Models of Magnetic Hysteresis Loops Useful for Technical Simulations Using Finite Elements Method (FEM) and Method of the Moments (MoM)

Roman Szewczyk(✉), Michał Nowicki,
and Katarzyna Rzeplińska-Rykała

Industrial Research Institute for Automation and Measurements,
Al. Jerozolimskie 202, 02-486 Warsaw, Poland
szewczyk@mchtr.pw.edu.pl

Abstract. The paper provides the analyse of three simplified models of hysteresis loop suitable for technical purposes. Models consider the coercive field and utilize linear approximation with saturation, Langevin equation as well as arcus tangent functions. Validation of the models was done on the experimental data from measurements of magnetic hysteresis loops of four different materials. Accuracy of the models is assessed quantitatively. Finally, the parameters for practical application of the models are presented from the point of view different magnetic materials used in modelling by the finite elements method or the method of the moments.

1 Introduction

Quantitative description of magnetic hysteresis loop is the one of the most sophisticated problem connected with contemporary physics of magnetic materials. Among recently developed models of the magnetic hysteresis, the most effective are Jiles-Atherton [1] model and Preisach model [2]. Both these models require solving of sophisticated equations. Moreover, the Jiles-Atherton model requires numerical integration in the case of anisotropic materials [3] as well as solving of ordinary differential equations (ODE), which may lead to different numerical problems, especially for high-permeability materials [4].

On the other hand, technical simulations oriented on finite element method or method of the moments don't require sophisticated analyses of the shape of the hysteresis loops. To be useful for technological simulations, the model of the magnetic hysteresis loop should provide fast and reliable reproduction of the shape of saturated magnetic hysteresis loops.

Approximation of magnetic hysteresis loops by the different mathematical functions were presented previously for specific cases [5, 6]. However, quantitative comparative analyses of efficiency of such approximation for modern magnetic materials was not presented. This paper is filling this gap, to enable effective application of simplified models of magnetic hysteresis loop in magnetostatic and magnetodynamic systems modelling for technical purposes.

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2 Proposed Simplified Models of Magnetic Hysteresis

For technical purposes, three models are proposed: linear model with saturation, model based on the Langevin equation as well as model utilizing the arcus tangent functions. In all three models, the coercive field H_c is considered in saturation hysteresis loops.

Linear model with saturation utilizes three parameters: coercive field H_c, relative permeability μ and saturation flux density B_s. This model is given by the set of following equations:

$$B(H) = \begin{cases} B_s \, when \, \mu\mu_0(H \pm H_c) > B_s \\ \mu\mu_0(H - H_c) \, when \, \frac{dH}{dt} > 0 \\ \mu\mu_0(H + H_c) \, when \, \frac{dH}{dt} < 0 \\ -B_s \, when \, \mu\mu_0(H \pm H_c) < B_s \end{cases} \tag{1}$$

where μ_0 is magnetic constant.

Langevin function based model also uses three parameters: H_c, B_s and a. This sigmodal-shaped function is given by the following set of equations [7]:

$$\mathrm{B(H)} = \begin{cases} \mathrm{B_s}\left(\coth\left(\frac{\mathrm{H-H_c}}{\mathrm{a}}\right) - \left(\frac{\mathrm{a}}{\mathrm{H-H_c}}\right)\right) \text{ when } \frac{\mathrm{dH}}{\mathrm{dt}} > 0 \\ \mathrm{B_s}\left(\coth\left(\frac{\mathrm{H+H_c}}{\mathrm{a}}\right) - \left(\frac{\mathrm{a}}{\mathrm{H+H_c}}\right)\right) \text{ when } \frac{\mathrm{dH}}{\mathrm{dt}} < 0 \end{cases} \tag{2}$$

In opposite to linear function and Langevin function based models, the arcus tangent function based model doesn't explicit specify saturation flux density. However, it is also based on the three parameters: H_c, μ and k. Model with hysteresis is specified by the following set of equations [8]:

$$B(H) = \begin{cases} \frac{\mu_0\mu}{k} \operatorname{atan}(H - H_c) \, when \, \frac{dH}{dt} > 0 \\ \frac{\mu_0\mu}{k} \operatorname{atan}(H + H_c) \, when \, \frac{dH}{dt} < 0 \end{cases} \tag{3}$$

As it can be seen, each model utilizes three parameters. However, the physical background of each parameter is different in the case of each models. Moreover, some of parameters, such as a in the arcus tangent model, doesn't have physical explanation and should be determined experimentally for each shape of hysteresis loop.

3 Tested Materials and Measuring Method

Experiments were performed on four different magnetic materials:

- anisotropic electrical steel M130-27 s magnetized in the easy axis direction. Sample was in the form of the Epstein frame [9],
- martensitic, stainless steel 3H13 for energetic purposes in the form of frame-shaped samples [10],
- manganese-zinc high permeability ferrite F801 in the form of ring-shaped samples,
- Fiemet-type nanocrystalline alloy in the form of ring-shaped samples.

Magnetic hysteresis loops were measured in quasi-static conditions by computer controlled hysteresis graph. Measurements were carried out in the room temperature.

4 Results of Modelling of Hysteresis Loops

Presented models were implemented with use of open-source OCTAVE 4.0 software. Parameters of them models were initially approximately determined on the base of its physical meaning (such as value of saturation flux density B_s or relative permeability μ). Next, the parameters of the models of hysteresis loops were identified during the optimisation process using a derivative-free Nelder and Mead simplex algorithm [11]. The target function F for optimization process was given by the following equation:

$$F = \sum_{i=1}^{n} \left(B_{model}(H_i) - B_{meas}(H_i)\right)^2 \tag{4}$$

where $B_{model}(H_i)$ were the results of the modelling and $B_{meas}(H_i)$ were the results of the experimental measurements of hysteresis loops, both for the value H_i of magnetizing field.

Results of the modelling of magnetic saturation hysteresis loops of four materials described in the Sect. 3 with use of linear function based model, Langevin function based model and arcus tangent function based model are presented in the Figs. 1, 2 and 3 respectively. Parameters of the models and assessment of accuracy of modelling is presented in the Table 1.

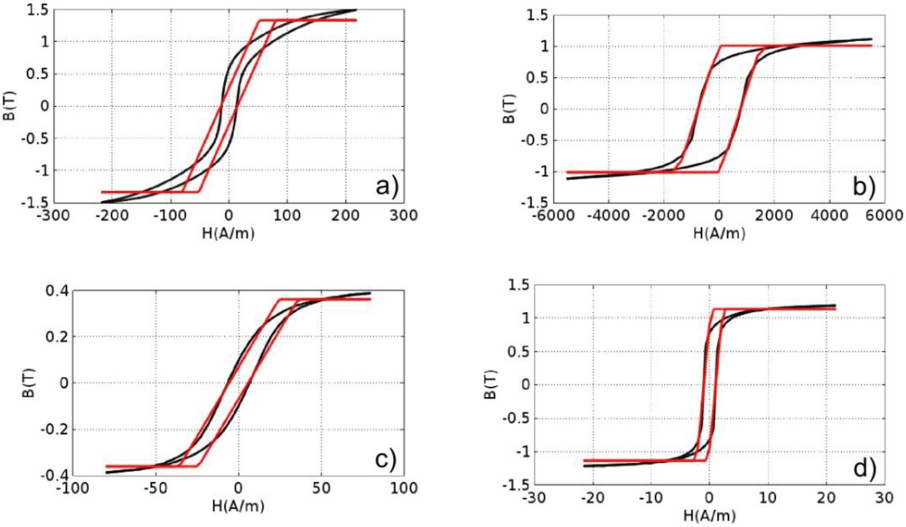

Fig. 1. Results of the modelling of saturation magnetic hysteresis loops using linear function based model for: (a) electrical steel M130-27 s, (b) martensitic steel 3H13, (c) Mn-Zn ferrite, (d) Finement-type nanocrystalline alloy

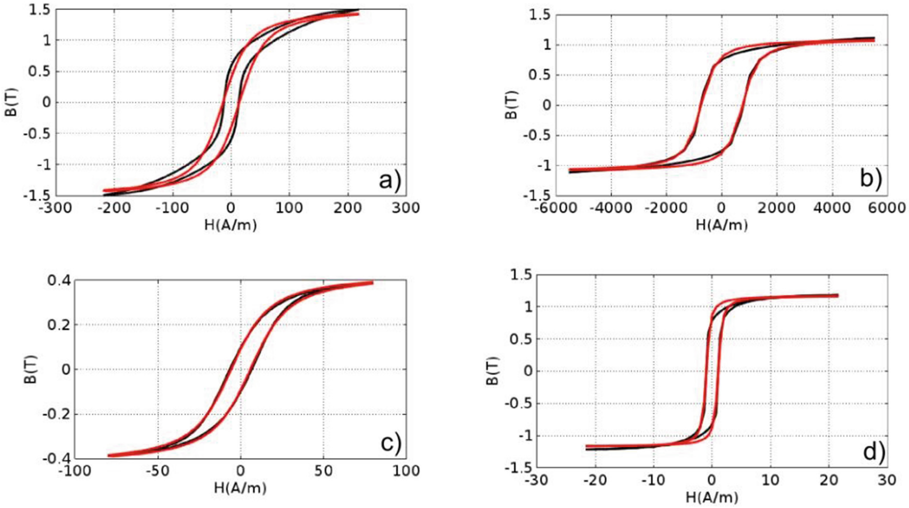

Fig. 2. Results of the modelling of saturation magnetic hysteresis loops using Langevin function based model for: (a) electrical steel M130-27 s, (b) martensitic steel 3H13, (c) Mn-Zn ferrite, (d) Finement-type nanocrystalline alloy

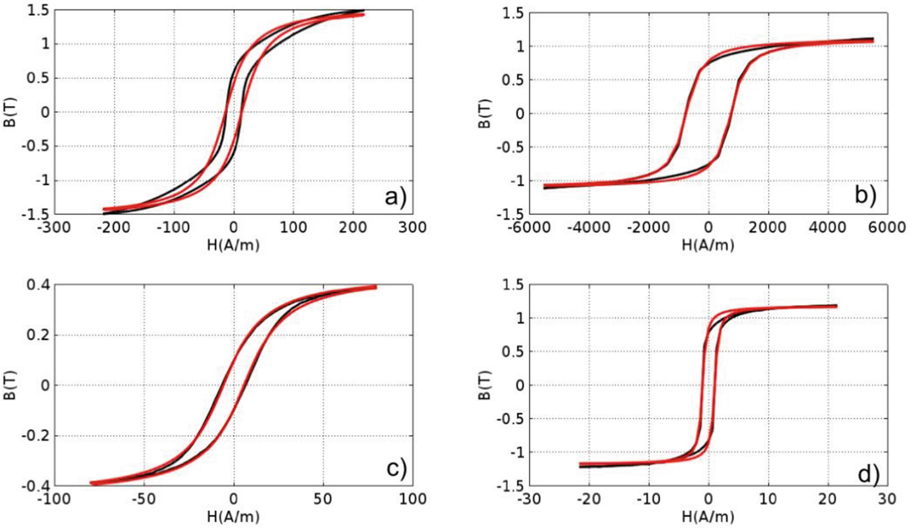

Fig. 3. Results of the modelling of saturation magnetic hysteresis loops using arcus tangent function based model for: (a) electrical steel M130-27 s, (b) martensitic steel 3H13, (c) Mn-Zn ferrite, (d) Finement-type nanocrystalline alloy

Table 1. Estimated parameters of three models of magnetic saturation hysteresis loops (R^2 – determination coefficient, e_{std} – average root mean squared error)

Parameter	Materials			
	Linear function based model			
	M130-27 s	3H13	Mn-Zn ferrite	Fimemet
B_s *(T)*	1.33	1.01	0.36	1.13
μ	16 112	1 099	9 549	792 085
H_c *(A/m)*	14.19	768	5.77	1.01
R^2	0.983	0.991	0.994	0.993
e_{std} *(%)*	5.6	4.6	3.2	5.3
Langevin function based model				
B_s *(T)*	1.54	1.11	0.44	1.18
a *(A/m)*	17.16	212	8.95	0.26
H_c *(A/m)*	13.89	747	5.80	1.02
R^2	0.995	0.998	0.9994	0.998
e_{std} *(%)*	3.24	1.91	0.77	2.71
Arcus tangent function based model				
μ	25 589	1 506	13 567	1 282 538
k	0.032	0.0026	0.0599	2.14
H_c *(A/m)*	13.84	748	5.81	1.02
R^2	0.996	0.9988	0.9993	0.9986
e_{std} *(%)*	2.97	1.70	0.93	2.48

5 Conclusions

Presented results indicate, that all three proposed with coercive field H_c is considered in saturation hysteresis loops: linear model with saturation, model based on the Langevin equation as well as model utilizing the arcus tangent functions are suitable from the point of view of finite elements method and method of the moments. In all three models, the determination parameter R^2 exceeds 0.98 for different types of magnetic materials.

On the other hand, in terms of both determination coefficient R^2 and the mean squared error e_{std}, the results of modelling indicate, that Langevin function based model as well as arcus tangent function based model gives better results than linear function based model. For this reason both Langevin function based model as well as arcus tangent function based models are more suitable for technical modelling, even if both of them are more sophisticated from computational point of view.

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