5

FLOW AND VACUUM PRODUCTION

The materials engineer must know not only how to measure flow, but also how to produce it. Some methods of producing flow and vacuum, such as gravity flow and suction lift, are well known, but there are many types of fans, pumps, and flow devices which are often required and used in materials laboratories and materials processing. Vacuum technology is a crucial part of materials processing and in materials science and engineering. Basic uses of vacuums include removal of gas molecules to permit travel of electrons, atoms, and ions through a required distance, removal of chemically active gases, decreasing the gas density to minimize heat transfer, refinement of melts, and controlled deposition of thin films on substrates. Keeping in mind that this is a textbook on transport phenomena, we have limited our presentation of vacuum technology to only several pages, and at that much of the presentation is qualitative and descriptive. We urge all readers involved in vacuum technology to immerse themselves in books devoted to this subject.[1,2]

Unfortunately the jargon and units used in engineering associated with pumps, fans, blowers and vacuum-producing devices can be rather strange, particularly in the U.S.A. where the use of English units still prevails. Accordingly, along with the working equations, we explain the jargon and non-S.I. units.

5.1 PUMPS

In general, pumps may be classified as either positive-displacement pumps or centrifugal pumps. We may subdivide positive-displacement pumps into reciprocal or rotary types, and centrifugal pumps into tangential- or axial-flow types.

Positive-displacement pumps deliver a definite volume of liquid at every stroke or revolution, regardless of the pressure and friction against which they are working. There are two common types of reciprocating positive-displacement pumps—the double-acting piston

[1]M. H. Hablanian, *High-Vacuum Technology*, Marcel Dekker, Inc., New York, NY, 1990.

[2]J. F. O'Hanlon, *A User's Guide to Vaccum Technology*, Second edition, John Wiley & Sons, New York, NY, 1989, pages 35-42.

D. R. Poirier and G. H. Geiger, *Transport Phenomena in Materials Processing*

pump and the single-acting plunger pump—and a large number of different designs of rotary positive-displacement pumps, the gear pump perhaps being the best known.

In general, positive-displacement pumps are employed where delivery of a relatively small volume of liquid against high pressure is required. In reciprocating pumps, as the piston withdraws, the fluid discharge ceases, and so the delivery is in pulses. In double-acting pumps, this pulsating characteristic is lessened.

In centrifugal pumps, the fluid enters axially at the suction connection, accelerates radially out along the blades of the impeller, and leaves the pump tangentially at a high velocity. Centrifugal pumps can have various types of vane configurations, each with its own combination of power losses due to circulatory flow, fluid friction, shock, and mechanical friction.

When using a positive-displacement pump, the major factor to consider when specifying the pump is the power required. The power (as well as other factors) is also important when specifying a centrifugal pump. To determine the power, we apply Eq. (4.11) to a system.

Consider the system with a pump shown in Fig. 5.1, and write Eq. (4.11) from the plane at level (3) to the discharge plane (1).

$$\left[\frac{P_1}{\rho_1} - \frac{P_3}{\rho_3}\right] + \left[\frac{\bar{V}_1^2}{2\beta_1} - \frac{\bar{V}_3^2}{2\beta_3}\right] + g(z_1 - z_3) + E_f = -M^* . \tag{5.1}$$

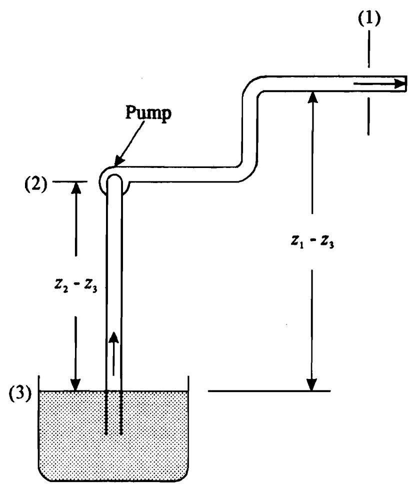


Fig. 5.1 Reference levels for the analysis of pump application.

If the frictional loss within the pump itself is accounted for by a mechanical efficiency Γ, then the work done *by the pump* is

$$\Gamma M_P^* = \left[\frac{P_1}{\rho_1} + \frac{\bar{V}_1^2}{2\beta_1} + gz_1\right] - \left[\frac{P_3}{\rho_3} + \frac{\bar{V}_3^2}{2\beta_3} + gz_3\right] + E_f . \tag{5.2}$$

In the nomenclature associated with pumps, the quantities in the brackets are called *heads*, *h*, E_f is the *friction head*, and so

$$M_P^* = \frac{\Delta h}{\Gamma} , \tag{5.3}$$

where Δh is the total head (i.e., the right side of Eq. (5.2)). Each head, as well as M_P^*, has the units of J kg^{-1} so that the power required by the pump from a motor (brake power) is given by

$$P_B = WM_P^* = \frac{W\Delta h}{\Gamma}, \tag{5.4}$$

where W is the mass flow rate in kg s^{-1} and P_B is in W. The power absorbed by the fluid (fluid power) is

$$P_f = W\Delta h \tag{5.5}$$

and, therefore,

$$\Gamma = \frac{P_f}{P_B}. \tag{5.6}$$

In the U.S.A., power ratings of pumps are given in *horsepower* (hp), whereas the ratings of the motors can be in either kW or hp. The conversion is 1 kW = 1.341 hp. Often the flow rate is given in gallons per minute (gpm), so the reader should also be familiar with the following conversion:

$$1 \text{ gallon (U.S.)} = 3.785 \times 10^{-3} \text{ m}^3.$$

In pumping liquids, there is a limit on the net positive *suction-head* that cannot be exceeded. This is set by the fact that if the dynamic pressure of the liquid ($P + \rho\overline{V}^2/2\beta$) falls below the vapor pressure of the liquid P_V, then the liquid vaporizes, and no liquid is drawn into the pump. This phenomenon is called *cavitation*. For example, consider the liquid just before it enters the pump at plane (2) in Fig. 5.1. In order to avoid cavitation,

$$\left[\frac{\overline{V}_2^2}{2\beta_2} + \frac{P_2}{\rho}\right] > \frac{P_V}{\rho}. \tag{5.7}$$

The pump lifts the liquid from the reservoir through a pipe, and Eq. (4.11), written between the reservoir surface (3), which is open to the atmosphere, and the suction entrance of the pump (2), assuming $\overline{V}_3 = 0$, is

$$\left[\frac{P_2}{\rho} - \frac{P_3}{\rho}\right] + \frac{\overline{V}_2^2}{2\beta_2} + g(z_2 - z_3) + E_f = 0. \tag{5.8}$$

From Eqs. (5.7) and (5.8), we obtain the following requirement in terms of the height of lift and the pressure at the reservoir:

$$\frac{P_3}{\rho} - g(z_2 - z_3) - E_f > \frac{P_V}{\rho}. \tag{5.9}$$

Therefore, the net positive suction-head defined below must indeed be positive:

$$h_{\text{nps}} = \left[\frac{P_3}{\rho} - \frac{P_V}{\rho}\right] - g(z_2 - z_3) - E_f > 0. \tag{5.10}$$

The efficiency of centrifugal pumps is a maximum at the design conditions, falling off on either side of them (Fig. 5.2). In selecting a centrifugal pump, one should consider its head-flow rate *characteristic curve*. The head is usually given in terms of feet of the fluid flowing through the pump, and indicates the height of a column of the liquid which could be pulled up through a frictionless pipe to the pump itself, assuming the limit imposed by cavitation, Eq. (5.10), is not exceeded. In practice, of course, the friction loss and any fitting or meter losses would have to be subtracted from this height.

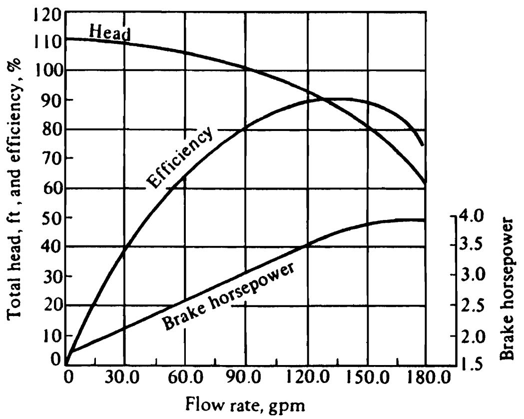
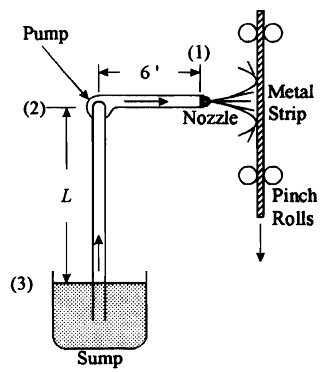


Fig. 5.2 Characteristic curves for a centrifugal pump.

When the head is expressed in feet, in reality the units are ft lb_f lb_m^{-1}, where lb_f and lb_m are the units for pound force and pound mass from the English system of units. A pound force is the force required to accelerate one pound mass by the acceleration of gravity (32.2 ft s^{-2}). In the following example, the reader is exposed to these units, as well as conversions to SI units.

Example 5.1 A strip of metal emerging from a set of rolls is to be cooled by means of a spray of water. If 60 gpm are required, and the pressure drop across the spray nozzle is 15 psi at this flow rate, is it possible to install the pump, as depicted to the right, if the characteristic curves given in Fig. 5.2 apply? Assume 3-in. welded steel-piping up to the pump and 1-in. welded steel-piping to the nozzle. The problem is specified in units commonly encountered in engineering situations involving pumps in the U.S.A.



Solution. From Fig. 5.2, we obtain the total head delivered by the pump, $-M^*$, which is 107 ft at 60 gpm. In reality, the units of the head are ft lb_f lb_m^{-1}.

Next we write Eq. (5.1) from planes (3) to (1). In this instance, $L = z_1 - z_3$, $\bar{V}_3 = 0$, $\beta_1 \simeq 1$, and $\rho_1 = \rho_3 = \rho = 1000 \text{ kg m}^{-3}$; therefore the mechanical energy equation is

$$\left[\frac{P_1 - P_3}{\rho}\right] + \frac{\bar{V}_1^2}{2} + gL + E_f = -M^*$$

Starting with $-M^* = 107 \text{ ft lb}_f \text{ lb}_m^{-1}$, we convert to SI units:

$$-M^* = \frac{107 \text{ ft lb}_f}{\text{lb}_m}\left|\frac{1.356 \text{ N m}}{1 \text{ ft lb}_f}\right|\frac{1 \text{ lb}_m}{0.4536 \text{ kg}} = 320 \text{ N m kg}^{-1}.$$

Now we proceed to evaluate the terms on the left side of the mechanical energy equation. The pressure drop across the nozzle is given as 15 psi. This is equivalent to

$$P_1 - P_3 = \frac{15 \text{ lb}_f}{\text{in}^2}\left|\frac{1^2 \text{ in}^2}{(0.0254)^2 \text{ m}^2}\right|\frac{4.448 \text{ N}}{1 \text{ lb}_f} = 1.034 \times 10^5 \text{ N m}^{-2}.$$

Thus

$$\frac{P_1 - P_3}{\rho} = \frac{1.034 \times 10^5 \text{ N}}{\text{m}^2}\left|\frac{\text{m}^3}{1{,}000 \text{ kg}}\right. = 103.4 \text{ N m kg}^{-1}.$$

At 60 gpm, we calculate the velocity in the 1-in. pipe (diameter equals 25.4 mm) as follows:

$$\bar{V}_1 = \frac{60 \text{ gallon}}{\text{min}}\left|\frac{3.785 \times 10^{-3} \text{ m}^3}{1 \text{ gallon}}\right|\frac{1 \text{ min}}{60 \text{ s}}\left|\frac{4}{\pi}\right|\frac{}{0.0254^2 \text{ m}^2}$$

$$\bar{V}_1 = 7.47 \text{ m s}^{-1}.$$

Similarly in the 3-in. pipe, we get

$$\bar{V} = 0.829 \text{ m s}^{-1}.$$

For now, we consider L (in m) to be an unknown. Therefore,

$$gL = \frac{9.807 \text{ m}}{\text{s}^2}\left|\frac{L \text{ m}}{}\right. = 9.807L \text{ m}^2 \text{ s}^{-2} = 9.807L \text{ N m kg}^{-1}.$$

Next we formulate E_f:

$$E_f = \underbrace{\tfrac{1}{2} e_f \bar{V}^2}_{\text{entrance loss}} + \underbrace{2f\left[\frac{L}{D}\right]\bar{V}^2}_{\text{3-in. pipe loss}} + \underbrace{2f\left[\frac{L}{D}\right]\bar{V}_1^2}_{\text{1-in. pipe loss}}.$$

From Figs. 4.4 and 4.5, we obtain:

$$e_f = (2)(0.4) = 0.8.$$

The Reynolds numbers are

$$\text{Re (1-in. pipe)} = \frac{D\,\bar{V}_1\,\rho}{\eta} = \frac{(0.0254)(7.47)(1000)}{(0.001)} = 1.90 \times 10^5$$

and similarly

$$\text{Re (3-in. pipe)} = 6.32 \times 10^4.$$

Now we seek the respective friction factors:

	Re	ε/D	f
1-in. pipe	1.9×10^5	0.0018	0.0050
3-in. pipe	6.3×10^4	0.0006	0.0055

With these friction factors, we calculate E_f:

$$E_f = \left[\frac{1}{2}\right](0.8)\left(0.829^2\right) + (2)(0.0055)\left[\frac{L}{0.0762}\right]\left(0.829^2\right)$$

$$+ (2)(0.0050)\left[\frac{1.829}{0.0254}\right]\left(7.47^2\right)$$

$$E_f = 40.46 + 0.09921L,\ \text{m}^2\ \text{s}^{-2}\ \text{or N m kg}^{-1}.$$

Collecting terms in the mechanical energy equation we have:

$$103.4 + \frac{7.47^2}{2} + 9.807L + 40.46 + \quad + 0.09921L = 320.$$

Therefore, $L = 14.96$ m.

We should be suspect of this result. We know from our everyday experience that when pumps are situated more than 34 ft (10.4 m) above water, there is cavitation and the water cannot be pumped.

In a more formal manner, let's examine the net positive suction head. We use E_f only between (3) and (2). Also, $(P_3 - P_V)/\rho \simeq P_3/\rho$ in this case. Therefore

$$h_{\text{nps}} = \frac{P_3}{\rho} - gL - E_f,$$

with

$$\frac{P_3}{\rho} = \frac{1\ \text{atm m}^3}{1000\ \text{kg}}\left|\frac{1.01325 \times 10^5\ \text{N}}{1\ \text{atm m}^2}\right. = 101.32\ \text{N m kg}^{-1}$$

$$gL = (9.807)(14.96) = 146.7\ \text{N m kg}^{-1}$$

and

$$E_f = (2)(0.0055)\left[\frac{14.96}{0.0762}\right]\left(0.829^2\right) = 1.48\ \text{N m kg}^{-1},$$

then

$$h_{nps} = -46.87 \text{ N m kg}^{-1}.$$

Cavitation will occur because $h_{nps} < 0$, and the installation will not work with $L = 14.96$ m. A solution is to decrease L to 6 m and to compensate for the reduction in friction loss, E_f, by adding a gate valve to the system between the pump and the nozzle.

With the gate valve, the friction losses are

$$E_f = \left[\frac{1}{2}\right](0.8)(0.829^2) + (2)(0.0055)\left[\frac{6}{0.0762}\right](0.829^2) + (2)(0.0050)\left[\frac{1.829}{0.0254}\right](7.47^2)$$

$$+ (2)(0.0050)\left[\frac{L_e}{D}\right]_v (7.47^2),$$

where $(L_e/D)_v$ is the equivalent length-to-diameter ratio for the gate valve. After collecting terms, we get

$$E_f = 41.05 + 0.558\left[\frac{L_e}{D}\right]_v, \text{ N m kg}^{-1}.$$

Since the total head at the desired flow rate is 320 N m kg^{-1}, the system will perform as desired when $(L_e/D)_v$ is such that

$$320 = 103.4 + \frac{7.47^2}{2} + (9.807)(6) + 41.05 + 0.558\left[\frac{L_e}{D}\right]_v$$

$$\left[\frac{L_e}{D}\right]_v = \frac{320 - 231.2}{0.558} = 159.$$

From Table 4.2 we see that the system will work with the gate valve closed about one-half. The reader can check to see whether the net positive suction head is positive.

5.2 FANS AND BLOWERS

Fans and blowers are used for moving gases. We employ fans when the pressure drop in the system to overcome is not larger than about 50 in. w.c.,* although volumes up to several hundred thousand cubic feet per minute (cfm) may be involved. Blowers and turboblowers, on the other hand, are used in situations where larger pressure drops occur.

As in the case of pumps, there are many different designs of fans—axial flow, propeller and cross-flow fans as well as centrifugal types with a variety of blade configurations. All manufacturers of gas-moving equipment supply characteristic curves describing the performance of their equipment under the specified operating conditions at the inlet, as, for example, in Fig. 5.3, which presents curves for both the static and total pressures and the

*The notation in. w.c., or inches water-column, is often used as the unit for pressure drop. Normal atmospheric pressure is 407.14 in. w.c.

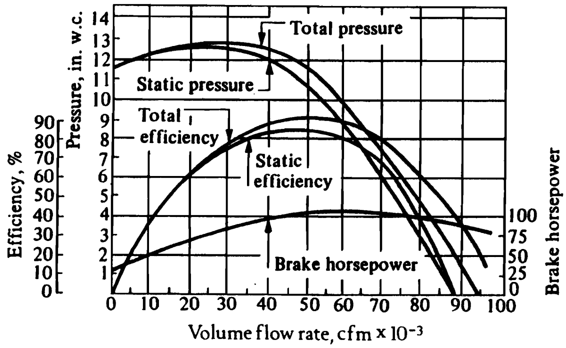


Fig. 5.3 Characteristic curves for a fan.

static and total efficiencies. If one wants to use a fan for moving a specified amount of gas against a given system resistance, it is necessary to know the static pressure developed by the fan at that flow rate.

Blowers are essentially constant-pressure machines with power consumption almost directly proportional to the volume delivered. Characteristic performance curves for a variable speed turboblower are given in Fig. 5.4. Pressure is in psig, which represents a gage pressure* in lb_f in^{-2}, and the speed of the blower is in revolutions per minute (rpm), and the volume flow rate is in cubic feet per minute (cfm). Blowers and turboblowers find their typical applications in large metallurgical processes, such as in producing the air blast for blast furnaces and smelting furnaces.

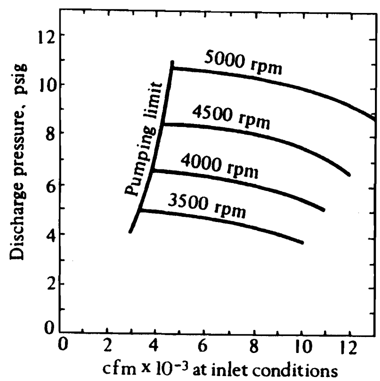


Fig. 5.4 Typical characteristic curves for a variable-speed blower.

Now, consider how the fan characteristics are measured. Figure 5.5(a) illustrates an appropriate system provided with a damper, so that the flow throughput may be varied. The Pitot tube measures the total pressure P_t, which is made up of the static pressure P_s (measured by the static pressure gage), and the dynamic pressure $\frac{1}{2}\rho \overline{V}^2$. By adjusting the damper to various positions, the values of P_t and P_s corresponding to the various flow rates can be

*The gage pressure is the pressure in excess of the ambient pressure.

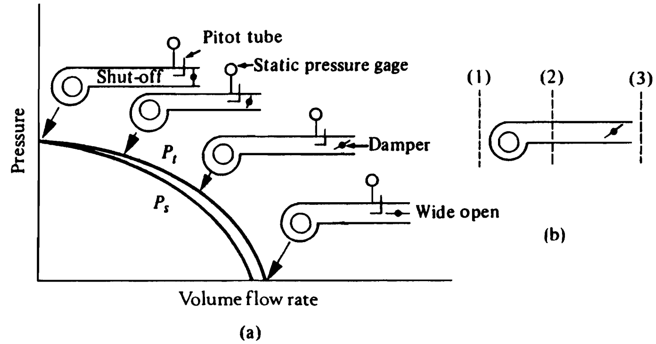


Fig. 5.5 Obtaining characteristic curves of a fan.

generated. Both of these values are relative to the inlet conditions, and are really pressure increases developed by the fan. At any flow rate, a theoretical horsepower may be calculated corresponding to either P_s or P_t, so that the definition of the *static efficiency*, Γ_s, as well as of the total efficiency, Γ, is based on the actual, or so-called brake horsepower.

The above statement can also be demonstrated by use of Bernoulli's equation applied to the system shown in Fig. 5.5(b). Between planes (1) and (3), the energy balance is simply

$$M^* + E_f = 0. \tag{5.11}$$

In this case, the system friction is composed of the resistance offered by the damper and the exit losses, that is, the friction between planes (2) and (3). The mechanical energy balance between planes (2) and (3) is then expressed

$$E_f = \left[\frac{P_2 - P_3}{\rho}\right] + \left[\frac{\bar{V}_2^2}{2}\right] \tag{5.12}$$

assuming $\beta = 1$.

Hence, by combining Eqs. (5.11) and (5.12), and noting P_1 - P_3, we arrive at

$$-M^* = \left[\frac{P_2 - P_1}{\rho}\right] + \left[\frac{\bar{V}_2^2}{2}\right], \tag{5.13}$$

or

$$-M^* \rho = (P_2 - P_1) + \left[\frac{\rho \bar{V}_2^2}{2}\right]. \tag{5.14}$$

Writing the above expression with the actual or brake horsepower P_B and the efficiency Γ, we obtain

$$\Gamma P_B = -M^* \rho Q = \left[(P_2 - P_1) + \frac{\rho \bar{V}_2^2}{2}\right] Q. \tag{5.15}$$

where Q is the volume flow rate.

Thus the total power delivered is based on the sum of the static pressure (P_2 - P_1) and the dynamic pressure ($\frac{1}{2}\rho \overline{V}_2^2$), or what we call the total pressure, as incorporated in brackets.

5.3 INTERACTIONS BETWEEN FANS OR PUMPS AND SYSTEMS

Because a fan is part of an overall system, the system as a whole determines the size of the fan required. For any system, there is a certain curve of volume flow rate versus system resistance or pressure drop (see Fig. 5.6). The reader should realize by now that the resistance usually increases as the square of the volume flow rate for a highly turbulent system. The system may contain any combination of duct work, beds of fluidized or packed solids, dust collectors, flues, etc. There is usually a specific volume throughput Q', required for proper operation of the process; this fixes the pressure drop resistance of the system, which in turn must be overcome by the fan or blower. Essentially then, the fan characteristic curve must intersect the system curve at the desired Q'-ΔP coordinate (the so-called *operating point*). The efficiency and required horsepower are then fixed. Note that a different fan with a different characteristic curve would place the operating point at a different position on the system curve, resulting in a flow different from Q'.

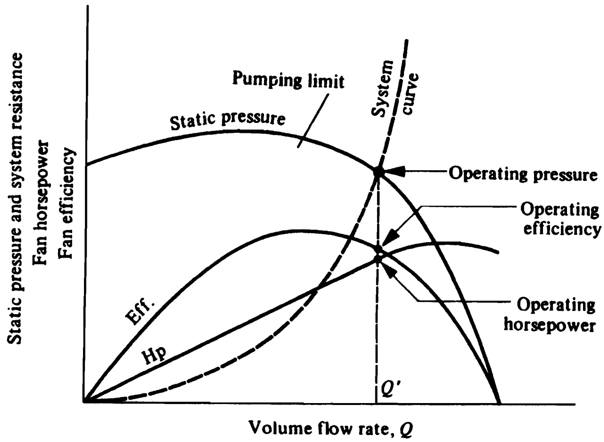


Fig. 5.6 Relationship between the fan curves and system curve. Often only the static pressure curve is supplied, so one assumes it is approximately the same as the total pressure curve.

The normal operating range for a fan is to the right of the peak of its pressure-volume curve. The so-called *pumping limit*, defines the furthest possible left-hand operating point. *Pumping* occurs when the fan is slowed down to the point where the static pressure created is less than the static pressure in the discharge line. At that point a flow reversal takes place. After this reversal, a momentary drop in the pressure in the discharge line occurs, and flow starts in a normal direction again. This pattern is repeated very rapidly and results in a pumping-type action, which may cause damage to the fan unless flow below this limit is avoided. It usually turns out that this point is reached at the maximum on the fan *total pressure* curve, which in turn is usually slightly to the right of the maximum of the static pressure curve.

It is not always possible to make desired changes in the operating point of a fan system. Changes in operating temperature or pressure drop require reevaluation of the system resistance curve, and if this changes, the operating point shifts, unless the fan is adjusted to keep Q' constant. Conversely, if it is desired to change Q', then the operating point has to be changed, and consequently the fan characteristic curve must be changed.

In order to estimate the effect of deviations from the specified conditions, or the effect of a change in fan size or speed, on the behavior of the fan or the gas, the so-called *fan laws* are used to adjust from one set of operating variables to another. The variables are given in Table 5.1.

Table 5.1 Variables for fan laws

Speed of rotation	n
Impeller diameter	D
Gas density	ρ
Static pressure	P_s
Power	P_B
Volume flow	Q

The fan laws are:

$$Q = k_1 D^3 n, \tag{5.16}$$

$$P_s = k_2 D^2 n^2 \rho, \tag{5.17}$$

$$P_B = k_3 D^5 n^3 \rho. \tag{5.18}$$

As an example of the use of the fan laws, if the density of the gas varies, and the speed and volume flow remain constant, then Eqs. (5.17) and (5.18) tell us that both the horsepower and the static pressure would be expected to vary *directly* with the density. The proportionality constants k_1, k_2, and k_3 are constants over a limited range, which should not deviate too far from a given point on the curve of pressure versus flow rate.

Changes in the pattern of flow of the gases entering the fan will also change the characteristic curve, since the calibration is performed with gases entering smoothly at right angles to the rotor. If the gases enter otherwise, then their flow through the fan will affect the characteristic curves.

It is best, therefore, to overrate fans somewhat on both the pressure and volume specifications, but not too much, because severe damping, if necessary, decreases the efficiency and wastes the available power. The problems that can arise from errors in matching the system and fan curves are illustrated in the following example.

Example 5.2 Part of an iron-ore pelletizing plant consists of a moving chain grate (carrying a bed of pellets subjected to a hot gases) and associated gas cleaning and exhaust equipment, as shown schematically in Fig. 5.7. The total pressure drop due to duct work, at the required flow of 320 000 cfm at 394 K, is 3.5 in. w.c., and, when added to the *calculated* design bed-resistance of 9.5 in. w.c. at the design flow, gives a total system *design resistance* of 13.0 in. w.c. The fan to be used in this system is then specified to meet a 320 000 cfm flow with a 13.0 in. w.c. static pressure drop (point O in Fig. 5.8).

Suppose, however, that when the plant is built, the *actual* pressure drop across the bed of pellets at the required flow is 21.0 in. w.c. This means that the actual system curve (I)

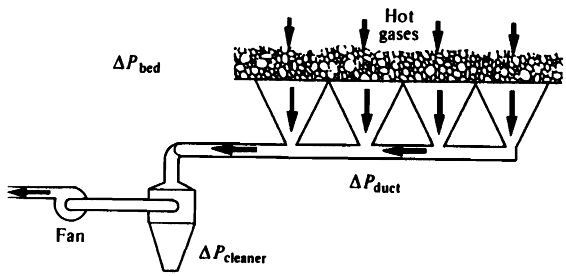


Fig. 5.7 Schematic diagram of an iron-ore pelletizing plant for Example 5.2.

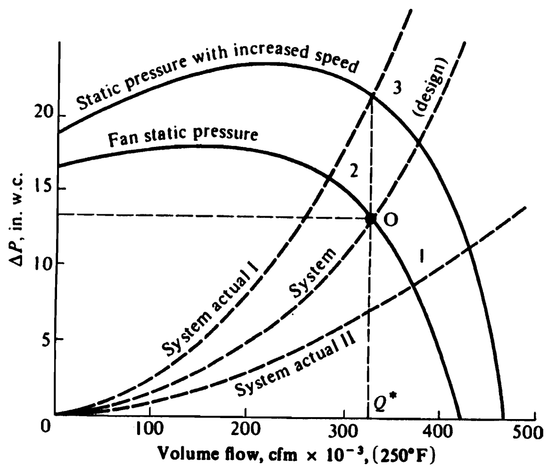


Fig. 5.8 Interaction between the system-resistance curves and fan curves for Example 5.2.

is different from the design system curve. Now the fan operates at point 2 (Fig. 5.8), and it pulls less than the required flow through the system (280 000 cfm). In this case, the only alternatives to regain a flow of 320 000 cfm are either to decrease the system resistance, or to change the characteristic curve by increasing the fan speed. In the first instance, one could decrease the system resistance by decreasing the bed height. If this is done, however, the lateral speed of the bed must be increased in order to keep the production rate at the design value. However, this may not be possible, because there is usually a minimum reaction time for the exposure of the solids to the hot gases, and the original design is usually close to that minimum. Changes in duct work may also be possible, but usually are not practical once the plant is built.

The other alternative—speeding up the fan until the operating condition is at point 3—is easier but may be very expensive. Reference to the fan laws shows us that the power required is proportional to the cube of the flow; that is, for constant fan size and gas density, $Q \propto n$, and $P_B \propto n^3$, so that $P_B \propto Q^3$. Therefore, for an increase of flow in the ratio

320 000/280 000 = 1.14, the ratio of the new horsepower required to the old will be $(1.14)^3 = 1.50$; in other words, a motor 50% more powerful is required. This may be quite expensive and could cause many problems in the electrical system. The ultimate solution is to buy a new fan.

If, on the other hand, the actual bed-resistance were, e.g., 7 in. w.c., then the fan would pull too much gas through the system, as at point 1 (Fig. 5.8). This can easily be corrected by increasing the system resistance through the use of dampers, until the system and fan curves coincide at the required flow and pressure drop, point O.

We have seen that the operating point of a system with a fan is given by the intersection of the system curve with the characteristic pressure curve of the fan. The same principle applies to the use of pumps where the operating curve is given by the intersection of the system curve with the characteristic head curve of the pump. In installations with more than one pump, this principle can lead to a rather surprising result; that the use of two equivalent pumps, rather than one, does not double the flow rate through the system. This is illustrated by the following example.

Example 5.3 Part of the process of regenerating spent hydrochloric acid from a continuous pickling line in a steel strip mill is shown in Fig. 5.9. When both pumps operate, the flow rate is 30 gpm (1.89×10^{-3} m^3 s^{-1}). What is the flow rate when one pump operates? The pumps are identical with the same characteristic head curve, Fig. 5.10. Assume that the flow is highly turbulent and that the "head" in the system is primarily because of friction in the system.

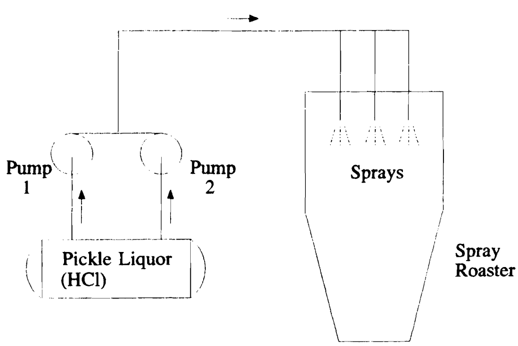


Fig. 5.9 System for Example 5.3.

Solution. With both operating, the pumps work against a head given by

$$h_{12} = E_f = KQ^2,$$

where h_{12} is the head overcome by both pumps 1 and 2, Q is the volume flow rate and K is a constant. We have assumed $E_f = KQ^2$ because the flow is turbulent. Therefore, the work done by one pump, when both are operating, must be

$$h_1 = \frac{h_{12}}{2} = \frac{KQ^2}{2},$$

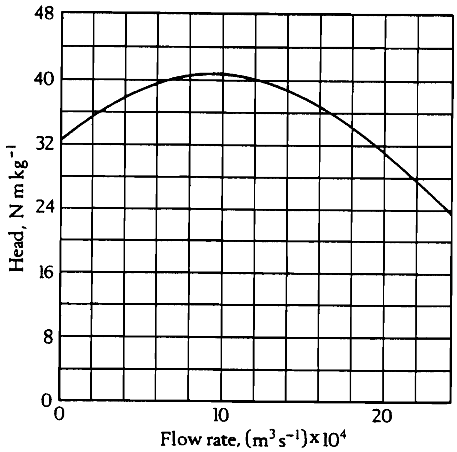


Fig. 5.10 Characteristic head curve for the pumps in Example 5.3.

with a flow rate of $Q/2$. Thus $Q/2 = 15$ gpm (9.45×10^{-4} m^3 s^{-1}) which gives an operating point at $h_l = 40.8$ N m kg^{-1} from Fig. 5.10. Therefore,

$$K = \frac{2}{} \left| \frac{40.8 \text{ N m}}{\text{kg}} \right| \frac{\text{s}^2}{\left(1.89 \times 10^{-3}\right)^2 \text{m}^6} = 2.28 \times 10^7 \text{ N s}^2 \text{ kg}^{-1} \text{ m}^{-5}.$$

With this value of K, we can plot the system curve on Fig. 5.10 and determine the operating point when one pump operates by itself.

$$h_l = KQ^2.$$

Q, m^3 s^{-1}	h_l, N m kg^{-1}
5×10^{-4}	5.7
10×10^{-4}	22.8
12.5×10^{-4}	35.6
15×10^{-4}	51.3

The curve generated by these calculated data intersects the characteristic head curve at $h = 39.2$ N m kg^{-1} and $Q = 13.2 \times 10^{-4}$ m^3 s^{-1} (21.0 gpm). Notice that with both pumps operating, the throughput is not doubled; in this example it increases by only 43%.

5.4 SUPERSONIC NOZZLES AND JET BEHAVIOR

During the past three decades, jets of gas have become important in several metallurgical processes; process engineers should therefore have a good understanding of the characteristics and behavior of jets, in order to make the best use of them. Applications of supersonic nozzles include injection of oxygen into molten steel baths for refining purposes, and inert gas jets used to atomize liquid metal streams into metal powders.

Consider the flow nozzle described in Fig. 5.11. If the area of the nozzle opening is considerably smaller than that of the approach pipe, then the velocity in the pipe is negligible (the gas is stagnant) with respect to the velocity at the nozzle opening, as long as the pressure in the pipe is at or above the *stagnation* pressure. The application of Bernoulli's equation to the adiabatic and frictionless flow of an ideal compressible gas under these conditions yields an equation for the velocity in the nozzle opening $\bar{V}_t$:

$$\bar{V}_t = \sqrt{\frac{2P_0}{\rho_0}\left[\frac{\gamma}{\gamma - 1}\right]\left[1 - \left[\frac{P_t}{P_0}\right]^{(\gamma - 1)/\gamma}\right]}, \tag{5.19}$$

where reference points are defined in Fig. 5.11 and $\gamma = C_P/C_V$. We may use this equation for a flow nozzle operating at subsonic velocities.

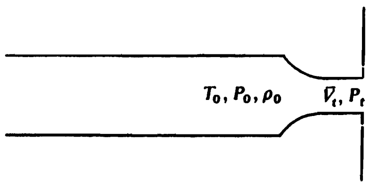


Fig. 5.11 Reference points for flow-nozzle equations.

Now the speed at which a compression-expansion wave passes through a medium, that is, the speed of sound V_s in that medium, is given by

$$V_s = \left[\frac{\partial P}{\partial \rho}\right]_S^{1/2}, \tag{5.20}$$

where the subscript on the partial derivative indicates constant entropy. For an ideal gas,

$$V_s = \left[\frac{\gamma P}{\rho}\right]^{1/2}. \tag{5.21}$$

The equation of momentum for one-dimensional flow is

$$\frac{dP}{\rho} + \bar{V}d\bar{V} = 0. \tag{5.22}$$

Substituting $dP = V_s^2 d\rho$ into Eq. (5.22), we get

$$V_s^2 \frac{d\rho}{\rho} + \bar{V}d\bar{V} = 0. \tag{5.23}$$

The continuity equation, which requires that the mass flow rate remain constant at all points in the nozzle

$$\frac{d\rho}{\rho} + \frac{d\bar{V}}{\bar{V}} + \frac{dA}{A} = 0, \tag{5.24}$$

must be satisfied along with Eq. (5.23), and so

$$\frac{d\bar{V}}{\bar{V}}\left[\frac{\bar{V}^2}{V_s^2} - 1\right] = \frac{dA}{A}. \tag{5.25}$$

The Mach number, M, is defined as $\bar{V}/V_s$, so that finally

$$\frac{d\bar{V}}{\bar{V}}\left(\mathrm{M}^2 - 1\right) = \frac{dA}{A}. \tag{5.26}$$

Now, if the velocity at any point in the nozzle is *less than* M = 1, and if the area of the nozzle decreases at that point (dA/A is negative), then the velocity increases at that point ($d\bar{V}/\bar{V}$ is positive).[†] At the throat $dA = 0$, so either $\mathrm{M} = 1$ or $d\bar{V}/\bar{V} = 0$. If $\mathrm{M} < 1$ at the throat, then $d\bar{V}/\bar{V}$ is zero and no further increase in velocity is possible, i.e., this is the maximum velocity which can be achieved.

If M = 1 at the throat, then we have reached the point at which supersonic flow can take place. By adding a diverging section on the end, and referring to Eq. (5.26), we see that if dA/A is positive, then either $d\bar{V}/\bar{V}$ will be positive, and a further increase to higher velocities is achieved in the diverging section, or the flow must come to a stop. The former case occurs as long as $P_t > P_{\text{exit}}$. A converging-diverging nozzle is shown in Fig. 5.12. It is usually called a *deLaval nozzle.*

For a converging nozzle, we obtain the conditions required to attain sonic velocity at the throat by using the energy relation for an ideal gas undergoing adiabatic flow:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\mathrm{M}^2. \tag{5.27}$$

When M = 1, sonic conditions exist denoted by an asterisk, and since the flow is also isentropic, substitution of the ideal gas relationship

$$\frac{P}{P_0} = \left[\frac{T}{T_0}\right]^{\gamma/(\gamma - 1)} = \left[\frac{\rho}{\rho_0}\right]^{\gamma} \tag{5.28}$$

gives the *critical pressure ratio*

$$\frac{P_t^*}{P_0} = \left[\frac{2}{\gamma + 1}\right]^{\gamma/(\gamma - 1)}. \tag{5.29}$$

This means that the ratio of throat pressure to reservoir pressure at the sonic flow condition is governed only by the value of γ. For air and oxygen, $P_t^*/P_0 = 0.528$. For pressure ratios between 1.00 and 0.528, the mass flow rate of the gas is $W_t = \rho_t \bar{V}_t A_t$, and using Eq. (5.28),

$$W_t = A_t \sqrt{\frac{2\rho_0 P_0 \gamma}{\gamma - 1}\left[1 - \left[\frac{P_t}{P_0}\right]^{(\gamma - 1)/\gamma}\right]}\left[\frac{P_t}{P_0}\right]^{1/\gamma}. \tag{5.30}$$

[†]The reader should consider whether or not M > 1 can be achieved in the converging portion of a nozzle.

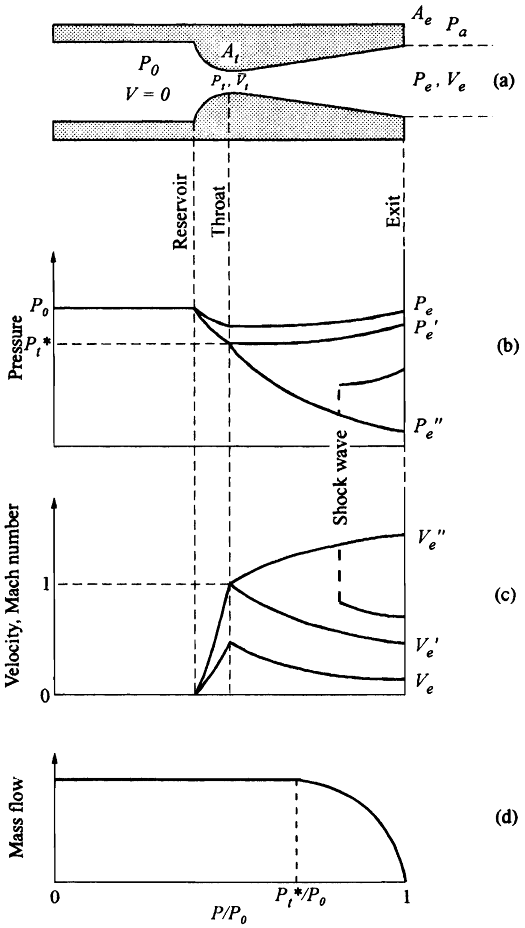


Fig. 5.12 Reference points and schematic internal conditions for various operating pressure ratios for converging-diverging nozzles.

Any further decrease in the ratio P_t/P_0 below that for the P_t^*/P_0, caused, for example, by increasing the reservoir pressure P_0, will not cause a further increase in the mass flow rate out of a converging nozzle, since the condition of isentropic flow is no longer satisfied beyond the nozzle. If $(P_t/P_0) \leq (P_t^*/P_0)$, then the nozzle is said to be *choked*, and the mass flow rate is

$$W_t^* = A_t \left[\rho_0 P_0 \gamma \left[\frac{2}{\gamma + 1} \right]^{(\gamma + 1)/(\gamma - 1)} \right]^{1/2} . \tag{5.31}$$

For the nozzle with the additional diverging section, the mass flow at sonic or supersonic conditions is still given by Eq. (5.31). However, since the velocity increases along the length of the diverging portion, the pressure correspondingly decreases from that at the throat. The problem is to determine how long this portion of the nozzle should be made in order to produce a jet with exit pressure P_e, equal to ambient pressure. Such a condition establishes the most effective condition for achieving the maximum supersonic jet length.

Up to this point, it has not been established what the absolute pressure at the throat, relative to the exit pressure, is. Several possible solutions arise, depending on the nozzle design. There are two values of P_t^*/P_e—and therefore two values of P_e, shown in Fig. 5.12(b), which result in isentropic shockless flow in the diverging portion of the nozzle for the conditions where M = 1 at the throat. The higher pressure P_e' will cause the flow to become subsonic again immediately at the throat, and will cause a decrease in velocity; the lower pressure P_e'' will allow supersonic flow throughout the nozzle. If the exit pressure is between P_e' and P_e'', $P_t = P_t^*$, then somewhere in the nozzle, a discontinuous transition from a lower to higher pressure (and simultaneously from supersonic to subsonic flow) occurs, as shown schematically in Fig. 5.12. This produces a *shock wave*.

If the nozzle design is such that $P_e = P_e''$, then we may proceed to calculate the exit velocity, using the equation

$$V_e = \sqrt{\left[\frac{2}{\gamma - 1}\right] \frac{\gamma P_0}{\rho_0} \left[1 - \left[\frac{P_e}{P_0}\right]^{(\gamma - 1)/\gamma}\right]} \tag{5.32}$$

or

$$\mathrm{M}_e^2 = \frac{2}{\gamma - 1} \left[\left[\frac{P_0}{P_e}\right]^{(\gamma - 1)/\gamma} - 1\right]. \tag{5.33}$$

The mass flow rate and throat area are still given by Eq. (5.31).

Now that we have the mass flow rate, we may calculate the area of the exit by means of a mass balance, since $W_t^* = W_e$,

$$A_t \sqrt{\rho_0 P_0 \gamma \left[\frac{2}{\gamma + 1}\right]^{(\gamma + 1)/(\gamma - 1)}} = A_e \sqrt{\rho_0 P_0 \gamma \left[\frac{2}{\gamma - 1}\right] \left[1 - \left[\frac{P_e}{P_0}\right]^{(\gamma - 1)/\gamma}\right]} \cdot \left[\frac{P_e}{P_0}\right]^{1/\gamma}, \tag{5.34}$$

$$\left[\frac{A_t}{A_e}\right]^2 = \left[\frac{2}{\gamma - 1}\right] \left[\frac{\gamma + 1}{2}\right]^{(\gamma + 1)/(\gamma - 1)} \left[\frac{P_e}{P_0}\right]^{2/\gamma} \left[1 - \left[\frac{P_e}{P_0}\right]^{(\gamma - 1)/\gamma}\right]. \tag{5.35}$$

Ideally, we want $P_e = P_a$ (the ambient pressure), since then the ultimate adiabatic expansion is reached, and the jet issues from the nozzle at atmospheric pressure. If P_e is less than P_a, the ambient atmosphere compresses the flowing jet and collapses it in a series of shock waves. If $P_e > P_a$, the jet continues to expand beyond the nozzle tip. In both non-ideal situations, the efficiency of conversion of the nozzle velocity to jet momentum decreases.

The angle of divergence of the nozzle is usually about 7° in order to avoid separation of flow from the nozzle walls.

Example 5.4 In the basic oxygen steelmaking process, we decarburize a bath of molten iron-carbon alloy with gaseous oxygen which is blown into the bath from a lance held above the bath. Determine the dimensions of the converging-diverging nozzle which are required to achieve a velocity of Mach 2 at the exit with an oxygen flow rate of 15 000 scfm.* What is the required driving pressure?

Solution. From Eq. (5.33), we can obtain P_0 by assuming that P_e = 1 atm to achieve the best behavior of the jet, and noting that γ for O_2 is 1.4:

$$\left[\frac{P_0}{P_e}\right]^{0.286} = \frac{2^2}{\left(\frac{2}{0.4}\right)} + 1.0,$$

and thus

$$\frac{P_0}{P_e} = 7.85.$$

For P_e = 1 standard atmosphere, $P_0 = 7.95 \times 10^5$ N m^{-2} (7.84 atm). Now we can make use of Eq. (5.35) to find the ratio A_t/A_e:

$$\left[\frac{A_t}{A_e}\right]^2 = \left[\frac{2}{0.4}\right]\left[\frac{2.4}{2}\right]^{2.4/0.4}\left[\frac{1}{7.84}\right]^{2/1.4}\left[1 - \left[\frac{1}{7.84}\right]^{0.4/1.4}\right]$$

$$\left[\frac{A_t}{A_e}\right] = 0.595.$$

Finally, we can calculate A_t from Eq. (5.31), since the nozzle is choked, as otherwise supersonic velocities could not be reached.

The mass flow is

$$W_t^* = 15\ 000 \text{ scfm} \times 0.089 \frac{\text{lb}_m}{\text{ft}^3} \times \frac{1 \text{ min}}{60 \text{ s}} = 22.2 \text{ lb}_m \text{ s}^{-1} = 10.1 \text{ kg s}^{-1}$$

$$10.1 = A_t \left\{(10.41)(7.95 \times 10^5)(1.4)\left[\frac{2}{2.4}\right]^6\right\}^{1/2} = 1.970 \times 10^3 A_t$$

$$A_t = 5.127 \times 10^{-3} \text{ m}^2$$

and

$$A_e = 8.617 \times 10^{-3} \text{ m}^2.$$

Thus, the throat diameter d_t = 80.8 mm, and the exit diameter d_e = 105 mm.

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*Standard cubic feet per minute. 1 m^3 s^{-1} = 2.1189×10^3 cfm.

As a jet exits from the nozzle, it entrains adjacent air, which in turn acts as a drag, creating turbulence. This slows some of the supersonically flowing gas to sonic and subsonic velocities and the supersonic core of the jet gradually decays, until at some distance from the nozzle the core disappears, and the entire jet is subsonic. Figure 5.13 shows the relationship between the exit velocity (in terms of Mach number) and the length of the supersonic core (in terms of either the ratio x/d_t, where x is the distance from the nozzle and d_t is the throat diameter, or the ratio x/d_e, where d_e is the exit diameter).

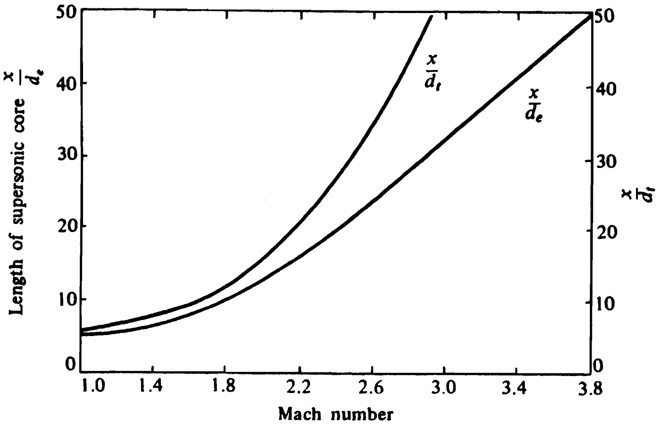

Fig. 5.13 Relationship between the length of supersonic core and exit Mach number.

We measure the overall spreading of the jet by the ratio r_0/r_t, where we define r_0 as the radius at which the velocity is one-half that at the center line. The typical velocity or impact pressure profile is that of a normal distribution about the center line. However, the spreading of the jet is minor until the supersonic core has decayed. Once this point has been reached, the jet expands at an included angle of about 18°. Figure 5.14 gives a dimensionless graph for determining the width of the jet as a function of the exit Mach number and the distance from the nozzle. We then take the *effective jet radius* r as $2r_0$, and can calculate it at any distance from the nozzle.

The increase in the mass flow of a subsonic jet due to the entrainment of the surrounding gas has been found to be directly proportional to the distance from the nozzle exit, according to the equation

$$W_x = W_e + k\sqrt{M_e\rho_e}\left[\frac{x}{d}\right], \tag{5.36}$$

where M_e = jet momentum at nozzle exit (= W_eV_e), x/d = distance from the nozzle exit, W_e = jet mass flow rate at nozzle exit, W_x = jet mass flow at x, ρ_a = density of the ambient gas.

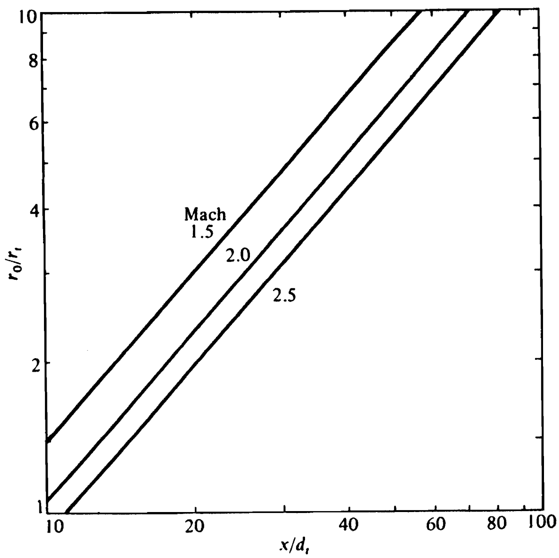

Fig. 5.14 Jet-spreading characteristics as a function of Mach number and distance from the nozzle. (Adapted from A. R. Anderson and F. R. Johns, *Jet Propulsion* **25**, 13 (1955).)

In supersonic jets, the entrainment in the region where supersonic flow predominates is less than in the subsonic region, where Eq. (5.36) is satisfactory. A very satisfactory representation of the increased jet mass for supersonic jets with exit velocities between Mach 1 and 2 is given by*

$$\frac{W_x - W_e}{W_e} = 9 \times 10^{-3}\sqrt{\frac{T_0}{T_a A_e}\left[1 + \frac{(\gamma - 1)M^2}{2}\right]^{-1}}\left[\left[\frac{x}{d_e}\right] - \left[\frac{x}{d_e}\right]_{\text{core}}\right], \qquad (5.37)$$

where T_0 = stagnation temperature, °R, T_a = ambient temperature, °R, and $(x/d_e)_{\text{core}}$ = length of the supersonic core determined from Fig. 5.13. (Degrees Rankine, °R, is an absolute temperature scale equal to 9/5 K.) This equation does *not* describe the increasing entrainment in the region from the nozzle to a distance of about 10 nozzle diameters downstream, but since this is not the region of usual interest and also the rate of increase is not very strong in this region, the equation is still useful. Note that as the ambient temperature increases, the entrainment decreases.

We should point out that the entrainment of the surrounding gases dilutes the jet-gas concentration. For example, the concentration of oxygen in cold jets of pure oxygen issuing into cold air, at a distance of 6 ft (1.8 m) from the nozzle, is 90% in the case of a Mach 3 nozzle versus 80% in the case of a subsonic nozzle with 25 psig (1.7 × 10^5 Pa) driving pressure.

*This equation is derived from data presented in the report by J. D. Kapner and Kun Li, *Mixing Phenomena of Turbulent Supersonic Jets*, American Iron and Steel Inst., June 26, 1967.

5.5 VACUUM PRODUCTION

In materials processing, vacuum technology plays an important role in processes such as vacuum annealing, vacuum deposition of coatings, vacuum melting, and vacuum degassing. In addition, vacuum equipment is a standard item in almost every materials laboratory. For these reasons, it is important that materials engineers be acquainted with the principles and operation of vacuum-producing equipment.

The most important variable in the design and specification of equipment for a vacuum system is the pressure which the pumping system must be able to maintain in the work chamber. In the U.S.A., the most commonly used unit of vacuum measure is the torr,* which is equivalent to the old millimeters of mercury. In European usage, the pressure is commonly expressed in millibars (mbar), with 1 bar equal to normal atmospheric pressure. Therefore, 1000 mbar = 760 torr = 1.0133×10^5 N m^{-2}. Figure 5.15 compares the various pressure scales, indicating the ranges of application of various types of vacuum gages and pumps.

Throughput (Q) is another term that is commonly used in vacuum technology. It is the quantity of gas, expressed as the volume of gas multiplied by its pressure, that passes a plane in a unit time. In SI units, throughput has units of Pa m^3 s^{-1}. Coincidentally these units are equivalent to W, so throughput can also be pictured as energy flow. We should keep in mind that throughput is not a mass flow, but of course if the temperature is known then mass flow rate can be calculated from throughput.

Another variable is the speed at which the desired pressure is reached. We define the *speed* S_p, of any type of vacuum pump by

$$S_p = Q/P \tag{5.38}$$

where P is the pressure at the inlet to the pump, and Q is the throughput at that point. Typical units for S_p are m^3 s^{-1}. We may use the same definition to express the pump-down speed of a system S_s, consisting of a working chamber, connecting duct work, and pumps, but the speed depends on the design of the rest of the system as well as on the pump itself.

A system, initially at atmospheric pressure, is first *roughed out* by either mechanical pumps or the first stage of an ejector system until a pressure is reached where vapor diffusion pumps, or further stages of an ejector system, become effective and can be used for final evacuation to the desired limiting pressure. During the initial roughing out, the gas density will be high enough, so that the mean free path of the gas is very small ($\sim 10^{-4}$ mm) compared with the dimensions of the conduit, and the flow rate of the gas is governed by the viscosity of the gas through the equations developed in previous chapters for turbulent and laminar flows. However, at low pressures, the density eventually reaches a value such that the mean free path is much greater than the conduit dimensions. At this point, viscosity is meaningless in determining the flow rate, and the gas flow becomes *molecular flow*. At an intermediate pressure, a transition flow regime is encountered when the mean free path is of the same order of magnitude as the equipment dimensions.

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*The name torr comes from E. Torricelli, a student of Galileo, who devised the first single stroke pump by inverting a closed tube of mercury into a dish containing mercury, creating a vacuum in the tube.

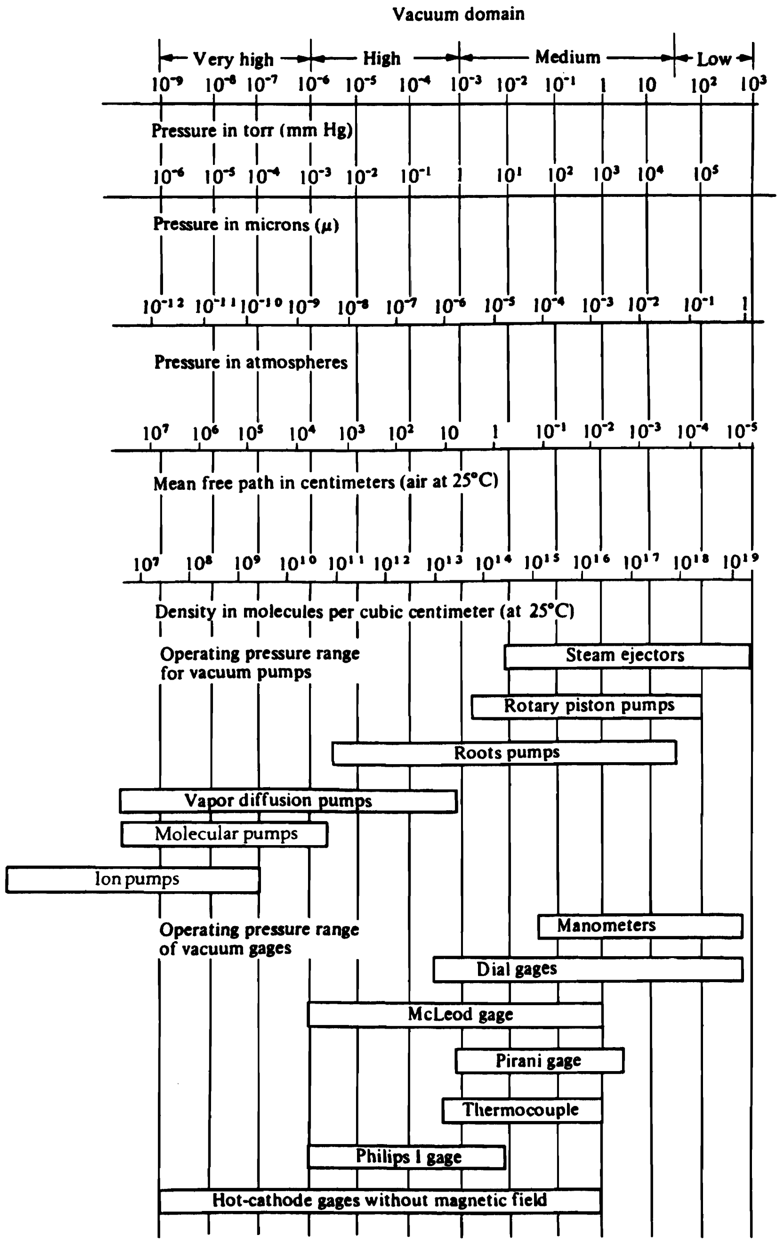


Fig. 5.15 Comparative pressure scales and ranges of application of various vacuum pumps and gages.

5.5.1 Molecular flow mechanics

In the *molecular flow* regime, the gas molecules move randomly, with a Maxwell-Boltzmann distribution of velocities, and collisions between molecules are rare compared to collisions with the walls. The only transfer of momentum is between molecules and the wall instead of from molecule to molecule. Net flow results from the statistical effect that the number of molecules leaving a given region is proportional to the number of molecules in the region, and the number reentering will be proportional to the number in the adjacent region. Knudsen developed the equations which govern the flow of gases through various geometries under molecular flow conditions. Consider a chamber with an aperture of area A, as in Fig. 5.16. The pressure on the downstream side of the aperture is P_2 and that in the chamber is P_1. If the concentration of gas molecules is given by n (molecules mm^{-3}), and the number hitting the wall is

$$Z = \frac{n\bar{V}}{4}, \text{ molecules mm}^{-2}\text{ s}^{-1}, \tag{5.39}$$

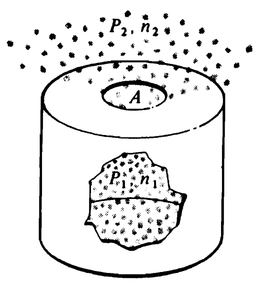


Fig. 5.16 Schematic chamber being evacuated from an internal pressure of P_1 to an external pressure of P_2 through an aperture of area A.

where $\bar{V}$ is the average molecular speed given in Eq. (1.4), then the frequency Z of gas molecules per unit area passing from the high-pressure side of the aperture through it, is

$$Z_1 = \frac{n_1}{2\sqrt{\pi}} \left[\frac{2\kappa_B T}{m}\right]^{1/2}.$$

The frequency from the low-pressure side back through the opening

$$Z_2 = \frac{n_2}{2\sqrt{\pi}} \left[\frac{2\kappa_B T}{m}\right]^{1/2},$$

yields a net frequency

$$Z_{net} = Z_1 - Z_2 = \frac{1}{2\sqrt{\pi}} \left[\frac{2\kappa_B T}{m}\right]^{1/2} (n_1 - n_2). \tag{5.40}$$

For a circular opening, the net rate at which molecules leave the chamber is

$$\dot{n} = \frac{\pi D^2}{4} Z_{net} = \frac{\sqrt{\pi}}{8} \left[\frac{2\kappa_B T}{m}\right]^{1/2} D^2 (n_1 - n_2). \tag{5.41}$$

Because $n = P/\kappa_B T$,

$$\dot{n} = \left[\frac{2\pi}{m\kappa_B T}\right]^{1/2} \frac{D^2}{8} (P_1 - P_2), \text{ molecules s}^{-1} \tag{5.42}$$

or

$$\dot{n} = \left[\frac{2\pi}{MRT}\right]^{1/2} \frac{D^2}{8} (P_1 - P_2), \text{ kmol s}^{-1} \tag{5.43}$$

where M is the molecular weight, kg kmol^{-1}, and R is the gas constant (R = 8.315 kJ kmol^{-1} K^{-1}).

The throughput for flow of molecules across the aperture can be written as

$$Q = C(P_1 - P_2), \tag{5.44}$$

where C is the *conductance* of the aperture. By the definition of throughput and using the ideal gas law, we can also write

$$Q = \dot{n}RT, \tag{5.45}$$

so that

$$C = \frac{\dot{n}RT}{P_1 - P_2}. \tag{5.46}$$

By combining Eqs. (5.43) and (5.46), we arrive at the conductance of an aperture:

$$C = \left[\frac{2\pi RT}{M}\right]^{1/2} \frac{D^2}{8}. \tag{5.47}$$

We can write Eq. (5.47) in a simpler form:

$$C = \frac{\overline{V}}{4} A, \tag{5.48}$$

where $\overline{V}$ is the Maxwellian speed of the molecules given by Eq. (1.4) and A is the area of the aperture. For other shapes (e.g., tubes), the conductance is usually given by Eq. (5.48) with a *transmission probability*, a, which is the probability that a molecule entering the tube will leave the tube at the other end. Thus, the conductance is found from

$$C = \frac{a\overline{V}}{4} A, \tag{5.49}$$

where A is the cross-sectional area of the tube. The conductance of an aperture of area A has a maximum value, and tubes and conduits of the same area will have a conductance less than the conductance of the aperture.

To describe molecular flow through long circular tubes, Knudsen derived

$$C = \frac{\pi \bar{V} D^3}{12L}, \tag{5.50}$$

or

$$a = \frac{4D}{3L}.$$

With this result, we can see that we must be careful in how we apply conductances. For example, Eq. (5.50) indicates that the conductance goes to infinity as L approaches zero, whereas the limit should be the value given by (5.48). In an attempt to deal with short tubes, it has been suggested that the aperture and the tube be considered as two flow resistors in series. When this is done

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}, \tag{5.51}$$

where C is for the short tube, C_1 is for the aperture, and C_2 is for the "tube," itself, given by Eq. (5.50). As $L/D \to 0$ this result reduces to Eq. (5.48), and as $L/D \to \infty$, it reduces to Eq. (5.50). However, it is only approximate for intermediate values of L/D and can be in error by as much as 15%. Actually, the Monte Carlo method to track molecular motions through various configurations has been applied to calculate the transmission probabilities. Many of these results are reviewed and presented by O'Hanlon.[3]

The conductance of an entire vacuum system composed of several different components may be approximately calculated by analogy with electrical circuits. Specifically,

$$\frac{1}{C_{\text{system}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \tag{5.52}$$

for a system with components 1, 2, 3, etc., in series, or, if the components are in parallel, then

$$C_{\text{system}} = C_1 + C_2 + C_3 + \ldots \tag{5.53}$$

Table 5.2 presents the conductances of several shapes.

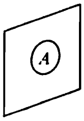
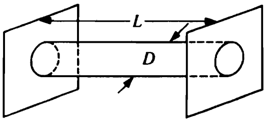
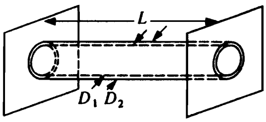
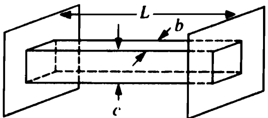
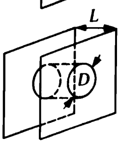
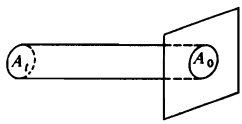
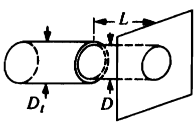
Example 5.4 A 150 mm diameter tube, with a length of 150 mm, connects a vacuum chamber and a vacuum pump. Which of the following modifications will give the greatest increase in conductance? (a) Decrease the length of the tube to 75 mm; (b) increase the diameter to 175 mm; (c) increase the length to 175 mm; (d) decrease the diameter to 125 mm.

Solution. We can immediately disregard modifications (c) and (d) because both result in an increase in L/D. The total conductance of the tube comprises the conductance of the aperture at the chamber-tube connection (C_1) and the tube itself (C_2). We combine Eqs. (5.48), (5.49) and (5.51):

$$\frac{1}{C} = \frac{4}{\bar{V}A} + \frac{4}{a\bar{V}A} = \frac{4}{\bar{V}A}\left[1 + \frac{1}{a}\right].$$

[3]J. F. O'Hanlon, *ibid.*

Table 5.2 Conductances of various geometric shapes for molecular flow*

Shape†	Conductance (L s^{-1})††	C, for air at 298 K
	$C = 3.64A\left(\frac{T}{M}\right)^{1/2}$	$= 11.7A$
	$C = 19.4\frac{A^2}{BL}\left(\frac{T}{M}\right)^{1/2}$	$= 12.2\frac{D^3}{L}$
	$C = 3.81\frac{D^3}{L}\left(\frac{T}{M}\right)^{1/2}$	$= 12.2\frac{(D_2 - D_1)^2(D_2 + D_1)}{L}$
	$C = 9.70\frac{b^2c^2}{(b + c)L}\left(\frac{T}{M}\right)^{1/2}$	$= 31.1\frac{b^2c^2}{(b + c)L}$
	$C = 2.85D^2\left(\frac{T}{M}\right)^{1/2}\left(\frac{1}{1 + 3L/4D}\right)$	$= 9.14\frac{D^2}{1 + 3L/4D}$
	$C = 3.64\left(\frac{T}{M}\right)^{1/2}\left(\frac{A}{1 - (A/A_t)}\right)$	$= \frac{11.7A_0}{1 - A_0/A_t}$
	$C = 3.81\frac{D^3}{L}\left(\frac{T}{M}\right)^{1/2}\left(\frac{1}{1 + \frac{4D}{3L}\left(1 - \frac{D^2}{D_t^2}\right)}\right)$	$= \frac{12.2D^3}{L\left[1 + \frac{4D}{3L}\left(1 - \frac{D^2}{D_t^2}\right)\right]}$

*This table is taken from J. M. Lafferty, *Techniques of High Vacuum*, General Electric Report No. 64-RL-3791G, 1964.

†The variables and their respective dimensions are

A = area, cm^2,
D = diameter, cm,
L,b,c = length dimensions, cm,
B = perimeter, cm,
T = absolute temperature, K,
M = molecular weight, g mol^{-1}

††1 L s^{-1} = 10^{-3} m^3 s^{-1}. L is liter.

With the original dimensions of the tube, the transmission probability is

$$a = \frac{4D}{3L} = \frac{4 \times 150}{3 \times 150} = \frac{4}{3}$$

so that

$$C = 0.571 \left[\frac{\bar{V}A}{4}\right].$$

For modification (a):

$$a = \frac{4 \times 150}{3 \times 75} = \frac{8}{3}$$

and

$$C_{(a)} = 0.727 \left[\frac{\bar{V}A}{4}\right].$$

For modification (b):

$$a = \frac{4 \times 175}{3 \times 150} = \frac{14}{9}$$

and

$$C_{(b)} = 0.607 \left[\frac{\bar{V}A}{4}\right].$$

The greatest increase is achieved by modification (a).

Pumping speed has the same units as conductance, but it should be noted that conductance implies a pressure gradient across a specific geometry. Pumping speed is simply the volume of gas flowing across any plane in a system per second, which is measured at the pressure existing at that particular plane.

The pump-down speed of a system depends on both the pump speed and the conductance of the connections. Refer to the schematic diagram of a vacuum melting system in Fig. 5.17. Because $P = Q/S$ at the inlet to the duct, $P_p = Q/S_p$ at the inlet to the pump, and $(P - P_p) = Q/C$ over the duct length, then

$$\frac{1}{S} - \frac{1}{S_p} = \frac{1}{C}, \tag{5.54}$$

or

$$S = S_p \left[\frac{1}{1 + S_p/C}\right]. \tag{5.55}$$

This means that the effective pump speed S of a system being evacuated by a pump with rated speed S_p, cannot exceed S_p or C, whichever is smaller. If the conductance of the duct is the same as the pump speed, then $S = S_p/2$. This should emphasize why it is desirable to make connections between the working chamber and the pump as short and as wide as possible.

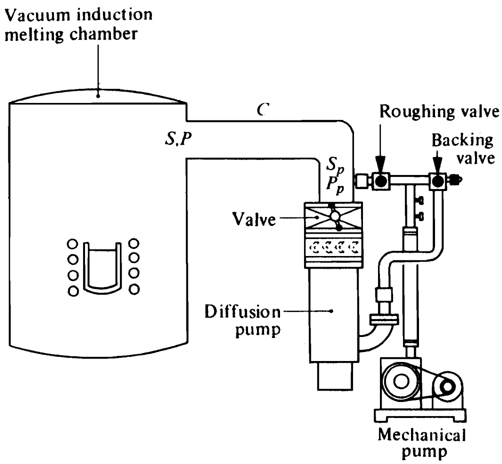


Fig. 5.17 A typical vacuum melting system.

5.5.2 Mechanical pumps

Most mechanical vacuum pumps are of the positive-displacement, rotary-piston type with sliding vanes, and sealed with oil. A small quantity of gas from the system is isolated, compressed, and discharged to the atmosphere with each rotation of the piston. These pumps have *intrinsic speeds*, S_0, ranging in value from 5×10^{-4} to 0.35 $m^3\ s^{-1}$.

There is a lower limit to the pressure that a pump may produce, known as the ultimate pressure P_a, at which point the speed drops to zero. This limit is determined by the amount of back leakage of a very small quantity of gas Q_a, which is nearly independent of pressure. At the inlet to the pump,

$$S_p = \frac{Q - Q_a}{P_p} = S_0 \left[1 - \frac{Q_a}{Q} \right] . \qquad (5.56)$$

At the ultimate pressure, $Q_a = Q$ and $S_0 = Q_a/P_a$. Therefore

$$S_p = S_0 \left[1 - \frac{P_a}{P_p} \right] . \qquad (5.57)$$

Substitution of Eq. (5.57) into Eq. (5.55), and elimination of S_p and P_p by the use of $PS = Q = (P - P_p)C$, leads to a more realistic value for the speed of the system

$$S = S_0 \left[\frac{1 - P_a/P}{1 + S_0/C} \right] , \qquad (5.58)$$

or

$$S = S' \left[1 - \frac{P_a}{P} \right] , \qquad (5.59)$$

where

$$S' = S_0 \left[\frac{1}{1 + S_0/C} \right].$$

Single-stage pumps have ultimate pressures in the range from 10^{-2}-10^{-3} torr. Two-stage mechanical pumps can reduce the ultimate pressure to 10^{-4}-10^{-5} torr. In order to remove condensable gases (especially water vapor), a trap, either cold or chemical, should be placed ahead of the pump. If a slightly lower pressure is desired, and if the quantity of gas is large, a so-called Roots pump may be used in conjunction with an oil-sealed mechanical forepump. This combination may produce an ultimate pressure of 10^{-6} torr. Typical characteristic curves for various types of mechanical pumps are shown in Fig. 5.18.

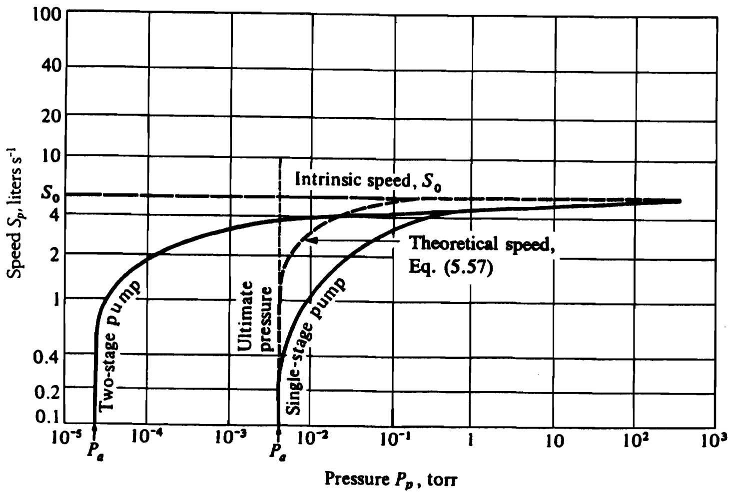


Fig. 5.18 The speed-pressure characteristics curves for a single- and two-stage rotary, oil-seated, vane-type pump.

5.5.3 Diffusion pumps

We apply the term diffusion pump to a jet pump which utilizes the vapor from low vapor-pressure liquids to impart increased momentum to the gas molecules being removed from the system, eventually forcing them out of the mouth of the discharge into a mechanical forepump. Figure 5.19 illustrates a typical diffusion pump. The pump liquid is heated in the boiler until its vapor pressure reaches an optimum value of about 1 torr. This vapor is carried to a nozzle from which it is ejected as a high-velocity jet directed away from the incoming gas and towards the wall of the pump. The gas molecules flow into the annular space between the wall and column by molecular diffusion. Some fraction, H, of the molecules that encounter the jet in the first stage is entrained into the jet and driven downstream with higher velocities than the molecules would normally have. The jet expands and eventually strikes the water-cooled wall, the working fluid condenses, and flows down the walls back to the boiler. In multiple-jet pumps, the gas molecules being pumped are caught in a succession of jet spray stages, and eventually ejected from the diffusion pump into the foreline.

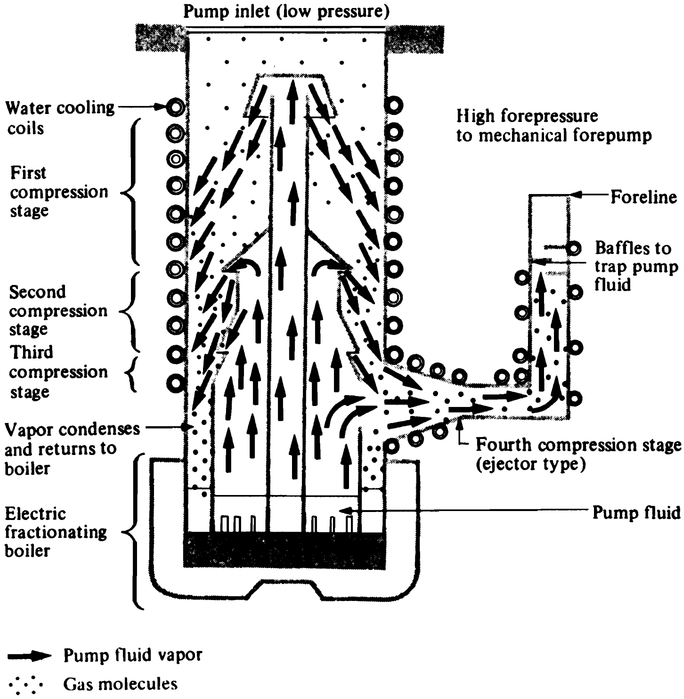


Fig. 5.19 Cross section of a typical vapor diffusion pump.

Since free molecular flow is needed for their successful operation, diffusion pumps usually operate at inlet pressures of 10^{-3} torr, or below. The compression ratios are not great enough to allow direct discharge to the atmosphere, so a relatively low forepressure must be maintained by a mechanical forepump, ultimately discharging to the atmosphere. For any diffusion pump there is a *limiting forepressure*, which is the pressure above which the boundary between the jet of pump vapor molecules and the randomly moving incoming gas molecules does not extend to the cold pump walls. In this situation, there is a direct connection between the high-vacuum and low-vacuum (forepump) sides of the jet, and effective pumping ceases. This pressure is typically of the order of 0.5 torr for multistage diffusion pumps and 0.05 torr for single-stage pumps.

We define the speed of diffusion pumps in the same way as for mechanical pumps. If A is the area of the pumping annulus, then the rate at which gas molecules are entrained by the jet is

$$HZA = \frac{HAn\bar{V}}{4}, \tag{5.60}$$

using the previously defined nomenclature. Then, the intrinsic pump speed S_0 is

$$S_0 = \frac{HZA}{n} = \frac{Q}{P_p} = \frac{HA\bar{V}}{4}, \tag{5.61}$$

and, substituting Eq. (1.4) for $\overline{V}$, we obtain

$$S_0 = \frac{H}{2\sqrt{\pi}} \left[\frac{2\kappa_B T}{m} \right]^{1/2} A, \tag{5.62}$$

or

$$S_0 = 3.64H \left[\frac{T}{M} \right]^{1/2} A, \text{ L s}^{-1}. \tag{5.63}$$

Specifically, for air at 293 K,

$$S_0 = 11.6(HA), \text{ L s}^{-1}, \tag{5.64}$$

where A is measured in cm^2. The coefficient H is a measure of the collection efficiency of the pump and is called the *Ho coefficient* (named after T. L. Ho). For most diffusion pumps the Ho coefficient is about 0.5. These equations imply that S_0 is independent of the pressure for diffusion pumps, and this is nearly true, as shown in Fig. 5.20.

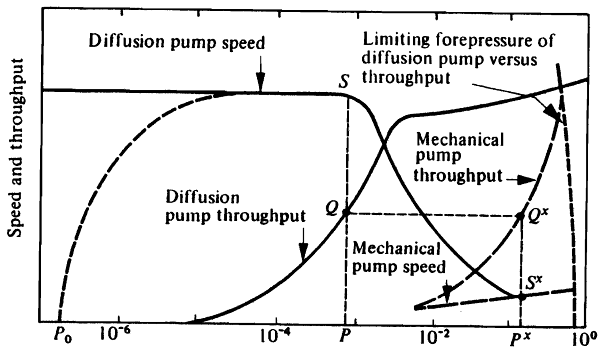


Fig. 5.20 Characteristic curve for a typical vacuum diffusion-pump, with the curves for a matching mechanical forepump included.

The ultimate pressures attainable with diffusion pumps depend on their design, including the number of stages, and also on the working fluid used. If we use mercury, a vapor trap, in the form of a baffle system externally cooled to low temperatures, must be placed between the diffusion pump and the system being evacuated in order to prevent back-diffusion of mercury vapor ($p_{Hg} = 1 \times 10^{-3}$ torr at 293 K). In this case, the effective speed of the pump is

$$S_p = S_0 \left[\frac{1}{1 + S_0/C_t} \right], \tag{5.65}$$

where C_t is the conductance of the trap. For well-designed traps, $S_p \approx S_0/2$. The ultimate pressures attainable with *well-trapped* mercury diffusion pumps are of the order of 10^{-6}-10^{-7} torr. More commonly, low-vapor-pressure hydrocarbon oils or silicone oils are used as the working fluids. They are capable of producing vacuums of 10^{-6} torr without traps.

5.5.4 Pumpdown time

Returning to our vacuum melting unit, the basic equation relating the change in pressure in the tank to the pumping speed of the system, is

$$PS = -V \frac{dP}{dt} + Q_l, \tag{5.66}$$

where P = pressure measured at a specified point in the system, S = speed at that point, V = system volume, and Q_l = additional gas flow made up of the leak rate, interior surface outgassing, and any process gases. In well-maintained systems with no process gas evolution, Q_l is eventually brought to a negligible level.

Solving Eq. (5.66) for S and equating with Eq. (5.59), we obtain

$$\frac{dP}{(P - P_a)} = -\frac{S'}{V}\, dt. \tag{5.67}$$

We find the pumpdown time by integrating this equation from the initial tank pressure P_1 to the final pressure P_2

$$t = \frac{V}{S'} \ln \left[\frac{P_1 - P_a}{P_2 - P_a}\right]. \tag{5.68}$$

However, because S' is a function of the conductance, it is also a function of pressure at the higher pressures where viscous flow occurs. Therefore, in order to use Eq. (5.68), our approach will have to be to add several values of t obtained by incrementing S' until it is no longer a function of pressure. Equation (5.68) is not completely accurate, but it is a good approximation down to a pressure of the order of 1 torr.

Finally, we must consider the interaction and matching of the forepump and diffusion pump. Figure 5.20 includes the performance curves for a diffusion pump and a mechanical pump. In operation, the forepump is turned on, and run alone until a forepump inlet pressure less than the limiting forepressure of the diffusion pump is reached, at which point the lower stages of the diffusion pump become operative. If the pressure at the diffusion pump inlet is P, then the throughput is at point Q, and since the throughput is the same at any instant for both pumps, then the forepump inlet pressure is P^x, its speed is S^x, and its throughput Q^x.

Because the speed and throughput required of mechanical pumps when employed as forepumps are often rather low values, their use in large systems for roughing out would require excessively long pumpdown times. Therefore separate mechanical roughing pumps are often used, and then turned off when the diffusion pump-forepump combination can be applied.

5.5.5 Ejectors

For very large systems, other types of vapor pumps, called *steam ejectors*, are used. Figure 5.21 shows a schematic diagram of such an ejector. The steam is made to pass through a converging-diverging nozzle, designed to reach Mach 2 or higher at the nozzle exit and, correspondingly, a very low pressure P_e. The inlet port design pressure is then this P_e, and the steam jet entrains gas molecules at this pressure, as it leaves the nozzle. Flow into the port is again due to statistical molecular flow, at the lower pressures. Once entrained, the gas and steam slow down and are compressed in the diffuser so that they may exit at an

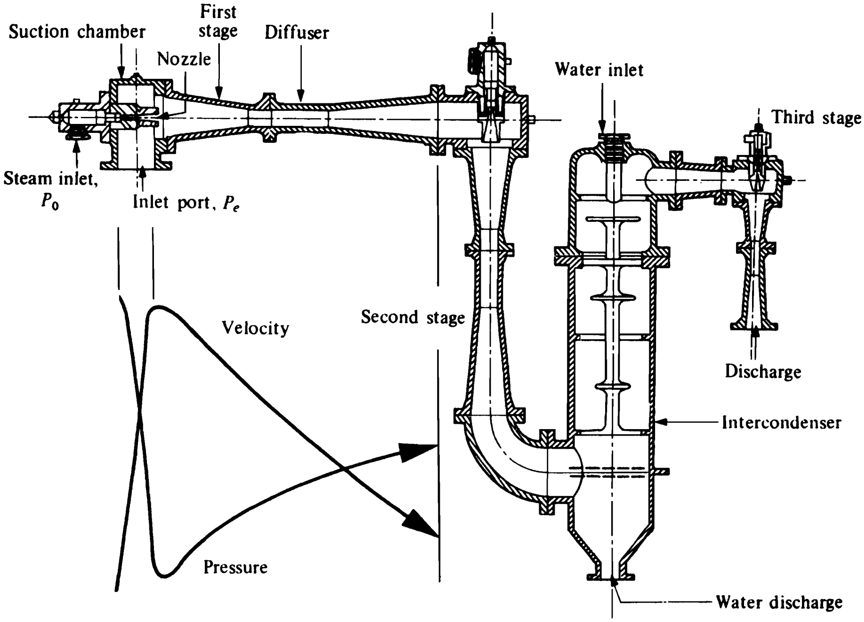


Fig. 5.21 Three-stage steam ejector with intercondenser. Schematic diagram of pressure-velocity relationships in the first-stage ejector is included.

exhaust pressure equal to the atmosphere, in the case of a single-stage pump, or at the design inlet pressure to the next stage in the case of a multiple-stage ejector. In order to lighten the load on the succeeding stages, condensers are often inserted between stages to remove the steam from the preceding stage.

The pumping capacity or throughput of a steam ejector is generally given in terms of pounds of dry air removed per hour. For comparison purposes

$$1 \text{ lb air at } 293 \text{ K hr}^{-1} \equiv 79.5 \text{ torr L s}^{-1}.$$

Figure 5.22 illustrates the range of pressures and throughputs obtainable with typical combinations of ejector stages and condensers.

5.5.6 Molecular pumps

We saw in Section 5.5.3 that diffusion pumps are often coupled with mechanical roughing pumps to achieve pressures that are typically 10^{-6} torr without traps. Molecular pumps can also be used to achieve similar pressures, although at somewhat more cost.

In molecular pumps, the gas molecules are imparted a velocity by momentum transfer from a fast-moving solid surface. In diffusion pumps, the gas molecules are entrapped and carried from the inlet to the discharge by the stream of vapor. In molecular pumps, collisions occur with the fast-moving surface. Pumps of this type can maintain pressures as low as 10^{-7} to 10^{-9} torr, while discharging at 10 to 40 torr (a considerably higher discharge pressure than tolerated by diffusion pumps).

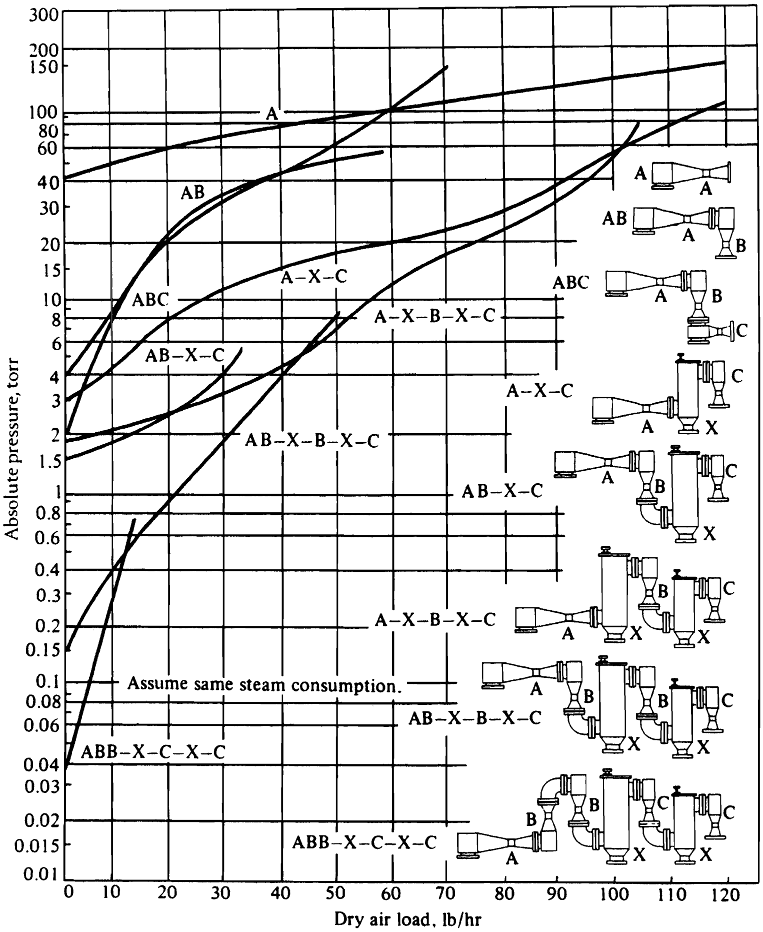


Fig. 5.22 Ranges of application of various steam-ejector (A, B, C)-intercondenser (X) combinations. (From an article by F. Berkeley, *Chem. Eng.* (April 1957), page 255.)

Advantages of molecular pumps include their installation in systems with smaller backing pumps than required when diffusion pumps are used. Backstreaming of oil molecules from the mechanical pump is lessened because the discharge pressure is relatively high. When large throughputs are needed, the turbomolecular pumps that are used are axial-flow turbines.

5.5.7 Gettering and ion pumps

The pressure region between 10^{-9} and 10^{-12} torr is often referred to as *ultrahigh vacuum*. At such low pressures, we start thinking about the number of molecules per unit volume rather than pressure as a force per unit area. Consequently, the amount of gas adsorbed on surfaces in the vacuum system is paramount, so maintaining a ultrahigh vacuum system requires the maintenance of ultraclean surfaces.

In an ultrahigh vacuum a surface can hold a large number of molecules compared to the number of molecules in the vacuum space. To effect a pumping action, a chemically active surface can be used to "get" molecules from the vacuum space by physisorption. The most commonly used material is titanium.

In the inset of Fig. 5.23, there is a titanium source that sublimes by electrical heating. The titanium atoms travel to the colder surrounding surface, where they condense and form a large surface of fresh titanium. In turn, the layer of fresh titanium forms stable compounds with chemically active gas molecules that strike the surface. The capture process is continuous so long as new layers are deposited.

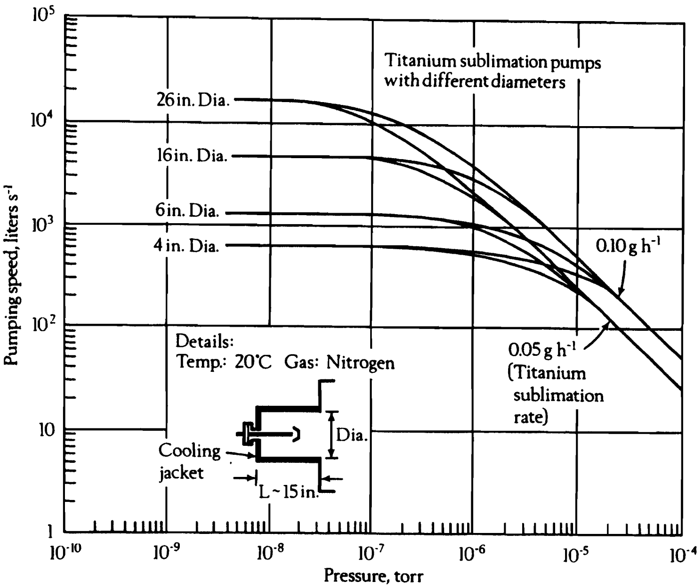


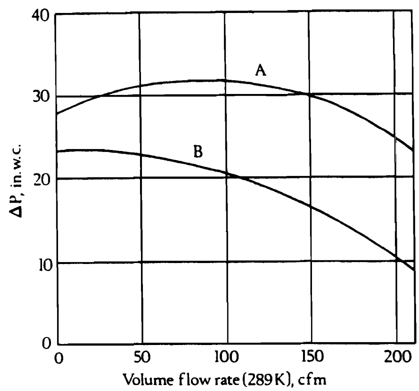
Fig. 5.23 Pumping speeds ot gettering pumps of various sizes. (From M. H. Hablanian, *High Vacuum Technology*, Marcel Dekker, Inc., New York, N.Y., 1990.)

So-called sputter-ion pumps can also be used to supply the layer of fresh titanium. The device consists of a cylinder of stainless steel (the anode) and plates of titanium (the cathode) positioned near the open ends of the cylinder. Electrons are emitted from the cathode because 5 to 7 kV are applied between the electrodes. The trajectories of these electrons are controlled by a magnet which causes the electrons to move in very long helical paths in order to improve the chances of collisions between the electrons and gas molecules. These collisions produce positively charged ions that travel to the cathode (titanium). This collision ejects titanium atoms (i.e., sputtering) which deposit on other surfaces in the system and capture other molecules.

The pumping speed characteristics of a sputter-ion pump are qualitatively similar to that shown for the diffusion pump in Fig. 5.20, with the major advantage of having an ultimate pressure (P_a) that is less than 10^{-11} torr. There is a range of intermediate pressures (10^{-7} to 10^{-5} torr), where the pumping speed is approximately constant. At lower pressures, the speed decreases to zero at P_a, and it decreases to very low values at higher pressures. Sputter-ion pumps have relatively low throughputs and should only be coupled with roughing pumps that can achieve 10^{-3} torr. The cathodes are eroded by sputtering, and they must be replaced periodically, which is not a simple matter.

PROBLEMS

5.1 Refer to Example 3.5 for the system characteristics and the desired operating point for flow through a sinter bed. The bed has a cross-sectional area of 0.189 m^2, and V_0 is a superficial velocity. You can select either fan A or fan B to blow the air. Their respective characteristic curves are given to the right.

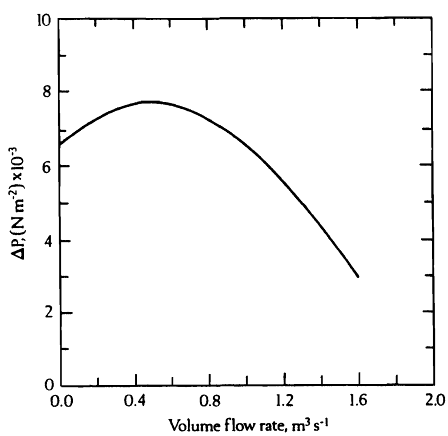


a) If you select fan A, what volume flow rate will be delivered through the sinter bed? Repeat for fan B. You may assume that the pressure drop through the sinter bed is proportional to the square of volume flow rate; i.e., $\Delta P = kQ^2$.

b) Which fan, A or B, is better suited for the bed of ore and coal which is discussed? Explain why.

c) What adaptation, if any, must be made to use the fan you have selected?

5.2 Refer to Problem 4.4. Suppose the characteristic curve of the fan is as shown to the right.



a) Assume that $\Delta P = kQ^2$ for the entire system and determine the operating point.

b) As particulates are collected, the pressure drop across the bag house increases. When the pressure drop increases by twenty percent, what will be the volume flow rate?

5.3 Two identical pumps are used to pump water from one reservoir to another whose level is 6.1 m higher than the first. When both pumps are operating the flow rate is 0.04 $m^3\ s^{-1}$. What is the flow rate when only one pump operates? Assume highly turbulent flow. The characteristic head curve for the pump is given by the following table.

Flow rate, $m^3\ s^{-1}$	Head, $N\ m\ kg^{-1}$
0	208
0.0057	211
0.0113	212
0.0170	211
0.0227	203
0.0340	178
0.0453	141
0.0566	99
0.0680	39
0.0736	15

5.4 Derive an equation for the conductance of a long straight tube. Assume viscous flow prevails and that the viscosity is given by Eq. (1.13).

a) Write the conductance in terms of viscosity.
b) Show that

$$\eta = \frac{m\bar{V}}{3\pi^{1/6}\sqrt{2}\,d^2},$$

where $\bar{V}$ is the Maxwellian speed of the molecules.
c) Write the conductance in terms of $\bar{V}$.

5.5 Compare the conductance for viscous flow in a long straight tube (from Problem 5.4c) to the conductance for molecular flow.

a) How does each vary with $\bar{V}$?
b) How does each vary with temperature?
c) For nitrogen at 300 K, what is the mean free path (see Eq. (1.5)) at normal atmospheric pressure (760 torr).

5.6 For nitrogen at 300 K, what is the minimum diameter of a long tube for viscous conduction at a) standard atmospheric pressure (760 torr). Repeat for b) 100 torr, c) 10 torr, d) 1 torr and e) 10^{-1} torr. [The criterion is $(\lambda/D) \geq 10$ where λ is the mean free path (see Eq. (1.5)).]

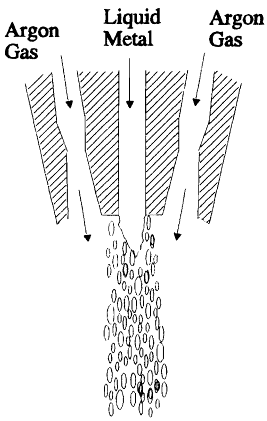
5.7 Consult Table 5.2 and obtain the conductance for two chambers connected by a tube with a diameter 250 mm and a length of 750 mm. Compare your result to the approximation given by Eq. (5.52). Assume that the gas is air at 298 K, with a molecular weight of 28.8 kg $kmol^{-1}$.

5.8 Consider the use of the two-stage pump of Fig. 5.18 that is connected to a chamber of 1 m^3 volume through a duct with an infinitely high conductance. Calculate the time to pumpdown to a) 10^{-2} torr and b) 10^{-4} torr.

5.9 A heat of steel (5×10^4 kg) is to be vacuum degassed from 5 ppm H_2 to 1 ppm H_2 and from 100 ppm N_2 to 75 ppm N_2 in 15 min. The steel is at 1873 K, and the chamber has 9 m^3 of space occupied by air after the top is closed with the ladle inside. At what pressure would you recommend operating the system? Calculate the throughputs of air, hydrogen and nitrogen that must be removed from the chamber. Consult Fig. 5.22 and specify a steam ejector to do the job.

5.10 An ultrahigh vacuum chamber (300 liters) is equipped with two pumping modes, one to achieve 10^{-4} torr and a titanium sublimation pump (Fig. 5.23) to achieve pressures below 10^{-4} torr. Assuming that nitrogen must be removed from the chamber, which pump(s) of Fig. 5.23 can be used if 10 minutes is an acceptable pumpdown time to go from 10^{-4} torr to 10^{-8} torr?

5.11 Supersonic nozzles are arranged circumferentially around a central orifice through which liquid metal is fed. The argon gas jets are focused on a point below the exit of the orifice, where they impinge on the metal stream to break it into fine droplets that solidify to microstructures of particular interest. It has been found that nozzle exit velocities on the order of Mach 3 are desirable. For a Mach 3 nozzle calculate the reservoir pressure P_0 needed, if the desired exit pressure is 1.0 atm and the flow rate of argon is 0.1 kg s^{-1}. What should the throat diameter, exit diameter and length of diverging section be? Assume $\gamma_{Ar} = 1.67$.



5.12 Derive Eq. (5.20).