Linear Stochastic Model Validation for Civil Engineering Structures Under Earthquakes

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Abstract. The autoregressive moving average exogenous (ARMAX) model validation of civil engineering structure under earthquake is developed in this paper. The Kanai-Tajimi and Clough-Penzien seismic models are developed. An identification process is used to estimate the polynomial parameters for unknown simulated seismic signal in order to take into account the soil-structure interaction (SSI) within the structural model. The results show that the ARMAX model presents an interesting representation for the linear stochastic systems in control point of view. Simulation tests using a single-degree-of-freedom structure are performed to show the efficiency of introducing the SSI, by identification, in the response of the structure under the seismic ground motion.

Keywords: Dynamics of structures · ARMAX model · Seismic ground motion · Soil structure interaction · Identification

1 Introduction

The impact of control theory in the different domains of engineering and applied sciences has become increasingly important in the last few decades, and the specialists of civil engineering structures are very interested in structural control against earthquakes. One of the important missions of structural control is to ensure the safety of structures and cities in large earthquakes. In spite of the unpredicted nature and the uncertainty of the seismic phenomenon, the structural control should provide the structure the possibility to control itself during perturbation [10, 17]. Vigorous researches on structural control have given more interesting solutions; some of them have already been adopted in actual building structures. These researches have also stimulated global and interdisciplinary activities giving rise to a wide variety of interesting work in many fields [3, 10].

It has been shown in the literature that in the last two decades, control devices and algorithms have been interesting to enhance the structural control performances [17]. The performance of such systems under environmental loads has improved greatly as a result of theoretical and experimental research and related development efforts [11].

Several models of the structures have been developed in the literature. *Kareem et al.* use state-space representation of the linear stochastic model for real-time model predictive control [11]. Whereas *Sheng-Guo Wang*, *Shafieezadeh et al.*, *Purohit and*

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Chandiramani use the state-space model to develop different strategies of optimal control [13, 15, 16]. Indeed, *Guenfaf and Allaoua* have presented an active control strategy using a state-space model to develop the linear quadratic controller for structural vibration control [6]. Also, *J. Awrejcewicz and P. Olejnik* have presented an active control law of buildings for the general concept of stabilization against external excitations. The problem was analyzed in a two case studies for not excited and externally loaded two degrees-of-freedom dynamical system by using state-space model [2, 17]. They developed a LQR algorithm with an augmented model introducing seismic excitation as external loading. An augmented state space model is used for calculating this control law. But the excitation dynamical model is not introduced in the system model and the obtained law is deduced from deterministic criterion [12].

Nonlinear model has been presented by *Guenfaf el al.* for a Generalized Minimum Variance Gain Scheduling Controller using Nonlinear Structural Systems under Seismic Ground Motion [7]. *Ying* also presented a nonlinear model for Stochastic Optimal Control of Structural Systems [18].

In the last few years there has been increased interest in the study of soil-structure interaction (SSI) effects on the structures subjected to active control. The dynamic response of massive structures, such as high-rise buildings and dams, may be influenced by soil-structure interaction as well as the characteristics of exciting loads and structures. The effect of soil-structure interaction is noticeable especially for stiff and massive structures resting on relatively soft ground. It may alter the dynamic characteristics of the structural response significantly. As a result, these interaction effects have to be considered in the dynamic analysis of structures [5].

Although, different investigations have been conducted to soil-structure interaction analysis, depending on the modeling method for the soil region. For earthquake resistant design of critical structures, a dynamic analysis, either response spectrum or time history is frequently required. To provide input excitations to structural models for sites with no strong ground motion data, it is necessary to generate artificial excitations. In practice, it has long been established that different earthquake records show different characteristics; this is due to parameters such as geological conditions of the site, distance from the source, fault mechanism, etc. Thus, the simulated earthquake records must have realistic duration, frequency content, and intensity, representing the physical conditions of the site. Therefore, it is reasonable to identify certain parameters to measure the resulting differences in the strong ground motion data [14].

2 Dynamic Model of the Structure

In this section, a motion equation is developed for a single degree of freedom (SDOF) structure under a seismic motion as illustrated in Fig. 1. First the following assumptions are considered:

- The structure is supposed to be a lumped mass m in the beam.
- The two vertical axes are weightless and inextensible in the vertical direction with spring constant $k/2$ each.

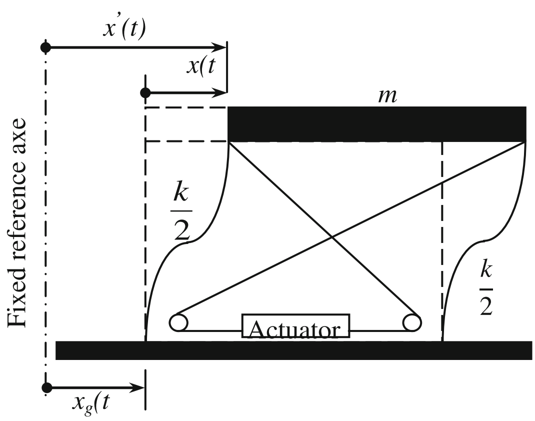

Fig. 1. Model of single degree of freedom structure

The essential physical properties of any linearly elastic structural system subjected to dynamic load include its mass, elastic properties and, energy-loss mechanism (damping), and the external source of excitation or loading. In the simplest model of

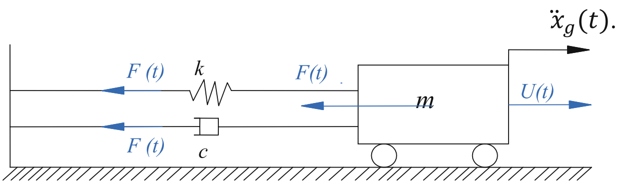

Fig. 2. Idealized SDOF structure model

SDOF system, each of these properties is assumed to be concentrated in a single physical element. The system as described is shown in Fig. 2 [4].

The entire mass m of this system is included in the rigid block. Rollers constrain this block so that it can move only in simple translation; thus the single displacement coordinate x completely defines its position. The elastic resistance of the displacement is provided by the weightless spring of stiffness k, while the energy-loss mechanism is represented by the damper c. The external-loading mechanism producing the dynamic response of this system is the time varying load $\ddot{x}_g(t)$.

The equation of motion for the system presented in Fig. 2 can be derived by directly expressing the equilibrium of all forces acting on the mass as follows:

$$m\ddot{x}'(t) + c\dot{x}(t) + kx(t) = u(t) \tag{1}$$

Where:

$$x(t)' = x(t) + x_g(t) \quad (2)$$

With $x_g(t)$ being the ground motion, and *u(t)* the external control force.

Using Eqs. (1) and (2), we have:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = u(t) - m\ddot{x}_g(t) \quad (3)$$

$\ddot{x}_g(t)$ is the ground acceleration.

Here, we developed the dynamic model of single-degree-of-freedom structure with its physical characteristics submitted to a seismic motion.

3 Seismic Dynamic Model

The aim of this work is to develop a model which takes into account the soil-structure

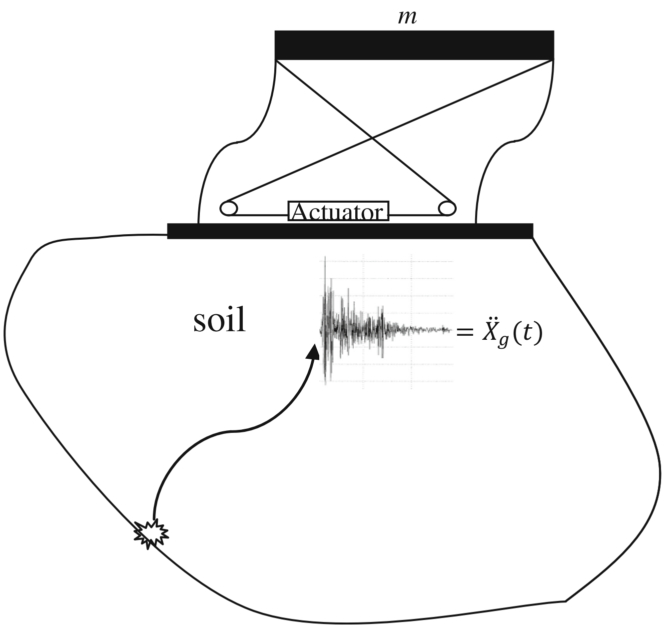

Fig. 3. Structural model under seismic motion

interaction in order to have a response which represents closely the displacement of the structure under the seismic motion (ε: the smaller the better) as shown in Fig. 3. For this, the ground motion model is given, so that the earthquake will be a white noise filtered by a function taken into account the specific soil parameters in where the structure is built as we can see it in Fig. 6.

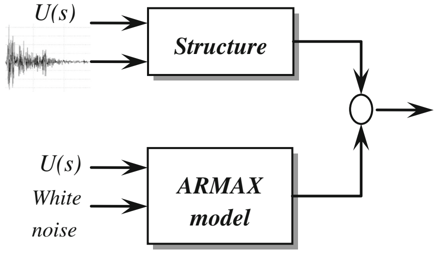

Fig. 4. Block diagram of comparing structural model with the ARMAX model

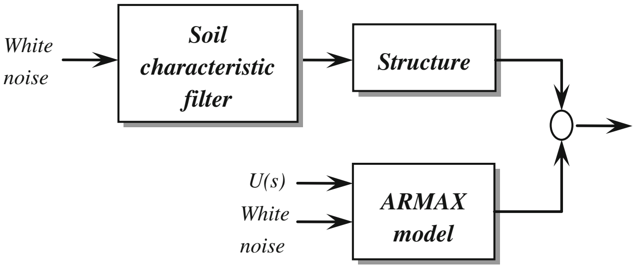

Fig. 5. Block diagram of comparing structural model with introducing soil characteristics (SSI) and the ARMAX model

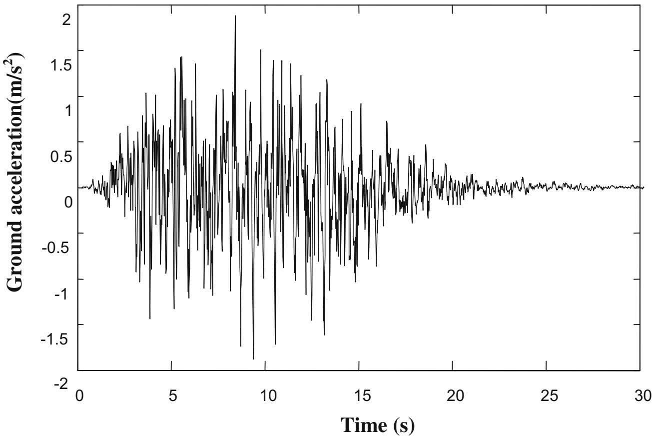

Fig. 6. Kanai-Tajimi seismic model simulated

To formulate an optimal problem, it is necessary to specify the process dynamics and its environment. It is assumed that the influence of the environment can be characterized by disturbances which are stochastic process [1]. As the system is linear, we can represent all disturbances as single disturbance acting on the output as shown in Figs. 4 and 5.

Kanai-Tajimi and Clough-Penzien have presented a seismic model based on a filtered white noise depending on soil characteristics.

The earthquake ground acceleration is modeled as a uniformly modulated non-stationary random process [10].

$$\ddot{x}_g(t) = \psi(t)\ddot{x}_s(t) \tag{4}$$

Where $\psi(t)$ a deterministic non-negative envelope is function and $\ddot{x}_s(t)$ is a stationary random process with zero mean and a Kanai-Tajimi power spectral density.

$$\Phi_g(\omega) = \left[\frac{1 + 4\xi_g^2(\frac{\omega}{\omega_g})^2}{(1 - \left(\frac{\omega}{\omega_g}\right)^2)^2 + 4\xi_g^2(\frac{\omega}{\omega_g})^2}\right] S_0^2 \tag{5}$$

Where ξ_g, ω_g filter parameters which depend on the site soil characteristics and S0 are is the constant spectral density of the white noise which reflects the seismic intensity [4]. Using such a second high-pass filter the Kanai-Tajimi spectrum is modified as follows to obtain the Clough-Penzien spectrum:

$$\Phi_c(\omega) = \left[\frac{1 + 4\xi_g^2(\frac{\omega}{\omega_g})^2}{(1 - \left(\frac{\omega}{\omega_g}\right)^2)^2 + 4\xi_g^2(\frac{\omega}{\omega_g})^2}\right]\left[\frac{(\frac{\omega}{\omega_c})^4}{(1 - \left(\frac{\omega}{\omega_c}\right)^2)^2 + 4\xi_c^2(\frac{\omega}{\omega_c})^2}\right] S_0^2 \tag{6}$$

A particular envelope function $\psi(t)$ given below will be used:

$$\psi(t) = \begin{cases} 0 & for\ t < 0 \\ \left(\frac{t}{t_1}\right)^2 & for\ 0 \leq t \leq t_1 \\ 1 & for\ t_1 \leq t \leq t_2 \\ exp[-a(t - t_2)] & for\ t \geq t_2 \end{cases} \tag{7}$$

Where t_1, t_2 and a are parameters that should be selected appropriately to reflect the shape and the duration of the earthquake ground acceleration. Numerical values of parameters are t1 = 3 s, t2 = 13 s, a = 0.26, ξ_g = 0.65, ω_g = 19 rad/s, S0 = 0.8*10 −2 m/s [7, 9] (Fig. 7).

4 ARMAX Model of the Structure

Determination of the ARMAX model of the structure under seismic excitation can be done using equation of motion (3).

After dividing this equation by m and introducing the notations below Eq. (3) becomes

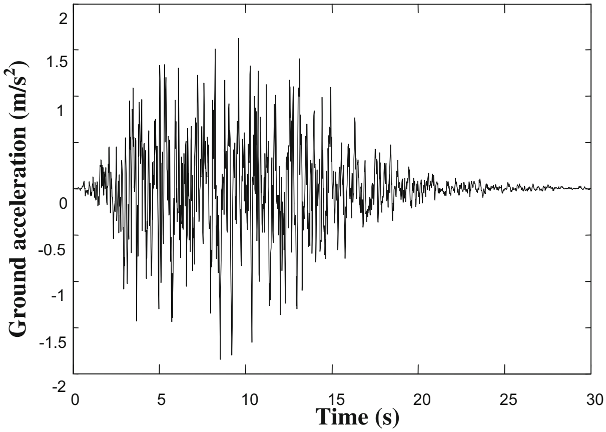

Fig. 7. Clough-Penzien seismic model simulated

$$\ddot{x}(t) + 2\xi\omega_0\dot{x}(t) + \omega_0^2 x(t) = \frac{1}{m}u(t) - \ddot{x}_g(t) \tag{8}$$

Applying Laplace transform to Eq. (8), we obtain

$$X(s) = \frac{\frac{1}{m}}{s^2 + 2\xi\omega_0 s + \omega_0^2}U(s) - \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2}\ddot{X}_g(s) \tag{9}$$

Where $X(s)$, $\ddot{X}_g(s)$ and $U(s)$ are the Laplace transform of *x(t),* $\ddot{x}_g(t)$ and *u(t)* respectively.

Depending on the model of seismic excitation, different ARMAX models can be obtained, and the following cases [8]:

The seismic excitation model is unknown or is not taken into consideration. Equation (9) has the form:

$$X(s) = \frac{H_{1N}(s)}{H_D(s)}U(s) + \frac{H_{2N}(s)}{H_D(s)}\ddot{X}_g(s) \tag{10}$$

The ARMAX model of the structure is obtained by discretization of Eq. (10) using computer programs (MATLAB R2010a).

$$x(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + \frac{C(q^{-1})}{A(q^{-1})}\ddot{x}_g(t) \tag{11}$$

Kanai-Tajimi model: In this case we take into consideration the SSI. The ground acceleration is described by the Kanai-Tajimi model as shown in Fig. 8 and in the following equation:

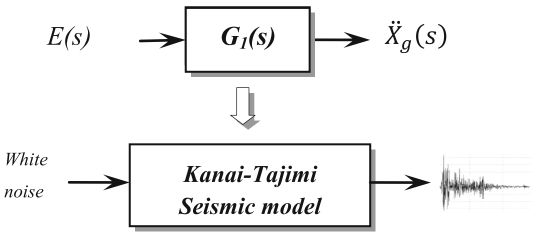

Fig. 8. Block diagram of the structural Kanai-Tajimi model.

$$\ddot{X}_g(s) = G_1(s)E(s) \tag{12}$$

Where

$$G_1(s) = \frac{G_{1N}(s)}{G_{1D}(s)} = \left(2\xi_g\omega_g s + \omega_g^2\right)/(s^2 + 2\xi_g\omega_g s + \omega_g^2)$$

And *E(s)* is the Laplace transform of white noise.

Clough-Penzien model: In this case the ground acceleration is described by Clough-Penzien model and introduced in the dynamic model of the structure as shown in Fig. 9. This excitation is described by the following equations.

$$\ddot{X}_g(s) = G_2(s)E(s) \tag{13}$$

Where $G_2(s) = \frac{G_{2N}(s)}{G_{2D}(s)} = \left(\frac{2\xi_g\omega_g s + \omega_g^2)}{s^2 + 2\xi_g\omega_g s + \omega_g^2}\right)\left(\frac{s^2}{s^2 + 2\xi_c\omega_c s + \omega_c^2}\right)$

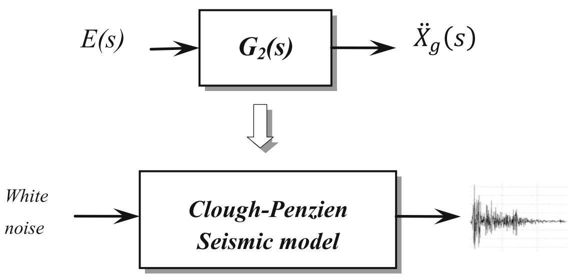

Fig. 9. Block diagram of the structural Clough-Penzien model.

It has been shown that the response without taking the SSI into account is not close to the structural one. Also, it has been presented a model taking wrong soil parameters into account, whereas, the best results has been given by the model response which included SSI with the right parameters [8].

5 ARMA Model Identification

According to the previous results, it is suitable to use an identification process to estimate the parameters of unknown earthquakes. We can find in the literature several identification algorithms such as ‘recursive least square’ (RLS) one, as represented in Fig. 10.

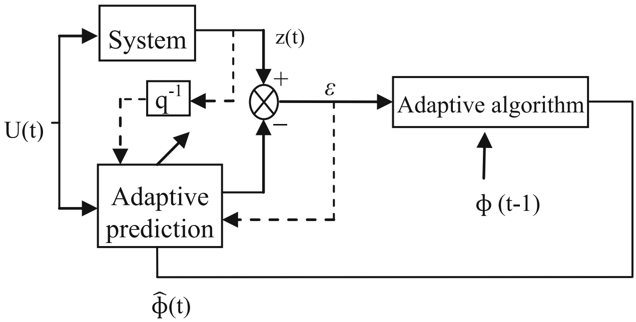

Fig. 10. Block diagram of the recursive identification method.

The recursive least square identification algorithm has been used to estimate the model parameters. It is based on the minimization of a quadratic criterion as follows:

$$J(t) = \sum\nolimits_{i=0}^{t} [z(i) - \hat{z}(i)]^2 = \sum\nolimits_{i=0}^{t} \varepsilon^2(t) \tag{14}$$

With
z(t) Actual output of the system, $\hat{z}$(t) Model output, ε Error.

$$\hat{z}(t) = \hat{\theta}^T(t)\phi(t-1) \tag{15}$$

With

$$\hat{\theta}^T(t) = [\widehat{a_1}, \ldots, \widehat{a_n}, \widehat{c_1}, \ldots, \widehat{c_l}] \tag{16}$$

$$\phi^T = [-z(t-1), .., -z(t-n),\ \varepsilon(t-1), .., \varepsilon(t-l)]$$

Where

$$J(t) = \sum\nolimits_{i=0}^{t} \left[z(i) - \hat{\theta}^T(i)\phi(i-1)\right]^2 \tag{17}$$

The minimization of J gives the parametric adaptation of recursive least square algorithm:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1) \quad (18)$$

$$F(t+1)^{-1} = F(t)^{-1} + \phi(t)\phi(t)^T \quad (19)$$

$$F(t+1) = F(t) - \frac{F(t)\phi(t)\phi(t)^T F(t)}{1+\phi(t)^T F(t)\phi(t)} \quad (20)$$

$$\varepsilon(t+1) = \frac{z(t+1) - \hat{\theta}^T(t)\phi(t)}{1+\phi(t)^T F(t)\phi(t)} \quad (21)$$

With F(t) is the adaptation of the gain matrix.

We consider in this paper a structure under an unknown earthquake. For this, a simulated seismic signal is carried out which is a linear combination between the Kanai-Tajimi and the Clough-Penzien models (described by Eqs. (12) and (13) respectively) as follows (Fig. 11):

$$\ddot{X}_n = 1/2(G_1(s) + G_2(s))E(s) \quad (22)$$

In order to show the effect of the SSI, we compare both the structural and the ARMA model responses following the cases:

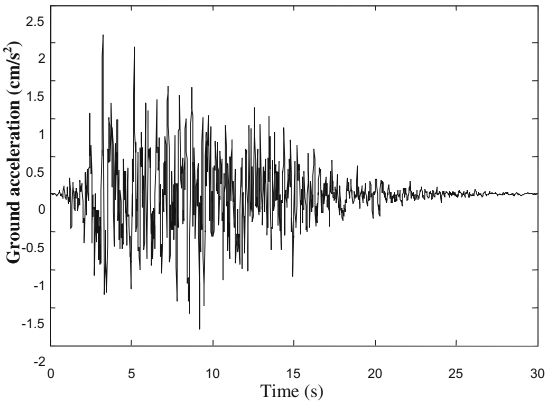

Fig. 11. Simulated earthquake ($\ddot{X}_n$)

Case 1: We compare the structural response model with the ARMA one described by Eq. (11), without introducing the soil characteristics (without considering SSI). Figure 12 shows the block diagram which calculates the error (ε) between the structural response and the ARMA model response with no control force (open loop).

Case 2: In this case, we have taken the estimated ARMA model with taking into account the soil-structure interaction. The soil characteristics had been identified by RLS algorithm and introduced in the structural model with no control force (U(s) = 0) as it is shown in Fig. 13.

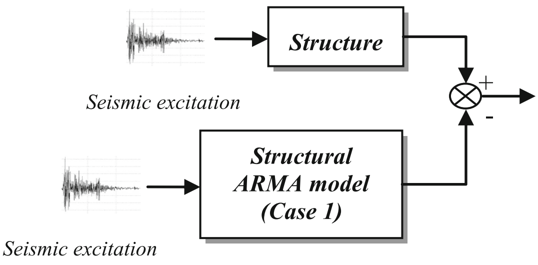

Fig. 12. Comparing model responses without considering soil-structure interaction (*Case 1*)

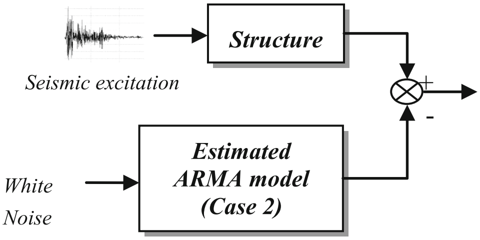

Fig. 13. Comparing model responses with considering soil-structure interaction (*Case 2*)

6 Simulation Results

A single-degree-of-freedom (SDOF) structure with the following structural properties is used [8, 9] m = 2921 kg, k = 1389kN/m, $\xi = 0.0124$. The sampling period Te = 0.02 s.

The Table 1 shows the error variance between structural response and the ARMAs models in different cases.

Table 1. Different error's variance

Cases	Case 1	Case 2
Errors variance ($\boldsymbol{\delta}^2$)	5.6524e-007	4.8807e-008

Parameters of ARMA model for the different cases are shown in the Table 2.

Simulations results have shown that the response without taking SSI into account is not close to the structural response as represented in Figs. 14 and 16. Whereas it is shown that the best results are obtained when we consider the soil characteristics (estimated model) as presented in Figs. 15 and 17.

Table 2. Parameters of ARMA models

	Case 1	Case 2
a_0	1	1
a_1	−1.803	−3.4003
a_2	0.9892	4.5455
a_3	–	−2.7995
a_4	–	0.6689
c_0	0	0
c_1	$-1.961*10^{-4}$	$0.2872*10^{-4}$
c_2	$-1.954*10^{-4}$	$0.8853*10^{-4}$
c_3	–	$-0.7033*10^{-4}$
c_4	–	$-0.2634*10^{-4}$

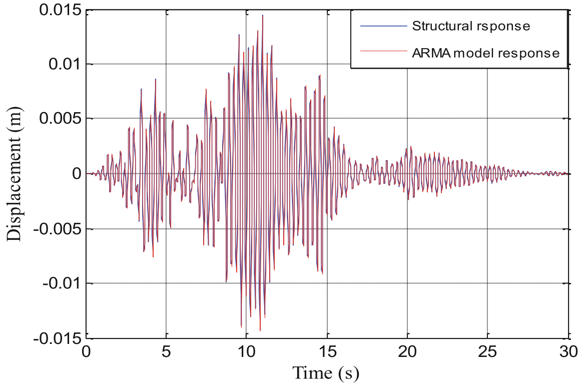

Fig. 14. Model responses under an unknown earthquake (Case 1).

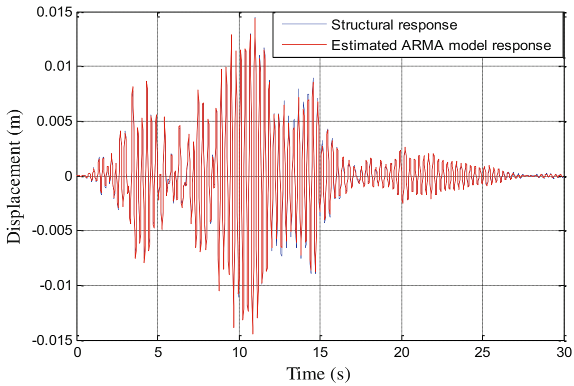

Fig. 15. Model responses under an unknown earthquake (Case 2).

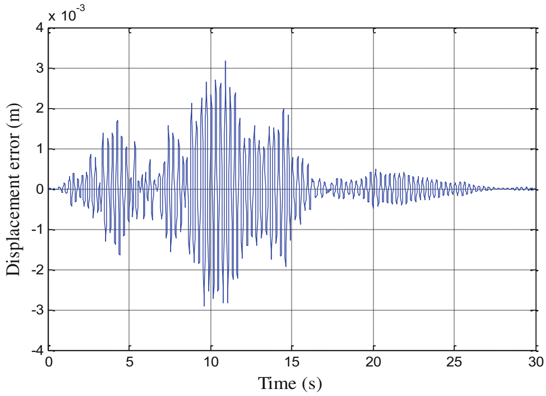

Fig. 16. Model error responses (Case 1).

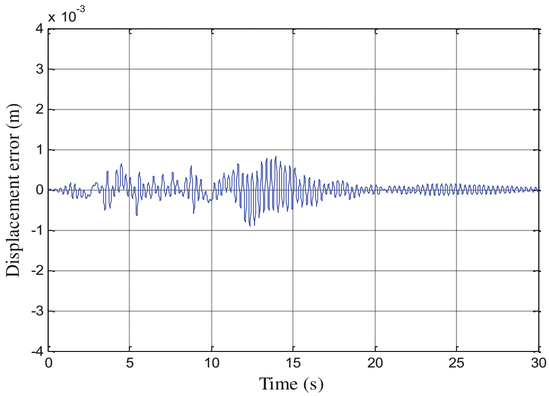

Fig. 17. Model error responses (Case 2).

7 Conclusions

The autoregressive moving average exogenous (ARMAX) of the structure has been developed in this paper. The Kanai-Tajimi and Clough-Penzien Seismic models have been presented and introduced in the dynamic of the structure. The earthquake signal identification study has been carried out and has given us the possibility to introduce the SSI within the structural ARMA model. We have chosen as example a structure built upon unknown soil. A simulated seismic signal has been formulated by a linear combination between Clough-Penzien and Kanai-Tajimi characteristics. The results leads us to conclude that the ARMA model with introducing SSI represents faithfully the complete structural model under a seismic ground motion.

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