

Local Livelock Analysis of Component-Based Models

Madiel S. Conserva Filho¹, Marcel Vinicius Medeiros Oliveira^{1(✉)},
Augusto Sampaio², and Ana Cavalcanti³

¹ Universidade Federal do Rio Grande do Norte, Natal, Brazil
madiel@ppgsc.ufrn.br, marcel@dimap.ufrn.br

² Universidade Federal de Pernambuco, Recife, Brazil

³ University of York, York, UK

Abstract. In previous work we have proposed a correct-by-construction approach for building deadlock-free CSP models. It contains a comprehensive set of composition rules that capture safe steps in the development of concurrent systems. In this paper, we extend that work by proposing and implementing a strategy for establishing livelock freedom based on constructive rules similar to those that ensure the absence of deadlock. Our method is based solely on the local analysis of the minimum sequences that lead the CSP model back to its initial state. The effectiveness of our livelock-analysis technique is demonstrated via three case studies. We compare the performance of our approach with that of two other techniques for livelock freedom verification: FDR2 and SLAP.

Keywords: Component-based systems · Local analysis · Livelock

1 Introduction

Component-based System Development (CBSD) has been used to deal with the increasing complexity of software. It focuses on the construction of systems from reusable and independent components [1]. Its correct application, however, relies on the trust in the behaviour of the components and in the emergent behaviour of the composed components because failures may arise if the composition does not preserve essential properties, especially in concurrent systems.

In [9], we have proposed a systematic design of CBSD that integrates components via asynchronous compositions, mediated by buffers, considering a grey-box style of composition [2], in which services that cannot be accessed by other components remain visible to the environment. This strategy is based on safe composition rules that guarantee, by construction, deadlock freedom. The absence of livelock is trivially ensured since the basic components are, by definition, livelock-free, and no operator that may introduce such a behaviour is used. The approach is underpinned by the process algebra CSP [4, 10], a well established formal notation for modelling and verifying concurrent systems. We

provided a component model, *BRIC*, that imposes constraints on the components and their interactions. Each component is represented by a tuple, where one of the elements is the behaviour of the system described as a CSP process.

This paper focuses on livelock analysis for asynchronous CSP models that perform black-box compositions. It defines a component notion that seems better aligned to CBSD, in which the internal services of components are hidden from its environment. This, however, may introduce livelock, a clearly undesirable behaviour. A system is livelock-free if there exists no state from which it may perform an infinite sequence of internal actions. The traditional livelock analysis performs a global analysis of an internal representation of a model as a labelled transition system, in order to verify that such a state cannot be reached [10]. This strategy is fully automated, for instance, in FDR2 [5]. One alternative is to make a static analysis of the syntactic structure of a system, proposing syntactic rules either to classify CSP systems as livelock-free or to report an inconclusive result. This strategy is implemented in SLAP [7]. Another promising strategy, which is the basis of compositional approaches, performs a local analysis that verifies only some parts of the system. It can identify problems before compositions, predicting, by construction, global properties based on known local properties of the composing components. Locality provides an alternative to circumvent the state explosion generated by the interaction of components and allows us to identify livelock before composition.

In this paper, we present a technique for constructing livelock free systems in *BRIC* using local analysis. We consider livelock freedom of *BRIC* components in the context of black-boxes rather than grey-boxes compositions adopted in [9]. We introduce side conditions that guarantee, by-construction, that the *BRIC* composition rules, which ensure deadlock freedom, also ensure livelock freedom. The verification of these conditions uses metadata that allow us to record partial results of verification, decreasing the overall analysis effort. Our strategy supports a systematic development that rules out designs with livelock. We consider two versions of *BRIC*: *BRIC**, in which asynchronicity is achieved using finite buffers, and *BRIC[∞]*, which uses infinite buffers. The possibility of introducing livelock is directly related to the finiteness of the buffer. We also present a comparative analysis of the performance of our strategy with respect to those implemented in FDR2 and in SLAP, based on three case studies.

In the next section, we introduce CSP. Section 3 presents the component model *BRIC* that defines the building blocks of our systematic development approach. In Sect. 4, we introduce our approach for livelock-free composition in *BRIC* based on local analysis. Its performance is evaluated in Sect. 5. Finally, we draw our conclusions, and discuss future work in Sect. 6.

2 CSP

CSP is one of the most important formalisms for modelling and verifying concurrent reactive systems. This process algebra can be used to describe systems as interacting components: independent entities called processes that interact with

each other exchanging atomic, instantaneous and synchronous messages, represented by events. The main CSP constructs used in this paper are presented below. Further information can be found in [4, 10].

There are two basic CSP processes: *SKIP* and *STOP*. The former represents the terminating process, and the latter deadlocks. The prefixing $c \rightarrow P$ is initially able to perform only the simple event c , and behaves like process P after that. Events may also be compound. For instance, $c.n$ is composed by the channel c and the value n . If we assume that the type of c is the set $\{1, 2\}$, the production $\{c\}$ returns the set of all events on c , $\{c.1, c.2\}$. Communications may be considered as outputs and inputs: $c!x$ represents an output on some channel c , and $c?x$ is the syntax for an input. The process $g \& P$ behaves as P if the predicate g is true. Otherwise, it behaves like *STOP*.

The process $P \square Q$ is an external choice between process P and Q : the environment needs to make the choice by communicating an initial event to one of the processes. When the environment has no control over the choice, we have an internal choice $P \sqcap Q$. The process $P; Q$ combines the processes P and Q in sequence. The process **if** b **then** P **else** Q behaves as P if b holds and as Q otherwise. The parallel composition $P \parallel_X Q$ synchronises P and Q on the events in the set X ; events that are not listed in X occur independently. The interleaving $P \parallel Q$ runs the processes independently.

The process $P[[a \leftarrow b]]$ behaves like P except that all occurrences of a in P are replaced by b . The hiding process $P \setminus X$ behaves like P , but all events in the set X are hidden and turned into internal actions, which are not visible to the environment. For example, $P = (a \rightarrow P) \setminus \{a\}$ is a divergent process that indefinitely performs the event a without communicating with its environment.

In order to illustrate some CSP constructs, we use a classical example of a concurrent system, the dining philosophers [10], which is used throughout this paper. It consists of philosophers that try to acquire a pair of shared forks in order to eat. The philosophers are sat at a table and there is a fork between each pair of philosophers. Each philosopher must pick up both forks before eating.

```
datatype EV = up | down
datatype LF = thinks | eats
channel fk1, fk2, pfk1, pfk2 : EV
channel life : LF
Fork = (fk1.up → fk1.down → Fork) □ (fk2.up → fk2.down → Fork)
Phil = life.thinks → pfk1.up → pfk2.up → life.eats →
      pfk1.down → pfk2.down → Phil
```

The process *Fork* ensures that two philosophers cannot hold a fork simultaneously. It offers a deterministic choice between the events $fk1.up$ and $fk2.up$, where $fk1$ and $fk2$ are channels of type *EV*. The process *Phil* represents the life cycle of a philosopher: before eating, the philosopher thinks and picks the forks up. After eating, the philosopher puts the forks down.

There are three well-established semantic models of CSP: traces (\mathcal{T}), stable failures (\mathcal{F}), and failures-divergences (\mathcal{FD}) [10]. The set $traces(P)$ contains all

possible sequences of events in which P can engage. The set $failures(P)$ contains all the failures of P , that is, pairs (s, X) , where s is a trace of P and X is a set of events which P may refuse after performing s . The failures-divergences is the most satisfactory model for analysing liveness properties of a CSP process. In \mathcal{FD} , a process P is represented by the pair $(failures_{\perp}(P), divergences(P))$. The set $failures_{\perp}(P)$ contains all failures of P , and additional failures that record that P can refuse anything after diverging. The set $divergences(P)$ contains all traces of P that lead it to a divergent behaviour and all extensions of those traces. A process P is divergence-free if, and only if, $divergences(P) = \emptyset$.

3 BRIC

The *BRIC* component model [9] has been originally proposed to ensure, by construction, the absence of deadlock. It is an algebra that has contracts as operands and composition rules as operators. A component contract, whose definition is presented below, is a tuple and encapsulates a component in *BRIC*.

Definition 1 (Component Contract). *A component contract $Ctr: \langle \mathcal{B}, \mathcal{R}, \mathcal{I}, \mathcal{C} \rangle$ comprises its behaviour \mathcal{B} , which is described as a restricted form of CSP process, I/O process, described below, a set of channels \mathcal{C} , a set of data types \mathcal{I} , and a total function $\mathcal{R} : \mathcal{C} \rightarrow \mathcal{I}$ from channels to their types.*

We use \mathcal{B}_{Ctr} , \mathcal{R}_{Ctr} , \mathcal{I}_{Ctr} and \mathcal{C}_{Ctr} to denote the elements of the contract Ctr . The behaviour \mathcal{B}_{Ctr} is represented by an I/O process, which is defined as follows, where we use α_P to denote the set of events that P can communicate.

Definition 2 (I/O Process). *We say a CSP process P is an I/O process if:*

- whenever $c.x \in \alpha_P$, then c is either an input or an output channel;
- P has infinite traces (but finite state space);
- P is divergence free;
- P is input deterministic, that is, after every trace of P , if a set of input events of P may be offered to the environment, they may not be refused by P after the same trace;
- P is strongly output decisive, that is, all choices (if any) among output events on a given channel in P are internal.

All channels of an I/O process are either input or output channels. I/O processes are also non-terminating processes but, for practical purposes in model checking, they have finite state spaces, and are divergence free. Input determinism and strong output decisiveness are not relevant in the context of livelock analysis. For this reason, we omit their formal definitions, which can be found in [8].

We illustrate the compositional development of *BRIC* with the construction of an asymmetric dining table with 2 philosophers and 2 forks. The behaviour of each philosopher and each fork is represented as a process $Phil_i$ or $Fork_i$, where $i \in \{1, 2\}$. The channels fk , pfk , both of type $ID.ID.EV$, and lf of type $ID.LF$,

where $ID : \{1, 2\}$, distinguish each philosopher and each fork, whose behaviours are described as an instantiation of *Phil* and *Fork* described in Sect. 2.

$$\begin{aligned} Fork_1 &= Fork \llbracket [fk1 \leftarrow fk.1.1, fk2 \leftarrow fk.1.2] \rrbracket \\ Fork_2 &= Fork \llbracket [fk1 \leftarrow fk.2.2, fk2 \leftarrow fk.2.1] \rrbracket \\ Phil_1 &= Phil \llbracket [lfe \leftarrow lf.1, pfk1 \leftarrow pfk.1.1, pfk2 \leftarrow pfk.2.1] \rrbracket \\ Phil_2 &= Phil \llbracket [lfe \leftarrow lf.2, pfk1 \leftarrow pfk.2.2, pfk2 \leftarrow pfk.1.2] \rrbracket \end{aligned}$$

As all forks and philosophers are represented by one process with indices on its channels, there is a separate definition for each component contract. For example, the contracts Ctr_{Fork_1} and Ctr_{Phil_1} are:

$$\begin{aligned} Ctr_{Fork_1} &= \langle Fork_1, \{fk.1.1 \rightarrow EV, fk.1.2 \rightarrow EV\}, \{EV\}, \{fk.1.1, fk.1.2\} \rangle \\ Ctr_{Phil_1} &= \langle Phil_1, \{lf.1 \rightarrow LF, pfk.1.1 \rightarrow EV, pfk.2.1 \rightarrow EV\}, \{LF, EV\}, \\ &\quad \{lf.1, pfk.1.1, pfk.2.1\} \rangle \end{aligned}$$

The contract Ctr_{Fork_1} has a behaviour defined by $Fork_1$, and two channels: $fk.1.1$ and $fk.1.2$, both of type EV . The behaviour of the contract Ctr_{Phil_1} is $Phil_1$. This contract has three channels, $lf.1$ of type LF , and $pfk.1.1$ and $pfk.2.1$ of type EV .

In *BRIC*, we have two types of component composition: binary composition and unary composition. The former is defined below. It provides an asynchronous interaction on channels ic and oc between two contracts Ctr_1 and Ctr_2 mediated by a (possibly infinite) bi-directional buffer ($BUFF_{IO}$).

Definition 3 (Asynchronous Binary Composition). *Let Ctr_1 and Ctr_2 be two distinct component contracts with disjoint sets of channels ($\mathcal{C}_{Ctr_1} \cap \mathcal{C}_{Ctr_2} = \emptyset$), and ic and oc be channels within \mathcal{C}_{Ctr_1} and \mathcal{C}_{Ctr_2} , respectively. The asynchronous binary composition of Ctr_1 and Ctr_2 is given by:*

$$Ctr_{1 \langle ic \rangle \succ \langle oc \rangle Ctr_2} = \langle (\mathcal{B}_{Ctr_1} \parallel \mathcal{B}_{Ctr_2}) \parallel_{\{ic, oc\}} BUFF_{IO}, \mathcal{R}_{Ctr_3}, \mathcal{I}_{Ctr_3}, \mathcal{C}_{Ctr_3} \rangle$$

where $\mathcal{C}_{Ctr_3} = (\mathcal{C}_{Ctr_1} \cup \mathcal{C}_{Ctr_2}) \setminus \{ic, oc\}$, $\mathcal{R}_{Ctr_3} = \mathcal{C}_{Ctr_3} \triangleleft (\mathcal{R}_{Ctr_1} \cup \mathcal{R}_{Ctr_2})$, and $\mathcal{I}_{Ctr_3} = \text{ran}(\mathcal{R}_{Ctr_3})$.

The behaviour of a binary composition is defined as the synchronisation of the behaviour of Ctr_1 and Ctr_2 via a (possibly infinite) bi-directional buffer. The channels used in the composition are not offered to the environment in further compositions (\mathcal{C}_{Ctr_3}). The operator \triangleleft stands for domain restriction and is used to restrict the mapping from channels to interfaces (\mathcal{R}_{Ctr_3}) and, furthermore, to restrict the set of interfaces of the resulting contract (\mathcal{I}_{Ctr_3}).

Unary compositions are used to assemble channels of a single component Ctr .

Definition 4 (Asynchronous Unary Composition). *Let Ctr be a component contract, and ic and oc be two distinct channels within \mathcal{C}_{Ctr} . The asynchronous unary composition of Ctr is defined as:*

$$Ctr \succ \left|_{\langle ic \rangle}^{\langle oc \rangle} = \langle (\mathcal{B}_{Ctr} \parallel_{\{ic, oc\}} BUFF_{IO}), \mathcal{R}_{Ctr}, \mathcal{I}_{Ctr}, \mathcal{C}_{Ctr} \rangle$$

where $\mathcal{C}_{Ctr} = (\mathcal{C}_{Ctr} \setminus \{ic, oc\})$, $\mathcal{R}_{Ctr} = \mathcal{C}_{Ctr} \triangleleft \mathcal{R}_{Ctr}$, and $\mathcal{I}_{Ctr} = \text{ran} \mathcal{R}_{Ctr}$.

The *BRIC* composition rules proposed to ensure deadlock freedom by construction are: interleave, communication, feedback and reflexive. The interleave composition aggregates two independent contracts such that, after composition, they do not communicate with each other.

Definition 5 (Interleave Composition). *Let Ctr_1 and Ctr_2 be two component contracts, such that $\mathcal{C}_{Ctr_1} \cap \mathcal{C}_{Ctr_2} = \emptyset$. The interleave composition of Ctr_1 and Ctr_2 is given by $Ctr_1 \llbracket \llbracket \llbracket Ctr_2 = Ctr_1 \langle \rangle \asymp \langle \rangle Ctr_2$.*

In this composition, components do not share any channel and no synchronisation is enforced. It is a particular kind of composition that involves no communication. In our example, philosophers and forks can be interleaved separately: $Forks = Ctr_{Fork_1} \llbracket \llbracket Ctr_{Fork_2}$ and $Phils = Ctr_{Phil_1} \llbracket \llbracket Ctr_{Phil_2}$. These compositions are valid since the contracts have disjoint channels.

The second rule is based on the traditional way to compose two components, attaching two components connecting two channels, one from each component. Here, Σ is the finite set of all events and $P \upharpoonright X = P \setminus (\Sigma \setminus X)$ restricts the behaviour of P to a set of events X by hiding all events but those in X .

Definition 6 (Communication Composition). *Let Ctr_1 and Ctr_2 be two component contracts, and ic and oc two channels, such that $ic \in \mathcal{C}_{Ctr_1}$ and $oc \in \mathcal{C}_{Ctr_2}$, $\mathcal{C}_{Ctr_1} \cap \mathcal{C}_{Ctr_2} = \emptyset$, and $\mathcal{B}_{Ctr_1} \upharpoonright \{ic\}$ and $\mathcal{B}_{Ctr_2} \upharpoonright \{oc\}$ are strong compatible. The communication composition of Ctr_1 and Ctr_2 is defined as*

$$Ctr_1[ic \leftrightarrow oc]Ctr_2 = Ctr_1 \langle ic \rangle \asymp \langle oc \rangle Ctr_2$$

The proviso of strong compatibility ensures that the outputs of each process are always accepted by the other process. Formally, considering that I_P^s and O_P^s denote the inputs and outputs of a process P after a trace s , respectively, P and Q are strong compatible if, and only if:

$$\forall s : traces(P) \cap traces(Q) \bullet (O_P^s \neq \emptyset \vee O_Q^s \neq \emptyset) \wedge O_P^s \subseteq I_Q^s \wedge O_Q^s \subseteq I_P^s$$

In our example, we are able to compose the contracts *Forks* and *Phils* using the communication composition: $PComm = Forks[fk.1.1 \leftrightarrow pfk.1.1]Phils$. The resulting contract includes all philosophers and forks. The remaining connections that are needed to complete the dining table require the connection of two channels of the same component. For this reason, *BRIC* also provides unary compositions that can be used for such connections and enables the construction of systems with cyclic topologies. Due to the existence of possible cycles, however, new conditions are required to preserve deadlock freedom.

The unary composition rules are feedback and reflexive. The feedback composition represents the simpler unary composition case, where two channels of the same component are assembled, but do not introduce a new cycle [9]. The requirement on the independence of the channels guarantees that no cycles are introduced. A channel c_1 is independent of a channel c_2 in a process when any communication on c_1 does not interfere with the communications on c_2 , and vice-versa; hence, both channels are independently offered to the environment.

Definition 7 (Feedback Composition). Let C_{tr} be a component contract, and ic and oc two communication channels from $\mathcal{C}_{C_{tr}}$ that are independent in $\mathcal{B}_{C_{tr}}$, and such that $\mathcal{B}_{C_{tr}} \upharpoonright ic$ and $\mathcal{B}_{C_{tr}} \upharpoonright oc$ are strong compatible. The feedback composition of C_{tr} hooking oc to ic is defined as $C_{tr}[oc \hookrightarrow ic] = C_{tr} \succ_{\langle oc \rangle}^{\langle ic \rangle}$.

The contract $PComm$ contains all forks and philosophers. The channels $fk.2.2$ and $pfk.1.2$, however, are independent in $PComm$ because they occur in the interleaved sub-components *Forks* and *Phils*, respectively. We may, therefore, connect these channels using feedback: $PFeed_1 = PComm[pfk.1.2 \hookrightarrow fk.2.2]$. The channels $fk.2.1$ and $pfk.2.1$ are also independent in $PFeed_1$. Intuitively, their connection do not introduce a cycle; we may, therefore, connect these channels using the feedback composition: $PFeed_2 = PFeed_1[pfk.2.1 \hookrightarrow fk.2.1]$.

The reflexive composition deals with more complex compositions that introduce cycles of dependencies in the topology of the system structure, some of which may be undesirable because they introduce divergence.

Definition 8 (Reflexive Composition). Let C_{tr} be a component contract, and ic and oc two communication channels from $\mathcal{C}_{C_{tr}}$ such that $\mathcal{B}_{C_{tr}} \upharpoonright \{ic, oc\}$ is buffering self-injection compatible. The reflexive composition is defined as $C_{tr}[ic \overset{\hookrightarrow}{\hookrightarrow} oc] = C_{tr} \succ_{\langle oc \rangle}^{\langle ic \rangle}$.

The definition of the reflexive composition is similar to that of the feedback composition. It, however, has a stronger proviso that requires buffering self-injection compatibility, which allows one to assembly two dependent channels of a process via a buffer, without introducing deadlocks. This property is similar to the notion of strong compatibility, except for the fact that two distinct channels of the same process must be compatible. Its formalisation can be found in [8].

In our example, we conclude the design of our system using the reflexive composition to connect channels $fk.1.2$ and $pfk.2.2$.

$$PSystem = PFeed_2[fk.1.2 \overset{\hookrightarrow}{\hookrightarrow} pfk.2.2]$$

This connection could not be achieved using feedback because the two channels are not independent in $PFeed_2$. Intuitively, their connection introduces a cycle that causes the dependence between these channels.

4 Livelock Analysis for *BRIC*

In the *BRIC* approach livelock is not an issue because the rules do not hide the composed channels in the CSP behaviour of the resulting contract; they are just removed from the communication channel set, preventing further compositions on them. This gives us a grey-box style of abstraction [2]. We extend the possibilities of performing compositions in *BRIC*, providing a constructive strategy to perform black-box compositions [11], where the components encapsulate functionality, increasing the abstraction level of the system.

In [9], the concept of livelock is not defined at the component contract level. We define the notion of livelock-free component contract that considers *BRIC*

components as black-boxes. For that, we consider the component behaviour and the communication channels that are in the component set of visible channels, which are eligible for future compositions. As a result, a component contract C_{tr} is livelock-free if the CSP process resulting from hiding all channels that are not in the set $\mathcal{C}_{C_{tr}}$ in the behaviour $\mathcal{B}_{C_{tr}}$ is divergence free.

Definition 9 (Livelock-free Component Contract). *A component contract $C_{tr} = \langle \mathcal{B}, \mathcal{R}, \mathcal{I}, \mathcal{C} \rangle$ is livelock-free if, and only if, divergences($\mathcal{B}_{C_{tr}} \upharpoonright \mathcal{C}_{C_{tr}}$) = \emptyset .*

In what follows, we present the definitions used in our livelock analysis technique and describe the local conditions that guarantee livelock-free \mathcal{BRIC} compositions at the component contract level. We make a clear distinction of asynchronous compositions via finite and infinite buffers because the finiteness of the buffer is relevant for detecting the possibility of livelock in asynchronous systems. We consider \mathcal{BRIC}^* , which achieves asynchronous compositions via finite buffers, and \mathcal{BRIC}^∞ , in which asynchronicity is achieved using infinite buffers.

4.1 Basic Definitions

A livelock-free contract never performs an infinite sequence of internal events without communicating with its environment. Hence, reasoning about divergences requires reasoning about infinite behaviours. Therefore, the first step of our approach identifies the infinite behaviours of a given component. The function $IP(P)$ returns the traces that lead a given process P to a recursion.

Definition 10 (Interaction Patterns). *Let P be a CSP process. The set of interaction patterns is defined as: $IP(P) = \{t : \text{traces}(P) \mid P \equiv_{\mathcal{FD}} (P/t)\}$.*

The process P/t (pronounced P after t) represents the behaviour of P after the trace t is performed. The set $IP(P)$ contains all traces of P after which the process (P/t) has the same failures and divergences of P : they are equivalent in the failures-divergences model. Hence, $IP(P)$ gives an infinite set of traces that leads the process P back to its initial state. In our example, the set of interaction patterns of $IP(\text{Fork}_1)$ contains the traces that lead this fork to a recursion:

$$\{\langle \text{fk.1.1.up}, \text{fk.1.1.down} \rangle, \langle \text{fk.1.2.up}, \text{fk.1.2.down} \rangle, \\ \langle \text{fk.1.1.up}, \text{fk.1.1.down}, \text{fk.1.1.up}, \text{fk.1.1.down} \rangle, \dots \}$$

This set is infinite. Our strategy, however, only needs the set of minimal interaction patterns, which only contains the traces that lead the process to its first recursion. In what follows, we use the function S° , which, given a set of traces S (in our case, interaction patterns), returns the concatenation closure on S , i.e., the set of all sequences we can obtain by taking any subset of traces from the original S and concatenating them together (possibly with repetitions).

$$S^\circ = \{t : \Sigma^* \mid (\exists ss : \text{seq}(\Sigma^*) \bullet \text{ran}(ss) \subseteq S \wedge t = \wedge / ss)\}$$

Here, Σ^* is the set of finite sequences of elements of Σ , $\text{seq}(\Sigma^*)$ is the set of finite sequences over Σ^* , \wedge/ss is the distributed concatenation of all the elements of the sequence of sequences ss , and $\text{ran}(ss)$ is the set of the elements of ss .

The set of Minimal Interaction Patterns of a process P , $MIP(P)$, is the minimal set from which we are able to generate the same traces that can be generated from $IP(P)$. Formally, it is a subset of any other subset of interaction patterns S of $IP(P)$, such that $S^\circ = IP(P)$.

Definition 11 (Minimal Interaction Patterns). *Let P be a CSP process. The set of minimal interaction patterns of P , $MIP(P)$, is a set such that*

$$(MIP(P))^\circ = IP(P) \text{ and } \forall S : \mathbb{P}(\Sigma^*) \mid S^\circ = IP(P) \bullet MIP(P) \subseteq S.$$

The following constructive proposition is based on the calculation of *traces* proposed by Roscoe [10]. It calculates the MIP for CSP processes that describe the behaviour of the basic components, which are strictly sequential (possibly with choices) with no hiding. Parallelism is achieved by composing component contracts using the composition rules. We also consider only tail recursion (and no mutual recursion), in which recursive calls may only happen after at least one visible event (guarded tail recursions). In what follows, we use N to denote the process name and \bar{P} to represent the CSP process expression that defines its behaviour. We also use W_1 and W_2 to denote CSP behaviours.

Proposition 1 (Minimal Interaction Patterns Calculation). *Let N be a process name, and \bar{P} its behaviour. Then $MIP(N)$ is given by $MIP_N(\bar{P})$:*

$$\begin{aligned} MIP_N(N) &= \{\langle \rangle\} \\ MIP_N(SKIP) &= MIP_N(STOP) = \{\} \\ MIP_N(c \rightarrow W_1) &= \{t : MIP_N(W_1); e : \{ \} c \} \bullet \langle e \rangle \wedge t\} \\ MIP_N(W_1 \square W_2) &= MIP_N(W_1 \sqcap W_2) = MIP_N(W_1) \cup MIP_N(W_2) \\ MIP_N(W_1[[R]]) &= \bigcup \{t : MIP_N(W_1) \bullet \text{ren}(t, R)\} \\ MIP_N(W_1; W_2) &= \left\{ \begin{array}{l} t_1 : \text{traces}(W_1); t_2 : MIP_N(W_2) \mid \text{last}(t_1) = \checkmark \\ \bullet \text{front}(t_1) \wedge t_2 \end{array} \right\} \\ MIP_N(g \& W_1) &= MIP_N(W_1) \\ MIP_N(\text{if } g \text{ then } W_1 \text{ else } W_2) &= MIP_N(W_1) \cup MIP_N(W_2) \end{aligned}$$

The sequence $\text{front}(t)$ contains all elements of the sequence t but the last one, $\text{last}(t)$ returns the last element of t , and the function $\text{ren}(t, R)$, presented below, applies the renaming relation on events R to the trace t . For functional renaming, this function returns a singleton set that contains a trace that corresponds to t but replaces every element in the domain of the renaming function by its image. However, relational renaming needs special care because it may turn simple prefixing into an external choice. By way of illustration, for $P = a \rightarrow P$, $P[[a \leftarrow b, a \leftarrow c]] = a \rightarrow P \square c \rightarrow P$. For this reason, the function ren presented below returns a set of traces and we need a distributed union (\bigcup) in the definition of MIP_N for renaming (see Proposition 1).

$$\begin{aligned} \text{ren}(\langle \rangle, R) &= \{\langle \rangle\} \\ \text{ren}(\langle e \rangle \hat{\ } t, R) &= \mathbf{if} \ e \in \text{dom}(R) \ \mathbf{then} \ \{e' : R[\{e\}]; \ s : \text{ren}(t, R) \bullet \langle e' \rangle \hat{\ } s\} \\ &\quad \mathbf{else} \ \{s : \text{ren}(t, R) \bullet \langle e \rangle \hat{\ } s\} \end{aligned}$$

In Proposition 1, when MIP_N is applied to N itself, the result is the empty sequence. With our assumption that the process is guarded tail recursive, this ensures that at this stage a minimum path is recorded. *SKIP* and *STOP* do not contain any *MIP* because they terminate (either successfully or not). The MIP_N of the prefix process $c \rightarrow W_1$ is formed by concatenating the sequence $\langle c \rangle$ to the front of the sequences of $MIP_N(W_1)$. The *MIP* of internal and external choices are the union of the MIP_N of the two operands. The *MIP* of $W_1[[R]]$ are those of W_1 replacing all occurrences of the events e in the domain of the renaming relation R by the relational image of $\{e\}$ in R . The $MIP_N(W_1; W_2)$ are the ones of W_2 prefixed by the traces of W_1 that lead to termination, but removing \checkmark . The calculation of the MIP_N of guarded processes $g \& W_1$ (and alternation $\mathbf{if} \ g \ \mathbf{then} \ W_1 \ \mathbf{else} \ W_2$) simply ignores the guard g and takes $MIP_N(W_1)$ (and $MIP_N(W_2)$) as the result. As a consequence, our approach may find false negatives because we consider interaction patterns which may not be feasible depending on the evaluation of g . For instance, if we consider a process $P = g \& a \rightarrow P$, our approach indicates the possibility of divergence in $P \setminus \{a\}$ because we do not analyse the value of g , which determines the existence of either a divergence or a deadlock.

In our example, the calculation of the minimum interaction patterns for $Fork_1$ and $Phil_1$ yields the following result.

$$\begin{aligned} MIP(Fork_1) &= \{\langle fk.1.1.up, fk.1.1.down \rangle, \langle fk.1.2.up, fk.1.2.down \rangle\} \\ MIP(Phil_1) &= \{\langle lf.1.thinks, pfk.1.1.up, pfk.2.1.up, lf.1.eats, \\ &\quad pfk.1.1.down, pfk.2.1.down \rangle\} \end{aligned}$$

We are now able to infer which channels can be used to compose a livelock-free contract in \mathcal{BRIC} . The function *Allowed* identifies all communication channels that can be individually hidden with no introduction of contract livelock.

Definition 12 (*Allowed*). *Let C_{ctr} be a livelock-free component contract. The set of communication channels of $\mathcal{C}_{C_{ctr}}$ that can be individually hidden with no introduction of divergence is given by $Allowed(C_{ctr})$ defined below:*

$$\begin{aligned} Allowed(C_{ctr}) &= \\ &\mathcal{C}_{C_{ctr}} \setminus \{c : \mathcal{C}_{C_{ctr}} \mid \exists s : MIP(\mathcal{B}_{C_{ctr}}) \bullet \text{ran}(s) \cap \text{evs}(\mathcal{C}_{C_{ctr}}) \subseteq \text{evs}(\{c\})\} \end{aligned}$$

The set $\text{evs}(cs) = \bigcup \{c : cs \bullet \{c\}\}$ contains all events produced by the channels in the set cs given as argument.

The set of *Allowed* channels of a given contract C_{ctr} contains all communication channels c , such that there is no $MIP(\mathcal{B}_{C_{ctr}})$ composed only by events on c . Using these channels on compositions does not introduce a contract livelock because even after individually hiding the communication on these channels, every member of $MIP(\mathcal{B}_{C_{ctr}})$ still has at least one further external communication on a different channel with the environment. In our example, the

sets of allowed channels are $Allowed(Ctr_{Phil_1}) = \{lf.1, pfk.1.1, pfk.2.1\}$ and $Allowed(Ctr_{Fork_1}) = \emptyset$. The latter is empty because every member of $MIP(Fork_1)$ either contains only interactions on $fk.1.1$ or only interactions on $fk.1.2$.

4.2 Conditions for Livelock Freedom in \mathcal{BRIC}^*

An interleave composition always results in a livelock-free contract, since the behaviour of both composing contracts are livelock-free by definition, and no communication channel is used in this composition. The proofs of the theorems in this paper can be found in [3].

In the communication composition via finite buffers, $Ctr_1[ic \leftrightarrow oc]^* Ctr_2$, a contract livelock may be introduced because we hide the channels ic and oc used in the composition, since they are removed from the set \mathcal{C} of the resulting component. There are, however, conditions under which this composition is safe.

For instance, we consider the composition $Ctr_{Fork_1}[fk.1.1 \leftrightarrow pfk.1.1]^* Ctr_{Phil_1}$ previously presented. Since the communication is asynchronous, after sending the events $fk.1.1.up$ and $fk.1.1.down$ to the buffer, $Fork_1$ recurses and may send such events to the buffer again before the first ones have been consumed by $Phils_1$ via $pfk.1.1.up$ and $pfk.1.1.down$. This, however, may be done only a finite number of times because the buffer is finite and, at some point, the communications on $pfk.1.1.up$ and $pfk.1.1.down$ will be enforced causing the occurrences, for instance, of the visible events $lf.1.thinks$ and $lf.1.eats$. This composition is, therefore, livelock-free. Along with the finiteness of the buffer, the fact that one of the connecting channels is in the corresponding set of allowed channels ($pfk.1.1 \in Allowed(Ctr_{Phils_1})$) guarantees a resulting livelock-free contract.

We establish below a condition that ensures that a contract livelock is not introduced in a communication composition in \mathcal{BRIC}^* .

Theorem 1 (Livelock-free Finite Communication Compositions). *Let Ctr_1 and Ctr_2 be two livelock-free component contracts, and ic and oc two channels in \mathcal{C}_{Ctr_1} and \mathcal{C}_{Ctr_2} , respectively. The composition $Ctr_1[ic \leftrightarrow oc]^* Ctr_2$ is livelock-free if $ic \in Allowed(Ctr_1)$ **or** $oc \in Allowed(Ctr_2)$.*

Regarding unary compositions, due to the finiteness of the buffer, we also only need to check if at least one of the communication channels used in the composition belongs to the set of *Allowed* channels of the contract.

Theorem 2 (Livelock-free Finite Unary Compositions). *Let Ctr be a livelock-free component contract, and ic and oc two channels in \mathcal{C}_{Ctr} . The compositions $Ctr[ic \hookrightarrow oc]^*$ and $Ctr[ic \dashv\rightarrow oc]^*$ are livelock-free if $ic \in Allowed(Ctr)$ **or** $oc \in Allowed(Ctr)$.*

We now turn our attention to the cases in which neither of the connecting channels are in the set of *Allowed*. For example, let us consider three simple livelock-free contracts Ctr_1 , Ctr_2 and Ctr_3 defined as follows.

$$\begin{aligned} C_1 &: \langle \mathcal{B}_{C_1}, \{a \rightarrow \mathbb{N}\}, \{\mathbb{N}\}, \{a\} \rangle, \text{ where } \mathcal{B}_{C_1} = a.1 \rightarrow a.2 \rightarrow \mathcal{B}_{C_1} \\ C_2 &: \langle \mathcal{B}_{C_2}, \{b \rightarrow \mathbb{N}\}, \{\mathbb{N}\}, \{b\} \rangle, \text{ where } \mathcal{B}_{C_2} = b.1 \rightarrow b.2 \rightarrow \mathcal{B}_{C_2} \\ C_3 &: \langle \mathcal{B}_{C_3}, \{c \rightarrow \mathbb{N}\}, \{\mathbb{N}\}, \{c\} \rangle, \text{ where } \mathcal{B}_{C_3} = c.2 \rightarrow c.3 \rightarrow \mathcal{B}_{C_3} \end{aligned}$$

The composition $Ctr_1[a \leftrightarrow c]Ctr_3$ is valid in \mathcal{BRIC} because a and c are strong compatible. However, neither a or c are allowed in the corresponding contracts; this composition yields a divergent contract. In general, however, this would not necessarily happen. For example, $Ctr_1[a \leftrightarrow b]Ctr_2$ would not introduce a contract livelock because the channels would not be able to synchronise. The \mathcal{BRIC} rules, however, require the connecting channels to be strong compatible, that is, at every state of a in \mathcal{B}_{Ctr_1} if $a.n$ is offered, then $b.n$ is also offered by \mathcal{B}_{Ctr_2} . In $Ctr_1[a \leftrightarrow b]Ctr_2$, a and b are not strong compatible. As a consequence of the strong compatibility requirement, there is no case in which neither of the connecting channels are in *Allowed* of their contracts and the \mathcal{BRIC} compositions result in a livelock-free component contract.

In \mathcal{BRIC}^∞ , the assumption that communications with the buffer will halt at some point because the buffer is full is no longer valid because the buffers are infinite. We, therefore, need stronger conditions to ensure livelock freedom.

4.3 Conditions for Livelock Freedom in \mathcal{BRIC}^∞

In the presence of infinite buffers, the conditions for safe compositions are necessarily stronger because one of the contracts may indefinitely interact with the buffer via the connecting channel. For example, let us revisit the example of Sect. 4.2 replacing the buffer by an infinite one. The communication composition $Ctr_{Fork_1}[fk.1.1 \leftrightarrow pfk.1.1]^\infty Ctr_{Phils_1}$ remains asynchronous. After sending $fk.1.1.up$ and $fk.1.1.down$ to the buffer, $Fork_1$ still recurses and may send such events to the buffer again before the first ones has been consumed by $Phils_1$ via $pfk.1.1.up$ and $pfk.1.1.down$. This, however, may now be done indefinitely because the buffer is infinite; there is no guarantee that $Phils_1$ ever consumes any message on $pfk.1.1.up$ and $pfk.1.1.down$ causing the occurrence, for instance, of the visible events $lf.1.thinks$ and $lf.1.eats$. For this reason, the divergence of $Fork_1$ affects the overall composition. Therefore, we need a stronger requirement to ensure contract livelock freedom in a communication composition in \mathcal{BRIC}^∞ .

Theorem 3 (Livelock-free Infinite Communication Compositions). *Let Ctr_1 and Ctr_2 be two livelock-free contracts, and ic and oc two channels in \mathcal{C}_{Ctr_1} and \mathcal{C}_{Ctr_2} , respectively. The composition $Ctr_1[ic \leftrightarrow oc]^\infty Ctr_2$ is livelock-free if $ic \in Allowed(Ctr_1)$ and $oc \in Allowed(Ctr_2)$.*

Regarding the unary compositions in \mathcal{BRIC}^∞ , we have to ensure that the pair of connecting channels can be hidden together. We define the function $Allowed_{Bin}(Ctr)$, which is similar to $Allowed(Ctr)$, but characterises all pairs of channels that can be hidden together without generating a contract livelock.

Definition 13 ($Allowed_{Bin}$). *Let Ctr be a livelock-free contract. The set of pairs of channels of \mathcal{C}_{Ctr} that can be hidden with no introduction of divergence is given by $Allowed_{Bin}(Ctr)$ defined as:*

$$Allowed_{Bin}(Ctr) = \{c_1, c_2 : \mathcal{C}_{Ctr} \mid \neg (\exists s : MIP(\mathcal{B}_{Ctr}) \bullet \text{ran}(s) \cap \text{evs}(\mathcal{C}_{Ctr}) \subseteq \{\} c_1, c_2 \})\}$$

For the same reason, the infiniteness of the buffers, unary compositions in \mathcal{BRIC}^∞ have a stronger condition for ensuring livelock freedom. We require both connecting channels to be allowed to be hidden together.

Theorem 4 (Livelock-free Infinite Unary Compositions). *Let Ctr be a livelock-free contract, and ic and oc two channels in \mathcal{C}_{Ctr} . The compositions $Ctr[ic \hookrightarrow oc]^\infty$ and $Ctr[ic \rightrightarrows oc]^\infty$ are livelock-free if $(ic, oc) \in Allowed_{Bin}(Ctr)$.*

The Theorems 1 to 4 establish the conditions under which we ensure that the result of any \mathcal{BRIC} composition is a livelock-free component contract.

In order to be able to perform further compositions using the resulting contracts in an efficient manner, we calculate the new MIP after every livelock-free composition. This information is stored in the contracts as metadata that aims at alleviating further verifications in our method for component composition.

4.4 Dealing with Metadata

The calculation of the $MIPs$ of composed components can be based on the function proposed in [10] that calculates the traces of a parallel composition as the combination of the traces of each argument process, where the synchronised events are shared and all other events are interleaved. In our strategy, however, we are not concerned with the MIP generated by the interleaving of the $MIPs$ because livelock can only be introduced by hiding events of a basic component.

For instance, using the merge from [10] to calculate the new $MIPs$ of the interleaving composition $Ctr_{Fork_1} \parallel Ctr_{Fork_2}$, we get all possible sequences resulting from merging $MIP(Fork_1)$ and $MIP(Fork_2)$:

$$\begin{aligned} & \{ \langle fk.1.1.up, fk.1.1.down, fk.2.2.up, fk.2.2.down \rangle \\ & \quad \langle fk.1.1.up, fk.2.2.up, fk.1.1.down, fk.2.2.down \rangle, \\ & \quad \langle fk.2.2.up, fk.2.2.up, fk.1.1.down, fk.1.1.down \rangle, \dots \} \end{aligned}$$

For any two minimum interaction patterns ip_1 and ip_2 from $MIP(Fork_1)$ and $MIP(Fork_2)$, respectively, this merge includes a large number of traces that communicate on the same channels from ip_1 and ip_2 , which only differ in the order of the events. This order, however, is not relevant for our strategy because, using \mathcal{BRIC} , further compositions like, for instance, with a contract Ctr_3 , will be made on a one channel to one channel basis. As a consequence, composing Ctr_3 with $Ctr_1 \parallel Ctr_2$ will be a communication between Ctr_3 with either Ctr_1 or Ctr_2 . Based on this analysis, we provide a variation of the merge function from [10]. This optimisation is extremely relevant to the scalability of our approach.

Definition 14 (Optimised Trace Merge). *Let xs be a set of events, x and x' denote members of xs , and y denote a typical member of $\Sigma \setminus xs$. The optimised trace merge is defined as follows.*

$$\langle \rangle \parallel_{xs}^{\langle s_0, t_0 \rangle} \langle \rangle = \{ \langle \rangle \} \quad (1)$$

$$\langle x \rangle \frown s \parallel_{xs}^{\langle s_0, t_0 \rangle} \langle \rangle = \{ u \mid u \in \langle x \rangle \frown s \parallel_{xs}^{\langle s_0, t_0 \rangle} t_0 \} \quad (2)$$

$$\langle \rangle \parallel_{xs}^{\langle s_0, t_0 \rangle} \langle x \rangle \frown t = \{ u \mid u \in s_0 \parallel_{xs}^{\langle s_0, t_0 \rangle} \langle x \rangle \frown t \} \quad (3)$$

$$\langle y \rangle \frown s \parallel_{xs}^{\langle s_0, t_0 \rangle} t = \{ \langle y \rangle \frown u \mid u \in s \parallel_{xs}^{\langle s_0, t_0 \rangle} t \} \quad (4)$$

$$s \parallel_{xs}^{\langle s_0, t_0 \rangle} \langle y \rangle \frown t = \{ \langle y \rangle \frown u \mid u \in s \parallel_{xs}^{\langle s_0, t_0 \rangle} t \} \quad (5)$$

$$\langle x \rangle \frown s \parallel_{xs}^{\langle s_0, t_0 \rangle} \langle x \rangle \frown t = \{ u \mid u \in s \parallel_{xs}^{\langle s_0, t_0 \rangle} t \} \quad (6)$$

$$\langle x \rangle \frown s \parallel_{xs}^{\langle s_0, t_0 \rangle} \langle x' \rangle \frown t = \{ \} \quad (7)$$

The differences between our definition for trace merging and that of [10] are: (1) Our merge function has the original traces s_0 and t_0 as arguments. This allows us to merge n concatenations of s_0 with m concatenations of t_0 ; (2) In the cases in which one side is willing to perform a synchronisation event x and the other side has finished (lines 2 and 3), we “reset” the side that has finished, enforcing at least one synchronisation on x and decreasing the size of one of the sequences by at least one; (3) In the cases in which one side is willing to perform an independent event (lines 4 and 5), we do not take all possible combinations of permuting the independent events for the reasons previously explained; and (4) In the cases in which the synchronisation is feasible (line 6), our merge function does not include the synchronised event in the result because they are hidden after composition. We define the merge function as follows.

Definition 15 (MIP Merge). *Let Ctr_1 and Ctr_2 be two livelock-free component contracts, ic and oc two communication channels in \mathcal{C}_{Ctr_1} and \mathcal{C}_{Ctr_2} , respectively, and x a fresh channel name. The MIP merge is defined as follows.*

$$\begin{aligned} MIPMerge(Ctr_1, Ctr_2, ic, oc) = & \\ & \{ s : MIP(\mathcal{B}_{Ctr_1}) \mid \{ \} ic \} \cap \text{ran}(s) = \emptyset \} \\ & \cup \{ t : MIP(\mathcal{B}_{Ctr_2}) \mid \{ \} oc \} \cap \text{ran}(t) = \emptyset \} \\ & \cup \cup \left\{ \begin{array}{l} s : MIP(\mathcal{B}_{Ctr_1}); t : MIP(\mathcal{B}_{Ctr_2}); sx, tx : \Sigma^* \\ \mid \{ \} ic \} \cap \text{ran}(s) \neq \emptyset \wedge \{ \} oc \} \cap \text{ran}(t) \neq \emptyset \\ \wedge sx \in \text{ren}(s, \{ v : \text{extensions}(ic) \bullet (ic.v, x.v) \}) \\ \wedge tx \in \text{ren}(t, \{ v : \text{extensions}(oc) \bullet (oc.v, x.v) \}) \\ \bullet sx \parallel_{\{ \} x}^{\langle sx, tx \rangle} tx \end{array} \right\} \end{aligned}$$

The resulting merge contains all MIPs from \mathcal{B}_{Ctr_1} and \mathcal{B}_{Ctr_2} that do not have events on the connecting channels ic and oc , respectively. The remaining MIPs are merged using the optimised trace merge. Before the merge, however, the MIPs have to be unified on the events of the connecting channels. For that, we use a fresh channel name x and the function ren to replace references to ic and oc in \mathcal{B}_{Ctr_1} and \mathcal{B}_{Ctr_2} , respectively, by x . The function $\text{extensions}(c)$ returns the values which will ‘complete’ the channel yielding an event [10].

Next, the metadata calculation for the binary operators is as follows.

Proposition 2 (Binary Composition Metadata). *Let Ctr_1 and Ctr_2 be two livelock-free component contracts and ic and oc two channels in \mathcal{C}_{Ctr_1} and \mathcal{C}_{Ctr_2} , respectively. The MIP of the binary compositions are defined as follows.*

$$\begin{aligned} MIP(Ctr_1 [|||] Ctr_2) &= MIP(\mathcal{B}_{Ctr_1}) \cup MIP(\mathcal{B}_{Ctr_2}) \\ MIP(Ctr_1[ic \leftrightarrow oc] Ctr_2) &= MIPMerge(Ctr_1, Ctr_2, ic, oc) \end{aligned}$$

Finally, we formalise the metadata calculation for the unary compositions.

Proposition 3 (Unary Composition Metadata). *Let Ctr be a livelock-free component contract and ic and oc two communication channels in \mathcal{C}_{Ctr} . The MIP of the unary compositions are presented as follows.*

$$\begin{aligned} MIP(Ctr[ic \hookrightarrow oc]) &= \{s : MIP(\mathcal{B}_{Ctr}) \bullet s \setminus \{ic, oc\}\} \\ MIP(Ctr[ic \rightleftarrows oc]) &= \{s : MIP(\mathcal{B}_{Ctr}) \bullet s \setminus \{ic, oc\}\} \end{aligned}$$

The calculation of the resulting MIP for unary compositions simply removes both connecting channels from the original MIP s.

5 Evaluation

In this section, we demonstrate that our constructive approach to build livelock free models can be applied in practice to large systems involving several compositions. We have developed three case studies: Milner’s scheduler [6], which schedules a number of tasks and can be modelled as a ring of cell processes synchronised pairwise, and two variations of the dining philosopher [10], a livelock-free version and a version in which we have deliberately included livelock. All case studies are developed using the *BRIC* methodology, hence, we worked with asynchronous versions of these three case studies.

For each case study, we provide a comparative analysis of three scenarios: the global analysis of FDR2, the static analysis of SLAP, and our local analysis. In these case studies, we have used a dedicated server with an 8 core Intel(R) Core(TM) i7-2600K, 16 GB of RAM and 160GB of SSD in an Ubuntu system. The CSP scripts of these case studies can be found at <http://goo.gl/mAZWXq>.

Table 1. Results of the livelock analysis for Milner’s scheduler in *BRIC**.

N	#	FDR2	SLAP (BDD)	SLAP (SAT)	LLA
5	5	0.123 s	0.045 s	4.196 s	0.177s
10	10	672.164 s	0.128 s	14.340 s	0.218 s
15	15	*	0.465 s	29.862 s	0.243 s
100	100	*	2428.308 s	**	0.559 s
1,000	1,000	*	*	**	3.959 s
3,000	3,000	*	*	**	7.578 s

Tables 1 and 2 summarise our results. The column N is the number of cells and philosophers for Milner’s scheduler and dining philosophers, respectively.

Table 2. Results of the livelock analysis for the dining philosophers in *BRIC**.

N	#	Livelock-free system				System with livelock			
		FDR2	SLAP (BDD)	SLAP (SAT)	LLA	FDR2	SLAP (BDD)	SLAP (SAT)	LLA
3	10	2.884s	0.342 s	2.114 s	0.219 s	0.941 s	0.252 s	1.224 s	0.215 s
10	38	*	51.708 s	383.884 s	0.303 s	*	26.259 s	149.091 s	0.297 s
100	398	*	*	*	0.778 s	*	*	**	0.769 s
1,000	3,988	*	*	*	3.888 s	*	*	**	3.431 s
10,000	39,988	*	*	*	206.689 s	*	*	**	185.209 s

The column # is the number of compositions, and the columns FDR2, SLAP and LLA present the time cost of the global analysis in FDR2, SLAP Static Analysis (using BDD and SAT), and our local analysis (LLA). The * indicates one hour timeout and ** indicates memory overflow.

The results show that FDR2 and SLAP are unable to deal with large asynchronous configurations. On the other hand, our method provided successful results of livelock analysis for 10,000 philosophers and 10,000 forks (20,000 CSP processes and 39,988 *BRIC** compositions) in less than 4 min. This proved to be a very promising result in dealing with complex and large systems.

6 Conclusion

In this paper, we propose a correct-by-construction approach for ensuring livelock freedom in *BRIC* models built using four composition rules. The development of this strategy is based on the minimum sequences that represent patterns of interactions after which the system recurses. Considering only these finite sequences, we are able to locally assert livelock freedom before integrating components. Furthermore, we use metadata for storing information that alleviate verification conditions during component composition. To perform this analysis in *BRIC*, we have provided a clear distinction of asynchronous compositions via finite and infinite buffers because the finiteness of the buffer is relevant for detecting the possibility of livelock in such systems.

We have used three case studies that demonstrate the scalability of our approach. For larger systems, the verification using FDR2 and SLAP may easily become costly and infeasible. On the other hand, our compositional livelock analysis seems promising as demonstrated in our case studies.

Our approach for local and compositional livelock analysis can still be improved. Parameters and non-tail recursion are not addressed here; they are, however, in our research agenda, which also includes additional case studies.

References

1. Beneken, G., Hammerschall, U., Broy, M., Cengarle, M., Jürjens, J., Rumpe, B., Schoenmakers, M.: Componentware - State of the Art 2003, October 2003
2. Bruin, H.: A grey-box approach to component composition. In: Czarnecki, K., Eisenecker, U.W. (eds.) GCSE 1999. LNCS, vol. 1799, pp. 195–209. Springer, Heidelberg (2000). doi:[10.1007/3-540-40048-6_15](https://doi.org/10.1007/3-540-40048-6_15)
3. Filho, M., Oliveira, M., Sampaio, A., Cavalcanti, A.: Local livelock analysis of component-based models. Technical report, UFRN 2(016). <http://goo.gl/zl1MQV>
4. Hoare, C.A.R.: Communicating Sequential Processes. Prentice-Hall, Upper Saddle River (1985)
5. Formal Systems Ltd.: FDR2: User Manual, version 2.94 (2012)
6. Milner, R.: Communication and Concurrency. Prentice-Hall, Upper Saddle River (1989)
7. Ouaknine, J., Palikareva, H., Roscoe, A.W., Worrell, J.: A static analysis framework for livelock freedom in CSP. *Log. Methods Comput. Sci.* **9**(3) (2013)
8. Ramos, R.T.: Systematic development of trustworthy component-based systems. Ph.D. thesis, Federal University of Pernambuco (2011)
9. Ramos, R., Sampaio, A., Mota, A.: Systematic development of trustworthy component systems. In: Cavalcanti, A., Dams, D.R. (eds.) FM 2009. LNCS, vol. 5850, pp. 140–156. Springer, Heidelberg (2009). doi:[10.1007/978-3-642-05089-3_10](https://doi.org/10.1007/978-3-642-05089-3_10)
10. Roscoe, A.W.: The Theory and Practice of Concurrency. Prentice-Hall Series in Computer Science. Prentice-Hall, Upper Saddle River (1998)
11. Soni, P., Ratti, N.: Analysis of component composition approaches. *Int. J. Comput. Sci. Commun. Eng.* **2**(1) (2013). ISSN: 2319-7080