Analysis of Agricultural Production in Asia and Measurement of Technical Efficiency Using Copula-Based Stochastic Frontier Quantile Model

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Abstract. The purpose of this paper is to evaluate the efficiency of agricultural production in Asia and analyze the production function of Asian countries. Methodologically, we employ the stochastic frontier model with the concern about dependency between two-sided error term and one-sided inefficiency. Likewise, we try to improve the performance of the standard stochastic frontier model by applying quantile regression to the frontier production function. Therefore, this paper introduces the model called Copula-based stochastic frontier quantile model as an alternative tool for this issue. The accuracy of this model is proved through a simulation study before applying to the agricultural production data of Asia.

Keywords: Stochastic frontier · Frontier production function · Quantile regression · Copula · Technical efficiency

1 Introduction

It is true that food drives the world and access to adequate food is one of the primary concerns for most people on the globe. With that in mind, Asia is well-known as the best area for growing staple food such as vegetable, fruit, and wheat, in which many Asian countries are on the lists of top agricultural producers and top exporters. This makes agriculture one of the largest and most significant sectors for Asian economy.

As agricultural production sector is a major driving force for Asias economic growth, we have to take into account the productive efficiency of this sector. Motivated by this reasoning, we attempt to analyze the agricultural production function of Asian countries and then examine the technical efficiencies of this region. Why do we need to consider the technical efficiencies? There are a variety of reasons that technical efficiencies are critical. For instance, technical efficiencies can lead to a rising in agricultural productivity without increasing the resource base, meaning that we are able to produce more output from the same

V.-N. Huynh et al. (Eds.): IUKM 2016, LNAI 9978, pp. 701–714, 2016.
DOI: 10.1007/978-3-319-49046-5_59

quantity of inputs. Likewise, the technical efficiency can bring about the producers competitiveness without increase in input factors (Bezat-Jarzebowska and Rembisz [1]). Most importantly, efficiency measurement is necessary not only for Asian countries, but also for other nations who aim to allocate effectively agricultural funds across heterogeneous farmers and maintain an adequate standard of living in rural communities (Kaditi and Nitsi [2]). This seems to make the technical efficiency important for nations, especially in the role of agricultural productivity growth.

The idea of technical efficiency was proposed by Farrell [3] through the use of frontier production function. His discovery spills over several extensions on estimation of the frontier production function as well as the measurement of technical efficiency. However, this paper takes into account a powerful model called stochastic frontier (SFM) which was proposed by Aigner et al. [4]. The main idea of this model is to find a linear relationship between output and input levels with two independent error terms representing inefficiency and the standard normal error, respectively. This model is widely used to assess technical efficiency of production units. The efficiency is simply measured by the parameters of the frontier production function; we calculate the distance between a country's actual level of production output and the maximum level of output given inputs, which is called the production frontier.

Apart from the use of the SFM, the study of Kaditi and Nitsi [2] pointed out that the SFM still makes a strong assumption for the functional form of the inefficiency distribution and is sensitive towards outliers, which in turn lead to a misspecification. Therefore, they employed a quantile regression as an alternative model to estimate the efficiency in agricultural sector. Various studies have followed this idea such as Duy [5], and Gregg and Rolfe [6]; they found that this approach is well-suited for efficiency estimations when they concern about heterogeneity in the different country-level data. Likewise, this approach also describes well the production of efficient producers or countries in different quantile level rather than on the average. With that in mind, this paper is trying to take the advantage of the quantile regression. But would rather not giving up using the SFM, we will put the use of quantile approach into the SFM and introduce the stochastic frontier quantile model as an alternative method for this issue. We believe that the quantile approach will provide new information to the SFM by estimating the whole percentile of production functions corresponding to different efficiency levels. (Bernini, Freo, and Gardini [7]).

Additionally, as the SFM contains two independent error terms i.e. the standard normal error and inefficiency, many studies concern about the validity of this independence assumption and try to explain it in many different ways. For example, Das [8] suggests that inefficiency at current time may depend on the noise at the previous time. Moreover, due to the misspecification of model, the standard normal error may contain some important variable, which in turn makes the inefficiency dependent. Therefore, this paper employs a well-known joint distribution function called copula to be a linkage between the two error terms as suggested by Smith [9] and Wiboonpongse et al. [10]. They empirically found that the independence assumption can be relaxed appropriately by the use of copulas which in turn allow us to explore the dependence structure of the error components.

Therefore, this paper suggests a new approach to the analysis of the frontier production function called Copula-based stochastic frontier quantile model which takes into account the impact of inputs on the production output and the efficiency scores of agriculture in different quantiles. In addition, this model also allows for the dependence between two error components through the ability of copula, which in turn makes this model more flexible and far from the grossly overestimates efficiency, as in the original SFM.

The outline of this paper is as follows. Section 2 we explain thoroughly about our proposed model and other necessary statistical properties related to our model including the basic idea of copula. Section 3 we discuss about the estimation technique and then some Monte Carlo experiments are reported Sect. 4. The empirical study of agricultural production in Asia is given in Sect. 5. Section 6 contains conclusion.

2 Methodology: An Introduction to the Stochastic Frontier Quantile Model

To construct the stochastic frontier quantile model (SFQM), three statistical approaches are considered: (i) the conventional stochastic frontier model (SFM), (ii) quantile regression with an Asymmetric Laplace distribution (ALD), and (iii) copula approach. These approaches will be discussed later.

As we mentioned previously, the SFM assumes the two error components, U and V, to have normal and positive distributions, respectively. These two errors represent noise and inefficiency of the SFM model. (Smith [9]) pointed out that U and V are dependent, so he suggested using copula to join these errors together to eliminate the weak independence assumption of the SFM. However, without considering heterogeneity in the country-level data, the SFM of (Smith [9]) may not be robust against outliers and cannot capture the extremes of distribution i.e. tail behavior of a probability distribution. Therefore, to overcome this problem, we abandon the normality assumption of the U in favor of the ALD; that is we extend the quantile regression to the SFM of (Smith [9]) and introduce a stochastic frontier quantile analysis (SFQM) model. Hence, this model becomes more flexible to the outlier and it can measure the relationship between output and input levels across efficiency quantiles. In addition, this model also provides the different slopes of parameters describing the production of Asian countries rather than average value.

2.1 Modelling the SFQM with Correlated Error Components

Consider a case of cross-section of countries; the stochastic frontier quantile model is given by the following equation where Y_i is the output level of the country i in which $i = 1, ..., I$, and X'_{ik} is a $I \times K$ matrix of K different input quantities of the $i - th$ country. The term β^ρ represents $(I \times K)$ matrix of estimated parameters of the input variables at quantile level denoted by ρ, such that $\rho \in [0, 1]$. The function $f(\cdot)$ is the imposed functional form of frontier such as the Cobb-Douglas production model. The term TE^ρ denotes technical efficiency across quantiles and E_i is the composed error term which will be discussed later.

$$\begin{aligned}
Y_i &= f({X'}_{ik}\beta^\rho) \cdot TE^\rho \\
Y_i &= {X'}_{ik}\beta^\rho + E_i, \quad i = 1, ..., I \\
E_i &= U_i - V_i \\
U &\sim ALD(0, \sigma_u^2, \rho) \\
V &\sim ALD(0, \sigma_u^2, \rho)
\end{aligned} \tag{1}$$

The conditional quantile of given an input matrix is measured using the conditional quantile function denoted by

$$Q_{\overline{y}_i} = (\rho | X_{ik}) = \beta(\rho){X'}_{ik} \tag{2}$$

In addition, the composed error term of the model is defined by $E_i = U_i - V_i$ where U_i is assigned as the Asymmetric Laplace distribution (ALD) with mean zero and variance σ_U^2 and V_i is a nonnegative random error that is truncated positive ALD with mean zero and variance σ_V^2 (see [11]). Therefore, when using ALD, we can get consistent estimation of the quantile function and obtain a different slope coefficients as well as the technical efficiency (TE) across different quantiles ρ. In the context of the SFQM, the technical efficiency or TE can be defined as the ratio of the observed output (Y_i) to the corresponding frontier output (Y_i^*) conditional on the levels of inputs used by the country at each quantile level. Thus, the technical efficiency across quantiles or TE^ρ is given by

$$TE^\rho = exp(X_{ik}\beta^\rho + V_i - U_i)/exp(X_{ik}\beta^\rho + V_i) \tag{3}$$

where U_i and V_i represent the noise and technical inefficiency, respectively. Most importantly, these two errors are assumed to be related in this case; therefore, the joint distribution of U_i and V_i then will be modelled by the copula approach, which in turn will be explained in the next section.

2.2 Copula Functions

By a theorem due to Sklar, copula is a powerful tool used for building multivariate distributions. Copula represents dependence structures among component variables. In this study, we consider the case of two variables that are U_i and V_i, with distribution functions F_1 and F_2, respectively. Suppose that both U_i and V_i are continuous, then the joint distribution function of a two-dimensional random vector and can be expressed as

$$H(u, v) = P(U \leq u, V \leq v) = P(F_1(U) \leq F_2(u), F_2(V) \leq F_2(v)) \tag{4}$$

Note that the marginal U_i and V_i are uniformly distributed on the interval [0,1]. The term $H(u,v)$ is a joint distribution of U_i and V_i evaluated at the point $(F_1(u), F_2(v)) \in [0,1]^2$. As such, it is of the form

$$H(u,v) = C(F_1(u), F_2(v)) \tag{5}$$

for some copula C, then the unique copula C is obtained as

$$C(u,v) = H(F_1^{-1}(u), F_2^{-1}(v)) \tag{6}$$

where F_i^{-1} is the quantile functions of marginal $i = 1, 2$; and u_i, v_i are uniform [0,1]. In summary, a (bivariate) copula is a (restriction of) bivariate distribution with uni-form marginal on [0,1]. Any joint distribution function can be built up from the marginal distributions and copula. The copula of a joint distribution can be extracted from the joint distribution.

2.3 Copula-Based Stochastic Frontier Quantile Model

In recent years, the validity of independence assumption between the two error components of stochastic frontier model has been questioned, particularly in the context of inefficiency in a dynamic setup, Das [8]. Two statisticians Burns [12] and Smith [9] suggest relaxing this weak independence assumption by using the ability of copula joining two marginal distributions of U_i and V_i, and they prove that the copula can work well as a joint distribution for this case. Hence, this study decides to employ the copula to join our two error components. The joint density of U_i and V_i can be derived by the copula function, $C(u,v)$ so that

$$F(U_i, V_i) = C(F_1(u), F_2(v)) \tag{7}$$

This bivariate distribution function $F(U_i, V_i)$ can be obtained using the marginal distribution function $F_1(u)$ and $F_2(v)$ of u and v, and bivariate copula function $C(u,v)$. The corresponding bivariate copula density function can be obtained by differentiating Eq. (6) with respect to u and v as follows

$$\begin{aligned} f(u,v) &= \frac{\partial^2}{\partial u \partial v} C(F_1(u), F_2(v)) \\ &= f_1(u) f_2(v) C_\theta(F_1(u), F_2(v)) \end{aligned} \tag{8}$$

As in Eq. (7), the terms $F_1(u)$ and $F_2(v)$ denote the probability density function (pdf) of U_i and V_i, respectively. The term $C_\theta(F_1(u), F_2(v))$ is the density function of the copula. Since the inefficient V_i cannot be obtained directly through the SFQM, this study employs the simulated likelihood function, which is the intractable integrals appearing in the likelihood functions. It is actually expectation of a well behaved function of random V_i. Thus, we transform (U_i, V_i) to be (E_i, V_i) where $E_i = U_i - V_i$. Thus we can rewrite the Eq. (7) as

$$f(u,v) = f(v, v+e) = (f_1(v) f_2(v+e) c(F_1(v), F_2(v+e)) \tag{9}$$

According to Smith [9], the pdf of is given by

$$\begin{aligned} f(e) &= \int_0^M f(v,e)du \\ &= E_v(f(V_i+E_i)c_\theta(F_1(v),F_2(v+e))) \end{aligned} \tag{10}$$

where

$$f(V_i+E_i) = \frac{p(1-p)}{\sigma_{(V_i-E_i)}} exp\left\{-\rho_p \frac{(V_i+E_i)}{\sigma_{(V_i-E_i)}}\right\} \tag{11}$$

where V_i is simulated from the positive truncated ALD with mean equal to zero and variance, σ_V^2. Consider the second density, the bivariate copula density for $V_i + E_i$ and V_i is contracted by either Elliptical copulas or Archimedean copula. In this study, we consider six copula functions namely, Normal copula, Student's t copula, Frank copula, Clayton copula, Gumbel copula, and Joe copula. The joint distribution or the copula function is uniform marginal thus the simulated $V_i + E_i$ and V_i are transformed by cumulative ALD and cumulative truncated ALD, respectively. One of the most important purposes of stochastic frontier analysis is to measure the technical efficiencies of countries at different quantile level (TE^ρ) based on the combination of input and the level of outputs. TE^ρ is the effectiveness of given set of inputs used to produce an output at each quantile level. It appears to be technically efficient if the countries produce a maximum output from the minimum of inputs. In fact, we cannot observe TE^ρ directly especially with different quantile levels, but by following the model of original TE given by Battese and Coelli [13], we are able to apply the quantile approach and copula to the original formula of TE. And hence, the formula of TE^ρ can be expressed as follows.

$$\begin{aligned} TE^\rho &= E(exp(-V_i)\,|E_i = e) \\ &= \frac{\sum_{i=1}^M exp(-V_i)f(V_i+E_i)c(F_i(V_i),F_2(V_i+E_i)\,|\theta)}{\sum_{i=1}^M f(V_i+E_i)c(F_i(V_i),F_2(V_i+E_i)\,|\theta)} \end{aligned} \tag{12}$$

where $(U = V_i + E_i) \sim ALD(0, \sigma_U^2, \rho)$ and $V \sim ALD^+(0, \sigma_V^2)$.

3 Estimation of the Copula-based SFQM

In general, the estimation of the parameters of a copula model is done by inference function for margins method or IFM. However, V_i cannot be observed and estimated by the univariate likelihood function, say $f(V_i + E_i)$. Therefore, the estimation in our model has to be necessarily based on the full likelihood in Eq. (9). As we mentioned before, the two error components U_i and V_i are assumed to be related and the joint distribution of and can be constructed by employing the copula approach. Here, we employ six well-known families of copula consisting of Gaussian, Student's t, Frank, Joe, Gumbel, and Clayton copulas in which a brief summary of the property of each copula family is described in [10].

As discussed in the works of Smith [9] and Das [8], the model faces with multiple integrals in the likelihood. Thus they suggested employing a simulated likelihood estimation function to obtain the asymptotically unbiased simulators for the integrals. Therefore, the exact likelihood functions based on a sample $\left\{V_i^R\right\}_{r=1}^R$ of size R,where $V_i^R = (V_1^R, ..., V_N^R)$, is expressed by

$$L(\beta_k^\rho, \sigma_{(V+E)}, \sigma_V, \theta) = \sum_{i=1}^{N} \left(\frac{1}{R} \sum_{j=1}^{R} \log f(V_{ij} + E_{ij}) c(v_{ij}, (v_j + e_j) \,|\theta\,) \right) \quad (13)$$

Then, the log likelihood function as shown in Eq. (13) will be maximized using the BFGS algorithm, which in turn makes the likelihood consistent for every quantiles. (Snchez, Lachos, Labra [14]).

4 Monte Carlo Simulation Study

In this section, we employ a simple Monte Carlo simulation study to evaluate primarily the performance and accuracy of our proposed model, which takes the form as Eq. (1). In practice, we generate data from the ALD and half ALD distributions and employ six copula families as described in the previous section to model the dependence structure of the two error components of the SFQM. We start with simulation of uniform u and v by setting the true copula dependency θ equal to 0.5 for Gaussian, Student's t, and Clayton copulas and equal to 3 for the rest copulas i.e. Gumbel, Joe, and Frank. For the case of Student's t copula, we set the true value of the additional degree of freedom vf equal to 4. Then, the obtained u and v are transformed into $U_i \sim \; ALD(0, \sigma_U, \rho)$ and $V_i \sim \; ALD^+(0, \sigma_V, \rho)$ by the quantile function of ALD and half-ALD, where $\sigma_U = 1$ and $\sigma_V = 0.5$. The covariates X_1 and X_2 of the output Y_i are randomly simulated from $uni(0, 2)$. We assume the true parameter for the intercept term α^ρ to be 1.5 and the coefficients β^ρ to be 2 and -2 for all quantiles $\rho = (0.25, 0.5, 0.75)$, and then generate data set n = 100. For each data set, we can observe the performance and accuracy of our proposed model by comparing the true parameters with the estimated parameters.

Table 1 shows the results of the Monte Carlo simulation investigating the maximum likelihood estimation of the Copula-based SFQM. We found that our model can perform very well through simulation study. It is found that the estimated parameters are very close to their true values with acceptable standard errors shown as the values in the braces. For example, in the case that we use Gaussian copula as a joint distribution for U and V, the estimated values of the intercept term α are 1.1055, 1.7249, and 1.6081 for quantile 0.25, 0.5, and 0.75, respectively, while the true value is 1.5 for all quantiles. The estimated coefficients β_1 are equal to 1.6946, 1.8084, and 1.5332 for different quantiles while the true value is equal to 2. This result is acceptable and the same for other copulas, therefore, the Monte Carlo simulation suggests that our proposed Copula-based SFQM is reasonably accurate.

Table 1. Simulation result

Copula	Gaussian			Student's t			Gumbel		
Parameter/Quantile	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
α	1.1055	1.7249	1.6081	1.1055	1.4154	1.5066	1.4173	1.5797	1.6831
	(0.1626)	(0.1734)	(0.1552)	(0.1626)	(0.0011)	(0.3336)	(0.3839)	(0.0747)	(0.5178)
β_1	1.6946	1.8084	1.5332	1.6943	1.8182	1.6376	1.9093	1.9566	1.7714
	(0.182)	(0.2755)	(0.2020)	(0.1820)	(0.4298)	(0.4359)	(0.3499)	(0.1666)	(0.3784)
β_2	−2.4778	−2.8275	−2.2651	−2.4778	−2.3800	−2.2684	−2.1553	-2.1102	-2.1441
	(0.4029)	(0.4523)	(0.1183)	(0.4029)	(0.1089)	(0.4982)	(0.3899)	(0.2815)	(0.1794)
σ_u	0.9959	1.0506	0.9841	0.1006	0.9760	1.0563	1.1106	1.0839	1.2877
	(0.0111)	(0.0977)	(0.1936)	(0.0741)	(0.1008)	(0.0076)	(0.1126)	(0.0301)	(0.1804)
σ_v	0.4645	0.6863	0.6909	0.4959	0.4383	0.7240	0.4273	0.4354	0.5918
	(0.2645)	(0.0336)	(0.0635)	(0.0119)	(0.0058)	(0.1128)	(0.1130)	(0.0428)	(0.0699)
θ	0.3555	0.5359	0.6113	0.4555	0.5809	0.5551	3.1728	4.2498	2.5983
	(0.1395)	(0.0819)	(0.0207)	(0.0395)	(0.0224)	(0.0081)	(0.0500)	(0.4288)	(0.0276)
Copula	Frank			Joe			Clayton		
Parameter/Quantile	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
α	1.8021	1.4844	1.8172	1.5313	1.3962	1.2315	1.2596	1.9652	1.7974
	(0.5793)	(0.0909)	(0.3652)	(0.1669)	(0.1049)	(0.0415)	(0.3714)	(0.3417)	(0.4397)
β_1	1.9164	1.7469	1.5253	2.0347	1.8012	1.7219	1.3307	1.8310	1.5333
	(0.1597)	(0.5266)	(0.1515)	(0.1338)	(0.1050)	(0.0448)	(0.1929)	(0.2864)	(0.2941)
β_2	−2.2897	−2.3334	−2.2191	−1.9108	−2.2988	−2.2577	−2.0191	−2.4506	−2.3424
	(0.6428)	(0.5568)	(0.3973)	(0.1164)	(0.0812)	(0.0391)	(0.3636)	(0.1867)	(0.6668)
σ_u	0.8298	0.9317	1.0467	0.8409	1.3523	0.9704	1.0381	1.0368	1.0694
	(0.1768)	(0.1075)	(0.1461)	(0.0934)	(0.0602)	(0.0452)	(0.1044)	(0.1054)	(0.1648)
σ_v	0.5291	0.4025	0.8604	0.2975	0.6641	0.4323	0.5029	0.5044	0.4878
	(0.0799)	(0.0344)	(0.0031)	(0.0503)	(0.0851)	(0.0304)	(0.0977)	(0.0811)	(0.0148)
θ	3.100	2.3677	2.2796	2.6488	2.5491	4.0894	0.6544	0.6405	0.4887
	(0.2065)	(0.3190)	(0.2142)	(0.1945)	(0.0549)	(0.0430)	(0.0009)	(0.0076)	(0.0768)

Note: We assume the true value for intercept term α^{ρ} for all quantiles $\rho = (0.25, 0.5, 0.75)$ to be 1.5, the coefficients β^{ρ} to be 2 and −2, and $\sigma_u = 1$ and $\sigma_v = 0.5$. The true copula dependency θ is equal to 0.5 for Gaussian, Student's t, and Clayton copulas but it is equal to 3 for Gumbel, Joe, and Frank copulas.

5 Empirical Results: Agricultural Production Model for Asia

This part presents the benchmark result of this paper. We analyze the agricultural production function of Asian countries using our proposed model copula-based stochastic frontier quantile model which has been proved to be accurate through the simulation study.

5.1 Dataset

Prior to the estimated result, this part begins with the brief explanation of the data used in this paper. This paper considers three important input variables for estimating Asian production function, including labor, fertilizer, and agricultural area. Since we consider a cross-section of countries, we then collect the data in year 2013 when the data of every country are the most perfect and latest, from World Bank database and Thomson Reuters DataStream, from Financial

Investment Center (FIC), Faculty of Economics, Chiang Mai University, covering 44 countries in Asia.

Production output (Y) refers to the crop production index which shows agricultural production for each year relative to the base period 2004-2006. It includes all crops except fodder crops and is calculated from the underlying values in international dollars, normalized to the base period 2004-2006.

Labor (L) refers to the rural population. Due to some limited access to the data, we are unable to get the exact number of labor working in the agricultural sector. So, we decide to use the number of rural population as defined by national statistical offices to represent this variable since we believe that people living in rural areas have high possibility of working in agriculture.

Agricultural area (A) refers to the share of land area that is arable, under permanent crops, and under permanent pastures.

Fertilizer (F) refers to the fertilizer consumption measured by the quantity of plant nutrients used per unit of arable land. The variable covers nitrogenous, potash, and phosphate fertilizers (including ground rock phosphate), except traditional nutrients such as animal and plant manures.

5.2 Model Specification

To analyze the factors affecting agricultural output and measure the technical efficiency of Asian production, we considered the following production model. The model primarily takes the form of Cobb-Douglas production function where labor (L), fertilizer (F), and agricultural area (A) are inputs.

$$Y_i = \alpha L_i^{\beta_1^\rho} F_i^{\beta_2^\rho} A_i^{\beta_3^\rho} \tag{14}$$

Then, we transform Eq. (14) into a translog production frontier which takes the form as

$$\ln Y_i = \alpha + \beta_1^\rho \ln L_i + \beta_2^\rho \ln F_i + \beta_3^\rho \ln A_i + U_i - V_i. \tag{15}$$

In this study, we consider three quantile levels that are $\rho = (0.25, 0.5, 0.75)$ to represent three groups of agricultural countries in Asia as classified by the 2008 World Development Report of World Bank, namely (1) agriculture-based, (2) transforming, and (3) urbanized countries, respectively.

5.3 Model Selection

As a sequential estimation method, the copula has to be selected before estimation. Therefore, this part is also about selecting a copula that is best-fit for the data. We employ the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to pick the best copula among the copula families that we concern, i.e. Gaussian, Students t, Clayton, Gumbel, Frank, and Joe. The results are presented in Table 2.

Table 2 shows the values of AIC and BIC for each Copula-based stochastic frontier quantile model, where the minimum values are bold numbers. According

Table 2. AIC and BIC for each Copula-based SFQM

Copula	Quantile level		
	0.25	0.5	0.75
Gaussian	610.47	4.25	3.42
	635.48	30.82	28.43
Student's t	734.21	6.32	8.83
	762.79	36.69	37.41
Clayton	885.55	3.39	6.13
	907.56	29.96	31.14
Gumbel	2.567	5.01	6.92
	27.57	31.57	31.93
Frank	1.11	5.85	5.97
	26.06	32.43	30.98
Joe	5.92	3.17	0.68
	32.49	29.74	25.68

to both criteria, the best model for quantile level 0.25 is the one based on the Frank copula where the values of AIC and BIC are equal to 1.11 and 26.06, respectively. However, the best models for quantile levels 0.5 and 0.75 are the one based on the Joe copula since it has the minimum values of AIC and BIC as shown in the table.

5.4 Estimated Results of Copula-Based SFQM

This part presents the benchmark result of this paper, where we estimate the stochastic frontier model based on the copulas we chose in the previous section. The results are presented in Table 3. Technically, it is found that the dependence between error components exists since the estimated parameters of the copulas θ are significant for all quantiles at the 1 % level. Apart from that technical consideration, we found some interesting point that the estimated parameters are not so different across quantile levels. This means the impact of inputs, i.e. labor, fertilizer, and agricultural area, on the agricultural output are quite the same for all Asian countries. For example, an additional 1 % of labor leads to 0.013 % increase in agricultural output at the 0.25-quantile, 0.015 % and 0.018 % at the 0.5 and 0.75 quantiles, respectively.

Fertilizer is found to affect significantly only the first quantile which represents a group of agriculture-based countries. This seems to make sense because the other groups which are transforming ($\rho = 0.5$) and urbanized ($\rho = 0.75$) countries are able to access to get high-tech agricultural innovation such as solar power, hydroponics, and aeroponics. These smart technologies help farmers get more output and improve their crops, which in turn make fertilizer exert less influence on agricultural product.

The last input, agricultural area, seems to create the largest impact on the output compared with other factors, but the impacts are not different across quantiles. We found that an additional 1 % of area brings about 0.038 % increase in agricultural output at the 0.25 and 0.5 quantiles. This result corresponds with many agricultural reports in which Asia uses a very large area to grow crops; the quantity of output depends essentially on the lands used.

Table 3. Estimated parameters and standard errors for Copula-based SFQM.

Parameter	Quantile level		
	0.25	0.5	0.75
α	4.335***	4.557***	4.708***
	(0.018)	(0.048)	(0.094)
β_1^ρ	0.013***	0.015***	0.018*
	(0.006)	(0.008)	(0.011)
β_2^ρ	0.022*	0.026	0.027
	(0.011)	(0.026)	(0.032)
β_3^ρ	0.038***	0.038***	0.037
	(0.011)	(0.013)	(0.026)
σ_U	0.037***	0.086***	0.233***
	(0.008)	(0.018)	(0.025)
σ_V	0.058***	0.010	0.022
	(0.003)	(0.023)	(0.022)
θ	1.164***	3.144***	5.558***
	(0.077)	(0.058)	(0.336)

Note: *, **, and *** denote rejections of the null hypothesis at the 10 %, 5 %, and 1 % significance levels, respectively.

Additionally, Fig. 1 is constructed to illustrate the position of each country. We aim to find out which quantile level that a country fits most based on different inputs. Each of the quantile levels has meaning (See Sect. 5.2); that is the 0.25-quantile means agriculture-based country, the 0.5-quantile means transforming country, and the 0.75-quantile means urbanized country. To visualize the position, we plot the data of each input, i.e. area (top left), fertilizer (top right), and labor (bottom), against the level of output (crop production). Note that the data are log-transformed. The dot lines refer to the quantile lines for the 0.25, 0.5, and 0.75 quantiles, which are estimated from the copula-based SFQM. Finally, the result from the copula dependence shows the significant positive correlation exists between noise and inefficiency.

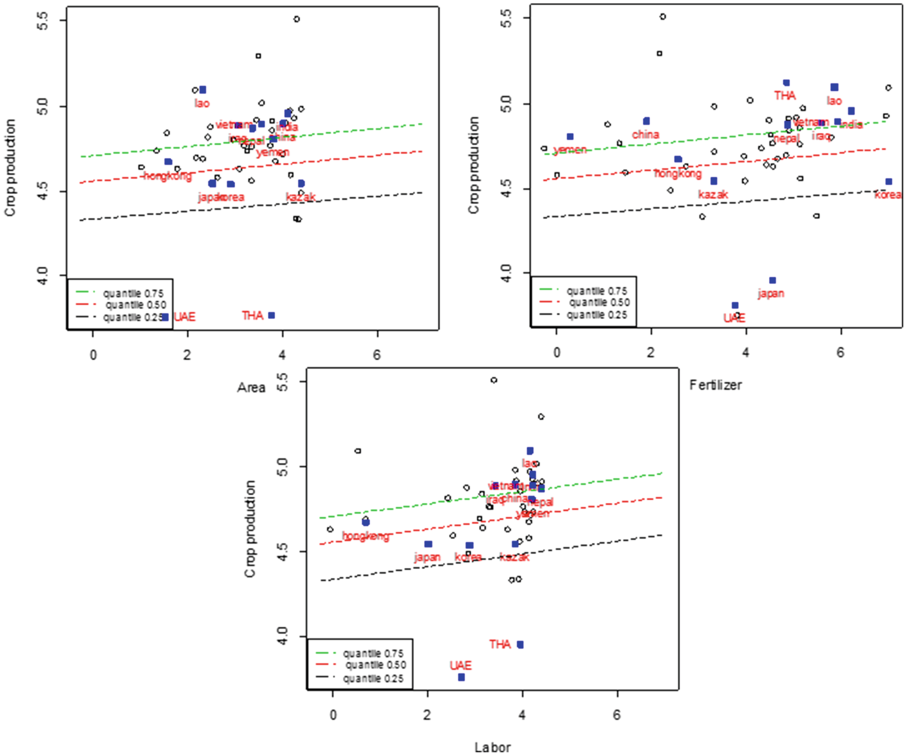

Fig. 1. Plot of production data fitting to quantile lines

5.5 Estimate of Technical Efficiency

This section presents the estimated result of the technical efficiency of Asian production. As we described in the end of Sect. 2.3, the technical efficiency is given by the ratio of observed output to maximum feasible output in which the value is equal to 1 means a country obtains the maximum feasible output. The value less than 1 refers to a shortfall of the observed output from maximum feasible output.

Figure 2 displays the values of technical efficiency at different quantiles, which are estimated by the copula-based SFQM. We found that the efficiencies are not the same for all quantile levels. The first quantile (0.25) representing the agriculture-based country has the lowest efficiency score in agricultural production where the range is 0.87 to 0.93 (average 0.90). The second and third quantiles representing the transforming and urbanized countries, respectively, have quite the same efficiency score with in the range around 0.91 to 0.99.

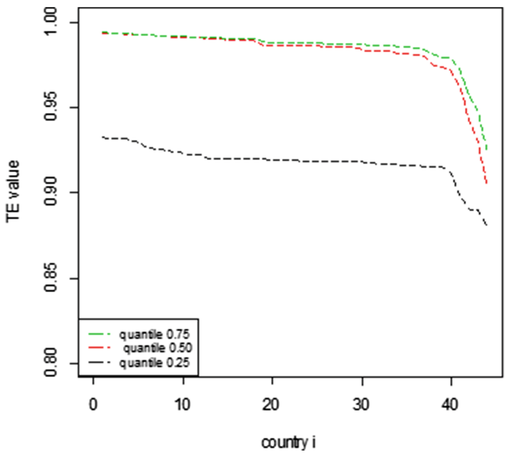

Fig. 2. Technical efficiency of Asian production at different quantiles

6 Conclusion

Since agricultural production sector has played a key role to the Asian economy, this paper attempts to analyze the agricultural production function of Asian countries and examine the technical efficiencies of this region. In methodology, we take advantage of the stochastic frontier model in terms of the dependency between the error term U and the non-negative inefficiency V. We employ the copulas to model this dependence structure, and then extend the quantile regression to the SFM with dependent error components to capture the tail behavior of a probability distribution. Therefore, we introduce the model named the Copula-based stochastic frontier quantile model as a frontier model for this issue.

To model the agricultural production in Asia, we consider labor, fertilizer, and agricultural area to be inputs and the crop production to be output. This methodology is applied to the agricultural data of 44 Asian countries. The results show that the impact of labor on production output is quite the same for all Asian countries, whereas fertilizer is found to have effect significantly only on the first quantile which represents a group of agriculture-based countries. Agricultural area creates the largest impact on output compared with other factors, but the effect sizes are not different across quantiles. For technical efficiency, we found that the 0.25-quantile has the lowest efficiency score in agricultural production within the range 0.87 to 0.93 while the other two quantiles have the efficiency score within the range 0.91 to 0.99. The overall results suggest that the agricultural sector in Asia is able to perform effectively in the long-run.

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