On the Structural and the Abstract in My Compositional Work

Clarence Barlow

Abstract From 1959 to 1969 I composed music as most others do and have done—by direct transference from the imagination to a musical instrument (in my case the piano) and from there to a written score. During this period, I found myself relying increasingly on traditionally structured techniques such as canon, fugue, dodecaphony, serialism and electronics. In 1970 I was struck for the first time by a mathematical rule-based idea for an ensemble piece, which necessitated my learning to program a computer. Since then I have composed over fifty works (half my total output) with computer help—works for piano, organ, chamber ensemble, orchestra and electronics. Of these fifty-odd pieces, about half are partially and sometimes wholly based on abstract mathematical principles. This paper describes eight of these pieces or relevant sections of them in varying detail.

For reasons of space, I have omitted a system of quantified harmony and meter I developed in 1978, used in several pieces *(Çoğluotobüsişletmesi, Variazioni, documissa '87, Orchideæ Ordinariæ, Otodeblu, Talkmaster's Choice, Amaludus, Estudio Siete and Für Simon Jonassohn-Stein)* and have written about (see references). Examples of the algebraic formulae used in these pieces are illustrated here below in Fig. 1 without explanation.

$$\xi(N) = 2\sum_{r=1}^{\infty}\left(\frac{n_r(p_r-1)^2}{p_r}\right)$$

$$\Downarrow$$

$$\mathcal{H}(P,Q) = \frac{\operatorname{sgn}\left(\xi(Q)-\xi(P)\right)}{\xi(P)+\xi(Q)}$$

$$\text{If } p=2, \text{ then } \Psi_p(n) = p-n\,;$$

$$\text{otherwise if } n=p-1, \text{ then } \Psi_p(n) = \left\lfloor p/4 \right\rfloor$$

$$\text{or else } \Psi_p(n) = \left\lfloor q+2\sqrt{\frac{q+1}{p}} \right\rfloor$$

$$\Downarrow$$

$$\Psi_z(n) = \sum_{r=0}^{z-1}\left(\prod_{i=0}^{z-r-1} p_i \Psi_{p_{z-r}}\left(1+\left(\left\lfloor 1+\frac{(n-2) \bmod \prod_{j=1}^{z} p_j}{\prod_{k=0}^{r} p_{z+1-k}}\right\rfloor \bmod p_{z-r}\right)\right)\right)$$

Fig. 1 Formulae for quantified harmony (*left*) and meter (*right*)

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1 *Cheltrovype* (1968–71) for Cello, Trombone, Vibraphone and Percussion

I was given the task of writing a piece for this instrumentation, from which the title derives, by my teacher Bernd Alois Zimmermann (1918–1970). Completed only after his untimely death, it consists of six parts, each of which is characterized by a totally different compositional technique. In Part V, the music for the three melody instruments follow a probabilistic pitch distribution system based on an exponential curve, a sine curve and a transformed parabola.

For Part V, I imagined the cello starting on a repeated low open-string C_2, occasionally interspersing the D-flat above it, then later adding the D, the E-flat, the E and so on chromatically upwards through a range of three and a half octaves to the note F_5 on the top line of the treble staff. At the same time, while keeping the lowest note at C_2, the pitch centroid of the melody gradually rose, reaching the highest note F_5 at the end. Sometime after the start of the cello, the trombone would enter with a repeated C_3, following the same procedure and reaching the same high F together with the cello. Somewhat later than the trombone, the vibraphone would follow suit, starting on a repeated C_4 (Middle C) and ending with the cello and the trombone on the same high F.

Right from the start, I realized that this music could not be composed spontaneously but would have to be subject to a set of rules, finally formalized in the shape of the formulae and corresponding curves shown in Fig. 2 at left. The lowest pitch is seen to be fixed at the MIDI value 36 ($=C_2$) and for the highest value a half-period of a sine curve was chosen. The pitch centroid was determined by an exponential curve for the most frequent pitch, simultaneously marking the peak of a transformed parabola curve in the y-z plane, not shown here. In this plane, this "parabola" has the value zero a half step below the lowest and a half step above the highest pitch, reaching its maximum at the most frequent pitch.

For the cello part, I decided to generate 500 notes, for each of which the probability of every pitch in the 42-half-step range from C_2 to F_5 (totalling $500 \times 42 = 21{,}000$ values) was to be determined by the transformed parabola. I first tried to do this with logarithmic tables (the year was 1970 and there were no electronic calculators), but soon gave it up due to the time-consuming nature of the process. A second attempt with a 50-pound electric office calculator proved to be also very time-consuming. This is what led me to learn to program in Fortran at the computer center of Cologne University; the cello part was complete within a week of my starting the Fortran course in the form of a 500-page table of probabilities, one page for the choice of each note.

The actual resulting notes were picked by the use of random numbers –see the dots in Fig. 2 at left– and written as a score, seen in Fig. 2 at right. The process was repeated for the trombone and the vibraphone with 222 and 115 notes, respectively, the range being 30 and 18 half-steps. According to my sources, Part V of *Cheltrovype* seems to have been the earliest computer music score composed in Germany.

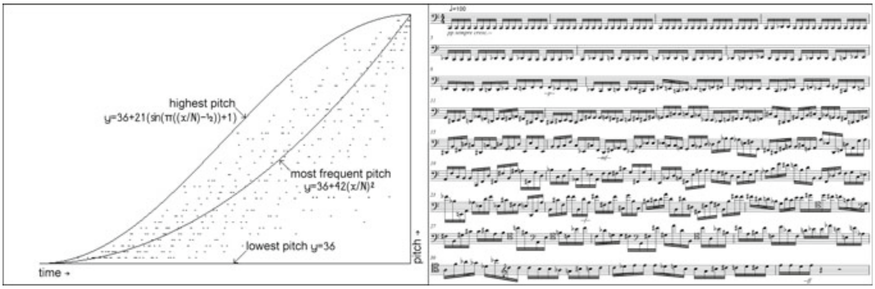

Fig. 2 Algebraic curves and formulae (*left*), score of cello part for Cheltrovype Part V (*right*)

2 *Sinophony II* (1969–72) for Eight-Channel Electronics

After having composed a four-channel analog electronic piece called *Sinophony* in 1970, consisting in the main of sine-tones (hence the title) but also containing noise bands, impulse-generated sounds and ring modulation, I decided in 1971 to compose a sequel consisting exclusively of sinusoids, not only as sound waves but also as form-shaping parameters, according to which theoretically infinitely many sine tones would move along predetermined sinusoidal paths in the domains of pitch, amplitude and duration. After fruitless attempts with the newly acquired ARP synthesizer in the electronic music studio of Cologne Music University (*Hochschule für Musik*), where I was a student, I drove two days to Stockholm, to the EMS Studio, where I worked at the PDP computer completely alone for two weeks during the Christmas period of 1972.

Figure 3 shows the function of the sine curve in shaping not only the sound wave, but its pitch, loudness and even its duration: in this last case the length of an event periodically increases and decreases within a time period fixed differently for various pitch groups in various tracks, the remainder of the period being occupied by silence.

Figure 4 shows a map of eight different tracks with time on the x axis, pitch on the y axis and the vertical width of each line reflecting the loudness. In some tracks the pitch remains constant (e.g. 1, 3 and 5), in which case individual pitches

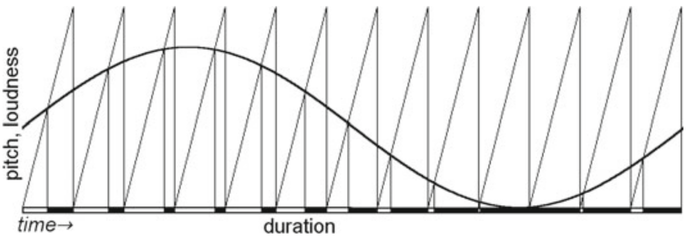

Fig. 3 A sine curve shaping the main parameters

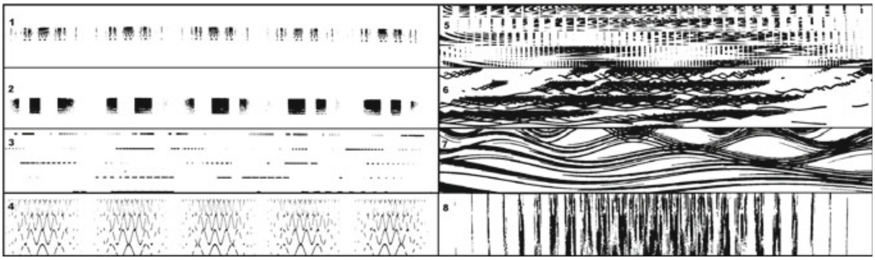

Fig. 4 A pitch-time-loudness map of the eight tracks of *Sinophony II*

are grouped in the form of an overtone series compressed to about $7/12$ths of the intervals of a normal harmonic series, i.e. the frequency of the nth partial relative to the fundamental f_1 in each group is given by $f_n = f_1 \cdot n^{\log(1.5)/\log(2)}$. Here the second partial is a perfect fifth above the first instead of an octave, all other intervals being compressed by the same factor. In track 7, each pitch group retains this relationship while undulating sinusoidally as a whole. In tracks 1 and 5 it is the loudness that undulates. Tracks 1 and 2 have only one pitch group each, tracks 3 and 4 only one sine tone per group and the remaining tracks have multiple groups with fundamentals spaced five perfect fifths apart and multiple sine tones in each. In tracks 1, 5 and 8, the duration undulates. Conceptually, *Sinophony II* has an infinite duration, frequency range and loudness ranging downwards from a fixed maximum value; for practical reasons, I factually generated about 800 sine tones within the range of 17 Hz to 17 KHz, 0 to −60 dB and with a total duration of 24’38”.

3 *Stochroma* (1972) for Solo Piano

In 1972 my composition teacher Karlheinz Stockhausen gave each of us in the class the task of writing a piano piece starting on the lowest note A, gradually increasing chromatically in range. I did not write the piece, but preferred to improvise on Stockhausen’s piano. He was not very impressed. Later I planned a conceptual piano piece in which pitch, loudness and duration are probabilistically determined, allowing duration and dynamic values to randomly and exponentially diverge (as powers of 0.5 and 2) from a central value to rare but great extremes (durations for instance range in seconds from the yocto to the yotta range and beyond in both directions). Figure 5 shows at left a short excerpt of the computer printout as sound number (there are 5000 sounds in total), pitches (German notation;‘----’ denotes silence), current range in half-steps above the lowest A, duration as multiples and divisions of powers of 2, and dynamics as degrees downwards from *fff* (=0). At right one sees a matching score on one of four systems notated in 2001 for an exhibition of conceptual art curated by composer Tom Johnson in the Queen Sofia Museum in Madrid.

Note the durations in the 2nd (4×2^{18} s) and 4th (17×2^{83} s) bars. Understandably, this piece has never been performed.

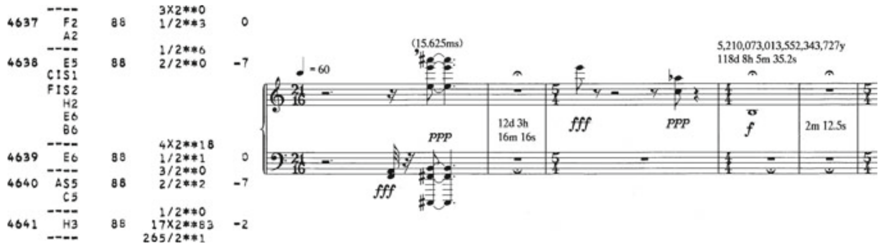

Fig. 5 *Stochroma*: part of page 267 of the 1972 computer printout (*left*) and the corresponding score (*right*)

4 *Bachanal* for Jim Tenney and Tom Johnson (1990) for Solo Piano

Once, discussing musically numerical issues with Tom Johnson in a Paris café, I told him of a discovery of mine: if the odd-numbered bits in a series of binary representations of the natural numbers were made negative, the resulting values would start not with 0, 1, 2, 3 ... but with 0, −1, +2, +1 .... This sequence expressed in half-steps corresponds to the notes of the B-A-C-H theme used ever since the composer J. S. Bach himself used it ("B" in German means B-flat in English and "H" means B-natural). But more than that, the next four values, instead of 5, 6, 7, 8, would now be −4, −5, −2, −3, again B-A-C-H transposed down a major 3rd. The next four (+8, +7, +10, +9) are again B-A-C-H transposed up 8 half-steps or two major 3rds. And the next four (+4, +3, +6, +5) are again B-A-C-H transposed up a major 3rd. These four sets of major 3rd transpositions (0, −1, +2, +1) are again in the form B-A-C-H. And so on. I used this phenomenon to generate *Bachanal* ("B-A-C-H analysis") while exponentially accelerating the process in such a way that the tempo of the higher-level transpositions (major 3rds, 10ths, 40ths etc.), bear a relationship to that of the first four notes. Figure 6 shows at left the transformation of the first 40 natural numbers into this "odd-bit negative" form, and also a pitch-time map of the accelerated notes.

Figure 7 shows the piece as a score.

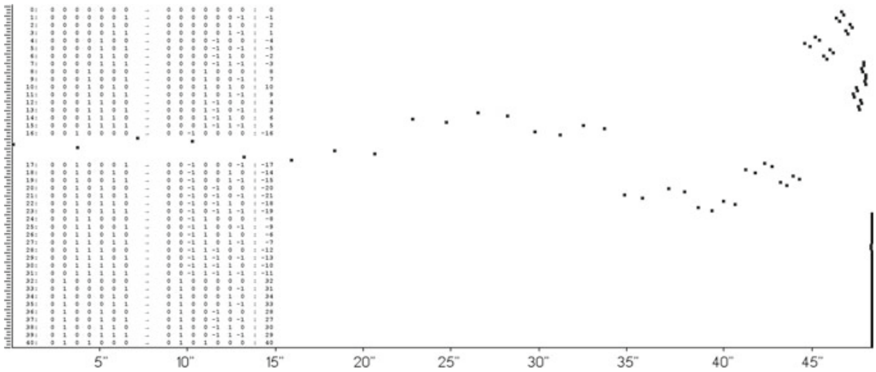

Fig. 6 Exemplified conversion of regular binary numbers into "odd-bit negatives", with a pitch-time map

Fig. 7 Score of *Bachanal for Jim Tenney and Tom Johnson*

5 *Piano Concerto #2* (1961–1998) for Piano and Orchestra

37 years lay between when I began and ended work on this piece. Its style in 1961, when I was 15, resembled European classical music of around 1800, whereas in 1963 its style had advanced to 1900, resembling Rachmaninov. By 1965, about 8 min from the beginning and 3 min before the end were complete. However my style in other pieces developed further, through Prokofiev and Hindemith to Schoenberg and Webern: at a loss as to how to continue the work, I laid it temporarily aside.

In 1975, finally notating that which had been done, I noticed an accelerando I had been unaware of between 3'48" (at MM 60) and 7'49" (at MM 119) where the music broke off, and another of the same rate (0.289%/s) but a much higher speed for about a minute after the recommencement of the music (MM 244–MM288). In 1998 I completed the piece, continuing the accelerando across the unfinished portion, which according to formulae I developed in 1975 for an accelerando (see Fig. 8), would be from bars 145 to 322, lasting 4'4".

$$s = SQ^{\frac{t}{T}}$$

$$n = \frac{TS(Q^{\frac{t}{T}}-1)}{\ln(Q)}$$

$$T = \frac{N\ln(Q)}{(S'-S)}$$

$$t = \frac{T\ln(s/S)}{\ln(Q)}$$

$$= \frac{T\ln((n\ln(Q)/TS)+1)}{\ln(Q)}$$

where S = initial tempo
S' = final tempo
$Q = S'/S$
T = total time
t = current time
[i.e. time at tempo s
or at beat n - see below]
s = tempo at time t
n = beat at time t
N = total number of beats
$\ln(x)$ = natural logarithm of x

Fig. 8 Formulae for acceleration/deceleration

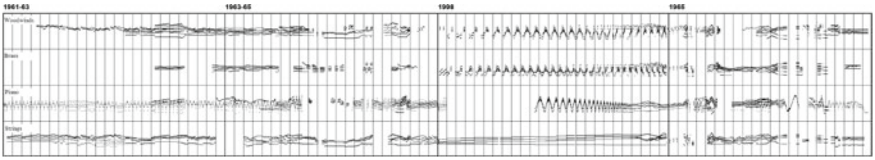

Fig. 9 Map of pitch (y) against time (x) for *Piano Concerto #2*

In this new section, different groups of instruments perform simultaneous but independent rising *accelerandi* and falling *decelerandi*, the pitch and the rhythm deriving from the shape of an inverted cosine. Figure 9 shows a pitch-time map of the piece. Notice the distinctly different shapes in the “1998” section.

6 *Les Ciseaux de Tom Johnson* (1998) for Chamber Ensemble

Written to celebrate Tom Johnson’s 60th birthday, this piece is based on the successive positions of six sets of three points derived from the name of the dedicatee, each set moving along a differently sized circle (see Fig. 10).

The letters T O M J O HN S O N were first plotted from left to right on an alphabetically upwards-reaching uniform grid. Six arbitrarily chosen three-letter sets TOM, SOJ, JNS, SON, MJH and OOO were then each transected by a circle, one of them a horizontal straight line through the OOO set. Next, each set was made to rotate in an anticlockwise manner along its circle by a distance equal to the segment of the circumference of the smallest circle SON subtending an angle of 4°. All sets move concurrently 90 times before SON returns to its original position. Each shifted

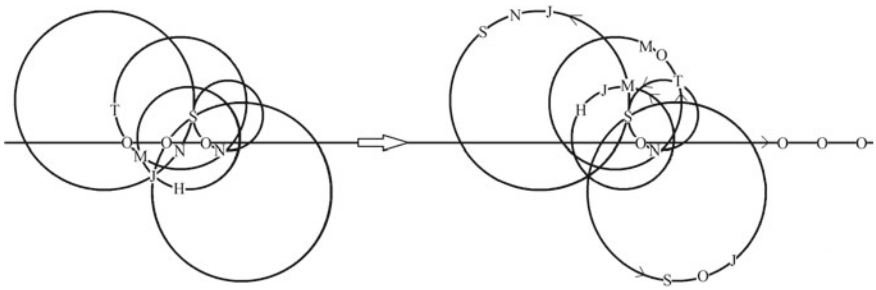

Fig. 10 Rotation states 1 (*left*) and 91 (*right*) of six three-letter sets along *six circles*

Fig. 11 The opening bars of *Les Ciseaux* de Tom Johnson

state is then scanned by a vertical line from left to right representing time, the letters it transected rendered by their height as pitch, yielding a total of 91 “mini-scores”. Figure 10 shows states 1 (left) and 91 (right). These scores are then overlapped such that the horizontal (time) distance between one OOO set and the next equals the distance between two successive ‘O’s in the set. The result is shown in Fig. 11 as a score excerpt comprising states 1, 2, and part of 3 – the repeated Middle Cs derived from the OOO set. The title, literally “the scissors of Tom Johnson” is a reference to his then age (“six-O”) and to the six O-shaped circles.

7 *“...or a Cherish’d Bard...”* (1999) for Solo Piano

This piece was written to celebrate the 50th birthday of the pianist Deborah Richards (who by the way premiered my *Piano Concerto #2* with the Icelandic Symphony Orchestra in Reykjavik in 2002). First, the letters DEB and AH were interpreted as German-named pitches and as hexadecimal numbers for the rhythms, yielding an infinitely long chain each, as shown in Fig. 12.

Fig. 12 DEB and AH pitch and rhythm chains, the basic material of *"...or a cherish'd bard..."*

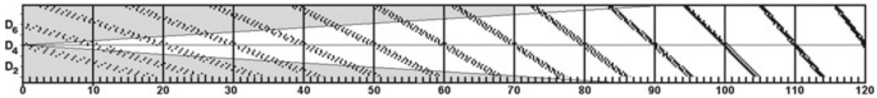

Fig. 13 Pitch-time map showing every tenth DEB chain in *"...or a cherish'd bard..."*

Fig. 14 Bar 93 of *"...or a cherish'd bard..."*

These chains were then repeated at a constant mutual time distance at their midpoints D4 and C♯4 respectively, but with a successively increasing gradient – see Fig. 13, in which every tenth DEB chain is shown for the full 120 bars over the full piano range. Additionally, a wedge-like filter encompasses an increasing number of pitches, as seen in the non-grey area of Fig. 13.

Finally, the probability that a note of the DEB chain is chosen for the piece was made to decrease continuously from 100 to 0% over the duration of the piece, while the complementary AH-chain probability increased from 0 to 100%, i.e. each note is taken from the DEB or AH chain. Since each chain derives from a different whole-tone scale, the music is whole-tone at the beginning and end, but chromatic in the middle. The title is an anagram of the dedicatee's name. Figure 14 shows bar 93 of the score with diamond-shaped note heads an octave higher (treble) or lower (bass) than written.

8 *Approximating Pi* (2007) for up to 16 Channels of Electronics

The Madhava–Leibniz converging series for the constant π begins thus:

$$\pi = 4\left(1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \tfrac{1}{9} - \ldots\right).$$

Figure 15 shows convergences #1 to #10 (left) and #29,991 to #30,000 (right) to 10 decimal places, with which the 4 billionth convergence finally reaches the correct ten places of π.

For this piece, each convergence is allocated a time window of 5040 samples (twice the lowest common multiple of the numbers 1–10), in which ten square wave partials of frequencies $8\frac{3}{4}n$ Hz and basic amplitude $2^\wedge d_n$ are set up, '$8\frac{3}{4}$' deriving from the 5040 samples, 'n' being the partial number and 'd_n' the nth digit in the convergence's decimal representation; e.g. for '3.141592654', the ten partials' basic amplitudes are 2^3, 2^1, 2^4, 2^1, 2^5, 2^9 etc., thereafter rescaled by the arbitrary sawtooth-spectral factor $2\pi/n$, where n is still the partial number. The convergences stabilize the digits from left to right to a value approaching π, the resultant timbre moving from

1	4.0000000000	29991	3.1411104826
2	2.6666666667	29992	3.1418308139
3	3.4666666667	29993	3.1411102231
4	2.8952380952	29994	3.1418310737
5	3.3396825397	29995	3.1411099631
6	2.9760461761	29996	3.1418313338
7	3.2837384837	29997	3.1411097029
8	3.0170718171	29998	3.1418315942
9	3.2523659347	29999	3.1411094422
10	3.0418396189	30000	3.1418318551

Fig. 15 Some π convergences (Madhava–Leibniz)

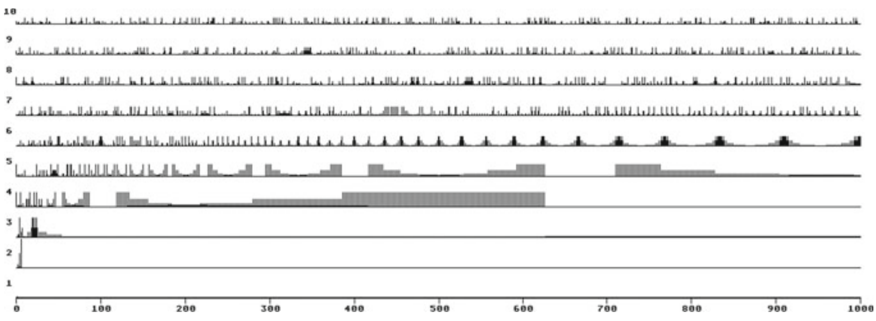

Fig. 16 The first 1000 convergences of the Madhava–Leibniz series as powers-of-2 spectral amplitudes

turbulence to constancy over a period of $4 \times 10^9 \times 5040 = 20.16 \times 10^{12}$ samples or about $14\frac{1}{2}$ years. The installation can be pitch-shifted (by sample dropping) and/or time-truncated. The fundamentals of the sixteen sound channels are transposed from $8\frac{3}{4}$ Hz to frequencies ranging from 9 to 402 times higher. Different versions with 2, 5, 8 and 16 channels have been realized, with durations ranging from about 8 to 74 min. Figure 16 shows the first 1000 convergences as spectral amplitudes in *Approximating Pi*.

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