Constrained Fuzzy Predictive Control Design Based on the PDC Approach

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Abstract. In this paper, we are considering a model based T-S fuzzy predictive control using LMI optimization. The purpose of T-S fuzzy predictive control law is to drive the state of the system to the original state. Adopting the PDC controller and using non quadratic case of the Lyapunov function to study the stability of the controlled systems were ensuring with the stabilizing controller. The stability is guaranteed based on the conditions expressed of terms of LMIs. In addition, input and output constraints of the fuzzy system are satisfied with the PDC controller. Where, the optimal solution has been obtained at each sampling time. The simulations results are show the effectiveness of this approach.

Keywords: Parallel distributed compensation PDC · Model predictive control (MPC) · Takagi-Sugeno (T-S) fuzzy systems · Linear matrix inequality (LMI)

1 Introduction

Constrained fuzzy model predictive control become among efficient techniques in control for its tolerance, and admiration of imposed constraints. MPC is based to use a model for the prediction of future behavior of the system [1]. A constrained optimal control problem is solved at each sampling instant in online MPC approaches; several schemes are offered to put ideas for adopting in online optimization for the control of medium and high speed systems [2–4]. Relaxed conditions in form of LMIs are introduced in [5]. This form is usually used for studying robustness and stability of fuzzy systems which are analyzing using Lyapunov function for the both cases: quadratic [6], and non-quadratic case for discrete time fuzzy systems [7].

The Lyapunov function is used to study the stability problems [8, 9] of optimization was employed and becomes the most using techniques to analysis stability, where fuzzy techniques are adopted for the optimization in MPC for nonlinear systems [10–14].

In addition, the fuzzy control law parallel distributed compensation (PDC) has been implemented during the last three decades [6, 15, 16]. This approach is based on

M. Chadli et al. (eds.), *Recent Advances in Electrical Engineering and Control Applications*, Lecture Notes in Electrical Engineering 411, DOI 10.1007/978-3-319-48929-2_11

quadratic Lyapunov function and becomes the most favorite control law has been applying for fuzzy systems.

In this work, we consider to study an optimal linear control law non quadratic based on the conception of model predictive control for discrete T-S model [14]. The main idea is to calculate the gains of the control law by solving the optimization of the LMIs constrained problem at each sampling time by correct some errors in [14], even in presence of extern uncertainties. The stability and also the robustness are ensured. The results show the effectiveness of the studying control law by stabilizing the constrained systems.

This paper is organized as follows: Sect. 2 introduces the notation and some preliminary results of the basic elements, MPC, T-S fuzzy system. Section 3 presents the proposed strategy to obtain control law for regulation of the closed-loop system. In Sect. 4, simulation results are presented.

2 Backgrounds

2.1 Model Predictive Control

Let us consider the following problem, which minimizes the following objective function in an infinite horizon [5]:

$$\min_{\mathrm{u(k+i/k)=F(x)x(k+i/k)}} \max_{\mathrm{i>0}} \mathrm{J}_{\infty}(\mathrm{k}) \tag{1}$$

$$\begin{aligned} &\mathrm{y_{h,min} \leq y_h(k+i/k) \leq y_{h,max}}, \quad \mathrm{i \geq 0}, \quad \mathrm{h = 1, 2, \ldots, q} \\ &u_{h,min} \leq u_h(k+i/k) \leq u_{h,max}, \quad i \geq 0, \quad h = 1, 2, \ldots, p \end{aligned}$$

$$J_\infty(k) = \sum\nolimits_{i=0}^{\infty} [X(k+i) + U(k+i)] \tag{2}$$

$$With \quad \begin{cases} X(k+i) = x^T(k+i/k) Q_0 x(k+i/k) \\ U(k+i) = u^T(k+i/k) R_0 u(k+i/k) \end{cases} \tag{3}$$

$Q_0 > 0$ *and* $R_0 > 0$, are two known weighting matrices.

2.2 Fuzzy Discrete Time T-S Model

Let us consider the following fuzzy discrete Time T-S system which represents a discrete time nonlinear system as follows:

$$\begin{aligned} &Rule\, \boldsymbol{i} : \boldsymbol{if} \;\; z_1(k) \; is \; M_{i1} \ldots \text{and} \; z_p(k) \; is \; M_{ip} \\ &\boldsymbol{then} \begin{cases} x(k+1) = A_i x(k) + B_i u(k) \qquad i = 1 \ldots r \\ \qquad y(k) = C_i x(k) \end{cases} \end{aligned} \tag{4}$$

With fuzzy discrete Time T-S model

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} h_i(z(k))(A_i x(k) + B_i u(k)) \\ y(k) = \sum_{i=1}^{r} h_i(z(k)) C_i x(k) \end{cases} \quad i = 1 \ldots r \tag{5}$$

And A_i, B_i *and* C_i are states matrices of system.

2.3 PDC Fuzzy Control Law

We use the PDC control law presented in [6]. Which describe and can writing as follows:

$$u(k) = -\left(\sum_{j=1}^{r} h_j(z(k)) F_j\right) x(k) = -\sum_{j=1}^{r} h_j(z(k)) Y_j G^{-1} x(k) \tag{6}$$

By substituting (6) in (5), the closed loop system is obtained as follows:

$$\begin{cases} x(k+1) = (A_z - B_z F_z) x(k) \\ y(k) = C_z x(k) \end{cases} \tag{7}$$

With:

$$A_z = \sum_{i=1}^{r} h_i(z(k)) A_i, B_z = \sum_{i=1}^{r} h_i(z(k)) B_i, C_z = \sum_{i=1}^{r} h_i(z(k)) C_i,$$
$$F_z = \sum_{j=1}^{r} h_j(z(k)) = \sum_{j=1}^{r} h_j(z(k)) Y_j G$$

3 Robust T-S Predictive Control Model Using PDC Controller

Theorem 1 [14].

Let us consider the constrained closed-loop system in (7) at time instant k, The equilibrium of the closed-loop discrete fuzzy model, given by (5), is globally asymptotically stable if there exists a matrix $P_i > 0$ define positive, Υ_{ij}, Y_j, G and $X_{ii} > 0$ and $X_{ij} = X_{ij}^T$, while:

$$\min_{\hat{P}_i, Y_j, G} \gamma \tag{8}$$

$$\begin{bmatrix} \gamma & x^T(k/k) \\ x(k/k) & \hat{P}_i \end{bmatrix} > 0 \tag{9}$$

$$\Upsilon_{ij} = \begin{bmatrix} G^T + G - \hat{P}_i & (A_iG - B_iY_j)^T & Y_j^T R_0 & G^T Q_0 \\ (A_iG - B_iY_j) & \hat{P}_i & 0 & 0 \\ R_0 Y_j & 0 & R_0 & 0 \\ Q_0 G & 0 & 0 & Q_0 \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} W & C_i(A_iG - B_iY_j) \\ (A_iG - B_iY_j)^T C_i^T & G^T + G - \hat{P}_i \end{bmatrix} > 0 \quad (11)$$

$$\begin{bmatrix} U & Y_j \\ Y_j^T & G^T + G - \hat{P}_i \end{bmatrix} > 0 \quad (12)$$

$$\Upsilon_{ij} > X_{ii} \quad i \in \{1, \ldots, r\} \quad (13)$$

$$\Upsilon_{ij} + \Upsilon_{ij} > X_{ij} + X_{ij}^T \quad i,j \in \{1, \ldots, r\}, \quad i<j \quad (14)$$

$$With: \quad X_l = \begin{bmatrix} 2X_{11} & (*) & (*) & (*) \\ X_{12} & 2X_{22} & \cdots & (*) \\ \vdots & \vdots & \ddots & \vdots \\ X_{1r} & X_{2r} & \cdots & 2X_{rr} \end{bmatrix} > 0 \quad (15)$$

Proof: Recall the closed-loop system in (7) and consider the following Non-quadratic Lyapunov function candidate:

$$V(x(k/k)) = x^T(k/k)\left(\sum_{i=1}^{r} h_i P_i\right)^{-1} x(k/k) \quad (16)$$

To ensure the stability of (4), it's necessary to satisfy the next inequalities:

$$V(x(k+i+1/k)) - V(x(k+i/k)) \leq -[X(k+i) + U(k+i)] \quad (17)$$

$$-V(x(k/k)) \leq -J_\infty(k) \quad (18)$$

We can write it:

$$\max_{A_i, B_i, i>0} J_\infty(k) \leq V(x(k/k)) \leq \gamma \quad (19)$$

While the problem of minimization become

$$\min_{\hat{P}_i, Y_j, G} \gamma \quad (20)$$

With:

$$x^T(k/k)\hat{P}_i^{-1}x(k/k) \le \gamma \Leftrightarrow \gamma - x^T(k/k)\hat{P}_i^{-1}x(k/k) \ge 0 \tag{21}$$

Using schur's complement to (21) we obtain:

$$\begin{bmatrix} \gamma & x^T(k/k) \\ x(k/k) & \hat{P}_i \end{bmatrix} > 0 \tag{22}$$

In the next, the PDC control law will be used to introduce more conditions that are ensuring the stability of system (5).

We have (17) it can be writing as:

$$\begin{aligned} & V(x(k+i+1/k)) - V(x(k+i/k)) \\ & \quad \le -\left[(x^T(k+i/k)Q_0x(k+i/k)) + (u^T(k+i/k)R_0u(k+i/k))\right] \end{aligned}$$

That can be writing as:

$$\begin{aligned} & [x^T(k+i+1/k)\hat{P}_i^{-1}x(k+i+1/k)] - [x^T(k+i/k)\hat{P}_i^{-1}x(k+i/k)] \\ & \quad < -\left[(x^T(k+i/k)Q_0x(k+i/k)) + (u^T(k+i/k)R_0u(k+i/k))\right] \end{aligned}$$

We replace $u(k+i/k)$ by (6):

$$\begin{aligned} & x^T(k+i/k)[x^T(k+1/k)\hat{P}_i^{-1}x(k+1/k) - \hat{P}_i^{-1}]x(k+i/k) \\ & \quad < -x^T(k+i/k)[Q_0 + G^{-T}Y_j^TR_0Y_jG^{-1}]x(k+i/k) \end{aligned}$$

With substitution of $x(k+1/k)$ by (7) we obtain:

$$\begin{aligned} & x^T(k+i/k)[(A_i - B_iY_jG^{-1})^T\hat{P}_i^{-1}(A_i - B_iY_jG^{-1}) - \hat{P}_i^{-1}]x(k+i/k) \\ & \quad < -x^T(k+i/k)[Q_0 + G^{-T}Y_j^TR_0Y_jG^{-1}]x(k+i/k) \end{aligned}$$

This inequality is equivalent to next inequality:

$$(A_i - B_iY_jG^{-1})^T\hat{P}_i^{-1}(A_i - B_iY_jG^{-1}) - \hat{P}_i^{-1} < -Q_0 - G^{-T}Y_j^TR_0Y_jG^{-1} \Leftrightarrow$$

We multiple in the left by G^T and by G in the right we get:

$$(A_iG - B_iY_j)^T\hat{P}_i^{-1}(A_iG - B_iY_j) - G^T\hat{P}_i^{-1}G < -G^TQ_0G - Y_j^TR_0Y_j \Leftrightarrow$$

Then we obtain:

$$G^T\hat{P}_i^{-1}G - (A_iG - B_iY_j)^T\hat{P}_i^{-1}(A_iG - B_iY_j) - G^TQ_0G - Y_j^TR_0Y_j > 0 \tag{23}$$

The term $G^T\hat{P}_i^{-1}G$, can be writing as follows:

$$\left(G^T-\hat{P}_i\right)\hat{P}_i^{-1}\left(G-\hat{P}_i\right)\geq 0 \Rightarrow$$

$$G^T\hat{P}_i^{-1}G-G^T\hat{P}_i^{-1}\hat{P}_i-\hat{P}_i\hat{P}_i^{-1}G+\hat{P}_i\hat{P}_i^{-1}\hat{P}_i\geq 0 \Leftrightarrow$$

$$G^T\hat{P}_i^{-1}G-G^T-G+\hat{P}_i\geq 0 \Leftrightarrow$$

$$G^T+G-\hat{P}_i\leq G^T\hat{P}_i^{-1}G \Leftrightarrow$$

$$G^T+G-\hat{P}_i\leq G^T\hat{P}_i^{-1}G \tag{24}$$

We hold (24) in (23):

$$G^T+G-\hat{P}_i-(A_iG-B_iY_j)^T\hat{P}_i^{-1}(A_iG-B_iY_j)-G^TQ_0G-Y_j^TR_0Y_j>0$$

With a small addition of Q_0 *and* R_0 matrices to the precedent inequality, we find:

$$\begin{aligned}&(G^T+G-\hat{P}_i)-(A_iG-B_iY_j)^T\hat{P}_i^{-1}(A_iG-B_iY_j)-Y_j^TR_0R_0^{-1}R_0Y_j\\&\quad-G^TQ_0Q_0^{-1}Q_0G>0\end{aligned} \tag{25}$$

Using generalized schur's complement and propriety in [12] to (25), we obtain:

$$\begin{bmatrix} G^T+G-\hat{P}_i & (A_iG-B_iY_j)^T & Y_j^TR_0 & G^TQ_0 \\ (A_iG-B_iY_j) & \hat{P}_i & 0 & 0 \\ R_0Y_j & 0 & R_0 & 0 \\ Q_0G & 0 & 0 & Q_0 \end{bmatrix} > 0 \tag{26}$$

The inequality (26) represents the LMI form of model fuzzy predictive control. Now, we also must put the constraints in the form of LMIs.

– Output Constraints

$$y_{h,min}\leq y_h(k+i/k)\leq y_{h,max},\quad i\geq 0,\quad h=1,2,\ldots,q$$

$$|y_h(k+i/k)|\leq y_{h,max},\quad i\geq 0,\quad h=1,2,\ldots,q$$

$$y_{max}=W$$

$$\|y(k+i/k)\|_{max}\triangleq\max_i y_i(k+i/k)$$

With (7), we can write:

$$\max_{i>0}\|y(k)\|_{max}\geq\max_{i>0}\left\|C_i(A_i-B_iY_jG^{-1})\hat{P}_i^{-1}x(k)\right\|_{max}$$

Using the LMI constraints in [5]. We obtain:

$$\begin{bmatrix} W & C_i(A_i - B_iY_jG^{-1}) \\ (A_i - B_iY_jG^{-1})^TC_i^T & \hat{P}_i \end{bmatrix} > 0$$

By using Congruence property with full rank matrix $\begin{bmatrix} I & 0 \\ 0 & G^T \end{bmatrix}$ gives:

$$\begin{bmatrix} W & C_i(A_iG - B_iY_j) \\ (A_iG - B_iY_j)^TC_i^T & G^T + G - \hat{P}_i \end{bmatrix} > 0 \tag{27}$$

– Input Constraints

$$u_{h,min} \le u_h(k+i/k) \le u_{h,max}, \quad i \ge 0, \quad h = 1, 2, \ldots, p$$

$$|u_h(k+i/k)| \le u_{h,max}, \quad i \ge 0, \quad h = 1, 2, \ldots, p$$

$$u_{max} = U$$

$$\|u(k+i/k)\|_{max} \triangleq \max_i u_i(k+i/k)$$

With (6), we can write:

$$\max_{i>0} \|u(k)\|_{max} \ge \max_{i>0} \left\| Y_jG^{-1}\hat{P}_i^{-1}x(k) \right\|_{max}$$

Use again the LMI constraints in [5]. We obtain:

$$\begin{bmatrix} U & (Y_jG^{-1}) \\ (Y_jG^{-1})^T & \hat{P}_i \end{bmatrix} > 0$$

By using Congruence property with full rank matrix $\begin{bmatrix} I & 0 \\ 0 & G^T \end{bmatrix}$ gives:

$$\begin{bmatrix} U & Y_j \\ Y_j^T & G^T + G - \hat{P}_i \end{bmatrix} > 0 \tag{28}$$

4 Simulation Results

In this section we present the design of conditions that ensure stability for nonlinear systems by the presented strategy, Constrained MPC for Fuzzy discrete time by PDC controller. Two examples are presented with and without uncertainties. The online set solutions were carried out using the YALMIP toolbox [17].

4.1 Example 1

The following system is taken from [18]:

$$\begin{cases} Rule1: if\ \boldsymbol{x}_1(\boldsymbol{k})\ is\ \boldsymbol{M}_1,\ then \begin{cases} \dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}_1 x(\boldsymbol{t}) + \boldsymbol{B}_1 u(\boldsymbol{t}) \\ y(\boldsymbol{t}) = \boldsymbol{C}_1 x(\boldsymbol{t}) \end{cases} \\ Rule2: if\ \boldsymbol{x}_1(\boldsymbol{k})\ is\ \boldsymbol{M}_2,\ then \begin{cases} \dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}_2 x(\boldsymbol{t}) + \boldsymbol{B}_2 u(\boldsymbol{t}) \\ y(\boldsymbol{t}) = \boldsymbol{C}_2 x(\boldsymbol{t}) \end{cases} \end{cases} \tag{29}$$

Using sector nonlinearity with the sampling time 1 s the T-S fuzzy discrete time system represents with:

$$\mathrm{A}_1 = \begin{bmatrix} 0.9504 & 0.9834 \\ -0.09834 & 0.9504 \end{bmatrix}, \quad \mathrm{A}_2 = \begin{bmatrix} 0.9635 & 0.6218 \\ -0.06218 & 0.3417 \end{bmatrix}$$

$$\mathrm{B}_1 = \begin{bmatrix} 0.4958 \\ 0.9834 \end{bmatrix}, \mathrm{B}_2 = \begin{bmatrix} 0.365 \\ 0.6218 \end{bmatrix}; \mathrm{C}_1 = [1 \quad 0], \mathrm{C}_2 = [1 \quad 0]$$

Weighting matrices and membership functions for rule 1 and rule 2 are:

$$\mathrm{Q}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathrm{R}_0 = 0.5, \quad \mathrm{M}_1(x_1(k)) = -x_2^2 + 1, \quad \mathrm{M}_2(x_2(k)) = x_2^2$$

Under the constraints: $-2 < y(k) < 2, \quad -0.5 < u(k) < 0.5$.

With the initials conditions are: $x_1(0) = 0, \quad x_2(0) = 0$.

The results in Figs. 1 and 2 shows that the conception of control law with corrections gives a better responses when we compare the results with that given in [15]. It is clearly that the stability is guarantee with respect of imposed constraints.

Also results in Fig. 3 show the robustness behavior of the study approach in presence of extern uncertainties.

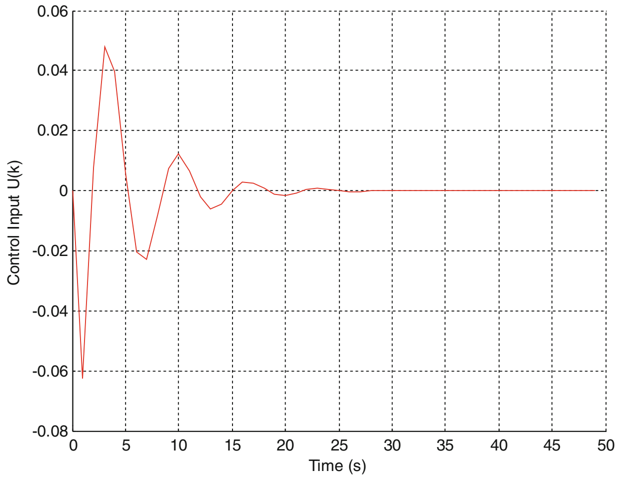

Fig. 1. Response of control input u(k)

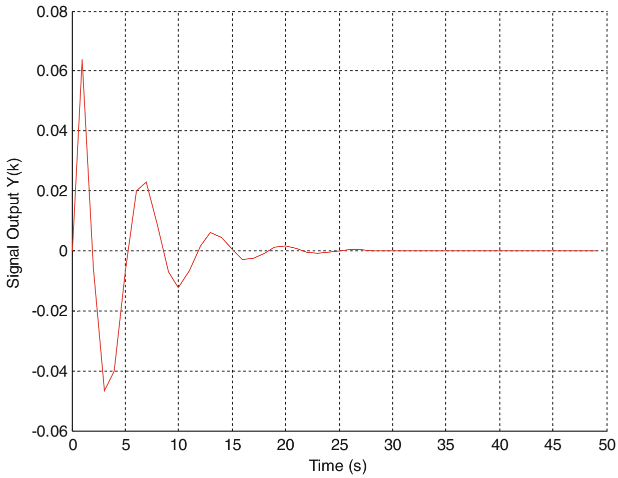

Fig. 2. Response of output signal y(k)

4.2 Example 2

The following system is taken from [14], with the states matrices:

$$A_1 = \begin{bmatrix} -0.5 & 2 \\ -0.1 & 1.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.19 & 0.5 \\ -0.1 & -1.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 4.1 \\ 4.8 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix}; \quad C_1 = [1 \quad 0.3], \quad C_2 = [0.8 \quad 0.2]$$

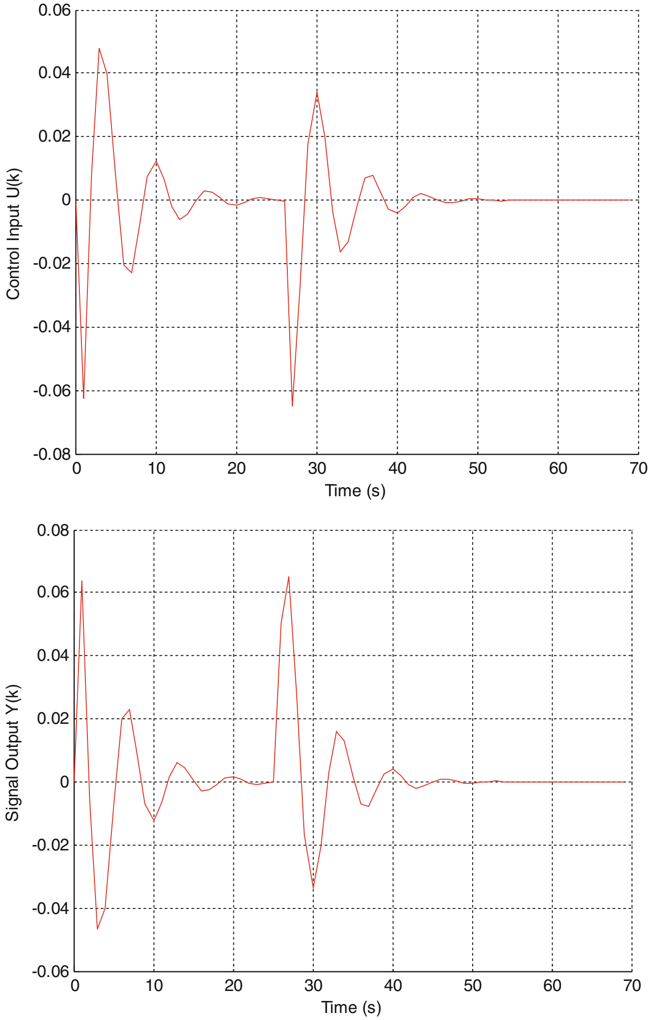

Fig. 3. Response of control input u(k) and output signal y(k)

Weighting matrices:

$$Q_0 = \begin{bmatrix} 0.8 & 0.1 \\ 0.1 & 0.95 \end{bmatrix}, \quad R_0 = 0.9.$$

And membership functions for rule 1 and rule 2 are:

$$M_1(x_1(k)) = \frac{1}{1 + \exp(-2x_1(k))}, \quad M_2(x_2(k)) = 1 - M_1(x_1(k))$$

Under the constraints: $-2.5 < y(k) < 2.5, \quad -1 < u(k) < 1.$
With the initials conditions are:

$$x_1(0) = -0.3, \quad x_2(0) = -1, \quad u(0) = -0.5, \quad y(0) = -0.5.$$

The results in Figs. 4 and 5 of the simulation show that the conception of corrected control law performance gives better results when we compare with [14]. The comparison is presented in Table 1.

Even in presence of extern uncertainties, results in Figs. 6 and 7 are showing that the robustness of this approach is ensured.

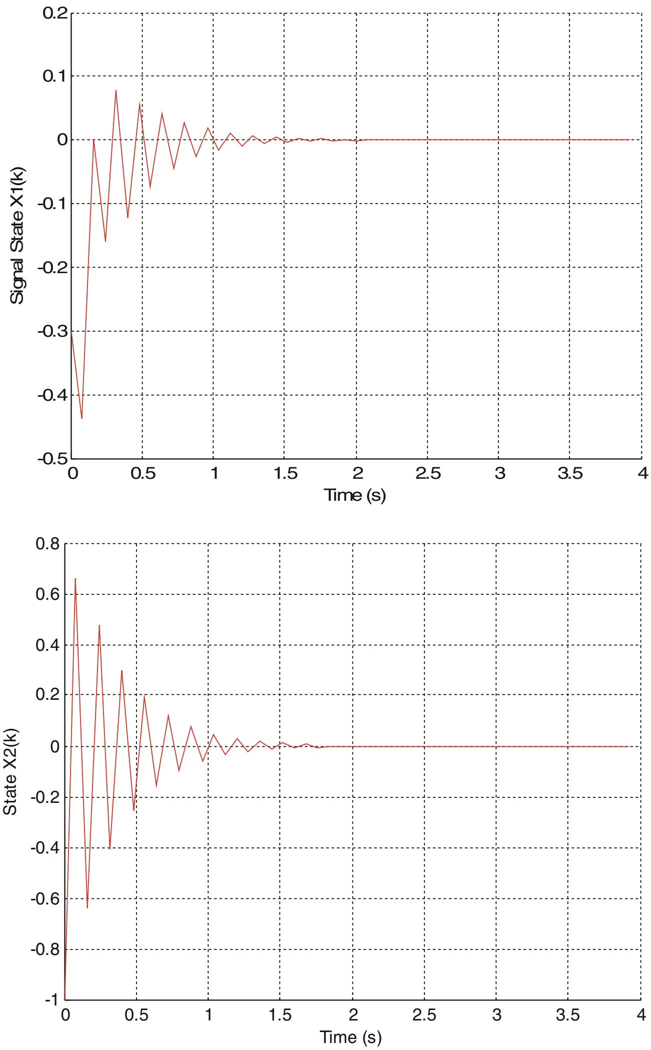

Fig. 4. Response of state X1(k) and X2(k)

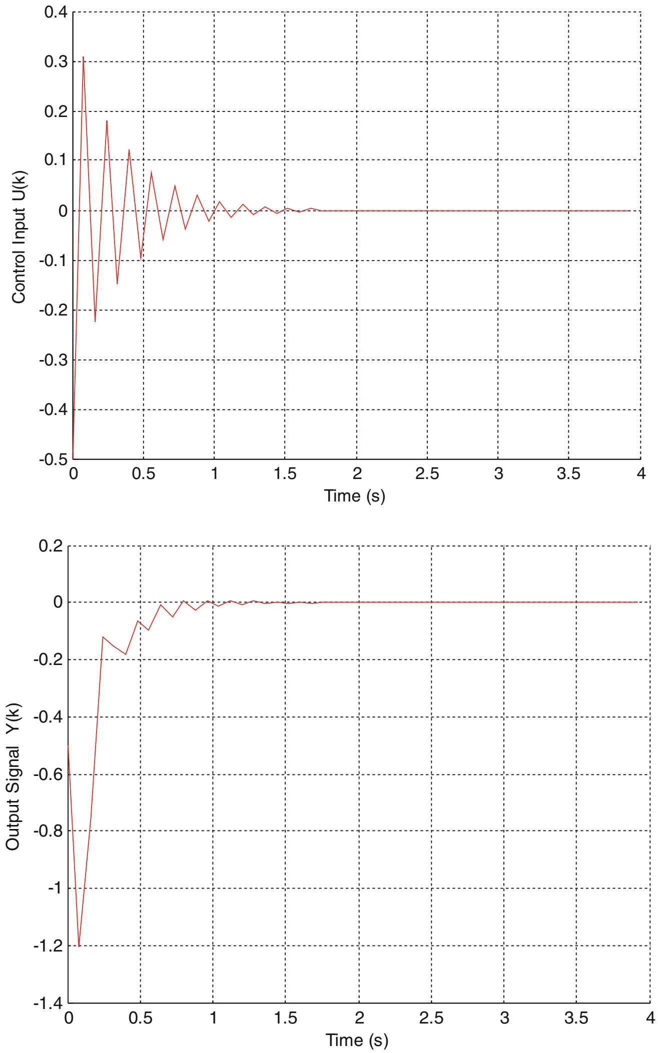

Fig. 5. Response of control input u(k) and output signal y(k)

Table 1. Comparison results with [14].

Comparison results	This paper	[14]
Chattering interval of x_1	−0.4–0.1	−3.3–3.2
Chattering interval of x_2	−0.7–0.7	−0.9–1.1
Chattering interval of u	−0.2–0.3	−0.9–0.7
Chattering interval of y	−1.2–0.1	−2.4–2.3
Convergence time at All	1.3–2.0	3.1–3.5

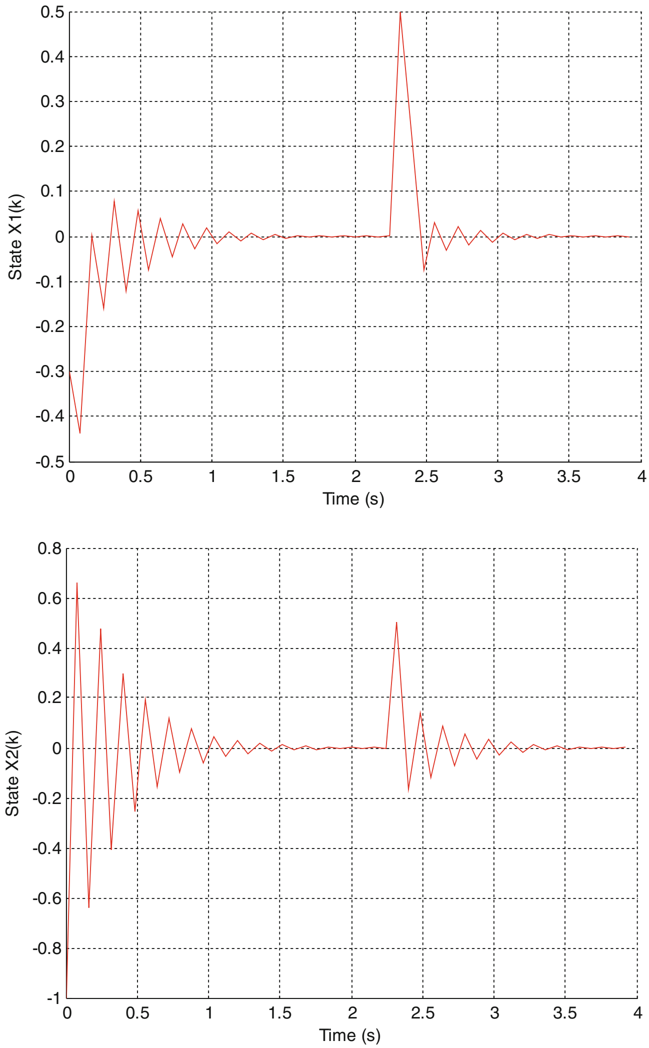

Fig. 6. Response of state X1(k) and X2(k)

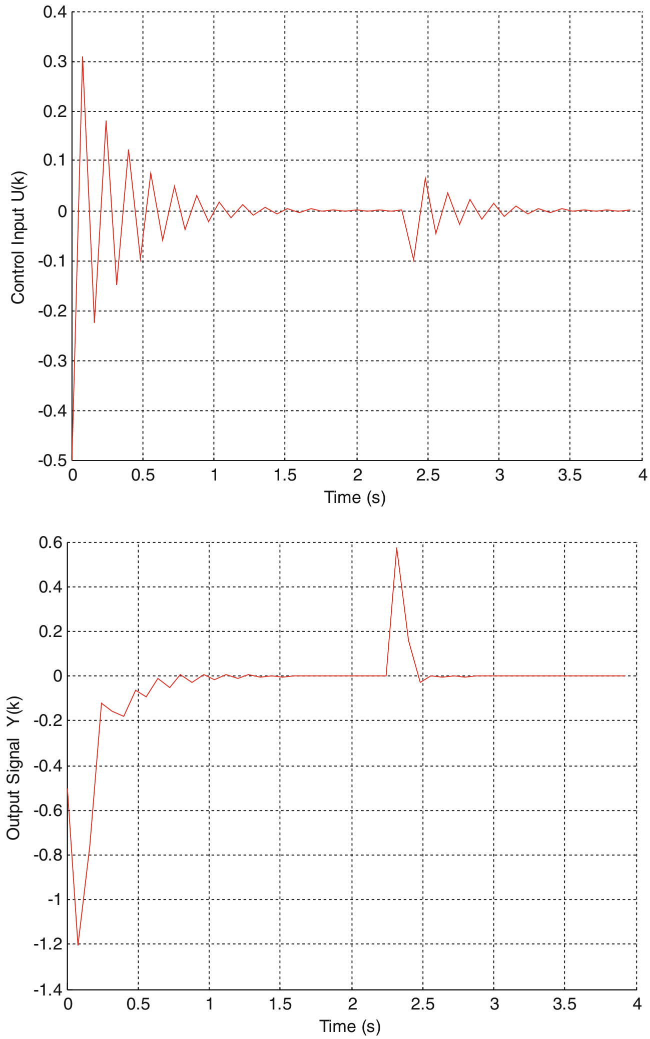

Fig. 7. Response of control input u(k) and output signal y(k)

5 Conclusion

In this paper, corrections of controller performance based T-S fuzzy predictive control under constraints was introduced. The PDC is designed with an infinite horizon predictive control; therefore the optimization problem with input constraint is transformed into constraint LMI problem. Thus LMI optimization is well suited for online implementation, which is essential for predictive control. The using of fuzzy controller PDC in this work shows good results for a class of nonlinear systems. Finally, the stability of the closed-loop system is guaranteed by the Lyapunov approach.

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