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CHARACTERIZATION OF METAL FLOW IN METALS PROCESSING BY A COMBINED APPROACH USING ADVANCED EXPERIMENTAL GRID PATTERN TECHNIQUES COUPLED WITH FE-ANALYSIS

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Abstract

A review is given regarding the latest advancement within the area of analysis and characterization of metal flow in deformational processing of metals.
Experimental grid pattern (GP) techniques have lately been developed to a very high level of sophistication. Thanks to this, the engineer now can determine the real metal flow in industrial or laboratory processes with high accuracy. Concurrently, software for FE-analysis has been developed to a very powerful tool that allows detailed observations to be made regarding metal flow on the computer in any industrial process.
The advanced experimental GP-techniques currently available are described and it is shown how they can be used to analyze metal flow inside and on the surface of the workpieces subjected to metal forming. It is also shown how accurate FE-models can reproduce the deformational behavior of the metal during the processing.
In this way useful information can be deduced, for instance it can be explained how typical process-related defects are created in a process, so that preventive measures can be taken to avoid the occurrence of such defects in an industrial environment.
Examples are shown of application of metal flow analysis in processing techniques like extrusion, rolling, forging, wiredrawing and sheet-metal forming.

Introduction

Many of the industrial metal forming process currently used were developed very early in the history. Reading about the history of the metal forming processes can for instance be found by searching the internet[1]. Modern rolling practice, for instance, can be attributed to the pioneering efforts of Henry Court who had a patent issued in 1783 on rolling using grooved rolls. He was not the first to use this manufacturing method, but he is considered the first to combine the use of many of the best features of various ironmaking and shaping processes at that time; thus he can be said[2] to be the “father of modern rolling”.
When it comes to extrusion of metals this method was also invented very early. In 1797 the first extrusion process for making of lead pipe was patented. It involved forcing preheated metal through a die via a hand driven plunger. Much later, in 1894, the use of the extrusion process was expanded to copper and brass[3] by Alexander Dick, largely the same way as extrusion is performed today, thus he is considered the inventor[4] of the modern hot extrusion process. Today, however, aluminum is the most commonly extruded metal.
Metal forming processes applied today have been developed to a very high level of sophistication compared to what was the case such as hundred years ago. The perhaps most

important recent development that has advanced the science of metal processing is the development of the Finite Element Method simulation technology[5] for metal forming purposes. The development of experimental GP-techniques to record metal flow[6] has also been important for the understanding of the mechanics of the metal forming processes. Such techniques are also called visio-plasticity techniques since they can visualize the velocity field causing the metal flow, and they are commonly used to check velocity fields, assumed theoretically or computed by FEA. In early work, use of model material for this purpose was very popular but, in addition, techniques to be applied on real materials have also been developed to high sophistication.
The use of a "combined experimental metal flow study and FE-simulation approach" for determining the real metal flow inside a metallic workpiece, or on it's surface, during a forming operation is to be discussed in depth in this article. Examples of application are to be shown for a number of specialized metal forming processes.

Grid pattern analysis for study of metal flow

An early study on metal flow in industrial extrusion is the investigation performed by Schweissgut[7]. He used the ***micro-structural method*** in order to try to explain how defects form in extrusion of copper. With the application of this technique he was roughly able to describe the formation mechanism of the so-called back-end defect, and why it occurs. The micro-structural method is not very precise because different regions of the billet are not precisely marked, and their final position after end of extrusion cannot be identified exactly. Because the material of the peripheral shear zone inside an extruding billet (in direct extrusion) is deformed heavily in shear, this materiel will develop a micro-structural appearance different from that of the material in the core, which deforms less in shear and more by elongation. For this reason the presence of material from different deformation zones inside the billet after outflow into the extrudate, or inside a partially extruded billet, can be identified approximately from the appearance of the grain-flow.
A better method is the so-called split-billet ***Inscribed Grid Pattern (IGP) technique***[8,9]**.** When this technique is used a billet is split longitudinally, and a GP is inscribed onto one longitudinal cut surface of half a billet, i.e., on its mid-section. The gridded and the ungridded halves are then added together, and the obtained aggregate is partially extruded. The billet is removed from the container, and is split along the mid-plane containing the GP. In this way the deformed pattern is revealed as it appears after extrusion. An early application of the IGP-technique is shown in Fig.1a) and b). Fig.1b) is a drawing of the experimental pattern. As the sketch depicts the most heavily deformed parts of the pattern (in the shear zone) has been erased by the deformation and is no longer visible.

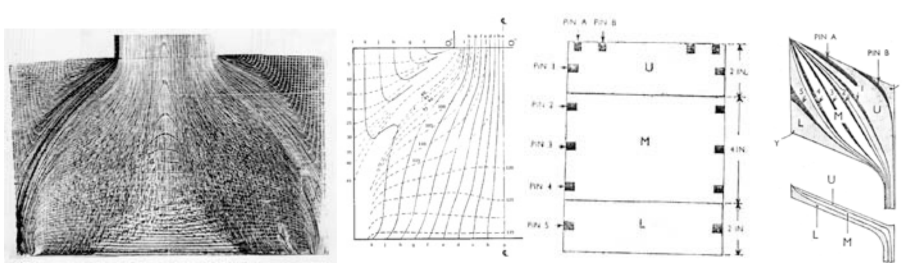
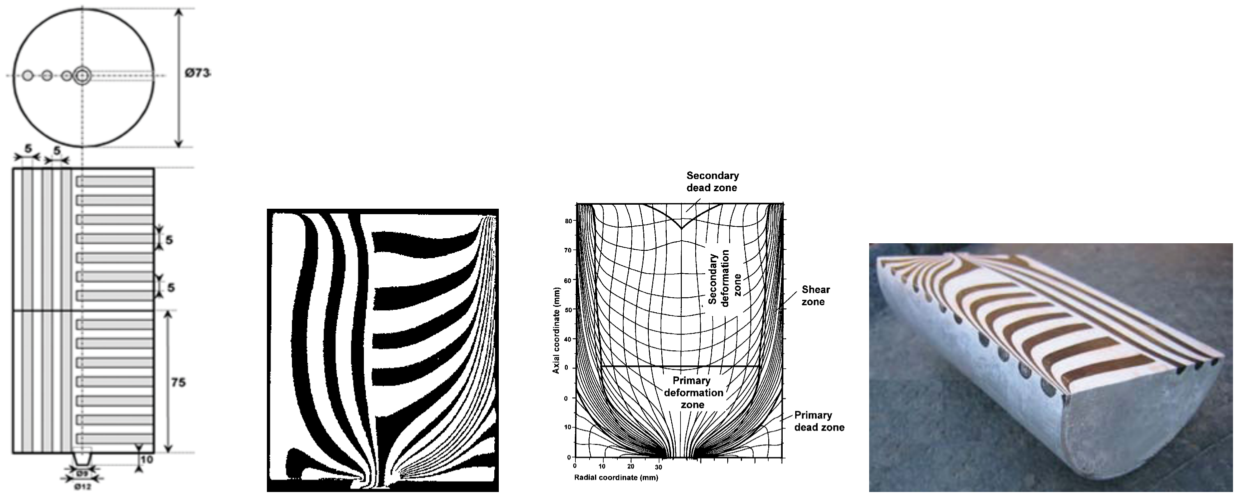


Fig.1. Characterizing metal flow in extrusion; a) Deformed IGP in Al-extrusion, b) Sketch of deformed pattern, c) Marker pins inserted into a Cu-billet and d) Appearance of marker pins in c) after tube extrusion.



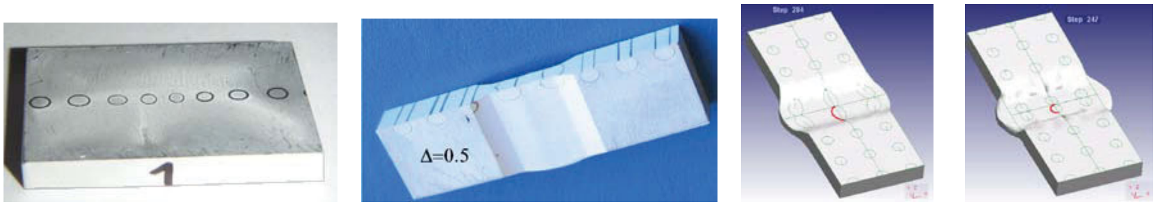
a) b) c) d)

Fig.2. PGP-technique; a) Longitudinally sectioned gridded billet showing appearance of pins, b) Stripe pattern inside the billet after partial extrusion, c) Stripe pattern transformed into GP, d) Appearance of ends of marker pins along boundary surface of the extruded billet.

Another method that has been much applied is the method of ***insertion of marker pins***[10]. Pins of a marker (or indicator) material with (approximately) the same flow stress as the billet material are inserted into holes drilled into the billet, for instance into its surface layer, as shown in Fig.1c), to trace metal flow of this layer. After end of forming, the partially extruded billet is removed from the container. When appropriately sectioned, eventually also after sectioning of the extrudate, it is possible to identify where the material of the surface layer has flown, if it has been extruded through the die opening or if it still remains inside the partially extruded billet.

This technique has been further refined into a marker ***Pin Grid Pattern (*PGP*) technique***. A pattern of marker material stripes are made by inserting pins in the symmetry plane of the extrusion billet, where metal flow is to be studied. Radial and axially oriented pins are kept at different sides of the mid-axis of the billet. After partial extrusion the internal deformed PGP is revealed by cutting, grinding and etching, as shown in Fig.2b). Because of symmetry, the stripes from the pins with one orientation can be rotated around the mid-axis of the billet, to be put on top of the other set of stripes. If the boundary lines of each stripe are recorded a similar pattern will be obtained as when the IGP-technique is used. Other investigators have also started to use the PGP-technique[11] because it has certain advantages over the IGP-technique, even though it is rather complex to use, and in addition, requires rather much labor and cost.

One advantage of the PGP-technique is that the appearance of the pins on the boundary interface between the workpiece and the tooling, i.e. on the surface of the partially extruded billet, can be identified as they appear when extrusion was terminated. This is if the tooling is made to allow removal of the workpiece from the die without deforming plastically during the removal operation. Another advantage over the IGP-technique is that a parting agent is not required to avoid pressure welding over the split surface. With the PGP-technique it is therefore easy to achieve full sticking friction conditions as commonly experienced in unlubricated Al-extrusion.



a) b) c) d)

Fig.3. SuRIT-technique to trace metal flow along workpiece tool interfaces in PSCT; a) Initial specimen with SuRIT-pattern, b) SuRIT-pattern revealed from deformed specimen, FEA-predicted flow of surface rings in case of c) frictionless and d) sticking friction conditions.

As a matter of fact, when the split-billet IGP-technique has been used for study of metal flow in Al-extrusion, the process has often been one with partial or full lubrication, even though intention was not to run lubricated. A third advantage of the PGP-technique is that the pattern does not get erased in heavily deformed regions inside the workpiece. If Fig.1b) is compared with Fig.2c) this difference becomes clear; the IGP-pattern has vanished in the shear zone ahead of the die in Fig.1b), while it is still present here when the PGP-technique is used.

As mentioned, the PGP-technique shows the appearance of the GP along boundary interfaces of a workpiece. If pins coated with contrast material are used, one will obtain a ***Surface Ring and Interior-Tubes***[12] ***(SuRIT)*** contrast material effect (shown in Fig.3 for plain-strain compression (PSC)). Then, on the surface of the specimen there will be a circle pattern, and in a section through the workpiece there will be a pattern of thin unidirectional stripes. This stripe pattern visualizes metal flow the same way as for the broader stripes obtained when contrast pins all made of marker material are used. The ring pattern on the surface of the specimen shown in Fig.3 is well suited for conducting inverse analysis using FEA to determine friction.

Finite Element Analysis (FEA)

Lately, the most important advancement in metal forming analysis has been the development and application of finite element analysis (FEA), i.e. use of the computerized finite element method (FEM). Recent progress in FEA, together with increasingly powerful computers, has permitted increased use of numerical modeling[5]. Hence, today it is possible to FEM-simulate the metal forming processes at various design stages. FEA in metal forming has made it possible to analyze a chosen process on the computer rather than using trial and error in the forming machine. Process simulation using FEA has given considerable cost and quality improvements in the industry. FEM simulation is an efficient tool for prediction of performance in industrial forming operations before dies are manufactured.

FEM programs can predict large deformations and thermally influenced material flow in metal forming. Thus they can provide detailed information about the forming mechanics. When a FEM model has been made for a particular forming application, the load requirement, velocity, strain rate, strain, and stress fields, etc., can easily be obtained for the considered process. Therefore, the current trend is toward increased application of FEA, for process simulation and optimization. For practical applications, the modeling techniques must, of course, describe experimental observations quantitatively with sufficient accuracy. To validate FEA-predicted metal flow, and assure that it is predicted correctly, the above described GP-techniques can be used.

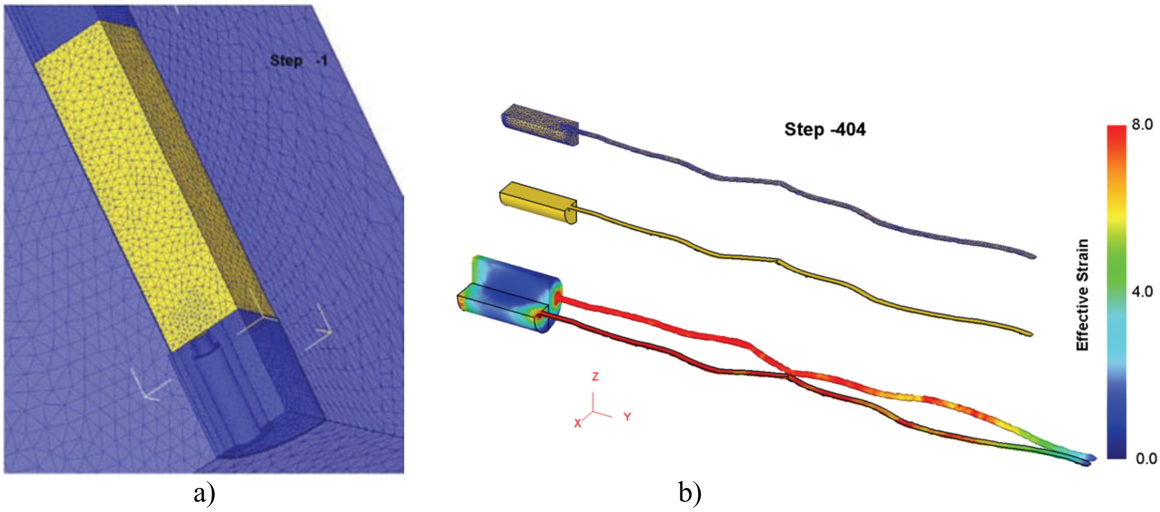


Fig.4. DEFORM™ three-dimensional FEM model of the two-hole extrusion process; a) Initial state with billet loaded into the container, and b) The model after extrusion to the same partial stage as in the experiment.

In FEM modeling the workpiece is represented by a collection of sub-domains called finite elements. The elements are bounded by sets of nodes. They define localized mass and stiffness properties of the model. Equations of equilibrium, in conjunction with applicable physical considerations such as compatibility and constitutive relations, are applied to each element, to construct a system of equations. The system is then solved to find unknown values using advanced numerical techniques. FEA is an approximate method, and its accuracy can be improved by increasing the number of elements used.

Metal flow in two-hole Al-extrusion from FEA supported by experimental PGP

Unlubricated two-hole rod-extrusion, when using a flat-faced die, is a rather simple extrusion process, even though it introduces much complexity in relation to axisymmetric extrusion. A series of unlubricated extrusion experiments were run many years ago[13] using the PGP-technique to characterize metal flow in this case. Two dies with holes of 9 mm diameter were used in the experiments, one with long (LHD =44.75 mm) and the other die with short center-to-center-distance (SHD=14.9 mm) between the holes. Extrusion was performed in a laboratory press. The used billets had 73 mm diameter and 150 mm length, and extrusion was performed inside a 75 mm diameter container. A billet and container temperature of 480°C was used and the ram speed was 5 mms^{-1}. The alloy considered here is AA6063.44. Two deformed grid patterns obtained in this investigation (after 14.3% partial extrusion) are shown at the top of Fig.5.
A FEM model was made of the extrusion process for each hole distance. The model[14] in case of the LHD is shown in Fig.4; a) is the initial state and b) is after partial extrusion. As the depicted here, because of the large extrusion ratio, the extruded rods become long even though an early extrusion stage is considered.
Fig.5 shows that there is good agreement between the deformed stripe pattern in simulation and the corresponding experimental PGP-pattern. This shows that the simulation model is able to mimic real metal flow in this extrusion process accurately.

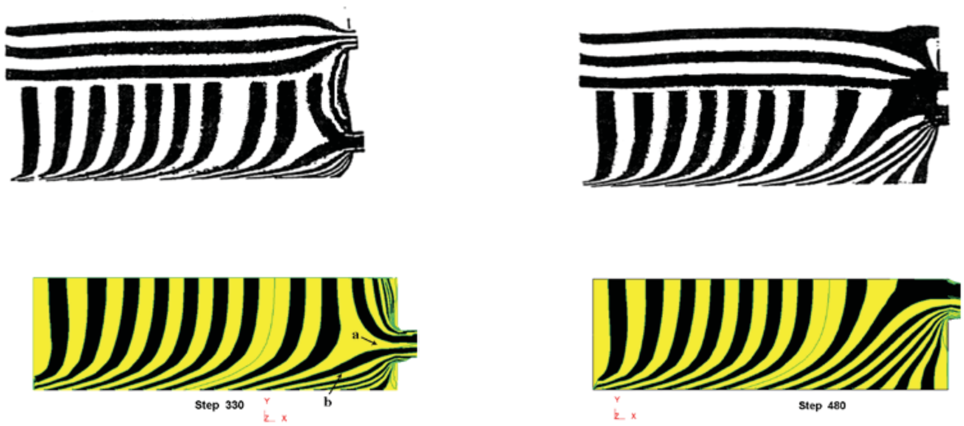


Fig.5. Comparison between experimental (at the top) and FEA-computed PGP (at the bottom).

Having confirmed that the FEM models predict real metal flow accurately, further studies were conducted using the two-hole extrusion model, to gain more knowledge about the mechanics of this process. One result obtained, in case of SHD, is that the metal flow inside most of the billet volume is quite similar to metal flow in single-hole extrusion with double hole size. But of course, this is not the case in the region near to the die. The flow near to the holes in two-hole extrusion is characterized by splitting of the material into two streams, instead of having one stream as in one-hole extrusion.

When extruding with the LHD the FEM model depicts a special metal flow phenomenon. It is perhaps best explained by referring to the three-dimensional effective strain-rate distribution picture shown in Fig.6, made in the post-processor of the simulation software. In position H (see figure), right above each hole, the FE-model predicts that a high strain-rate region will be present. Thus, due to the placement of the die holes close the container wall, the metal flow in this case is split into sepearate streams some distance above the die. Each stream is then directed outwards away from the centre of the billet towards the container wall where the flow becomes highly speeded up. Because of this, the shear deformation becomes much more localized here ($\dot{\bar{\varepsilon}} \approx 1s^{-1}$) than else where along the container wall ($\dot{\bar{\varepsilon}} \approx 0.1s^{-1}$), see Fig.6b). Moreover, when looking at this position in the experiment with greater attenion, one could actually see that the material of the billet at the container wall in this position in reality is characterized by sub-surface shear.

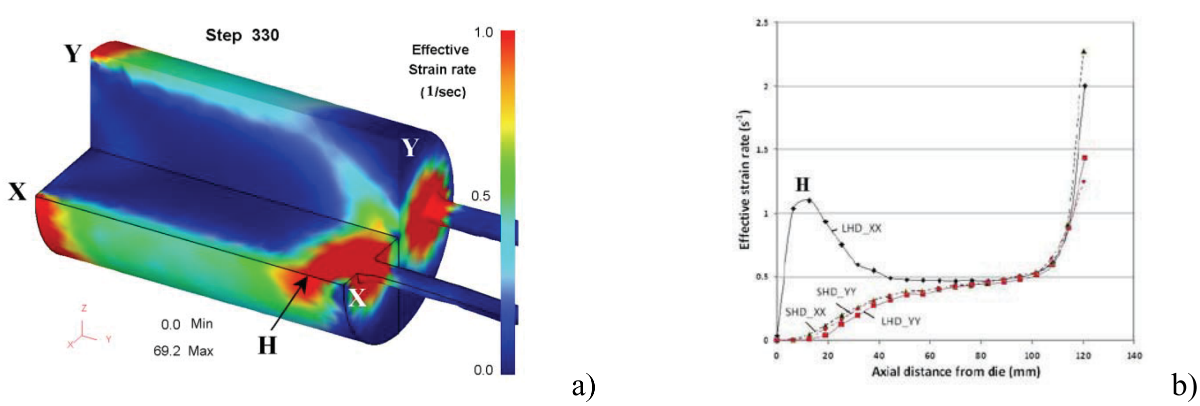
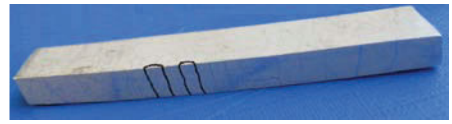
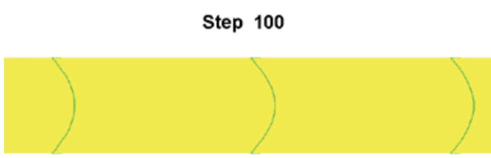


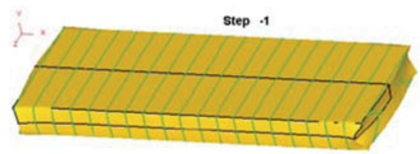
Fig.6. Results from FE-simulation; a) Effective strain-rate distribution inside the billet in LHD-case; and b) Distribution of the effective strain over the interface of the billet/the container wall along the lines XX and YY for both hole distances.

a) b)

c) d)





m04

e) f)

Fig.7. SuRIT-pattern used to reveal deformation in rolling. a-b) Half of experimental specimen showing appearance of pattern, c-d) Details from a-b) showing deformation of initial vertical lines across specimen thickness, and e-f) Corresponding patterns as c-d) obtained in FEA of rolling schedule.

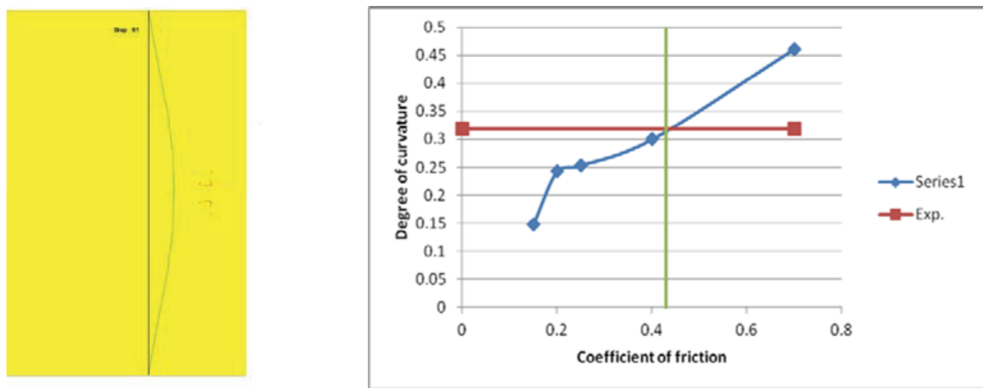
Inverse analysis to determine friction in cold rolling of aluminum

Use of the SuRIT-pattern for estimation of the friction in rolling will now be considered. Small slabs with this kind of pattern were made of an aluminum alloy by means of extrusion as described in connection with Fig.3. A number of identical specimens with initial dimensions t=10, w=20 and l=40 mm were rolled down at room temperature to thinner gauge at a low rolling speed of 40 mms^{-1} in a laboratory rolling mill using a roll with 200 mm diameter. Liquid paraffin was used as lubricant during the rolling process. The following rolling schedule was used: t_0=10, t_1=6.97 (st.1), t_2=5.45 (st.2) and final thickness t_3=3.64 mm (st.3). One specimen was rolled down to step 2, while two other specimens were rolled down to final gauge.

The rolled specimens were sectioned along their axial mid-plane to reveal the appearance of the final deformed grid patterns. Two of the sectioned specimens with the deformed pattern are shown in Fig.7a-b). The curvatures after rolling of what were initially straight vertical grid lines are shown as magnified details in Fig.7c-d).

Both the experiments and FE-models of the rolling schedule showed that vertical straight grid lines oriented across the thickness of the specimen would remain approximately straight during the first two rolling steps of the process, see Fig.7d). However, when the slab was rolled a third step these lines would deform into curved shape. This means that the surface layers of the slab lag behind the material in the core that becomes squeezed backwards in relation to the rolling direction.

The FE-model of the last rolling step (St.3) was run multiple times for different numerical values of the friction coefficient. In the simulation models the curvature of the line was predicted to depend on friction in the range $0.15 > \mu > 0.40$ as shown in Fig.8a-b). When this diagram is applied for estimating the friction coefficient in the rolling experiment (by inverse analysis) one obtains the result; $\mu=0.32$.



a) b)

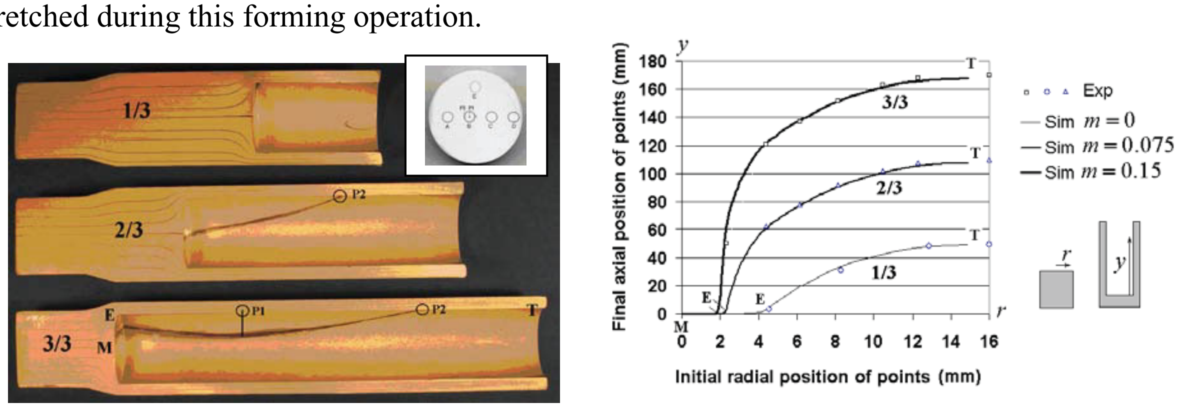
Fig.8. Friction dependent curving of initially straight vertical grid line across the thickness of a rolled slab. a) Picture from FEA showing curved line and b) Degree of curvature of stripe vs. μ as predicted by FEA.

If we assume that the FE-model reproduces the real rolling conditions well, we have this way by means of the inverse analysis been able to determine the actual friction coefficient in this laboratory rolling operation. What we have done is to apply the SuRIT-pattern inside the slab for "in-situ" determination of the friction in the last step in the considered rolling experiment.

Friction dependent metal flow in forging processes[15]

In-depth studies of different forging processes have also been conducted in our department, such as cold backward impact extrusion of a hollow cup, and hot forging of a straight forging with a flange-and-rib cross-section.

In backward cup-extrusion the investigation did include two different cup geometries. In one metal flow study a SuRIT-pattern was used on the ends of the slug, see Fig.9a). As the punch head penetrates into the slug during forming the slug material will be pressed backward out through the slot between the punch and the container die. In our particular case a component with a high cup wall was formed. In this case the penetration depth of the punch head is large. Therefore, the surface initially touching the punch head face will be heavily deformed and stretched during this forming operation.



a) b)

Fig.9. Friction dependent flow of the surface layer of a slug over the punch head in backward cup-extrusion: a) Appearance of SuRIT-pattern in experiment[16] and b) Simulation compared with experimental results to find friction over the punch head by inverse analysis.

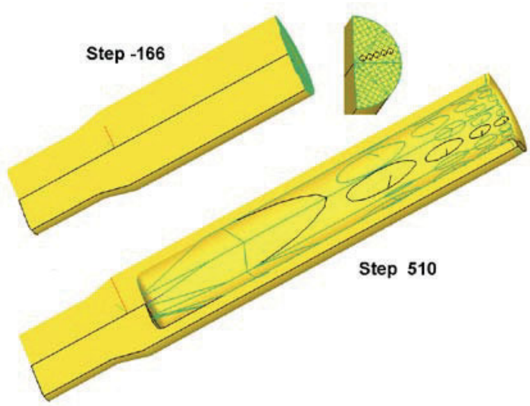


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Fig.10. FEA of the backward cup-extrusion process showing initial and final appearance of FEA-predicted surface ring pattern (m=0) of end-face of slug penetrated by the punch head.

A FE-model of the process was made and a number of simulations were run for different interfacial friction values. The stretching of the end face of the specimen over the punch head was quantified from the SuRIT-pattern used in the experiment. The same was done in FE-simulation (see Fig.10) for different values of the friction factor. A graphical representation of obtained data is shown in Fig.9b).

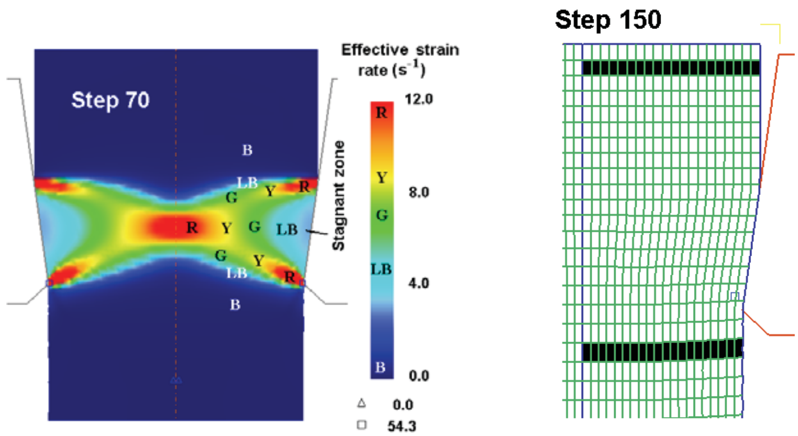
Equality between the simulation and the experiment with respect to surface expansion was obtained when the friction factor *m* was increased step-wise throughout the process. In the beginning *m* was set equal to zero (the first 1/3rd of the process), then during further forming it was increased to *m*=0.075 (up to 2/3^{rds} partial forming) and in the last 1/3rd stage it was increased to *m*=0.15. In this way, by inverse modeling using FEA combined with GP experiments, we were able to determine friction in this forming operation.

After this, the FEM model was used in a further study to characterize the metal flow in the cup-extrusion process. Results of the metal flow study are presented elsewhere[15].

Deformation in wiredrawing[17]

An extensive study was conducted on wiredrawing long time ago by Wistreich[18] who measured friction coefficients in some laboratory copper wiredrawing experiments using split drawing dies with different cone-shape. At our department we have successfully reproduced this experimental work by FEA. The FEA-predicted deformational behavior in low-friction drawing (μ=0.028) of wire in a die with geometry characterized by 8° half die cone angle, and a reduction of 0.20 (as investigated by Wistreich and commonly used in industry), is shown in Fig.11a). The simulation model predicts that in this case a characteristic deformation-cross will be present inside the plastic zone, and, in addition, there will be a dead (or stagnant) zone of metal adjacent to the cone wall of the die.

Thus the deformation history in this drawing operation is predicted quite different for the surface layer and the core of the wire. While the material in the core is subjected to high strain rates in a short time interval upon passage through the middle of the deformation cross, the surface material of the wire that flows through the deformation cross near to the die, experiences two separate strain-rate peaks, one at the inlet side, and the other at the outlet side from the dead zone. The FEA-predicted distortion of an internal GP inside the wire in this drawing situation is very small, see Fig.11b). Characteristically, however, the core of the wire is predicted to be dragged faster through the die than the corresponding surface material of the wire, which lags behind.



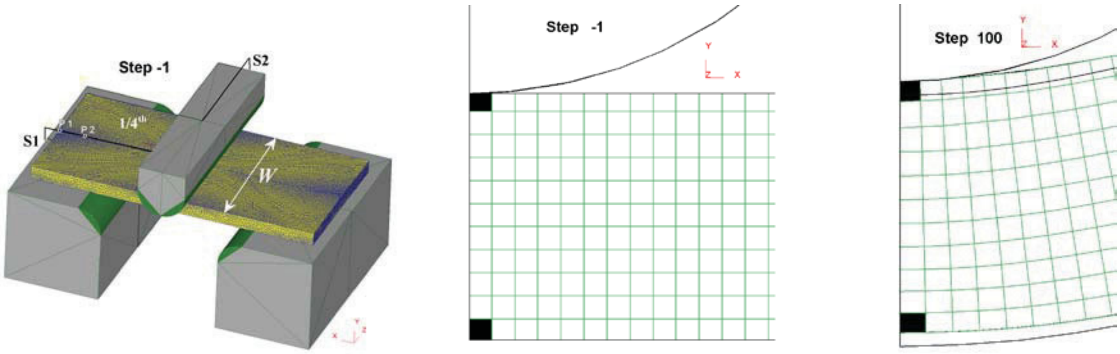
a) b)

Fig.11. Deformational conditions in wiredrawing of Cu through a die with industrial-like geometrical shape.

A FEA-study of friction effects on the final dome shape of an initially plane disk after flow through the die is to be performed in future work. It is expected the dome shape of such a disc will increase with increasing friction. Thus, straight grid lines across the thickness of the wire can be used for "in-situ" study of friction in wiredrawing, i.e. the inverse FE-analysis approach can easily be used in this case also to determine friction.

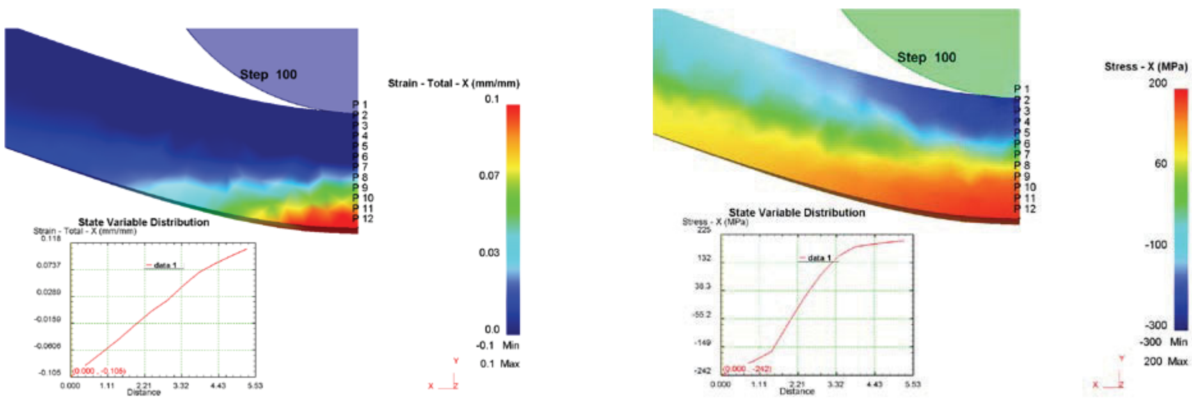
Sheetmetal forming by bending[19]

Sharp radius bending of sheet metal has been studied experimentally in our laboratory in a three-point bending rig. The bending operation has also been modeled by FEA as a three-dimensional process, because it is known that there will be three-dimensional effects of deformation near the edges of the specimen during bending. Visio-plasticity studies have not been performed by us, but the FE-model has been used in order to try to understand predicted deformations in the middle of the bend. The FEM model is shown in Fig.12a) and the results of the GP FEA-study are shown Fig.12b-c). As Fig.12c) shows there is bigger distortions subjected to a material element situated at the inside (or the outside) of the bend than correspondingly in the wiredrawing operation dealt with in the previous section. In future studies to validate the predicted deformations here the SuRIT-pattern can be a candidate for visio-plasticity analysis.



a) b) c)

Fig.12. FEA-model of three-point bending laboratory process; a) Overview over bending configuration, b) and c) Predicted distortions in GP introduced into the bend.



a) b)

Fig.12. Conditions in the middle of a sharp-radius bend in three-point bending as predicted by FEA; a) Strain distribution over the thickness of the bend and b) Stress distribution the same place.

FEA-predicted strain and stress distributions (in the x-direction) of the material in the bend have been shown in Fig.12. Moreover, this data have been plotted into the graph shown in Fig.13 in order to compare with simple strain-stress calculations when assuming no thinning and a uniaxial stress state in the bend. As one can see from this figure in this case there is not good agreement between the FEA-predictions and simple theory.

Discussion

Through a number of cases, it has been shown here how FEA combined with GPA can be applied for the purpose of studying the deformational conditions in metal processing. With use of internal grid patterns inside the workpiece one can characterize the internal distortions inside the metal, and with patterns on the surface one can determine the movement of the surface layers. The GP-techniques reveal the real plastic deformations of the material, and when combined with FEA one can arrive at an estimate of various forming parameters, such as the interfacial friction between tooling and workpiece. One should, however, be aware that when inverse analysis is performed this way, the accuracy of the results may not be good if inaccurate FE-models are used, or if some of the input data into the models are not accurate (as for instance flow stress data of the workpiece material).

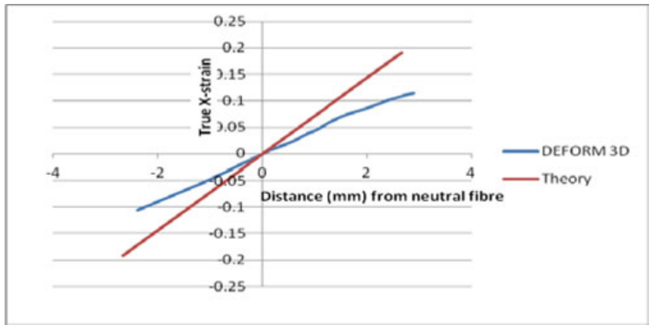


Fig.12. Strain distribution across bend in sharp-radius bending as predicted by FEA and simple idealized theory.

Conclusion

With the most recent developments within Finite Element Method simulation technology the metal former of today is equipped with a very strong tool for conducting accurate analysis of metal forming operations, before the operation is realized in the forming machine. In addition experimental grid pattern techniques are available which can be applied in order to determine the real deformation behavior in an industrial forming environment.
By a combined approach using FEA coupled with grid pattern studies one can obtain important "in-situ" information regarding conditions in an actual metal forming process. For instance, one can this way characterize metal flow, or determine the friction conditions or other important forming parameters.

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