Fatigue of Materials II Advances and Emergences in Understanding
Edited by: T.S. Srivatsan, M. Ashraf Imam, and R. Srinivasan
TMS (The Minerals, Metals & Materials Society), 2013

FUNDAMENTALS OF FATIGUE CRACK INITIATION and PROPAGATION: SOME THOUGHTS

C. S. Pande[1]
Materials Science and Technology Division
Naval Research Laboratory
4555 Overlook Avenue S.W.
Washington DC 20375-5343, U. S. A.

Abstract

The prediction of fatigue properties of structural materials is rightly recognized as one of the most important engineering problem. Therefore a basic understanding of the fundamental nature of fatigue crack initiation and growth in metals has long been a major scientific challenge starting with the first dislocation model of fatigue crack growth of Bilby et al. in 1963. For this purpose understanding the process of emission of dislocations from cracks, and determining precise expressions for the size of the plastic zone size, the crack-tip opening displacement and the energy release rate of the cracks are some of the major technical challenges. In this short paper we comment briefly on some of our important recent results obtained theoretically and by in-situ TEM studies and discuss how they may contribute to the understanding of the phenomena

Keywords: fatigue, metallic materials, dislocations, cracks shielding.

Introduction

The progressive failure of a material by the incipient growth of flaws under cyclically varying stress is termed as fatigue and has over the years accounted for a vast majority of in-service failures in structures and components in the domain of aerospace engineering (airframe structures), in civil engineering (off-shore platforms, buildings, bridges), and in mechanical engineering. Such a failure can be a consequence of pure mechanical loading, aggressive environments (corrosion-fatigue) or elevated temperatures (creep-fatigue). The fatigue failure process can be categorized into the following discrete, yet mutually related and interactive, phenomena involving (i) cyclic plastic deformation prior to crack initiation, (ii) initiation of one or more microscopic cracks, (iii) growth and eventual coalescence of the microscopic cracks to form one or more macroscopic cracks, (iv) subsequent growth or propagation of both the microscopic and macroscopic cracks, and (v) final catastrophic failure [1].

Five fundamental questions need to answered in the study of the fatigue phenomenon are: (i) how cracks are produced? (ii) Where do the dislocations come from? (iii) How do the cracks interact with the dislocations, spanning both mobile and immobile? (iv) How cracks are able to propagate at loads far less than that needed for fracture? and (v) What is the effect and/or contribution of intrinsic microstructural features, such as: voids, interfaces, grain boundaries, and second-phase particles? Once these questions have been addressed, we will be in a position to use this knowledge to predict the fatigue behavior in a variety of materials and structures.

Crack Nucleation:

Crack initiation is the formation process of fine microscopic cracks having less than detectable length, say 0.001 inch or 0.025 mm. Over the years, research in the general area of metal fatigue and cyclic deformation has led to several hypotheses for fatigue crack initiation [2-15]. Most investigators generally agree that the fatigue cracks initiated at or near singularities on or just below the surfaces of metals. Such singularities may be inclusions, embrittled grain boundaries, sharp scratches, pits and slip bands [16]. However, subsurface nucleation has also been observed in metals having a strong adherent surface oxide, which retards crack initiation at the external surface [17-20]

It is now known that dislocations are convincingly involved both in the initiation and propagation of cracks in metallic materials. Basinski and Basinski [21] found that cracks could easily nucleate at the thin persistent slip bands (PSB's) that are generated immediately prior to fracture initiation. Mott [22] in 1958 suggested that vacancies are generated immediately below the surface and gradually accumulate, progressively grow under the influence of repeated loading and eventually coalesce to form fine microscopic cracks immediately below the surface. Antonopoulos and co-workers [23] extended the idea put forth by Mott [22] and proposed a model based on vacancy dipoles, which develop in the persistent slip band. Essmann and co-workers [24] in their independent study used similar ideas for the nucleation of fine microscopic cracks. However, it was Neumann [25] who developed and put forth a model based on an activation of two operating slip systems. Thus, it appears that the nucleation process of cracks is now fairly well understood for a wide spectrum of metals and we will not comment on this any further.

Dislocation Emission from Cracks

We have shown that once a crack has been nucleated or artificially produced, say by incorporating a notch, it acts as a source of dislocations. Several researchers have over the years in their experiments observed cracks that emitted dislocations at its tip. Ohr and coworkers [26-28] made an observation of the distribution of dislocations in the plastic zone during *in situ* tensile deformation of thin films obtained from the bulk in an electron microscope. A somewhat surprising result was that a careful observation of the region immediately ahead of the crack tip revealed this region to be free of dislocations. They called this region as Dislocation Free Zone (DFZ). Park and coworkers [29] and subsequently few others have made similar observations. Ding and coworkers [30] showed that by in-situ straining of pure tin solder foils in a transmission electron microscope the dislocations emanated from the blunted main crack tip and dislocation free zone were formed between the crack tip and the emitted dislocations. Pande (C. S. Pande, unpublished work) in his study obtained direct evidence of (i) dislocations being emitted at the crack tip, (ii) internal stresses due to a crack, and (iii) dislocation free zone, by straining a (111) oriented copper specimen (thinned from the bulk) using an in-situ transmission electron microscope.

Crack-Dislocation Interaction(s)

The next step is to be in a position to put to effective use this knowledge with the prime objective of predicting well-documented fatigue behavior. For an excellent summary of the fatigue phenomenon to including experimental observations, models and related theories reference is made to the material presented and discussed by Suresh [31]. Obviously, the first step towards both enabling and enriching our understanding of the fatigue phenomenon at the microscopic level is the need for an investigation of crack-dislocation interactions.

Most interesting work during the last few years has been development of discrete modeling approach by Pippan and coworkers [32-35], Deshpande and co-workers [36], and continuum modeling by Mastorakos and Zbib [37]. Hitherto, most of the documented studies related to modeling of the fatigue phenomenon have been two-dimensional in nature. However, Mastorakas and Zbib [37] used a three-dimensional analysis, which is more appropriate and realistic to facilitate a better understanding of the fatigue phenomenon. Several discrete dislocation simulations have been done in the past [32-52]. It has been found and documented that the threshold behavior can be related to the discrete nature of plastic deformation. Several fractographic features, to include abnormal striation spacing and zigzag propagation of the crack at low growth rates, can also be convincingly explained.

Masumura, Pande and Chou [53] have examined the case of two arrays of discrete edge dislocations in the presence of a semi-infinite crack for an elasto-static formulation for isotropic materials utilizing complex variable functions, they developed the forces that were required for equilibrium. Their analysis provided the following information: (i) number of dislocations to be in equilibrium with the crack at a given load, and (ii) the size of the dislocation free zone.

The resulting analysis also provided the following:

(i) A measure of the size of the plastic zone.
(ii) An estimate of the critical threshold for further dislocation emission.
(iii) The magnitude of dislocation shielding of the crack.

These results accord well with the results of dislocation analysis reported by Lin and Thomson [54]. However, the method of Lin and Thomson cannot provide an expression for the size of the dislocation free zone because they replace the array of dislocations in front of the crack by two super dislocations.

In principle, the problem treated by Masumura and co-workers [53] can also be studied using the approximation of continuously distributed dislocations (CDDs). A continuum distribution of dislocations has been used in the past to study dislocation-crack interactions based upon the mathematical methods of singular integral equations. Bilby, Cottrell and Swinden (BCS) [55] modeled crack tip interaction using continuously

Distributed dislocations in lieu of a discrete number of dislocations. They were able to obtain an equilibrium distribution of dislocations both at and immediately around the crack tip (represented as a distribution of continuous dislocations). Their results were similar to that obtained and provided by Dugdale [56]. The analysis of BCS corresponded to the angle (θ) being 0 and size of the dislocation free zone being zero.

There are several problems that use the BCS model for studying the fatigue phenomenon and over the years various modifications have been suggested. A recent model using continuously distributed dislocations (CDDs) was put forth by Du and co-workers [57]. They investigated the dislocation free zone model for a symmetrical Mode I crack. Both the crack and two symmetrical plastic zones (inclined at angle θ the crack plane) by continuously distributed dislocations whose equilibrium position is considered by using a set of singular integral equations. For $\theta = 0$ the model easily reduces to that put forth by Chang and Ohr [58]. Although Chang and Ohr obtained closed form expressions for the various parameters, the model cannot be safely used for studying Mode I since they took the angle to be equal to zero. In contrast, Du and co-workers [57] consider the case of $\theta > 0$ but were not able to obtain any closed form solutions and were forced to resort to numerical computations for obtaining meaningful results.

The results of Masumura and co-workers [53] should be compared with that of Lin and Thomson [54]. The concept of a super dislocation was used by them to obtain a measure of K_D, provided a realistic measure of the mean position (r_m) could be easily determined. The super-dislocation is often used for classical dislocation pileups, but may not be a good approximation for an inverted pileup of dislocations. Some of the results obtained by Masumura and co-workers [53] are quite similar to those obtained by Lin and Thomson [54], thus providing a convincing verification of their test results. However, it should be noted that Masumura and co-workers [53] provide other information that is not possible in their treatment and this refers to the size of the plastic zone as a function of various loading conditions. They found a certain minimum value of k^* to be necessary for the emission of dislocations and its concurrent movement away from the crack. It is as yet not clear if this value of k^* can be associated with one of the two fatigue thresholds [59]. For further details the interested reader is referred to the paper by C. S. Pande that is contained in this volume.

Pippan and Weinhandl [60] pointed out that the simulation studies may open up many unanswered questions. These include the following:

(a) The effects of environment.
(b) A more detailed analysis of the three-dimensional nature of the crack.

(c) The role of internal dislocation sources and the crack tip as a source of dislocations.

(d) A transition from the blunting/re-sharpening crack propagation to a cleavage dominated process.

Crack Propagation

Many factors influence the rate of growth or propagation of fatigue cracks in metals, their alloy counterparts and composites based on metal matrices. During fatigue the driving force for crack growth is less than the driving force required for the same crack to grow under conditions of monotonic loading. A variety of theories have been proposed to rationalize fatigue crack behavior in the Paris law regime. In some cases, e.g. [61, 62], it is presumed that the fatigue crack growth rate is proportional to the cyclic crack opening displacement (COD) that implies a Paris exponent of two. The damage accumulation models give rise to a Paris exponent of four [63, 64]. More recent continuum plasticity-based models have been developed as detailed by Nguyen and coworkers [65] and Tvergaard, and Hutchinson [66], which can lead to a wider range of behavior.

Two structural factors that tend to affect or influence the glide processes taking place during crack growth are: (i) crystallographic orientation, and (ii) stacking fault energy. The effects of crystallographic orientation have been studied for single crystals of aluminum, and thus related to stacking fault energy with specific reference to polycrystalline copper [67]. Since the early mechanism proposed by Mott [22] is believed to contain the basic elements involved in crack initiation, a modification was made to include the primary growth stage [67]. The mechanism of growth in the second stage was strongly influenced by the normal stresses and the presence of substructure [68, 69]. Explanations dealing with the growth of crack along the sub-boundaries have associated this tendency with preferred crackling within the sub-boundaries themselves. Examination of thin films of aluminum in an electron microscope after cyclic loading [70] have shown that the boundaries are not regular arrays of dislocations as in the case of a simple tilt or simple twist boundary, but instead they consisted of complex tangles of dislocations.

We now consider some more recent studies pertaining to the crack propagation mechanisms. Jono and co-workers [71] observed growth behavior of the fatigue crack and slip deformation both at and near the crack tip by using an atomic force microscope. For a grain-orientated 3% silicon iron under conditions of constant amplitude loading, they found that in the lower ΔK region there was only one preferential slip system of this material in operation and the fatigue crack tended to grow along the slip plane. Constraints in slip deformation due to cyclic strain hardening resulted in either crack arrest or crack branching. However, in the high ΔK region two preferential slip systems operated simultaneously to an almost identical extent and the fatigue crack tended to grow in a direction perpendicular to the far-field load axis. The slip distance in one complete load cycle was measured quantitatively using the image processing technique. This observation is significant and points to an acceptable mode of fatigue crack propagation. Ihara and Tanaka [72] developed a mechanism for Mode I fatigue-crack propagation, using an idea which involved the initiation and opening of the cleavage-mode crack. Their approach used a stochastic damage-accumulation model for gradual propagation of the fatigue crack. The calculated results for da/dN (crack growth rate per cycle) appeared to agree with the experimental data.

Once a microcrack or a void is produced in front of the crack it can be shown mathematically that the stress intensity needed for crack propagation is noticeably reduced. The reduction in stress intensity is substantial when the microscopic crack or void is close to the original crack (unpublished work by N. Louat and C. S. Pande).
In our opinion a detailed mechanism of crack propagation is one of the major challenge in the study of fatigue. Below we provide some new speculative ideas on this topic that is currently being in the process of development by others and us.

It is now well established that a maximum applied stress intensity factor, K_{max}, must exceed a certain value K_{th}, before crack propagation is possible. Another aspect of the problem is the reduction in fatigue crack growth and its eventual stoppage if DeltaK (the difference between K_{max} and K_{min}) falls below a threshold value. Thus, as clearly documented by Vasudevan *et al.* [73] in their recent review of a large body of fatigue data, there are two separate constraints or threshold conditions on fatigue growth. At high K_{max}, must be above some critical value and at low K_{max}, K_{max} must exceed some critical value K_{th} for a crack to propagate at a given rate. Both conditions must be satisfied at all times.

Experimental results determining these thresholds for various parameters such as microstructure, load ratio R ($R = K_{min} / K_{max}$), load history, overloading and environmental is now available. Hence any model of fatigue must address these issues in a quantitative fashion.

Our aim is to provide a quantitative description of these thresholds and to relate them to various parameters mentioned above. The model for fatigue that we want to propose is shown schematically below:

The micro crack of length *c* is potentially moving to the right under the action of the applied stresses. We first show that the crack cannot move until K_{max} exceeds a certain value. We also assume that during the fatigue process, a small micro crack of size *a* is created in front of the macro crack at a distance *delta* away. Both *a* and *delta* are expected to be small. When the cracks move they eventually join together, thus sharpening the macro crack tip for further advance by the repetition of this process. The validity of this model needs to be considered in detail and should be checked experimentally.

We first calculate, using this model the minimum threshold on K_{max} for the macro crack to move in absence of the micro crack and then calculate the effect of the minicrack on this value.
For simplicity, we initially consider only Mode I loading and the macro crack for mathematical purposes will be considered to be semi-infinite. Once the crack moves, the micro crack joins the macro crack, sharpening it in the process.

Under dynamic conditions (*i.e.,* crack moving), it can be shown that a relation exists between the fracture energy, T, and the stress intensity factor, K. The relation is

$$T = \frac{1}{E} B K^2, \tag{1}$$

where B is a function of crack velocity v only and E is Young's modulus and K is related to the crack length c and the stress field, by the usual relation,

$$\sigma_{ij} \approx K\ c\ \frac{K_o}{\sqrt{2\pi r}}, \tag{2}$$

where K_o is a function of angle crack is moving, all of these parameters may be a function of velocity. For small velocities, it can be shown that the function of K and B can be resolved as a product of a velocity term and a static term ($v = 0$). Thus,

$$K = k(v) K^s(c) \tag{3}$$

and

$$B = (1 - \frac{v}{v_o}). \tag{4}$$

where v_o is the maximum velocity (= acoustic) and superscript "s" denotes the static term ($v = 0$), *i.e.*, independent of velocity. We also assume that T is independent of velocity.

Using equations (1), (3) and (4),

$$v = \frac{dc}{dt} \approx v_o \left[1 - \frac{E\,T}{\left(K^s\right)^2} \right] \tag{5}$$

where $v = (dc/dt)$ is crack velocity. Equation (5) shows that

$$v = 0 \ \text{ if } \left(K^s\right)^2 = E\,T \tag{6}$$

i.e., no crack motion if $K^s < (E\,T)^{1/2}$. In this calculation, K^s threshold is entirely dependent on the material parameters E and T and not much affected by microstructure*, etc*. This will be modified somewhat in the presence of the micro crack.

Calculations show that in the presence of a micro crack K^s threshold will increase from the value given by equation (6). A detailed calculation shows that the K^s value can more than double, making it easier for the crack to propagate.

The presence of the micro crack is then responsible for crack to propagate in fatigue, which it would not do otherwise. The threshold value in the presence of the micro crack is given by

$$K_F^s \approx K^s\left[\frac{\Delta + a}{\Delta}\,\frac{E_o(1-\frac{\Delta}{\Delta+a})}{K_o(1-\frac{\Delta}{\Delta+a})}\right] \tag{7}$$

where E_o and K_o are complete elliptic integrals of the first and second, respectively. Similarly that a second threshold is necessary. This one is required to initiate a micro crack that is used in the calculation discussed above. Needless to say these ideas need to be developed further.

Both the applied K field and the intrinsic friction stress affect the dislocation shielding. The interaction between these two parameters is inhomogeneous and non-linear and such scaling becomes an important issue. The concept of a super dislocation is useful to obtain a measure of K_D, provided that a realistic measure of the mean position ρ_m can be determined. The use of an inverted pile-up may not be a good approximation to these arrays where crack-dislocation interaction plays an important role.
We find that a certain minimum value of K^* is necessary for dislocation emission and its movement away from the crack. It is not yet clear if this value of K^* can be associated with one of the two fatigue thresholds.

Role of Surfaces

Recently, Pande Masumura and Chou (unpublished) considered the role of boundary displacements of a dislocation in the process of propagation of fatigue cracks. It was noticed during that investigations that information on boundary displacements produced by defects such as dislocations seems to be extremely sparse. They present (1) some supplementary remarks on the basis of the link between the harmonic functions and the method of images, and (2) a set of explicit expressions for the displacement field of an edge dislocation interaction with a boundary surface. They proceed by presenting a simple proof for the link between the harmonic functions and image field. The proof given by them seems simple, almost trivial, but it is new. It was not needed by early workers of electrostatics since their main interest was in spherical harmonics. But for defect fields in general, one needs a quantitative basis for the link in order to obtain stress functions which are not always harmonic. Surfaces and interfaces modify the stress fields of dislocations and their contributions should be carefully taken into account.

Concluding Remarks

Recently, the development and emergence of several analytical techniques has definitely stimulated the possibility of arriving at a basic understanding of crack initiation processes. It is truly hoped that further research along these lines coupled with theory and simulations will soon lead to a much better understanding of the complex crack initiation phenomenon and subsequent growth or propagation of the crack through the microstructure.

ACKNOWLEDGEMENTS

This work is based partly on an earlier review by the author and his coworkers. (C.S. Pande Tirumalai Srivatsan, Ashraf Imam,, Crack- dislocation interaction and its implication for a fundamental understanding of Fatigue Proceedings: International Symposium on Fatigue of Materials Advances and Emergences in Understanding Editor(s): Tirumalai Srivatsan, Ashraf Imam) and is supported by the Office of Naval Research. Finally are grateful to Dr. Robert A. Masumura for his substantial contributions to this work.

References

1. M. E. Fine: *Metallurgical Transactions* 11A, 1980, 365.
2. N. Thompson, and N. J. Wadsworth: Metal Fatigue Advanced Metal Physics (Philosophical Magazine Supplement 7, 1958, 72-82.
3. D. H. Avery and W. A. Backofen: Fracture of Solids (edited: D. C. Drucker and J. J. Gilman), Interscience Publishers, New York, USA, 1963, p. 339.
4. J. C. Grosskreutz and P. Waldow: *Acta Metallurgica*, Vol. 11, 1963, p. 717-727.
5. K. U. Snowden: *Acta Metallurgica*, Vol. 11, 1963, pp. 675-685.
6. C. Laird and G. C. Smith: *Philosophical Magazine*, Vol. 8, 1963, 1945-1955.
7. J. C. Grosskreutz: Sagamore Materials Conference: Fatigue: An Interdisciplinary Approach edited by J.J. Byre and N. L. Reed and V. Weiss, Syracuse University Press, 1964, p. 27-35.
8. J. C. Grosskreutz and G. G. Shaw: Fracture 1969 Proceedings of the Second International Conference on Fracture, Brighton, 12969, Chapman and Hall Publishers, London, UK, 1969, pp. 602-622.
9. W. J. Plumbridge and D. A. Ryder: *Metallurgical Reviews*, 136, 1969, pp. 119-139.
10. J. C. Grosskreutz: Phys. Status Solidi, 1970.
11. D. J. Duquette: Corrosion Fatigue: Chemistry, Mechanics and Microstructure, NACE 2, edited by O. F. Devereux, 1972, pp. 12-22.
12. S. Suresh and R. O. Ritchie: *International Metals Reviews*, 29 (6), 1984.
13. H. Mughrabi, R. Wang, K. Differt and U. Essmar: Fatigue Crack Initiation hy Cyclic Slip Irreversibility in High Cycle Fatigue," in Quantitative Measurement of Fatigue Damage, Dearborn, Michigan, 1982.
14. T. S. Srivatsan and T. S. Sudarshan: *Journal of Materials Science*, Vol. 23, 1988, 1521-1533.

15 A. S. Basinski, R. Pascual and S. J. Basinski: *Acta Metallurgica*, 31, 1983, 591-601.

16. M. E. Fine and R. O. Ritchie: Fatigue and Microstructure (edited by M. Meshii) American Society for Metals, Metals Park, Ohio, USDA, 1979, pp. 245-255.
17. P. H. Frith: *Journal of Iron and Steel Institute*, 159, 1948, 385-390.
18. W. A. Wood: Fatigue in Aircraft Structures, Academic Press, New York, USA, 1956.
19. H. E. Frankel, J. A. Bennett and W. A. Pennington: *Transactions of ASM*, 52, 1960, pp. 643-663.
20. W. A. Wood, S. Coustand and K. R. Sargart: *Acta Metallurgica*, 11, 1963, pp. 643-653.
21. Z. S. Basinski and S. J. Basinski. Low amplitude fatigue of copper single crystals-III. PSB sections. *Acta Metallurgica*, 33:1319–1327, 1985.
22. N. F. Mott. A theory of the origin of fatigue cracks. *Acta Metallurgica*, 6:195–197, 1958.
23. J. G. Antonopoulos, L. M. Brown and A. T. Winter, *Philosophical Magazine* 34, 549-563, 1976.

24. K. Differt, U. Essmann, and H. Mughrabi. A model of extrusions and intrusions in fatigued metals - II. surface roughening by random irreversible slip. *Philosophical Magazine A*, 54:237–258, 1986.
25. P. Neumann. Coarse slip model of fatigue. *Acta Metallurgica*, 17:1219–1225, 1969.
26. S. Kobayashi and S. M. Ohr, 37th Ann. Proc. Electron Microscopy Soc. of America", G.W. Bailey, Ed., San Antonio, TX, 1979, pp424-425.
27. S. Kobayashi and S. M. Ohr, "In-situ Fracture Experiments in B.C.C. Metals", *Philosophical Magazine,* 1980:A42: 763-772.
28. S. Kobayashi and S. M. Ohr, *Scripta Met*. "In-situ Observations of the Formation of Plastic Zone Ahead of a Crack Tip in Copper", *Scripta Metallurgica*, 1981:15:343-348.
29. C. G. Park C. S. Lee and Y. W.Chang Mechanical behaviour of materials Volume4, Proceedings of the 6th International Conference, Kyoto, Japan, July 29-Aug. 2, 1991. Vol. 4 (A93-40776 16-39), p. 3-9.
30. Ying Ding, Chunqing Wang, Mingyu Li and Weiqiang Wang, *Materials Science and Engineering B* 127 (2006) 62–69.
31. S. Suresh, "Fatigue of Materials", 2nd Edition, Cambridge University Press, Cambridge.
32. R. Pippan, *Acta Metall Mater* 39 (1991), pp. 255–262.
33. R. Pippan, *International Journal of Fracture,* 58 (1992), pp. 305–318.
34. F. O. Riemelmoser, R. Pippan and H. P. Stüwe, *International Journal of Fracture,* 85 (1997), pp. 157–168.
35. F. O. Riemelmoser, R. Pippan and H. P. Stüwe, *Acta Materialia* 46 (1998), pp. 1793–1799.
36. W. A. Curtin, V. S. Deshpande, A. Needleman, E. Van der Giessen, M. Wallin, International Journal of Fatigue 32 (2010) 1511–1520.
37. I. N. Mastorakos and H. M. Zbib, *Journal of Metals*, April 2008, p 59.
38. A. J. Wilkinson and S. G. Roberts, *Scripta Mater* 35 (1996), pp. 1365–1371.
39. A. J. Wilkinson, S. G. Roberts and P. B. Hirsch, *Acta Metall Mater* 46 (1998), pp. 379–390.
40. V. Doquet, *Fracture Engineering Materials Structures* 21 (1998), pp. 661–672.
41. E. van der Giessen, V. S. Deshpande, R. P. Cleveringa and A. Needleman, *J Mechanics Physics of Solids* 49 (2001), pp. 2133–2153.
42. V. S. Deshpande, A. Needleman, E. van der Giessen, *Acta Materialia* 49 (2001), pp. 3189–3203.
43. R. Pippan, R. O. Riemelmoser, H. Weinhandl and H. Kreuzer, *Philosophical Magazine* 82 (2002), pp. 3299–3309.
44. R. Pippan and R. O. Riemelmoser, *Z Metallkunde* 86 (1995), pp. 823–826.
45. F. O. Riemelmoser, P. Gumbsch and R. Pippan, *Material Transactions* 42 (2001), p. 2.
46. F. O. Riemelmoser, R. Pippan and H. P. Stüwe, *Acta Materialia* 46 (1998), p. 1793.
47. P. Hansson and S. Melin, *Engineering Fracture Mechanics* 75 (2008), pp. 1400–1411.
48. C. Bjerkén and S. Melin, *International Journal Fatigue*, 25 (2003), pp. 559–566.

49. C. Bjerkén and S. Melin, *Engineering Fracture Mechanics* 71 (2004), pp. 2215–2227.
50. P. Hansson and S. Melin, *International Journal of Fatigue* 27 (2005), pp. 347–356.
51. S. Groh, S. Olarnrithinum, W. A. Curtin, A. Needleman, V. S. Deshpande and E. van der Giessen, *Philosophical Magazine*, 88 (2008), pp. 3565–3583.
52. B. Kunkler, O. Duber, P. Koster, U. Krupp, C. P. Fritzen and H. J. Christ, *Engineering Fracture Mechanics*, 75 (2008), pp. 715–725.
53. R. A. Masumura, C. S. Pande and Y. T. Chou, International Journal of Fatigue, Volume 27, Issues 10-12, October-December 2005, Pages 1170-1174.
54. I. H. Lin and R. Thomson, "Cleavage, Dislocation Emission, and Shielding for Cracks under General Loading", *Acta Metallurgica,* 1986:34:187-206.
55. B. A. Bilby, A. H. Cottrell, and K. H. Swinden, "The Spread of Plastic Yield From a Notch", *Proc. R. Soc. London. Ser.*, 1963:A272:304.
56. D. S. Dugdale, *Journal Mechanics Physics of Solids*, 1960:8:100.
57. K. -L. Du, K. L, J. -B. Lu, and R. -W. Li, "A Continuous Dislocation Model of Mode I Crack", 15th ASCE Engineering Mechanics Conference, Columbia University, NY, 2002, pp.1-8.
58. S. J. Chang and S. M. Ohr, "Dislocation-free Zone Model of Fracture", *Journal Applied. Physics*, 1981:52:7174-7181.
59. A. K. Vasudevan, K. Sadananda and G. Glinka, "Critical Parameters for Fatigue Damage", *International Journal of Fatigue*, 2001:23:S39-S53.
60. R. Pippon and H. Weinhandl, International Journal of Fatigue 32 (2010) 1503–1510.
61. C. Laird, G. C. Smith, Crack propagation in high stress fatigue. Philos Mag, 1962; 7:847–57.
62. F. A. McClintock: Plasticity aspects of fracture. In: H. Liebowitz, editor. Fracture: an advanced treatise, vol. 3. New York: Academic Press; 1971. p. 47–225.
63. J. Weertman:. "Rate of growth of fatigue cracks calculated from the theory of infinitesimal dislocations distributed on a crack plane. In*ternational Journal of Fracture*, 1966; 2:460–7.
64. J. R. Rice Mechanics of crack tip deformation and extension by fatigue. Fatigue Crack Propagation, ASTM STP 1974; 415: 247–309.
65. O. Nguyen, A. Repetto, M. Ortiz, A. Radovitzky, A cohesive model of fatigue crack growth. *International Journal of Facture* 2001; 110: 351–69.
66. V. Tvergaard, J.W. Hutchinson. Crack growth per cycle by blunting and void growth. In: A. F. Blom, editor. Proceedings of the 8th international Conference on Fracture. vol. 1. West Midlands (UK): EMAS; 2002. p. 107–16.
67. A. J. McEvily and R. C. Boettner: *Acta Metallurgica*, Vol. 11, 1963, 725-743.
68. J. C. Grosskreutz: *Journal of Applied Physics*, 33, 1962, 1787.
69. J. Holden: *Philosophical Magazine*, 6, 1961, 547-557.
70. J. C. Grisskreutz: Proceedings of the Fifth International Congress for Electron Microscopy, Academic Press, New York, 1962, J-9.

71. M. Jono, A. Sugata and Y. Uematsu, Atomic force microscopy and the mechanism of fatigue crack growth, *Fatigue Fracture Engineering Materials Structures,* 24 (2001), pp. 831–842.
72. C. Ihara and T. Tanaka, *Engineering Fracture Mechanics*, Volume 74, 2007, pp 1488-1498.
73 A.K. Vasudevan, K. Sadananda and G.Glinka, “Critical Parameters for Fatigue Damage”, *Int. J. of Fatigue*, 2001:23:S39-S53.