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ROLE OF THE PLASTIC FLOW OF THE MATRIX ON YIELDING AND VOID EVOLUTION OF POROUS SOLIDS

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Abstract

In this paper, it is shown that yielding and void evolution in a porous metallic material is strongly influenced by the particularities of the plastic flow of the matrix. This is demonstrated by comparing the effective response of porous solids for which the matrix is described by Tresca and von Mises yield criterion, respectively. The effective response of the porous solid is calculated analytically using rigorous limit analysis theorems and upscaling techniques. Analysis is conducted for both tensile and compressive axisymmetric loading scenarios and spherical void geometry. For the first time it is demonstrated that if the matrix plastic response is governed by Tresca yield criterion, the overall response is softer, the combined effects of pressure and the third-invariant on yielding being much stronger than in a porous solid with von Mises matrix. Furthermore, the rate of void growth or collapse is much faster in a porous solid with Tresca matrix.

Introduction

It is generally accepted that ductile failure in metals is due to the nucleation, growth, and coalescence of voids [1]. Voids evolve due to the plastic deformation of the surrounding solid material. Thus, it is essential to understand the influence of the yield criterion used to describe the plastic flow of the matrix (void-free material) on the response of the porous solid.. However, within the last decade most of the efforts have been devoted to the description of the effects of void geometry on the dilatational response and much less attention has been paid to understanding the role played by the plastic flow of the matrix. Indeed, in most of the available models for ductile damage, the matrix is described by the von Mises yield criterion (e.g. Gurson [3] and its various extensions such as Tvergaard and Needleman [4], Cazacu et al [5] etc.). Very recently, using rigorous limit-analysis theorems and upscaling techniques Cazacu et al [6] have derived an yield criterion for a porous solid for which the matrix's plastic behavior is governed by Tresca's yield criterion.

The aim of this paper is to investigate how the particularities of the plastic flow of the matrix affect the mechanical response of porous solids containing randomly distributed spherical voids. Specifically, we compare the effective response of porous solids for which the plastic behavior of the matrix is described by Tresca and von Mises yield criterion, respectively. To this end, we calculate for axisymmetric loading conditions the respective micro-plastic (local) dissipation. It is important to note that we do not adopt any of the approximations that are generally made (e.g. Gurson, [3]) in evaluating the respective local plastic dissipations. Thus, for the first time it is possible to account for the specificities of the plastic flow in the fully dense material on the yield criterion of the porous solid (Section 2). It is demonstrated that the combined effects of pressure and the third-invariant of the stress deviator on yielding are much more pronounced in a porous solid with matrix obeying Tresca criterion than in a porous solid with matrix governed by the von Mises criterion. Furthermore, for the first time the effects of the plastic flow of the matrix on void evolution are analyzed. For axisymmetric loadings corresponding to positive triaxialities, it is shown that in a porous solid with Tresca matrix the rate of void growth is much faster than in a porous solid with von Mises matrix. For example, for a triaxiality T= 2, the rate of void growth is twice as fast as that predicted by Gurson's [3] criterion. For negative triaxialities, the rate of void collapse is also sensitive to the plastic flow of the matrix, Gurson's [3] criterion predicting the slowest rate. However, for negative triaxialities the differences in void evolution are less pronounced than in the case of axisymmetric loadings at fixed positive triaxialities.

Influence of the yield criterion of the matrix on the mechanical response of a porous solid containing spherical voids

Beginning with the pioneering work of Rice and Tracey [7]andGurson [3], kinematic homogenization in conjunction with Hill-Mandel lemma [8] has proven to be a rigorous upscaling method for deriving the macroscopic plastic potential of porous metallic materials. For example, using this approach, Gurson [3] developed one of the most widely used criteria for porous solids. Its expression is:

$$\Phi = \left(\frac{\Sigma_e}{\sigma_T} \right)^2 + 2f \cosh\left(\frac{3\,\Sigma_m}{2\sigma_T} \right) - 1 - f^2 = 0, \qquad (1)$$

where f is the porosity, Σ_e is the von Mises effective stress, Σ_m is the mean stress, and σ_T is the tensile yield stress of the fully-dense material. In this paper, we will also use this approach to study how the particularities of the plastic flow of the matrix affect the effective response of the porous solids both in terms of yielding and void evolution. Specifically, we compare the response of porous solids for which the plastic behavior of the matrix is described by Tresca and von Mises yield criterion, respectively. In either case, the porous solid is supposed to contain randomly distributed spherical voids, hence a representative volume element (RVE) is a hollow sphere of inner radius, a, and outer radius, $b = a f^{-1/3}$. The analysis is done using the incompressible and isotropic local velocity field **v,** proposed by Rice and Tracey [7], which is consistent with boundary conditions of uniform strain rate, i.e. :

$$\mathbf{v} = \frac{b^3}{r^2} D_m \mathbf{e}_r + \left[D'_{11} \left(\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 \right) + D'_{33} \left(\mathbf{e}_3 \otimes \mathbf{e}_3 \right) \right] \mathbf{x}, \qquad (2)$$

In Eq. (2), ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) are the unit vectors of a Cartesian coordinate system, $\mathbf{D}$ is the imposed macroscopic strain rate tensor, which is supposed to be constant and axisymmetric; $\mathbf{D}'$ is the deviator of $\mathbf{D}$ while $D_m = \mathrm{tr}\,\mathbf{D}/3$. If the plastic behavior of the matrix is governed by the Von Mises criterion, the plastic dissipation corresponding to the local strain rate field $\mathbf{d} = \left(\nabla\mathbf{v} + \nabla\mathbf{v}^T \right)/2$ is:

$$\pi(\mathbf{d})^{\text{Mises}} = \sigma_T \sqrt{2\left(d_I^2 + d_{II}^2 + d_{III}^2 \right)/3} = \sigma_T \sqrt{4\, D_m^2 \left(b/r \right)^6 + 4 D_{11}'^2 + 2 D'_{11} D_m \left(b/r \right)^3 \left(1 + 3\cos 2\theta \right)}, \qquad (3)$$

while in the case when the matrix obeys Tresca's criterion, the local plastic dissipation is

$$\pi(\mathbf{d})^{\text{Tresca}} = \sigma_T \left(|d_I| + |d_{II}| + |d_{III}| \right)/2, \qquad (4)$$

where d_I, d_{II}, d_{III} are the principal values of the strain rate tensor **d**, σ_T is the uniaxial yield in tension of the matrix, and r and θ are spherical coordinates. Because Tresca's yield criterion depends on both invariants of the local stress deviator, the corresponding plastic dissipation depends on both the sign and ordering of the principal values of **d**. Thus, unlike in the case when plastic flow is governed by the von Mises criterion, the expression of the local plastic dissipation in terms of the invariants of the imposed strain rate tensor **D** is not unique. For example, if $D_m \geq 0$ and $D'_{11} \geq 0$:

$$\pi(\mathbf{d})^{\text{Tresca}} = \sigma_T \left[\frac{1}{2}\sqrt{D_m^2 (b/r)^6 + D_{11}'^2 + 2D_{11}' D_m (b/r)^3} + \frac{3}{2}\sqrt{D_m^2 (b/r)^6 + 2D_{11}' D_m (b/r)^3 \cos(2\theta) + D_{11}'^2} \right] \tag{5}$$

(for the expressions of the local plastic dissipation corresponding to all other loading cases, see Cazacu et al., 2013b). Comparison between Eq.(3) and Eq.(5) shows the strong differences between the plastic dissipation associated with the von Mises and Tresca criterion, respectively.
As already mentioned, in the literature it is usually assumed that coupling between shear and mean stress effects can be neglected (see [4,7]). For example, in his analysis Gurson considered that in the expression of $\pi(\mathbf{d})^{\text{Mises}}$ (see Eq. (4)) the *"cross-term"* $D_m D'_{11}$ can be neglected and proposed the following truncated form of the local plastic dissipation:

$$\pi(\mathbf{d})^{\text{Mises}} \cong \sigma_T \sqrt{4\, D_m^2 (b/r)^6 + 4 D_{11}'^2} \ .$$

Let analyze the implications of adopting the same simplifying hypothesis when estimating the plastic dissipation associated with Tresca's yield criterion. It can be easily seen that if we use the same approximation, Eq. (5) reduces to:

$$\pi(\mathbf{d})^{\text{Tresca}} \cong \sigma_T \sqrt{4\, D_m^2 (b/r)^6 + 4 D_{11}'^2} \ . \tag{6}$$

It is thus clearly demonstrated that neglecting the cross-term $D_m D'_{11}$ amounts to erasing the specificities of the plastic flow of the matrix, the resulting effective yield criterion for the porous solid being the same i.e. Gurson [3] (Eq. (1)). In other words, the approximation leads to the same effective yield criterion for the porous solid irrespective of the plastic behavior of the matrix i.e. whether the response is described by Tresca criterion, which incorporates dependence on both the second and third invariant of the deviator of the local stress, or by von Mises which describes only the influence of the second invariant. The macroscopic plastic potential of the porous solid can be further obtained by integrating over the RVE the local plastic dissipation [11]. Very recently, Cazacu et al. [5-6] has shown that in the case when the plastic behavior in the matrix is described by von Mises or Tresca criteria the respective integrals can be calculated analytically. In this paper, for the first time the void evolution predicted by both criteria is investigated. For sake of conciseness only the expression of the yield criterion for Tresca matrix obtained using Eq.(5) of $\pi(\mathbf{d})^{\text{Tresca}}$ is briefly recalled. Let denote by $u = 2|D_m|/D_e$ the macroscopic strain-rate triaxiality, $\mathbf{\Sigma}$ the effective (macroscopic) stress at yielding and $J_3^{\Sigma} = tr(\Sigma')^3/3$ the third-invariant of the stress deviator, $\mathbf{\Sigma}'$. The parametric representation of the yield surface of the porous solid with Tresca matrix is:

(a) For stress states such that $\Sigma_m \geq 0$ and $J_3^{\Sigma} \leq 0$:
(a1) For $u < f$:

$$
\begin{cases}
\Sigma_m / \sigma_T = \dfrac{1-f}{12u} + \dfrac{1}{24}\dfrac{16u\sqrt{uf}+6uf+f^2+9u^2}{u\sqrt{uf}}\ln\left(\dfrac{\sqrt{u}+\sqrt{f}}{\sqrt{f}-\sqrt{u}}\right) \\
\qquad -\dfrac{1}{24}\dfrac{6u+16u^{3/2}+1+9u^2}{u^{3/2}}\ln\left(\dfrac{\sqrt{u}+1}{1-\sqrt{u}}\right)+\dfrac{4}{3}\ln\left(\dfrac{\sqrt{f}-\sqrt{u}}{\sqrt{f}\left(1-\sqrt{u}\right)}\right) \\
\Sigma_e / \sigma_T = -\dfrac{1}{8}\left(5(f-1)+\dfrac{3}{2}\dfrac{u^2-2uf+f^2}{\sqrt{uf}}\ln\left(\dfrac{\sqrt{u}+\sqrt{f}}{\sqrt{f}-\sqrt{u}}\right)-\dfrac{3}{2}\dfrac{\left(u^2-2u+1\right)}{\sqrt{u}}\ln\left(\dfrac{\sqrt{u}+1}{1-\sqrt{u}}\right)\right)
\end{cases} \tag{7a}
$$

(a2) For $f < u < 1$:

$$
\begin{cases}
\dfrac{\Sigma_m}{\sigma_T} = \dfrac{1-f}{12u} + \dfrac{1}{24}\dfrac{16u\sqrt{uf}+6uf+f^2+9u^2}{u\sqrt{uf}}\ln\left(\dfrac{\sqrt{u}+\sqrt{f}}{\sqrt{u}-\sqrt{f}}\right) \\
\qquad -\dfrac{1}{24}\dfrac{6u+16u^{3/2}+1+9u^2}{u^{3/2}}\ln\left(\dfrac{\sqrt{u}+1}{1-\sqrt{u}}\right)+\dfrac{4}{3}\ln\left(\dfrac{\sqrt{u}-\sqrt{f}}{\sqrt{f}\left(1-\sqrt{u}\right)}\right) \\
\dfrac{\Sigma_e}{\sigma_T} = -\dfrac{1}{8}\left(5(f-1)+\dfrac{3}{2}\dfrac{u^2-2fu+f^2}{\sqrt{uf}}\ln\left(\dfrac{\sqrt{u}+\sqrt{f}}{\sqrt{u}-\sqrt{f}}\right)-\dfrac{3}{2}\dfrac{u^2-2u+1}{\sqrt{u}}\ln\left(\dfrac{1+\sqrt{u}}{1-\sqrt{u}}\right)\right)
\end{cases} \tag{7b}
$$

a3) For $u \geq 1$:

$$
\begin{cases}
\dfrac{\Sigma_m}{\sigma_T} = \dfrac{1-f}{12u} + \dfrac{1}{24}\dfrac{16u\sqrt{uf}+6uf+f^2+9u^2}{u\sqrt{uf}}\ln\left(\dfrac{\sqrt{u}+\sqrt{f}}{\sqrt{u}-\sqrt{f}}\right) \\
\qquad -\dfrac{1}{24}\dfrac{6u+16u^{3/2}+1+9u^2}{u^{3/2}}\ln\left(\dfrac{\sqrt{u}+1}{\sqrt{u}-1}\right)+\dfrac{4}{3}\ln\left(\dfrac{\sqrt{u}-\sqrt{f}}{\sqrt{f}\left(\sqrt{u}-1\right)}\right) \\
\dfrac{\Sigma_e}{\sigma_T} = -\dfrac{1}{8}\left(5(f-1)+\dfrac{3}{2}\dfrac{u^2-2fu+f^2}{\sqrt{uf}}\ln\left(\dfrac{\sqrt{u}+\sqrt{f}}{\sqrt{u}-\sqrt{f}}\right)-\dfrac{3}{2}\dfrac{u^2-2u+1}{\sqrt{u}}\ln\left(\dfrac{1+\sqrt{u}}{\sqrt{u}-1}\right)\right)
\end{cases} \tag{7c}
$$

(b) For stress states such that $\Sigma_m \geq 0$ and $J_3^{\Sigma} \geq 0$:

(b1) For $u \leq f$:

$$
\begin{cases}
\dfrac{\Sigma_m}{\sigma_T} = \dfrac{1-f}{12u} + \dfrac{1}{24}\dfrac{\left(9u^2-6fu+f^2\right)}{u\sqrt{uf}}\arctan\left(\dfrac{2\sqrt{uf}}{f-u}\right)-\dfrac{1}{24}\dfrac{\left(9u^2-6u+1\right)}{u^{3/2}}\arctan\left(\dfrac{2\sqrt{u}}{1-u}\right)+\dfrac{2}{3}\ln\left(f\dfrac{u+1}{u+f}\right) \\
\dfrac{\Sigma_e}{\sigma_T} = -\dfrac{1}{8}\left(5(f-1)+\dfrac{3}{2}\dfrac{u^2+2uf+f^2}{\sqrt{uf}}\arctan\left(\dfrac{2\sqrt{uf}}{f-u}\right)-\dfrac{3}{2}\dfrac{u^2+2u+1}{\sqrt{u}}\arctan\left(\dfrac{2\sqrt{u}}{1-u}\right)\right)
\end{cases} \tag{7d}
$$

b) For $f < u < 1$

$$
\begin{cases}
\dfrac{\Sigma_m}{\sigma_T} = \dfrac{1+f}{12u} + \dfrac{1}{24}\dfrac{9u^2-6uf+f^2}{u\sqrt{uf}}\arcsin\left(\dfrac{2\sqrt{uf}}{u+f}\right)-16\ln(2)-2 \\
-\dfrac{1}{24}\dfrac{\left(9u^2-6u+1\right)}{u^{3/2}}\arctan\left(\dfrac{2\sqrt{u}}{1-u}\right)+\dfrac{2}{3}\ln\left(\dfrac{(u+f)(u+1)}{f}\right) \\
\dfrac{\Sigma_e}{\sigma_T} = -\dfrac{1}{8}\left(10u-5(f+1)+\dfrac{3}{2}\dfrac{\left(u^2+2uf+f^2\right)}{\sqrt{uf}}\arcsin\left(\dfrac{2\sqrt{uf}}{u+f}\right)-\dfrac{3}{2}\dfrac{\left(u^2+2u+1\right)}{\sqrt{u}}\arctan\left(\dfrac{2\sqrt{u}}{1-u}\right)\right)
\end{cases} \tag{7e}
$$

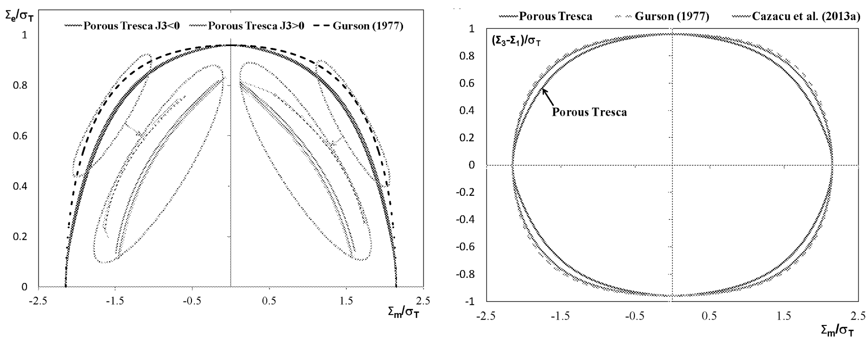
c) For $u \geq 1$:

$$\begin{cases} \dfrac{\Sigma_m}{\sigma_T} = \dfrac{f-1}{12u} + \dfrac{1}{24}\dfrac{9u^2-6uf+f^2}{u\sqrt{uf}}\arcsin\left(\dfrac{2\sqrt{uf}}{u+f}\right) - \dfrac{1}{24}\dfrac{9u^2-6u+1}{u^{3/2}}\arcsin\left(\dfrac{2\sqrt{u}}{u+1}\right) + \dfrac{2}{3}\ln\left(\dfrac{u+f}{f(u+1)}\right) \\ \dfrac{\Sigma_e}{\sigma_T} = -\dfrac{1}{8}\left(5(1-f) + \dfrac{3}{2}\dfrac{\left(u^2+2uf+f^2\right)}{\sqrt{uf}}\arcsin\left(\dfrac{2\sqrt{uf}}{u+f}\right) - \dfrac{3}{2}\dfrac{\left(u^2+2u+1\right)}{\sqrt{u}}\arcsin\left(\dfrac{2\sqrt{u}}{u+1}\right)\right) \end{cases} \quad (7f)$$

It is worth noting that the local plastic dissipation $\pi(\mathbf{d})^{Tresca}$ is an even function of the local strain rate tensor **d**. Thus, the effective plastic potential of the porous solid with Tresca matrix is an even function of **D** and the yield surface is an even function of the stress tensor, Σ. It follows that:

$$\begin{aligned} &\frac{\Sigma_m}{\sigma_T} = -\left.\frac{\Sigma_m}{\sigma_T}\right|_{J_3^\Sigma \leq 0,\ \Sigma_m \geq 0} \;; \quad \frac{\Sigma_e}{\sigma_T} = \left.\frac{\Sigma_e}{\sigma_T}\right|_{J_3^\Sigma \leq 0,\ \Sigma_m \geq 0} \\ &\frac{\Sigma_m}{\sigma_T} = -\left.\frac{\Sigma_m}{\sigma_T}\right|_{J_3^\Sigma \geq 0,\ \Sigma_m \geq 0} \;; \quad \frac{\Sigma_e}{\sigma_T} = \left.\frac{\Sigma_e}{\sigma_T}\right|_{J_3^\Sigma \geq 0,\ \Sigma_m \geq 0} \end{aligned} \quad (7g)$$

This criterion will be designated in the following as porous Tresca. As an example, in Fig. 1 are shown the yield curves according to the porous Tresca criterion (Eq. (7)) corresponding to $J_3^\Sigma \geq 0$ and $J_3^\Sigma \leq 0$, respectively, and the Gurson yield surface (Eq. (1)) corresponding to the same porosity, f = 0.04. In contrast with Gurson's criterion, the yield locus depends on the signs of the mean stress and J_3^Σ. For stress states with tensile mean stress, $\Sigma_m \geq 0$ the response is softer for $J_3^\Sigma \geq 0$ than for $J_3^\Sigma \leq 0$, while the reverse holds true for $\Sigma_m \leq 0$. It is worth noting that the yield surface of the porous material is smooth although its matrix is governed by Tresca yield criterion (i.e. the presence of voids "smooth out" the corners of Tresca's criterion). Because no approximations were made when calculating the local plastic dissipation, the yield criterion of the porous solid has a very specific dependence on the signs of the mean stress and J_3^Σ.To better assess this effect, the porous Tresca yield surface (Eq.(7)), the yield surface for porous solids with von Mises matrix proposed by Cazacu et al [5] along with Gurson's [3] yield surface corresponding to the same porosity (f = 0.04) are represented in the plane (Σ_1-Σ_3, Σ_m), where Σ_1 denotes the axial stress and Σ_3 the lateral stress, the mean stress being $\Sigma_m = (\Sigma_1 + 2\Sigma_3)/3$ (see Fig.1(b)). It is clearly seen the strong influence of the particularities of the plastic flow of the matrix on yielding of the porous solid. In particular, the third-invariant effects on yielding are much stronger for a porous solid with matrix obeying Tresca criterion than for a porous solid with matrix obeying von Mises criterion. Furthermore, for stress triaxialities $T = \Sigma_m/\Sigma_e$ different from zero or infinity, the response of the porous Tresca material (Eq. (7)) is softer than that of a porous solid with Von Mises matrix according to Cazacu et al. [5] criterion. Note that Gurson [3] is an upper bound for both criteria, the difference in yielding between the porous Tresca and Gurson's [3] being very strong. This very strong influence of the plastic flow of the matrix on yielding of the porous solid can be easily explained by comparing $\pi(\mathbf{d})^{Tresca}$ (Eq.(5)), $\pi(\mathbf{d})^{Mises}$ (Eq.(3)) and the truncated expression of $\pi(\mathbf{d})^{Mises}$ used by Gurson [3] (same as Eq. (7)). Obviously, all criteria coincide for T = 0 (i.e. purely deviatoric loadings) when the yield limit is given by $\Sigma_e = |\Sigma_1 - \Sigma_3| = \sigma_T(1-f)$ or for T = ∞ (i.e. purely hydrostatic loadings) when the yield limit is $\Sigma_m = \pm(2\sigma_T/3)\ln f$. Note also that the curvature of the yield surface of a porous solid depends strongly on the criterion that governs the plastic behavior of the matrix. This implies a marked difference in void evolution that will be further examined in the next section.



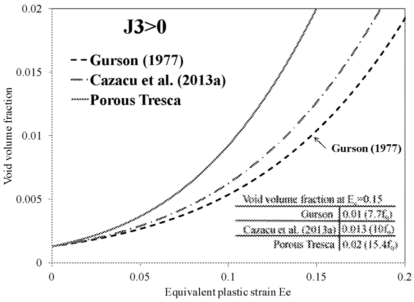
(a) (b)

Figure 1. Yield surface of the porous solid according to the porous Tresca criterion (Eq. (7)) for axisymmetric stress states for which $J_3^\Sigma \leq 0$ and $J_3^\Sigma \geq 0$, respectively, in comparison with Gurson's [3] for the same porosity (f= 0.04): (a) (Σ_e,Σ_m) plane, (b) (Σ_1-Σ_3, Σ_m) plane.

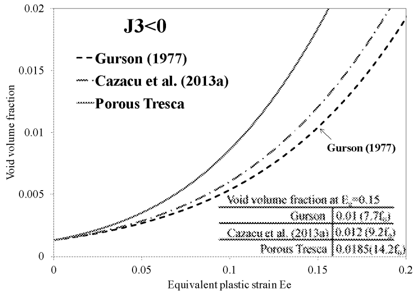
Effect of the plastic flow of the matrix on void evolution

The influence of the yield criterion describing the plastic flow of the matrix on void evolution in the porous solid is investigated by comparing the predictions of the porous Tresca criterion, and that of Cazacu et al. [5] and Gurson's [3] (matrix modeled by von Mises yield criterion). Fig.2 show the predicted void evolution as a function of the effective macroscopic equivalent strain, E_e for axisymmetric loadings at fixed triaxiality, T=2 corresponding to $\Sigma_1 \geq \Sigma_3$ (i.e. $J_3^\Sigma \leq 0$) and $\Sigma_1 \leq \Sigma_3$ (i.e. $J_3^\Sigma \geq 0$), respectively. The initial porosity is f_0 = 0.0013. Irrespective of the sign of the third-invariant, the rate of void growth is much faster in a porous solid with Tresca matrix than in a porous solid with von Mises matrix. Furthermore, because Gurson's [3] criterion is an upper bound for both criteria, it predicts the slowest void growth. The differences in the rate of void growth are significant. For example, for $J_3^\Sigma \geq 0$ at a macroscopic equivalent plastic strain E_e = 0.15, Gurson [3] predicts that the porosity f = 0.01, Cazacu et al. (2013) predicts that f = 0.013 while according to the porous Tresca criterion (Eq. (7)) f =0.02. These results also show that neglecting the coupling between shear and mean stress in the expression of the local plastic dissipation amounts not only to erasing the particularities of the plastic flow of the matrix (see Eq. (7)) but also to a drastic underestimation of the rate of void growth. The same conclusions can be drawn by analyzing the predicted void evolution for T=2 and $J_3^\Sigma \leq 0$ (Fig. 2b). Since both Cazacu et al. [5] and the porous Tresca criterion involve a very specific dependence on the signs of the mean stress and the third-stress invariant, J_3^Σ, void evolution depends on the sign of J_3^Σ. Indeed, comparison between the results presented in Fig2(a) and Fig. 2(b) shows that these criteria predict an influence of the third invariant of the stress deviator on void growth, the rate of void growth being faster for $J_3^\Sigma \geq 0$ (Fig. 2a) than for $J_3^\Sigma \leq 0$ (Fig. 2b). According to both criteria, at an equivalent plastic strain E_e= 0.15, the void volume fraction is almost 8% higher for $J_3^\Sigma \geq 0$

than for $J_3^\Sigma \le 0$. Obviously, since Gurson's [3] does not account for couplings between mean stress and shear stresses it cannot capture the influence of the sign of J_3^Σ on void growth.



(a)



(b)

Figure 2. Comparison between the evolution of the void volume fraction with equivalent strain E_e for fixed stress triaxiality T = 2 predicted by Gurson's [3], Cazacu et al. [5], and the porous Tresca criterion (Eq.(7)); initial porosity, f_0 = 0.0013 : (a) loadings such that $\Sigma_1 = \Sigma_2 \le \Sigma_3$ (i.e. $J_3^\Sigma \ge 0$) and (b) loadings such that $\Sigma_1 = \Sigma_2 \ge \Sigma_3$ (i.e. $J_3^\Sigma \le 0$).

Figure 3 shows the evolution of the void volume fraction as a function of the effective equivalent strain, E_e for axisymmetric loadings and negative stress triaxiality, T = -2, the initial porosity being set to f_0=0.05. Due to the fact that the mean stress, Σ_m, is negative (compression) void collapse occurs. Note that the rate of void collapse is much faster in a porous solid with Tresca matrix than in a porous solid with von Mises matrix. Furthermore, as demonstrated previously, Gurson's [3] criterion is an upper bound for both criteria and as such predicts the slowest rate of void collapse. For example, at Ee = 0.15, the void volume fraction according to the porous Tresca criterion, Cazacu et al. [5], and Gurson [3] are: f = 0.0062, f = 0.0087, and f = 0.0099, respectively. It is very interesting to note that the influence of the plastic flow of the matrix is stronger on void growth than on void collapse.

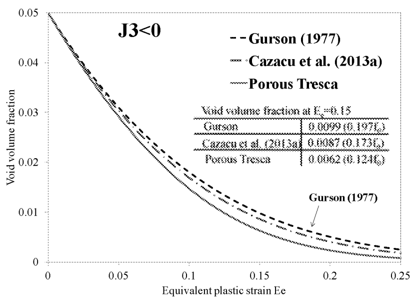


Figure 3. Comparison between the void volume fraction evolution with equivalent strain E_e for axisymmetric stress states such that $J_3^\Sigma \le 0$ and stress triaxiality T = -2 according to Gurson's [3], Cazacu et al. [5], and the porous Tresca criterion (Eq.(7)); initial porosity, f_0 = 0.05.

Summary and Conclusions

In this paper, it was shown that the plastic flow of the matrix has a strong influence on yielding and void evolution of porous solids containing randomly distributed spherical voids. This was demonstrated by comparing the effective response of porous solids for which the matrix is described by Tresca and von Mises yield criterion. For the first time, analysis was conducted for both tensile and compressive axisymmetric loading scenarios. The main findings are:

- While in the literature it is assumed that coupling between shear and mean stress effects can be neglected (see Gurson, [3]) we demonstrated that this approximation leads to the same effective yield criterion for the porous solid irrespective of the plastic behavior of the matrix i.e. whether the response is described by Tresca criterion, which incorporates dependence on both the second and third invariant of the deviator of the local stress, or by von Mises which describes only the influence of the second invariant.
- For stress triaxialities different from zero or infinity, the response of the porous Tresca material (Eq. (7)) is softer than that of a porous solid with Von Mises matrix. In particular, the difference in yielding between the porous Tresca and Gurson's [3] is very strong (see Fig. 1).
- Third-invariant effects on yielding are much stronger in a porous solid with matrix obeying Tresca criterion than in a porous solid with matrix obeying von Mises criterion (see also Fig.2).
- For axisymmetric loadings corresponding to positive stress triaxialities, it was shown that the rate of void growth is much faster in a porous solid with Tresca matrix than in a porous solid with von Mises matrix. For example, for a triaxiality T= 2, the rate of void growth is twice as fast than that predicted by Gurson's [3] criterion.
- For negative triaxialities, the rate of void collapse is also sensitive to the plastic flow of the matrix, Gurson's [3] criterion predicting the slowest rate. However, the differences in void evolution are less pronounced than for positive triaxialities.

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