

On the Expressiveness of Temporal Equilibrium Logic

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Abstract. We investigate expressiveness issues of Temporal Equilibrium Logic (TEL), a promising nonmonotonic logical framework for temporal reasoning. TEL shares the syntax of standard linear temporal logic LTL, but its semantics is an orthogonal combination of the LTL semantics with the nonmonotonic semantics of Equilibrium Logic. We establish that TEL is more expressive than LTL, and captures a strict subclass of ω -regular languages. We illustrate the expressive power of TEL by showing that LTL-conformant planning, which is not expressible in LTL, can be instead expressed in TEL. Additionally, we provide a systematic study of the expressiveness comparison between the LTL semantics and the TEL semantics for various natural syntactical fragments.

1 Introduction

Answer Set Programming (ASP) is now well established as a successful paradigm for declarative programming, with its roots in the fields of knowledge representation (KR), logic programming, and nonmonotonic reasoning (NMR) [3]. An adequate and well-known logical foundation for ASP is provided by *Equilibrium Logic* [19, 20], a nonmonotonic extension of the superintuitionistic logic of *here-and-there* (HT) [17]. This provides useful logical tools for the metatheory of ASP and a framework for defining extensions of the basic ASP language, for example to arbitrary propositional and first-order theories, to languages with intensional functions, and to hybrid theories that combine classical and rule-based reasoning [7, 10, 14, 21].

ASP has been applied to a wide range of temporal reasoning problems, including prediction, planning, diagnosis and verification. However, since it is not an intrinsically temporal formalism, it suffers some important limitations. Most ASP solvers deal with finite domains, which hampers the solution of temporal reasoning problems dealing with unbounded time, like proving the non-existence of a plan. Temporal scenarios dealing with unbounded time are typically best suited for modal temporal logics, a fundamental framework for the specification

An authors' online version of this paper is available at <https://www.dropbox.com/s/x0fnjzhjwira780/TEL%20Expression.pdf?dl=0>. Its appendix includes proofs that are omitted here for lack of space.

of the dynamic behavior of reactive systems. However, standard modal temporal logics, such as propositional linear-time temporal logic LTL [22], are not designed to deal with many issues in KR. These logics (like classical logics) have a monotonic consequence relation, meaning that adding a formula to a theory never produces a reduction of its set of consequences. A monotonic logic cannot handle various commonsense reasoning tasks such as reasoning by default.

Temporal Equilibrium Logic (TEL). TEL was proposed by Cabalar and Vega [8] as a nonmonotonic temporal logic, able to capture temporal reasoning problems not representable in ASP. It is apparently the only nonmonotonic extension of a standard modal temporal logic (viz. LTL) that does not use additional operators or constructions.

TEL shares the syntax of standard LTL, but its semantics is an orthogonal combination of the LTL semantics with the nonmonotonic semantics of Equilibrium Logic. As for Equilibrium Logic, the non-monotonic semantics of TEL is based on a selection criterion (a kind of minimization) among the models of the intermediate monotonic temporal logic of Here-and-There (THT), a combination of LTL and the propositional superintuitionistic logic of Here-and-There (HT).

Many works have been dedicated to the theoretical study of TEL, and some tools have been developed for computing models of temporal programs under TEL semantics (see e.g. [5]). Theoretical key results include the use of TEL to translate action languages [8], an automata-theoretic approach for checking the existence of TEL models [4], a decidable criterion for proving strong equivalence of two TEL theories [6], and a systematic study of the computational cost of TEL satisfiability [2] (a problem which is in general EXPSPACE-complete).

Our Contribution. We investigate expressiveness issues for the TEL framework. It is known [4] that like LTL, TEL allows to specify only ω -regular temporal properties. As a first contribution, we show that TEL is in general more expressive than LTL. In particular, the class of TEL-definable languages strictly includes the class of LTL-definable languages and is strictly included in the class of ω -regular languages. We also illustrate the expressive power of TEL by considering the problem of finding conformant plans for temporal goals in dynamic systems in the presence of incomplete information¹ when the goal and the system behavior are specified in LTL [9]. We show that this problem, which is not expressible in LTL [9], can be instead expressed in TEL.

As an additional non-trivial theoretical contribution, we provide a systematic study of the expressiveness comparison between the LTL semantics and the TEL semantics for various natural syntactical fragments. The considered fragments are obtained by restricting the set of allowed temporal modalities and/or by imposing a bound on the nesting depth of temporal modalities. The expressive power of LTL semantics for these fragments has been made relatively clear by numerous researchers. Thus, since for some of these fragments, TEL satisfiability is known to be relatively tractable [2], the aim is also to understand what kind of temporal reasoning problems can be captured by these fragments under the TEL

¹ On both the initial situation and on the full effects of actions.

semantics. Furthermore, we consider the class of splittable temporal programs [5], a TEL fragment which is known to be LTL-expressible and for which a solver has been implemented [5]. We show that a slight syntactical generalization of this fragment, obtained by relaxing a constraint on the use of temporal literals in the dynamic rules (intuitively ensuring that “the past does not depend on the future”), already leads to a fragment more expressive than LTL.

Some of the expressiveness results obtained also point to a peculiar difference between LTL and TEL: due to the interpretation of the implication connective, in TEL, a temporal modality cannot be expressed in terms of its ‘dual’ modality. Thus, in TEL, dual temporal modalities, such as F (‘eventually’) and G (‘always’), need to be considered independently from one another. This is illustrated by one of our results: while for the syntactical fragment whose allowed temporal modalities are F and X (‘next’), the TEL semantics is less expressive than the LTL semantics, for the dual fragment, the TEL semantics already allows one to express non-LTL-definable requirements.

Related Work. Several research areas of AI have combined modal temporal logics with formalisms from knowledge representation for reasoning about actions and planning (see e.g. [12]). Combinations of NMR with modal logics designed for temporal reasoning are much more infrequent in the literature. The few exceptions are typically modal action languages with a nonmonotonic semantics defined under some syntactical restrictions. Recently, an alternative to TEL has been introduced, namely, Temporal Answer Sets (TAS), which relies on dynamic linear-time temporal logic [16], a modal approach more expressive than LTL. However, while the non-monotonic semantics of TEL covers any arbitrary theory in the syntax of LTL, TAS uses a syntactic transformation that is only defined for theories with a rather restricted syntax. A framework unifying TEL and TAS has been proposed in [1].

2 Preliminaries

Let \mathbb{N} be the set of natural numbers and for all $i, j \in \mathbb{N}$, let $[i, j] := \{h \in \mathbb{N} \mid i \leq h \leq j\}$. For an infinite word w over some alphabet and for all $i \geq 0$, $w(i)$ is the i^{th} symbol of w . Let P and P' be two disjoint finite sets of atomic propositions. Given an infinite word w over 2^P and an infinite word w' over $2^{P'}$, $w \oplus w'$ denotes the infinite word over $2^{P \cup P'}$ given by $w(0) \cup w'(0), w(1) \cup w'(1), \dots$, and $w \oplus P'$ denotes the infinite word over $2^{P \cup P'}$ given by $w(0) \cup P', w(1) \cup P', \dots$. A proposition p is *flat in w* if $p \in w(i)$ for all $i \geq 0$. Note that each proposition $p' \in P'$ is flat in $w \oplus P'$. We extend the operator \oplus to ω -languages \mathcal{L} over 2^P in the obvious way: $\mathcal{L} \oplus P'$ denotes the ω -language over $2^{P \cup P'}$ consisting of the infinite words of the form $w \oplus P'$ where $w \in \mathcal{L}$.

2.1 Temporal Equilibrium Logic

We recall the framework of Temporal Equilibrium Logic (TEL) [8]. TEL is defined by first introducing a monotonic and intermediate version of standard linear temporal logic LTL [22], the so-called logic of *Temporal Here-and-There* (THT) [8].

The nonmonotonic semantics of TEL is then defined by introducing a criterion for selecting models of THT.

Syntax and Semantics of THT. While the syntax of THT coincides with that of LTL, the semantics of THT is instead an orthogonal combination of the superintuitionistic propositional logic of Here-and-There (HT) [17] and LTL. Fix a finite set P of atomic propositions. The set of THT formulas φ over P is defined by the following abstract syntax.

$$\varphi := p \mid \perp \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \rightarrow \psi \mid X\varphi \mid \varphi U\psi \mid \varphi R\psi$$

where $p \in P$ and X , U , and R , are the standard ‘next’, ‘until’, and ‘release’ temporal modalities. Negation is defined as $\neg\varphi \stackrel{\text{def}}{=} \varphi \rightarrow \perp$ while $\top \stackrel{\text{def}}{=} \neg\perp$. The classical temporal operators G (‘always’) and F (‘eventually’) can be defined in terms of U and R as follows: $F\varphi \stackrel{\text{def}}{=} \top U\varphi$ and $G\varphi \stackrel{\text{def}}{=} \perp R\varphi$. The size $|\varphi|$ of a formula φ is the number of distinct subformulas of φ . The *temporal depth* of φ is the maximum number of nested temporal modalities in φ .

Recall that LTL over P is interpreted on infinite words over 2^P , called in the following *LTL interpretations*. By contrast, the semantics of THT is defined in terms of infinite words over $2^P \times 2^P$, which can also be viewed as pairs of LTL-interpretations. Formally, a *THT interpretation* is a pair $M = (H, T)$ consisting of two LTL interpretations: H (the ‘here’ interpretation) and T (the ‘there’ interpretation) such that

$$\text{for all } i \geq 0, H(i) \subseteq T(i)$$

Intuitively, $H(i)$ represents the set of propositions which are true at position i , while $T(i) \setminus H(i)$ is the set of propositions which *may* be true (i.e. which are not falsified in an intuitionistic sense). A THT interpretation $M = (H, T)$ is said to be *total* whenever $H = T$. In the following, for *interpretation*, we mean a THT interpretation. Given an interpretation $M = (H, T)$, a position $i \geq 0$, and a THT formula φ , the satisfaction relation $M, i \models \varphi$ is inductively defined as follows:

$$\begin{aligned} M, i &\not\models \perp \\ M, i &\models p && \Leftrightarrow p \in H(i) \\ M, i &\models \varphi \vee \psi && \Leftrightarrow \text{either } M, i \models \varphi \text{ or } M, i \models \psi \\ M, i &\models \varphi \wedge \psi && \Leftrightarrow M, i \models \varphi \text{ and } M, i \models \psi \\ M, i &\models \varphi \rightarrow \psi && \Leftrightarrow \text{for all } H' \in \{H, T\}, \text{ either } (H', T), i \not\models \varphi \text{ or } (H', T), i \models \psi \\ M, i &\models X\varphi && \Leftrightarrow M, i+1 \models \varphi \\ M, i &\models \varphi U\psi && \Leftrightarrow \text{there is } j \geq i \text{ so that } M, j \models \psi \text{ and for all } k \in [i, j-1], M, k \models \varphi \\ M, i &\models \varphi R\psi && \Leftrightarrow \text{for all } j \geq i, \text{ either } M, j \models \psi \text{ or } M, k \models \varphi \text{ for some } k \in [i, j-1] \end{aligned}$$

We say that M is a (THT) model of φ , written $M \models \varphi$, whenever $M, 0 \models \varphi$. A THT formula φ is THT *satisfiable* if it admits a THT model. A formula φ is THT *valid* if every interpretation M is a THT model of φ . Note that the semantics of THT is defined similarly to that of LTL except for the clause for the implication connective \rightarrow which must be checked in both the components H and T of M . As a consequence $M, i \not\models \varphi$ does not correspond to $M, i \models \neg\varphi$ (i.e., $M, i \models \neg\varphi$ implies

that $M, i \not\models \varphi$, but the converse direction does not hold in general). However, if we restrict the semantics to total interpretations, $(T, T) \models \varphi$ corresponds to the satisfaction relation $T \models \varphi$ in LTL. More precisely, the LTL models T of φ correspond to the total interpretations (T, T) which are THT models of φ . With regard to THT validity, a THT valid formula is also an LTL valid formula, but the converse in general does not hold. For example, the *excluded middle* axiom $\varphi \vee \neg\varphi$ is not a valid THT formula since, as highlighted above, for an interpretation $M = (H, T)$, $M \not\models \varphi$ does not imply that $M \models \neg\varphi$. Similarly, the temporal formulas $F\varphi \leftrightarrow \neg G\neg\varphi$ and $\varphi_1 U \varphi_2 \leftrightarrow \neg\varphi_1 R \neg\varphi_2$, which are well-known valid LTL formulas (and allow to express, in LTL, a temporal modality in terms of its dual modality), are not THT valid formulas. Thus, in THT, dual temporal modalities, like F and G , or U and R , need to be considered independently from one another. The following proposition summarizes some observations made above, where we use \models_{LTL} to denote the satisfaction relation in LTL.

Proposition 1. *Let (H, T) be an interpretation and φ be a THT formula.*

1. *If $(H, T), i \models \varphi$, then $(T, T), i \models \varphi$ (for all $i \geq 0$).*
2. *$(H, T), i \models \neg\varphi$ iff $(T, T), i \models \neg\varphi$ (for all $i \geq 0$).*
3. *$(T, T) \models \varphi$ iff $T \models_{\text{LTL}} \varphi$.*

The non-monotonic logic TEL. This logic is obtained from THT by restricting the semantics to a subclass of models of the given formula, called *temporal equilibrium models*. For LTL interpretations H and T , $H \sqsubseteq T$ means that $H(i) \subseteq T(i)$ for all $i \geq 0$, and $H \sqsubset T$ means that $H \sqsubseteq T$ and $H \neq T$.

Definition 1 (Temporal Equilibrium Model). *Given a THT formula φ , a (temporal) equilibrium model of φ is a total model (T, T) of φ satisfying the following minimality requirement: whenever $H \sqsubset T$, then $(H, T) \not\models \varphi$.*

If we restrict the syntax to HT formulas (i.e., THT formulas where no temporal modality is allowed) and the semantics to HT interpretations $(H(0), T(0))$, then (non-temporal) equilibrium models coincide with stable models of answer set programs in their most general form [13]. In particular, the interpretation of negation is that of default negation in logic programming: formula $\neg\varphi$ holds (φ is false by default) if there is no evidence regarding φ , i.e., φ cannot be derived by the rules of the logic program. As a first example, let us consider the THT formula φ given by $\varphi = G(\neg p \rightarrow Xp)$. Its intuitive meaning corresponds to the first-order logic program consisting of rules of the form $p(s(X)) \leftarrow \text{not} p(X)$, where time has been reified as an extra parameter $X = 0, s(0), s(s(0)), \dots$. Thus, at any time instant, if there is no evidence regarding p , then p will become true at the next instant. Initially, we have no evidence regarding p , so this will imply Xp . To derive XXp , the only possibility would be the rule $\neg Xp \rightarrow XXp$, an instance of φ . As the body of this rule is false, XXp becomes false by default, and so on. It is easy to see that the unique equilibrium model of φ is $((\emptyset\{p\})^\omega, (\emptyset\{p\})^\omega)$.

Note that an LTL satisfiable formula may have no temporal stable model. As an example, consider the formula φ given by $\varphi = G(\neg Xp \rightarrow p) \wedge G(Xp \rightarrow p)$. The

unique LTL model is $\mathbb{T} = \{p\}^\omega$. However, (\mathbb{T}, \mathbb{T}) is not an equilibrium model of φ , since the interpretation (\mathbb{H}, \mathbb{T}) , where $\mathbb{H} = (\emptyset)^\omega$ is a THT model of φ .

For a THT formula φ , we denote by $\mathcal{L}_{\text{TEL}}(\varphi)$ (resp., $\mathcal{L}_{\text{LTL}}(\varphi)$) the ω -language over 2^P consisting of the LTL interpretations \mathbb{T} such that (\mathbb{T}, \mathbb{T}) is an equilibrium model of φ (resp., \mathbb{T} is an LTL model of φ). Note that by Proposition 1, $\mathcal{L}_{\text{TEL}}(\varphi) \subseteq \mathcal{L}_{\text{LTL}}(\varphi)$. A TEL language (resp., LTL language) is an ω -language of the form $\mathcal{L}_{\text{TEL}}(\varphi)$ (resp., $\mathcal{L}_{\text{LTL}}(\varphi)$) for some THT formula φ . We now observe the following.

Remark 1. LTL-definable languages are TEL-definable.

Indeed, by Proposition 1, the set of LTL models of a THT formula φ over P corresponds to the set of TEL models of $\varphi \wedge \psi_{\text{Tot}}(P)$, where formula $\psi_{\text{Tot}}(P)$ (we exploit this formula in many parts of the paper) captures, under the THT semantics, the total interpretations over P .

$$\psi_{\text{Tot}}(P) := \bigwedge_{p \in P} \mathbf{G}(p \vee \neg p)$$

Next, we observe that like LTL, the class of languages definable by TEL is strictly included in the class of ω -regular languages. Indeed, by [4], every TEL language is effectively ω -regular. Moreover, let us consider the ω -regular language $\mathcal{L}_{\text{even}}$ consisting of the LTL interpretations \mathbb{T} over $P = \{a\}$ of the form $\emptyset^{2n} \cdot \{a\} \cdot \emptyset^\omega$ for some $n \geq 0$ (where the maximal prefix preceding the unique a -position has even length). One can trivially check that $\mathcal{L}_{\text{even}}$ is not TEL definable. Hence:

Proposition 2. *The class of TEL languages is strictly included in the class of ω -regular languages.*

2.2 Problems Investigated and Summary of the Main Results

In this paper, we compare the expressive power of the LTL semantics and the TEL semantics for full THT and various syntactical THT fragments.

In particular, we consider the syntactical fragments of THT obtained by restricting the set of allowed temporal modalities and/or by bounding the temporal depth. Formally, given $O_1, O_2, \dots \in \{X, F, G, U, R\}$, we denote by $\text{THT}(O_1, O_2, \dots)$ the fragment of THT for which only the temporal modalities O_1, O_2, \dots are allowed. For $k \geq 0$, $\text{THT}_k(O_1, O_2, \dots)$ denotes the fragment of $\text{THT}(O_1, O_2, \dots)$ where the temporal depth is at most k . We write nothing for k when no bound is imposed. For instance, $\text{THT}_2(\mathbf{G})$ denotes the fragment where the unique allowed temporal modality is \mathbf{G} and the temporal depth is at most 2. We also consider a syntactical fragment of THT, we call *splittable* THT, corresponding to a generalization of splittable temporal programs introduced in [5]. A *temporal literal* is either an ordinary literal or a literal preceded by the

next operator X . A splittable THT formula is a conjunction of formulas of the following types:

- *Initial rules*: a formula of the form $B \rightarrow H$, where B is a conjunction of temporal literals and H is a disjunction of temporal literals.
- *Dynamic rules*: formulas of the form $G r$, where r is an initial rule.
- *Constraints*: formulas of the form $\neg\varphi$ for arbitrary THT formulas φ (such formulas impose constraints only on the ‘there’ part of an interpretation).

THT fragments under TEL semantics	LTL	
THT, THT(X, R), splittable THT	$>$	Theorem 6
THT(X, U), THT _{k+1} (X, U) ($k \geq 1$)	\perp	Theorem 6
THT(U), THT _{k+2} (U) ($k \geq 1$)	\perp	Theorem 6
THT _{k+1} (X, R), THT(R), THT _{k+2} (R) ($k \geq 1$)	\perp	Theorem 6
THT(X, G), THT _{k+1} (X, G) ($k \geq 1$)	$\not\leq$	Proposition 4
THT(X, F)	$<$	Theorem 3
THT(F, G)	$<$	Theorem 2
THT ₁	$<$	Theorem 5

Fig. 1. Expressive comparison between TEL fragments and full LTL

For two THT fragments \mathcal{F} and \mathcal{F}' and $\mathcal{S}, \mathcal{S}' \in \{\text{LTL}, \text{TEL}\}$, we say that \mathcal{F} under the \mathcal{S} -semantics is subsumed by \mathcal{F}' under the \mathcal{S}' -semantics, written $(\mathcal{F})_{\mathcal{S}} \leq (\mathcal{F}')_{\mathcal{S}'}$, if for each \mathcal{F} -formula φ , there is a \mathcal{F}' -formula φ' s.t. $\mathcal{L}_{\mathcal{S}'}(\varphi') = \mathcal{L}_{\mathcal{S}}(\varphi)$. Moreover, \mathcal{F}' under the \mathcal{S}' -semantics is more expressive than \mathcal{F} under the \mathcal{S} -semantics, denoted by $(\mathcal{F})_{\mathcal{S}} < (\mathcal{F}')_{\mathcal{S}'}$, if $(\mathcal{F})_{\mathcal{S}} \leq (\mathcal{F}')_{\mathcal{S}'}$ but not $(\mathcal{F}')_{\mathcal{S}'} \leq (\mathcal{F})_{\mathcal{S}}$. Additionally, we say that \mathcal{F} under the \mathcal{S} -semantics is expressively incomparable with \mathcal{F}' under the \mathcal{S}' -semantics, written $(\mathcal{F}')_{\mathcal{S}'} \perp (\mathcal{F})_{\mathcal{S}}$, if neither $(\mathcal{F})_{\mathcal{S}} \leq (\mathcal{F}')_{\mathcal{S}'}$ nor $(\mathcal{F}')_{\mathcal{S}'} \leq (\mathcal{F})_{\mathcal{S}}$. Sometime, we simply write LTL to mean $(\text{THT})_{\text{LTL}}$.

Figure 1 summarises some of the obtained results concerning the expressiveness comparison between the considered THT fragments under the TEL semantics and full THT under the LTL semantics.

3 Expressing LTL-conformant Planning in TEL

In this section, we illustrate the expressive power of TEL by showing that the LTL-conformant planning problem considered in [9], which is not expressible in LTL [9], can be instead expressed in TEL. Some other approaches in ASP for the formalization of conformant planning can be reformulated in the LTL-conformant planning framework such as the one based on Gelfond’s action language [15].

In the context of reasoning about actions and planning, we consider a setting where we have incomplete information on the dynamic system and the knowledge about the system is represented in LTL. In particular, the system is described by introducing a set of atomic facts, called fluents, whose truth value changes as the system evolves, and by specifying through LTL the effects of actions on such a set of facts. Thus, we consider two disjoint finite sets of atomic propositions: F – the set of fluents – and A – the set of actions. The behavior of the given system is specified by an LTL formula φ_s over $A \cup F$ which describes the set of possible evolutions of the system, each of which is represented as an infinite sequence of situations, where transitions from one situation to the next are caused by actions. Note that with this formalization, we may have incomplete information both on the initial situation and on the actual effects of actions so that, given a sequence of actions, we will have multiple possible evolutions, one of which is the actual one. The LTL-conformant planning problem consists in constructing a plan, i.e. a sequence of actions that guarantees the satisfaction of a temporal goal expressed in LTL whenever the conditions specified by φ_s are satisfied.

Formally, the LTL-conformant planning problem is the problem of finding, given two LTL formulas φ_s and φ_g over $A \cup F$ (representing the system specification and the temporal goal, respectively), an infinite sequence $\top_A = \{a_0\}, \{a_1\}, \dots$ of actions such that for all LTL interpretations \top_F over F (i.e. for all the possible infinite sequences of truth assignments to fluents), it holds that

$$\top_A \oplus \top_F \models \varphi_s \rightarrow \varphi_g$$

Let $Con(\varphi_s, \varphi_g)$ be the set of such conformant plans \top_A . Such a set cannot be in general expressed in LTL [9]. Here, we show that, unless an additional set of flat propositions, $Con(\varphi_s, \varphi_g)$ can be instead expressed in TEL. We construct in *linear-time* a THT formula φ_{con} whose set of equilibrium models corresponds to $Con(\varphi_s, \varphi_g) \oplus F'$, where $F' = F \cup \{u\}$, and u is a fresh (dummy) proposition non in $A \cup F$.

Before defining φ_{con} , we need additional definitions. A THT formula is in negation normal form (*NNF*) if the implication connective occurs only as negation and, additionally, negation is applied only to atomic propositions. By using De Morgan’s laws, the duality between \cup and \cap , and the fact that in LTL, $\xi_1 \rightarrow \xi_2$ can be rewritten as $\neg \xi_1 \vee \xi_2$, we can convert the THT formula $\neg(\varphi_s \rightarrow \varphi_g)$ into a THT formula $\bar{\psi}_{sg}$ in *NNF* having the same set of LTL models.

Let $K_u(\bar{\psi}_{sg})$ be the THT formula obtained from the *NNF* formula $\bar{\psi}_{sg}$ by replacing each occurrence of a negative literal $\neg p$ with $p \rightarrow u$. Intuitively, $p \rightarrow u$ is used to express negation on the ‘here’ part H of an interpretation (H, \top) such that u is flat in \top and $u \notin H(i)$ for all $i \geq 0$. Formally, one can easily show by structural induction that for such an interpretation, $(H, \top) \models K_u(\bar{\psi}_{sg})$ iff $H \models_{\text{LTL}} \bar{\psi}_{sg}$. Hence, $(H, \top) \models K_u(\bar{\psi}_{sg})$ iff $H \not\models_{\text{LTL}} \varphi_s \rightarrow \varphi_g$.

The THT formula φ_{con} over $A \cup F'$ is then defined as follows:

$$\varphi_{con} := G \left(\bigvee_{a \in A} (a \wedge \bigwedge_{a' \in A \setminus \{a\}} \neg a') \right) \wedge (\psi_{Tot}(A \cup F') \rightarrow \bigwedge_{p \in F'} Gp) \wedge (Fu \rightarrow \psi_{Tot}(A \cup F')) \wedge (u \vee K_u(\bar{\psi}_{sg}))$$

The first conjunct captures the THT interpretations (\mathbf{H}, \mathbf{T}) such that \mathbf{H} and \mathbf{T} agree over the set A of actions, and exactly one action occurs at any timestamp. The second and third conjuncts ensure that every proposition in $F' = F \cup \{u\}$ is flat in \mathbf{T} and whenever $\mathbf{H} \neq \mathbf{T}$, $u \notin \mathbf{H}(i)$ for every $i \geq 0$. Finally, the last conjunct is fulfilled iff whenever $\mathbf{H} \neq \mathbf{T}$, $\mathbf{H} \models_{\text{LTL}} K_u(\overline{\psi}_{g_s})$. Formally, the following holds, which proves the result (for details see the online version of this paper at <https://www.dropbox.com/s/x0fnjzhjwira780/TEL%20Expression.pdf?dl=0>).

Claim. $\mathcal{L}_{\text{TEL}}(\varphi_{\text{con}}) = \text{Con}(\varphi_s, \varphi_g) \oplus F'$.

4 Maximal Fragments Expressible in LTL

In this section, we individuate maximal THT fragments which under the TEL semantics are subsumed by full LTL.

The fragment THT(F, G). We show that full THT under the LTL semantics is more expressive than the fragment THT(F, G) under the TEL semantics. On the other hand, we additionally establish that for the considered fragment, the TEL semantics is more expressive than the LTL semantics. For the first result, we exploit a well-known characterization of the ω -regular languages which are LTL-expressible [18, 24]. In the following, we also consider *finite* (THT) interpretations which are non-empty prefixes of (THT) interpretations.

Definition 2 (*N-stutter Closure* [18, 24]). *For $N \geq 1$, an ω -language \mathcal{L} over an alphabet Σ is N -stutter closed if for all finite words x, y, u, w and infinite words v over Σ ,*

$$\begin{aligned} u \cdot w^N \cdot v \in \mathcal{L} &\text{ iff } u \cdot w^{N+1} \cdot v \in \mathcal{L} \\ x \cdot (u \cdot w^N \cdot y)^\omega \in \mathcal{L} &\text{ iff } x \cdot (u \cdot w^{N+1} \cdot y)^\omega \in \mathcal{L} \end{aligned}$$

Proposition 3 ([18, 24]). *If \mathcal{L} is an ω -regular language over 2^P which is N -stutter closed for some $N \geq 1$, then \mathcal{L} is LTL-expressible.*

For $\varphi \in \text{THT}(\mathbf{F}, \mathbf{G})$, let $N_\varphi := n^2(h+1)^{2^n}$, where $n = 2^{2^{|P|}}$ and h is the temporal depth of φ . We demonstrate that the language $\mathcal{L}_{\text{TEL}}(\varphi)$ is N_φ -stutter closed. For this, we use an additional notion, we call *h-bisimilarity*.

Definition 3 (*h-bisimilarity*). *Let w and w' be two finite words over an alphabet Σ and i and i' be two positions of w and w' , respectively. Given $h \geq 0$, (w, i) and (w', i') are h -bisimilar if $w(i) = w'(i')$ and whenever $h > 0$, then:*

- *for all $i \leq j < |w|$ (resp., $i' \leq j' < |w'|$), there exists $i'' \leq j'' < |w'|$ (resp., $i \leq j < |w|$) such that (w, j) and (w', j') are $(h-1)$ -bisimilar.*

We say that w and w' are h -bisimilar if $(w, 0)$ and $(w', 0)$ are h -bisimilar.

For each $h \geq 0$, a formula in $\text{THT}_h(\mathbf{F}, \mathbf{G})$ cannot distinguish under the THT semantics two interpretations where one is obtained from the other one by replacing finite segments with h -bisimilar ones. Formally, we establish the following result.

Lemma 1. *Let $h \geq 0$, $\varphi \in \text{THT}_h(F, G)$, and N and N' be two finite h -bisimilar interpretations. For all finite interpretations M_1, M_2, M_3 and infinite interpretations M_4 ,*

$$\begin{aligned} M_1 N M_4 \models \varphi &\text{ iff } M_1 N' M_4 \models \varphi \\ M_1 (M_2 N M_3)^\omega \models \varphi &\text{ iff } M_1 (M_2 N' M_3)^\omega \models \varphi \end{aligned}$$

The following lemma is based on a counting argument, asserts that for all $h \geq 1$ and finite interpretations consisting of concatenations of N segments, where $N \geq n^2(h+1)^{2^n}$ and $n = 2^{2^{|P|}}$, there always exists a segment whose removal or pumping preserves h -bisimilarity.

Lemma 2. *Let $h \geq 1$ and M be a finite interpretation of the form $M = M_1 \dots M_N$ such that $N \geq n^2(h+1)^{2^n}$, where $n = 2^{2^{|P|}}$. Then,*

- for some $j \in [1, N]$, M and $M' = M_1 \dots M_{j-1} \cdot M_{j+1} \dots M_N$ are h -bisimilar. Moreover, M' is non-total if M is non-total.
- for some $j \in [1, N]$, M and $M_1 \dots M_j \cdot M_j \cdot M_{j+1} \dots M_N$ are h -bisimilar.

By Lemmas 1 and 2, we deduce the desired result.

Theorem 1. *For each $\varphi \in \text{THT}(F, G)$, $\mathcal{L}_{\text{TEL}}(\varphi)$ is N_φ -stutter closed.*

Proof. Let $\varphi \in \text{THT}(F, G)$ and h be the temporal height of φ . We assume that $h > 0$ (otherwise, the result is obvious). Recall that $N_\varphi = n^2(h+1)^{2^n}$, where $n = 2^{2^{|P|}}$. Let T and T' be two LTL interpretations such that

$$\begin{aligned} \mathsf{T} &= u \cdot w^N \cdot v \text{ and } \mathsf{T}' = u \cdot w^{N+1} \cdot v \\ (\text{resp., } \mathsf{T} &= x \cdot (u \cdot w^N \cdot y)^\omega \text{ and } \mathsf{T}' = x \cdot (u \cdot w^{N+1} \cdot y)^\omega) \end{aligned}$$

for some finite words x, u, y, w and infinite words v , where $N \geq N_\varphi$ and $n = 2^{2^{|P|}}$. We show that $\mathsf{T} \in \mathcal{L}_{\text{TEL}}(\varphi)$ iff $\mathsf{T}' \in \mathcal{L}_{\text{TEL}}(\varphi)$. By Lemma 2, w^N and w^{N+1} are h -bisimilar. Thus, by Lemma 1, (T, T) is a THT model of φ iff $(\mathsf{T}', \mathsf{T}')$ is a THT model of φ . We prove the following, hence, the result follows:

1. for all $H \sqsubset \mathsf{T}$, there is $H' \sqsubset \mathsf{T}'$ such that $(H, \mathsf{T}) \models \varphi$ iff $(H', \mathsf{T}') \models \varphi$.
2. for all $H' \sqsubset \mathsf{T}'$, there is $H \sqsubset \mathsf{T}$ such that $(H, \mathsf{T}) \models \varphi$ iff $(H', \mathsf{T}') \models \varphi$.

We focus on Condition 1 (the proof of Condition 2 being similar). Let $H \sqsubset \mathsf{T}$ and $M = (H, \mathsf{T})$. Assume that $\mathsf{T} = u \cdot w^N \cdot v$ (the other case, where $\mathsf{T} = x \cdot (u \cdot w^N \cdot y)^\omega$, being similar). Then, M can be written in the form $M = M_1 N_1 \dots N_N M_2$ such that $|M_1| = |u|$ and $|N_i| = |w|$ for all $i \in [1, N]$. By Lemma 2, there exists $j \in [1, N]$ such that $N_1 \dots N_N$ is h -bisimilar to $N_1 \dots N_j \cdot N_j N_{j+1} \dots N_N$. Let $M' = M_1 N_1 \dots N_j N_j N_{j+1} \dots N_N M_2$. Since M is non-total, M' is non-total too, and by Lemma 1, $M \models \varphi$ iff $M' \models \varphi$. Moreover, since $\mathsf{T}' = u \cdot w^{N+1} \cdot v$, the non-total interpretation M' is of the form (H', T') , and we are done. \square

We now establish the main result for the fragment $\text{THT}(F, G)$.

Theorem 2. *$(\text{THT}(F, G))_{\text{TEL}} < \text{LTL}$ and $(\text{THT}(F, G))_{\text{TEL}} > (\text{THT}(F, G))_{\text{LTL}}$.*

Proof. One can easily show that the LTL-expressible ω -language consisting of the LTL interpretation $\emptyset \cdot \{a\} \cdot \emptyset^\omega$ cannot be expressed by any $\text{THT}(\mathbf{F}, \mathbf{G})$ formula under the TEL semantics. Thus, since TEL languages are ω -regular, by Proposition 3 and Theorem 1, we obtain that $(\text{THT}(\mathbf{F}, \mathbf{G}))_{\text{TEL}} < \text{LTL}$. For the second part of the theorem, first, we observe that for a $\text{THT}(\mathbf{F}, \mathbf{G})$ formula φ , the set of LTL models of φ corresponds to the set of TEL models of the $\text{THT}(\mathbf{F}, \mathbf{G})$ formula $\varphi \wedge \psi_{\text{Tot}}(P)$. Hence, $(\text{THT}(\mathbf{F}, \mathbf{G}))_{\text{TEL}} \geq (\text{THT}(\mathbf{F}, \mathbf{G}))_{\text{LTL}}$. It remains to show that $(\text{THT}(\mathbf{F}, \mathbf{G}))_{\text{TEL}} \not\leq (\text{THT}(\mathbf{F}, \mathbf{G}))_{\text{LTL}}$. For this, let $P = \{b, u\}$ and $(\mathbf{T}_1, \mathbf{T}_1)$ and $(\mathbf{T}_2, \mathbf{T}_2)$ be two total interpretations defined as follows: $\mathbf{T}_1 = \{u\}\{b, u\}^2\{u\}^\omega$ and $\mathbf{T}_2 = \{u\}\{b, u\}\{u\}^\omega$. No $\text{THT}(\mathbf{F}, \mathbf{G})$ formula can distinguish \mathbf{T}_1 and \mathbf{T}_2 under the LTL semantics. On the other hand, we show that there exists a $\text{THT}_1(\mathbf{F}, \mathbf{G})$ formula φ such that $(\mathbf{T}_2, \mathbf{T}_2)$ is a TEL model of φ , and $(\mathbf{T}_1, \mathbf{T}_1)$ is not.

$$\text{Let } \varphi := \mathbf{G}(\neg \neg u) \wedge \mathbf{F}b \wedge (\mathbf{F}u \rightarrow \psi_{\text{Tot}}(P)) \wedge \mathbf{F}((b \rightarrow u) \wedge \neg \neg b)$$

Under the THT semantics, the first three conjuncts capture the interpretations (\mathbf{H}, \mathbf{T}) such that (i) for all $i \geq 0$, $u \in \mathbf{T}(i)$, (ii) if $\mathbf{H} \neq \mathbf{T}$, then for all $i \geq 0$, $u \notin \mathbf{H}(i)$, and (iii) there is $h \geq 0$ such that $b \in \mathbf{H}(h)$. Additionally, the fourth conjunct is fulfilled whenever either $\mathbf{T} = \mathbf{H}$, or there is $k \geq 0$ such that $b \notin \mathbf{H}(k)$ and $b \in \mathbf{T}(k)$. It easily follows that the set of TEL models of φ is $\{u\}^* \cdot \{b, u\} \cdot \{u\}^\omega$, where there is exactly one occurrence of $\{b, u\}$, and the result follows. \square

The fragment $\text{THT}(\mathbf{X}, \mathbf{F})$. For the fragment $\text{THT}(\mathbf{X}, \mathbf{F})$, we crucially use the following known result [2], where for a total interpretation (\mathbf{T}, \mathbf{T}) , a position $i \geq 0$ is *non-empty* in (\mathbf{T}, \mathbf{T}) if $\mathbf{T}(i) \neq \emptyset$.

Lemma 3 ([2]). *Let φ be a $\text{THT}(\mathbf{X}, \mathbf{F})$ formula. Then, every equilibrium model of φ has at most $|\varphi|^2$ non-empty positions.*

Since there are $\text{THT}(\mathbf{X}, \mathbf{F})$ formulas whose LTL models contain infinite occurrences of non-empty positions (for example, the formula $\neg \mathbf{F} \neg p$), by Lemma 3 we easily deduce the following result.

Theorem 3. *Given a $\text{THT}(\mathbf{X}, \mathbf{F})$ formula φ , one can build a $\text{THT}(\mathbf{X}, \mathbf{F})$ formula ψ such that $\mathcal{L}_{\text{LTL}}(\psi) = \mathcal{L}_{\text{TEL}}(\varphi)$. Moreover, $(\text{THT}(\mathbf{X}, \mathbf{F}))_{\text{TEL}} < (\text{THT}(\mathbf{X}, \mathbf{F}))_{\text{LTL}}$.*

The fragment THT_1 . For the fragment THT_1 , where there is no nesting of temporal modalities, we first establish the following result.

Theorem 4. *Given a THT_1 formula φ , one can construct a THT formula whose LTL models correspond to the TEL models of φ .*

Sketched Proof. For the fixed finite set P of atomic propositions, it is possible to define an equivalence relation of *finite* index on total interpretations such that the following holds: (1) each equivalence class C is finitely representable and no THT_1 formula over P can distinguish elements of C under the TEL semantics, (2) given an equivalence class C and a THT_1 formula φ over P , one can effectively

check whether C is associated with TEL models of φ , and (3) each equivalence class C is effectively LTL-characterizable. \square

The construction in Theorem 4 cannot be done remaining in THT_1 . Indeed, the following holds.

Theorem 5. $(\text{THT}_1)_{\text{TEL}} < \text{LTL}$ and $(\text{THT}_1)_{\text{TEL}} > (\text{THT}_1)_{\text{LTL}}$.

Proof. Let us consider the LTL-expressible ω -language consisting of the LTL interpretation $\emptyset^3 \cdot \{a\} \cdot \emptyset^\omega$. One can easily show that such a language cannot be expressed by any THT_1 formula under the TEL semantics. Thus, by Theorem 4, we obtain that $(\text{THT}_1)_{\text{TEL}} < \text{LTL}$. For the second part of the theorem, first, we observe that for a THT_1 formula φ , the set of LTL models of φ corresponds to the set of TEL models of the THT_1 formula $\varphi \wedge \psi_{\text{Tot}}(P)$. Hence, $(\text{THT}_1)_{\text{TEL}} \geq (\text{THT}_1)_{\text{LTL}}$. It remains to show that there exists a THT_1 formula whose set of TEL models cannot be captured by any THT_1 formula under the LTL semantics. For this, let $P = \{b, u\}$, $\text{T}_1 = \{u\}\{b, u\}^2\{u\}^\omega$, and $\text{T}_2 = \{u\}\{b, u\}\{u\}^\omega$. Evidently, no THT_1 formula can distinguish T_1 and T_2 under the LTL semantics. On the other hand, by the proof of Theorem 2, there exists a $\text{THT}_1(\text{F}, \text{G})$ formula φ such that (T_2, T_2) is a TEL model of φ , and (T_1, T_1) is not. Hence, the result follows. \square

5 TEL Fragments Non-subsumed by LTL

In this section, we derive an almost complete picture of the TEL fragments (w.r.t. the considered THT syntactical hierarchy) which are expressively incomparable with LTL. We also show that under the TEL semantics, the fragment $\text{THT}(\text{X}, \text{R})$ and splittable THT are more expressive than LTL. We conclude the section by providing a characterization of ω -regular languages in terms of TEL languages.

We first individuate minimal THT fragments which under the TEL semantics are not subsumed by LTL.

Proposition 4. *Let \mathcal{F} denote any of the following THT fragments: $\text{THT}_2(\text{X}, \text{U})$, $\text{THT}_3(\text{U})$, $\text{THT}_3(\text{R})$, and splittable $\text{THT}_2(\text{X}, \text{G})$. Then $(\mathcal{F})_{\text{TEL}} \not\leq \text{LTL}$.*

Proof. Here, we focus on splittable $\text{THT}_2(\text{X}, \text{G})$ (for the other fragments, see the authors' full paper online). Let \mathcal{L}_{odd} be the ω -regular language given by

$$\mathcal{L}_{\text{odd}} := \{\text{T} \mid \text{T} = \{a, b, u\}^{2n+1} \emptyset^\omega \text{ for some } n > 0\}$$

where a, b , and u are distinct atomic propositions. One can easily show that \mathcal{L}_{odd} is not LTL expressible (see e.g. [11]). We exhibit a splittable $\text{THT}_2(\text{X}, \text{G})$ formula φ_{odd} over $P = \{a, b, u\}$ whose set of TEL models corresponds to \mathcal{L}_{odd} .

Formula φ_{odd} is the conjunction of the following three splittable $\text{THT}_2(\text{X}, \text{G})$ formulas, where $\psi_\emptyset := \neg a \wedge \neg b \wedge \neg u$ characterizes the empty positions of the “there” interpretation.

$$\neg \neg u \wedge \neg \neg \text{G}(\psi_\emptyset \vee (a \wedge b \wedge u)) \wedge \neg \neg \text{G}(\psi_\emptyset \rightarrow \text{X}\psi_\emptyset) \quad (1)$$

$$G(u \rightarrow a \wedge b) \wedge G(Xu \rightarrow u) \wedge G(u \rightarrow Xu \vee X\neg u) \quad (2)$$

$$a \wedge G(a \wedge b \rightarrow u) \wedge G(a \rightarrow Xb) \wedge G(b \rightarrow Xa \vee X\neg u) \quad (3)$$

Formula (1), which is a conjunction of constraints in a splittable THT formula, captures the interpretations (H, T) such that $T \in \{a, b, u\}^+ \emptyset^\omega$. Formula (2) additionally ensures that whenever $H \neq T$, then $u \notin H(i)$ for all positions i . Finally, formula (3) requires that whenever $H \neq T$, the prefix of H corresponding to the *slice* of T (i.e., the maximal prefix of T which does not contain empty positions) is in $(\{a\}\{b\})^+$. This last condition can be satisfied iff the length of the slice of T is even. Hence, it easily follows that the TEL language of φ_{odd} is exactly \mathcal{L}_{odd} , and we are done. \square

We now establish the main results of Sect. 5.

Theorem 6. *The following holds, where $k \geq 2$ and $\mathcal{O} \in \{U, R\}$:*

1. $(THT(X, U))_{TEL} \perp LTL$ and $(THT_k(X, U))_{TEL} \perp LTL$;
2. $(THT(X, R))_{TEL} > LTL$ and $(THT_k(X, R))_{TEL} \perp LTL$;
3. $(THT(\mathcal{O}))_{TEL} \perp LTL$ and $(THT_{k+1}(\mathcal{O}))_{TEL} \perp LTL$;
4. $(THT_k(X, R))_{TEL} > (THT_k(X, R))_{LTL}$ and $(THT_{k-1})_{TEL} > (THT_{k-1})_{LTL}$;
5. (splittable THT) $_{TEL} > LTL$.

Proof. In [2], it is shown that every TEL model of a $THT(X, U)$ has a finite set of non-empty positions. Since there are THT formulas whose LTL models have infinitely many non-empty positions, by Proposition 4, Properties 1 and 3 with $\mathcal{O} = U$ follow. One can trivially check that the LTL-expressible ω -language $\mathcal{L} = \emptyset\{a\}^\omega$ is not expressible in the fragment $THT(R)$ under the TEL semantics. Hence, by Proposition 4, Property 3 for the case $\mathcal{O} = R$ follows as well. For Properties 2 and 4, let $n \geq 1$ and \mathcal{L}_n be the LTL-expressible ω -language consisting of the LTL interpretation $\emptyset^n \cdot \{a\} \cdot \emptyset^\omega$. One can easily check that no THT_h formula with $h < n$ can capture \mathcal{L}_n under the TEL semantics. Since THT is expressively equivalent to $THT(X, R)$ under the LTL semantics and for all $h \geq 1$ and $\varphi \in THT_h(X, R)$, the set of LTL models of φ corresponds to the set of TEL models of the $THT_h(X, R)$ formula $\varphi \wedge \psi_{Tot}(P)$, by Theorem 5 and Proposition 4, Properties 2 and 4 follows. Finally, for Property 5, we exploit Proposition 4 and the fact that the set of LTL models of a THT formula φ corresponds to the set of TEL models of the splittable THT formula $\neg\neg\varphi \wedge \psi_{Tot}(P)$. \square

We conclude this section by showing that TEL languages capture in a *weak sense* the full class of ω -regular languages. In fact, this weak equivalence, as formalized by the following Theorem 7, is similar to the well-known equivalence between ω -regular languages and ω -languages defined by formulas of Quantified propositional LTL (QLTL)[23], where for capturing a given ω -regular language over 2^P by a QLTL-formula, one needs to use quantification over additional propositions not in P . Intuitively, flat propositions in TEL play the role of bounded propositions in QLTL.

Theorem 7. *Let \mathcal{L} be an ω -language over 2^P . Then, \mathcal{L} is ω -regular iff there exists a finite set Q disjoint from P such that $\mathcal{L} \oplus Q$ is a TEL language.*

6 Conclusion

We have provided a systematic study of the expressiveness comparison between the LTL semantics and the TEL semantics for various natural THT syntactical fragments. Some interesting questions remain open: for example, we don't know whether the *TEL* semantics of the fragment THT(F, G, X) is able to capture full LTL. Additionally, it is well-known that the class of LTL-definable languages is algebraically robust, being, in particular, closed under all boolean operations. It is an intriguing open question whether the same holds for the class of TEL-definable languages.

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