

Adaptive Backstepping Control Using Combined Direct and Indirect σ -Modification Adaptation

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Abstract. In this paper, by using the dynamic surface control technique, an adaptive backstepping controller using combined direct and indirect σ -modification adaptation is proposed for a class of parametric strict-feedback systems. In this approach, a σ -modification parameter adaptation law that combines direct and indirect update laws is proposed. At first, the x -swapping identifier with a gradient-type update law is presented for a class of parametric strict-feedback nonlinear systems. Next, the main steps of the controller design for a class of nonlinear systems in parametric strict-feedback form are described. The closed-loop error dynamics is shown to be globally stable by using the Lyapunov stability approach. Finally, simulation results for a single-link flexible-joint robot manipulator are given to illustrate the tracking performance of the proposed adaptive control scheme.

Keywords: Backstepping control · Direct and indirect adaptive control · Adaptive dynamic surface control · Lyapunov stability · Flexible joint manipulators

1 Introduction

Backstepping has been a powerful method for synthesizing adaptive controllers for the class of nonlinear systems with linearly parameterized uncertainties [1]. The uncertainties are assumed to be linear in the unknown constant parameters [2]. The adaptive backstepping control techniques have been found to be particularly useful for controlling parametric strict-feedback nonlinear systems [3], which achieve boundedness of the closed-loop states and convergence of the tracking error to zero. However, adaptive backstepping control can result in overparametrization and adaptation laws differentiations [3], a significant drawback that can be eliminated by introducing tuning functions [4]. For nonlinear systems with parametric lower-triangular form, several adaptive approaches were also presented in [5].

Though, backstepping technique has become one of the most popular design methods for a large class of single-input single-output (SISO) nonlinear systems [3, 5].

A drawback in the traditional backstepping technique is the problem of “explosion of complexity” [2, 6, 7]. That is, the complexity of the controller grows drastically as the system order increases [1]. This problem is caused by the repeated differentiations of certain nonlinear functions such as virtual controls [2, 6, 7]. In [6], a procedure to deal with this problem for the non-adaptive case has been presented for a class of strict-feedback nonlinear systems, and it is called dynamic surface control (DSC). This problem is eliminated by introducing a first-order filtering of the synthetic virtual control input at each step of the traditional backstepping approach [1, 6–8]. In [2], authors are extending this technique to the adaptive control approach and it is called adaptive dynamic surface control.

The methodology proposed in this paper is an extension of the ideas presented in [9–11] for the adaptive backstepping design. This paper presents a new approach that combines direct and indirect σ -modification adaptation mechanism for adaptive backstepping control of parametric strict-feedback nonlinear systems. In fact, the tracking error based parameter adaptation law of the direct adaptive backstepping control with DSC [2, 11] will be combined with an identification error based parameter adaptation law of the indirect adaptive backstepping control [5, 11–14]. The combined adaptive law is introduced in order to achieve better parameter estimation and hence better tracking performance. The stability analysis of the closed-loop system is performed by using the Lyapunov stability theorem.

This paper is organized as follows. In Sect. 2, the identification based x -swapping is provided. The combined direct/indirect adaptive backstepping control with DSC is presented in Sect. 3. The stability analysis of the closed-loop system is given in Sect. 4. In Sect. 5, numerical example for a single-link flexible-joint robot manipulator is used to demonstrate the effectiveness of the proposed approach. Conclusion is contained in Sect. 6.

2 Identification Based x -Swapping

The goal of a swapping filter is to transform a dynamic parametric model into a static form, such that standard parameter estimation algorithms can be used. The term swapping describes the fact that the order of the transfer function describing the dynamics and the time varying parameter error $\tilde{\theta}$ is exchanged [14]. Two types of swapping schemes are presented in [5, 12–14], the z -swapping-based identifier derived from the tracking error model and the x -swapping-based identifier derived from the state dynamics [14]. Each of these two swapping-based identifiers allows application of gradient and least squares update laws. In this paper we use the gradient update law. To illustrate the x -swapping-based identifier procedure, we consider the following nonlinear system in parametric x -model [5, 12–14]

$$\dot{x}_i = f_i(x, u) + F_i^T(x, u)\theta_i \quad (1)$$

where

$$\begin{cases} f_i(x, u) = x_{i+1}, i = 1, \dots, n-1 \\ f_n(x, u) = u \end{cases} \quad (2)$$

$$F_i^T(x, u) = \varphi_i^T(\bar{x}_i), \bar{x}_i = [x_1 \quad x_2 \quad \dots \quad x_i]^T, i = 1, \dots, n$$

$\theta_i \in \mathbb{R}^{p_i}$.

Then, we introduce the following two filters

$$\dot{\Omega}_{0i} = A_i(x, t)(\Omega_{0i} + x_i) - f_i(x, u), \Omega_{0i} \in \mathbb{R} \quad (3)$$

$$\dot{\Omega}_i^T = A_i(x, t)\Omega_i^T + F_i^T(x, u), \Omega_i \in \mathbb{R}^{p_i} \quad (4)$$

where, $i = 1, \dots, n$, and $A_i(x, t) < 0$ is a negative definite matrix for each x continuous in t . We define the estimation error vector as

$$e_i = x_i + \Omega_{0i} - \Omega_i^T \hat{\theta}_i, e_i \in \mathbb{R} \quad (5)$$

with $\hat{\theta}_i$ the estimate of θ_i and let

$$\tilde{e}_i = x_i + \Omega_{0i} - \Omega_i^T \theta_i, \tilde{e}_i \in \mathbb{R} \quad (6)$$

Then, we obtain

$$e_i = \Omega_i^T \tilde{\theta}_i + \tilde{e}_i \quad (7)$$

The error signal \tilde{e}_i satisfies

$$\dot{\tilde{e}}_i = \dot{x}_i + \dot{\Omega}_{0i} - \dot{\Omega}_i^T \theta_i = A_i(x, t)\tilde{e}_i \quad (8)$$

To guarantee boundedness of Ω_i when $F_i(x, u)$ grows unbounded, a particular choice of $A_i(x, t)$ is made [5, 14]

$$A_i(x, t) = A_{0i} - \lambda_i F_i^T(x, u)F_i(x, u)P_i \quad (9)$$

where $\lambda_i > 0$ and A_{0i} is an arbitrary constant matrix satisfying

$$P_i A_{0i} + A_{0i}^T P_i = -I, P_i = P_i^T > 0 \quad (10)$$

The update law for $\hat{\theta}_i$ employs the estimation error e_i and the filtered regressor Ω_i . The gradient update law is given by

$$\dot{\hat{\theta}}_i = \Gamma_i \frac{\Omega_i e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}}, \Gamma_i = \Gamma_i^T > 0, v_i \geq 0 \quad (11)$$

where, $i = 1, \dots, n$.

To establish the identifier properties, let $[0, t_f)$ be the maximal interval of existence of solutions of (1), the x -swapping filters (3) and (4), and the gradient update law (11). Then for $v_i \geq 0$ the following properties hold [5, 12–14]

- (i) $\tilde{\theta}_i \in L_\infty[0, t_f)$
- (ii) $e_i \in L_2[0, t_f) \cap L_\infty[0, t_f)$
- (iii) $\dot{\hat{\theta}}_i \in L_2[0, t_f) \cap L_\infty[0, t_f)$

We consider the following Lyapunov function

$$V_i = \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \tilde{z}_i^T P_i \tilde{z}_i \quad (12)$$

Along of Eqs. (8) and (11), the derivative of the Lyapunov function (12) is

$$\begin{aligned} \dot{V}_i &= \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i + \dot{\tilde{z}}_i^T P_i \tilde{z}_i + \tilde{z}_i^T P_i \dot{\tilde{z}}_i \\ &= \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i + \tilde{z}_i^T A_i^T(x, t) P_i \tilde{z}_i + \tilde{z}_i^T P_i A_i(x, t) \tilde{z}_i \\ &\leq -\tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i - \tilde{z}_i^T \tilde{z}_i = -\frac{\tilde{\theta}_i^T \Omega_i e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}} - \tilde{z}_i^T \tilde{z}_i \\ &= -\frac{e_i^T e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}} + \frac{e_i^T}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}} \tilde{z}_i - \tilde{z}_i^T \tilde{z}_i \\ &\leq -\frac{3}{4} \frac{e_i^T e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}} - \frac{1}{4} \frac{e_i^T e_i}{(1 + v_i \text{tr}\{\Omega_i^T \Omega_i\})^2} + \frac{e_i^T}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}} \tilde{z}_i - \tilde{z}_i^T \tilde{z}_i \\ &= -\frac{3}{4} \frac{e_i^T e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}} - \left(\frac{e_i}{2(1 + v_i \text{tr}\{\Omega_i^T \Omega_i\})} - \tilde{z}_i \right)^T \left(\frac{e_i}{2(1 + v_i \text{tr}\{\Omega_i^T \Omega_i\})} - \tilde{z}_i \right) \\ &\leq -\frac{3}{4} \frac{e_i^T e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}}, i = 1, \dots, n \end{aligned} \quad (13)$$

Since \dot{V}_i is negative semi-definite, one has $\tilde{\theta}_i \in L_\infty[0, t_f)$. From $e_i = \Omega_i^T \tilde{\theta}_i + \tilde{z}_i$ and the boundedness of Ω_i , one concludes that e_i and $\dot{\hat{\theta}}_i \in L_2[0, t_f) \cap L_\infty[0, t_f)$.

3 Direct/Indirect Adaptive Backstepping Control with DSC

In the direct/indirect adaptive backstepping control with DSC procedure, the control law and the parameter estimation are not separated. In this paper, the parameter update law for $\hat{\theta}_i$ combine gradient-type update laws based on the x -swapping identifier and tracking error based update laws. The control objective is to achieve the asymptotic tracking of a reference signal y_r by x_1 . The reference signal y_r and its derivatives $\dot{y}_r, \dots, y_r^{(n)}$ are assumed piecewise continuous and bounded. In the following, we

describe the main steps of the controller design for the nonlinear system in parametric strict-feedback form

$$\begin{aligned}\dot{x}_1 &= x_2 + \varphi_1^T(x_1)\theta_1 \\ \dot{x}_i &= x_{i+1} + \varphi_i^T(\bar{x}_i)\theta_i, i = 2, \dots, n-1 \\ \dot{x}_n &= u + \varphi_n^T(x)\theta_n\end{aligned}\quad (14)$$

where, $x = [x_1 \ x_2 \ \dots \ x_n]^T \in \mathbb{R}^n$ and $u \in \mathbb{R}$ are the state variables vector and the input of the system, respectively. $\theta_i \in \mathbb{R}^{p_i}$ are unknown constant parameter vectors, $\bar{x}_i = [x_2 \ x_3 \ \dots \ x_i]^T$. The nonlinear functions $\varphi_i^T(\bar{x}_i) : \mathbb{R}^i \rightarrow \mathbb{R}^{p_i}$ are known.

Step 1 ($i = 1$)

The first surface is defined by $S_1 = x_1 - x_{1d}$, and its time derivative is given by

$$\dot{S}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 + \varphi_1^T(x_1)\theta_1 - \dot{x}_{1d}\quad (15)$$

we choose \bar{x}_2 to drive S_1 towards zero with

$$\bar{x}_2 = -\varphi_1^T(x_1)\hat{\theta}_1 + \dot{x}_{1d} - K_1 S_1\quad (16)$$

we pass \bar{x}_2 through a first order filter, with time constant τ_2 , to obtain x_{2d}

$$\tau_2 \dot{x}_{2d} + x_{2d} = \bar{x}_2, x_{2d}(0) = \bar{x}_2(0)\quad (17)$$

$$\dot{x}_{2d} = \frac{1}{\tau_2} \left(-x_{2d} - \varphi_1^T(x_1)\hat{\theta}_1 + \dot{x}_{1d} - K_1 S_1 \right)\quad (18)$$

Step i ($i = 2, \dots, n-1$)

The i^{th} surface is defined by $S_i = x_i - x_{id}$, and its time derivative is given by

$$\dot{S}_i = \dot{x}_i - \dot{x}_{id} = x_{i+1} + \varphi_i^T(\bar{x}_i)\theta_i - \dot{x}_{id}\quad (19)$$

we choose \bar{x}_{i+1} to drive S_i towards zero with

$$\bar{x}_{i+1} = -\varphi_i^T(\bar{x}_i)\hat{\theta}_i + \dot{x}_{id} - K_i S_i\quad (20)$$

we pass \bar{x}_{i+1} through a first order filter, with time constant τ_{i+1} , to obtain x_{i+1d}

$$\tau_{i+1} \dot{x}_{i+1d} + x_{i+1d} = \bar{x}_{i+1}, x_{i+1d}(0) = \bar{x}_{i+1}(0)\quad (21)$$

$$\dot{x}_{i+1d} = \frac{1}{\tau_{i+1}} \left(-x_{i+1d} - \varphi_i^T(\bar{x}_i)\hat{\theta}_i + \dot{x}_{id} - K_i S_i \right)\quad (22)$$

Step n

The n^{th} surface is defined by $S_n = x_n - x_{nd}$, and its time derivative is given by

$$\dot{S}_n = \dot{x}_n - \dot{x}_{nd} = u + \varphi_n^T(x)\theta_n - \dot{x}_{nd} \quad (23)$$

we choose the control input u to drive S_n towards zero with

$$u = -\varphi_n^T(x)\hat{\theta}_n + \dot{x}_{nd} - K_n S_n \quad (24)$$

The update laws (direct part) for the parameter estimates are given by [2, 11]

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \bar{\Gamma}_1 S_1 \varphi_1(x_1) \\ \dot{\hat{\theta}}_i &= \bar{\Gamma}_i S_i \varphi_i(\bar{x}_i), i = 2, \dots, n-1 \\ \dot{\hat{\theta}}_n &= \bar{\Gamma}_n S_n \varphi_n(x) \end{aligned} \quad (25)$$

where, $\bar{\Gamma}_i > 0 (i = 1, \dots, n)$ are design parameters that can be adjusted for the rate of convergence of the parameter estimates.

Let us introduce the following two filters

$$\dot{\Omega}_{0i} = A_i(x, t)(\Omega_{0i} + x_i) - f_i(x, u), \Omega_{0i} \in \mathbb{R} \quad (26)$$

$$\dot{\Omega}_i^T = A_i(x, t)\Omega_i^T + F_i^T(x, u), \Omega_i \in \mathbb{R}^{p_i} \quad (27)$$

where, $i = 1, \dots, n$, and

$$A_i(x, t) = A_{0i} - \lambda_i F_i^T(x, u)F_i(x, u)P_i \quad (28)$$

where $\lambda_i > 0$ and A_{0i} is an arbitrary constant matrix satisfying

$$P_i A_{0i} + A_{0i}^T P_i = -I, P_i = P_i^T > 0 \quad (29)$$

The gradient update law (indirect part) is given by [5, 11–14]

$$\dot{\hat{\theta}}_i = \Gamma_i \frac{\Omega_i e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}}, \Gamma_i = \Gamma_i^T > 0, v_i \geq 0 \quad (30)$$

where, $i = 1, \dots, n$ and $e_i = x_i + \Omega_{0i} - \Omega_i^T \hat{\theta}_i, e_i \in \mathbb{R}$.

Now we propose the following combined direct and indirect σ -modification adaptation law [11]

$$\dot{\hat{\theta}}_i = \bar{\Gamma}_i S_i \varphi_i(\bar{x}_i) - \bar{\Gamma}_i \sigma_i (\hat{\theta}_i - \bar{\theta}_i), \bar{\Gamma}_i > 0 \quad (31)$$

where, $i = 1, \dots, n$, σ_i is a small positive constant, $\bar{\Gamma}_i$ is a positive definite constant matrix and $\bar{\theta}_i$ is computed with the gradient method as follows

$$\dot{\tilde{\theta}}_i = \Gamma_i \frac{\Omega_i e_i}{1 + v_i \text{tr}\{\Omega_i^T \Omega_i\}}, \Gamma_i = \Gamma_i^T > 0, v_i \geq 0 \quad (32)$$

where, Γ_i is a positive definite constant matrix and $e_i = x_i + \Omega_{0i} - \Omega_i^T \tilde{\theta}_i$, $e_i \in \mathbb{R}$.

4 Stability Analysis

We define the boundary layer error as [2, 11, 15]

$$y_i = x_{id} - \bar{x}_i, i = 2, \dots, n \quad (33)$$

and the parameter estimate errors as

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i, i = 1, 2, \dots, n \quad (34)$$

Then the closed-loop dynamics can be expressed in terms of the surfaces S_i , the boundary layer errors y_i , and the parameter estimate errors $\tilde{\theta}_i$.

The dynamics of the surfaces are expressed, for $i = 1$, as

$$\begin{aligned} \dot{S}_1 &= \dot{x}_1 - \dot{x}_{1d} = x_2 + \varphi_1^T(x_1)\theta_1 - \dot{x}_{1d} \\ &= S_2 + x_{2d} + \varphi_1^T(x_1)\theta_1 - \dot{x}_{1d} \\ &= S_2 + y_2 + \bar{x}_2 + \varphi_1^T(x_1)\theta_1 - \dot{x}_{1d} \\ &= S_2 + y_2 - K_1 S_1 + \varphi_1^T(x_1)\tilde{\theta}_1 \end{aligned} \quad (35)$$

For $i = 2, \dots, n-1$

$$\begin{aligned} \dot{S}_i &= \dot{x}_i - \dot{x}_{id} = x_{i+1} + \varphi_i^T(\bar{x}_i)\theta_i - \dot{x}_{id} \\ &= S_{i+1} + x_{i+1d} + \varphi_i^T(\bar{x}_i)\theta_i - \dot{x}_{id} \\ &= S_{i+1} + y_{i+1} + \bar{x}_{i+1} + \varphi_i^T(\bar{x}_i)\theta_i - \dot{x}_{id} \\ &= S_{i+1} + y_{i+1} - K_i S_i + \varphi_i^T(\bar{x}_i)\tilde{\theta}_i \end{aligned} \quad (36)$$

For $i = n$

$$\dot{S}_n = \dot{x}_n - \dot{x}_{nd} = u + \varphi_n^T(x)\theta_n - \dot{x}_{nd} = -K_n S_n + \varphi_n^T(x)\tilde{\theta}_n \quad (37)$$

The dynamics of the boundary layer errors y_i are expressed, for $i = 2$, as

$$\begin{aligned} \dot{y}_2 &= \dot{x}_{2d} - \dot{\bar{x}}_2 = \frac{1}{\tau_2} \left(-x_{2d} - \varphi_1^T(x_1)\hat{\theta}_1 + \dot{x}_{1d} - K_1 S_1 \right) - \dot{\bar{x}}_2 \\ &= \frac{1}{\tau_2} (-x_{2d} + \bar{x}_2) - \dot{\bar{x}}_2 = \frac{1}{\tau_2} (-y_2 - \bar{x}_2 + \bar{x}_2) - \dot{\bar{x}}_2 = -\frac{1}{\tau_2} y_2 - \dot{\bar{x}}_2 \end{aligned} \quad (38)$$

For $i = 3, \dots, n$

$$\begin{aligned}\dot{y}_i &= \dot{x}_{id} - \dot{\bar{x}}_i = \frac{1}{\tau_i} \left(-x_{id} - \varphi_{i-1}^T(\bar{x}_{i-1}) \hat{\theta}_{i-1} + \dot{x}_{i-1d} - K_{i-1} S_{i-1} \right) - \dot{\bar{x}}_i \\ &= \frac{1}{\tau_i} (-x_{id} + \bar{x}_i) - \dot{\bar{x}}_i = \frac{1}{\tau_i} (-y_i - \bar{x}_i + \bar{x}_i) - \dot{\bar{x}}_i = -\frac{1}{\tau_i} y_i - \dot{\bar{x}}_i\end{aligned}\quad (39)$$

Let us consider the following Lyapunov function

$$V = \sum_{i=1}^n V_{is} + \sum_{i=2}^n V_{iy} + \sum_{i=1}^n V_i \quad (40)$$

where

$$V_{is} = \frac{1}{2} S_i^2, V_{iy} = \frac{1}{2} y_i^2, V_i = \frac{1}{2} \tilde{\theta}_i^T \bar{\Gamma}_i^{-1} \tilde{\theta}_i \quad (41)$$

Then, one has

$$\begin{aligned}\dot{V}_{is} &= S_i \dot{S}_i = S_i S_{i+1} + S_i y_{i+1} - K_i S_i^2 + S_i \varphi_i^T(\bar{x}_i) \tilde{\theta}_i \\ &\leq |S_i| |S_{i+1}| + |S_i| |y_{i+1}| - K_i S_i^2 + S_i \varphi_i^T(\bar{x}_i) \tilde{\theta}_i \\ &\leq (1 - K_i) S_i^2 + \frac{1}{2} S_{i+1}^2 + \frac{1}{2} y_{i+1}^2 + S_i \varphi_i^T(\bar{x}_i) \tilde{\theta}_i, i = 1, \dots, n-1\end{aligned}\quad (42)$$

$$\dot{V}_{ns} = S_n \dot{S}_n = -K_n S_n^2 + S_n \varphi_n^T(x) \tilde{\theta}_n \leq -K_n S_n^2 + S_n \varphi_n^T(x) \tilde{\theta}_n \quad (43)$$

We assume that, $|y_i \dot{\bar{x}}_i| \leq M_{1i} y_i^2 + M_{2i} S_i^2 + \delta_i^2$, where, M_{1i} and M_{2i} are positive constants, and δ_i are bounded functions, then, we can write

$$\begin{aligned}\dot{V}_{iy} &= y_i \dot{y}_i = -\frac{1}{\tau_i} y_i^2 + y_i \dot{\bar{x}}_i \leq -\frac{1}{\tau_i} y_i^2 + |y_i \dot{\bar{x}}_i| \\ &\leq -\frac{1}{\tau_i} y_i^2 + M_{1i} y_i^2 + M_{2i} S_i^2 + \delta_i^2, i = 2, \dots, n\end{aligned}\quad (44)$$

$$\begin{aligned}\dot{V}_i &= \tilde{\theta}_i^T \bar{\Gamma}_i^{-1} \dot{\tilde{\theta}}_i = -\tilde{\theta}_i^T \bar{\Gamma}_i^{-1} \dot{\tilde{\theta}}_i \\ &= -\tilde{\theta}_i^T \bar{\Gamma}_i^{-1} \left(\bar{\Gamma}_i S_i \varphi_i(\bar{x}_i) - \bar{\Gamma}_i \sigma_i(\hat{\theta}_i - \bar{\theta}_i) \right) \\ &= -\tilde{\theta}_i^T S_i \varphi_i(\bar{x}_i) + \tilde{\theta}_i^T \sigma_i(\hat{\theta}_i - \bar{\theta}_i), i = 1, \dots, n\end{aligned}\quad (45)$$

One has: $\theta_i - \bar{\theta}_i$ is bounded, thus: $e_{\theta_i} = \theta_i - \bar{\theta}_i$ is bounded, $\bar{\theta}_i = \theta_i - e_{\theta_i}$ and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

$$\begin{aligned}
\dot{V}_i &= -\tilde{\theta}_i^T S_i \varphi_i(\bar{x}_i) + \tilde{\theta}_i^T \sigma_i (\hat{\theta}_i - \theta_i + e_{\theta_i}) \\
&= -\tilde{\theta}_i^T S_i \varphi_i(\bar{x}_i) - \sigma_i \tilde{\theta}_i^T \tilde{\theta}_i + \sigma_i \tilde{\theta}_i^T e_{\theta_i} \\
&\leq -\tilde{\theta}_i^T S_i \varphi_i(\bar{x}_i) - \sigma_i \tilde{\theta}_i^T \tilde{\theta}_i + \frac{\sigma_i}{2} \tilde{\theta}_i^T \tilde{\theta}_i + \frac{\sigma_i}{2} e_{\theta_i}^T e_{\theta_i} \\
&\leq -\tilde{\theta}_i^T S_i \varphi_i(\bar{x}_i) - \frac{\sigma_i}{2} \tilde{\theta}_i^T \tilde{\theta}_i + \frac{\sigma_i}{2} e_{\theta_i}^T e_{\theta_i}
\end{aligned} \tag{46}$$

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^n \dot{V}_{is} + \sum_{i=2}^n \dot{V}_{iy} + \sum_{i=1}^n \dot{V}_i \\
&\leq \sum_{i=1}^{n-1} \left[(1 - K_i) S_i^2 + \frac{1}{2} S_{i+1}^2 + \frac{1}{2} y_{i+1}^2 + S_i \varphi_i^T(\bar{x}_i) \tilde{\theta}_i \right] - K_n S_n^2 + S_n \varphi_n^T(x) \tilde{\theta}_n \\
&\quad + \sum_{i=2}^n \left(-\frac{1}{\tau_i} y_i^2 + M_{1i} y_i^2 + M_{2i} S_i^2 + \delta_i^2 \right) - \sum_{i=1}^n \tilde{\theta}_i^T S_i \varphi_i(\bar{x}_i) - \sum_{i=1}^n \frac{\sigma_i}{2} \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{i=1}^n \frac{\sigma_i}{2} e_{\theta_i}^T e_{\theta_i} \\
&\leq -(K_1 - 1) S_1^2 - \sum_{i=2}^{n-1} \left(K_i - M_{2i} - \frac{3}{2} \right) S_i^2 - \left(K_n - M_{2n} - \frac{1}{2} \right) S_n^2 - \sum_{i=2}^n \left(\frac{1}{\tau_i} - \frac{1}{2} - M_{1i} \right) y_i^2 \\
&\quad + \sum_{i=2}^n \delta_i^2 - \sum_{i=1}^n \frac{\sigma_i}{2} \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{i=1}^n \frac{\sigma_i}{2} e_{\theta_i}^T e_{\theta_i}
\end{aligned} \tag{47}$$

If we assume that, $\delta_i \in L_\infty$, $e_{\theta_i} \in L_\infty$, $K_1 > 1$, $K_i > M_{2i} + \frac{3}{2}$, $K_n > M_{2n} + \frac{1}{2}$ and $\frac{1}{\tau_i} > \frac{1}{2} + M_{1i}$, we obtain the boundedness of all signals S_i , y_i and $\tilde{\theta}_i$. Moreover, the surface S_i can be made arbitrarily small by adjusting the design parameters K_i .

5 Numerical Example

Consider the single-link flexible-joint robot shown in Fig. 1. The dynamic model of this robot is given as follow [16, 17]

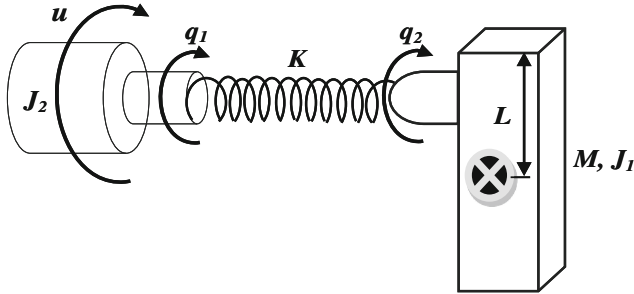


Fig. 1. Single link flexible joint robot.

$$\begin{aligned} J_1 \ddot{q}_1 + MgL \sin(q_1) + K(q_1 - q_2) &= 0 \\ J_2 \ddot{q}_2 - K(q_1 - q_2) &= u \end{aligned} \quad (48)$$

where u is the input torque. J_1 and J_2 are the inertias of the link and the motor respectively. M is the link mass. g is the gravity. L is the link length. K is the stiffness. q_1 and q_2 are the angular positions of the link and the motor shaft, respectively.

Let the state variables defined as follows: $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$ and $x_4 = \dot{q}_2$, and its dynamic model becomes

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(x_1, x_3) + g_1 x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(x_1, x_3) + g_2 u \end{aligned} \quad (49)$$

with

$$\begin{aligned} f_1(x_1, x_3) &= -\frac{MgL}{J_1} \sin(x_1) - \frac{K}{J_1} x_1, g_1 = \frac{K}{J_1} \\ f_2(x_1, x_3) &= \frac{K}{J_2} (x_1 - x_3), g_2 = \frac{1}{J_2} \end{aligned} \quad (50)$$

The single-link flexible-joint robot model used in this paper is given by (49) where the parameter values are given in Table 1 [17].

For the numerical simulation, the unknown parameter θ_1 of the system is selected as $\theta_1 = MgL$. Our objective is to force the output of the system to follow the reference trajectory given by: $y_d = 0.1 \sin(t)$.

We choose \bar{x}_2 , \bar{x}_3 and \bar{x}_4 to drive S_1 , S_2 and S_3 towards zero with

$$\bar{x}_2 = \dot{x}_{1d} - K_1 S_1 = \tau_2 \dot{x}_{2d} + x_{2d}, x_{2d}(0) = \bar{x}_2(0) \quad (51)$$

$$\dot{x}_{2d} = \frac{1}{\tau_2} (-x_{2d} + \dot{x}_{1d} - K_1 S_1) \quad (52)$$

Table 1. Single-link flexible-joint robot model parameters.

Symbols	Values	Units
g	9, 81	$[m/s^2]$
M	1	$[kg]$
L	1	$[m]$
J_1	0.4	$[kg.m^2]$
J_2	0.02	$[kg.m^2]$
K	100	$[N.m/rad]$

$$\bar{x}_3 = \frac{1}{g_1} \left(\frac{\hat{\theta}_1}{J_1} \sin(x_1) + \frac{K}{J_1} x_1 + \dot{x}_{2d} - K_2 S_2 \right) = \tau_3 \dot{x}_{3d} + x_{3d}, x_{3d}(0) = \bar{x}_3(0) \quad (53)$$

$$\dot{x}_{3d} = \frac{1}{\tau_3} \left(-x_{3d} + \frac{1}{g_1} \left(\frac{\hat{\theta}_1}{J_1} \sin(x_1) + \frac{K}{J_1} x_1 + \dot{x}_{2d} - K_2 S_2 \right) \right) \quad (54)$$

$$\bar{x}_4 = \dot{x}_{3d} - K_3 S_3 = \tau_4 \dot{x}_{4d} + x_{4d}, x_{4d}(0) = \bar{x}_4(0) \quad (55)$$

$$\dot{x}_{4d} = \frac{1}{\tau_4} (-x_{4d} + \dot{x}_{3d} - K_3 S_3) \quad (56)$$

We choose the control u to drive S_4 towards zero with

$$u = \frac{1}{g_2} \left(-\frac{K}{J_2} (x_1 - x_3) + \dot{x}_{4d} - K_4 S_4 \right) \quad (57)$$

For the swapping-based identifier we use the following filters

$$\dot{\Omega}_0 = \left(-\frac{1}{2} - \lambda \left(-\frac{\sin(x_1)}{J_1} \right)^2 \right) (\Omega_0 + x_2) + \frac{K}{J_1} x_1 - g_1 x_3 \quad (58)$$

$$\dot{\Omega}^T = \left(-\frac{1}{2} - \lambda \left(-\frac{\sin(x_1)}{J_1} \right)^2 \right) \Omega^T - \frac{\sin(x_1)}{J_1} \quad (59)$$

The combined direct and indirect σ -modification adaptation law is given by

$$\dot{\hat{\theta}}_1 = -\frac{\bar{\Gamma}}{J_1} S_2 \sin(x_1) - \bar{\Gamma} \sigma (\hat{\theta}_1 - \bar{\theta}), \bar{\Gamma} > 0 \quad (60)$$

where, $\bar{\theta}$ is computed with the gradient method as

$$\dot{\bar{\theta}} = \Gamma \frac{\Omega e}{1 + \text{vtr}\{\Omega^T \Omega\}}, \Gamma = \Gamma^T > 0, v \geq 0. \quad (61)$$

where, $e = x_2 + \Omega_0 - \Omega^T \bar{\theta}$.

The selected initial conditions are: $x(0) = [0.1 \ 0 \ 0.1 + \frac{MgL}{K} \sin(0.1) \ 0]^T$, $\hat{\theta}_1(0) = \bar{\theta}(0) = 0$ and $\Omega_0(0) = \Omega^T(0) = 0$. The design parameters are selected as follows: $K_1 = 1$, $K_2 = 80$, $K_3 = 10$, $K_4 = 100$, $\bar{\Gamma} = 15$, $\sigma = 0.5$, $\Gamma = 20$, $\tau_2 = \tau_3 = \tau_4 = 0.009$ and $\lambda = v = 0.1$.

Numerical simulation results are shown in Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11. Figures 2, 3, 4 and 5 show actual and desired trajectories of the angular position and velocity of the link and the motor shaft. Figures 6, 7, 8 and 9 show the trajectories of the surfaces. Figure 10 shows the trajectory of the control input signal u . Figure 11

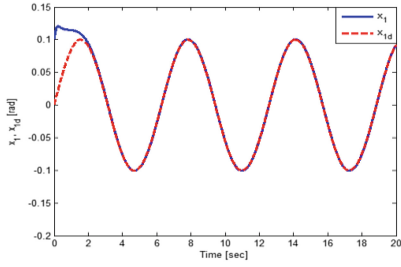


Fig. 2. Angular position of the link: actual x_1 (“—”) and desired x_{1d} (“- -”).

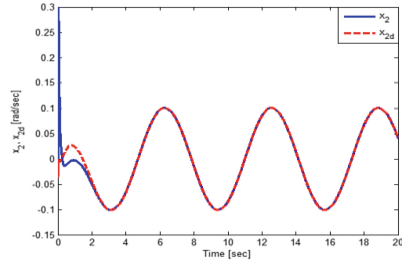


Fig. 3. Angular velocity of the link: actual x_2 (“—”) and desired x_{2d} (“- -”).

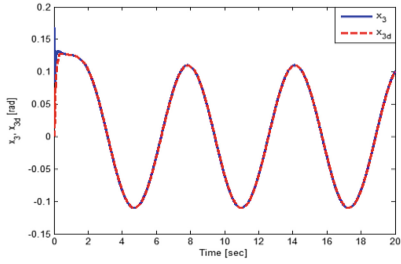


Fig. 4. Angular position of the motor shaft: actual x_3 (“—”) and desired x_{3d} (“- -”).

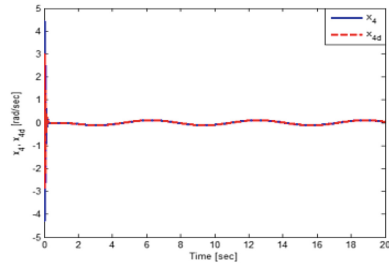


Fig. 5. Angular velocity of the motor shaft: actual x_4 (“—”) and desired x_{4d} (“- -”).

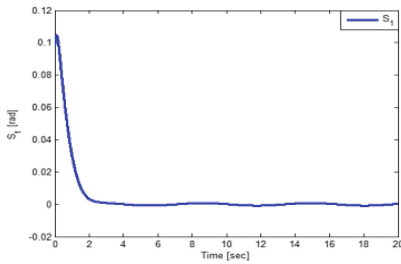


Fig. 6. Surface S_1 .

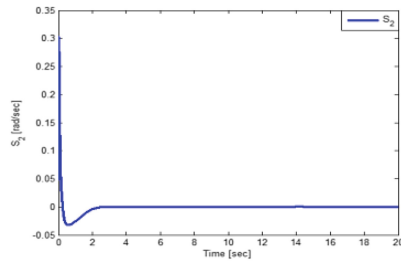
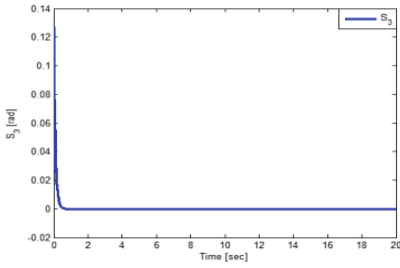
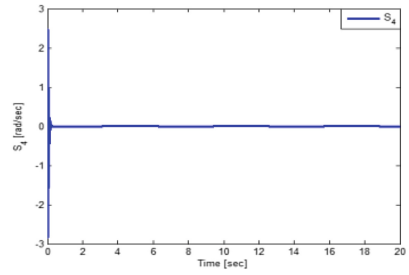
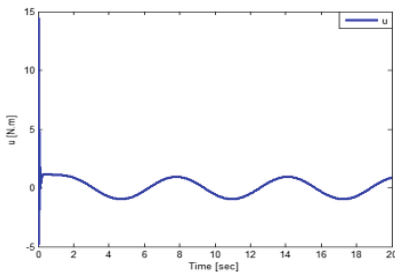
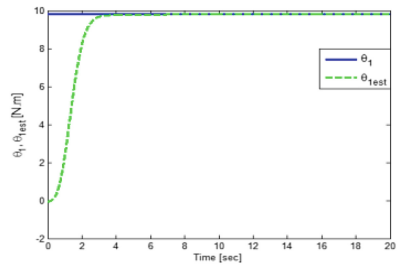


Fig. 7. Surface S_2 .

shows the trajectory of the estimated parameter $\hat{\theta}_1$. From these results, we observe that the actual trajectories converge towards the desired trajectories, and that the errors converge to zero and the estimated parameter $\hat{\theta}_1$ converge towards θ_1 . We can see that the results show that the proposed method has good tracking performance.

Fig. 8. Surface S_3 .Fig. 9. Surface S_4 .Fig. 10. Control input u .Fig. 11. Estimated parameter $\hat{\theta}_1$.

6 Conclusion

In this paper, adaptive backstepping control using combined direct and indirect σ -modification adaptation based gradient update law is designed using the DSC technique for a class of parametric strict-feedback nonlinear systems. The proposed approach eliminates the problem of explosion of complexity of the traditional backstepping approach. Stability analysis shows that the uniform ultimate boundedness of all signals in the closed-loop system can be guaranteed, and the tracking error can be made arbitrarily small by adjusting the control design parameters. Numerical simulation results demonstrate the effectiveness of the proposed approach.

References

1. Sun, G., Wang, D., Li, X., Peng, Z.: A DSC approach to adaptive neural network tracking control for pure-feedback nonlinear systems. *Appl. Math. Comput.* **219**(11), 6224–6235 (2013)
2. Yip, P.P., Hedrick, J.K.: Adaptive dynamic surface control: a simplified algorithm for adaptive backstepping control of nonlinear systems. *Int. J. Control* **71**(5), 959–979 (1998)
3. Kanellakopoulos, I., Kokotovic, P.V., Morse, A.S.: Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Trans. Autom. Control* **36**(11), 1241–1253 (1991)

4. Krstic, M., Kanellakopoulos, I., Kokotovic, P.V.: Adaptive nonlinear control without overparametrization. *Syst. Control Lett. North-Holland* **19**(3), 177–185 (1992)
5. Krstic, M., Kanellakopoulos, I., Kokotovic, P.V.: *Nonlinear and Adaptive Control Design*. Wiley-Interscience Publication, New York (1995)
6. Swaroop, D., Gerdes, J.C., Yip, P.P., Hedrick, J.K.: Dynamic surface control of nonlinear systems. In: *Proceedings of the American Control Conference*, Albuquerque, New Mexico, pp. 3028–3034 (1997)
7. Swaroop, D., Hedrick, J.K., Yip, P.P., Gerdes, J.C.: Dynamic surface control for a class of nonlinear systems. *IEEE Trans. Autom. Control* **45**(10), 1893–1899 (2000)
8. Zhang, T.P., Ge, S.S.: Adaptive dynamic surface control of nonlinear systems with unknown dead zone in pure feedback form. *Automatica* **44**(7), 1895–1903 (2008)
9. Soukkou, Y., Labiod, S.: Adaptive backstepping control using combined direct and indirect adaptation based gradient update law for a single-link flexible-joint robot. In: *5th International Conference on Electrical Engineering and Automatic Control*, Setif, Algeria, 24–26 November 2013
10. Soukkou, Y., Labiod, S.: Adaptive backstepping control using combined direct and indirect adaptation based least squares update law for a single-link flexible-joint robot. In: *Proceedings of the 3rd International Conference on Industrial Engineering and Manufacturing*, Batna, Algeria, pp. 272–279, 11–13 May 2014
11. Soukkou, Y.: *Commande adaptative par backstepping d'une classe des systèmes non linéaires incertains*. Magister Thesis, University of Jijel (2014)
12. Krstic, M., Kokotovic, P.V.: Adaptive nonlinear control with nonlinear swapping. In: *Proceedings of the 32nd Conference on Decision and Control*, San Antonio, Texas (1993)
13. Krstic, M., Kokotovic, P.V.: Adaptive nonlinear design with controller-identifier separation and swapping. *IEEE Trans. Autom. Control* **40**(3), 426–440 (1995)
14. Oort, E.R.V.: *Adaptive backstepping control and safety analysis for modern fighter aircraft*. Ph.D. Thesis, Delft University of Technology, Netherlands (2011)
15. Hou, M.Z., Duan, G.R.: Robust adaptive dynamic surface control of uncertain nonlinear systems. *Int. J. Control Autom. Syst.* **9**(1), 161–168 (2011)
16. Li, Y., Tong, S., Li, T.: Fuzzy adaptive dynamic surface control for a single link flexible joint robot. *Nonlinear Dyn.* **70**(3), 2035–2048 (2012)
17. Nicosia, S., Tomei, P.: A tracking controller flexible joint robots using only link position feedback. *IEEE Trans. Autom. Control* **40**(5), 885–890 (1995)