertaining* and *persuading*. In ascertaining, a learner pursues an explanation that will be perceived by herself as convincing, while persuading is the practice of convincing others of the reasonableness of an explanation. These processes are interrelated as one of the criteria for a self-ascertaining explanation can be its anticipated power to persuade others.

Next we introduce four explanation schemes (see Harel and Sowder 2008, and Toulmin 1958 for compatible ideas of proof schemes and warrants), which are generalized reasoning for a mathematical convention that can turn an explanation to self-ascertaining and can be employed for persuading others. The schemes are illustrated with possible answers to Questions 1–6. Question 7 is addressed separately afterwards.

Arbitrary Choice This reasoning indicates that no clear preference for a particular convention can be suggested and alternatives could have been chosen instead. For instance, this reasoning can account for a counter-clockwise direction for measuring angles in the Cartesian plane instead of a clockwise direction (see Question 1).

Consistency This reasoning can account for the conventions regarding definitions of concepts in particular cases, cases which do not follow from a general definition. The convention for these cases is extrapolated from the cases that fit the general definition, and extrapolation is aimed to avoid mathematical inconsistencies. For instance, in Question 2, since $n!$ is defined as $n! = n\cdot(n-1)\cdot\ldots\cdot2\cdot1$ for a natural number n, then $n! = \frac{(n+1)!}{n+1}$. Extending this property to 0 results in $0! = \frac{1!}{1} = 1$.

Convenience This reasoning can account for the conventional definitions of concepts aimed at making mathematics that follows from this definition more conve-

Have you noticed that superscript (-1) is used to indicate reciprocals (5^{-1}) and inverse functions ($f^{-1}(x)$)? Why do you think this is the case? Suggest a convincing explanation for this convention and be prepared to persuade your peers in your explanation.

Fig. 17.1 The (-1) task

nient. For instance, in Question 3 calculating lengths of segments in orthogonal coordinate system is easier than in a non-orthogonal one. In Question 4, if the basis of the function $f(x) = a^x$ would have been allowed to be negative, the function would have lost its powerful properties, such as being defined for all real numbers, positiveness, continuity, monotonicity.

Conceptual Connection This reasoning can be employed for explaining the conventional names and symbols of mathematical concepts by relating them with other, more general, mathematical concepts. For answering Question 5, it can be suggested that odd/even numbers are key elements in some properties of odd/even functions. For instance, Maclaurin series of odd/even functions correspondingly contain only odd/even powers of the variable (Sinitsky et al. 2011). In Question 6 the symbol "$|\cdot|$" can be associated with the modulus of complex numbers and defined as $|a + i{\cdot}b| = \sqrt{a^2 + b^2}$, when a and b are real numbers. Accordingly, a possible explanation for the convention is that an absolute value is a particular case of the modulus for real numbers: $|a| = |a + 0{\cdot}i| = \sqrt{a^2}$.

This paper is concerned with tasks that invite learners to **c**reate **e**xplanations for **m**athematical **c**onventions (CEMC tasks). The task in Fig. 17.1 (*the (-1) Task* in what follows) is an example of such a task and it is based on Question 7. We claim that CEMC tasks provide opportunities to enhance learners' *conceptual knowledge* via active engagement with concepts. In this paper conceptual knowledge is seen as a "comprehension of mathematical concepts, operations and relations" (Kilpatrick et al. 2001, p. 5).

In the next section we analyze the CEMC tasks through the lens of several constructs used in mathematics education. This is followed by snapshots of responses of mathematics teachers who engaged with the (-1) Task. These snapshots illustrate possible realizations of the claimed learning opportunities. We conclude by suggesting how CEMC tasks can be used in research and practice.

17.2 Theoretical Foundation

In this section we analyse CEMC tasks with the lens of structure of attention (Mason 2008) and the lens of uncertainty (Zaslavsky 2005). We use these theoretical lenses for sketching a possible path of active engagement with mathematical concepts.

17.2.1 CEMC Tasks Through the Lens of Structure of Attention

Mason (2008) argues that learning new mathematics through tasks is deeply related to what is in the learners' focus of attention and how it is attended. While the focus of attention can shift, Mason distinguishes among five structures of attention: *holding wholes*, when a whole structure is in the focus; *discerning details*, when a particular element of the whole is attended; *recognizing relationships*, when connections between the discerned details are attended; *perceiving properties*, when the discernment of details is driven towards generalization of their property or connections between them; and *reasoning on the basis of perceived properties*, when details are discerned as a result of an a priory established connection or property.

One of the implications of Mason's theory is that directing learners' focus and structure of attention is necessary for sense-making and internalization of ideas intended by a teacher or task designer. CEMC tasks direct learners' attention towards a particular mathematical convention and requests them to explain it. For instance, in the (-1) Task, the attention is drawn to the fact that the same symbol of "$\cdot^{-1}$" is used for denoting reciprocals and inverse functions. The intended idea is the conceptual relation between the two uses of the symbol, as both point to the inverse element in a group structure. To recall, a group element (a^{-1}) is considered to be an inverse of another group element (a), if a binary operation ($\times$) between them results in an identity element of the group (I). Symbolically: $a \times a^{-1} = a^{-1} \times a = I$. Accordingly, reciprocals are a code name for inverse elements in a group of real numbers (without zero) with an operation of multiplication and the identity element of 1; inverse functions stand for inverse elements in a group of bijective functions (with an appropriate choice of domains) with an operation of composition and the identity element of $f(x) = x$.

17.2.2 CEMC Tasks Through the Lens of Uncertainty

Zaslavsky (2005) argues that tasks that elicit learners' uncertainty in the mathematical validity of a claim, problem-solving method, conclusion or outcome can facilitate learning of mathematics. The researcher roots her arguments in the conflict theory, variations of which were acknowledged by many scholars (e.g. Dewey 1933; Festinger 1957; Piaget 1985). Generally speaking, scholars agree that when an individual is experiencing uncertainty (cognitive conflict or disequilibrium, in terms of Piaget; dissonance, in terms of Festinger; perplexity, confusion or doubt, in terms of Dewey) she will be motivated to modify something in her ways of acting and thinking in attempt to escape from this situation.

CEMC tasks are expected to elicit uncertainties related to the existence of an explanation for a convention and related to competing explanations. The uncertainties related to the existence of an explanation are expected to be particularly intense among learners who are not familiar with the tasks. The lack of experience in reasoning about conventions can evoke an initial explanation that conventions are chosen arbitrarily. However, this explanation is in tension with a common perception of mathematics as a logical discipline where every decision can be sustained (e.g. Schoenfeld 1989). Moreover, in some conventions the similarities in the name of the concepts (such as in Question 5) or a common symbol (such as in the (-1) Task) hint that there exists a more convincing explanation than an arbitrary choice.

Explanations can compete with each other in being perceived as more assertive and persuasive by suggesting different arguments for the same convention (cf. Zaslavsky 2005, for competing claims). For instance, the convention in Question 2 was previously explained by extrapolating the property of $n! = \frac{(n+1)!}{n+1}$ to zero. Alternatively, factorial of a natural number n can be defined as the number of permutations in a set with n different objects. Extrapolation of n to zero results in an empty set, which is unique. Accordingly, there is only one permutation for an empty set. The convention in Question 5 was previously explained with Maclaurin series. An alternative explanation can rely on the fact that the oddness and evenness of monomial functions (i.e. $f(x) = ax^n$ when n is a natural number or zero) corresponds with the oddness and evenness of the exponent n. The request of the CEMS tasks to come up with a self-ascertaining and persuading explanation for a mathematical convention elicits uncertainty in choosing among competing explanations. Some examples of uncertainties regarding existence of explanations and competing explanations that can emerge when learners are engaged in the (-1) Task are presented in the snapshots section.

17.2.3 Possible Path for Active Engagement with Concepts Through CEMC Tasks

We consider the processes of ascertaining and persuading to be central for learners' resolutions of uncertainties evoked by CEMC tasks. As a response to the request to come up with explanations that will be convincing to the learner as well as to others, learners are expected to engage in explorations of the concepts involved in the convention. Accordingly, at the *ascertaining phase* of CEMC tasks, learners can consider alternative definitions of the concept(s) appearing in the convention, the properties of these concepts and connections between them (see presented explanations of the conventions in Questions 1–5 again). In some cases, explanations can be based on the concepts that are not mentioned in the convention (see Question 6) and stem from tertiary mathematics (see the (-1) Task). Creating such explanations can involve reading mathematical literature and sense-making of new concepts. The *persuading phase* of CEMC tasks is aimed at sharing and discussing competing explanations, and distinguishing among reasoning schemes (e.g., arbitrary choice, consistency, convenience, conceptual connection).

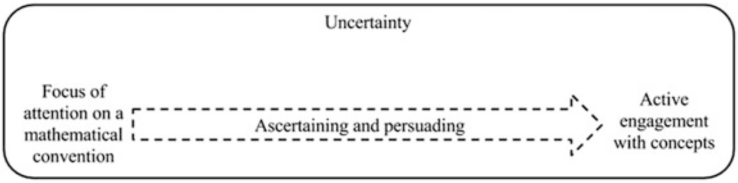

Fig. 17.2 Possible path of active engagement with mathematical concepts through CEMC tasks

Following through the ascertaining and persuading phases can lead learners to an active engagement with mathematical concepts and result in the emergence of explanation(s) intended by a teacher or task designer (see Fig. 17.2). In this way, CEMC tasks are designed to create a-didactical situations where learners may obtain new mathematical knowledge without intentional directions being given (Brousseau 1997). We use the dotted line for the arrow in Fig. 17.2 to symbolize that learners' ascertaining and persuading does not ensure the intended engagement.

17.3 Snapshots on Teachers' Responses to the (-1) Task

We frequently use the (-1) Task in master's courses in teacher education programs and professional development workshops. We invite teachers to respond to the task in writing and to create explanations with which they are satisfied. We allocate at least a week for the task completion, so there is no pressure of time in seeking explanations. After that, the explanations are shared in a classroom discussion.

This section contains snapshots on responses of two experienced teachers who worked on the (-1) Task in a problem-solving course, during the last term of their teacher education program. We chose to present the responses of the particular teachers (Sophia and Ezra, pseudonyms) as they exemplify the variety of teachers' approaches to the task. Our comments on teachers' responses were driven by the question "How does teachers' conceptual knowledge evolve whilst resolving the uncertainty elicited by the (-1) Task?"

17.3.1 Self-sufficient Response of Sophia

17.3.1.1 Synopsis of the Response

In reflecting on the task, Sophia (as well as all other teachers) indicated that while being quite familiar with reciprocals and inverse functions, she has never noted that these concepts are denoted by the same symbol, and consequently, she did not consider any connections between them. Sophia said that after reading the task, she got a feeling that the explanation is concealed in advanced courses that she studied when majoring in mathematics. She recalled topics in which a superscript (-1) appeared. Eventually, she recalled concepts from the Abstract Algebra course which assisted her in formulating the following explanation:

> Multiplicative inverses in a group/ring/field are consistent with function inverses in the sense that applying the inverse operation/function twice returns the original element.

17.3.1.2 Comments

We presented the response of Sophia as an example of a learner whose mathematical knowledge was sufficient for creating the intended explanation at the ascertaining phase: Sophia interpreted the symbol of "$(\cdot)^{-1}$" as a hint for a conceptual connection between reciprocals and inverse functions. In *recognizing relations*, she activated her advanced mathematical knowledge in abstract algebra. Browsing the course topics in a search for the symbol of "$(\cdot)^{-1}$" indicates that her attention was structured by *holding wholes*. Eventually, she discerned the overarching concept of "inverse elements" that appeared in the topics of "group/ring/field". Her explanation is based on a property of an inverse element that can be represented symbolically as $\left(a^{-1}\right)^{-1} = a$. The explanation evidences Sophia's realization of the conceptual connection between reciprocals and inverse functions, a realization that was not obvious for her at the beginning.

17.3.2 Help-Seeking Response of Ezra

17.3.2.1 Synopsis of Response

Ezra indicated that while initially attending the (-1) Task, he could not come up with any connection that explained the common symbol. At some point, he decided to look for ideas on the Internet. Ezra started with an etymology dictionary in attempt to find the source of the word "inverse". He explained that he learned this technique in a linguistics course. Ezra found that "inverse" comes from the Latin "inversus", which is "turned upside down" or "overturned". He focused on the procedures for finding reciprocals and inverse functions and came up with the "reversing" explanation: for finding a reciprocal of a fraction, the numerator and denominator should be reversed; for finding an inverse function of $y = f(x)$, the variables exchange their names and then the created equation should be solved for y. In the obtained equation x's are on the left side and y's are on the right side, so he called it "a reversed equation". However, Ezra addressed this explanation as being personally unsatisfactory and said:

> I couldn't come up with something mathematical that can really explain this fact [using the same symbol]. So I decided to search for an explanation in some mathematical websites.

In his search, Ezra encountered the paper of Even (1992), which discussed the "undoing" explanation. He said that he liked this explanation and formulated it in the following way:

> An inverse is something that will return you to the starting point. Let's say I pushed the wrong button on a calculator and multiplied by 5. For correcting this, I need to divide by 5, which is multiplying by 1 over 5. The same goes with functions: applying an inverse function undoes the effect of the original function.

In the discussion Ezra shared with his classmates the deficiencies that Even (1992) found for this "undoing" explanation.[1] He also referred to this explanation as being qualitative, in the sense that it revealed to him "a bigger picture" of connections between reciprocals and inverse functions without going into mathematical details. Due to the lack of these details, he perceived this explanation as being "not mathematical enough"; and thus, he readily considered alternative explanations provided by his classmates. As a response to Sophia's explanation, Ezra said that it was very abstract and that it was good that he had found the "undoing" explanation beforehand, as it helped him to make sense of the situation.

17.3.2.2 Comments

We presented the response of Ezra to illustrate how the uncertainty evoked by the task can engage a learner in a search for the new mathematical knowledge and intensify the need for discussion. In the ascertaining phase, Ezra reached to web-outlets for help and came up with the "reversing" and "undoing" explanations. The former explanation was inspired by the linguistic meaning of the word "inverse" and it was based on the *discerned similarities* in the procedures for determining a reciprocal of a fraction and an inverse of a function. However, Ezra considered this explanation as not being self-ascertaining. We suggest that when creating this explanation, his reasoning was *based on the perceived property*, which assumed the existence of conceptual connection.

Ezra's latter explanation of "undoing" was formulated based on the mathematics education paper and it can be symbolically represented as $a^{-1} \times a = I$. Although Ezra perceived this explanation as self-ascertaining, he anticipated that it will not be fully persuasive for his classmates as "not sufficiently mathematical". Apparently, the deficiencies of the explanation identified by Even (1992) intensified Ezra's perception. However, the "undoing" explanation turned to be useful for internalizing

[1]Even (1992) argued that while "undoing" is helpful in understanding the concept of inverse functions, one should not be limited to this approach as it may result in mathematical difficulties. For instance, she found that perceiving the root functions as "undoing" of power functions hindered about one third of the teachers who participated in her study from detecting an inverse to exponential functions. Additional misconception that might emerge is that all functions have inverses.

Sophia's explanation. Namely, the "undoing" and "group/ring/field" explanations completed each other by filling "the bigger picture" with mathematical details.

17.4 Concluding Remarks

The claim of this paper is that CEMC tasks provide opportunities to enhance learners' conceptual knowledge. We addressed this claim theoretically and exemplified empirically. In the presented snapshots learners enhanced their conceptual knowledge as a result of an episodic intervention with a CEMC task. Systematic variation of every aspect of the setting in which the task was addressed can turn into a fascinating research avenue. Possible outcomes of these avenues can contain characterization of the relations between the mathematics conventions being explained and mathematical knowledge being involved, as well as identification of the roles of learners' mathematical backgrounds and social interactions in creating explanations. In addition, it is interesting to explore the impact of task formulations on learners' responses. For instance, the (-1) Task can be reformulated without mentioning reciprocals (see a possible reformulation Fig. 17.3).

The uncertainty that CEMC tasks elicit among learners who are not familiar with this activity can expose some of the classroom socio-mathematical norms, which become tangible when there is a deviation from them (Yackel and Cobb 1996). On the other hand, recurrent engagement of a classroom in CEMC tasks can provide opportunities for cultivating intended sociomathematical norms. In this way, CEMC task can turn to powerful research and pedagogical tool, the potential of which goes beyond conceptual knowledge.

Mathematical conventions are usually presented as part of an introduction to a topic, and they are taken for granted afterwards. Students, who are newcomers to mathematics, can require explanations for establishing particular mathematical conventions. The presented cases of Sophia and Ezra show that providing convincing explanations can be challenging even for experienced teachers because they are used to perceiving these conventions as unquestionable norms in the mathematics community. Working on CEMC tasks prepares teachers for these contingent events (see Rowland and Turner 2007, for contingency component of teacher knowledge).

Suggest a convincing explanation for the decision to use the symbol of $(\cdot)^{-1}$ for denoting inverse functions. Be prepared to persuade your peers in your explanation.

Fig. 17.3 Reformulations of the (-1) task

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Chapter 18
Criteria for Identifying Students as Exceptional in a Mathematical Camp for "Gifted" Students

Rachel Hess-Green and Einat Heyd-Metzuyanim

Abstract The present paper examines the criteria for identifying students as mathematically gifted or exceptionally bright in a mathematical summer camp for advanced high-school students. We employ discourse analysis tools to analyze and classify instructors' stories about students as told in private staff meetings. The findings show seven categories for evaluation criteria, some of which were purely social/affective, others mathematical. We use the explication of these criteria to examine a case of an exceptionally valued participant that expressed significant emotional distress during a mathematical argument with her instructor.

18.1 Introduction

Identity and competence are both important concepts in educational research (Boaler and Greeno 2000; Gresalfi et al. 2009; Sfard and Prusak 2005). However, identity and construction of competence of mathematically gifted students has scarcely been researched. This may be because of the myth that giftedness is an inherent cognitive trait. In this paper we concentrate on the way that identities, or stories about exceptionally talented students participating in a mathematical summer camp, can shed light on the social construction of giftedness in mathematics, its interaction with mathematical actions of students, and the process by which students are initiated into the academic mathematical community.

18.2 Theoretical Background

18.2.1 Camps for Gifted Students

Programs for mathematically gifted students are varied (Koichu and Andzans 2009). The majority of these programs are supported by institutions that are interested

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in developing the future generations of mathematics researchers and technology professionals. In recent years, spotlight has turned to the emotional and social needs of these students (Reis 1995; Richards et al. 2003). Some of the first programs for mathematically gifted students that were mentioned in the literature were mathematical competitions for adolescents (Wieschenberg 1990). These programs were a milestone for many young mathematicians and encouraged them to study mathematics. Afterwards, mathematical camps were developed as preparation for mathematical competitions and for mathematical enrichment (Koichu and Andzans 2009). This kind of platform also connects young adults who share similar interests, enabling them to create friendships and collaborate with students even after the program. Leaning on a socio-cultural perspective that views learning as becoming a participant in a certain community, Barab and Hay (2001) studied a science camp for middle school students held at a university. They characterized the learning in this camp as apprenticeship learning. In it, the students gained experience in doing science next to practicing scientists and developed a rich language in which to conduct discussions about science. This research pointed to the opportunities the institution provides for learning and for community among students, and provides an indicator of the potential of looking at mathematics programs for gifted students through the socio-cultural lens.

18.2.2 Identity and Emotion

A central concept that can help in capturing the social and affective aspects of learning is that of identity. Researchers using this term have emphasized that learning is not just acquisition of skills and bits of knowledge; it is also a process of becoming a certain person (Wenger 1998). Since we aim to capture processes related to identity as manifested in discourse, we found Sfard and Prusak's (2005) communicational definition of identity useful: a collections of stories about a person that are reifying, endorsable and significant. The reifying quality comes with the use of verbs such as be, have or can rather than do, and with the adverbs always, never, etc. Another way to reify is by using adjectives instead of verbs such as "she is clever" instead of "she solved the question in a great way". Heyd-Metzuyanim and Sfard (2012) added to this definition a conceptual toolset for examining processes of identity construction in classroom talk. They pointed to the fact that while learning mathematics, students do not just participate in the mathematical discourse (or mathematize), they also participate in an *identifying* discourse whose main goal is to produce narratives about oneself and others.

A central tenet in this framework is that 1st person identities (what students say about themselves) is often a result of individualizing 3rd person stories—stories told by significant others about who we are. In an educational institution, the significant narrators are often the teachers or instructors. In addition, when staff members identify a student we derive from these identifying stories the evaluation criteria that are normative for the institution. For example, teachers will respond differently to the question: Who is a capable student in mathematics? And their response may be a result of institutional culture. In one institution, teachers may answer “quick creative student thinking outside the box”, while another institution would glorify “careful and accurate student who works in an orderly manner” (Hempel-Jorgensen 2009).

In schools, evaluation is often very explicit, mostly in the form of grades which are (at least supposed to be) based on clear criteria. Even there, different classrooms construct competence in different ways (Gresalfi et al. 2009). One, very common, school version of competence involves what students need to know or do in order to be “correct”. But there can be other versions. For example, sharing mistakes or misunderstandings can be a useful learning activity, and the person who has shared a mistaken idea can be considered to be competent. Thus evaluation criteria form part of the norms that the students are expected to align with and these norms reflect institutional expectations and goals. The examination of evaluation criteria in a relatively non-formalized setting such as a mathematical camp provides an opportunity to examine the process by which students identified as “gifted” are initiated into the academic mathematical community. It also forms a structure to examine possible reasons for students’ success or failure in such settings, as a function of their success in aligning themselves with these norms.

18.3 The Context for the Present Case Study

This study forms a portion of an ethnographical study that has been taking place over the last 2.5 years and which has examined three sessions (2013, 2014, 2015) of a 2-week-long mathematical camp for high school students during summer vacations. The curriculum of the camp focuses mainly on number theory at a Bachelor’s degree level. In addition, students engage in social activities, spending all their days and nights in the camp. Most learning activities take place in small groups with four to five students in each group with instructors that advise the students and monitor their development. The first author was one of the instructors in the camps and was tutoring one group in each of them. The overall goal of the research is to understand the ways by which the camp provides an opportunity for students to enter the academic mathematical community, emphasising the affective and social aspects of this process. This camp represents a kind of academic mathematical community because the curriculum is based on an academic course in number theory emphasizing proving and construction of definitions and claims. Also the staff of this camp stressed the importance of mathematical writing as a communication tool, and the dean of mathematics faculty of the camp emphasized the camp as developing the next generation of mathematicians.

In the present article, we concentrate on the ways in which the camp staff identified students as competent or even unique and “brilliant”. This forms a sub-goal in our quest to understand what the institutional expectations are and how students align or fail to align with them.

To illustrate the motivation for this goal we shortly describe an episode (described in more detail in Hess-Green and Heyd-Metzuyanim (2015)), which took place in the camp of summer 2013. In it, Jasmine (Pseudonym), one of the most highly-evaluated students in that camp (as indicated by the 1st prize she got in the final competition of "successful students" at the end of the camp) volunteered to present her solution to an advanced problem that was given the previous day. The question was:

Prove that for each number $n! + 2, n! + 3, \ldots, n! + n$ *there is a prime divisor that does not divide any other number from this set.*

None of the other students had solved this problem before. Jasmine was the only one who claimed to have solved it. In the process of writing on the board, the instructor asked Jasmine questions about her claims. This developed into mathematical argumentation in which the instructor (the first author) questioned Jasmine's claims and Jasmine attempted to defend them. The interaction was full of emotional expressions on the part of Jasmine (such as giggling, and signs of distress and embarrassment) and apologetic remarks made by the instructor, aimed at calming Jasmine down. Additional evidence for Jasmine's distress was seen after the session in a spontaneous meeting where Jasmine told the instructor: "I think you embarrassed me in front of the whole class, I stood helpless". To explain her discontent with the instructor's conduct she said to another student "I'm standing at the board, proving a problem, and every second she's interfering with it…. But it was true too. I proved it. I proved it completely and you (instructor) didn't believe (it)!"

Though Jasmine's proof was not valid mathematically, she was expecting the instructor "to believe" her simply because she said she had proved it to herself at home. Jasmine's expressions of distress and embarrassment made it clear she was experiencing a gap between her current performance and what she thought were the expectations from her. It seemed there was a gap between the norms she was accustomed to (perhaps from former school experiences) and the norms practiced in the camp. Since Jasmine eventually overcame these difficulties and was one of the students who was described as the most successful in the camp, we were curious to reveal criteria that may have been used to evaluate her as such an outstanding student, and what in that specific unpleasant episode was the misalignment between her performance at the board and the expectations of the instructor. More generally, this case turned our attention to the complex social and evaluative processes that are involved in instructors' identification of students as exceptionally gifted. We therefore turned to a systematic mapping of the ways students were evaluated in the next camp (2014) as a way to capture institutional norms and incidents of alignment/misalignment of students with these norms.

The question that we asked was—what were the criteria for evaluation and how was mathematical competence constructed in the camp. We shall return to what this mapping could show us in the case of Jasmine in the discussion.

18.4 Methods

All classes in the camp as well as the daily staff meetings and the social activities were videotaped. All video tapes have been mapped according to activities and interesting events connected with identity and affect. A portion of these episodes have been transcribed to enable closer analysis of the interactional processes taking place in them. To unearth the instructors' criteria for evaluating students we used the transcripts of staff meetings. Staff meetings are usually a non-accessible place for outsiders, a "behind the scenes" location in an institution (Goffman 1978). They are a site in which important identity construction processes take place, in particular, 3rd person identity construction (Sfard and Prusak 2005) in which stories about the students are being formed without students having access to this private talk.

Our process of analysis proceeded as follows. We first marked in the transcripts of the staff meeting all the stories that refer to students and connect to norms of participation. For example, the following is a story of an instructor about Shai, telling of an incident in which she posed a problem to the group: "then Shai not just solved it (the question) but also said that 75 is not the "upper bound" but rather the bound is 97 and in 98 I found a counter example". Afterwards the instructor said about Shai "**he is really sweet and really clever**". The underlined portion which we call an "explanation" refers to mathematical stories that led to identity statements. The part in bold is a statement that is an identifying statement, including an attribute of the student.

For the present analysis, we mapped the staff meetings of the summer of 2014. Since the case of Jasmine was taken from the summer of 2013 we also returned to transcripts from 2013 to make sure that similar categories were present there as well.

Based on Heyd-Metzuyanim and Sfard (2012), we classified the stories into three types: Stories about activities, like "she solved that question beautifully". Reifying stories, like "she is smart", and mixture of the two previous types, like "she solved that question beautifully; she is so smart!"

When we give an example from discourse like "she is smart"$_{[\text{day2inst3}]}$ the code refers to day 2, instructor number 3.

18.4.1 Method for Classifying Students as "Successful" vs. "Unsuccessful"

Our first challenge in capturing the ways that staff constructed competence was classifying who, in fact, was identified as a successful student in the camp. This was because the camp did not have any formal measures (such as tests or grades) to assess students. The only process of assessment was related to the end-of-camp ceremony, in which a few students were declared as the winners, based on their performance during the camp. Much of the evaluative talk during the staff meetings was geared towards deciding who this winner would eventually be. Yet aside from the winners, other students did not receive a clear label or grade. Therefore, it was necessary to come up with an analytical method that would enable us to point to stories of successful vs. non-successful students as authored by the staff instructors.

The analysis of all the separate stories about students showed that these stories showed a developmental trajectory from stories about actions to reifying stories including attributes of students. A classification of all the 3rd person identification statements during 8 days of staff meetings revealed that the stories with explanations decreased as time went on, whereas the stories without explanations increased as the camp progressed. This process of reification into 3rd person identity stories allowed us to claim that instructors, indeed, formed stable identity stories about their students during the camp and shared those with each other. We then marked the students described as outstanding and those which the stories about them led to positive labels.

The next step was to examine what the criteria were for the construction of these stories and the labeling of students as exceptionally talented. We did this by going back to the explanations and to stories of actions that were told about the "exceptional" vs. the "non-exceptional" students.

18.5 Findings

We divide the criteria that we have identified into two general categories: criteria having to do with social, affective or non-mathematical aspects of students' activity and criteria having to do with specific attributes of mathematical activity.

Not every criterion was used to evaluate students that were labelled as outstanding, but these criteria always provided a positive story. In each criterion we describe its meaning, give an example from the discourse transcript and provide an opposing example from the students described as regular or "not outstanding" students.

18.5.1 Social/Affective Criteria

1. **Independently solving questions and exercises**

   In contrast to many current mathematics education programs that stress group work and collaboration, in this camp solving a problem independently was highly valued. This was especially prominent when students solved difficult questions on their own. For example, an instructor said "when I give a clue in class he just leaves the class, that's how serious he is."$_{\text{[day2inst3]}}$ The justification for the "serious" attribute was that a student was so insistent on solving problems on his own that he would shut out any possible clues or help from others. Conversely, students that were not labelled as outstanding were described as needing the assistance of the instructor "it is difficult for them to reach the result by themselves, they do understand (if they are shown the solution).I believe for Shalom it is even more difficult to reach an answer by himself"[day2inst3].

2. **Solving quickly and perseverance**

   Time was an important parameter in the Camp and quick solutions gained marked appreciation. Some examples are: "Yaniv flies over the material, he is really smart"[d4i3l13] or "He completed the exercises before everyone else"[day5inst2]. Another was, "They tried to solve this question for about an hour or so, and tried different ways to solve it, but did not succeed. Within 5 min, some students from her (other instructor) group came and solved it."[day8inst4]. Notably, the last story was told about two *groups* of students. One was identified as a "weak" group and one as an excelling group. Thus criteria for evaluation were sometimes employed on groups and not just on individual students.

   Interestingly, the opposite of quick solutions, perseverance in dealing with problems for long periods of time, was also used sometimes as a sign for exceptional students. For example "Shai thought a lot about the exercises but could not solve anything. Nevertheless he was fine with it, he said it's hard but I'll deal with it"[day8inst2].

3. **Engaging with mathematics for one's own satisfaction**

   Students who spent extra hours for studying were highly evaluated. Mostly, these activities were described as driven by students "enjoyment" or intrinsic motivation. Examples were "They are solving exercises for enjoyment at home"[day3inst2], or "They talk about math also in their free time"[day2inst4] or "They sat down in breaks and solved after evening activity, they go to the dorms and continue to look for solutions"[day1inst3].

4. **Solving on the board**

   Solving exercises on the board was one of the highlights of the camp's activities. The staff's rationale for this was that a solution on the board develops a culture of peer reviewing and improves mathematical writing and mathematical reasoning. The activity of solving on the board also served to identify certain students as more capable and courageous than others. For example, "he went to the board and solved (the problem) beautifully"[day3inst2] or by contrast "your students were happy because they did not have enough time to go to the board"[day8inst2] which implied that these students (that were described as weaker) were afraid, or anxious of presenting their solutions on the board.

18.5.2 Mathematical Criteria

So far, we have looked at criteria that could be used in any content area. Yet as common as these were in the evaluative stories about students, those were the mathematical stories that were most important in discriminating between the "regular" students and the exceptional ones.

5. **Solving advanced questions**

   Each day, the students received a worksheet with problems ordered according to a difficulty level that was decided by the mathematical advisor of the staff (a professor at the institute hosting the Camp). The staff did not take part in the rating of difficulty levels of these problems; they were simply taken for granted. Yet they formed an important tool for assessing students' success and comparing them with each other. Some advanced questions were assumed to be solvable only by the exceptionally talented students. For example "Shai is the one that solved the advance question on the board?" or "Yaniv is in the advanced!"$_{[day3inst1]}$ (by "the advanced" meaning that he has reached the advanced exercises). Students were also evaluated according to their emotional reaction to these "advanced" questions. "Oren is working on the advanced, the rest are deterred by it."$_{[day8inst4]}$. In fact, the use of "the advanced" in place of "the advanced questions" signaled that these problems had special significance in the Camp and formed a potent tool for classifying students' activity and identifying them according to it.
6. **Asking questions and generalization**

   An important mathematical action that was highly valued in the camp was asking questions that moved away from or generalized the original task. This was not only valued as showing inquisitiveness and curiosity but also as adhering to a mathematical norm in which asking progressively similar questions that vary with relation to only one parameter helps draw the limits of a certain mathematical principle. For example, when given a problem on primitive numbers (types of numbers used in modulo arithmetic with prime numbers), students were asked to find the primitive numbers in Z_{13}^{*}.Yaron played with the numbers and found that their inverse are also primitives. He then asked himself if that is a coincidence. While describing this event to her colleagues, the instructor said "He (Yaron) found that if a number is a primitive number then the inverse is also primitive"$_{[day8inst2]}$.
7. **Brilliancies**

   Exceptional solutions or ideas, which we termed "brilliancies" were an important yet one of the most elusive labels that were given to students' activity. Close examination of instances where students' solutions were labeled as exceptional or talked about excitedly between the instructors revealed these were mostly solutions that connected different subject domains together (such as combinatorics and number theory). Another form of "brilliancy" could be seen in this story: "Shai did it (solved the problem)! And he invented the concept of quadratic residues"$_{[day8inst2]}$. Whether Shai indeed "invented" the concept or just used a solution that reminded the instructor of a concept they knew from another mathematical domain is not clear. In any case, it was not the solution the instructor had expected, an apparently important feature of solutions identified as brilliant. More generally, instructors often assessed students' solutions by comparing them to standard Bachelor degree curriculum. Those solutions that were perceived as exceeding standard Bachelor-level solutions were exceptionally valued.

18.6 Discussion

In this paper we asked what the criteria were according to which students were evaluated and identified as exceptionally talented in a summer camp for "gifted" students. We found that they could be divided into social/affective and mathematical criteria. The social/affective were independence, quick solutions vs. perseverance, engaging with mathematics for one's own satisfaction and solving on the board. The mathematical criteria were solving advanced questions, searching generalizations and brilliant solutions.

We searched for the Camp's staff criteria of evaluation in order to gain insight into processes of becoming valued participants in this mathematical community, and these evaluation criteria can shed light on social construction in this community.To exemplify what this insight can offer, we now turn back to Jasmine's case and examine it in relation to these criteria.

Close examination of Jasmine's actions during the "argumentation on the board" episode, reveals that, in fact, she was adhering to most of these norms or meta-rules practiced in the Camp's community. The problem she claimed to have solved was solved independently. Indeed, she repeated several times that she had "solved it at home". This "solving at home" also adhered to another criterion of evaluating students: that of spending one's own free time on problems. The question that she chose to present on the board was one of the "advanced" questions, thus aligning herself with the criteria that valued specifically those questions. The main conflict between Jasmine's actions on the board and the expectations of her instructor had to do with her mathematical justifications. However, our examination of the total set of identifying narratives in staff meetings reveals that even this conflict could be explained by paying attention to the practiced norms. In fact, the "brilliant" solutions were rarely examined very deeply. Mostly, instructors evaluated them pretty quickly and intuitively as correct and exceptional. This does not mean these solutions were wrong. Our examination of some of them after the camp had ended revealed they were mostly correct. It simply points to the fact that Jasmine had reasons to expect her solution to be accepted without much inquiry.

We originally started our quest into the norms of the Camp believing that Jasmine was simply in a process of aligning herself with new norms and leaving old (school) ones. After the current analysis, it became clear the story is more complex and that Jasmine's emotional distress may have been a product of some incoherence in the norms of the community itself.

One of the challenges of beginnings in a new environment is in understanding "the rules of the game", which are the ways of behaving and the expectations of the institution (Goffman 1978). These expectations can be found by looking at the criteria for evaluation that we found above. When there was a gap between the expectations of the institution and the way Jasmine acted, we found many emotional expressions. This emotional episode was both a sign of temporary misalignment and a stage in Jasmine's growth towards a more central role in this mathematical community.

More generally, we found that the goals of the mathematical camp were not just educational, in the sense of enriching the students' mathematical knowledge. The camp had also diagnostic goals, whereby the academic mathematicians attempted to identify potentially promising students, those who may populate the next generation in the field. With relation to the social construction of giftedness, we saw this process of identifying students as a facet of such "gifted identities" construction.. By focusing on the criteria for success and for a student's promising potential, we showed what the community of academic mathematicians, at least in the case explored, considers as ideals for such "giftedness". This study thus continues studies that have shown how mathematical identities are constructed in activity and discourse (Bishop 2012; Heyd-Metzuyanim 2013). However, to our knowledge, it is the first to show this process in exceptionally successful students.

Practically, the contribution of this paper is in the explication of criteria for evaluation of gifted or promising students, which often remain tacit in the mathematical community. We hope these criteria can be used in order to understand the evaluation criteria of other such programs that do not yet have clear criteria for the evaluation of giftedness or promising potential of academic young mathematicians.

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Chapter 19
Identity and Rationality in Classroom Discussion: Developing and Testing an Analytical Toolkit

Laura Branchetti and Francesca Morselli

Abstract This contribution originates from a joint project aimed at networking theoretical tools and employ them to better understand teaching and learning episodes. Combining the construct of identity and that of rational behavior, we investigate episodes from classroom discussions, showing the interplay between students' identities and rational behavior, and the crucial role of the teacher in reinforcing student's identities.

19.1 Introduction

This paper is part of a ongoing project aimed at networking theoretical tools (the construct of identity and that of rational behavior) to reach a better understanding of teaching and learning episodes. In particular, we chose the combining strategy, which consists in "looking at the same phenomenon from different theoretical perspectives as a method for deepening insights on the phenomenon" (Prediger et al. 2008, p. 172). In a former research (Branchetti and Morselli 2016) we studied group works in mathematics classes. In this contribution we turn to another phase of the teaching and learning process, namely when group work has to be shared with the school-mates and the teacher during a classroom discussion.

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19.2 Theoretical Background

19.2.1 Classroom Interaction

We rely on a sociocultural perspective, according to which the learning of mathematics takes place in a social context through interactions. We also consider culture a decisive factor in the discussions, since it may orient individuals' interaction in the classroom (Radford 2006, 2011), and in particular teachers' interventions (Radford 2006). Students' interaction in a small group is presented by Radford (2011) as a complex process in which students are involved at many levels, not only at the cognitive one. The processes of objectification (students align their thoughts with culture) and subjectification (a thinking and becoming process of being-with-others mediated by alterity) that take place in the teamwork are mediated by culture (Radford 2008). In line with Radford we consider interactions as potential catalyzers or, conversely, obstacles, in the learning processes of the individuals involved in the discussion. The role of the teacher in classroom discussions was deeply studied also by Bartolini Bussi (1996), who elaborated a theoretical framework to analyze different kind of discussions and strategies of interventions of teachers.

19.2.2 Identity

There are at least two macro-categories of approaches to the investigation of the role of identity in mathematics education: "local" studies, that investigate the evolution of a classroom discussion in stages or brief periods, and "longitudinal" studies, in which students' behaviors are investigated in longer periods. As Gomez-Chacon and Ma (2011, p. 2) stresses: "it is not enough to observe and know the stages in the process of emotional shifts or changes during problem solving ("local affective dimension"). It is also not enough to detect cognitive processes associated with positive or negative emotions. We need to contextualize their emotional reactions within the social reality which gives rise to them. The "global affective dimension" is understood as a result of the paths followed by the individual in the local affective dimension. These paths are established with the cognitive system and they contribute to the construction of the general structures of one's self concept as well as beliefs about mathematics and the learning of mathematics. [...] Identity is understood as a structured joining of elements which permits the individual to define himself/herself in a situation of interaction and to act as a social agent." Although we agree with these important remarks and we keep in mind that the behaviors we observe depend on past experiences that structured the students' identities in the classroom, we will carry on the first kind of analysis, since we are interested more in the local effects of intertwined factors that influence rather than in exploring how in general identities end to be structured by the practice. In particular we investigate with a networked framework the limits and potentialities of group works and of discussion of group results with the peers and the teacher, in order to find out factors that make them effective or not in the mathematics "social learning processes". The investigation of this dimension of the teaching-learning processes has been carried out by different research groups all over the world. One of the most relevant perspective for our work was developed by Sfard and her research group and deals with the role of identities in the classroom activities. Sfard and Prusak (2005, p. 1) defined identity "a set of reifying, significant, endorsable stories about a person." This definition is deeply related to the commognitive perspective (Sfard 2008), whose cores are the notions of thinking and communicating. Since thinking is a form of human doing, it can only develop as a collective patterned activity: "Thinking is an individualized version of (interpersonal) communicating." (Sfard 2008, p. 81). Heyd-Metzuyanim, in collaboration with Sfard, continued the work opening up to other influences and deepening some aspects. In her paper published in 2009 she distinguished the different ways of interacting of each student in terms of individuality, in particular in a mathematics group work, so as "to point out how identity and emotional processes influence the effectiveness of learning. Subjectifying may help in mathematizing or obstruct it" (Heyd-Metzuyanim 2009, p. 2). The subjectification process is linked both theoretically and operationally to the identity construction process and to the mathematizing activity in group work. Heyd-Metzuyanim framed also mathematizing and subjectifying in the commognitive perspective: mathematizing is communicating about mathematical objects, subjectifying is communicating about participants of the discourse. Identities stories can talk about the way in which a person relates to the mathematics and so can influence the participation in the teamwork, the engagement, and definitively, success or failure in mathematics activities. The author looked at verbal and non-verbal acts of subjectification, distinguishing participation and membership. Then she classified the acts clarifying whether they are identifying processes or not. Identifying utterances (verbal or non-verbal) are "those that signal that the identifier considers a given feature of the identified person as permanent and significant." (Heyd-Metzuyanim 2009, p. 2). The prototypical cases of different aspects of the relation between subjectifying, mathematizing and identifying are exemplified in the quoted paper by Heyd-Metzuyanim (2009). In a further work, Heyd-Metzuyanim (2013) employs the commognitive framework to analyze teacher-individual interactions and argues that in some cases interaction is non-productive and turns into a co-construction of the student's identity of failure. The study sheds a new light on the role of the teacher in interaction with students, since he/she plays a role not only in the mathematizing process, but also in the identifying one. Moreover, the study brings to the fore the existence of different forms of participation to the mathematical discourse, namely acting "as if" she were participant into the discourse, pretending to mathematize but, in reality just pursuing the designated identity of participant. These findings, concerning the crucial role of the teacher in co-constructing identity and the alternative forms of participation, will also help us to frame our reflection.

19.2.3 Rationality

The construct of rationality was developed by Habermas (1998) in reference to discursive practice and later adapted to mathematical activity (see: Morselli and Boero 2009, for the special case of mathematical proving; Boero et al. 2010 for its integration with Toulmin's model and its use for classroom implementation). According to Habermas, rational behaviour may be seen as three interrelated dimensions: epistemic dimension (related to the control of the propositions and their chaining), teleological dimension (related to the conscious choice of tools to achieve the goal of the activity) and communicative one (related to the conscious choice of suitable means of communication within a given community). In the case of mathematics, fostering students' approach to argumentation and proof as a rational behavior means promoting the students' acquisition of basic content knowledge, but also the ability to manage (from a logical and linguistic point of view) the reasoning steps and their enchaining and the ability to communicate the arguments in an understandable way, thus taking into account three interrelated dimensions:

- "an epistemic aspect, consisting in the conscious validation of statements according to shared premises and legitimate ways of reasoning [. . .];
- a teleological aspect, inherent in the problem solving character of proving, and the conscious choices to be made in order to obtain the aimed product;
- a communicative aspect: the conscious adhering to rules that ensure both the possibility of communicating steps of reasoning, and the conformity of the products (proofs) to standards in a given mathematical culture" (Morselli and Boero 2009, p. 100)

As outlined in our previous study (Branchetti and Morselli 2016), when dealing with peer interaction, communicative dimension plays a crucial role, as well as epistemic rationality, which is linked to the possibility of changing opinion:

> "Someone is irrational if she puts forward her beliefs dogmatically, clinging to them although she sees that she cannot justify them. In order to qualify a belief as rational, it is sufficient that it can be held to be true on the basis of good reasons in the relevant context of justification - that is, that it can be accepted rationally. The rationality of a judgment does not imply its truth but merely its justified acceptability in a given context" (Habermas 1998, p. 310).

In the subsequent part we will briefly summarize our previous results concerning rationality and identity during group work as a special case of peer interaction.

19.2.4 Identity and Rationality: Our Former Study

In a former study (Branchetti and Morselli 2016) we analyzed a group of middle school students (grade 6) dealing with some questions concerning negative numbers. They worked in group so as to produce a shared answer to the question posed by the teacher. The analysis in terms of subjectification and identity revealed some recurrent utterances and behaviors. The first observed phenomenon concerns students' different ways of participating (or non-participating) to the group work. This issue is crucial because participation affects personal concept development. Another issue concerns agreement (or lack of agreement) and the different reactions of students when their mates do not agree with them. Our working hypothesis was that dimensions of rationality may help to understand such phenomena. First of all, teleological rationality may refer to different goals; furthermore some interventions are clearly on communicative or epistemic level. Combining the two analysis, we suggested that

> "individual participation or resistance to participation and also membership or non-membership may be described in terms of dimensions of rationality: if individual interventions are on different levels (epistemic vs communicative), it seems very difficult to reach an agreement. If a dimension prevails, some students can avoid to participate. Moreover, individuals may have different aims and act accordingly (teleological rationality), may consider the epistemic dimension or not, and this may affect individual/collective conceptual change " (Branchetti and Morselli 2016, p. 1151).

Accordingly, we claimed the need of taking into account all the three dimensions of rationality and we proposed the mismatch between dimensions (different students focus on different dimensions) as a possible source of difficulty during group-work. To sum up, students' interaction in group work (without the teacher's interventions) may be affected by social dynamics that lead students to look for a "forced agreement" that may cause the loss of constructed knowledge because of a negative interaction with the pairs due to identifying and/or subjectifying acts or because of a difference in Habermas' prevailing dimension in the discussion. The two potential causes may not be disjointed but rather interconnected. In this contribution we turn to another kind of activity (the moment when a group presents the solution to the whole class) to test the transferability of such conclusions and also to refine and adapt the theoretical tools at disposal, in a situation where also the teacher plays a role.

19.3 Context

The teaching experiment we refer to was carried out in a lower secondary school (grade 7) in the north-west of Italy. The teaching experiment is part of a bigger data collection concerning a joint research work on the development of argumentative competences (Levenson and Morselli 2014). The task sequence was inspired by a formative assessment unit of the MARS project (http://map.mathshell.org/materials/lessons.php). The mathematical content at issue was ratio as a way of comparing quantities. Here is the task proposed to the students:

> Guglielmo loves organizing parties with his friends. When he and his friends get together, Guglielmo makes a fizzy orange drink by mixing orange juice with soda. On Friday, Guglielmo makes 7 liters of fizzy orange by mixing 3 liters of orange juice with 4 liters

of soda. On Saturday, Guglielmo makes 9 liters of fizzy orange by mixing 4 liters of orange juice with 5 liters of soda. Does the fizzy orange on Saturday taste the same as Friday's fizzy orange, or different? If you think it tastes the same, explain how you can tell. If you think it tastes different, does it taste more or less orange? Explain how you know.

The students worked individually, afterwards (and before any feedback by the teacher) they were asked to work in small groups (three to four students), share their solutions and, if possible, to reach a common agreement. Afterwards, there was a balance discussion, where the students of each group had to report to all the classmates the solution and convince them of its validity.

19.4 Data Analysis

In this contribution we focus on the moment when a group of four middle-high level students (Francesco, Elena, Giacomo, Nicolò) report its solution. Due to space constraints, we will propose only some excerpts. In the first one, Francesco goes to the Interactive Whiteboard and exposes the group solution to the whole class.

1. Francesco: Anyway the answer is yes, for us they taste the same, because in order to compare them, to see if they are the same or not, I did the least common multiple, then.. (writing the solution to on the interactive whiteboard)...
2. Francesco. But if we look at this and this (the fraction for saturday) 1/63 is here (points at the orange) while the other 1/63 is here (points to the quantity of soda of friday). Then they balance [...]
3. Teacher: because you say: on saturday we have 1/63 of orange more than on friday.
4. Francesco: yes.
5. Teacher: right?
6. Francesco: right.
7. Teacher: but orange or soda?
8. Francesco: well... on friday soda and on saturday orange.
9. Observer: then, do they taste the same or not?
10. Francesco: yes...
11. Teacher (going to the interactive whiteboard): then, think to your reasoning... you said that here (she points at the orange of friday and saturday) there is a difference of 1/63 and here (she points to the soda of friday and saturday)?
12. Francesco: the same.
13. Teacher: the same. So, now draw your conclusion from that point.
14. Francesco: then..
15. Teacher: I don't understand the conclusion. You said: here it is one more.
16. Nicolò: ah, teacher!
17. Francesco: I am not able to explain this, the difference is always the same.

In line 1, Francesco proposes the explanation in terms of fractions. In line 3, the teacher reformulates the explanation given by Francesco. The interactions between the teacher and the student Francesco reflect two needs for the teacher: involving all the students into the discussion, making as much clear as possible the explanation of the group of Francesco, and leading the students of the group to realize that the explanation does not work and needs to be amended. Some reformulations of the teacher are at first perceived by Francesco as requests to clarify the explanation (communicative level), while the final aim of the teacher is to bring to the fore that the method of fractions leads to the opposite conclusion (the taste is not the same) (epistemic level). In line 11 the teacher reformulates Francesco'explanation, with the aim (more and more explicitly at epistemic level) of making Francesco revise his reasoning. We may note that the teacher speaks to Francesco showing assurance. In lines 13 and 15, the teacher intervenes on the epistemic level. Francesco is still on his position, claiming he is not able to explain it in another way. We may note that in this part, the other students of the group and the other classmates seem to be out of the interaction. Nicolò (line 17) tries to get into the discussion, but nor the teacher and Francesco linsten at him. If we look at the verbal acts, we may note that at the very beginning, when expressing the solution, Francesco uses the plural pronoun (for us). Anyway, when expressing the explanation for the solution, Franesco turns to the singular pronoun (I did). Also the teacher talks to him using the singular pronoun, even if Francesco is supposed to report a group solution. Immediately after, Nicolò succeeds in intervening and reports the group explanation ("Yes, the taste is the same, because, indeed, if there it is one more, for us the taste is the same because anyway the 28 is added to the 35. Up (he means on the row of saturday) the 36 is added to 27 and…that is to say, you get the same"), as established during the group-work. His aim is to support the group from a communicative point of view (making the solution clear to the classmates). At this point both Francesco and Nicolò seem "lost" in a pure arithmetic game, where having the same result (the sum is 63) is perceived as a warrant for the fact of having the same taste. In terms of interaction, we may observe that it is the observer to encourage Nicolò to talk, while the teacher is still focused on the interaction with Francesco. Nicolò's intervention is not taken into consideration by Francesco and the teacher, who go on with their interaction. Immediately after, also Elena is invited to intervene, but she renounces to talk ("No, it is that…he explained it better and I gave up! (laughing)"), identifying herself as less good in maths than Francesco. Afterwards, thanks to the interventions (questions) of the teacher, Francesco finds out that something does not work and proposes a new solution. Elena rapidly changes her mind, grasping the new solution and succeeding also in re-explaining it to the mates. Nicolò, on the contrary, does not agree with the new solution and distances himself from Francesco and Elena. At first he expresses his doubts, but his explanation is disturbed by Elena, who makes gestures to signify that Nicolò is wasting time ("Teacher, I mean. The taste remains the same because…stop for a moment! (speaking to Elena) After the orange is added, then the taste remains the same"). We may say that Elena identifies Nicolò as less good in maths than Francesco, thus as not deserving the same attention than Francesco. The teacher seems to agree, or at least she does not reproach Elena. Elena tells again the new explanation, but Nicolò does not accept it.

18. Elena: in one case, on friday, there is 1/63 soda more, in comparison to saturday, and on saturday there is 1/63 more orange.
19. Nicolò: so, there is always a 1/63 difference. Nicolò: indeed I had not written this thing, I had written another thing, anyway…
20. Teacher: what did you write?
21. Nicolò: I had written as the others, that there was a difference of 1 liter (he laughs; also Elena laughs).
22. Teacher: and how did you convince of their…
23. Nicolò: because after I had seen… anyway… because at the beginning they had said that it (the taste) was the same then I had convinced myself…

Nicolò's interventions brings to the fore that the group solution was not a really agreed solution: the group had reached an agreement in terms of final answer (the taste is the same) but not in terms of explanation (difference of 1 liter versus fractions). Nicolò had accepted the explanation with fractions just because it initially led to the agreed solution (same taste). Now that the method leads to the opposite conclusion, he is no more ready to accept it. While Francesco and Elena changed their mind in order to accept the solution given by the trusted method of fraction, Nicolò refuses the method in order to keep the (intuitive) solution.

19.5 Discussion and Preliminary Conclusions

The analysis carried out through the lens of identity, that we showed just reporting some sentences, highlights differences in students' identities from the point of view of the relationship with mathematics, the classmates and the teacher. The most of the data that we can categorize as identifying acts are verbal and indirect, but they are so recurrent to allow us to consider them as significant. Francesco results the most considered as good in mathematics, both by the teacher and by the groupmates. We highlighted the way the teacher speaks to him, taking into account his answers, turning sometimes from the group to him. This has also an effect in terms of participation: Francesco is the most involved in the discussion. Also Elena recognizes his reliability. Nicolò is identified, directly and indirectly, as less influent in the group than Francesco, both by Elena and the teacher. The teacher identifies him indirectly and through non verbal acts, when quite she ignores his interventions and nods to Elena who's making fun of him, while Elena is more direct. Looking at teachers' intervention we can also see identifying subjectification acts in terms of rationality. In the first part of the discussion we may see the intertwining of the epistemic and communicative dimensions. At first the teacher intervenes at the communicative level, but then the interventions become more and more epistemic. When the interventions turn to be clearly at epistemic level ("Now draw your conclusion from that point"; "I don't understand the conclusion"), Francesco is able to revise his explanation. Being ready to revise the explanation, when it does not work anymore, is connected to epistemic rationality. Nicolò's interventions are very interesting form the point of view of rationality. His initial agreement with the mates, based on the final conclusion (but not on the common kind of explanation) may be read in terms of lacks at epistemic level, as well as his disagreement towards the new explanation and solution proposed by Francesco. When the method of fractions leads to a new conclusion (different tastes), which is against the former one, Nicolò refuses the method, rather than changing the final conclusion. The different identities may have influenced the group work evolution, since, as Nicolò said, the group turned to Francesco's choice of using fractions without a deep comprehension of the method itself. The choice redirected all the groupmates' strategies towards an approach they didn't master very well. This social dynamic lead the group to present a solution that was not a group solution (Branchetti and Morselli 2016), rather a "forced agreement" based on Francesco's epistemic identity and, possibly, Elena's communicative one. Nicolò's claim (line 23) sounds very interesting in this sense. The internal dynamics that underlie the group work emerged during the class discussion, which confirms to be a crucial moment, not only for establishing a common class solution, but also for giving individual contributions and voices, that had disappeared during group work, to appear again. Without this discussion maybe Nicolò would have just reached a superficial understanding of the problem, and Francesco himself would have conserved a wrong idea, convinced of a wrong argumentation by the agreement of the group. The analysis brings to the fore the crucial role of the teacher. The teachers' behavior may contribute to reinforce the students' identities through indirect or direct, verbal or not verbal acts, and so it may influence also students' future participation in group works and other mathematical activities. Furthermore, the rationality levels of teachers' intervention may depend on the identity of the students.

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Chapter 20
Developing an Analyzing Tool for Dynamic Mathematics-Related Student Interaction Regarding Affect, Cognition and Participation

Laura Tuohilampi

Abstract In this study, a video excerpt of two boys working on a mathematical open-ended problem is discussed. In the video, affective and social factors influence and partly constrain development of logical thinking. Analyzing such an episode is challenging, as appropriate tools are few. This study elaborates the video excerpt to find out what affective, cognitive and social phenomena exist in the episode, aiming to develop an analyzing tool for such purpose. In addition, a framework called Patterns of Participation will be adapted to test its usefulness to the analysis. As a result, it was found out that many essential features of the episode were revealed through that framework. However, it was suggested to include theories of emotions, student engagement and positioning to make the analysis more profound.

20.1 Introduction

Research in the affective domain has been calling for more socially oriented studies (Skott 2015; Tuohilampi et al. 2016; Lerman 2000). This is because by focusing on individuals' affect structures without much context (e.g. when affect is measured using questionnaires to cover individual affective traits solely), it has been ignored the role and the possible impact of expectations, atmosphere, tasks and participation to social processes, to mention just a few contextual factors. Nevertheless, to include contextual factors has so far being challenging, as the tools are still under development, the concepts and implications have remained blurry, and the frameworks to combine affect and cognition are few. To date, the affect is typically researched distinct from cognition, probably to some extent as a result of the lack of appropriate methodologies. Yet Clarke (2015), reported that researchers in the field have challenged the dichotomy, and there is widespread discussion about the complex and intimate connections between the two. It seems that there is a need for methodologies that could consider the dynamic interaction including multiple

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dimensions: social, affective and cognitive orientations that guide or constrain the actions in learning situations.

In this study, I will use a video lesson of two 5th grade boys working on a mathematical open-ended problem. In the video it looks like the boys differ in their degree of mathematical and social determination, and that the more confident one seems to be the more self-absorbed. When working on the problem, the boy that seems less determined starts to reach a solution, but this gets ignored by the other boy. Instead of arguing, the boy with the solution accepts the other boy's suggestions, and ultimately agrees with the other boy about an unprepared solution. This is an interesting episode, and makes one wonder if there are mechanisms that make affective social responses in the learning environment outweigh cognitive thinking? As claimed earlier, the tools to analyze such an episode are few, so in this study I elaborate the video excerpt aiming to build an analyzing tool for that purpose. I will draw from grounded theory to find out what affective, cognitive and social phenomena exist in the episode, and adapt a framework called *Patterns of Participation* (PoP), developed by Skott (e.g. 2015) to test the framework's possibilities in recognizing the essential information of the episode.

20.2 Theoretical Framework

The deterioration of mathematics related affect is a worldwide problem, and the phenomenon has been studied extensively. Nevertheless, the focus has mostly been on the individual factors (such as self-competence, enjoyment, or goal orientations; see Chamberlin 2010) which furthermore have been measured unreliably (Tuohilampi et al. 2014). Such factors connect poorly to cognitive development (see meta-analysis in Ma and Kishor 1997) and give only superficial basis for the subsequent analyses. What follows is that we lack tools for deep analyses about the dynamics between affect and cognition.

The affect and cognition seem to intertwine with no clear causation. Barnes (2015) suggested in her study that cognitive moments of understanding can generate a significant change in students' mathematical conceptual knowledge and further in their emotional trait towards mathematics. In her study she also stated that in order to achieve the moments of understanding, the students need to be given engaging tasks. Thus, there is a need for emotionally engaging starting point wherein cognitive changes may emerge, and emotional changes may follow or happen concurrently. The idea was supported by a study of Morselli and Sabena (2015), wherein it was seen that having emotional ups and downs (e.g. being inspired—getting confused—succeeding) was an effective way to improve cognitive understanding. The engagement—the emotionally enabling starting point—is further related to the learning atmosphere that is socially set. In a study of Tuohilampi et al. (2015), it was argued that students need to be aware of the socially acceptable norms in their learning environment in order to get their social needs fulfilled (or social disappointments prevented). These norms frame what kind of actions are socially accepted during mathematics classes, and thus frame the quality and amount of efforts the students will make.

The need to consider context has emerged from the empirical findings claiming that the beliefs people hold (espoused beliefs) are not necessarily congruent with the enactment of those beliefs (enacted beliefs). The idea is that the context frames or even constrains what beliefs are to be followed and to what extent this is to be done. In study undertaken by Tuohilampi et al. (2015) the context was acknowledged, but not merely as a basis for the individual beliefs, nor as solely a reification of the individual beliefs. The social context (e.g. learning atmosphere or the interaction between students and teacher) was seen as something in which the subjects participate: the learning atmosphere may appear boring, and in such a classroom the students are likely to get social reflections that indicate boredom in their surroundings. Yet, the students may individually hold, for example, feelings of interests or beliefs of mathematics being enjoyable. The idea of participating in the social context is somewhat compatible with the theoretical model of Skott (2015), who has introduced Patterns of Participation (PoP) as a tool to perceive contextual factors. Skott, who draws from social practice theory, calls for a shift from mental reifications to participation in different social practices. Skott argues that the inconsistencies between the espoused and enacted beliefs has been understood to be a result of other goals that need to be highlighted, and claims this approach making belief holders to look like incongruent and unpredictable. In Skott's view, it is not (only) that the beliefs differ, or that the beliefs exist in separate (possibly contradictory) clusters, but that the experiences the people have gained in different situations, context and interactions differ as well. Thus, in Skott's perspective the possible inconsistencies researchers may notify between espoused and enacted beliefs are not necessarily a sign of beliefs' high situatedness (that the context impacts strongly how and why particular beliefs result in certain actions) or low stability (that the beliefs impact the actions, but in a varied way as they vary themselves). Skott's idea is to see the beliefs and the activity being in a connected process, the beliefs being formulated within the activities that are colored by past experiences. In Clarke's (2015) words Skott tries to solve the problem of causation by shifting the connection into identity. This way you see experiences, beliefs and actions as intertwining dynamically together, which makes the thinking of causation actually irrelevant. However, having the question of causation included or not, Skott's framework of participation seems like being a useful frame in order to recognize whether students working with a mathematical task in a group (or in a pair, as in my case) participate processes for example with other students, with their mathematical knowledge, or with self-posited or teacher settled expectancies, and what could be the result of such patterns of participation.

The framework of Skott does not give direct answers about how to connect cognitive aspects—in my case mathematical thinking—to affect structure. However, it seems like a useful basis in developing a tool to analyze an episode consisting of social, cognitive and affective dimensions, especially as Skott's model seems not be restricted only to affective patterns. In this study, the model will serve as the ground of the analysis. In order to develop a tool to analyze what kinds of mechanisms make affective social responses in the learning environment outweigh logical thinking, we need to know *how the patterns students participate can be used to explain a combination of social, affective and cognitive dynamic interaction during mathematical problem solving?*

20.3 Method

In this study, I will use a video data that was gathered within the research project that aimed to develop mathematics learning during primary school years in Finland and Chile (see further description of the project in Laine et al. 2012). Here, I focus on two Finnish students' problem solving episode that was filmed during academic year 2012–2013 in a public school in region near to Helsinki. The examinees were 5th graders (approximately 11 years old) at the time of the filming, and at that time they had participated the intervention including monthly problem solving lesson for more than 2 years.

In the filmed problem solving lesson, the students were working on a task called Patch exercise. In the exercise, the prompt was: *You have a patch for farming different kinds of plants. Your patch is shaped like a rectangle and its sides are 10 and 5 m long. Make different designs how to farm your patch. You should take into account the distances the different plants need when you put them into ground. How do you decide what plants to include? Notice also that you should choose at least one plant from each group (plants, bushes and seeds). A list of plants was given with their spatial needs, e.g. garden pea (distance between seeds 4, 5 cm, distance between rows 40 cm).* The students had 45 min to work with the problem, and they were instructed to work in pairs. From the video you can see the facial expressions and gestures of the boys, and you can hear their talking. Yet, you cannot see the task or its development. However, the outputs of the students were collected, so you can see there what the students produced during the lesson.

In the first phase of the analysis, I made a content log of the video. Using the log and the original film I tried to search what was essential in the episode, bearing in mind the framework of Skott. I drew from grounded theory, listing the essential features and making categorizes and connections with them. After that, I pursued to recognize how the found features related to the two frameworks. Finally, I constructed the features' relationships based on what type of participation they possessed. See Table 20.1 to get the idea of the analysis.

20.4 Results

The episode seemed to consist of (a) the students' mathematical thinking, (b) the affective factors of the students, (c) the style of the discussion, (d) the roles and the positioning the students give or take (e.g. negotiator, proposer or innovator as roles;

Table 20.1 Excerpts of the episode and their analysis in chronological order

Transcript	Act	Actor	Pattern
(35.27) Boy 2 to the teacher: Boy 1's brains are really working, but mine are not	Boy 1's brains are really working, but mine are not	Boy 2	Posititioning
Teacher to boy 2: Have you explained your thinking to boy 2? Could you do it now?	(Not considered in this analysis)	(Teacher)	(Not considered in this analysis)
Boy 1: I cannot explain. My brains are about to explode! (Starts to list numbers in some kind of sequence)	I cannot explain My brains are about to explode!	Boy 1 Boy 1	Shared mathematical thinking Own mathematical thinking
Boy 1: I was so close to the solution! (Keeps on listing number in some kind of a sequence)	I was so close to the solution! Listing numbers (aloud)	Boy 1 Boy 1	Own mathematical thinking Shared mathematical thinking/Own mathematical thinking

defender or commander as positioning), (e) the profiles of the students, and (g) the patterns the students participate.

The style of the discussion (c) and the roles and the positioning the students give or take (d) can be seen as two different sides of the same phenomenon, as the discussion style reflects the students' roles and positions and vice versa. Further, the mathematical thinking (a), the affective factors (b) and the roles, positioning and discussion styles (c) and (d) can be seen as patterns the students participate: they need to consider their own (as well as others') logic, their own (as well as others') affect structures (aspirations, emotions, self-competence, self-confidence etc.), and they need to consider the social interaction that emerges from and appears in roles, positioning and discussion styles, which are further intertwined in the affect structures the students possess. Finally, the profiles of the students can also be seen as affective: the goals, values and aspirations representing its motivational dimension, the self-beliefs representing its cognitive dimension and feelings and temperature representing its emotional dimension. The essential features of the episode can be placed under different patterns the students participate, and the patterns of participation illustrated in Tables 20.2 and 20.3 can be formulated.

According to Boy 1's PoP, he participates intensively with his own mathematical thinking and ideas. He tries to find out what needs to be done in the task, and he seems particularly interested in finding out the solution. He participates only loosely with the shared mathematical thinking that would be required in such a collaborative process (he gives no justifications or criticism to his thinking, he turns down another boy's suggestions, and he works rather individually). Boy 1 participates

Table 20.2 The PoP of the more determined boy (Boy 1)

Pattern	Participation (excerpts)
Own mathematical thinking	Boy 1 (to no one specifically): *I'm just about to get to the solution!* Boy 1 (to no one specifically): *My brains are about to explode!*
Shared mathematical thinking	Boy 1 to Boy 2: *I cannot explain* Boy 1 to Boy 2: *No, it cannot be that* *Be quite I'm thinking!*
Mathematics needed in the task	Pattern appears in continuing efforts the Boy 1 is making to find the solution
The teacher	Teacher to Boy 1: *Can you tell it?* Teacher (friendly) to Boy 1: *Haven't you explained your thinking to the Boy 2?* Teacher to Boy 1 in whole class situation: *What should we do now, do you have a suggestion?*
Another boy (Boy 2)	Boy 1 to Boy 2: *Can you give me an eraser?* Boy 1 to Boy 2: *What plant do we take next?*
Other classmates	Pattern appears in Boy 1's working style and gestures: not much physically oriented towards others
Orientation (motivational dimension of affect)	Boy 1 (to no one specifically at the end of the lesson, when no one was about to ask about the solution any more): *Now I realized it all!* Boy 1 to Boy 2: *Don't show the paper to anyone!*
Self-competence (cognitive dimension of affect)	Boy 1 to Boy 2: *Now I got it!* Boy 1 to Boy 2: *Don't interrupt... Oh no, I got confused again!*

quite intensively with the teacher pattern: the teacher gives much space and voice to him (also in whole class situations); he is little criticized by the teacher; and he is used as a sort of a competent assistant by the teacher. Boy 1's attitude towards Boy 2 seems like scatterbrained. He interacts only loosely with Boy 2, who seems more like a useful servant to him, yet in a rather neutral way (the two boys seem to have a positive or at least not a negative relationship). The pattern with other classmates looks like somewhat distant. No one in the classroom interrupts him, and he does not pay much attention to others. However, as Boy 1 seems to be performance oriented, the significant others are certainly important. The performance orientation appears in his strive to find the solution and his willingness to announce that he have found it (even though this did not actually happen) as soon as it is clear that no one will ask him for any justifications. He has high aspiration to solve the problem, not merely because of his performance orientation, but also as he seems to consider himself competent and thus capable to work appropriately with the mathematics needed (the pattern with self-competence) Boy 2's PoP shows him to participate intensively with shared mathematical thinking: he criticizes his own suggestions, he asks for more justifications from Boy 1, and makes propositions regarding the task and the possible

Table 20.3 The PoP of the less determined boy (Boy 2)

Pattern	Participation (excerpts)
Own mathematical thinking	Boy 2 to Boy 1 (about Boy 1's incorrect proposition): Oh yes that's true Boy 2 to Boy 1 (about Boy 1's incorrect proposition): That might work Boy 2 to Boy 1 (when Boy 2 starts to reach the correct solution): Could it be... Can you think it through considering... Boy 2 to Boy 1 (when Boy 2 starts to reach the correct solution): Listen to me, I guess you can find it out by...
Shared mathematical thinking	Boy 2 to Boy 1: So, what is it you are now trying to solve? Boy 2 to Boy 1: If we have time, can we still take peas? Boy 2 to Boy 1: Can you now explain your theory?
Mathematics needed in the task	Boy 2 to Boy 1: What should we do here? Boy 2 to Boy 1: Would it be possible to find it out by...
The teacher	Pattern appears for example when Boy 1, who does not have an eraser, asks an eraser from Boy 2, who neither has one. Boy 1 calls out the teacher to give a new one to Boy 2, telling Boy 2 has lost his. The teacher gives the eraser without a comment
Another boy (Boy 1)	Boy 2 to Boy 1: I hear nothing if you talk to that direction Boy 2 to Boy 1: So what is that you want to do now? Boy 2 to Boy 1: I promise to keep silent now
Orientation (motivational dimension of affect)	Socially/contextually oriented: Pattern appears in the drive to meet the expectations of the task, the learning goals and the social circumstances

ways to think. When it comes to his own mathematical thinking, his participation is low especially at the beginning of the episode, as he approves Boy 1's suggestions even if they were unjustified or unprepared.

Yet, the participation with his own mathematical thinking increases when he starts to reach a logical solution. He participates with the mathematics needed in the task, as he tries to find out what needs to be done, especially what is the appropriate process in the problem (what mathematics is needed?). The relationship with the teacher seems to be rather disconnected (not negative, but perhaps just neutral). The teacher does not ask much from the boy 2, nor does the Boy 2 ask much from the teacher. The pattern with Boy 1 is highly intensive. Boy 2 is striving to co-operate with Boy 1. Boy 2 is not necessarily searching for acknowledgement to himself or his ideas, but for a collaboration and a fulfillment of the mission. Boy 2's actions towards Boy 1 are friendly and appreciative. Boy 2 is open to surroundings and he reflects much what happens around. He orients himself physically to others' directions, especially towards Boy 1. The challenge to Boy 2 is that he needs to balance between two contradicting goals: how to co-operate with the Boy 1's incorrect or unprepared mathematical ideas and Boy 1's determination, and how to progress with the task and acknowledge his own logic. The overarching goal for him seems like to be flexible with whatever requirements. It is not that Boy 2 seems like having a low self-esteem; his functioning looks like natural for him, and to be flexible and meet the expectations seem to be important and easy for him in a self-evident way.

20.5 Discussion

This study aimed to develop a tool to analyze what kinds of mechanisms make affective social responses in the learning environment overrule logical thinking. The patterns students participate was used to explain how a combination of social, affective and cognitive dimensions interact during mathematical problem solving.

Patterns of Participation (PoP), developed by Skott (2015) was here considered as a collector of the essential features of the two students problem solving episode. The essential features, namely, (a) the students' mathematical thinking, (b) the affective factors of the students, (c) the style of the discussion, (d) the roles and the positioning the students give or take (e.g. negotiator, proposer or innovator as roles; defender or commander as positioning), (e) the profiles the students have and (g) the patterns the students participate were reconstructed in a way where features (a)–(e) were placed under (g); all seen as patterns that student participate.

To put the essential features under the PoP seemed to reveal mostly what was crucial in the episode: one can recognize more or less the students' profiles, discussion styles, their engagement in different processes (e.g. in mathematical thinking) and their engagement with different people (e.g. with teacher and with each other), their way of acting, and their orientations. However, it was difficult to analyze emotions within the framework. For example, performance orientation of Boy 1 can be a result of certain emotional trait, and the attempts to hang on with the unjustified ideas a result of particular emotional states during the problem solving process. Also to recognize and evaluate the significances of certain actions, whether own or others, was difficult. To adapt also theories of student engagement could be useful for that purpose. The students' positioning was not clearly visible in the framework of PoP. For example, it was difficult to decide how to use the PoP as the only tool to consider the positioning of Boy 2 when he told to the teacher: "Boy 1's brains are really working, but mine are not" (when Boy 1 had not even been exactly correct). Finally, the context was reachable only when it related to students. In the video, it was clearly present that the teacher had difficulties in understanding the mathematics of the task. This seemed to have one reason of why Boy 1 held the incorrect ideas. However, the teacher's understanding was not easily included to any of the students' patterns. Still, this problem can be solved by accepting that further contextual factors, such as the teacher's competence in this case, can be patterns that students participate as well, probably in a varying degree of distance.

This study gives a starting point to the development of an analyzing tools that could combine multidimensional student interaction in the context of mathematics. The already existing framework was here proven to be useful basis, and its deficiencies became visible. In future, further frameworks can be adapted to make the analyzing tool more complete.

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Part IV
Emerging Themes in Affect-Related Research: Engagement, Fear, Perfectionism... and Assessment

Chapter 21
Motivating Desires for Classroom Engagement in the Learning of Mathematics

Gerald A. Goldin

Abstract The study of students' in-the-moment classroom engagement with mathematics entails consideration of several domains dynamically influencing each other in complex ways, including: cognition and metacognition, motivation, affect and meta-affect, and social interactions with the teacher and with peers. One feature of such engagement is the student's immediate motivating desire or goal, which in a way can incorporate aspects of all of these domains. This paper considers some related theoretical ideas, including the diversity of motivating desires that can occur, how they embody cognition, affect and social interactions, their role in characterizing structures of engagement, and the development of a survey instrument for their study.

21.1 The Complexity and Importance of In-the-Moment Engagement for Mathematics Learning

The term *engagement* suggests an *object* of engagement, toward which the student's *attention* is focused (for example, engagement with mathematics, with a specific mathematical activity, or with other students in the context of doing mathematics). It suggests also that interaction occurs between the student and his or her environment, involving the object of engagement, and that this interaction *matters* to the student. Engagement (in all disciplines) is considered as fundamental to learning outcomes. It is regarded as a complex, multidimensional construct (e.g. Shernoff 2013).

Mathematical engagement in particular has been characterized as involving cognitive, affective, and behavioral dimensions (Fredricks et al. 2004). But to understand it one must also consider the student's motivation (i.e., the *conative* dimension), the social and cultural context, and the student's social interactions (Middleton et al. 2017). Research on mathematical engagement has frequently addressed these dimensions via study of characteristics of students and their class-

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room environments—i.e., student's traits, including their motivational orientations, and classroom norms.

Engagement with mathematics has itself been understood and operationalized either as a trait characterizing individual students as more or less engaged, or alternatively as an in-the-moment phenomenon, which is my main focus in this discussion. The reasons for this choice of focus are that in-the-moment engagement mediates learning as it occurs, and is open to the direct influence of the classroom teacher. In-the-moment engagement is highly variable, as many factors interact dynamically with each other: both the object of engagement and the student's immediate goals can shift quickly during mathematical activity—e.g., the student's attention and purpose might change from solving the task at hand to conversing with a fellow-student about an unrelated topic, to helping another student understand a mathematical concept, to gaining recognition by the teacher of the student's ideas. Likewise emotions during engagement can change quickly—e.g., from worry to relief; from frustration to anger to joy (Op 't Eynde 2006). In short, in-the-moment classroom engagement with mathematics is extraordinarily complex (e.g. Alston et al. 2007; Op 't Eynde 2007).

In earlier research (Goldin et al. 2011; Verner et al. 2013; Lake and Nardi 2014) the idea of an *engagement structure* was proposed and elaborated, as a way to capture some of this complexity by characterizing patterns that recur when people engage with mathematics in social contexts. Engagement structures are posited as psychological structures that develop nearly universally in individuals, and seen as constellations of behavioral, affective, and social aspects. A structure is evoked or activated by the situation as the person experiences it. Activation of an engagement structure in a mathematics classroom begins with the student's experience of a *motivating desire*.

21.1.1 Engagement Structures and Motivating Desires

A number of simultaneous, intertwined, mutually interacting strands (many of which are traditionally considered separately in the mathematics education literature) constitute each engagement structure: (a) the motivating desire experienced by the person as a goal or objective in the immediate situation; the *reason* for engagement; (b) characteristic patterns of behavior, including social interactions, oriented toward fulfilling the desire; (c) sequences of emotional states, or affective pathways, occurring in response to the evolving situation; (d) the person's expressions of affect, including facial expressions, "body language," exclamations and interjections, laughter, etc. (e) meanings and implications encoded by the person's emotions; (f) the person's meta-affect, including feelings about feelings and regulation of emotion; (g) "self-talk"—the thoughts or "inner voice" experienced by the person; (h) interactions of the dynamically changing strands with the person's beliefs and values; (i) interactions with the person's orientations and traits of the personality; and (j) interactions with mathematical cognition, including problem-solving strategies and heuristics.

Engagement structures have also been termed "archetypal affective structures" (Goldin et al. 2007; Schorr et al. 2010), highlighting their conjectured universality and their predominantly affective components (desire, emotions, expressions of affect, meanings of emotions, meta-affect, beliefs and values). Nine engagement structures described earlier (Goldin et al. 2011) involve (and are named for) motivating desires that occur often in mathematics classrooms:

- the desire to complete an assigned task—"Get The Job Done" (GTJD)
- the desire to exhibit one's mathematical ability, and have it recognized or acknowledged—"Look How Smart I Am" (LHSIA)
- the desire to obtain a payoff (intrinsic or extrinsic)—"Check This Out" (CTO)
- the desire to enter and maintain the experience of doing mathematics—"I'm Really Into This" (IRIT), or *flow* (Csikszentmihalyi 1990)
- the desire to restore one's status, to "save face" after being challenged—"Don't Disrespect Me" (DDM)
- the desire to avoid any possible conflict, disagreement, or disapproval—"Stay Out Of Trouble" (SOOT)
- the desire to correct a perceived slight or inequity—"It's Not Fair" (INF)
- the desire to explain a mathematical procedure or concept to another student—"Let Me Teach You" (LMTY)
- the desire to look as if one is doing the mathematics, paying attention, or working, without actually being engaged in it—"Pseudo-Engagement" (PE)

Verner et al. (2013) describe an engagement structure inferred from observations of a multicultural group of mathematics teachers studying the geometry of ornamental designs originating in different cultures:

- the desire for acknowledgment of one's cultural heritage as it occurs in a mathematical context—"Acknowledge My Culture" (AMC)

In ongoing research at Rutgers University, we have identified several additional possible engagement structures through the examination and analysis of middle school students in mathematics classes working in small groups on challenging tasks. The motivating desires around which these possible structures are organized include:

- the desire to obtain help or support in solving a mathematical problem or understanding the mathematics—"Help Me" (HM)
- the desire to be the center of attention—"Focus On Me" (FOM)
- the desire to be held highly in the opinion or caring of other students or teacher—"Value Me" (VM)
- the desire to avoid notice or attention—"Don't Notice Me" (DNM)
- the desire for it to be recognized that one is right, and the other person wrong—"I'm Right You're Wrong" (IRYW)

- the desire to escape from the current social environment—“I Want Out” (IWO)
- the desire to interrupt the ongoing mathematical activity of others in the class—“Stop The Class” (STC)

When a particular engagement structure is active in someone, the motivating desire evokes characteristic behaviors, social interactions, thoughts, emotions, meta-affect, and so forth, as described in earlier research. The value of these descriptions lies in the *stability* of the construct as a recognizable, recurrent pattern displayed by many students in mathematics classrooms (and other contexts, too). This parallels the value of describing cognitive structures and how they develop in learners. A language that distinguishes motivating desires and ensuing patterns can help teachers interpret, understand, and influence students’ in-the-moment mathematical engagement.

21.2 Motivating Desires Serving Other Motivating Desires

A short discussion of these motivating desires may help elucidate the question of how to distinguish one from another, and the related question of whether the corresponding engagement structures are best regarded as psychologically distinct from each other.

Consider for example the four engagement structures, “Value Me (VM),” “Focus On Me (FOM),” “Look How Smart I Am (LHSIA),” and “I’m Right You’re Wrong (IRYW).” The desire to be at the center of attention (FOM) may serve the student psychologically as *one specific way* to be valued (VM). The desire to look smart (LHSIA) may plausibly be regarded as an instantiation of the desire to be the center of attention (FOM), as showing off one’s mathematical ability and having it recognized may be experienced as a way to receive attention. IRYW may, in turn, serve as one way to look smart (LHSIA). But IRYW may also serve a psychological need to *dominate* another person, in a way that LHSIA does not. Alternatively, IRYW may occur in the context of serving the need to straighten out the student’s own understanding of the mathematics, to demonstrate convincingly the validity of that understanding given that it has been challenged by someone else. In these situations, IRYW does not necessarily make the student the center of attention; FOM is not the overarching desire.

“Don’t Notice Me” (DNM) may be a way to “Stay Out Of Trouble” (SOOT), or it may be an expression of modesty or shyness. “Stop The Class” (STC) may be a way to fulfill the motivating desire to escape doing mathematics, “I Want Out” (IWO), or a way to become the center of attention (FOM), or it may address a psychological need to disrupt the productive or enjoyable activities of others stemming from social alienation. By distinguishing different motivating desires as they occur in the moment, it becomes possible to understand how one engagement structure may be activated *in service of another*—in much the same way that cognitive structures or schemas may "call on" other such structures during mathematical reasoning and problem solving.

Because an engagement structure consists not only of a motivating desire, but also of behaviors and social interactions, thoughts, emotions, etc. interacting dynamically. I would suggest that the question of whether it is useful to distinguish two structures or to regard them as essentially the same depends not only on the similarities between the desires, but on how parallel the resulting characteristic patterns are found to be.

21.2.1 Meta-conation

The concept of meta-conation has been suggested as a way to understand self-regulatory activity pertaining to volition and desire (Snow 1996). When desires serving other desires, this suggests that a meta-conative perspective may be useful. Students regulate their desires, and they pursue them in service of other desires. They also have desires *about* their desires, desires about their emotions about their desires, and so forth. A student may sometimes be motivated to change or abandon a desire.

I would like to suggest that meta-conation, in important ways distinct from meta-affect and meta-cognition, deserves study in its own right in mathematics education research.

21.2.2 Branching of Engagement Structures

When a second motivating desire becomes active in service of an original desire, it is possible that as the situation evolves—as the engagement structures play themselves out—the first desire is supplanted entirely. In a sense, one active engagement structure *branches* into another. This suggests the value to engagement research of identifying possible *branch points*; that is, moments in a scenario where such a transition between active engagement structures is likely to occur.

For example, a student working with a small group of peers may want to impress other students with her mathematical ability (LHSIA), and set out to exposit a mathematical idea or procedure. As other students begin to question her, she desires to help the other students understand what she is saying (LMTY), so they can better appreciate her ability. At this moment, the second motivating desire (to explain the mathematics) is evoked in service of the first (to impress others with how smart she is). But the student may become engaged with the others' questions, their appreciation of her explanations, and their apparent learning in such a way that her desire to impress them becomes subsidiary, or even forgotten. Through social interactions, her motivating desire to teach the others fully supplants her desire to show off her own ability. The consequent affect may then be quite different.

Rossman (2013) explores the relationship between these two engagement structures through the analysis of classroom audio and video data, identifying characteristic behavioral features. Evidently it would be a difficult research undertaking to infer still more complex occurrences, such as motivating desire branchings. This would probably require combining considerable fine-grained observation and analysis with retrospective, stimulated-recall interviews and other techniques.

Nevertheless rapid, in-the-moment changes in motivating desires are likely to be an important feature of the complexity of mathematical engagement, and deserve attention. Of course, this leads in a direction quite different from the traditional study of students' longer-term traits and achievement orientations.

21.2.3 Motivating Desires Serving Psychological Needs

The characterization of engagement structures as archetypal suggests that they develop in individuals in response to fundamental psychological needs. Goldin et al. (2011) point to *needs of the personality* (Murray 2008) behind the various motivating desires. For example, the desire evoking the engagement structure "Don't Disrespect Me" (DDM) is to restore status or a sense of respect after being challenged, which may address the fundamental need Murray calls *infravoidance:* "to avoid conditions which may lead to belittlement" (Murray, p. 192, quoted in Goldin et al., p. 553).

In this way, one may place the discussion of motivating desires within the wider context of theories of personality.

Against the background of universal or near-universal human needs, the immediate social and psychological context of classroom mathematics as appraised by the student offers opportunities to fulfill one or more needs. Then the desire to attain a specific goal is aroused. But the interpretation and appraisal of the context depends, of course, on the student's attitudes, beliefs, and values; and the *threshold* for arousal of a particular motivating desire depends on the student's mood, personality traits, and motivational orientations. Thus different students may be motivated by different desires in similar circumstances on a particular occasion, and the same student is likely to experience a variety of different motivating desires in similar circumstances on different occasions.

21.2.4 Some Characteristics of Motivating Desires

It is of interest to try to classify the identified motivating desires according to various characteristics frequently considered in work on motivation.

Some engagement structures are *overtly mathematical*; i.e., the goal is specific in some way to the mathematical content. These include: GTJD, LHSIA, CTO (when the payoff is intrinsic), IRIT, LMTY and IRYW.

Some engagement structures stem from motivating desires which may be classified as involving *learning* or *mastery* goals. These include GTJD, CTO (intrinsic payoff), IRIT and LMTY.

Other structures stem from desires which may be classified as centered on *ego* or *performance* goals, including LHSIA, CTO (extrinsic payoff), DDM, SOOT, INF, PE, FOM, IRYW and VM.

Most of the motivating desires identified have some explicit *social* aspect; e.g., belonging, recognition, respect, equity, generosity, social avoidance or withdrawal. Such desires are associated with the engagement structures LHSIA, DDM, SOOT, INF, LMTY, PE, VMC, HM, FOM, VM, DNM, IRYW, IWO and STC.

Some of the motivating desires involve *approach* goals, including those evoking GTJD, LHSIA, CTO, IRIT, DDM, LMTY, AMC, HM, FOM, VM, IRYW and STC. Others involve *avoidance* goals, including those evoking SOOT, PE, DNM and IWO.

Many of the motivating desires mentioned tend to *productive mathematical* engagement, at least in most contexts. However, it is our perspective that those which may seem irrelevant or detrimental to such engagement nevertheless serve *adaptive* functions for the individual. These are especially important for us to understand. "Pseudo-engagement" (PE), for instance, may serve as an adaptive way for a student to alleviate boredom and avoid embarrassment. For someone who is unable to understand the mathematics being discussed, it is likely to be a less productive engagement structure than, for example, "Help Me." For a gifted, creative student who is engaged in an activity different from the one led by the teacher—this could be an individual mathematical exploration, an artistic sketch, a poem, an imaginative excursion—PE may enable that engagement without creating a distraction for the rest of the class.

Thus we do *not* see some motivating desires or engagement structures as "good" and others as "bad." Rather, their appropriateness or inappropriateness depends on the situation and the educational goals. At times, standing up for self-respect or assertively demonstrating one's abilities ("ego goals") can enhance mathematical self-confidence and foster productive engagement. At other times, these desires may be inappropriate and impede meaningful learning. Focusing on in-the-moment engagement allows us to steer clear of overly-broad generalizations, and distinguish context-dependent effects on the quality of mathematical engagement.

21.2.5 Structural Aspects of Motivating Desires

I prefer the term "motivating desire" to "goal" for a few reasons, although sometimes the two are used interchangeably. In the problem solving literature, a "goal" is usually understood as a state whose characterization is cognitive; it is assumed that the (mathematical) problem consists of attaining such a goal state. Certainly the formulation of a motivating desire does entail cognitive appraisal. But "desire" also has more affective connotations, suggesting either (a) an object of desire (the "goal"), together with some emotional feelings—perhaps anticipatory emotions—associated with the goal; or else (b) an aversion, a desire to avoid something, together with associated emotional feelings. Furthermore, the object of desire may simultaneously be a *situation* (e.g., one in which the student's mathematical ability has been recognized and praised) and an *affective state* (e.g., the feeling of pleasure and self-worth associated with such recognition and praise). The possibility of experiencing *desire for an emotional state* already introduces a meta-affective aspect to the engagement.

The modifier "motivating" is explicitly conative, suggesting that the desire is more than an idle wish—it results in behavior, social interactions, thoughts, and emotions. The depth or intensity of the desire in the moment becomes a relevant consideration.

Motivating desire refers here to *immediate* desire. The goal of doing well on next week's quiz is a relatively short-term goal; that of becoming a medical doctor is much longer-term; and of course goals of both these sorts influence students' mathematical motivations. Middleton et al. (2014), in their discussion of *perseverance*, discuss mathematical goals with regard to their specificity, their nearness to the present state of the learner, their focus, and their character as approach vs. avoidance. In the present discussion of engagement structures, our focus is on *desire in the "here and now"*—the student wants to master *this* concept, complete *this* task, impress the rest of the class in the next few minutes, help a friend who is sitting in the next seat, or avoid notice by the teacher right now.

We have described engagement structures as involving dynamical interactions of a motivating desire, behavior including social interactions, emotions and meta-affect, cognition, and other factors. But we see that the motivating desire itself is *already* a complex construct. It stems from a deeper, fundamental human need. It involves cognitive appraisal that the situation, in context, provides an opportunity for meeting the need. The individual's traits, orientations, and beliefs influence that appraisal, and set a threshold for the desire to occur. The desire takes an object in the form of a type of situation, experience, or outcome, typically social in nature, or else takes as its goal the avoidance of some such situation. The motivating desire is conative, leading to choices by the individual. And it is profoundly affective, being accompanied by emotions and/or anticipatory emotions, with the desire itself very likely directed toward (or away from) an emotional state as well as a situation.

Many of the specifics of this discussion are compatible with research on *situational interest* (e.g., Hidi et al. 2004); but a wider discussion is beyond the scope of the current article.

Each motivating desire discussed here has its own structure of reasons for being. If the construct is to be useful, it is important that there be some commonalities among students in whom the specific desire arises—contexts fostering its occurrence, and the consequent thoughts, emotions, social interactions, and so forth comprising the corresponding engagement structure. The best-researched models for students' mathematical motivation (e.g. Pekrun 2006) allow some potential insight into what some of those commonalities may be.

21.3 Motivating Desires and Survey Instrumentation for the Study of Engagement Structures

In addition to the qualitative analysis of videotaped classroom episodes, our group at Rutgers University has been developing some descriptive survey instruments designed to be administered to middle school mathematics classes at the end of an activity involving group activity. The purpose of these surveys is to ascertain the occurrence of various motivating desires, and the activation of corresponding engagement structures, during the course of classroom activity. They are referred to as RIME (Rutgers University Inventory of Mathematical Engagement).

Survey questions are given at the end of a typical class using electronic devices, with follow-up questions contingent on some of the responses. Compared with experience sampling methods, this has the advantage that the mathematical engagement is not interrupted; rather, students are asked immediately afterward about the desires, thoughts, actions, and emotions they recall having during the class activity.

The limitations, of course, are substantial. It is not known that the reported thoughts, actions, or emotions occur in connection with each other, or in connection with the reported motivating desire. There are also limitations to students' recall after an activity is over. Moreover, although anonymity is assured, some students may be reluctant to acknowledge desires, thoughts, or behaviors which are uncomfortable for them, or which might elicit disapproval.

Survey questions have been validated with respect to scenarios descriptive of each engagement structure in a subset of the above-discussed list. They fall in the following categories, with a small number of questions per category for each engagement structure included in the survey:

- Contexts: "What math class was like today"
- Motivating desires: "What I wanted today in math class"
- Behaviors: "What I did or tried to do in math class today"
- Thoughts or self-talk: "Thoughts I had today during math class"
- Emotions in context: "Feelings I had during math class today"
- Outcomes: "What happened by the end of math class today"
- Outcome emotions: "By the end of class today, I felt . . . "

The time available for survey administration allows three engagement structures to be fully explored per survey. Alternatively, a broader survey restricted to contexts and motivating desires can assess the wider spectrum of such desires as they occur in a class of students, providing aggregate information. A more full description of RIME is intended for subsequent publication.

21.4 Value for Mathematics Teachers

Research identifying students' motivating desires during mathematical activity, and describing characteristic thoughts, emotions, behaviors, social interactions, etc. that these desires evoke, can be of great value to teachers. It becomes possible to recognize and discuss observed patterns of engagement and disengagement explicitly. It is then natural to explore teaching strategies eliciting a variety of appropriate, in-the-moment motivating desires, and resulting in desirable patterns of mathematical engagement.

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Chapter 22
What Are Students Afraid of When They Say They Are Afraid of Mathematics?

Barbara Pieronkiewicz

Abstract Students' fear about and reluctance toward mathematics have been a great concern to mathematics educators for many decades. The problem is such well known that one may hardly believe there is anything left to say. Sometimes, however, to see some new aspects of the object that seems to be well known, it is good to consider it from another angle. Thus, in this paper some data obtained from a questionnaire administered a few years ago to 149 Polish students from different school levels are discussed from the theoretical perspective that has not been considered in the original study. For the current analysis I use the dimensional ontology laws (Frankl, The will to meaning: foundations and applications of logotherapy. Penguin, London, 2014) and the transgressive concept of man (Kozielecki, Koncepcja transgresyjna człowieka. PWN, Warszawa, 1987).

22.1 Theoretical Framework

Central to all research on affect in mathematics education are the many shades of human affect occurring in relation to mathematical activity, and having a significant impact on the cognitive processing of mathematical data. Underneath the affect, however, there is always a human being whom the researcher sees through the lens of a particular psychological perspective he takes. Let me then first bring a little bit closer the perspective that stands behind my approach.

22.1.1 Dimensional Ontology Laws and Man's Search for Meaning

In this paper I adopt a multi-dimensional perspective proposed by Frankl (2014), to whom a human being is an entity consisting of body (soma), mind (psyche)

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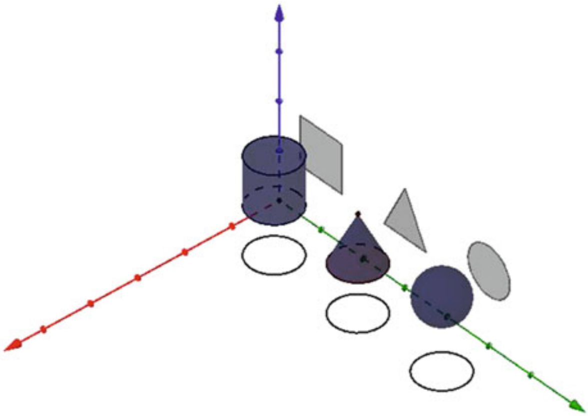

Fig. 22.1 Graphic representation of two dimensional ontology laws combined

and spirit (noetic core). The central premise of Frankl is that the complexity of human behavior, cognition and affect cannot be understood if we reduce the multidimensional human being to either of these dimensions exclusively. Frankl came to believe that integrating apparently contradictory models of the person was possible only through a multi-perspective approach encompassing what is seen from different vantage points. The "dimensional ontology" laws formulated by Frankl capture this idea by using an analogy to dimensions known from geometry. Figure 22.1 presents my combination of two well known existing graphic representations illustrating two laws separately. The laws are the following:

Law I: One and the same phenomenon projected out of its own dimension into dimensions lower than its own is depicted in such a way, that the individual pictures contradict one another.

Law II: Different phenomena projected out of their own dimension into one dimension lower than their own are depicted in such a manner, that the pictures are ambiguous (Frankl 2014).

Dimensional ontology laws have many possible applications. They may serve as lenses through which one sees the diversity of the sciences and the urgent need for interdisciplinary approaches toward educational problems. For example, mathematics does not provide the answers that would help us to encompass and understand all determinants of the process of its learning. In order to do so, in our research we need to take a multi-disciplinary perspective to support our attempts.

Looking at this picture, a teacher may recognize the diversity in his classroom, where in a group of low achieving students some might be highly gifted, yet suffering from an underachievement syndrome, and others not. In this paper I use Frankl's approach to highlight the fact that there can be a variety of reasons behind commonly shared student declarations such as "I don't like mathematics" or "I'm afraid of mathematics". In a broader sense, Frankl points out that it is possible to find differences, where on the surface level there seem to exist only similarities. What also enriches this approach is an insightful conclusion emerging from what the author experienced during years spent in a concentration camp. Watching people in extreme, hopeless and dehumanizing reality, he noticed that their ultimate drive was to find the meaning in life, no matter how difficult the circumstances were. Even when they had to go through the unavoidable pain and suffering, these tormented entities were still able to transmogrify the external misery by finding islands of hope, faith and humanity. According to Frankl (1985), the attitude toward life circumstances is, above all else, a matter of the choice people make. People have free will to consciously decide how they are going to act when facing obstacles. They can also make the decision on who they choose to become.

22.1.2 The Transgressive Concept of Man and Affective Transgression

Another psychological perspective I take to reflect upon the phenomena of students' fear about and reluctance toward mathematics is the transgressive concept of man (Kozielecki 1987). The concept of psychological transgression refers to crossing personal boundaries and subverting limitations, both of which play an important role in everyone's life. To Kozielecki, a man is a multidimensional, self-directed, expansive creature who intentionally crosses physical, social or symbolic boundaries, as well as his own psychological limitations.

The possibility of being transgressive is in all human beings and can be applied both to cognition (Semadeni 2015) and to the affective domain (Pieronkiewicz 2015). People are potentially able to reflect on their emotions, attitudes, beliefs and values, and provide constructive treatment when finding something counterproductive or maladaptive that needs to be repaired. Not everyone takes the challenge (not everyone is capable) of having deep insight into one's affect. In that sense, some people have *transgressive affect*, whereas some other not. Pieronkiewicz (2015), referring to the transgressive concept of man proposes focusing on the process of *change in affect* itself. She also defines *affective transgression in the learning of mathematics* as an intentional process of overcoming personal affective barriers that preclude one's mathematical growth and development. The process is a psychological, individual and constructive transgression toward oneself. To transgress one's affective limitations, the person needs to have some insight into his or her emotions and has to be aware of the belief system he or she holds. Finally, the necessary condition for transgression to happen is that the person expresses the will to change, believing that the changes are good and possible. The abiding alternation of affect requires involving and developing meta-affective competencies. Meta-affect defined as "affect about affect, affect about and within cognition about affect, and the individual's monitoring of affect through cognition (thinking about the direction of one's feelings) and/or further affect"

(DeBellis and Goldin 2006, p. 136), is considered to be the trigger and a very powerful tool needed for affective transgression to proceed.

This theoretical perspective, seen in the light of an assertion by Ernest (2004), reveals one more advantage:

> Many people have come to feel that mathematics is cold, hard, uncaring, impersonal, rule-driven, fixed and stereotypically masculine. Evidently there is a strong parallel between the absolutist conception of mathematics, the negative popular view of mathematics, and separated values. Likewise, a second parallel exists between the fallibilist conception of mathematics, connected values and the humanistic image of mathematics promoted by modern progressive mathematics education as accessible, personally relevant and creative (Ernest 2004).

Among all components of affect, values seem to be least-attended. Meanwhile, the dimensional ontology laws and the transgressive concept of man contribute two essential and universal values that might be of interest to students. These are: man's search for meaning (under all circumstances and, perhaps, in all activities), or in a broader sense "the meaning of it all" (Vinner 2013); and the developmental value of continuous struggle in challenging one's limitations. These values can be reached and discussed in the mathematics classroom.

22.2 Research

The data for this study were gathered in the year 2005. The selection of participants was determined by two prerequisite conditions: the investigator wanted to include in her study samples representing all school levels, and the decisive factor for choosing classes was whether the investigator had previously known the mathematics teachers of these classes. The latter requirement was imposed in order to make it possible for the investigator to better understand students' responses by referring to her previous knowledge about those teachers and their ways of teaching. The original main aim of this research was to come to know the views of mathematics that students had, and to identify factors potentially causing students' fear about mathematics. A questionnaire comprised of 12 questions was administered to 149 school students (42—primary school, grades 5–6; 48—middle school, grades 2–3; 59—secondary school, two first grades, named in this paper 1 and 1* respectively). Questionnaires, assigned and collected in each class by the investigator, were filled by the subjects anonymously. Questions chosen to the questionnaire covered a wide range of students' experiences related to the learning of mathematics: interest in mathematics, attitude toward mathematics teacher, parents' support in learning the subject, time spent on learning mathematics and so forth. In the present paper, ten years after the original study was conducted, I examine students' responses once again. This time, however, I look at the data through the lenses of dimensional ontology laws and the transgressive concept of man. Analysing and interpreting the data, I seek to answer two questions:

1. What is it that students do not like when they say they do not like mathematics?
2. What are students afraid of when they say they are afraid of mathematics?

Due to page limitations I only present here some answers to the following open questions from the questionnaire used in the study:

- Q4: In a few sentences describe how you feel during your math classes at school.
- Q5: What is your attitude towards mathematics? Do you like it or not? Why is that?
- Q6: If you like math very much or you just can't stand it—do you remember when and how this started for you?

All of the statements written by the students in the original study have been read carefully one more time. The current analysis pertains to the affective memories of mathematics related experiences that students referred to in their answers. The analysis relies on the assumption (Bruner 1986) that students' short narratives are their subjective re-presentations and reconstructions of the past events, rather than objective reports from the real situations. They do not give the researcher a precise information of what happened, however, they do carry the information of how the students remembered some facts and what meaning they have assigned to the events they are describing.

Ten years ago, my attempt was to grasp the whole spectrum of different answers that students gave. The answers were grouped into two major groups of statements revealing either positive or negative attitude toward mathematics. Then, the analysis made it possible to identify some factors potentially causing students' fear about mathematics and, on the other hand, some factors that could have influenced the formation of students' positive views of mathematics. The results thus obtained were consistent with other presently well known studies (e.g. Varsho and Harrison 2009). Nevertheless, the study helped to better understand not only the specificity of a few local populations of students, but also to learn more about the teachers being described by their pupils. There seemed to be nothing more to find out about the subjects and the central problem of this study. Trying to look at this old problem from another angle, however, I chose to use the perspective of the two dimensional ontology laws and pay attention to the differences apparent among seemingly similar answers given by those subjects who revealed their negative attitudes (Di Martino and Zan 2010) toward mathematics. Looking at the data through the new lenses, I realized that in the original study I missed considering a very, if not the most, important dimension of meta-affective competencies of the subjects. In fact, this dimension could be crucial, if I would have ever wanted to change the negative views of mathematics in case of any students. Ten years ago, I was sure that the most significant difference between students' justifications of their fear or reluctance toward mathematics laid in the content of their statements. Currently, searching for the differences where previously I saw only similarities, I was able to acknowledge the differentiation of the emotional nature and depth of students' answers. Some students gave very superficial responses, but some others reported on their very personal, intimate past experiences with mathematics. To outline the differences apparent in the scope of subjects' answers, below I present and discuss a collection of exemplar statements that illustrate the variety of emotional depth levels.

22.3 Data Analysis

22.3.1 Fear of/Reluctance Toward Mathematics

Careful reading of the collected answers reveals that students declare they "don't like/hate mathematics" far more frequently than they refer to their fear described in terms of: nervousness, being stressed, or being frightened. Asked for reasons of their feelings ("don't like/hate") about mathematics, some students unwittingly express their beliefs, giving—with no additional comments—answers like:

> I don't like math since I was born. (grade 3)
> I'm a humanist. (grade 2)

The authors of the above statements do not blame mathematics for the way they experience it, but they also do not take the responsibility for the state of affairs. There are some other, external to and independent of the students, but at the same time not necessarily inherent in mathematics, reasons standing behind the feeling of dislike. Perhaps, these students hold the widespread belief that to be good in mathematics one needs to have a mathematical brain, or that the mathematical talent depends on genes.

Many students do blame their current math teachers for evoking negative emotions toward the subject:

> I don't like math because the teacher doesn't want to bring it closer to us, she is only doing her job. (grade 6)
> The teacher makes this subject disgusting to me! (grade 5)
> Although I like this subject, when I think about having math classes with our teacher, I don't want to go there. (grade 5)

Common expressions such as "I don't like mathematics" seem to represent the surface level of the fear/reluctance toward the subject.

22.3.2 Fear of/Reluctance Toward Doing Mathematics

Deeper analysis of students answers reveals that what many students are trying to communicate is rather "I don't like/I'm afraid of *doing* mathematics" or "I don't like what happens during math classes/what our teacher does to us". Even answers like:

> I rather don't like math, because it is demanding and time consuming. (grade 3)

describe unwillingness toward engagement and effort, rather than reluctance toward the subject itself.

Perhaps, students who wrote:

> I don't like math because I don't understand it. I view math as a nonsensical subject. (grade 3)
> I don't like math because I'm not interested in it at all. (grade 3)

inform us that all of their previous mathematics teachers neither have successfully shown them the beauty of mathematics and relational understanding of the subject, nor succeeded in raising the mathematical curiosity (respectively). It would be, thus, understandable that these students refuse being engaged in mathematics classes. Paradoxically, it is the right thing for a person to do, not to get involved in something that has no *meaningful* sense. According to Frankl (1985) people are equal in their search for the meaning and purpose in life. Maybe, in order to "convert" these students to mathematics, the teacher should address some value related issues first?

Answers of the first two kinds stop over the superficial level. What is being expressed, rarely relates personally to the student. Authors of the given examples do not reveal their emotional responsiveness to the mathematical activity.

22.3.3 Fear of Failure

A still closer look at the answers reveals that what students don't like or are afraid of is not actually doing mathematics itself, but the *experience of failure*, which they *predict* before they start doing anything:

> What I like best is starting a new theme. Then I tell myself: this I will know! Nevertheless, I usually don't make it. . . : ((grade 6)

Experiencing failure evokes many negative emotions like sadness, disappointment or shame. These emotions are not only hurting the student in the present moment. They also bring to mind some negative memories from the past.

22.3.4 Fear of Experiencing Emotional Pain

Underneath the fear of failure are usually some past painful experiences (i.e. disappointment, underappreciated efforts, feelings of humiliation and shame) that students still keep an affective memory of.

> This feeling comes from a primary school when the teacher offended me; now I laugh at this, but then that was a nightmare. (grade 2)
> It was in the 4th grade. I came to classes with material learned by rote. Surprisingly, I got 2 [note: Polish equivalent of D]. I got nervous and started studying every day. Since that day I always had jitters. (grade 5)

I hate math when I have to go to the blackboard, because I think my class would think of me as a jerk who knows nothing, and they would be laughing at me (grade 6)

Teachers' perceived disregard and injustice induce fear and sense of humiliation, and may instigate effort avoidance and hatred toward mathematics:

Our teacher disregards students. (grade 5)
When someone at the blackboard is not making out, she looks at me and my friend and smiles; her "smile" seems to be telling you "You're gonna be next." (grade 1*)
I'm always prepared for the lesson, I have private lessons and my tutor sees that I know, but not Mrs X. She doesn't want to give me a chance; she wants me to stumble and she always gives me a lower grade. (grade 1*)
She is very angry when we don't know something and to punish us, she promises us a short test from the material we don't understand and she's doing this on purpose. (grade 1*)
The atmosphere in her classes is unpleasant, I'm nervous and I'm afraid that if I speak, I'm gonna be criticized. (grade 1*)

What permeates a considerable number of students' responses is the desire to be important to and noticed by the teacher, the deeply grounded need to be respected, validated and appreciated combined with the fear of humiliation, the need to be assessed accordingly to one's effort, not one's previous achievements, and the need of experiencing success:

She criticizes me and she takes all hope for a good grade away from me. (grade 6)
I started hating math from the second fall of the first year of middle school; despite the huge effort I've put in learning, I didn't succeed in getting 5 at the end. I broke down, as I always was very good at math. (grade 2)
I don't like mathematics very much, because every time, even if I learn it thoroughly, I get only "+3 [note: Polish equivalent of C+]". (grade 6)
I do like our teacher and respect her a lot, but I don't like when getting only 3 [note: Polish equivalent of C] for my answers, when I think I deserve 4 [note: Polish equivalent of B]. I don't know why, I never get "-4", but instead always "+3". (grade 6)

Students responses cited in this section carry very intimate and personal information. They reveal those aspects of the learning of mathematics, that usually remain invisible in the classroom, like for instance, students' vulnerability to being hurt. It is worth highlighting that in the above statements subjects refer to some concrete events from the past, that probably brought a great load of emotions. To write such a statement, each of the students had to scan his or her memory and then choose a single event from the scope of all recollected math related experiences. Students often report on some subjectively traumatizing past events that seem to have objectively crippled their mathematical potential for many years. Because of the emotional nature of these reports, I assume, that they express the personal meaning students assigned to their individual experiences.

22.3.5 Fear of Letting Oneself Feel His True Feelings and Fear of Losing Self-consistency

The last level of emotional depth is only hypothetical, as none of the subjects' answers has reached that level. This is not surprising. Such a deep level could be probably, if ever, reached during an individual interview, if the environment of openness and trust would have been established first. And, of course, to give such an insightful comment on his or her emotional experiences, the student would need to have a heightened self-awareness, which is rather rare among youngsters.

Some psychologists, however, state that in order to heal emotional pain, one needs to acknowledge it first and invite it up into his awareness. People do not want to work out the pain because they have learned to "avoid their primary emotions and often need permission to feel" (Greenberg and Rhodes 1991, p. 47). People in general are afraid of losing self-consistency, while surprisingly, it is the motivation to avoid negative affect that leads to dysfunctions in the self-regulation system (Curtis 1991). The aim of the self is to remain stable, but paradoxically, it is through the process of destabilization that change and newness occur. It seems that avoiding mathematics and using the "I don't like math" red herring only maintains and reinforces affective memory of previous hurts and keeps the problem neither faced, nor resolved.

22.4 Concluding Remarks

What I just described refers to the conscious level of students' memory. But what about the unconscious mind? What possible impact, if any, could implicit memories from the past have on students' current math related affect? Some studies show that people can experience emotional states or emotional behaviours with no awareness of why some emotions occur in specific circumstances. Probably one of the most famous experiments was performed by a Genevan neurologist and psychologist, Edouard Claparède who:

> pricked an unsuspecting Korsakoff's syndrome patient with a pin hidden in his hand - an event that caused her quite a bit of distress. Claparède subsequently left the room, and returned after the patient had regained her composure. Upon questioning, she failed to recognize Claparède, and had no recollection of the unfortunate incident that had just transpired between them. Nevertheless, she refused to shake his hand. When asked why, she replied, "Sometimes people hide pins in their hands" (Eich and Macaulay 2000, p. 38).

Many more anecdotes as well as formal studies (i.e. Nemiah 1979; Tobias et al. 1992) provide an evidence that emotional responses "can persist even though one does not know how they originated" (ibid., p. 39) and thus, they may serve as indicators of implicit memory. The importance of these findings comes up when we realize how many things we are not aware of. In fact, we are even not aware of what we are aware of, and what we are not. The more things we are aware of, however, the more control we can exercise over our behaviour and decisions we make in order to guide our lives. Referring this conclusion to the study examining students' fear of or reluctance toward mathematics, I assume that among those students who reported their experiences on the superficial level, there were some entities who—due to the processes like repression or dissociation, or maybe just mere forgetting—lost the memory of past events which have caused the anxiety, fear or reluctance toward mathematics they experience currently. From the perspective of the transgressive concept of man, to be ready to "transgress oneself", that is to overcome one's affective limitations, a person needs to have some mastery in recognizing and naming his or her emotions, attitudes and beliefs first. The more one is aware of one's affect, the more plausible it is that he or she would attempt to change its counterproductive components. Meanwhile, many low-achieving students who manifest intense emotional responses to mathematics, instead of being asked about the details of their math related affect, are too often written off by their teachers. Labelled as "hopeless cases" they are not given the chance to transgress themselves and, as a result, fulfill their intellectual potential.

The insight one may get into his inner world of emotions and emotional processing of the data requires, however, developing meta-affective competencies. DeBellis and Goldin (2006) state that "the development of powerful affective and meta-affective structures, (…) may turn out to be keys that unlock mathematical power in learners" (p. 145). The authors contribute to the discussion on math anxiety a very essential remark, namely, that the most important goal in mathematics education is *not* to "eliminate frustration, remove fear and anxiety, or make mathematical activity consistently easy and fun" (p. 137), but rather to develop meta-affective competencies that would help the students to transmogrify the obstacles and difficulties they encounter into positive experiences. This envoy is consistent with Frankl's (1985) word of advice, with which I shall end: "When we are no longer able to change a situation, we are challenged to change ourselves" (p. 135).

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Chapter 23
What Is Perfectionism in Mathematical Task Solving?

Lovisa Sumpter

Abstract This paper explores what perfectionism can be in mathematics education and how it can be indicated in student's task solving. Four dimensions of perfectionism were identified in one student's work: Excessive Concern over Mistakes, High Personal Standards, Doubts about Actions, and Need for Organization. The literature review show that the number of papers in mathematics education and perfectionism are rather low despite a vast quantity of research in perfectionism. Therefore, it is suggested that this is a research area with great potential, especially regarding the large numbers of students expressing stress.

23.1 Introduction

Stress is a common problem in Swedish secondary schools (The Swedish National Agency for Education 2013a). The amount of students at lower and upper secondary school who reports that they 'most often' or 'always' feel stressed is around 40% and this number has been unchanged between the years 2000–2012. There are both differences between different programmes (where programmes preparing for university studies have more students reporting stress compare to vocational programmes) and gender differences (48% of the girls report stress compared to 27% of the boys). Although the most common cause to stress is homework, as a shared second place together with tests, we find expectations on oneself and on one's schoolwork. Four out of ten students report they this is a cause of stress once a week or more often. This is an intrinsic generated stress. Compared to parents' expectations, an extrinsic generated stress, fewer students (20%) indicate this as a source. It seems that intrinsic negative stress is the bigger villain in the piece.

But at the same time, according to PISA 2012, Swedish students are among those who are the least anxious regarding mathematics together with students from Denmark, Finland, Iceland, and the Netherlands (The Swedish National Agency for Education 2013b). Anxiety and negative attitude towards mathematics are otherwise

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both considered being "a growing barrier for many children" (Geist 2010, p. 24), and mathematics anxiety seems to have negative impact on the cognitive process since it interrupts the working memory (Ashcraft 2002). Swedish students report more positive intrinsic motivation (such as "I am interested in what I learn in mathematics") and positive extrinsic motivation (such as "It is useful for me to learn mathematics since it improves my future prospects"). Motivation, as concluded by Ryan and Deci (2000) is highly important since it basically is the engine: it produces. However, looking closer at the PISA data there has been an increase of the students reporting different levels of anxiety (The Swedish National Agency for Education 2013b), and when studying the questions posed to the students, we can see that the PISA questionnaire is more of a blunt tool compared to the national attitudes surveys. So even though it is the 'right' type of motivation that has been marked, we still don't know how much it mirrors their conceptions or how much it is 'a correct way of responding'. From PISA, we cannot say so much about stress and expectations and studying the national surveys, we don't know what is mathematics specific.

This becomes more complicated since both stress and expectations have several causes and dimensions which means that there are several factors involved. One of these factors is perfectionism, a factor with high impact on behaviour, thinking and actions (Hewitt and Flett 1993; Hollender 1965). Still, as a factor in educational setting in particular mathematics education little is known where researchers even argue that

> perfectionism and its consequences potentially influence the course of career decision-making and persistence in STEM and correspondingly provide as yet untapped targets for intervention (Rice et al. 2013a, p. 125).

Perfectionism seems to influence in mathematics education, but little is known. This paper has two aims. The first aim is to seek understanding about perfectionism and what could it be in mathematics education. This will be answered by a literature review. The second aim is to see how perfectionism could be indicated in a mathematical task solving session. The research question is: What dimensions of perfectionism can be indicated in one student's mathematical task solving?

23.2 Background

I will here first present a brief introduction to perfectionism, and then a review of research about perfectionism and mathematics education.

23.2.1 Perfectionism

This is a well researched area in psychology and I can only scratch the surface. According to Campbell and Di Paula (2002), perfectionism is a set of self-beliefs that exists in the self-concept. They are important not just to other beliefs that are part of the self-concept, but also to one's motivation and goal-pursuit. With a focus on mathematics education, I here define beliefs as "individual's understandings that shape the ways that the individual conceptualizes and engages in mathematical behaviour generating and appearing as thoughts in mind" (Sumpter 2013, p. 1118). Emotions are, together with motivation, closely connected to beliefs such that together with other affective factors, they determine interest, willingness, and persistence (Dogan 2010). These affective factors also interplay: emotions may both establish and strengthen beliefs (Mercer 2010).

It should be stressed that perfectionism in itself does not automatically has to be negative; perfectionism can also be positive (Lundh 2004). Perfectionism becomes a problem when the strive for perfection excludes the acceptance of non-perfection. Another division of perfectionism is between 'passive' perfectionism, which is maladaptive (evaluative concern), and 'active' perfectionism which would be an adaptive (achievement striving) form (Hollender 1965; Adkins and Parker 1996). Both these divisions share a common a starting point, a consideration what is 'healthy' and 'unhealthy' for the individual.

There are several definitions of perfectionism, many of them multidimensional, and I will here present two. The first definition is a result of the work of Hewitt and Flett (1993). According to them, there are three main types of perfectionism: (1) self-oriented perfectionism where there is a tendency to set standards for yourself that are unrealistic and impossible to attain; (2) other-oriented perfectionism where there is an inclination to project high standards on others; and, (3) socially prescribed perfectionism where there is a belief that others have expectations on you, expectations that are impossible to meet. These three categories are rather big and it is plausible to think that there are several sub-categories. Another multidimensional definition is provided by Frost et al. (1990). They suggest there are six dimensions of perfectionism: (1) Excessive Concern over Mistakes thinking it is extremely important not to make mistakes; (2) High Personal Standards where the individual set high standards and high expectations; (3) Doubts about Actions which involves feelings that you may not have completed a task correctly; (4) Need for Organization that could be expressed in fussiness about neatness, order and how things are organised; (5) High Parental Expectations which could be viewed as an origin of perfectionism; and, (6) High Parental Criticism which could also be viewed as an origin. What these two definitions have in common is that both talk about beliefs that could be seen as motivational beliefs (c.f. Sumpter 2013). However, perfectionism is not 'just' a set of motivational beliefs since it includes standards and expectations that are not only high but also very difficult or even impossible to meet, and that these standards and expectations becomes a problem for the individual following (Lundh 2004). Therefore, it is crucial to separate perfectionism from high competency and successfulness (Frost et al. 1990). Also, research has shown that instead of viewing healthy perfectionism and unhealthy perfectionism as opposite poles, there are indications that they are independent constructs (Parker et al. 2001). This would imply that there is a need to identify which dimensions of perfectionism are indicated to be able to say whether or not is positive/negative, adaptive/maladaptive, or healthy/unhealthy perfectionism. Saying this, perfectionism is often linked to depression and/ or anxiety disorders (Hewitt et al. 2002) making it a serious problem independent of different degrees or variations of it.

23.2.2 Perfectionism and Mathematics Education

If the research body about perfectionism is great, the number of studies about perfectionism and mathematics education is much smaller. I started by doing a literature search. The data were generated from the ERIC database, August 2015. The search terms were 'mathematics'/'math'/'maths', 'education', and 'perfectionism'. The number of papers resulting from these searches was eight, and after excluding some papers since they were not relevant to the topic (e.g. about science anxiety), I was left with six papers. Most of these papers are about perfectionism and performance and it seems that perfectionism have an impact on performance; Tsui and Mazzocco (2007) found that mathematically gifted children (sixth graders) that expressed higher level of either math anxiety or perfectionism had smaller difference between timed tests versus untimed tests when compared to children with low levels of math anxiety or perfectionism. However, this study does not look at stress or how the children experienced the test situation. Also, the students were mathematically gifted which means that perfectionism and math anxiety can work as an intrinsic motivation in test situations, but still be a negative factor. A few studies compare students from gifted and regular programs. A Canadian study concluded that perfectionism was unrelated to levels of reading and mathematics achievement except for students in the gifted program where they could find a positive association between perfectionism and mathematical achievement (Stornelli et al. 2009). In the same study, they also report a relationship between perfectionism and fear and sadness. This could imply that even though perfectionism is linked to mathematical achievement for some children, it could well be a negative factor. This is further stressed by a study looking at Czech students where the strongest result was the connection between migraines and high personal standards (Parker et al. 2001). Otherwise, perfectionism was more a problem for the typical students than among the students that were considered mathematically gifted. Therefore, we cannot conclude that perfectionism is only a problem, or a helping factor, for mathematically gifted children.

There are some indications about gender differences. One Australian study reports that girls in year 10 had higher mathematics anxiety than boys (Moore 2010), in line with the Swedish reports (The Swedish National Agency for Education 2013a). Also, the results showed that students with higher levels of passive, maladaptive, perfectionism had higher mathematics anxiety and higher writing anxiety than students with lower levels of passive perfectionism (Moore 2010). And, there was an interaction between gender and active (adaptive) perfectionism among girls: anxiety levels decreased as a function of increased active perfectionism, but only in mathematics. The results were not replicated for writing anxiety. Other gender differences have been observed regarding grade-point averages (GPAs) and students doing STEM majors (Rice et al. 2013a). Looking at the control groups, men had higher grade-point averages (GPAs) at low levels of self-critical perfectionism than they had at higher levels of perfectionism. This could be contrasted with women's results: the GPAs were high when self-critical perfectionism was high, but low when self-critical perfectionism was low. One study looking only at women and STEM-courses concluded that women that are maladaptive perfectionists were at risk to perform worse in STEM courses that normally are male dominated, whereas women that are adaptively perfectionistic performed well in those courses (Rice et al. 2013a).

23.3 Method

To answer the research question, I have re-analysed a set of data that was collected for a study looking at beliefs indicated in students' mathematical reasoning (see Sumpter 2013). The students selected were all four from the Natural Science program, the most mathematical intense one in the Swedish school system, and they had just finished their third course in mathematics (C-level) including differentiation (functions and graphs) and just started the next course (D-level) where they just started studying differential equations. The students were picked out by their teacher based on the criteria that the students should not have extremely good or poor results. In this study, one student named Ella, expressed several beliefs about motivation and expectations both during the task solving session but also in the two interviews that were made. In this present paper, the data are analysed using deductive thematic analysis using the six dimension given by Frost et al. (1990) as themes. The aim of the analysis is to see if the different dimensions appear at all more than to see of often they appear. Since the data was not collected with the aim to search specifically for perfectionist beliefs, not all dimensions may not be covered simply due to the method. Just as in Sumpter (2013), I will here talk about *Beliefs Indications* (BI) since I can only analyse what Ella express. (For more information about BI and the data collection regarding, see Sumpter 2013.) It should be stressed that the data was not collected for studying this specific topic but for studying what arguments students gave for different choices made in a task solving session. Also, there are no specific interview questions focusing on perfectionism. Therefore, I cannot say Ella is a perfectionist in an adaptive/maladaptive or healthy/unhealthy way. I can only talk about explicit statements that Ella made and see them as an indication of a dimension of perfectionism. The results are still interesting if they can be part of an understanding how perfectionism could be expressed in mathematical task solving.

23.4 Results

Ella is trying to solve following task: Find the largest and smallest values of the function $y = 7 + 3x - x^2$ on the interval $[-1, 5]$. This is a routine task, the easiest of the three picked out for the task solving session, and similar ones could be found in the students' textbook.

Ella says, even before the camera starts recording, that she is nervous and she doesn't like to make errors.

Interviewer: Why don't you like to make errors?
Ella: It is hard.
Interviewer: It is hard. On/For you?
Ella: Yes. I think I should be able to do it [correct]. I had good grades on both the A-level and the B-level. I passed with special distinction [the highest grade] on both A and B, but now everything feels so hard.

Ella says, even before the camera starts recording, that she is nervous and she doesn't like to make errors. In this passage, Elle express both *Excessive Concern over Mistakes* and *High Personal Standards*. Both these dimensions of perfectionism are considered self-oriented.

In the second interview, she explains why she wants to follow an algorithm, especially on a test:

Ella: I like to do things that I know works. When I'm doing some exams and so. Otherwise it doesn't matter. Then you can try anything you like.
Interviewer: Ok. So if you do an exam…?
Ella: Then I want to, because I know that this is what they accept. The right method, and then I know I'll get some points.

Ella confirms that when solving a specific task, there is a specific algorithm that she prefers to use motivated by compensation (passing the exam). There are also indications of a cognitive intrinsic motivational belief saying that she doesn't trust her own reasoning enough to use it on an exam. She explains why this is true at home but not on a test in school:

Ella: It is hard. That is because I'm scared of getting bad result on the test. Because I don't want to have bad grades. Maybe not a good way of thinking. I just really want to do well, I want to have good grades so I can get to [study] what I want to later, if I want to continue my studies later on. Then it is more important to do in the way they [the teachers] want you to do instead of what you might dare to do.

This is confirmed in the questionnaire where she fully agrees to the statement 'The purpose of mathematics education is that the teacher tell you which methods you should use to solve certain tasks.' Later on in the second interview, Ella comes back to this issue:

Ella: I trust that I understand those methods they have taught us. It is just my own [methods] I don't completely trust. Most of the times what they say are true.

These indicated beliefs about safety/security and their connection to motivational beliefs and/or active goals are once again confirmed when she continues:

Ella: I'm scared of that I'm going to do poorly. If you have tried a method at home, then you don't know if the teachers think it is weird. If it will do or not.

She ends by motivating why she prefers to study mathematics instead of other courses, stressing both intrinsic cognitive motivational belief and a personal belief about expectations when saying "I rather do maths and so. Then I know I will pass."

Ella makes a decision to proceed with the problem solving saying: "Ok, I'll do it.". She reads the task and says: "I have to write it like this, have to do it". This is an indication of a *Need for Organization*. However, there are no more data from the interviews to support this dimension.

Ella writes $y = 7 + 3x - x^2$ [...] She continues writing $[-1, 5]$ and says:

Ella: Shall I differentiate or what? I don't understand what I should do. It's probably that. [silence]

This could be an indication of *Doubts about Actions*. Later in the task solving session, Ella will express more that support such a dimension.

Interviewer: What is your first thought?
Ella: I'm thinking that I should ... yeah, differentiate it first.
Interviewer: Because... ? Why do you think that?
Ella: Well ... [I] know that we normally do that. That is the first thing you do when you have a task where you should put values [in a formula] I assume.
Interviewer: How come you think about differentiation? What made you think... ?
Ella: That is because it says x and like x^2. Then it is good to differentiate.

[talk about the interval] Ella writes: $y = 3 - 2x$.

Ella: Shall I differentiate or what? I don't understand what I should do. It's probably that. [silence] That is what you get if you differentiate. You take away the number and then... Then I'm not sure if I should put in that [the end points for the interval] in the first formula or in the differentiation formula. Because I can't remember. But I can try.

Again, Ella expresses doubt about what to do, what she should do. She continues:

Ella: You can... I think that you can just put in everything [all the values], but that is going to take a long time.
Interviewer: You mean, put in $-1, 0, 1, \ldots$?
Ella: I think that I was taught how to do it in an easier way. That is if you should get the maximum and the minimum values, maybe. This becomes hard when I don't know what to do. How I should do it. I'm thinking about different things. We just started with something new today. And that is a lot of things.

Ella emphasizes yet again 'how she should do it'.

Ella: Because if you calculate maximum and minimum then I don't get ... or ... it was nothing.
Interviewer: If you calculate maximum and minimum then you don't get what?
Ella: Then I get these two. I guess. That interval. I don't know how I should use it...
Interviewer: How you should use it?
Ella: Mmm. I don't know. [silence]

Ella: But if I try to put in [the values] in the formula. See if that works. Ugh! I don’t like this. When it is like this. [silence]

Again, doubt is linked with negative emotions. This passage could also be an indication of *Excessive Concern over Mistakes*, or more correctly a concern about to do something that is not ‘correct’.

Eight minutes has passed and Ella is getting more emotional upset. She acts nervously, waving her hand. This is why the interviewer makes the decision to encourage Ella to continue her solving attempt but with some guidance. During the guidance Ella says that “because if you differentiate then you don’t get a second degree function. You get that [points at $y' = 3 - 2x$.] and then … it feels like I’m lost.”

Ella laughs, also nervously. The interviewer asks her:

Interviewer: Do you want to try to solve some of the other tasks?
Ella: I want to do it [to solve this task]. I don’t want to do it [in this way], that I start with one thing, and do it half- done, and then I take the next thing and do it half-done. Because, then it is like this, that it is unfinished, and that doesn’t feel good.

Again, Ella talks about *High Personal Standards*: to finish what you have started. It could also be a *Need for Organization* in terms of doing things in the right order. Shortly after this passage, the task solving session was ended.

23.5 Discussion

In Ella’s task solving session, four of Frost et al. (1990) six dimensions were indicated: *Excessive Concern over Mistakes*, *High Personal Standards*, *Doubts about Actions* and *Need for Organization*. Two dimensions were not found, *High parental Expectations* and *High Parental criticism*. That could however be a result of the data collection; the study from the data was taken from did not specifically aim to study these dimensions and in task solving sessions at school, lab situation or not, most often parents are not explicitly involved. It could well be that Ella would express such beliefs or opposite ones if been asked about it or if the interviews have been taking place in a different context. It is also not clear whether or not it is healthy/unhealthy or maladaptive/adaptive perfectionism that is indicated, and I will therefore not make any conclusions regarding divisions of that kind. However, there are several signals that Ella did struggle with the ‘non-perfection’, as Lundh (2004) describes it, of the task-solving: it didn’t go smoothly, she was not in control of each step, and she expressed several statement that have negative emotions interconnected similar to what Mercer (2010) describe. Ella starts by saying that she has been doing very well, but “now everything feels so hard”. It seems as if she at least might have been one of the high-achieving students’ that have been reported in previous studies where some dimensions of perfectionism are helpful (Stornelli et al. 2009; Tsui and Mazzocco 2007). However, in these studies it is not completely clear whether these students are gifted and high-achievers, or if they are considered gifted based on other measures than grades and points on test, or gifted independent of achievement. The two studies from Rice and colleagues (Rice et al. 2013a,b) looked at the connection perfectionism and achievement the other way around and performance can be related to different types of perfectionism. Just as Frost et al. (1990) concluded, we need to find out more about the relationship between the components high standards and achievement to say whether it is a case of perfectionism or not.

Even though perfectionism is a well researched area in psychology, the literature review show there are very few studies in mathematics education and there are no qualitative studies at all. All of the papers in the literature review looked at perfectionism in relation of some sort of achievement of a test. None have looked for instance at individual differences, how perfectionism hinder or help different students, or when perfectionism appears in mathematics education. So even if previous research indicates a relationship between achievement in mathematics and perfectionism, we don't know how this relationship works or how it is created. And, if we see unhealthy perfectionism as part of a negative affective sphere, and anxiety and negative attitude towards mathematics are growing problems (Geist 2010; The Swedish National Agency for Education 2013a,b), this is an area we need to understand more about.

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Chapter 24
Gender Differences Concerning Pupils' Beliefs on Teaching Methods and Mathematical Worldviews at Lower Secondary Schools

Boris Girnat

Abstract This article documents the development of a questionnaire concerning pupils' beliefs on teaching methods and mathematical worldviews. A representative poll leaded to some remarkable gender differences that are reported here as a first application of this questionnaire. These differences can be seen in more instructivist and less apply-oriented attitudes of the female group and more constructivist, process- and applied-oriented and less instructivist attitudes of the male group. Additionally, the constructivist and instructivist scales correlate positively only in the male group.

24.1 Pupil's Beliefs on Teaching Methods and Worldviews

The Swiss Conference of Cantonal Ministers of Education is planning a nation-wide assessment of basic competencies in mathematics at the end of compulsory education in grade 9 (cf. EDK 2015). This assessment is intended to take place in 2016. The School of Teachers Education Northwestern Switzerland is the leading house for constructing the mathematical tasks of the performance test and for a part of the context questionnaire. This questionnaire will contain a socio-demographic part and a "mathematical" part focused on pupils' attitudes, affects, emotions, and self-efficacy concerning the teaching and learning of mathematics. These kinds of pupils' beliefs are described e.g. by McLeod (1992) in summary. Studies like Schoenfeld (1989) indicate that they have an explorative power for pupils' mathematical performance. Hence, it became a common standard to accompany a mathematical performance test by a context questionnaire containing the topics mentioned above.

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The Swiss questionnaire will contain some scales that were already used in studies like TIMSS and PISA and that are mainly focused on self-efficacy and the motivational and emotional aspects of pupils' beliefs, affects, and attitudes (cf. e.g. OECD 2013, pp. 184–186 and 194). My intention was to broaden this questionnaire by two new topics: (a) pupils' preferences in teaching methods and (b) pupils' beliefs on mathematical worldviews. To create suitable scales, I carried out a first pretest in fall 2014 with 256 participants (cf. Girnat 2015). In spring 2015, I conducted a second larger and representative pretest with 956 participants to overhaul the scales and to include more covariates like gender, the type of school, and school marks. In this article, I give an overview on the scales and the findings of the second pretest related to gender differences. The focus is set on gender differences, since these differences are in general very typical for many aspects of mathematics education (cf. Gallagher and Kaufman 2005, Pajares and Graham 1999, for self-efficacy, motivation constructs, and mathematics performance; and cf. Stipek and Gralinski 1991, for achievement-related beliefs and emotional responses).

24.2 Setting Up the Scales

The basic idea for creating scales to measure pupils' preferences in teaching method is the antagonism of instructivist and constructivist teaching methods. According to Duit (1995), constructivist learning theories are based on the assumption that learning is a learner's active construction of knowledge related to his prior experiences and convictions. Insofar, constructivist learning environments are characterised by properties that are supposed to enforce these construction processes like pupil-centeredness, autonomy, inclusion of the pupils' prior knowledge, social negotiations of meanings, and possibilities to explore and discover insights by self-directed activities. Instructivist environments, on contrary, are marked by teacher-centeredness and a mostly passive understanding of the pupils' learning process focused on understanding and re-enacting teachers' explanations or examples and getting routine by solving series of similar tasks.

The first pretest in fall 2014 was designed to examine the factorial structure of 22 items that were intended to express typical aspects of instructivist and constructivist teaching methods. The participants were prompted to rate the items on a six step Likert scale of agreement/disagreement. The question was how useful they regarded the teaching method expressed by the items to learn mathematics. This is a difference to common scales on teaching methods where the question is what teaching methods are used in the classroom, and not how pupils value these methods (cf. e.g. OECD 2013, p. 194). After collecting the data, an exploratory factor analysis was carried out (cf. Tabachnick and Fidell 2001), following Horn's parallel analysis to determine the number of factors to extract (cf. Horn and Engstrom 1979). I used the psych package (Revelle 2015) with R (R Core Team 2014). The explorative factor analysis leaded to a five factor solution (cf. Girnat 2015): As expected, the instructivist items form a single scale (called instrlearn in the following), but the constructivist items were arranged into four different ones: learning by exploration and discovery (disclearn), using real-world situations to understand mathematics (realref), social learning and learning in groups (soclearn), and pupils' autonomous choice of tasks and topics (autlearn). The last factor autlearn was dropped, since there were no significant relations to the pupils' properties, like gender or marks, observable. Afterwards, some new items were created to gain almost the same amount of items for each scale in the second pretest. The following list contains all the items used in the second pretest. The items of the first pretest are marked by an *.

disclearn_1) I love to puzzle out solutions to tasks. I also love to solve tasks by trial and error.
disclearn_2*) I like tasks and problems that encourage me to discover different mathematical insights by myself.
disclearn_3) It's exciting when we discover how to solve a task on our own before the teacher has explained it to us.
disclearn_4*) In mathematics you can discover a lot on your own.
disclearn_5) In mathematics, you can come up with creative solutions without theoretical background knowledge.
disclearn_6*) In mathematics, you can fiddle and puzzle out a lot on your own. This is the best way to come to a solution.
disclearn_7) If you are working on a mathematical problem, you often come up with new insights spontaneously and automatically.
instrlearn_1) It is important that our teacher provides us with consistent rules, methods and notations, and that everyone then follows these precisely.
instrlearn_2*) I think it's useful to solve a lot of tasks in order to understand a method correctly.
instrlearn_3*) I learn mathematics well, if the teacher first demonstrates a new method and we then repeat this method with several tasks.
instrlearn_4*) I want to see rules and examples that show me how to solve my tasks.
instrlearn_5*) Doing exercises should be based on training the exact same method again and again until we can all handle the task.
instrlearn_6*) It's best if the teacher first demonstrates the solution of a task and we repeat his method step by step to solve the task afterwards.
instrlearn_7*) The teacher should present mathematical topics and methods to us. He shouldn't encourage us to discover them on our own.
soclearn_1*) I learn mathematics well, if we collaborate in groups to solve a problem and develop our own solution.
soclearn_2*) I prefer it if we as pupils explain to each other how to solve a task rather than the teacher explaining to us how to do it.
soclearn_3) I like to work in pairs or bigger teams.
soclearn_4*) I often understand a mathematical topic first, if I discuss it with classmates or colleagues.
soclearn_5) I understand mathematics better if we collaborate in groups as opposed to being shown by the teacher on the blackboard.
soclearn_6) I learn a lot when I work on a task together with other classmates.
realref_1*) When we introduce a new mathematical theme, I like to start with a real situation from everyday life and then explore the mathematical theme in this context.
realref_2*) I find it interesting to solve everyday life problems using mathematics.

realref_3*) Mathematical task don't need to be related to everyday life. I don't need such illustrations.
realref_4*) Tasks should always be related to everyday life. They shouldn't only relate to pure mathematics.
realref_5*) A mathematical theme only makes sense to me, if I can see how it helps to solve real-life problems from everyday contexts.
realref_6) Mathematical tasks should always be related to reality.

The second part of scales concerning mathematical worldviews is based on the ideas of Grigutsch et al. (1998). They introduced four basic dimensions to describe a teacher's mathematical worldview: The formalism aspect (concerning rigour, logic, deduction, formalism, and technical terms as typical characteristics of mathematics); the apply aspect (stressing the practical use of mathematics in everyday life, in the professional world, and for society); the process aspect (underlining the creativity of doing mathematics); and the scheme aspect (related to a standpoint that regards mathematics as a bound of rules, algorithms, and prescriptions to be followed). The items of the second part of my questionnaire are developed according to Grigutsch et al., but there were three differences: (1) the process aspect was integrated into the scale disclearn (as items 4, 5, 6, 7), since this aspect seems to be more related to the learning of mathematics than to its nature; (2) the "formalism aspect" was renamed into "system aspect", since formalism seems just to be one of its parts; (3) most of the items were linguistically simplified and they were adapted to the pupils' horizon of mathematical experiences. This seemed to be necessary, since the original items were created to investigate teachers' mathematical worldviews. The following list contains all the items used in the second pretest (the items already used in the first pretest are again marked with an *). The participants were asked to rate these items on a six step Likert scale of agreement/disagreement. The question was how strong they agreed that the content expressed by an item was a characteristic property of mathematics.

applyasp_1*) Mathematical knowledge is important for everyday life.
applyasp_2*) Mathematics is necessary for many occupations.
applyasp_3*) In mathematics education, we often deal with topics that have no practical use.
applyasp_4*) Many mathematical themes are of practical use.
applyasp_5*) Mathematics is important to our society.
applyasp_6*) Without mathematics, you won't get far.
schemasp_1*) You have to follow the teacher's examples and sample solutions exactly to manage your tasks successfully.
schemasp_2*) Ideally, the solution of a task looks the same in every pupil's exercise book.
schemasp_3*) To solve mathematical tasks successfully depends on having learnt the right methods off by heart. Otherwise you'll get lost.
schemasp_4) In mathematics education, it is most important to learn predefined ways of solving problems off by heart and to apply them correctly.
schemasp_5*) It's impossible to invent mathematics on your own. You depend on having mathematics shown and explained to you.
schemasp_6) You'll only be able to learn mathematics if someone explains you its methods and you imitate them.
systasp_1) It's necessary to understand mathematical methods. It's not enough just to apply them.

systasp_2*) A solution has to be written down in a formally correct notation to be correct.
systasp_3*) All parts of mathematics are systematically linked to each other.
systasp_4*) You have to be able to think logically and to justify theorems if you do mathematics.
systasp_5*) In mathematics it's important to use technical terms and conventional notations.
systasp_6) In order to understand new themes it is important to understand previous ones.
systasp_7) In mathematics, it is indispensible to know exactly what symbols and technical terms stand for.

24.3 Rechecking the Scales

The second pretest was used to recheck the scales. This was carried out in two steps: At first, an exploratory factor analysis was used to see if the result of the first pretest could be reproduced. Aside from two problematic items (realref_1 and applyasp_3), the number of factors and the assignment of the items to the factors could be reproduced. Secondly, for each factor or latent variable, a confirmatory factor analysis was carried out (cf. Brown 2006), using the lavaan package (Rosseel 2012) with R. In several cases, there was evidence that the one factor solution was not the best possibility and that it might advisable to split the single factor in two ones. Table 24.1 contains the fit indices for the measurement models. Due to the ordinal nature of the questionnaire's data, the diagonally weighted least squares method with a correction for the means and variances (WLSMV) was used to estimate the parameters and to set up the test statistics (cf. Beaujean 2014, p. 98 for the WLSMV method, and pp. 153–166 for the fit indices). In some cases, a two factor solution seems to be the better alternative. If so, both the single and the two factor solution are reported.

As Table 24.1 shows, all of the two factor solutions have got substantially better fit indices than the single factor solutions. However, statistical properties should never be the only reasons to prefer one model above the other. The choice of a model has also to be based on content to be valid. In the five cases of "split" factors, the two factor solutions also seem to contribute an enhancement with regard to the content: (1) The two factors of disclearn separate the state and trait aspect of learning by discovery (implying that it was no good idea to combine the attitudes to teaching methods with mathematical worldviews); (2) the items of instrlearn_a single this aspect of instructivism out that is related to repetitive exercises, whereas instrlearn_b addresses the instructions of the teachers; (3) soclearn_a is related to social arrangements in general, while soclearn_b stresses the communicative learning effect of social situations; (4) the difference between schemasp_a and _b is that schemasp_b specifically addresses the technique "learning by heart", whereas schemasp_a is more general; (5) systasp_a expresses the logical and systematic aspect of mathematics, whereas systasp_b is related to formal correctness. Concerning these analyses, the two factor solutions are to prefer.

Table 24.1 Fit indices of the confirmatory factor analysis of the measuring models

Scale	χ^2	df	p value χ^2	CFI	RMSEA	RMSEA <0.05	SRMR	Cor
disclearn	57.352	14	0.000	0.992	0.057	0.198	0.044	—
disclearn_a (1, 2, 3) disclearn_b (4, 5, 6, 7)	12.427	21	0.493	1.000	0.000	1.000	0.020	0.835
instrlearn	46.911	14	0.000	0.986	0.050	0.471	0.046	—
instrlearn_a (2, 3, 5) instrlearn_b (1, 4, 6, 7)	20.108	21	0.093	0.997	0.024	0.988	0.030	0.793
soclearn	41.192	9	0.000	0.986	0.062	0.134	0.046	–
soclearn_a (1,3) soclearn_b (2,4,5)	7.591	10	0.108	0.997	0.031	0.794	0.023	0.731
realref (2, 4, 5, 6)	1.102	5	0.576	0.995	0.000	0.929	0.010	–
applyasp	9.980	9	0.352	1.000	0.011	0.993	0.022	–
schemasp	27.617	9	0.001	0.990	0.047	0.559	0.037	–
schemasp_a (1, 2, 5, 6) schemasp_b (3, 4)	11.154	8	0.193	0.998	0.021	0.972	0.024	0.809
systasp	49.450	14	0.000	0.987	0.052	0.380	0.052	–
systasp_a (1, 3, 4, 6) systasp_b (2, 5, 7)	19.408	13	0.111	0.998	0.023	0.990	0.033	0.805

24.4 Gender Difference I: The Means

To investigate group difference between the means of latent variables, the first step consists in checking the strong or scalar invariance of the measurement models, i.e. that the loadings and intercepts can be treated as equal in all groups (cf. Beaujean 2014, pp. 61–69). For this task, the R package semTools (semTools Contributors 2015) was used. In every case, the strong invariance was given.

Table 24.2 contains the mean differences. To calculate the differences, all latent variables were standardised and the female group was set to be the reference group. Therefore, the female group always has zero as its mean, and the mean of the male group directly expresses the difference to the mean of the female group. Since the latent variables are standardised, the differences can be interpreted as effect sizes using the thumb rule that 0.2 indicates a small, 0.5 a medium, and 0.8 a strong effect

Table 24.2 Mean differences (using the female group as the reference group)

Scale	Mean difference male	Standard error of difference	*p* value mean difference	Correlation female	Correlation male
disclearn_a	0.685***	0.095	0.000	0.795	0.873
disclearn_b	0.296***	0.081	0.000		
instrlearn_a	−0.417***	0.081	0.000	0.750	0.851
instrlearn_b	−0.200**	0.075	0.007		
soclearn_a	0.179*	0.077	0.020	0.665	0.789
soclearn_b	0.115	0.076	0.131		
realref	0.200**	0.072	0.006	–	–
applyasp	0.285***	0.076	0.000	–	–
schemasp_a	0.172*	0.076	0.023	0.817	0.790
schemasp_b	0.162*	0.076	0.033		
systasp_a	−0.107	0.073	0.144	0.766	0.847
systasp_b	0.012	0.074	0.872		

The mean differences are marked with the usual asterisks to indicate the significance levels (* stands for $p \leq 0.05$, ** for $p \leq 0.01$, and *** for $p \leq 0.001$).

(cf. Cohen 1988). In case of the "split" factors, the table also contains correlation between the two factors separated for each group.

Nine of the twelve mean differences are significant; remarkably, the differences concerning the system aspect are not. Among the significant differences, the strongest effects are observable in the field of the constructivism/instructivism dichotomy: The male group prefer the explorative activities (disclearn_a) much more than the female group. The difference between disclearn_a and disclearn_b is interesting: disclearn_b stands for the conviction that mathematics "in general" is a field of creativity and discovery. In this case, the gender difference is small. Disclearn_a expresses the willingness to explore and discover mathematical insights on your own, i.e. including the motivational background and the cognitive, emotional or motivational obstacles like anxiety or low self-efficacy. In this case, the gender difference is the highest one observed in this study. This difference seems to be similar to the state/trait distinction in psychology, also regarded as relevant for mathematical beliefs (cf. Stipek and Gralinski 1991). Furthermore, the female group estimates instructivist teaching method higher than the male group. The more relevant difference can be located in the scale that expresses "learning by repetitive exercises" (instrlearn_a), but also the female group prefers the teachers' instructive and explaining activities (instrlearn_b). The effects on real-world connections (realref and applyasp), scheme aspect and learning in groups (soclearn_a) are smaller, but still significant. With one exception (schemasp), the correlation between the "split" factors are lower within the female group. This indicates that the female perceptions of teaching methods and mathematical worldviews is more "fine-grained" than the male ones, i.e. female pupils discriminate these beliefs more precisely.

24.5 Gender Difference II: Correlations

Table 24.3 contains the covariances between the latent variables of the scales. The covariances are separately reported for the female and male group, with the first value being the covariance of the female group. Since all the latent variables are standardised, the covariances can be understood as correlations. Bold entries indicate significance at least on 0.05 level. The most interesting results are located in the first two columns: These columns contains the correlations between the two "discovery scales" and the other ones. It is remarkable that "learning by repetition" (instrlearn_a) correlates positively with both "disclearn" scales, i.e. this type of instructivist learning is not seen as opposed to constructivism, but as an addition (more by the male pupils than by the female ones). This is different in case of instrlearn_b, the teacher-centered explanations: The female group perceive this teaching method as opposed (−0.177) or neutral (0.044) to constructivist discovery, whereas the male group understands it as slightly (0.167) or remarkably (0.407) supportive. Furthermore, learning by discussing in groups (soclearn_b) correlates to constructivist discovery within the male group (0.306), but not within the female one (0.087), similar in case of the two scheme aspects with slightly positive correlations in the male group, but not in the female one. These results are remarkable, since in literature (cf. Duit 1995, see above) constructivist and instructivist teaching methods are normally seen as being opposed to each other. The correlations reported here, however, indicates that especially the male pupils regard these methods as additions to each other, and not as antipodes. The female pupils see them partly as neutral and partly also as additions.

Overall, the general predominance of positive correlations does not indicate that it might be possible to divide the scales into two parts as the theoretical literature of teaching methods typically suggests: a more constructivist part (disclearn_a/b, soclearn_a/b, realref, and applyasp) and a more instructivist one (instrlearn_a/b, schemasp_a/b, and systasp_a/b). This result may advice to examine the pupils' perceptions of teaching methods more intensively and to compare the results to theoretical expectations.

24.6 Reflection and Further Research

As stated above, the scales presented in this paper are intended to be used in a context questionnaire accompanying a performance test. Primarily this context allows examining the potential of these scales. Two questions are of great interest: (1) How are the relations between these scales and the results of the performance test? (2) How are the relations to the other context scales (mostly related to emotional and motivational issues)? Possibly the scales presented here can identify causes of emotions and motivation or can operate as mediators to raise or decrease these affects.

Table 24.3 Covariances between the standardised latent variables, separated by gender

Scale	dl_a	dl_b	il_a	il_b	sl_a	sl_b	rr	aa	sc_a	sc_b	sy_a
disclearn_b	**0.775**										
	0.914										
instrlearn_a	**0.273**	**0.374**									
	0.501	**0.550**									
instrlearn_b	**−0.177**	0.044	**0.634**								
	0.167	**0.407**	**0.774**								
soclearn_a	**0.210**	**0.291**	**0.323**	**0.279**							
	0.399	**0.497**	**0.396**	**0.311**							
soclearn_b	0.087	0.122	**0.300**	**0.271**	**0.638**						
	0.306	**0.365**	**0.362**	**0.325**	**0.779**						
realref	**0.154**	**0.245**	**0.464**	**0.417**	**0.503**	**0.441**					
	0.459	**0.463**	**0.496**	**0.449**	**0.570**	**0.587**					
applyasp	**0.484**	**0.468**	**0.442**	**0.212**	**0.334**	**0.231**	**0.543**				
	0.659	**0.670**	**0.562**	**0.403**	**0.441**	**0.392**	**0.722**				
schemasp_a	−0.073	0.017	**0.437**	**0.709**	**0.475**	**0.373**	**0.576**	**0.371**			
	0.225	**0.303**	**0.631**	**0.726**	**0.443**	**0.574**	**0.636**	**0.637**			
schemasp_b	−0.035	0.099	**0.415**	**0.639**	**0.355**	**0.403**	**0.528**	**0.340**	**0.801**		
	0.279	**0.345**	**0.582**	**0.591**	**0.530**	**0.534**	**0.657**	**0.590**	**0.795**		
systasp_a	**0.335**	**0.411**	**0.630**	**0.336**	**0.396**	**0.306**	**0.649**	**0.662**	**0.490**	**0.438**	
	0.585	**0.656**	**0.747**	**0.518**	**0.530**	**0.433**	**0.614**	**0.873**	**0.614**	**0.636**	
systasp_b	**0.279**	**0.340**	**0.562**	**0.444**	**0.382**	**0.337**	**0.645**	**0.617**	**0.601**	**0.586**	**0.765**
	0.541	**0.586**	**0.614**	**0.480**	**0.455**	**0.403**	**0.678**	**0.803**	**0.713**	**0.703**	**0.854**

But even not concerning further investigations, the most remarkable findings of the study presented here are the indications that there are some strong and significant gender differences in this field of beliefs: (1) The most important mean differences can be located in the perceptions of instructivist and constructivist teaching methods. These differences indicate instructivist and less apply-oriented attitudes of the female pupils and more constructivist, process- and applied-oriented and less instructivist attitudes of the male group. (2) The gender differences in the correlation matrix indicate that the perception of “the whole situation” established by teaching methods and mathematical worldviews is in some aspects quite different. Within the male group, the correlations are positive without exception, i.e. that male pupils perceive different teaching methods and mathematical worldviews more as complements than as opposites. Concerning the female group, the situation is gradually different, insofar the two “discovery scales” of a constructivist view have zero or negative correlations with some scales that expresses more instructivist points of view.

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Chapter 25
"Every Time I Fell Down (Made a Mistake), I Could Get Up (Correct)": Affective Factors in Formative Assessment Practices with Classroom Connected Technologies

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Abstract In this contribution we analyse data coming from the research project FaSMEd, which aims at investigating the role of technologically enhanced formative assessment methods in raising the attainment levels of low-achieving students. Our working hypothesis is that low attainment is also linked to affective factors and that, consequently, these factors should be taken into account when planning interventions and when evaluating their effectiveness. We report our first steps towards the analysis of the experiments in terms of affect, drawing some preliminary conclusions on the students' attitude towards the project and outlining further research developments.

25.1 Introduction

This contribution comes from our experience within the European project FaSMEd—Formative Assessment in Science and Mathematics Education (FP7, project number 612337), carried out since January 2014. The project focuses on the use of technology in formative assessment classroom practices in ways that allow teachers to respond to the emerging needs of low achieving learners in mathematics and science, so that they are better motivated in their learning of these subjects. Outcomes will inform the development of a toolkit aimed at supporting teachers in the activation of effective formative assessment practices in mathematics and science through the use of different technologies.

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The project involves nine partners form Europe and South Africa. Starting from the common frame concerning formative assessment, each partner carried out specific choices concerning the focus of FA (on individuals or on groups), the technology to use and the kind of activities to carry out.

In this paper we refer to the experience in the Italian classrooms. Our working hypothesis is that low attainment is also linked to affective factors and that, consequently, affective factors should be taken into account when planning interventions and when evaluating their effectiveness. We report our first steps towards the analysis of the experiments in terms of affect. First of all, we introduce the FaSMEd project and our methodological choices to introduce the context within which students worked and better highlight the effects of this context in terms of affect. Then we present the theoretical tools chosen for the affective factors (the constructs of attitude towards mathematics and motivation), the data at disposal and the first results.

25.2 The FaSMEd Project: The Theoretical Background and Our Methodological Choices

In the FaSMEd project, formative assessment (FA) is intended as a method of teaching where

> [. . .] evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited (Black and Wiliam 2009, p. 7).

Evidence about student achievement can be collected and exploited in different moments of the learning process and with different purposes by the three main agents involved in this process: the teacher, the learner and the peers. Wiliam and Thompson (2007, adapted from Ramaprasad 1983) highlight three crucial processes in learning and teaching: (a) establishing where learners are in their learning; (b) establishing where learners are going; (c) establishing how to get there.

Black and Wiliam (2009, from Wiliam and Thompson 2007) further conceptualise formative assessment as consisting of five key strategies:

1. Clarifying and sharing learning intentions and criteria for success;
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
3. Providing feedback that moves learners forward;
4. Activating students as instructional resources for one another;
5. Activating students as the owners of their own learning.

The final aim of the FaSMEd project is to study ways to use technology to improve formative assessment. Even if affect is not at the core of the project, within the Turin Unit we are convinced of the importance of affective factors for the learning of mathematics, which therefore must be considered when planning the activities and the use of new technologies. More specifically, we make the key assumption that low achievement is linked to lack of basic competences, but also to affective and metacognitive factors. Furthermore, we argue that during class activities, it is important to enable students develop ongoing reflections on their learning processes and make their thinking visible (Collins et al. 1989) through their sharing it with the teacher and the classmates.

Connected classroom technologies (Irving 2006; Roschelle et al. 2004; Shirley et al. 2011) seem promising to these aims because they both enable to share the ongoing and final productions of the students and to collect their opinions during the activities and at the end of them. More specifically, we chose a connected classroom software (IDM-TClass) that allows the teacher to: (a) show, to one or more students, the teacher's screen and also other students' screens; (b) distribute documents to students and collect documents from the students' tablets; (c) create different kinds of tests and have a real-time visualization of the correct and the wrong answers; (d) create instant polls and immediately show their results to the whole class. Moreover, the students' written production can be displayed (through the data projector or the interactive whiteboard), compared and discussed.

25.3 Attitude and Motivation in Formative Assessment Activities with Technology

In order to consider students' affect in formative assessment activities within the FaSMEd project, we will focus on their attitude and their motivation.

Di Martino and Zan (2010, 2014) developed a three dimensional model for describing students' **attitude** towards mathematics. As they underline, this theoretical characterization "*takes into account students' viewpoints about their own experiences with mathematics, i.e., a definition of attitude closely related to practice*" (Di Martino and Zan 2014, p. 575). The resulting three-dimensional model of attitude (TMA model) features attitude towards mathematics by three strictly interrelated dimensions: emotions related to mathematics, vision of mathematics, perceived competence in mathematics. Drawing on this definition, we will investigate students' perceived competence in mathematics within the FaSMEd experience, vision of mathematics within FaSMEd experience, emotional disposition towards the FaSMEd experience.

The Fasmed project points out the need of helping low achieving learners in mathematics and science "*so that they are better motivated in their learning*" (FaSMEd Document of Work, Part B, p. 2), putting an emphasis on motivation.

Motivation is a crucial construct in research in mathematics education. Motivation "*reflects personal preferences and explains choices. […] Motivation varies from very local preferences (This would be a perfect moment for a cappuccino) to a variety of different levels of goals (I want to solve this task, I want my peers to think that I am clever) and very global needs such as needs for nutrition and social belonging*" (Hannula 2011, p. 42). Middleton and Spanias (1999) define motivations as "*reasons individuals have for behaving in a given manner in a given situation*" (p. 66). Motivation may be intrinsic (when students engage in the task they consider learning important, and because they enjoy learning) or extrinsic (when students engage in tasks to obtain rewards or avoid punishment). Middleton and Spanias observe that motivations towards mathematics are deeply linked to mathematical self confidence (Bandura 1997) and develop early, under the influence of teachers' actions and attitudes; thus, creating interesting didactical contexts may improve students' motivations.

Following Hannula (2006), we can also consider a key distinction between "state" and "trait" aspects of motivation. The state aspect of motivation refers to needs, values, desires and motivational orientations, while the trait aspect of motivation refers to active goals during mathematical activity. Hannula (2006) claims that research should focus more on the *trait* aspect of motivation, in order to understand why a student chooses not to put effort in one activity, and to look for ways to promote a desired motivational state in students. Hannula also highlights that motivation is only partially conscious. Motivation may manifest itself in emotion, cognition or behaviour.

Key concepts linked to motivation are goals and needs: needs are more general than goals and are influenced by students' beliefs about self and mathematics, as well as by the school context and the sociomathematical norms. For instance, Hannula (2006) points out that a student may feel a need for competency and, consequently, the goal of solving tasks efficiently; or a student could have a "social" need, and the consequent goal could be to work collaboratively.

As pointed out, our research project concerns the use of technology for promoting formative assessment in mathematics. Only few studies addressed the issue of affect in technology-enhanced mathematical teaching and learning. Among them, we refer to Galbraith and Haines (1998), who propose to "disentangle" and analyse in depth two classes of affective factors: affect concerning mathematics and affect concerning technology. In the second class they include computer confidence ("*students [. . .] believe they can master computer procedures required of them [. . .] and in cases of mistakes in computer work are confident of resolving the problem themselves*", p. 278) and computer motivation ("*students [. . .] find computers make learning more enjoyable*", p. 278). We will take the study of Galbraith and Haines (1998) as a reference for our reflection, even if a key difference concerns the kind of technology at issue: in their study, technology refers to specific software for the teaching and learning of mathematics, while in our case technology is designed to be used in a classroom for managing the interaction between students and teacher.

25.4 The Context of the Project

In Italy the FaSMEd Project involves 19 teachers from three different clusters of schools located in the North-West of the country. 12 of them work in primary school (grades 4–5) and the other 7 in lower secondary school (grades 6–7). All the teachers work on the same mathematical topic: functions and their different representations (symbolic representations, tables, graphs).

Low-achievers attend regular classes with the other students, since there is no streaming and there are only mixed ability classes. Low-achievers are identified through the classroom teachers' assessment.

25.4.1 The Activities

We integrated the use of connected classroom technologies within a set of activities coming from different sources. Among them, the ArAl Units, which are models of sequences of didactic paths developed within the project "ArAl—Arithmetic pathways towards favouring pre-algebraic thinking" (Malara and Navarra 2003; Cusi et al. 2011).

Students work in pairs. As a general methodological choice, each pair is formed by students of the same level, in order to avoid that low-achievers rely only upon other students and to foster their active involvement in the activities. Each pair has one tablet (with IDM-TClass software) at disposal.

25.4.2 Data and Research Questions

Data at disposal are video-recordings of classroom activities, field notes of the observers, teacher interviews/questionnaires at the end of each activity, students' interviews/questionnaires at the end of the first cycle of activities (about 20 h). Here is the list of the questions for the written questionnaire (for interviews, the same questions served as a script):

1. What did you learn from the FASMED activities?
2. What did you like most in the FASMED activities? Why?
3. Is there something you did not like in the FASMED activities? What? Why?
4. Which question/task did you find most difficult? Why?
5. Did technology help you to understand better whether your way of reasoning was correct or not? If yes, how did technology help you? If not, why didn't it help you?
6. Did technology help you to understand how to improve or correct your answer? If yes, how did technology help you? If not, why didn't it help you?
7. Did technology help you to understand better how your classmates reason? If yes, how did technology help you? If not, why didn't it help you?

Data were collected for the aim of gathering a wide range of information on students' experience during the activities, and to foster different levels of students' reflections: meta-level reflections about the learning results of the activities (question 1) and the main sources of difficulties that students had to face (question

4); reflections on the affective aspects involved in students' learning (questions 2 and 3); reflections on the actual role played by technology in supporting, at the students' level, the crucial processes 'establishing where learners are in their learning' (question 5) and 'establishing how to get there' (question 6), and, at the peer's level, the activation of students 'as instructional resources for one another' (question 7).

In this paper, we study affective factors emerging when students reflect on their experiences within the FaSMEd Project. More specifically, we tackle the following issues:

- What can we observe about students' attitude towards the FaSMEd Project?
- Which aspects are influencing students' motivation during the FaSMEd Project?

25.5 Data Analysis

Among the data at disposal, we focus on the students' questionnaires and interviews at the end of the first cycle of activities. The data analysis is qualitative and mainly based on recurrent trends of answers. In order to give an insight into data, results are illustrated by excerpts from questionnaires and interviews. Also some non-recurrent, but significant and peculiar answers are reported and commented.

Focusing on students' **attitude toward the FaSMEd Project activities** and following Di Martino and Zan (2010, 2014), we organize results around three dimensions: **perceived competence within the project, view of mathematics activity within the project, emotional disposition towards the project.** When appropriate, we highlight the link to the formative assessment strategies described by Black and Wiliam (2009).

Answering question 1, some students declare an **increased perceived competence** in thinking and understanding:

> In many cases it is easier than expected and my mind opened (*Fred*)
> I learnt to reason with my mind and not that of my classmate, I learnt things I was not able to do. (*Marty*)

Fred and Marty's answers may be linked to strategy 5 (*Activating students as the owners of their own learning*). Also Sam's sentence may be read in terms of increased competence and in connection to strategy 5:

> Every time I fell down (made a mistake), I could get up (correct) (*Sam*)

The answers of many students show a connection between the better way of reasoning, as perceived by the students, and their working in collaboration within the project:

> We learnt to reason. [. . .]. To think with the other. (*Rina*)
> Discussing in the class it is easier to understand whether you observation is right or wrong (*Ilda*)

> I liked collaborating with my mate because we had different ideas and opinions, we came to a solution (*Nic*)
> I learnt to reason with my mind and not that of my classmate, I learnt things I was not able to do (*Tina*)

These sentences immediately recall the fourth FA strategy "*Activating students as instructional resources for one another*" (Black and Wiliam 2009). The following reflections, by Kim and Asia, highlight how the students benefitted from this strategy:

> It is easier to work in couple because if I don't understand there is the classmate who helps me (*Kim*)
> Yes, it helped somehow for everything. I had Lory and Lory had me (*Asia*)

Asia, a low-achieving student, seems not so able to say *with respect to what* she was helped, but with very expressive words she indicates *who* has helped her, underlining the importance of having her mate as a support, and being herself a support.

Few students, conversely, report about their low perceived competence within the FaSMEd activities. What strikes is the mix of high motivation, on the one hand, and negative emotions felt when experiencing a lack of improvement during the activities, on the other hand. This tension is palpable in Emi, for instance:

> I learnt that: I did not understand, unfortunately, and I'm very sorry because I would have liked to learn how to solve problems. I would have bee very happy if I had learnt how to solve problems with this project (*Emi*)

From the answers to question 1 we also grasp the influence of the project on the students' **vision of the mathematical activity**, which comes to be seen as a living process, where different procedures are possible, mistakes may happen (and can be useful), reasoning is more important than remembering and applying, solving a maths problem can take much time.

> I learnt that there can be more than one answer to a question in the problems (*William*)
> I learnt that maths is not only matter of calculations, but also reasoning and shortcuts to do the worksheets better (*Lisa*)
> I learnt that for all the tasks there is not a given time, it is sufficient to do them and reason well. And it doesn't matter whether you are goog or not, help are always important (*Bea*)
> If I reason and think well, with no rush, I can answer also to the most difficult questions (*Lina*)

From the answers to questions 3 and 4 we have information about the **emotional disposition** towards the project. Most students remarked a positive disposition. Concerning reasons for positive emotional disposition, students refer sometimes to the fact of working with new technologies. This could be linked to *computer motivation*, and may act for them as a strong *extrinsic* motivation:

> Using tablets. We are modern guys then we like using technological devices (*Lena*)
> I liked using tablets, because doing maths with the tablets was a wonderful sensation (*Ricky*)

In the activities with the tablet, pairs of students were asked to write and send their agreed answer to the teacher, whenever they felt ready. Students underline to have appreciated to have the time for thinking, thus revealing the need for an own safe space for thinking.

> Even if you do not understand, maybe you ask [. . .] I don't understand immediately what they tell me, and when I ask there is somebody who says: what? Don't you understand? But now, having the tablet, I can ask help [to my mate, to the teacher] and that person can not intervene, making me feeling more idiot than what I am, and I understand more (*Ilda*)
> It helped me because it made me reason quietly (*Alex*)

Students also stress that thinking within the pair/group before answering mainly give the possibility of 'thinking together'. It is therefore evident that the social aspect of the activities results the prevalent reason for positive disposition towards the project, as outlined in this excerpt:

> I liked the most when we started thinking what to understand before answering, so, if it was not clear to you what to write on the tablet, the friend could help you to answer. Because in this way you know there is somebody on your side. (*Vic*)

Besides group-work, some students focus on the classroom discussions, where they could express their ideas in front of the whole class. In this case, we may say that discussion is a motivating activity because it meets the *social need*.

> I liked the most when somebody speaks and everybody listens, because you feel important, you feel part of a group (*Lexi*)

Also some low-achieving students show to appreciate collective discussions, such as Ludovica and Enzo. In their answers we also see an instantiation of the FA process "*Establishing where learners are in their learning*" and of the FA strategies "*Providing feedback that moves learners forward*" and "*Engineering effective classroom discussions that elicit evidence of students' understanding*".

> Using the tablet is very helpful because, once you have finished, you sent your work. The teachers shared it with the Interactive White Board and everybody could verify it and in this way you understood why you had done a mistake (*Ludo*)
> I liked doing polls to understand who preferred the idea of somebody or that of somebody else, because it helped me to understand the right or wrong motivations (*Enzo*)

Conversely, few students declared that they did not like to be asked to participate actively to the discussion. They were probably influenced by low self-confidence and fear of failure.

> I didn't like to answer, to read my answer in the discussion, because if we made a mistake I felt worried (*Rak*)

Other students say they did not like the fact of working in pairs, due to the composition of the couples. In order to highlight how FA processes support low-achievers in their learning, in fact, we decided to create homogeneous couples. Low-achievers, therefore, worked in pairs together. In the words of Gila we see the effect of low self-confidence. Conversely, in the words of Debby we see the negative effect of high self-confidence, which turns into lack of disposition to work collaboratively:

> I did not like the couples. For instance me and Claire (dud and dud)" (*Gila*)
> I would have liked to be alone because when I spoke they contradicted me immediately, even when I was right. (*Debby*)

25.6 Discussion and Preliminary Conclusions

In this contribution, starting from the description of the FaSMEd project on the use of technologies for enhancing formative assessment, we analysed students' final reflections and declarations on the project, looking for emerging affective factors. More specifically, we organized our analysis around the concept of attitude looked for possible links with motivation, on the one hand, and employed formative assessment strategies, on the other hand.

As a general result, we found a positive attitude towards the project and its activities.

We observed

- a good level of perceived competence, influenced by FA strategies, especially strategy 3 (Providing feedback that moves learners forward) and 4 (Activating students as instructional resources for one another);
- a widespread positive emotional disposition, linked to the methodological choices, such as group work and discussion; these methodological choices were strictly related to the goal of activating FA strategies;
- a problem solving view of Mathematics, possibly fostered by the FaSMEd activities (unfortunately, we do not have data on the students view of Mathematics before our intervention cases).

Further work is needed to explicitly connect affective aspects with the different FA strategies. Moreover, a future step of our work will be to highlight the connections between the different levels of feedback provided to students and the effects of this feedback from the affective point of view.

Concerning the use of technology, we may note a general computer motivation. The challenge for the project is to make the use of tablets not a mere matter of extrinsic motivation, but a source of intrinsic motivation. The analysis of interviews and questionnaires allowed to highlight that this can be done when the use of technology is coupled with specific methodological choices, such as group work and classroom discussions that may help meeting specific needs, such as the need for an *own safe space for thinking* and the *social need*.

Besides attitude towards the project, we obtained some information on attitude towards mathematics. When answering to question 1, one student explicitly said that thanks to the project he improved in mathematics:

> With FaSMEd I got very much better in mathematics (*Frank*)

Our hypothesis is that such an improvement was at first an improvement in attitude towards mathematics. Conversely, Danny reported a negative attitude to mathematics (during the group interviews, he answered “No” to the direct question “*Do you like maths?*”) that did not improve, even if he liked the project, as highlighted in his answers to questions 2 and 3:

> The same [attitude], it is not that you understand better. I still don’t like it.

The issue of long-term effect of the project, in terms of improvement in understanding and attitude towards mathematics, is still open and should be addressed with additional data collection and analysis.

Finally, we could study teachers’ attitudes, in particular teachers’ attitude towards the project (with specific reference to their attitude towards new technologies), since it may have an influence on students’ answers and processes.

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Chapter 26
Teachers' Affect Towards the External Standardised Assessment of Students' Mathematical Competencies

Giulia Signorini

Abstract International and national standardized assessments of students' mathematical competencies have an increasing role in the educational policies. They not only assess if the educational standards have been reached by students, but in a certain sense they determine what is considered particularly relevant as educational outcome, and therefore they can affect teachers' educational choices. This influence on teachers' choices depends on teachers' opinions and feelings about the standardized assessment of their students' mathematical competencies. Within this frame, we carried out a large narrative research about teachers' emotions and beliefs towards the Italian standardized assessment of students' mathematical competencies, comparing the differences between school levels and discussing their educational relevance.

26.1 Introduction

In the last 20 years we have seen the growth and the diffusion of several external evaluation systems for the assessment of students' mathematical learning, both at international level (OCSE-PISA, TIMSS) and nationally (as testified by the presence of national standardized tests in many countries inside and outside Europe[1]). Their results are often used to certify the quality of the educational system and to orientate political choices regarding school reforms (Pons 2012; Mangez and Hilgers 2012). In a certain sense, these tests—according to their specific goals—determine what is relevant as educational outcome and how it is possible to assess students' levels;

[1]National evaluation systems are present, for example, in Australia (NAPLAN), Canada (FSA), Italy (INVALSI), Germany (VERA), Malaysia (PMR), USA (NAEP) and in many other countries.

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so it is not surprising that the standardized assessments used by the main evaluation systems and the interpretation of their results have become an hot topic in education.

The main lines of discussion concern their equity (Boaler 2003), the reliability of the information that this kind of tests can (and cannot) give (Bodin 2005; Stobart 2005) and focus mainly on students, on possible causes of their successes and failures (Andrews et al. 2014; Wijaya et al. 2014) and the factors that influence their performance (Papanastasiou 2000). Less attention, however, is paid to teachers and their opinions about external evaluation system.

Nevertheless, a large amount of literature highlights the influence of teachers' emotions and beliefs on their educational choices (Thompson 1992; Philipp 2007). We believe that teachers' point of view about the external evaluation can deeply affect the quality of the impact that external evaluation has on the teaching practices, for example shifting from positive effects (such as the fostering of problem-solving activities in the classroom) to negative ones (e.g., the spread of mere teaching-to-test activities), and also influencing the effectiveness of the evaluation system itself, for example by loading the tests emotionally for students (distorting, therefore, the results of the surveys). Within this framework, in the Italian context, we have carried out a research project about teachers' attitudes towards the external evaluation system for the assessment of students' mathematical learning promoted by INVALSI (National Institute for the Assessment of the Educational and Instructional System).

In this paper, we discuss teachers' emotions towards the national standardized assessment of students' mathematical competencies and the reasons declared by teachers in order to explain their feelings. More in general, the project will allow us to bring out teachers' beliefs about external standardized assessment of mathematical competencies.

26.2 The Italian Context

Founded in the early 2000s as a result of an intense cultural and political debate on the issue of the external evaluation of the educational system, since 2008 the Italian INVALSI institute develops tests, administered every year, that are census and involve all the students attending the grades 2, 5, 8, 10. For each grade, the problems within the tests try to involve real problematic situations and often require a problem-solving approach, essentially sharing the PISA framework (OECD 2014). The problems are created by math teachers of the same school level, following a precise framework based on the curricular National Indications issued by the Ministry of Education, and they are the same for all high schools in Italy, even if they follow different ministerial curricula. In each school tests are administered by the teachers themselves, except for some random classes where they are administered by external examiners (the national average is calculated only on the results of these classes). Once the tests have ended, teachers correct the tests following a correction grid provided by the institute (except for the previous random classes, where also the correction is done by an external examiner) and then each school sends its results to INVALSI. The results of the tests are not public and do not affect the marks of students, except for grade 8 (where the result of INVALSI tests is included in the final marks of the students). After a few months, INVALSI returns to the schools many statistical data concerning the levels of students' learning and the comparison with the national average and other schools with similar characteristics (numbers of students, social environment, . . .).

INVALSI explicitly claims that its tests are not designed to verify the individual student's level, but they are standardized tests designed to detect levels of learning in a global way, according to classrooms, schools and school levels. Therefore, the declared aim of the INVALSI institute is to evaluate the national educational system taken as a whole, according to the goals fixed by the Italian National Standards, in order to promote the pursuing of these goals and to improve the quality of the national school system.

Nevertheless, the present situation in Italy is unsmiling. In all school levels, since their introduction, the INVALSI tests are often viewed with suspicion and hostility by teachers, students and also parents. During the test days, strikes are not rare. Moreover, from a didactical point of view, it seems evident the growth of questionable teaching-to-test activities in all school levels.

26.3 Method and Rationale

26.3.1 Collection of Data

At the beginning of the new millennium, following the shift of the research in mathematic education from a measurement approach to an interpretivist one, the research about affect emphasizes the potential of the use of narratives methods for its aims. A number of studies were carried out using essays, diaries, written open questionnaires or oral interviews (Di Martino and Zan 2015).

Following this trend and within an interpretivist paradigm, we decided to study teachers' emotions towards the Italian external evaluation system and the main causes of these emotions carrying out a qualitative research and developing an on-line open questionnaire addressed to all the mathematics teachers in every school level. In line with the methodological choices of recent studies about teachers' affect (Coppola et al. 2013), we set the questionnaire as voluntary and anonymous.

The choices of the research instruments and how to use them are not neutral. In this case, these choices reflect our belief that the variety of possible answers coming from this method is an irreplaceable value for the purpose of our study. According to Cohen et al. (2007, p. 249):

> It is open-ended responses that might contain the "germs" of information that otherwise might not have been caught in the questionnaire (. . .) An open-ended question can catch the authenticity, richness, depth of response, honesty and candor which are the hallmarks of qualitative data.

Moreover, the choices made are coherent with our goals. We know that the collected answers constitute a convenience sample, i.e. not fixed on a statistical basis, but we want to describe, interpret and understand a phenomenon, and we are not interested in *measuring* it. On the other hand we firmly believe that the a-priori definition of a statistical sample would be very questionable in this case and however it is impossible with the "anonymous-approach" that we consider essential in an investigation about sensitive issues.

The developed questionnaire contains 28 questions, 15 of which are open in order to allow everyone to choose what he feels is the most important to say, with the words he feels are the best. The questions concerned background information (four closed questions), emotions (four open questions), teachers' point of view on the problems proposed in the tests (five questions, two of which are open), their perception of the goals (three open questions), strengths and weaknesses of the evaluation system (four questions, two of which are open), didactical practices in relation to the tests (four questions, two of which are open) and general satisfaction about the evaluation system (four questions, two of which are open), asking them what kind of changes they would do in order to make it more compliant to their opinions.

In the last page of the questionnaire the teachers were asked to share with us their e-mail if they agreed to participate in a second phase of the research, which has been realized carrying out not-anonymous interviews.

26.3.2 Analysis of Data

The large number of open questions allowed each participant to express his opinion, identifying what he considers the most important and arguing his positions with the words that he considers the most appropriate. Consequently, the study of such answers is extremely interesting but requires a specific analysis methodology. We chose to follow an analytical approach (Demaziére and Dubar 1997): the text is analysed in order to systematically produce sense starting from people's words.

Within the paradigm of the grounded theory (Glaser and Strauss 1967), in order to better understand the causes of teachers' emotions towards the external evaluation, we have labelled the empirical material with no pre-constructed codes, trying to do an analysis as richest as possible by giving relevant codes and taking into account the interpretative nuances. The result is the a posteriori construction of a set of general categories, properties and relationships, often intertwined and related to each other.

The analysis of the data coming from the whole questionnaire is not yet complete, in this paper we will discuss the analysis to the answers related to emotions and their reasons.

26.4 Results and Discussion

A first important result of our study is the participation of the Italian teachers to our survey, beyond any expectations: about 2000 math teachers have filled our open questionnaire, confirming that we investigated about an hot topic for the educational community and in particular that teachers desire to express their own point of view about the external evaluation of mathematical competencies. The answers collected are almost equally distributed among the three school levels: about 32% teachers from primary school, 32% from middle school and 36% from high school.

Also the voluntary participation to the second not-anonymous phase—more demanding in terms of time and engagement—has been remarkable: among the 2000 teachers who filled our questionnaire, more than 800 have agreed to participate in the interview-stage.

The analysis of the answers to the first question of our questionnaire ("Which is the first emotion that come into your mind if you think about INVALSI?") highlights, within the words chosen by our respondents, the presence of emotions that describe a *positive* relationship with the test (*curiosity, interest, satisfaction*) but a prevalence of emotions that describe a *negative* relationship. In this latter case, it is interesting that we can recognize two different typologies: the emotions that evoke a negative judgment about the relevance of the test (*useless*, *boredom*, *indifference*); and the emotions that evoke a strong personal emotional involvement (*anxiety*, *stress*, *anger, frustration, apprehension*).

The wide range of emotions emerged and the number of their occurrence are represented in Fig. 26.1, realized with a TagCloud program: the size of the words in this image is proportional to their frequency within the sample.

The most widespread emotion used by the teachers who filled our questionnaire is *anxiety*, with 240 teachers indicating it as the first emotion that comes into their mind when they think about INVALSI.

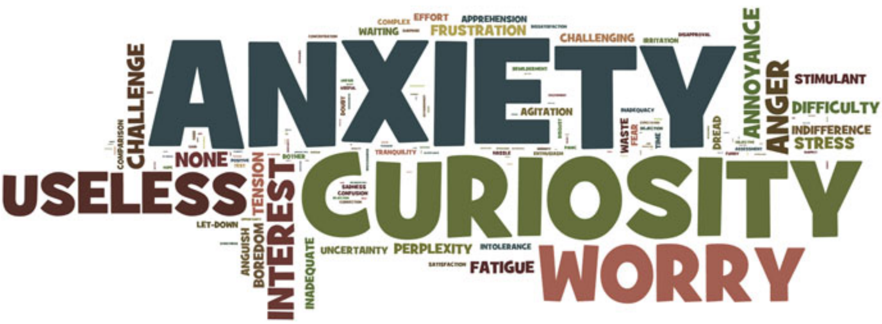

Fig. 26.1 Teachers' answers to the question: "*which is the first emotion that come into your mind if you think about INVALSI?*"

For space limitations, the analysis of the causes will be illustrated only for this emotion (the most recurrent emotion between those reported by our large sample), but similar considerations can be made also for other emotions.

Focusing our attention on who chooses anxiety as the answer to question 1, we will recognize and distinguish three main typologies of causes of teachers' anxiety. We want to underline that the distinctions between these typologies are not always so perfectly sharp as it could appear while reading the following overview: in this perspective, we believe that the categorization is interesting because highlights, classifies and describes the main reasons that teachers reported to explain their emotions towards the external standardized assessment.

It is interesting that the three categories that we recognized involve the three different components of the teaching-learning process represented in the *classic* educational triangle: student, knowledge and teacher.

In particular, we recognize the following three main different categories of causes for anxiety associated to external assessment: causes related to what and how students' competencies are assessed, causes related to the effect of the tests on classroom practices and causes related to the perceived assessment of teachers' skills. In the following we will briefly discuss each of these.

26.4.1 Causes Related to What and How Students' Competencies Are Assessed

Within this category we can recognize two subcategories: causes related to *justice* and causes related to *contents*.

In "causes related to *justice*" we collect all the answers in which teachers complain about the fact that external evaluation is someway unfair with their students. Those teachers typically refer to the fact that external evaluation does not take into account students' efforts, the progresses they have been able to make from their individual starting point, the difficulties they have managed to overcome. Examples of this kind of answers are the following:

> The tests are standardized and do not take into account the specificities of the social context. Any fair comparison cannot be separated from the analysis of the socio-cultural environment in which students live.
>
> The tests do not take into account the initial situation of the students, their learning process, their individual situations.
>
> Tests do not show the systematic work done in the classroom. Often best students get disappointing results and less able students get higher marks (maybe by randomly guessing or by practical sense).
>
> I think tests are not appropriate to detect the mathematical preparation of students because the questions are based primarily on logical skills, discomforting diligent students with less mathematical intelligence. There are students who struggle to make logical operations but they succeed in acquiring mathematical techniques thanks to their efforts. The tests penalize these students.

It is interesting to observe that the last typology of criticism is very common among the answers related to *justice*. Many teachers perceive as unfair the fact that a smart student who usually does not participate in the classroom lessons can gain better marks than a student with some difficulties, but who tries daily to overcome them. A special issue concerns primary school teachers, who often complain about the fact that tests can not assess the actual competencies of their students since the unusual setting in which the evaluation takes place (fixed time, no chance to ask the teacher for help…) provokes in the younger students negative affective reactions that influence their performance.

In "causes related to *contents*", instead, we collect the answers in which teachers complain about the fact that external evaluation is not able to take into account the topics actually addressed in the classroom. For example:

> Often the arguments do not fit the actual curricula but, above all, the way of posing the questions does not match the way the teachers pose them into the classroom. Therefore it often happens that a student who knows how to do one thing, if the test is presented in a different way, gets wrong.
> The problems in the tests are completely different from those contained in the textbooks. They do not highlight the training of students.
> The setting and the structure of closed-questions are too different from the tests traditionally performed at school. They need to think a test that is in line with what teachers do in classrooms everyday.

In a certain sense, *justice* and *contents* subcategories are intertwined in the point of view of many high school teachers that disagree about the fact that the tests for the grade 10 (end of the compulsory school age in Italy) are not distinguished according to the typology of school, despite the fact that the different schools follow different ministerial curricula.

26.4.2 Causes Related to the Effect of the Tests on Classroom Practices

We include in this category the answers in which teachers declare that they are anxious because the presence of an external evaluation affects their teaching practice in a negative way, or anyway limits their *freedom* of teaching.

Examples of this kind of answers are the following:

> The tests are likely to interfere excessively with the teaching, inducing an ad hoc training at the expense of flexibility and reflection.
> You cannot try to standardize teaching, you must take into account the social context.
> Because sometimes I don't manage to explain topics that students then find into the problems of the tests.

It emerges as many teachers (especially primary school teachers) complain that the external evaluation systems force them to look at the *quantity* rather than the *quality* of education: it is fundamental to cover all the topics included in the curriculum, rather than to consider the specific needs of each class. The idea is that the external tests force the teachers to speed up their traditional teaching program, without the possibility to consider and to develop didactical actions for who remains behind.

Moreover, many respondents underline that the presence of (and the relevance given by the system to) external standardized assessments *forces* teachers to promote teaching-to-test activities in classroom. This point of view seems to be more frequent among middle and high school teachers. They indicate that—in their opinion—tests have a clear culpability into the dangerous shift from relevant educational goals related to the development of competencies to performance goals related to the attainment of good results in the tests.

26.4.3 Causes Related to the Perceived Assessment of Teachers' Skills

This category collects the answers of respondents who indicate as the reason for their anxiety the perception that external assessments have the hidden goal of assessing teachers' efficacy.

Teachers included in this category perceive negatively this kind of personal evaluation because they disagree with the idea that teachers' efficacy can be measured comparing students' outcomes (for example, some high school teachers complain about the fact that a large number of students do not engage in the resolution of the test since it does not affect their marks), but also because this kind of evaluation would not be able to take into account the different social and cultural context in which teachers work, as well as the different characteristics and aptitudes of their students:

> I'm beginning to think it may be a way to say that teachers are not up to their job.
> I'm afraid of having worse performance than my colleagues.
> I think it is not right that results obtained from different classes, different schools, different regions, are used to draw conclusions about the validity of a teacher.

The negative results are considered an implicit criticism to the teacher and it is demeaning towards those who do their job seriously. Tests do not take into account all the variables that come into play in the process of teaching and learning.

In these answers it is clear the aversion towards the possibility that a sort of *teachers' ranking* can be developed using the tests results. It is important to say that in Italy this is not done officially, and INVALSI institute repeatedly declares that its evaluation should not be a teacher evaluation (as it is, instead, in other countries). Nevertheless, even in Italy it is not rare that such a ranking is locally done within schools and that teachers with the worst results are put under pressure by their superiors, also risking to enter into competition with other colleagues (a danger more often perceived by primary and middle school teachers).

26.5 Conclusions

Given the nature of our study, we believe that in these conclusions it is important to discuss a posteriori two main interrelated topics: the goodness of the methodology adopted and the significance of the results obtained.

As explained in the method section, we developed an open questionnaire because we are convinced that the suggestion of ready-made answers by the researchers within closed questions (both in the content and formulation of the eventual multiple choices) and the decision to force the identification of the teachers (in a very complicated situation as the one present in Italy) would have provided us less and less reliable information. After collecting and analysing our data, we believe that the methodological choices we have done, in the awareness of their limitations, allowed us to incentivize the amazing teachers' participation and to gather a wide range of shades of teachers' opinions in a genuine way.

Concerning the analysis of the answers related to teachers' emotions towards external evaluations, our study highlights different positions among teachers (in this paper we have focused on the most widespread of these emotions—anxiety—that is a "negative" emotion, but among the answers to the questionnaire many interesting "positive" emotions emerge too). It emerges a prevalence of negative emotions and the presence of emotions showing a strong personal involvement.

Moreover, discussing the causes declared to explain anxiety—chosen as an exemplary emotion—we describe a common and interesting phenomenon: one single emotion can be evoked by different reasons, according to the teachers' values and involving different dimensions of the teaching and learning process.

As a consequence, we may observe that these results also show some teachers' beliefs on external evaluation systems and on mathematics itself. For example, teachers belonging to the last category (teachers fearing a personal assessment by the evaluation system) share the firmly belief that the goal of the tests is to evaluate teachers, even if it is explicitly declared that this is not the aim of the external evaluation system. Teachers complaining about the fact that external evaluations force them to standardize their didactical practices seem to give more value to individualization and adaptation to the context rather than to achieving the same educational goals in every school. Finally, teachers who are included in the first category (teachers who perceive the external evaluation as unfair for their students) seem to share the same idea of success in mathematics: in order to be a successful student in mathematics it is more important to be able to overcome difficulties (regardless of the final result) than to reach the fixed standard without efforts; thus, for them, an external evaluation that assesses only the achievement of fixed standards is a wrong evaluation. For almost each of these categories we can also find the opposite point of view among teachers who have filled the questionnaire, and it is interesting to observe that the perceived assessment of teachers' skills appears as the most frequent reason for both negative and positive emotions that evoke a strong personal emotional involvement.

From the respondents' answers some differences also emerge between school levels, in particular between primary school teachers, who seem more focused on the affective impact of the standard assessment, and high school teachers, more interested in the equity of the evaluation.

To be aware of these distinctions, both in the emotions evoked among teachers and in the different causes of these emotions (and the different implicit value systems and beliefs they subtend), could be the fundamental step in order to plan effective interventions to foster the positive attitudes and fight the negative ones, with the aim to improve the didactical potential of external evaluation systems.

In the second phase of the research project teachers who voluntarily shared their e-mail in the online questionnaire have been interviewed. During the analysis of the data from this second phase we plan to go deeper in the study of the most relevant profiles of teachers' attitudes and to explore some interesting issues about the main factors that seem to influence these attitudes.

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Chapter 27
Conclusion

Peter Liljedahl

On the last day at MAVI 21 Lovisa Sumpter gave a presentation that began with a story about how her talk came out of a talk that I had given at MAVI 20. This sort of things happens at MAVI often. MAVI is informal enough and small enough that we can start a presentation by talking about each other's past research. MAVI also has a consistent enough participation base that when Lovisa began her presentation she was not only telling a story but invoking a memory—a memory that many of us share.

So, what stories will people be telling at MAVI 22? What connection will they be making to MAVI 21's presentations? In this conclusion I will attempt to anticipate this connections by looking closely at the forward looking statements and questions each author posed to us as, either explicitly or implicitly, within their papers.

Cusi, Morselli & Sabena, in their look at formative assessment strategies and their connection to affective issues were explicit in their acknowledgement that further work in this area is needed—especially if we are to begin to have an eventual effect on teachers' practice. Goldin, likewise, signals the need for further work in understanding observed patterns of engagement and disengagement and to inform these observations through a greater understanding of students' in-the-moment motivating desires, thoughts, emotions, and social interactions. Girnat's, in his research, found that there are strong and significant gender differences concerning constructivist versus instructivist attitudes about teaching mathematics. Although not explicitly stating that more work is needed, such results are an implicit call to all of us to work to understand better this phenomenon. The same is true of Sumpter's interesting look at the relationship between perfectionism—a rising issue in education in general and mathematics education in particular—and achievement. That there is a connection is one thing, But Sumpter intimates that how it is created

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and how this relationship works is still an open question. Aderonke also looked at achievement, but more specifically in relation students' interest and attitudes, as well as its impact on enrolment. Although Aderonke is talking specifically about Nigeria, these relationships are something that we should all be concerned about.

Beswick in her research on teachers' perceptions of their best and worst mathematics students wonders what the results of asking them about their average students. Such a question, she conjectures, would give us insights into how teachers' characterisations of good and bad students guides their teaching plans. Staying with work on teachers, Branchetti & Morselli conclude that teachers' behaviours, through direct or indirect verbal or nonverbal acts, may impact students' identities, and thus their participation in classroom activity. Not only is may an invitation to confirm this hypothesis, but also to drill deeper into both teachers' behaviours and students' identities. Eichler, Bräunling & Männer found a consistency between teachers' beliefs and classroom practices. Through their lens of central and peripheral beliefs, coupled with a consideration of mathematics-related and social contexts, they were able to explain the practices of two primary teachers. These results stand in stark contrast to prior research on the relationship between teachers' beliefs and practices and, as such, invites us to look closer at both teaching contexts and teachers' beliefs. Their acknowledgement that there findings may be limited by the fact that they considered only two cases is a further invitation to extend this research. Levenson & Barkai, in looking at the prospective primary teachers' beliefs about the role of explanations in the teaching of mathematics found that the prospective teachers viewed explanations as playing various roles in the classroom. They were left wondering, however, how strongly these prospective teachers hold their various views. They also noticed that the emotional dimensions did not emerge in their research. Finally, they intimate that a questionnaire based on their results would be useful for future investigations.

> Results of this study may assist in building a questionnaire that would allow for a more detailed and focused investigation of beliefs related to the roles of explanations and their relationship to beliefs regarding mathematics and its teaching. (Levenson & Barkai)

Lake, meanwhile, looked at the role of play in the secondary mathematics classroom found that play and playfulness, in their many varied forms, can be very powerful tools for engaging students and teachers alike. Such conclusions are provocative in their own right, but the introduction of a new construct to the MAVI community offers us new lens to look at teaching practice. On the other hand, Fahlström, in his study on teachers' conceptions of the role of physical environment on student learning, offers us a new lens through which to view teachers' beliefs about the teaching of mathematics.

Liljedahl & Rouleau and Liljedahl both looked at contexts that may motivate teacher change. Liljedahl looked specifically at a case of teacher collaborative design which resulted in massive changes to the participating teachers' practice. From this he hypothesised that"these catalysing experiences were effective only in that they occurred within a context that was largely unfamiliar to the teachers" (Liljedahl) implicitly inviting this invariant to be investigated in other contexts.

Meanwhile Rouleau & Liljedahl, in their look at teacher's tensions challenge the notion that teachers live with, rather than resolve, tensions. In so doing, they offer us all a question to ponder—can tensions fuel a desire to change practice and if so, can tensions be used to these ends.

Tirosh, Tsamir, Levinson, Barkai & Tabach, in their look at the relationship between preservice teachers' self-efficacy and knowledge found that teachers had lower self-efficacy in defining repeating patterns as compared to extending and continuing repeating patterns. Although prior research has shown that self-efficacy is not always correlated with knowledge, these results indicate that there is something more subtle happening here and invite a deeper look at what it is that is different in the act of defining a concept as opposed to operating with a concept.

Ahtee, Näveri & Pehkonen, in looking at teacher activity through students' drawings—a methodology that has been discussed at MAVI many times—offer us an insight that is worth looking into. Not only are students' drawings a window into teacher actions, but also into the varying ways in which students experience the same event. This insight invites a myriad of methodological possibilities for us to think about and experiment with.

Both Palmér & Karlsson and Tuohilampi look at student problem solving. Palmér & Karlsson focused specifically conceptions of problem solving and found that primary school students have very different images of problem solving. On the one hand, this is not surprising. On the other hand, this calls on all of us to think deeply about the implications that this has on our use of methodologies relying on students' and teachers' self-reporting of problem solving. Tuohilampi, avoiding this complexity, use the framework of Patterns of Participation to analyse video of two boys working on an open-ended problem in an effort to discern the role of social factors in influencing logical thinking. Although she found that the framework was effective in revealing several essential features of the interaction, she suggests augmenting the framework to include theories of emotions, engagement, and positioning—thus, opening the door for further work.

Ten of the papers presented at MAVI 21, based on their results, make explicit recommendation for interventions for teachers, parents, classrooms, programs, and policy. Beginning with teachers, Andrà, in looking at teachers' autobiographical narratives, concluded that teachers' use of metaphors reveal the structure of their beliefs and, as such, recommends that "metaphors should not be taught of as pure mental constructs, but as grounded in our physical body and in our social experience of the world" (Andrà). Beswick suggests that teachers may need help developing proficiencies in struggling students. Brunetto & Kontorovich learned "that integration of technology in the classroom does not necessarily promote learner-centered environment", implying that interventions around the pairing of technology and learner-centred are needed. Heyd-Metzuyanim, in looking at two teachers attempting to implement reform-type teaching suggest that teachers need to be prepared to deal with student emotions as well as their own feelings of empathy towards frustrated students. González, in his work, came to the conclusion that in order for teachers to engage students with decision-making tasks and foster decision-making skills they will need both their pre-service and in-service educational opportunities to work on these ideas.

With respect to the remaining intervention recommendations, Albersmann & Bosse, in their look at the inconsistencies of parents' beliefs about the teaching and learning of mathematics, suggest that parental programs should help parents recognize the relevance of constructivist learning approaches and the role autonomy supportive behaviour. Kontorovich & Zazkis recommend recurrent invitations for the class to create explanations of mathematical conventions (CECM) as a way to foster intended sociomathematical norms. Hess-Green and Heyd-Metzuyanim offer criteria for the evaluation of giftedness and Aderonke recommends policy changes in Nigeria to foster stronger ties between Mathematics and Science.

Finally, there were two authors whose research questions were so far reaching that their results only addressed a small portion of the answer. First, Pieronkiewicz asked the question *what are students afraid of?* Fear is not a new construct to affect research. Regardless, this is a fundamental question for all of us to ponder with much work remaining to be done. Second, Signorini uses affect as a lens to look at teachers' relationship with external assessment. Given that there are so many aspects to the affective dimension this leaves much work yet to be done.

The 26 papers presented at MAVI 21 covered a broad swatch of affective research in mathematics and mathematics education. As many questions as they answered, however, their work raised even more. But they left us with new frameworks, new constructs, and new lenses to look at this work to answer these questions.

I do not know what papers will be presented at MAVI 22 or subsequent MAVI's. But if history is any indicator there will be a couple of papers picking up on the conversation from MAVI 21.