

Adaptive Robust Ability of High Order Sliding Mode Control for a 3-D Overhead Crane System

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Abstract. Traditionally, 3-D overhead crane systems are widely used in industry and automatic operation would reduce the risk. It is difficult to precisely position the payload in overhead crane due to the lack of actuators in this system. This paper develops an adaptive robust ability of high – order sliding mode controller (HOSMC). The finite time stability of the closed-loop system is proved without traditional Lyapunov theory. The results based on suitable second-order sliding surface and super – twisting controller. Simulation studies are performed to demonstrate the validity of the proposed control scheme.

1 Introduction

Over the past three decades, extensive research has been performed toward high performance load transportation of the overhead cranes operation. The crane is naturally an under-actuated mechanical system, in which the number of independent actuators (inputs) is less than the degree of freedom (outputs) to be controlled. So that in order to meet high performance control requirements is difficult task, naming the suitable motion speed with accurate load positioning and maintaining small low swings [1]. A number of control approaches have been suggested and mainly based on nonlinear dynamic models developed for 3D overhead cranes in adaptive – robust control [2–6]. In [4], a complete nonlinear dynamic model of an overhead crane has been proposed, and the backstepping technique that achieves 3D position control and anti-sway control simultaneously is derived in a unified control scheme under parameter variations. In [6], a nonlinear controller for payload positioning and swing suppression of overhead crane systems has been proposed and the stability analysis was performed under much less strict assumptions. Although the numerical complexity of model predictive control (MPC), a new control approach for anti-swing tracking control of a 3D overhead crane based on MPC and computed torque control is pointed out in [3], including external disturbances on the actuators driving the crane. In [13], a novel adaptive control scheme with the use of the tuning function, including both the cart motion and the swing angle dynamics is designed to ensure the stability of the closed-loop system. Some researchers implemented image sensing to measure the swing angle of an overhead crane. This work proposes the use of

visual tracking technology create the rapid and smooth load motion [8]. Sliding mode control (SMC) is capable of controlling nonlinear system with parameter uncertainties (such as weight, moment of inertial of payload, . . .) and external disturbances (such as winds), and fuzzy logic control (FLC) is independent of system model. Park et al. [7] proposed an adaptive fuzzy sliding-mode control (AFSMC) law for the trajectory tracking of 2-D overhead crane systems, subject not only to system uncertainties but also to actuator nonlinearity of the dead-zone type. In [11], an adaptive sliding mode fuzzy control algorithm based on combining SMC's robustness and FLC's independence of system model is derived. However, it is difficult to apply SMC for mechanical systems because of the sensitivity of these systems to chattering. Higher-order sliding mode control (HOSMC) can overcome this phenomenon by confining the switching control to the higher derivatives of the control variable. Bartolini et al. [9] proposed a control scheme guarantees a fast and precise load transfer and the swing suppression, based on second-order sliding surface. In [12], 2nd-order and 3rd-order sliding mode differentiators are used and actuator fault diagnosis schemes are derived to achieve fault detection and isolation. In [15], super-twisting algorithm is one of the development of high-order sliding mode control with attractive properties: finite convergence time, disturbance rejection. In [13], an separation principle output feedback controller for a class of MIMO nonlinear systems has been implemented based on HOSMC. Pisano et al. [16] proposed an anti-swing control law, which is based on the super-twisting approach for the 3-dimensional overhead crane. Le Anh Tuan et al. [2] developed an adaptive version of sliding mode controller for 3D overhead cranes. However, the Barbalat's Lemma - based proof presented in that paper was incorrect because of the lack of uniformly continuous property of. Moreover, the condition of matrices and α in sliding surface's expression was not pointed out clearly. In this paper, we propose a solution for the above problems based on uncertain model and the finite convergence time of super - twisting controller is estimated (Fig. 1).

Heading level	
l	The adjustable rope length (m)
x, z	The displacements of trolley and bridge, respectively (m)
ϕ, θ	The cargo swing angles (rad)
m_c, m_b, m_t, m_l	Equivalent masses of the cargo, trolley, bridge and hoist, respectively (kg)
g	Gravity acceleration (m/s^2)
u_b, u_t, u_l	The external forces driving bridge, trolley and cargo hoist, respectively (N)

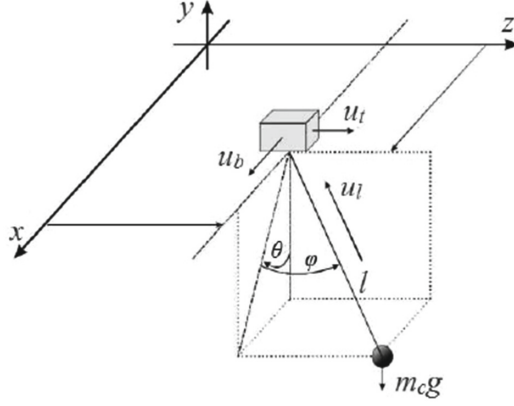


Fig. 1. 3D crane physical model [16].

2 Main Contents

2.1 3D Crane Model

The motion equations of 3D overhead crane system are created by using Lagrange's equation and can be represented in the following matrix form [2]:

$$M(q, \Theta)\ddot{q} + C(q, \dot{q}, \Theta) + g(q, \Theta) = u' \quad (1)$$

where $M(q, \Theta)$ is the inertia matrix, $C(q, \dot{q}, \Theta)$ is the Centripetal-Coriolis term, $g(q, \Theta)$ is vector due to gravity, and $u' \in \mathbb{R}^{n \times n}$ is the input vector. The details of the above dynamics are given in the following expressions: $q = [z \ x \ l \ \varphi \ \theta]^T$ $u' = [u_b \ u_t \ u_l \ 0 \ 0]^T$.

$$M = \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} & m_{15} \\ 0 & m_{22} & m_{23} & 0 & m_{25} \\ m_{31} & m_{32} & m_{33} & 0 & 0 \\ m_{41} & 0 & 0 & m_{44} & 0 \\ m_{51} & m_{52} & 0 & 0 & m_{55} \end{bmatrix} \left\{ \begin{array}{l} m_{11} = m_t + m_b + m_c, \\ m_{13} = m_{31} = m_c \sin \varphi \cos \theta, \\ m_{14} = m_{41} = m_c l \cos \varphi \cos \theta, \\ m_{15} = m_{51} = -m_c l \sin \varphi \sin \theta, \\ m_{22} = m_t + m_c, \\ m_{23} = m_{32} = m_c \sin \theta, \\ m_{25} = m_{52} = m_c l \cos \theta, \\ m_{33} = m_l + m_c, \\ m_{44} = m_c l^2 \cos^2 \theta, \\ m_{55} = m_c l^2 \end{array} \right. \quad (2)$$

$$C = \begin{bmatrix} 0 & 0 & c_{13} & c_{14} & c_{15} \\ 0 & 0 & c_{23} & 0 & c_{25} \\ 0 & 0 & 0 & c_{34} & c_{35} \\ 0 & 0 & c_{43} & c_{44} & c_{45} \\ 0 & 0 & c_{53} & c_{54} & c_{55} \end{bmatrix} \begin{cases} c_{13} = m_c \cos \varphi \cos \theta \dot{\varphi} - m_c \sin \varphi \sin \theta \dot{\theta}, \\ c_{14} = m_c \cos \varphi \cos \theta \dot{l} - m_c l \cos \varphi \sin \theta \dot{\theta} - m_c l \sin \varphi \cos \theta \dot{\varphi}, \\ c_{15} = -m_c l \cos \varphi \sin \theta \dot{\varphi} - m_c \sin \varphi \sin \theta \dot{l} - m_c l \sin \varphi \cos \theta \dot{\theta}, \\ c_{23} = m_c \cos \theta \dot{\theta}, \\ c_{25} = m_c \cos \theta \dot{l} - m_c l \sin \theta \dot{\theta}, \\ c_{34} = -m_c l \cos^2 \theta \dot{\varphi}, \\ c_{35} = -m_c l \dot{\theta}, \\ c_{43} = m_c l \cos^2 \theta \dot{\varphi}, \\ c_{44} = m_c l \cos^2 \theta \dot{l}, \\ c_{45} = -m_c l^2 \cos \theta \sin \theta \dot{\varphi}, \\ c_{53} = m_c l \dot{\theta}, \\ c_{54} = m_c l^2 \cos \theta \sin \theta \dot{\varphi}, \\ c_{55} = m_c \dot{l}. \end{cases} \quad (3)$$

$$g = [0 \ 0 \ g_3 \ g_4 \ g_5]^T \begin{cases} g_3 = -m_c g \cos \varphi \cos \theta, \\ g_4 = m_c g l \sin \varphi \cos \theta, \\ g_5 = m_c g l \cos \varphi \cos \theta. \end{cases} \quad (4)$$

In this paper, the dynamic Eq. (5) is presented to account for the parameter uncertainties and external disturbances:

$$M(q, d)\ddot{q} + C(q, \dot{q}, d) + g(q, d) = \begin{bmatrix} u + \xi(q, \dot{q}, \ddot{q}, t) \\ 0 \end{bmatrix} \quad (5)$$

Where Θ is the vector of unknown constant parameters, d is the estimates for Θ and $\xi(q, \dot{q}, \ddot{q}) = n(t) + \Lambda$: $n(t)$ is the vector of external disturbances; $\Lambda = (M(q, \Theta)\ddot{q} + C(q, \dot{q}, \Theta)\dot{q} + g(q, \Theta)) - (M(q, d)\ddot{q} + C(q, \dot{q}, d)\dot{q} + q(q, d))$ is estimation error vector of this model.

2.2 High-Order Sliding Mode Controller Design

The dynamic equation of an overhead crane (5) can be rewritten in following form by separating the model into actuated and un-actuated part 1:

$$\overline{M}(q)\ddot{q}_1 + \overline{C}_1(q, \dot{q})\dot{q}_1 + \overline{C}_2(q, \dot{q})\dot{q}_2 + \overline{g}(q) = u + \xi \quad (6)$$

where: $q_1 = q_a = [z \ x \ l]^T$ (actuated states) $q_2 = q_u = [\varphi \ \theta]^T$ (unactuated states) $u = [u_b \ u_t \ u_l]^T$.

The tracking control structure is designed through high-order sliding mode control based on second-order sliding surface as follows:

$$\begin{cases} s = \dot{q}_1 + \lambda + \tilde{q}_1 + \alpha q_2 = 0 \\ \dot{s} = \ddot{q}_1 + \lambda \dot{q}_1 + \alpha \dot{q}_2 = 0 \end{cases} \quad (7)$$

where $\tilde{q}_1 = q_1 - q_{1d}$; $\tilde{q}_2 = q_2 - q_{2d}$;

$$\lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}; \alpha = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \\ 0 & 0 \end{bmatrix}.$$

Let us now find the control input $u_N = u_{eq} + \underline{\Delta}$ to stabilize sliding surface in finite time. The equivalent input u_{eq} is obtained from $\dot{s} + \lambda s = 0$:

$$u_{eq} = \overline{C}_1(q, \dot{q})\dot{q}_1 + \overline{C}_2(q, \dot{q})\dot{q}_2 + g(q) - \overline{M}(q)(2\lambda\dot{q}_1 + \lambda^2\tilde{q}_1 + \alpha\dot{q}_2 + \lambda\alpha q_2) \quad (8)$$

So that, the control input $u_N = u_{eq} + \underline{\Delta}$ obtain the following result:

$$\dot{s} + \lambda s = \overline{M}(q)^{-1}\underline{\Delta} \quad (9)$$

If $\underline{\Delta} = -K.sgn(s)$ with $K = diag(k_1, k_2, k_3)$ and $k_i > \delta(\forall i = 1, 2, 3)$ are sufficiently large constants with $\delta = \sup \xi(q, \dot{q}, \ddot{q}, t)$ then it is found that s converge to 0 faster than the root of following equation:

$$\dot{s} + \overline{M}(q)^{-1}Ksgn(s) = 0 \quad (10)$$

Therefore s converge to 0 in finite time and combine with (9), we obtain s, \dot{s} converge to 0 in finite time.

Remark 1: The stability of high order sliding surface is established in [2]. However, in order to achieve a tracking performance based on sliding mode control, we need to guarantee s, \dot{s} converge to 0 in finite time even with parameter uncertainties and external disturbances.

The identification of two metrics λ, α to make \tilde{q}_1, \tilde{q}_2 converge to 0 are implemented by considering the stability of following nonlinear system:

$$\dot{x} = \begin{bmatrix} -\lambda x_1 - \alpha x_2 \\ x_3 \\ h(x) \end{bmatrix} = f(x) \quad (11)$$

$$x = [x_1 \ x_2 \ x_3]^T = [\tilde{q}_1 \ q_2 \ \dot{q}_3]^T$$

$$\begin{aligned} h_1(x) &= -\frac{\cos \varphi}{l \cos \theta} \lambda_1^2 \tilde{z} + \frac{\dot{\varphi}}{l} \lambda_3 \tilde{l} - \frac{\cos \varphi}{l \cos \theta} \lambda_1 \alpha_1 \varphi + \left(\frac{\cos \varphi}{l \cos \theta} \alpha - \frac{\dot{l}}{l} + \tan \theta \dot{\theta} \right) \varphi \\ &\quad + \tan \theta \dot{\varphi} \dot{\theta} - \frac{g \sin \varphi}{l \cos \theta} \\ h_2(x) &= \frac{\sin \varphi \sin \theta}{l} \lambda_1^2 \tilde{z} - \frac{\cos \theta}{l} \lambda_2^2 \tilde{x} + \frac{\dot{\theta}}{l} \lambda_3 \tilde{\lambda} - \frac{\cos \theta}{l} \lambda_2 \alpha_2 \theta - \frac{\sin \varphi \sin \theta}{l} \alpha_1 \dot{\varphi} \\ &\quad - \cos \theta \sin \theta \dot{\varphi}^2 + \frac{\cos \theta \alpha_2 - \dot{l}}{l} \dot{\theta} - \frac{q \cos \varphi \sin \theta}{l} \end{aligned}$$

The linearization of (11) around the equilibrium 0 is given by (12):

$$\dot{x} = Ax \quad (12)$$

where

$$A = \left. \frac{\partial f}{\partial x} \right|_{x=0} = \begin{bmatrix} -\lambda & -\alpha & 0_{3 \times 2} \\ 0_{2 \times 3} & 0_{2 \times 2} & I_{2 \times 2} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad (13)$$

In [14], the system (10) is local stable if this equivalent nonlinear system is also stable. After some manipulations, we get:

$$|sI_7 - A| = (s + \lambda_3) \left(s^3 + \left(\lambda_1 - \frac{\alpha_1}{l_d} \right) s^2 + \frac{g}{l_d} s + \frac{\lambda_1 g}{l_d} \right) \left(s^3 + \left(\lambda_2 - \frac{\alpha_2}{l_d} \right) s^2 + \frac{g}{l_d} s + \frac{\lambda_2 g}{l_d} \right)$$

Let:

$$P(s) = \left(s^3 + \left(\lambda_1 - \frac{\alpha_1}{l_d} \right) s^2 + \frac{g}{l_d} s + \frac{\lambda_1 g}{l_d} \right)$$

$$Q(s) = \left(s^3 + \left(\lambda_2 - \frac{\alpha_2}{l_d} \right) s^2 + \frac{g}{l_d} s + \frac{\lambda_2 g}{l_d} \right)$$

$P(s)$ and $Q(s)$ are Hurwitz polynomials if the following conditions are satisfied using Routh–Hurwitz criteria:

$$\lambda_1 - \frac{\alpha_1}{l_d} > 0; \frac{g}{l_d} > 0; \frac{\lambda_1 g}{l_d} > 0; \left(\lambda_1 - \frac{\alpha_1}{l_d} \right) \frac{g}{l_d} > \frac{\lambda_1 g}{l_d}$$

$$\lambda_2 - \frac{\alpha_2}{l_d} > 0; \frac{g}{l_d} > 0; \frac{\lambda_2 g}{l_d} > 0; \left(\lambda_2 - \frac{\alpha_2}{l_d} \right) \frac{g}{l_d} > \frac{\lambda_2 g}{l_d}$$
(14)

For a general result, the proposed control law parameters satisfying (15) will guarantee the stability of the 3D Crane system.

$$\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0, \alpha_1 < 0, \alpha_2 < 0$$
(15)

Remark 2: Note that the control input $u_N = u_{eq} + \underline{\Delta}$ (8, 9) is not enough for complete control signal. It is necessary to choose two suitable matrices λ, α (14, 15) and it has not been pointed out in [2].

2.3 Super-Twisting Controller Design

The motion of the 3-D overhead crane (1) can be simplified by assuming that the load swing angles ϕ, θ are small enough, it can be simplified as:

$$\ddot{x} = -\frac{D_x}{M_x} \dot{x} + \frac{1}{M_x} u_b; \ddot{\theta} = \frac{D_x}{M_x l} \dot{x} - \frac{1}{M_x l} u_b$$
(16)

$$\ddot{y} = -\frac{D_y}{M_y} \dot{y} + \frac{1}{M_y} u_t; \ddot{\phi} = \frac{D_y}{M_y l} \dot{y} - \frac{1}{M_y l} u_t; \ddot{l} = g - \frac{D_l}{m} \dot{l} + \frac{1}{m} u_l$$
(17)

In [16], the tracking control structure is designed through the following three dimensional sliding variable:

$$s = \begin{bmatrix} s_x \\ s_y \\ s_l \end{bmatrix} = \begin{bmatrix} \dot{x} - \dot{x}^r + c_x(x - x^r) - k_x \theta_x \\ \dot{y} - \dot{y}^r + c_y(y - y^r) - k_y \theta_y \\ \dot{l} - \dot{l}^r + c_l(l - l^r) \end{bmatrix}$$
(18)

where c_x, c_y, c_l, k_x, k_y positive constants. So that, the sliding dynamics along the model (16, 17) is:

$$\frac{d}{dt}s = \begin{bmatrix} a_1(x, \dot{x}, t) + b_1(x, \dot{x}, t)u_b \\ a_2(y, \dot{y}, t) + b_2(y, \dot{y}, t)u_t \\ a_3(l, \dot{l}, t) + b_3(l, \dot{l}, t)u_l \end{bmatrix} \quad (19)$$

The control inputs u_b, u_t, u_l can be obtained as follows [16]:

$$\begin{aligned} u_{b,t,l} &= -\lambda_i \sqrt{|s_i|} \text{sign}(s_i) + \omega_i \\ \dot{\omega}_i &= -\alpha_i \text{sign}(s_i) \\ (i \in x, y, l) \end{aligned} \quad (20)$$

where λ_i and α_i are sufficiently large constants.

From (19) and combine with the results (Levant 2005), the convergence time is estimated as follows:

$$\begin{aligned} T &\leq \sum \frac{|\dot{x}_i|}{K_m \alpha - C} \\ (x_i = x, y, l; |\dot{a}| + U_m \left| \dot{b} \right| \leq C; 0 \leq K_m \leq b(t, x) \leq K_M; \left| \frac{a}{b} \right| < q U_M; 0 < q < 1) \end{aligned} \quad (21)$$

Remark 3: The estimation of convergence time has not been pointed out in [16] and (21) is the additional result in finite time stability.

2.4 Verification by Simulation

In this section, simulations via MATLAB/Simulink are performed to verify the validity of controller (8, 9) and (20) with 2 suitable matrices λ, α satisfying (14, 15):

$$\lambda_1 = \lambda_2 = 0.75; \lambda_3 = 1; \alpha_1 = \alpha_2 = -4$$

The matrix K is chosen big enough for a fast convergence to sliding surface :

$$K = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The simulation results are shown in Figs. 2, 3 and 4. All components of sliding surface variable are controlled to converge to 0 in a finite time (Figs. 2 and 4). Hence, it is not only rapid to track with short settling time and no overshoot of the actuated states (Figs. 2, 3 and 4), but also two unactuated cargo-swing angles are also maintained small (Figs. 2, 3 and 4).

Besides, there is a small difference between parameters used for controller and plant to prove the robustness of the proposed sliding mode control law. These parameters are listed in the following table:

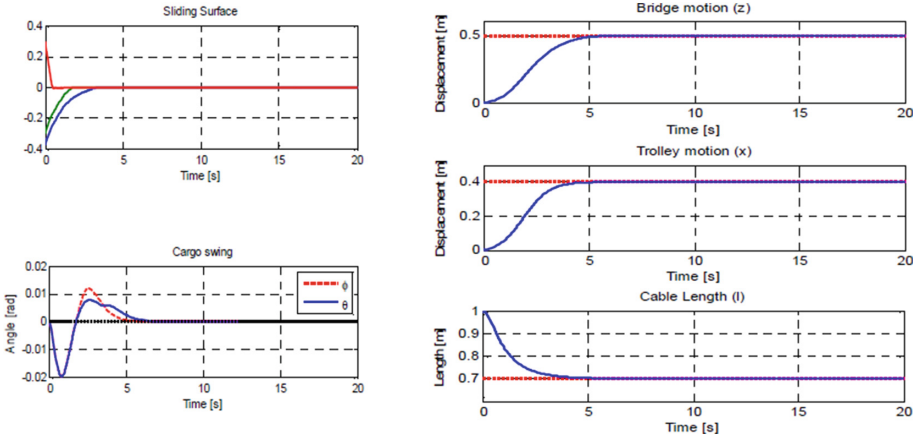


Fig. 2. Sliding surface s , Actuated states and Un-actuated states.

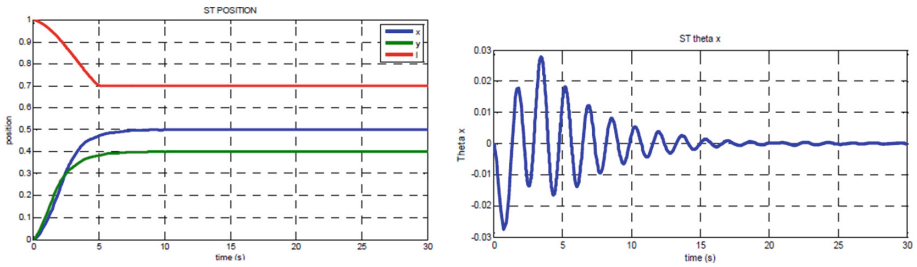


Fig. 3. Actuated states and Un-actuated states (Super – Twisting Controller).

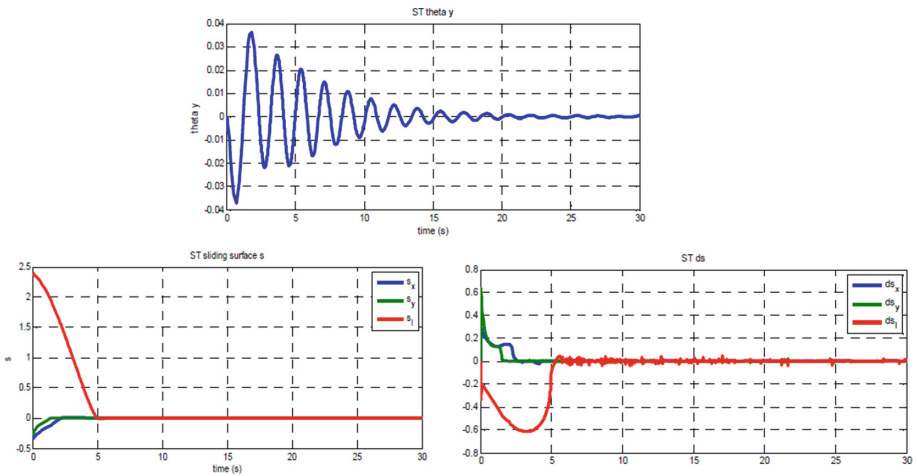


Fig. 4. Un-actuated states, sliding surface s , time derivative of the sliding surface s (Super – Twisting Controller).

Parameters	Unit	Controller	Plant
m_c	Kg	0.85	0.85
m_b	Kg	7	6.9
m_t	Kg	5	4.8
m_l	Kg	2	2.1
b_t	Kg/s	20	21
b_b	Kg/s	30	31.5
b_r	Kg/s	50	46.5

3 Conclusion

In this paper, tracking ability and finite time stability is analyzed, refer to differential equation without traditional Lyapunov theory and the use of two suitable matrices in the second-order sliding surface is proposed. It is theoretically proved that asymptotic tracking of the payload position and regulation of the swing angle can be achieved robustly despite the parameter uncertainties, external disturbance. Good results in offline simulation showed the effectiveness of the proposed theoretical development in this work.

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