# **Chapter 8 Convection Bond Graphs for Thermodynamic Systems**

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### **8.1 Introduction**

In his development of bond graphs, Henry Paynter focused on systems most familiar to engineers who dealt with the dynamics of mechanical and electrical systems, in effect postponing consideration of thermodynamic systems which are inherently more difficult. Nevertheless, he envisioned these graphs to express the dynamics of general physical systems in a graphical language based on the principles of energy. Thermodynamics is the science of energy, so the author believes that inclusion of thermodynamic systems into the scope of bond graphs is necessary for these graphs to assume the status of a universal language for the dynamics of physical systems. The challenge is that conventional bonds with their single effort and single flow variables are very awkward when dealing with the flows of highly compressible fluids.

One result of the enormous complexity of thermodynamic systems has been emphasis on steady-state behavior. Many engineers and scientists devote their careers to expanding knowledge of heat transfer or the thermodynamic and transport properties of the substances employed in thermodynamic systems. Since bond graphs are used almost exclusively for dynamic behavior, this means that few people active in the field are conversant with thermodynamics. In order to add the additional layer of dynamics to thermodynamic analysis effectively requires a willingness to forgo a degree of accuracy in the treatment of the steady-state behavior, which the author long ago accepted as a necessary compromise. It is now time for the bond graph community to roll up its sleeves and dig in to the field of thermodynamic systems, with the aim of expediting the approximate modeling and simulation of their dynamic behavior, which can be crucial for design success.

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W. Borutzky (ed.), *Bond Graphs for Modelling, Control and Fault*

The physical property of entropy and its corollary exergy—sometimes called "availability"—is essential to thermodynamic systems, but not to the mechanical and electrical systems originally addressed by Paynter. Attempts to shoehorn thermodynamic models with entropy into conventional bond graphs have resulted, in the authors opinion, in discouraging and unnecessary complexity. The author's approach is to extend the bond graphs themselves by allowing a new kind of bond called a convection bond  $[1, 5]$  $[1, 5]$  $[1, 5]$ , and new elements which employ these bonds, often in conjunction with the traditional—"simple"—bonds. The result can be called general bond graphs; bond graphs with no convection bonds become a subset of these, which could be called "traditional" bond graphs. The presentation below starts with an introduction to convection bonds and elements that employ them, and proceeds to the determination of the properties of thermodynamic substances before addressing practical systems.

THERMOSIM® [\[8\]](#page-34-2), which employs a graphical interface called BondGrapher, is available without charge on the author's website [\[9\]](#page-34-3), also on which a radically revised Users' Manual should become available before this book is published.

#### <span id="page-1-0"></span>**8.2 General Bond Graphs**

The conventional or simple bond was conceived to represent the interconnection between two physical entities so as to represent the "effort" and "flow" variables which when multiplied together give the instantaneous power being transferred. Examples include force and velocity, electric voltage and current, and pressure and—for incompressible fluids—the volume flow rate. Four such pairs are needed to completely represent the flow of a compressible pure substance through a port, three of which are significant: pressure and the volume flow rate for the "flow power," internal energy and the mass flow rate for the transport of internal energy, and kinetic energy and the mass flow rate for the transport of kinetic energy. The fourth pair is absolute temperature and entropy flux for heat conduction through the port. This fourth mode is so small, due to a small temperature gradient, it is almost universally ignored in the examples given in thermodynamics textbooks, and the author also ignores it. Heat transfer through conduction from the fluid to its surroundings, transverse to the flow, is a totally different issue, however; it is central to the operation of nearly all thermodynamic systems.

What the author has done is combine the three significant modes of energy propagation for flow through a port, with its three bonds, into a single bond with *two* independent efforts and one flow, replacing six bond variables with only three. A great simplification results. The flow is the mass flow rate, *m*. Ignoring for the moment the sometimes small mode for transport of kinetic energy, the power factor effort is the enthalpy of the fluid, *h*. Enthalpy is the sum of the internal energy, *u*, and the product of the absolute pressure, *P*, and the specific volume, *v*, or inverse of density. The first mode gives the transport of internal energy and the second gives the pressure-flow power. According to the state postulate of thermodynamics, the <span id="page-2-0"></span>**Fig. 8.1** The convection bond

state of a pure substance can be expressed as a function of any pair of independent intensive properties; enthalpy is used here as one of them. The other one most commonly used is the pressure or the stagnation pressure, because it is the causal effort. Other pairs of intensive properties also substitute for particular purposes; many are routinely computed. Entropy, *s*, is one of them; it plays a particularly important role in characterizing how much of the power is available as work—the "exergy." The transport of kinetic energy is added to the other two modes simply by replacing the nominal enthalpy with the stagnation enthalpy. The difference between the two can be computed readily using in part the bond variable of mass flow rate.

The resulting "convection bond" is distinguished from a simple bond by the use of a second bond line, drawn dashed as shown in Fig. [8.1.](#page-2-0) The use of convection bonds implies the introduction of a small number of new bond graph elements, most of which have both conventional and convection ports. From the perspective of thermodynamics, conventional bonds could be considered to be a special case of convection bonds in which only the power factor effort variable is used and the flow variable is allowed to be other than a mass flow rate.

20 bond graph elements are employed. The conventional elements include the effort source (Se), the flow source (Sf), the resistance (*R*), the compliance (*C*), the inertance  $(I)$ , the transformer  $(T \text{ or } TF)$ , the gyrator  $(G \text{ or } GY)$ , the common-effort junction (0), and the common-flow junction (1). Causal stokes are employed in the same manner as with simple bonds. Linearized analysis is not attempted in view of the highly nonlinear behavior of the primary applications. Elements that may include convection ports are discussed in the following section, along with others that have simple ports involving heat conduction. Relevant information not cited elsewhere herein is also available  $[2, 7]$  $[2, 7]$  $[2, 7]$ , including a PowerPoint survey  $[6]$  that predates BondGrapher.

For lack of space this chapter does not discuss the multiport compliance Cf element [\[7\]](#page-34-5), which in THERMOSIM has a unique ability to deduce differential equations when the only information provided by the user is relations for stored energy. This is most relevant to electromechanical systems, which are not the focus here.

#### *8.2.1 The CS Element*

The compliance element *C* in non-thermodynamic systems represents a storage of energy as a function of the time integral of the flow on the attached bond, the state variable called its displacement. A compliance element for a chamber containing a potential two-phase mixture of a liquid and a vapor has energy that is a function



<span id="page-3-0"></span>**Fig. 8.2** The CS element for a thermodynamic system comprising a compressible fluid in a control volume or chamber



of two state variables, for practical reasons the specific volume of the fluid and its absolute temperature. This chamber is represented by the CS element, shown in Fig. [8.2.](#page-3-0) It is the workhorse of thermodynamic system models.

In the THERMOSIM® simulation package, zero, one, or two convection bonds or fluid channels—may be attached to the element. The causal stokes on these bonds are always at the remote ends of their bonds; the mass flows are causal inputs and the pressure is a causal output, but not the only causal output. In addition, if the volume of the chamber is changeable, as by the motion of a piston, a simple bond is drawn with the effort  $P$  and flow  $V$ , which is the rate of change of the volume. Here the pressure is again the causal output—the only one. Finally, any number of heat conduction bonds may be attached. These are also represented by simple bonds, but are distinguished from the volume change bond by their causal stokes which are always adjacent to the element. The efforts on these bonds are the external temperatures which when compared with the fluid temperature drives the heat conduction. The thermal "resistance" to these heat flows are part of the CS element itself; the element is technically a macro element that contains both reversible elements and irreversible elements—and is so represented in the author's textbook—but the user does not need to know this. In keeping with the traditional use of power-factoring variables for simple bonds, the flow on a heat conduction bonds is the flux of entropy that it represents. Since most people prefer to deal with heat rates directly, however, the author employs the popular notation of quasi-bond graphs, labeling the flow as the heat transfer itself.

There are three state variables associated with the CS element: the mass of the fluid therein, the volume of the chamber, and the absolute temperature of the fluid. For purposes of this chapter they are called *mi*, *Vi*, and *Ti*, respectively. Note that the specific volume of the fluid,  $v_i$ , equals the ratio  $V_i/m$ . These are used because most available analytical relations for thermodynamic states are functions of *vi* and *Ti*.

<span id="page-3-2"></span>The constitutive equations for these three variables are as follows:

<span id="page-3-1"></span>
$$
\frac{dm_i}{dt} = \dot{m}_{\text{ini}} - \dot{m}_{\text{out}} \tag{8.1}
$$

$m_i$	Mass flow rate (with various subscripts), kg/s
t	Time, seconds
$V_i$	Volume of the <i>i</i> th CS element, $m3$
$m_i$	Mass of the fluid in the <i>i</i> th CS element, kg
$T_i$	Temperature of the fluid in the <i>i</i> th CS element, K
$P_i$	Pressure of the fluid in the <i>i</i> th CS element (either static or dynamic), Pa
QQ <sub>i</sub>	Net heat flow into the <i>i</i> th CS element, J/s
$v_i$	Specific volume of the fluid in the ith CS element, equal to $V_i/m_i$ m <sup>3</sup> /kg
$h_i$	Enthalpy of the fluid in the <i>i</i> th CS element, J/kg
$du dT_i$	Gradient of internal energy with respect to temperature, J/kg/K
$dudv_i$	Gradient of the internal energy with respect to specific volume, $J/m3$

<span id="page-4-2"></span>**Table 8.1** Meanings of symbols in Eqs. [\(8.1\)](#page-3-1), [\(8.2\)](#page-3-2), and [\(8.3\)](#page-4-0)



<span id="page-4-1"></span>**Fig. 8.3** Model of a steam catapult for an aircraft on a carrier, including engine thrust

$$
\frac{dV_i}{dt} = fb_i \qquad \text{(the externally causal flow)} \tag{8.2}
$$
\n
$$
\frac{dT_i}{dt} = \left[ \text{QQ}_i + (P_i + \text{dudv}_i) \left( v_i + \frac{dm_i}{dt} - Vd_i \right) \right] / (m_i \text{dudT}_i) + \left[ (h_{\text{ini}} - h_{\text{out}}) \dot{m}_{\text{ini}} - (h_{\text{out}} - h_i) \dot{m}_{\text{out}} \right] / (m_i \text{dudT}_i) \tag{8.3}
$$

<span id="page-4-0"></span>The symbols as used here are defined in Table 8.1. Implementing them properly in THERMOSIM with the relations to all the other elements in a system requires a couple 1000 lines of code and a more sophisticated nomenclature in the principal simulation program *convec2.m*, a very different story from conventional bond graphs and bond graph elements. A major reason for the complexity is the frequent need to employ analytical Runge–Kutta iterations when the two known independent thermodynamic variables are a different pair, such as pressure and enthalpy, or entropy and pressure.

A very simple example of the use of a CS element is for a model of a steam catapult such as used on an aircraft carrier, shown in Fig. [8.3.](#page-4-1) Since the fluid system which contains the steam has no active fluid ports or significant heat conduction during the takeoff run, the only bond to the CS element is the one for the expansion of the chamber. The CS element is the only non-traditional part of the model. Convection bonds and elements can be mixed with simple bonds and elements of course following certain rules.

$$
S_e \frac{P_v h}{m} \rightarrow \text{RS}_c \frac{P_v h}{m} \rightarrow \text{CS}
$$

<span id="page-5-0"></span>**Fig. 8.4** A tank being charged with a fluid from a source through a restriction

#### *8.2.2 The RSc Element*

A model for an independent source of fluid—in effect a chamber of infinite size connected to a fluid chamber through a restriction such as valve or an orifice is shown in Fig. [8.4.](#page-5-0) The source is called the convection Se element, sometimes written as Sec. The restriction is called the convection RSc element, which is highly irreversible despite the fact that the power flows on the two bonds are identical and equal to hm. In THERMOSIM the RSc element is treated as an equivalent orifice with upstream flow that is reversible but an expansion that is not. The fluid upstream and/or downstream could be a saturated mixture of vapor and liquid, which introduces a complication because the effect of gravity on whichever chamber is upstream—has the higher pressure—may separate the liquid from the vapor, depending on the elevation of the physical port. One also does not know in general from which end the fluid is emitted; it may even change in the middle of a run. Therefore, the user must choose one of the following nine possibilities:

- (1) No gravity separation for liquid or vapor. (This is the default choice.)
- (2) Only vapor passes into the input side (for forward flow); there is no restriction for flow passing into the output side (for reverse flow).
- (3) Only liquid passes into the input side (for forward flow); there is no restriction for flow passing into the output side (for reverse flow).
- (4) Only vapor passes into the output side (for reverse flow); there is no restriction for flow passing into the input side (for forward flow)
- (5) Only liquid passes into the output side (for reverse flow); there is no restriction for flow passing into the input side (for forward flow).
- (6) Only vapor passes in both directions.
- (7) Only liquid passes into the input side (for forward flow); only vapor passes into the output side (for reverse flow).
- (8) Only vapor passes into the input side (for forward flow); only liquid passes into the output side (for reverse flow).
- (9) Only liquid passes in both directions.

THERMOSIM employs a special function file *convecRS2.m* to compute the mass flow rate and enthalpy of the flow through a general compressible fluid through a vena contracta, which the user characterizes by an equivalent orifice area. The flow upstream of the vena contracta is assumed to be adiabatic and isentropic (reversible with constant entropy). The expanding flow downstream of the vena contracta is assumed to have the downstream pressure, and therefore is highly irreversible. The flow at the throat can be subsonic or choked. The function file has as independent <span id="page-6-0"></span>**Fig. 8.5** The work-to-heat element RSr

Force, torque or voltage<br>
velocity or electrical current<br>
RS<sub>r</sub>  $\frac{\theta}{s}$  (q)

<span id="page-6-1"></span>**Fig. 8.6** The RSh element for heat conduction

 $\frac{\theta_1}{\dot{s}_1 \cdot (q)}$  RS<sub>h</sub>  $\frac{\theta_2}{\dot{s}_2 \cdot (q)}$ 

arguments, in addition to the area and the gravity separation index, the upstream and downstream pressures and 14 different thermodynamic properties of the upstream fluid. Over 200 lines of code are required, plus calls to ancillary files, to compute the local properties of the substance within the element.

#### *8.2.3 The RSr Element*

The traditional use of bond graphs assumes all elements except the one-port "resistance" *R* conserve energy. The *R* element "dissipates" energy, making the thermal energy that it creates unavailable to the rest of the system. Several authors have remedied this limitation by replacing the *R* element with the two-port workto-heat element RS element, or as given in THERMOSIM, the RSr element. In Fig. [8.5,](#page-6-0) the mechanical or electrical power on the left-hand bond that is converted to heat emerges on the right-hand bond as heat. As above, the effort variable on the heat conduction bond is absolute temperature, and the true flow variable is the entropy flux. Nevertheless, to satisfy the common practice, the flow on the heat conduction bond is treated in THERMOSIM as heat flux itself—given as *q*, the symbol commonly used to heat flux in thermodynamics, which is not to be confused the use of the same symbol in conventional bond graphs to represent generalized displacement—consistent with the use of quasi-bond graphs. This is not only more familiar to most engineers and scientists, but also more likely corresponds to the use of linear constitutive relations with approximate constant parameters.

## *8.2.4 The RSh Element*

A version of the RS element, called in THERMOSIM the RSh element, is also used to represent the temperature and entropy changes that occur when heat is conducted through a thermal "resistance." This element is pictured in Fig. [8.6.](#page-6-1) Note that all the RS elements create entropy without changing the power. Note also that the causal strokes shown are mandatory, and therefore automatically applied by BondGrapher.

<span id="page-7-1"></span><span id="page-7-0"></span>

#### *8.2.5 The Heat Source Elements: Se, Sf, and Sq*

The conventional source elements Se and Sf can be used for thermal sources, with the variables shown in Fig. [8.7.](#page-7-0) In THERMOSIM a variation of the thermal Sf source, called the Sq source, also shown, is sometimes employed to solve a very special problem.

Its difference with Sf is merely the placement of the causal stroke, which in fact is a false placement; the heat flow is still the actual causal variable. The sole purpose for using this element is to allow a heat source element to be bonded to a CS element without the bond being confused with a pressure-velocity bond.

# *8.2.6 The Ct Element*

A particular combination of heat conduction bonds and elements occurs so often that it is designated in THERMOSIM by a special shorthand element. This is the Ct macro element which represents heat being passed between the fluid in a CS element—or from an IRS element, described below—and the walls of a chamber or a fluid channel, and also heat being passed between these walls and the environment. In the first of two implementations, shown in Fig. [8.8,](#page-7-1) the environment is given a particular temperature, and the thermal resistance thereto is represented by an RSh element. In the second implementation, shown in Fig. [8.9,](#page-8-0) the environment comprises a heat source, such as from an electrical heater. In both cases the vertical bonds are connected to the CS or IRS element, and the thermal compliance of the walls is represented by the *C* element. Both combinations are represented by the Ct element shown at the left in Fig. [8.9.](#page-8-0)

# *8.2.7 The 0S Element*

The 0S element shown in Fig. [8.10](#page-8-1) represents a fluid junction, or pipe tee. In THERMOSIM the junction is restricted to three channels or convection bonds. This



<span id="page-8-0"></span>**Fig. 8.9** The second combination of element designated by the Ct element, and the Ct notation itself

<span id="page-8-1"></span>**Fig. 8.10** The OS junction for three joined fluid channels

element is like a conventional 0-junction in that the total input mass flow equals the total output mass flow, and the three pressures are equal. If the flow is diverging, the effort variables from the single input channel are conveyed to the output channels, and the junction is thermodynamically reversible. If, rather, the flows from two channels are merging, the two input flows may have different temperatures, and irreversible mixing occurs, which means a loss in available energy (exergy). The flux of enthalpy which emerges is set equal to the sum of the two input enthalpy fluxes.

When there are more than one 0S-junctions in a system model, complex issues of their relative status require about 300 lines of code to sort out, but all the user needs to know is that some restrictions apply to allowed bond graph structures.

#### *8.2.8 Example System*

An example that illustrates the use of the various elements presented above is given in Fig. [8.11.](#page-9-0) This is a piston-cylinder air compressor with a tank and a mechanical load. The piston is driven from the mechanical source Sf, and the air enters from its atmospheric source Se. The cylinder is represented by the left-most CS element; the RS (RSc) elements on its two sides represent the input and output valves, which can be programmed to operate as a function of the position of the piston or in response to the various pressures so as to keep the flow passing from left to right. The flow bifurcates at the fluid tee represented by the 0S element, to pass to the large tank as given by the right-hand CS element, or toward the load as programmed by the user who sets the behavior of the load valve given by the right-most RS (RSc) element.

 $P_1 h_2$   $\uparrow h_2$ <br>  $\uparrow h_2$ <br>  $\uparrow h_3$ <br>  $\downarrow h_3$ 



<span id="page-9-0"></span>

The air in the load tank gets hot. Heat is conducted to the walls of the tank, where it is stored, and from there to the environment; these features are all represented in the macro Ct element. The output piston and cylinder is represented by the CS element toward the upper-right. The attached  $T$  element transforms the volume change rate to velocity. The weight of the load is represented by the uppermost Se element, its mass or inertia by the *I* element, and an effective dashpot by the *R* element. The transformer element *T* near the bottom of the bond graph transforms the rate of change of the piston volume to velocity, and the RSr element there uses this velocity, *v*, and the temperature of the 0-junction,  $\theta_c$ , to compute the friction force on the piston,  $F$ , and the heat generated thereby,  $q_f$ . Some of this heat goes to the environment Se through the thermal resistance RSh, some goes to heat the walls of the cylinder—which gets quite hot—and some passes back into the air in the cylinder.

#### *8.2.9 The 1- and 1S-Junction Elements with Convection Bonds*

The only work interaction between convection and non-convection parts of a system presented thus far is through the volume change bond of the CS element. The 1Sjunction and the generalized 1-junction perform the other work interactions.

The traditional 1-junction is a mainstay of non-convection bond graphs; it can have any number of bonds which are forced to share a common flow. In THER-MOSIM it is restricted to three bonds; additional junctions are used when needed. Precisely two of the bonds are allowed to be convection bonds, leaving the third as a simple bond. Since the flow on convection bonds is always a mass flow, this means that all the flow on all three bonds of any such 1-junction must be the same mass flow. Further, *the junction is defined so as to represent thermodynamically reversible behavior*, so the entropies on its two convection bonds must equal one another. Since energy is also conserved, as with the traditional 1-junction, the effort on the simple or mechanical bond equals the difference between the enthalpies of the two convection bonds—assuming their power convention arrows are aligned the same way.

Sometimes one desires to have a common-flow junction with two convection bonds having equal pressures rather than equal entropies. This is the definition of 1S-junction, which thereby becomes thermodynamically irreversible. Use of the 1 and 1S-junctions is illustrated by the model for the heart of a positive-displacement compressor shown in Fig. [8.12.](#page-11-0)

Part (a) of the figure shows an ideal or lossless compressor (100 % volumetric and adiabatic efficiencies) with torque *M* and angular velocity  $\dot{\phi}$ . The modulus of the transformer  $T_m$  is the ratio of the mass flow to the angular velocity, which is the product of the density of the entering fluid times the volumetric displacement per radian or rotation. Part (b) shows how to represent compressors with less than 100 % volumetric and adiabatic efficiencies. The modulus of the transformer *T* can



<span id="page-11-0"></span>**Fig. 8.12** Bond graphs (**a**) and (**b**) for a positive-displacement compressor or pump

<span id="page-11-1"></span>**Fig. 8.13** The IRS element



be shown to be  $T = \eta_{\nu}/\eta_{\text{ad}}$ , where  $\eta_{\nu}$  is the volumetric efficiency and  $\eta_{\text{ad}}$  is the adiabatic efficiency. The modulus of the transformer  $T_s$  can be shown to be  $T_s$  =  $\eta_v / (1 - \eta_{\rm ad}).$ 

<span id="page-11-2"></span>Note that elements in THERMOSIM containing the letter *S* in their names are defined to be at least potentially irreversible.

#### *8.2.10 The IRS Element*

The final element with convection ports is the IRS element, first introduced in 2003 [\[4\]](#page-34-7) with a major update in 2016 [\[10\]](#page-34-8). This represents a fluid channel with fluid friction, inertia and heat transfer to its surroundings—normally the walls. Such channels are the principal way engineers achieve critical heat transfer in the systems they design; the most common are round tubes. There are two versions of this element in THERMOSIM, one for nominal liquids such as water or a hydraulic fluid, and a second for general compressible substances. Both are represented by the bond graph notation shown in Fig. [8.13,](#page-11-1) and employ the momentum of the stream as its single state flow variable. The multiple heat conduction bonds shown are for the general case. Often, however, none or only one such bond is used.

The principal parameters are the length of the channel and its area, which is assumed to be uniform. In keeping with the philosophy that lumped bond graph elements should use approximate constitutive relations in order to achieve

a reasonable compromise between accuracy and computational efficiency, the wall friction models assume an equivalent steady or equilibrium velocity profile across the channel. Further, a round cross-section—that is a tube—or its equivalent using the concept of a hydraulic diameter is assumed. Implementing this requires computation of the effective Reynolds number of the flow—or for two-phase flow, separate Reynolds numbers for the liquid and vapor phases. This is turn requires knowledge of the viscosity or viscosities of the fluid. For purposes of relating mass flow to the momentum, a slug-flow approximation is employed, so that the mass flow is taken as the ratio of the momentum to the length. The rate of change of the momentum is proportional to the pressure difference between the two ends of the element minus the pressure drop due to wall shear, with a correction for the difference between the pressures associated with the momentum fluxes at the two ends.

The compliance of the fluid in the channel is not included within the IRS element, but rather is modeled by CS elements that may be placed at its ends. This means that when compressibility effects are especially important—often they are not—such as when wave propagation effects are of interest, multiple IRS elements interspersed with CS elements should be used. Normally, the compliance of the fluid in the tube is split between the two adjacent CS elements by assigning half of its volume to each.

The simple bonds in Fig. [8.13](#page-11-1) are for the heat conduction to the surroundings, as with the CS element. It may be attached to a Ct macro element or a more primitive combination of heat conduction bonds and elements. Many scientists and engineers devote their careers to modeling such heat transfer. The author [\[10\]](#page-34-8) has selected what he hopes are the most useful but reasonably simple set of heat-transfer correlations available in the literature, but the user may prefer to substitute others.

The IRS model for liquid media is simpler than that for general compressible substances, and is vastly simpler when there is no heat conduction bond or only one. The hydraulic system pictured schematically in Fig. [8.14](#page-13-0) with the corresponding bond graph in Fig. [8.15](#page-13-1) illustrates its use when there is one heat conduction bond. BondGrapher, which is introduced in Sect. [8.4,](#page-18-0) created this figure in which the selected element CS22 is highlighted with color.

Note the incorporation of the model in Fig. [8.12](#page-11-0) for the pump, allowing for adiabatic and volumetric efficiencies less than 100 %. The two fluid tees are represented by the two 0S elements. The elements CS18 and CS36 represent the hydraulic sides of the low-pressure and high-pressure accumulators, respectively, while CS22 and CS31 represent the nitrogen sides. Se23 and Se33 represent the temperature of the environment with which the accumulators interact, while R21 and R27 represent resistances associated with the motion of the accumulator interfaces, which are necessary for computational causality purposes although they are too small to have a significant effect. CS6 and CS9 represent small fixed volumes along the fluid lines to accommodate the small effects of compressibility and to satisfy computational causality. The friction and inertia effects of the fluid lines are represented by IRS7, IRS10, and IRS16; the latter two are quite long—4 m and also exchange heat with their surroundings through the use of elements Ct34



<span id="page-13-0"></span>Fig. 8.14 Schematic for the hydraulic system (using fluid power symbols)



<span id="page-13-1"></span>Fig. 8.15 Bond graph for the hydraulic system

and Ct35. RS25 models a spring-loaded check valve that partially opens when the upstream, pressure exceeds 20 MPa, preventing overpressure. RS12 is a valve that opens when the user wishes to move the double-sided piston-cylinder actuator, which operates with a hydraulic fluid. Further discussion including simulation results is given in Sect. [8.4.3.](#page-26-0)

For long channels the properties of the fluid and the wall temperatures may vary considerably from one end to the other. This is particularly true when the fluid is a highly compressible substance, but also applies to pure liquids with significant

<span id="page-14-0"></span>

<span id="page-14-1"></span>**Fig. 8.17** Bond graph for the hot water shower system

transport time from one end to the other and significant thermal interaction with the channel walls. The author models such situations by segmenting the channel, as shown in Fig. [8.16.](#page-14-0) Each such segment has one of the heat conduction bonds shown in Fig. [8.13](#page-11-1) which is assumed to be attached to a Ct element. Each Ct element also has its thermal compliance and state variable for the average temperature of the wall at that location. The users draws only one Ct element with the BondGrapher interface, however; the rest are inferred by a designation of the number of the segments  $N(I)$ , where  $I$  is the index number of the IRS element, avoiding what otherwise could be a very messy bond graph. The user assigns the overall thermal compliance of the tube to this element, represented here as *C*; the program automatically divides *C* by *N* to give the compliance of the individual segments.

Along with the *N* state variables for the wall temperatures of the segments there are an additional *N* state variables for the average fluid temperatures within the segments. Thus the IRS element has a total of  $2N + 1$  state variables, the 1 being for the momentum or mass flow rate. The wall shear is also computed separately for each segment, but this requires no further state variables. Temperatures of the wall and the fluid are computed for both the segments and at the interfaces.

A person wants to take a quick shower; how long does he have to wait for the hot water emerging from a hot water heater 10 m away to reach him, at location Sec6, and of necessity also warm the walls of the copper pipe? A bond graph model for this system is shown in Fig. [8.17.](#page-14-1) The hot water emerges from the element Sec1. Cold water also emerges from Sec8 to mix with the hot water at pipe tee 0S5. The valves from the two sources, RSc4 and RSc7, are abruptly opened at time zero. The element IRS2 is the long pipe modeled with 20 segments, and CS3 is a small volume representing the mixing of new water with old and the slight compressibility of the water in the pipe, and satisfying causal requirements. This example is continued, with simulation results, in Sect. [8.4.4.](#page-27-0)



<span id="page-15-0"></span>**Fig. 8.18** A model for a fluid channel containing a compressible fluid using unsegmented IRS elements

When a channel contains a compressible fluid, the state of that fluid can change radically along its length, potentially being a liquid at one end, a saturated mixture in the middle, and a pure vapor at the other end. A proper lumped model of this channel contains a long cascade of alternating lumped IRS and CS elements, part of which is shown in Fig. [8.18;](#page-15-0) the CS elements represent the fluid compliance and the IRS elements represent the wall friction and inertia and thermal interaction with the walls of the channel via the attached single Ct elements. An especially difficult situation arises when the segment of fluid line represented by the element is partly a compressed liquid and partly a saturated mixture, since the properties of the fluid in these two phases are radically different. The length of the cascade would have to be exceedingly long to accommodate this occurrence, imposing a huge computational burden. A radical structural simplification is proposed to reduce this complexity without overly compromising accuracy.

Experience has shown that the mass flow rates approach equilibrium vastly quicker than the temperatures of the fluid and the channel wall. This is associated with the fact that the effect of the mechanical compliance—the compressibility of the fluid—is effectively over an order of magnitude smaller than the effect of the thermal compliance—heat capacity—of the walls. This suggests approximating a channel with at least a modest span of fluid states by a single IRS segmented element with its uniform mass flow rate, allowing the heat transfer to the walls and the state of the fluid to vary along its length. The resulting number of IRS elements needed to represent a channel adequately should be reduced radically, sometimes to only one.

The first-generation IRS element required the user to specify the thermal conductance between the wall and the environment, and was implemented only for refrigerant R12. Further, the temperature of the tube wall was not allowed to vary quite properly along its length in response to varying heat-transfer rates. A final difficult problem was the danger that the phase of the fluid would change in the middle of a segment to or from liquid or to or from vapor—the other state being a saturated mixture—producing gross errors in key properties that determine the heat transfer for that segment. To combat this frequent occurrence, the transition was anticipated and the segment sizes adjusted so that the transition would occur very near the end of a segment. This caused great complication and on some occasions even caused the program to abort.

The new segmentation also has a key advantage of allowing efficient treatment of the problem of sudden changes in phase in the middle of a segment. It also allows a sudden change from boiling to condensation to be treated efficiently.

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The calculations for the IRS element are coded in the 1000-plus-line script file *IRS4.m*, which also calls the function file *viscosity.m* for the thermal conductivities as well as the viscosity, and the function file *boiling.m*. The file *IRS4.m* is called by the main simulation program for potentially multiphase fluids, *convec2.m*. The differential equation for the state (or temperature) for the wall segment  $k$  is

$$
\frac{dx_k}{dt} = \left(T_e - T_{\text{seg}}(I,k)\right) * H\left(I,k\right) - Q_{ls}\left(I,k\right) \tag{8.4}
$$

in which  $H(I, k)$  is the thermal conductance of the "thermal resistance" of the Ct element for the *k*th segment. This conductance is based on the properties of the environmental—or "alternative" fluid—and certain properties. It is computed in a new relatively short function file *CtH.m* also called by *IRS4.m*, which can also be called to compute the heat transfer associated with CS elements.

The differential equation for the temperature of the fluid in a segment is

$$
\frac{dx_{N+k}}{dt} = ((T_e - x_{k+N}(I,k))\dot{m} + Q_{I2}(I,k)/dh/dt)/(m/N)
$$
(8.5)

in which *I* is the index of the IRS element, *k* is the index of the segment,  $Q_{I2}$  is the heat flux from the wall to the fluid for the segment, *dh*/*dt* is the specific heat of the fluid, and  $m/N$  is the mass of the fluid in one segment. This version of the equation has been specialized for pure liquids to speed up execution with negligible error.

CS elements are nominally considered to represent physical volumes with large enough dimensions that the interior velocities of the fluid are small enough not to produce dynamic heads of significance. For example, a CS element bonded to an 0Sjunction is assumed to give that junction a static pressure common to all of its bonds. The hydraulic system above is an example. IRS elements, by contrast, nominally have such a small cross-sectional area that the dynamic head is indeed significant. What does it mean, then, when a CS element is bonded to an IRS element? For the nominal case, including the hydraulic system, it means that the interior area of the CS element through which fluid is passing is presumed to be significantly larger than that of the attached IRS element. Clearly, however, that is not the case for the cascade of IRS and CS elements depicted in Fig. [8.18.](#page-15-0) In effect, a different kind of CS element is needed there, which produces a total dynamic pressure on the joining bond rather than a static pressure. This is called a "total-pressure CS element," for which the dynamic head on the connecting bond is simply added to the static pressure to give the proper total pressure. It is necessary for the user to designate that this is what he wants, however; BondGrapher has been modified to ask the modeler's choice, with the default being the "static pressure CS element."

# **8.3 Thermodynamic and Physical Properties of Common Substances**

Starting in 1999 [\[3\]](#page-34-9) and continuing until the present  $[3, 5, 8, 9]$  $[3, 5, 8, 9]$  $[3, 5, 8, 9]$  $[3, 5, 8, 9]$  $[3, 5, 8, 9]$  $[3, 5, 8, 9]$  $[3, 5, 8, 9]$  the author has developed practical code for modeling the properties of gaseous, liquid, and saturated mixtures thereof of 30 common pure substances that are identified as potentially being in the vapor state or saturated mixture state—ammonia, argon, butane, carbon dioxide, cesium, ethane, ethylene, helium, heptane, hexane, hydrogen, isobutene, isopentane, mercury, methane, neon, nitrogen, octane, oxygen, propane, propyl alcohol, propylene, the refrigerants 12, 13, 134a, 22, 23, 502, and 503, rubidium, and water. Prominent amongst his sources is the work of Reynolds [\[12\]](#page-34-10). Gaseous air is treated as a mixture of nitrogen, oxygen, and argon. He has also focused on four additional substances identified as liquids—liquid water and the hydraulic fluids MIL H 5606, MIL PRF 27601, and MIL PRF 83282.

In the spirit of a compromise between accuracy and complexity he has avoided use of the most accurate models available for some substances, particularly for water where lifetimes of effort have produced very accurate but extremely complex models that would drastically retard the speed of simulation, favoring instead a model that was once considered generally excellent. He has also forgone some available accuracy in the region of the critical point; should this especially complex region become critical, making an improvement ought to be considered.

Any pair of independent properties are necessary and sufficient to specify the state of a pure substance. For the pure liquids, the most practical pair is temperature and pressure. For the general substances, as is pointed out in Sect. [2.1,](http://dx.doi.org/10.1007/978-3-319-47434-2) the pair that is effectively mandated by the available data is temperature and specific volume the inverse of mass density. This use of specific volume can be problematical in the compressed liquid state, however, since liquids have nearly constant specific volume. Depending on the substance, approximate values may not be computed unless one specifies a value for the bulk modulus, which is an option.

Analytic models taken from the literature are used to compute the dependent thermodynamic properties, which include enthalpy, internal energy, specific heat, entropy, the quality of saturated mixtures—the author extends the definition of quality to make it useful also for the compressed liquid and vapor states—specific volume, enthalpy and entropy of a saturated liquid at the given temperature, and these same properties for a saturated vapor at the given temperature. The list is of course expanded by absolute pressure for the general substances and mass density for the pure liquids. Further, simulation programs used by THERMOSIM employ several Runge–Kutta iterations that are expedited if the gradients of the nonsaturation dependent properties with respect to the two independent properties are also available in analytic form. The author therefore derived analytical derivatives of the available empirical relations for pressure, internal energy, enthalpy and entropy, plus the gradient of quality with respect to temperature, adding nine more dependent variables to the list for the general substance. When the phase changes between pure liquid and saturated vapor, these gradients change abruptly by orders

of magnitude, requiring special treatment. Abrupt changes in heat-transfer and wall friction domains also are cause for special smoothing.

Separate simulation programs are employed for systems in which the principal fluid is a liquid or is a general substance.

The transport properties of viscosity and thermal conductivity are also needed if one is to compute heat transfer and fluid friction. Critically important dimensionless groups including Reynolds and Nusselt numbers depend on these. Analytical expressions for viscosity and thermal conductivity as functions of temperature and specific volume are difficult to find or deduce, unfortunately, so at this writing the author has only limited results. He managed to convert into the needed form some raw data for the four liquids listed above. For general substances he has results for water, nitrogen, oxygen, argon, air as a vapor, and an approximate form for the refrigerant R12. It is not easy to find appropriate correlations in the literature; much more work remains. See also Sect. [8.5](#page-32-0) below.

#### <span id="page-18-0"></span>**8.4 The Graphical Interface BondGrapher and Examples**

Each system model is coded in a MATLAB<sup>®</sup> system file that, when called by the user, directs the desired dynamic simulation. In practice, this system file is most conveniently produced with the help of a graphical interface called BondGrapher<sup>®</sup>, which was written by the author's son, Scott A. Brown. All the files needed to carry out the simulation except for BondGrapher, assuming one has access to MATLAB, are freely downloadable from the author's website [\[9\]](#page-34-3). The prospective user should also download the extensive Users' Manual (make sure it is Version 4), which provides the details and many examples of the procedure. BondGrapher includes help files, but only the ones that pertain to drawing bond graphs are useful as of this writing. To acquire BondGrapher, which also is free for the foreseeable future, one should email the author and include a commitment not to transmit the program to a third party without permission.

The structure of the computational part of the THERMOSIM package is suggested in Fig. [8.19.](#page-19-0)

#### *8.4.1 BondGrapher and the Steam Catapult System*

BondGrapher has a main screen and a screen called *Global Options*. The main screen with the bond graph for the aircraft catapult of Fig. [8.3](#page-4-1) drawn is shown in Fig. [8.20.](#page-4-2) The graph may have been entered directly, or downloaded from an existing system file created and saved earlier using a name of the user's choice. The procedure to draw a new bond graph involves placing symbols for elements over node locations, and placing bonds between nodes by clicking on the element at one end of the bond and then double-clicking the node at the other end. The



<span id="page-19-0"></span>**Fig. 8.19** Structure of the THERMOSIM package



<span id="page-19-1"></span>**Fig. 8.20** The main screen of BondGrapher showing the aircraft catapult of Fig. [8.3](#page-4-1)

symbols (Fig. [8.20\)](#page-4-2). The main screen of BondGrapher showing the aircraft catapult of Fig. [8.3](#page-4-1) may be chosen by typing their first letter, and hitting the space bar if more than one symbol begins with that letter. Alternatively, one can check the desired element on a list that is available on the left side of the screen. The program knows from the choice of elements whether a bond should be a simple one or a convection one. The program almost always knows where to place the causal strokes. The power-convention half arrow is placed according to the order of the chosen elements. The user can make changes by clicking where indicated at the left side of the screen. Also, bonds can be activated with another click. With a little practice this procedure is very fast. It is suggested that one first sketch the desired graph on a sheet of paper, to make sure it does not have confusing crossing bonds and otherwise is easy to read. One can always move the elements around with a simple click-and-drag operation. The elements and bonds are numbered, separately, in the order they have been selected; the numbers can be changed. The bond graph image can be moved by clicking and dragging on any location not too near a node. Also, the size of the graph may be changed; the author's preferred method is to use the mouse wheel. The Instructors' Manual gives more detail and many examples. One may print out the bond graph or the entire screen at any time.

Constant parameter values for elements may be inserted by clicking on the element and then clicking on the *Parameter* tab on the left side of the screen. The CS element with this tab is chosen in the example. If the element requires parameters, one or two corresponding boxes appear into which one may place either direct values or symbols for the indicated parameters, to be given numerical values later. One similarly indicates the values of any initial conditions by clicking of the *initial state* tab. When no value is entered, the default value of zero is assumed. The constitutive relations entered this way for  $R$ ,  $C$ ,  $I$ , and  $RS<sub>r</sub>$  elements may be either linear or satisfy the power-law relation

```
Output variable = parameter \cdot sign (input variable) \cdot abs(input variable)<sup>exponent</sup>
```
An exponent set to one directs a linear relation. The  $R$  and  $RS<sub>r</sub>$  elements give coulomb friction when the exponent is set equal to 0. Elements other than these three allow only linear relations unless information is inserted elsewhere, as described below.

The macro element Ct and the element IRS are special cases in which three parameters must be chosen. The third parameter is entered where a second initial condition for the bond would normally be placed. There are two versions of the Ct element, as described above. The version with the Se reservoir is chosen by placing the heat conduction coefficient of its RSh element, with units of W/K, in the first parameter position. The version with the Sf reservoir is chosen by placing 0 there. In both cases the second position is used for the thermal compliance, in J/K. The third parameter is the temperature of the Se, in K, or the heat rate of the Sf, in W. The first initial state is the actual heat in the compliance, which is the product of its compliance and its temperature, with units of Joules. The third parameter for the IRS element is the number of segments, *N*, into which the channel is divided.

The simulation program setup by BondGrapher saves the values for the temperature and the specific volume—or density for pure liquids—for any bonds with fluid flow, after each time step. These vectors, called *Tc* and *vc*, help start iterations for the subsequent time step. Trial values for these are needed for the very first step, but usually any values not orders of magnitude incorrect will work. Therefore, default



<span id="page-21-0"></span>**Fig. 8.21** The Global Options page for the example of the airplane steam catapult

values are given. Should these fail, one can change them by clicking on one of the relevant bonds, and click on *initial state* in the *selected bond* group on the left side of the screen. The user is asked if he would like to use that value for all of the relevant bonds; "yes" is usually the best answer.

At any point one may click of the *view error report* box located toward the lower left of the screen. This report points out any violations of the rules or essential omissions you need to correct before running a simulation. It will also observe issues over which only the user knows the correctness of, such as whether parameters or initial conditions still set at zero are what is intended.

The *Global Options* screen is displayed by clicking on the *Options* button toward the lower-right corner of the main screen. The example for the aircraft catapult is shown in Fig. [8.21.](#page-19-1) If the system employs one or two fluids, the choice is made on the left side of the screen: the catapult uses water (in the wet steam region). In general, a second "alternate" fluid is also allowed for regions that cannot mix physically with the "main" substance, such as for the gas in an accumulator used in a hydraulic system, or for the thermal reservoir, for example, included within a Ct macro element that is attached to a CS element or an IRS element. One also chooses the integrator to be employed by MATLAB; the integrator ode45 is most commonly chosen for non-thermodynamic systems, and ode23 often appears to be more robust for thermodynamic systems.

Another choice selected on the left side of the *Global Options* screen is how long each time segment in a run should be, and how many time segments are to be run. If the numbers of the "first step" and the "last step" are both 1, as with the catapult, there is only one time segment and the "time step" is its duration in seconds. The simulation stores the history of the states through the most recent time step in a file *TSIMdata.mat*, which the MATLAB command *load TSIMdata* allows the user to access. If the "first step" is chosen to be 1 and the "last step" is a larger than the "first step," the states at the ends of each time segment are stored in a file *dataX.mat*, which can be accessed by the command *load dataX*. This provides a data compression feature that may be attractive if a run would otherwise produce data files that are longer than one would prefer. In practice, however, the most useful feature of the data compression mode is the ability to interrupt a simulation at any point, look at the results up to that point, and then resume the simulation. This is also helpful if the simulation is very slow, or if one wishes to change one or more parameters in the middle of an overall run. Following an interruption in a simulation and commanding *load dataX*, one can determine the number of the last time segment completed with the command *size(time)* or*size(state)*. To resume the simulation, one enters this number as the "first step" of the continuation.

Finally, the bottom of the left side of the Global Options page allows the user to choose the relative and absolute tolerance of the simulation, and the initial step size. See MATLAB *help* for detailed information.

The Steam Catapult system uses only one of the six tabs on the right side of the Global Options page, that called *Custom*. This tab is used to provide the values of any constants indicated elsewhere only by symbols. The only such symbol for this system is the temperaureT00, used in the definition of Tc. Other uses of the right side of the Global Options page are discussed in the context of other systems below.

After running the system file, you may enter MATLAB commands to plot, print, or save whatever results you wish. Selected results from the simulation of the steam catapult system are shown in Fig. [8.22.](#page-21-0)

# *8.4.2 The Piston-Cylinder air Compressor and Programming Nonlinearities and Special Functions*

The main page of the BondGrapher that produced the MATLAB system file  $aircompsys2.m$  is shown in Fig. [8.23,](#page-23-0) with the element *C* and the parameter tab clicked. On the screen the element is highlighted in color. The exponent of the constitutive relation for this compliance is given as 1, indicating a linear relation, of several either directly given or computed in the *Custom tab* on the right side of the Global Options page, as shown in Fig. [8.24,](#page-23-1) using standard MATLAB coding.

There are four more tabs to the right of the Custom tab. The most important of these are the *Parameter* tab and the *Extra variable* tab, which are discussed presently. First, however, it is necessary to know something about a matrix called *el(i, j)*.



<span id="page-23-0"></span>**Fig. 8.22** Selected results of the steam catapult system



<span id="page-23-1"></span>**Fig. 8.23** Main page of BondGrapher for the piston-cylinder air compressor



<span id="page-24-0"></span>**Fig. 8.24** Global Options page of BondGrapher for the air compressor system

The matrix *el* is the repository of the order and content of all the elements in a system model. It is a prominent part of the system file created by BondGrapher. It has one row for each element, in the order of their element numbers. Each row has three members. The first member is a code number designating which of the 20 types the element is. The other two members are both zero for the four junctions. Their content for the remaining 16 element types is given in a table in Sect. [1.6](http://dx.doi.org/10.1007/978-3-319-47434-2) of the Users' Manual (version 4) which is too large to reproduce here.

In the *Parameter* tab the parameters are defined by the user in one of the following forms, in which *X* refers to the quality of the fluid and *Pb* refers to the pressure on bonds:

$$
el (i, 2) = function (t, x, P, X, h, s, Pb) \text{ (for multiphase substances)}
$$
\n
$$
or el (i, 2) = function (t, x, Pb, h, s) \text{ (for liquids)}
$$
\n
$$
el (i, 3) = function (t, x, P, X, h, s, Pb) \text{ (for multiphase substances)}
$$
\n
$$
or el (i, 3) = function (t, x, Pb, h, s) \text{ (for liquids)}
$$

Here *i* is the number of the bond and the 2 or 3 refers to the parameter as identified in the table. Look at the *Parameter* tab for the air compressor system given in Fig. [8.25.](#page-24-0) Its first line specifies the sinusoidally varying angular velocity of the input shaft; the table says that the value of the flow for Sf elements is given in the third position,



<span id="page-25-0"></span>**Fig. 8.25** The *Parameter* and *Extra Variable* tabs for the air compressor system

explaining the number 3. Its fourth and fifth lines specify how the openings and closings of the inlet and outlet spring-loaded check valves for the cylinder depend on the pressures on either side of the valves; the table says that the area of RSc elements is also given in the third position. The last line specifies the times at which the valve to the load (another RSc element) is opened and closed (to achieve a desired motion of the load). The third line satisfies a requirement for certain cases of RSr elements dictated by a singular peculiarity of THERMOSIM; the causal input velocity to the element is specified explicitly. This is required when the velocity is not a direct function of a state variable but rather comes from a transformer or a gyrator, since the relations for *T* and *G* elements are not invoked before the input velocity is needed. The number 10 there is the number of the Sf element in the bond graph.

"Extra variables" are most commonly used to allow non-state variables to be drawn or otherwise displayed or recorded. For example, one may wish to plot a *pressure* on a *bond* that is not available as a state variable of some CS element. A second use is to compute the integral of a variable that itself is not already a state variable, for example, the *displacement* associated with an inertance that is not already a state variable because no associated compliance element is assigned, or the total mass that passes through a convection bond.

The *time derivatives* of these variables are coded by the user in the *Extra variable* tab in the form

 $f(x)$  = function of *t*, *x*, eb, fb, Pb, Tb, Xb, Q, PW, NRe, vb, fgb, vfb, P, *X* 

where  $i$  ranges from 1 to the total number of such variables. The arguments pressure, Pb, quality, Xb, effort, eb, flow fb, specific volumes, vb, vgb, and vfb, and temperature, Tb, refer to bonds, and use their index numbers. The heat flux *Q* is associated with IRS and CS elements, the friction pressure drop PW and Reynolds number NRe are associated with the IRS elements, and the pressure *P* and quality *X* refer to the CS elements. (When the fluid is air, Xb refers to the water component only.) The needed indices are given on the bond graph drawn by BondGrapher, assuming the proper boxes are checked. Indices are also needed for the components of any elements of the state vector, *x*, which may be used.

In addition to entering the actual code for the extra variables, the *number* of them must be entered into the indicated box on the left of the *Options* screen.

For the air compressor system it was desired to plot five variables as functions of time that cannot be directly represented as functions of the state extra variables: the flows on the bonds 16 and 1, and the pressures on bonds 1, 5, and 18. The resulting *Extra variable* tab is also shown in Fig. [8.25.](#page-24-0)

Direct plotting of an "extra" variable satisfies the "second use" mentioned above. To plot or recover a variable for the primary use, its *time derivative* must be computed using the differences between successive values. This is done using the MATLAB "diff" function, as described the Manual.

#### <span id="page-26-0"></span>*8.4.3 BondGrapher and the Hydraulic System*

The main page of BondGrapher for the hydraulic system, given above in Fig. [8.15,](#page-13-1) shows that element CS22 has been clicked—it is given in color—and that the initial state tab is also clicked. This element represents the gas side of the lowpressure accumulator. Symbols are entered to represent the initial volume of the gas—nitrogen, since it is not flammable—and its initial temperature and mass. The chamber has no convection bond attached to it, so the mass of the gas is constant. Its volume decreases at the same rate as the volume of the liquid side of the accumulator increases (and vice versa), as mandated by the volume-change bond. Its temperature changes due to heat conduction with the environment through the heat conduction bond.

The *Global Options* page with the *Custom* tab clicked is shown in Fig. [8.26.](#page-25-0) The left side of this page includes the choices of the two fluids, the MATLAB integrator, and the size of the time steps and their number for the compressed data feature that was used. The values of the constant parameters are entered into the *Custom* tab.

The values of the non-constant parameters are entered into the *Parameter* tab of the *Global Options* page, as shown in Fig. [8.27.](#page-27-1) There, the relief valve as represented by RS25 is shut—has a zero orifice area—when the hydraulic pressure is less than 20 MPa. It is opened for times less than 3 s and more than 6 s into the run, as set by the user. Between these two times this valve opens by an amount proportional to the pressure difference across it, similar to the relief valve.



<span id="page-27-1"></span>**Fig. 8.26** The *Global Options* page with the *Custom* tab for the hydraulic system



<span id="page-27-2"></span>**Fig. 8.27** The *Parameter* tab for the hydraulic system

<span id="page-27-0"></span>Saving the BondGrapher file with the name *hydraulicsys2.m* produces a MAT-LAB file which, when run, produces results that can be plotted using conventional MATLAB commands. Selected results are shown in Fig. [8.28.](#page-27-2)



<span id="page-28-0"></span>**Fig. 8.28** Selected results from the simulation of the hydraulic actuator system

 $\ddot{ }$ 



<span id="page-29-0"></span>**Fig. 8.29** The Global Options screen with the Custom tab for the shower system

#### *8.4.4 BondGrapher and Segmented IRS Elements*

For segmented IRS elements for liquids the user must provide the initial temperatures of the *N* wall temperatures and *N* fluid temperatures. This is done in the *Custom* tab of the *Global Options* screen. This screen is shown in Fig. [8.29](#page-28-0) for the shower problem introduced in Sect. [8.2.10](#page-11-2) and Fig. [8.17.](#page-14-1) Note that all these initial temperatures are set equal to the environmental temperature (Te). The only higher initial temperature is that of the water heater, which is set 44 K higher in the parameter tab for Sec1 on the main screen of BondGrapher. The Custom tab also establishes the values of all the parameters. The inner diameter of the copper pipe is 0.01 m, its outer diameter is 0.012 m, and its length is 10 m. The effective orifice area of the two valves is one-fifth that of the inside of the pipe.

The main simulation program for pure liquids is *convecliq4.m*, which calls the script file *IRSliq4.m* to handle whatever IRS elements are present. This latter program also calls the function file *CtH.m* to treat the implicit Ct elements. For general highly compressible substances the respective files are *convec4.m, IRS4.m* and (again) *CtH.m*.

Selected results of a simulation of the start-up transient are shown in Fig. [8.30.](#page-29-0) For the specified valve settings, the mass flow rate reaches its equilibrium value of about 0.13 kg/s in under 0.2 s, giving a fluid velocity of about 1.7 m/s. Thus it takes about 6 s for a particle of water to traverse the pipe. It takes longer for the output stream to reach its equilibrium value, partly since the walls of the pipe absorb heat and partly because of the mixing in the pipe as represented by the CS volume which is set to equal half of the total volume of the pipe. The output temperature felt by the person showering was computed using the extra variable option; the temperature is that of bond 5, or Tb5. The computed rises in the temperatures of the water and the tube are slower than an exact analysis would give, and demonstrate some oscillation, due to the finite sizes of the segments, but the filtering effect of the fluid compliance is greater. The final temperature of the shower is lukewarm, despite the two valves

 $C$  is the latter of  $C$ 



<span id="page-30-0"></span>**Fig. 8.30** Temperature results for the hot water shower system

having the same area, because of the large pressure drop through the pipe from turbulent viscous friction. The cold-water valve should have its area reduced.

The temperatures of the tube wall segments are close to those of the water because the heat capacity of the walls is considerably smaller than that of the water, and the conduction rates are strong. The situation is very different, and therefore of greater importance, for highly compressible substances.

As of this writing, the author is updating the simulation files for IRS elements with highly compressible fluids that have produced preliminary results [\[10\]](#page-34-8). He expects to post corrected results on his website [\[9\]](#page-34-3) before this book is published. The tentative results strongly suggest, for highly compressible fluids and especially those spanning the saturated mixture region, that fluid inertia plays a small role compared to that from fluid compliance coupled with heat transfer and wall friction. The inertia also increases execution time. One conclusion is that a new RS element should be developed that eliminates the inertia while retaining the other phenomena. Such an element does not appear to be particularly useful for pure liquids, however, because of their higher density and vastly smaller compressibility.

The analysis for highly compressible general substances employs the Models A and B shown in Figs. [8.31](#page-30-0) and [8.32,](#page-31-0) with cryogenic nitrogen in the liquid, saturated mixture, and pure vapor phases. The two models each have a total of 48 segments, which for Model B, means 12 for each of its four IRS elements. The more elaborate Model B is used to illustrate the effects of the fluid compliance in a channel. The volumes of each of its three CS chambers are taken at one-quarter the volume of the entire tube. These are specified to be "total pressure" CS elements, as per the discussion at the end of Sect. [8.2](#page-1-0) above.

The author presented models for a heat pump in 1999 [\[3\]](#page-34-9) and in his textbook [\[5\]](#page-34-1); the latter is reproduced in Fig. [8.33.](#page-31-1) The analysis there employed the now

<span id="page-31-0"></span>

<span id="page-31-1"></span>**Fig. 8.32** Model B of the same channel



**Fig. 8.33** Bond graph model of a heat pump system using IRS elements

discontinued first generation of the IRS element for compressible fluid flow, and is on the author's agenda for updating using the new IRS element.

The right side of this bond graph shows a compressor driven by an induction motor represented by Se23 and rotational inertia represented by I25 and transfer and 1- and 1S-junction elements that impart less than 100 % volumetric and adiabatic efficiencies, as with other systems above. The critical heat transfers take place on the inside and outside of tubes, represented by the four IRS elements and their attached Ct elements. The part of the evaporator that conducts heat—and exhibits fluid inertia and critical wall shear friction—is represented by CS1, IRS2, Ct12 which has the cold side thermal reservoir, and CS3. An electrical heater—used to insure that vapor enters the compressor—is modeled by the element Sq18. The thermal energy storage in the fluid there is represented by the elements CS1 and CS2, which split the volume of the tube equally. The evaporator is represented by the elements CS6, IRS7, Ct16, CS8, IRS9, Ct15, CS10, IRS11, Ct14, and CS12; the Ct elements there have the hot-side thermal reservoir. The expansion device is a simple orifice-type restriction, represented by the element RSc13.

Assuming the compressed-data option is used, THERMOSIM records time in the *time* vector, the momentum of the flow (product of length and mass flow rate), the state of the CS element, the temperatures and the thermal displacements of the wall (product of thermal compliance and temperatures) for each segment in the *state* matrix, the history of the wall temperatures for the interfaces in the *historyTwj* array, the history of the fluid temperatures for the segments in the *historyTfj* array, the history of the overall pressure drop due to wall shear in the *historyPWc* matrix, and the history of the overall heat-transfer rate between the fluid and the wall in the *historyQQc* matrix. Any number of other variables can be computed using the "extra variables" option.

# <span id="page-32-0"></span>**8.5 Correlations for Viscosity, Thermal Conductivity, and Heat Transfer**

The program *viscosity.m* addresses the thermal conductivity and viscosity of water, argon, nitrogen, oxygen, gaseous air, and somewhat restricted results for refrigerant R12. Three hydraulic fluids and liquid water are addressed separately. As of this writing the viscosity and thermal conductivity of many other fluids remain to be inserted into THERMOSIM; it is not easy to find appropriate correlations in the literature.

Heat transfer is an enormously complicated subject for which many people have devoted entire careers. For purposes of THERMOSIM, only a few commonly used approximations for equilibrium conditions have been incorporated for general use; the user can introduce others. Relations for heat conduction for simple shapes such as cylinders and tubes have been coded for several systems, using the Custom or Parameter tabs. The Parameter tab has also been used for convection correlations. Correlations for twelve different convection modes are incorporated into the program *IRS4.m* for internal and external convective heat transfer in tubes, which can also be borrowed for use with CS elements. Which applies depends on whether the fluid is a pure liquid, a mixture of liquid and vapor or a pure vapor, the Reynolds number of the flow and sometimes on the possible two phases of the flow, and whether there is heating or cooling. The correlations and their sources are listed [\[10\]](#page-34-8); the user may wish to substitute different correlations. Perhaps the most complex cases is boiling, for which the program has been adapted from a widely usable approximation given by Kandlikar [\[11\]](#page-34-11). Application is limited so far to only a few substances. A short function file *CtH.m* expedites approximating the external convective heat transfer between a tubular structure and the environment.

### **8.6 Conclusions**

Convection bonds are non-traditional bonds having two efforts rather than one in order to represent simply and effectively the flow of a compressible fluid, particularly a highly compressible fluid. Their use is compatible with simple bonds, and introduces new bond graph elements that typically employ both types of bonds. THERMOSIM is a simulation package for the dynamic simulation of physical systems based on bond graphs with or without convection bonds. Its use is expedited by an exceptional graphical interface called BondGrapher, which the author himself could not have written.

THERMOSIM efficiently computes fairly accurately the thermodynamic properties of 35 different substances, using where necessary iterations based on analytic expressions for the gradients of properties derived by the author from available models of those substances. Nevertheless, for some practical models the necessary complexity of the software can make execution rather slow. The program also computes transport properties of several substances necessary for the calculation of fluid friction and heat transfer, but the list is far from complete.

Most systems that can be treated with other bond graph modeling and simulation software can also be treated by THERMOSIM, including models with activated bonds. Analysis of linear or linearized models such as the production of linear transfer functions or frequency-response analysis not included, however. The objective is simply the dynamic simulation of models, since the systems of primary interest are so highly nonlinear that linearized models are of little utility.

As of this writing the IRS element has been fully developed only for use with pure liquids; complete implementation for general substances should become available before this book is published. The results of the preliminary study of IRS elements for general substances nevertheless suggest that a new convection-bond element should be developed, similar to the IRS element but without the inertia. This conclusion does not apply to liquids.

One unique feature of THERMOSIM for non-thermodynamic systems which is of particular value to modeling and simulating electromechanical systems is its ability to deduce automatically the constitutive equations for nonlinear multiport compliances defined merely by expressions for their stored energy [\[7\]](#page-34-5). The Symbolic Math Toolbox of MATLAB is used.

The author is nearing the end of his career, and would welcome the interest of anyone who would like to investigate the possibility of contributing to THERMOSIM or eventually succeeding as its manager. One needn't be a thermodynamicist—the author is not—but have some interest in the subject.

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