# Proportion, Perception, Speculation: Relationship Between Numbers and Music in the Construction of a Contemporary Pythagoreanism

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**Abstract** This investigation is a departure point for understanding what Pythagoreanism can mean today, how can harmony be conceived at several time scales and what might a hierarchical model of form together with an algebra of perception entail for music composition. The study of qualitative aspects of music through mathematics is made by taking James Tenney's theory of musical form together with Alain Badiou's 'objective phenomenology' in order to imagine new ways of composing music.

## 1 Qualitative Numbers

The tradition commonly ascribed to as Pythagoreanism can refer to various doctrines, groups of people, disciplines and genealogies of research lead by common problems. Of special interest for contemporary musical and harmonic research is the relationship between perceptual qualities and numbers. From this standpoint, the tradition that bears this name does not begin in ancient Greece nor is it limited to a single culture or lineage, going back as far as we know through Egypt, Mesopotamia, India, China, passing also through Native American cultures as well as going forward through Semitic cultures, Scholastic philosophy and further on to mainstream modernity and involving musicians, philosophers, mathematicians and other kinds of inventors and eccentric characters not limited, as is commonly portrayed, to a single gender.

It is interesting, both as inspiration and point of departure, to think what a renewed Pythagoreanism might involve. Music composition might seem like a natural terrain for this to happen, a position that can preserve the speculative hallmark of this lineage due to its synthetic and artistic objectives, while also dealing with some of the problems that are commonly associated with this stance. Firstly, there is no interest in the mystical or sectarian facets typified by the transmigration of souls and the so-called school of *akousmatikoi*. There is also the misconstrued image of Pythagorean

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movements as expressing eschatological mentalities of intellectual (male) elites, a sociological prejudice which need not be an issue in this context. The main difficulty, however, is the danger of overextending claims based on contingent aspects of mathematical models, leading to unwarranted conclusions. The topics must therefore be approached carefully and with a pinch of skepticism to avoid extravagant speculation, while at the same time taking the ideas at the heart of scientific Pythagoreanism seriously. This aspect is embodied by the other Pythagorean school, that of the *mathematikoi*, epitomized by Archytas of Tarentum and particularly by the science of harmonics, that branch of philosophy that showed that sensible (audible) nature can be cognized through mathematical means ('means' acquiring here a double sense of 'procedure' as well as mathematical partitions and averages) [1].

In particular, to contemporize harmonics involves confronting a problem that afflicts Pythagoreanism from the inside, namely, the status of the disproportions that resist incorporation into mathematical notation. As illustrated by the discovery of the incommensurability of irrational numbers, there are aspects of reality which resist incorporation into quantitative units or ciphers, exceeding laws and explanations, be they audible but not recordable phenomena, mathematical 'monsters', logical paradoxes, etc.[1] Exceptions have always been part of mathematics and still today apparently trivial problems from arithmetic quickly lead to the frontiers of knowledge. I don't believe creativity or the lived aspect of musical experience can be mastered by mathematical models, nor is that the main reason for delving into the subject matter. In contemporary mathematics numbers are far less central or foundational than abstract structures, processes and articulations between spaces, so the conception of quantity and the problem of the 'unspeakable' may acquire new perspectives. Furthermore, there is no desire to tame or integrate these residues into ever more complex mathematics (although that happens in certain cases) but to acknowledge them as irreducible marks of contingency, as productive gaps, without that implying the collapse of the orientational role of the whole edifice, which need not be totalizing in its ambit(ion).

The transcription between the sensible and intelligible realms carries a fascination that persists, after centuries of thought and progress in science, in the process of abstraction, the measurement of qualities, the ability to go beyond what is intuitive and the aesthetic implications of formalization. Instead of the Aristotelian definition of Pythagoreanism as the principle that everything is governed by number, i.e., being *is* number, our focus is on the principle 'number is a bridge between matter and psyche' [3]. Other features are also relevant for today's harmonics such as micro-macro relationships or the string as a model, an important invariant throughout Pythagoreanism which provides links between the continuous and the discrete, between mathematics, physics and music, as well as with theories of perception. The spirit behind the proposal of a rehabilitation of Pythagoreanism necessarily commits us to a rethinking what sonorous number can mean in music and the openings this might bring to composition.

[1] For an account of these problems and their history see [2].

## 2 Harmonic Duality

Harmonic materials in music have two independent but intertwined aspects operating simultaneously: proportionality and pitch-distance. The former relates to intervallic 'characters', while the latter concerns features such as 'high', 'low', 'bright', 'dull', and corresponds to timbre, 'not timbre in the sense of spectrum, but timbre in the sense of regular pitch perception, coloured' [4]. The proportional facet comprises relations between whole numbers, concerns fundamental pitches and does not take timbre and register into account. The timbral aspect, on the other hand, involves register, spectral constitution and its main qualitative effect is sensory dissonance. Harmonicity, which does not always coincide with consonance, is proportionality's main perceptual quality.

These two aspects are entangled and their prominence and balance differs according to musical styles, performance practices, tunings, timbres and so forth.[2] They have a parallel in the division between mathematical and empirical schools in Greek harmonics as well as with the dichotomy between the discrete (arithmetic proportions, the Pythagorean approach) and the continuous (geometric pitch-distance, the Aristoxenian approach). This relationship between music and mathematics goes in two directions, as when integer harmonic means are discovered as solutions to musical problems, and, conversely, when properties of mathematical objects arise in sensory qualities, as it happens, for example, when prime numbers are understood as generating the fundamental types of harmonic intervals.

The link between numbers and sounds occurs in connection with measurement. Proportionality is indirect, having features that lie beyond the senses, pertaining to the intellect and mediated by experimental apparatuses (such as the monochord). It is empirical more than merely sensory, as is the case with pitch distance, where the relation is directly phenomenal. I agree with Michael Pisaro when he states that "perception tends to make a continuum out of the world: (our ears) are better at finding continuity than at finding fissures" [7]. Information from the world is 'folded' by perception into a qualitative immediate continuum which can be understood in terms of log morphisms that compress extensive physical sound quanta into intensive qualia (with different transfer functions for sound parameters such as pitch, intensity, timbre, time integration and so on). Proportionality can be modeled in the multidimensional harmonic space of Euler tonal lattices consisting of discrete nodes that map to linear pitch space in such ways that what is near in harmonic space does not coincide with what is near in pitch-distance space.[3] In terms of numerical structures, proportions are enfoldings of $\mathbb{N}$ into $\mathbb{Q}$, while timbre is a folding of the multidimensional space of waves inside $\mathbb{R}$. The log morphisms mediating these foldings switch the algebra of intervals from multiplicative ratios to additive distances, exchanging the identity element from 1 to 0. They wrap diverse multiples into single magnitudes.

[2] For more details on this entanglement and its compositional uses see [5]. The seminal research on the two facets of harmony initially took shape in the fascinating book [6].

[3] An octave or a fifth, for instance, lie close to a given pitch in harmonic space, while in 12 tone equal temperament, a semitone, which is harmonically relatively far, would be the nearest interval.

This harmonic dichotomy can also be seen as a question of orthogonality. Fourier analysis relates multidimensionality and multiple degrees of freedom to linear information: proportionality as multi-dimensionality is projected into linear pitch-height. Harmonic space projects into pitch space in the same way as a waveform, which can be depicted as a surface on the $\mathbb{C}$ plane in terms of vectors, phases, poles, etc., folds into a single pitch with timbre. Timbre space operates in an intuitionist way, with no excluded third and no order relation. By way of the Fourier transform, n-dimensional orthogonality is projected into a linear scalar number (the fundamental) with a timbral qualia (represented by the coefficients of its partials).

A question arises: can this duality that belongs to intervallic perception be extended past the timescale of immediate harmony and beyond the attribute of pitch?

## 3 Time Scales

The present interest in mapping out connections and analogies between mathematics and perceptual fields in music follows composer James Tenney's theory of musical form as a nested hierarchy of polyphonies, with a discrete/continuous polarity within each level: a morphological, continuous, facet of audible parametric contours in time, and a structural, discontinuous aspect of relations between parts and between parts and wholes (see [8, 9]). A mereology of perceptual fields, of interlocking objects, where at each scale there is an assimilation of differences under determined identities which are different from those at other scales. The underpinnings for realizing this synthesis are prompted by philosopher Alain Badiou's *objective phenomenology*, a particular philosophical reading of set-theoretical and categorical ideas (see [10, 11]), where set theory provides the *noumenal* material for the construction of phenomenal *logics* which govern differentiations into orders and degrees of intensities.

Tenney approached the question of form and content through these temporal scales, where forms at a given level become the content at the next higher one. In terms of sets, the elements of a level are composed from the powerset of elements at its next lower domain, *urelements* which become the (quantitative-multiple) forms that coalesce into (qualitative-unitary) matter at a higher scale. The *logic* at a particular level determines the nuances, qualities and thresholds of appearing by means of an *appearance* function that attributes to each pair of elements of the underlying constituting set, an element of a set whose elements represent degrees of relation. Each level operates through a specific *protocol of differentiation* that captures the multiples that appear in music through a network of differences and identities.

Perceptual constraints make the relative scale of each temporal domain have a specific quality to it, yielding three main strata, notwithstanding the fact that each piece of music produces its own context with any number of extra intermediate levels. These main qualities of levels correspond to the micro, meso and macro scales. There is a strong *Gestalt* of the clang at the micro to meso level, its qualities resulting from the contracting of vibrations into a single qualitative state of timbre, color, note, etc.

The meso level of sequences involves comparative memory, and the macro level of piece constitutes the architectural form of a work.

Levels are inaccessible among each other, finitude is relative to a model (in reference to Skolem's *finitization of infinity*). Urelements are always finite with respect to their powerset, forming a chain which in principle goes both up and down indefinitely. Material is extensional, perception intensional at the next higher strata, which marks the horizon of inaccessibility, the potential infinity from the point of view interior to this level, outside of which it is intensionally accessed. What is quantitative becomes qualitative 'at the limit', when a finite, countable set is seen from the 'point of view' of its inaccessible powerset.

What do these levels suggest from the different perspectives of music, psychology and mathematics?

## 4 Levels of Perception

The following table shows some categories that help us understand music in terms of levels. Between content and context lies the principal playground for music. There are the scales of elements, clangs, sequences and piece according to Tenney's view, next to which we can see musical notions and some continuous and discrete qualitative kinds parallel to them. The highest level of context has to do both with the general atmosphere and the space of the situation that surrounds a piece, as well as to the style and genre of groups of pieces. Analogous to atmosphere is some kind of macro structure associated with long term pieces as well as to sociological aspects of music making. Below follow form, morphology, profile and timbre in continuous formations, as well as architecture, structure, pattern and proportion in discreteness. These categories are not fixed, as it is difficult to imagine sufficiently general concepts that adapt to all kinds of music, which after sufficient reflection can be replaced by other categories that can better trace them. This table should be seen as a starting point for further research, both theoretical and musical (Fig. 1).

The last two columns show psychological processes corresponding to each level and next to it are their comparable relevant mathematical fields: in the first level we find the tools of harmony (harmonic arithmetic, means, harmonic space and metrics, etc.) that can also relate to networks of proportionality in durations; there is also the tools of logarithmic pitch with its compressions, expansions and rotations as well as the 'smooth space' of non metric durations. To give an example of how these discontinuous/continuous polarities rely on each other while being epistemically independent, think how the same continuous morphological profile can convey very different perceptual information depending on which subjacent discrete pitch grids are used to realize them.

The next level involves combinatorics (groups, graphs, knots, etc.) of patterns of units integrated from the lowest level. The corresponding continuous field could

| scale | level | music | quality continuous/discrete | | psyche | math |
|---|---|---|---|---|---|---|
| micro | ***content*** | | | | subliminal stimulation | |
| | ***element*** | material | timbre | proportion | sensory perception | $\mathbb{Q} \leftrightarrow \mathbb{R}$, harmonic space, Fourier |
| meso | ***clang*** | rhythm / texture | profile | pattern | proprioception / echoic memory | combinatorics / math morphology |
| | ***sequence*** | method / technique | morphology | structure | working memory | processes / networks |
| macro | ***piece*** | aesthetic model / narrative | form / drama | unity / architecture | episodic memory | interactions / schemes |
| | ***context*** | space/place styles/genres | atmosphere | global/social structure | '*autonoetic*' memory / social memory | |

**Fig. 1** Table of time scales according to different perspectives. The *level* column contains Tenney's categories, the next one general musical ones, followed by qualitative musical aspects (divided into continuous/discrete), psychological processes and mathematical structures that could pertain to each level

relate to the mathematics of morphology and the continuous mutability of profiles.[4] At the next higher level, the fields we suggest have to do with setting up processes and networks of relations, interactions of already given musical forms that might be studied through categories and morphisms, comprehending, at a higher level of abstraction, relations among the structures and parametric spaces that have been presented at lower levels. This can go further into the macro levels of drama and narrative organization (both directional and non directional) in consonance with the overall aesthetic models that pertain to atmosphere and style.[5]

Statistical measures at each level can also explain and be useful to generate the distribution of musical structures. Tenney proposes the idea of *ergodic* form to explain non directional morphologies while directed processes can be grasped in terms of parametric densities and limits.

Musical material should be understood as a hypothesis, not just as an inert 'stuff' manipulated arbitrarily by forces of thought, but also having a say in the process of creation, getting to kick back and impose constraints. Abstraction is a back and forth process where thought, imagination and matter (however immaterial musical and sonorous materials might seem) influence each other. Matter is not pre-constituted but must be understood as information is gathered while it is manipulated, thereby making up the material during the process: its constructibility becomes isomorphic

[4] In the wake of Tenney, two of his colleagues have proceeded to study each of these aspects in turn through morphological [12] and structural metrics [13].

[5] Some suggestions in this direction are pursued in [14].

to its understanding. This approach to creativity is experimental in the sense that it requires an empirical intervention and is not reduced to pure speculation or the imposition of abstract ideas on passive, maleable material. Art can be considered a form of engineering,[6] and the approach we take by means of an algebraic phenomenology of sorts means that art can also reverse-engineer the conditions of its reception and production.

## 5 Objective Phenomenology

Tenney's theory comes from phenomenology and Badiou sets up a phenomenology based on the theory of locales, providing tools for complementing and going beyond the *Gestalt* principles of similarity, continuation, closure, proximity as well as the figure/ground dichotomy. Phenomenology, as the theory of appearing and objectivity, concerns relations between qualitative differences and an order structure that provides the unity through which a manifold is unified as an object. When a thing (a set) is localized in a world, this is because the elements of the set are inscribed within a distribution of degrees over all the differences that appear in this world, the *disposition of the infinite hues of a concrete world* ([16], p. 51). An object is a multiple associated with an evaluation of the identities and differences immanent to it. There are many types of orders and, consequently, many possibilities for the logical organization of worlds. Two worlds with the same things can be absolutely different from each other because their transcendental gradings are different. There is always in a world a certain number of limits to appearing's intensity.

Lets give a short summary of Badiou's theory[7]:

- Beings can be determined in their abstract form by the pure multiple of set theory, as the infinite composition of elements with a metaphysical stopping point at the void set.
- Elements compose localized entities in relation to each other within situations or 'worlds'.
- Beings can appear in different situations while being ontologically the same; a multiple co-belongs in general to many worlds.
- The appearance of an entity implies differences with itself and with other entities through degrees of gradation: a relational network.
- A transcendental is the operator set that allows giving meaning to the 'more or less' of identities and differences in a determinate world.
- The scale of evaluation of appearing depends on the situation. These degrees do not depend on any subject or consciousness. There is no privileged frame of reference (listener, performer, composer, for instance). The conditions for experience of a subject are not pre-given, there is no subjective receptivity nor constitution because

[6] For more on this interesting topic of abstraction and material, see [15].

[7] It is beyond the scope of this article to provide a comprehensive account of Badiou's theory. For more information see [10, 11, 16].

the transcendental is an intrinsic constitution of being as it belongs to the same world as the appearances.
- Transcendentals are local, there are many of them, 'difference is differentiated'.

The algebraic structure of a world, corresponding to a Heyting algebra,[8] is the following:

- The set $A$ that ontologically subtends a situation, the 'material' set.
- A transcendental $T$, the set of degrees of appearance.
- Degrees of identity $\mathbf{Id}(\alpha, \beta) = p$; $\alpha, \beta \in A$, $p \in T$.
- Order relation, $\leq$, obeying reflexive, transitive and antisymmetric axioms.
- Minimal degree, $\mu$.
- Conjunction, $\cap$.
- Envelope, $\sum B$; regions defined by a predicate over intensities of subsets $B$ of $A$.
- $\cap$ is distributive with regard to $\sum$.
- Dependence of degrees, $p \Rightarrow q$; the degree of connection between two intensities.
- Every degree admits a reverse and $p \cap \neg p = \mu$.
- Maximal degree, $M = \neg\mu$.
- The phenomenon of $a$ relative to $A$:
  $\Phi(a/A) = \{a, [\mathbf{Id}(a, x_1), \mathbf{Id}(a, x_2), \ldots, \mathbf{Id}(a, x_\alpha), \ldots]/x_\alpha \in A\}$; the set of $a$ and the degrees of appearing of all $x$'s which co-appear with $a$ in $A$.
- Degrees of existence, $\mathbf{E}x = \mathbf{Id}(x, x)$; the extent to which $x$ appears in a world.
- Phenomenal components, $\pi(x) = p$; identity function with respect to a fixed degree.
- Atoms of appearing (phenomenal identities).
- Objects, $(A, \mathbf{Id})$, a support set together with a transcendental indexing.
  - Localizations, $a\lceil p = \pi(x) \cap p$; a local decomposition from the spectrum of intensities.
  - Compatibility, $a \ddagger b$, $a\lceil \mathbf{E}b = b\lceil \mathbf{E}a$; atomic equality through reciprocal localization on existences: $a$ and $b$ are compatible if they are in 'the same zone of existence'.
- Proper inexistent of an object, $\varnothing_A$; $a \in A$ inexists if $\mathbf{E}a = \mu$. An element $a$ of an object is said to be its inexistent if its being is attested but its existence is not. Every object admits of one (and only one) inexistent.
- Transcendental functor, $\mathbf{F}A(p) = \{x/x \in A \text{ and } \mathbf{E}x = p\}$.

There is a lot of work to do in order to think through what this structure can mean musically, both from the point of view of understanding perceptual cues as well as to generate new ones.

Maybe the transcendental functor, which marks the territories or the 'retroaction of appearing on being' ([11], p. 221), associating to every element $p$ of $T$ that part of $A$ composed of $x$'s such that $\mathbf{E}x = p$, could be interpreted as a connection from

[8] To further our investigations we might turn away from Badiou towards more mathematically oriented literature. Also, Badiou's theory of change and the Event is not directly relevant to our purposes (although it is not incompatible either).

the percepts to the conditions of production of those percepts, between listener and producer, referring the phenomenal realm back to the material facts of music, an operation common in music composition. On the other hand, the inexistent, which is local and 'testifies, in the sphere of appearance, for the contingency of being-there' (*ibid*, p. 324), i.e., is present materially but does not appear, is a measure for what can happen to a world, and can thus become pivotal in delineating the unexpected, contingent and open, so this approach is compatible with musical indeterminacy.[9]

Especially interesting is the positing of atoms of appearance by way of language propositions which function conceptually at a high level of aesthetic abstraction, also showing how this model can be adapted to simple settings which do not need insinuate any explicit use of mathematics. Atoms can also be defined to posit appearances by way of arbitrary functions, sampled material, descriptions rather than definitions, both real and fictional (recordings, data for sonification, random distributions, patterns, algorithms in general, translations from other media or disciplines, and so on). Following Zalamea [17], the former kind can be said to be *eidal* in mode, while the latter are *quidditial*. Any degree of mixture between eidal and quiddital modes is possible.

It is not immediately obvious what the reverse of a particular sound may be. It has to do with absence but not necessarily with silence. In a sense it is all that begins when a thing (a sound) 'ends', i.e., its form (and in sound this implies abstracting time spatially along with other attributes). It is what contains the sound, the region of the world that envelops it. There are many possible concrete ways in which reverses can happen in music, for example in remainder sonorities, as in Alvin Lucier's *Slices* for cello and orchestra (2007), where a large orchestral cluster is punctuated note by note with 'holes' by the solo cello.

Envelopes, conjunctions, dependencies, phenomenal components, atoms, compatibilities and inexistents have to be imagined for music in general as well as for specific pieces.

## 6 Analysis and Synthesis

Badiou's theory does not explicitly deal with mereologies and here the meeting with Tenney might prove fruitful. Atoms and appearance functions can be 'plugged in' to each temporal scale, either as perceptual transfer functions or as the extension of concepts, properties or metaphors the give rise to gestalt-like forms over a ground. They can be defined both from the top-down and from the bottom-up, in any order and at any level, engendering hierarchies whose structure can change over time. These functions determine what musical variables are to be taken into account and the arbitrariness of it means that they do not have to be tied to traditional musical parameters, can appear from a variety of perspectives (listener, performer, author, situation, etc.) and incorporate multi-modal media (other 'senses': visual, performa-

[9] I'm interested in establishing collaborations both with musimathicians as well as with mathemusicians in order to find out what can be made of these ideas.

tive, theatrical, etc.). Crossing the natural/cultural distinction, musical experiences can be launched that have political consequences in that they are driving forces for a subject which is not presupposed.

It is a question of dual eidal ascents and quiddital descents: the expansion and dilation of 'the entanglement between thought and matter, the intelligible and the sensible' ([15], p. 19) (synthesis), as well as the contraction of thought to a point (what is it like to 'be' a grain of sound?) (analysis).[10] The creative process becomes a coming and going between real and ideal as diverse forms of transit between multiple material and conceptual strata, where the notion of parameter becomes enlarged to a perceptual field, a phase space or manifold with intrinsic properties (curvature, orientability, symmetries, connectivities boundaries, etc.).

Acknowledging and intervening in the interweaving between the continuous and the discrete can impart more depth and dimensionality to these parameter fields. Inner periodicities encode within degrees of intensity 'hues' that stand out from other saliences; contours can have qualities that go beyond the up/down and long/short dimensions of morphology, adding breadth to spatial relationships and alluding to the vertical from within the horizontal.

As much as this algebra permits thinking music and perception in terms of mathematical structures, it can also be used inversely to imagine what these structures might imply as applied to music: to imagine transcendentals with arbitrary limits and conditions of individuation and appearance going against the grain of intuition. This is the speculative aspect, where instead of modeling nature and music mathematically, new musical thresholds can be imagined out of the mathematical structures that prescribe intersections involving quanta and qualia indiscriminately.

Finally, there is also the perspective from the totality, where the reciprocal actions between levels and their morphologies, in loops between qualities and time scales, take place to produce something which is more than the sum of the parts: a harmony.

## References

1. Barker, A.: Harmonics in Classical Greece. Cambridge University Press, Cambridge (2007)
2. Heller-Roazen, D.: The Fifth Hammer. Pythagoras and the Disharmony of the World, Zone Books, New York
3. Watkins, M.: Prime evolution (interview). Collapse **1**(1), 93–189 (2006)
4. Gilmore, B.: Clarence Barlow interviewed by Bob Gilmore, Amsterdam, 1st August 2007, Paris Atlantic Magazine (2015). www.paristransatlantic.com/magazine/interviews/barlow.html
5. Lach, J.S.: Harmonic Duality. From interval ratios and pitch distance to spectra and sensory dissonance, Leiden University (2012)
6. Barlow, C.: Bus Journey to Parametron. Feedback Papers, Cologne (1981)
7. Pisaro, M.: Continuum Unbound (notes to CD). Gravity Wave, Jersey City (2014)
8. Tenney, J.: Form in 20th century music. In: Vinton, J. (ed.) Dictionary of Contemporary Music. E.P. Dutton, New York (1974)

[10] "As the thought experiment is fully immersed within the material system, it permits the abstracting force to assume material behaviors and new generative schema otherwise unavailable to an isolated account of thought trapped in naive intuitions of itself."[15], p. 24.

9. Tenney, J.: Meta-Hodos and META Meta-Hodos. Frog Peak Music, Lebanon (1988)
10. Badiou, A.: Mathematics of the transcendental. Bloomsbury Academic, New York (2014)
11. Badiou, A.: Logics of Worlds: Being and Event. Bloomsbury Academic, New York (2014)
12. Polansky, L.: Morphological metrics. J. New Music Res. **25**(4), 289–368 (1986)
13. Winter, M.: Structural Metrics: An Epistemology. University of California, Santa Barbara (2010)
14. Vriezen, S.: Action Time, The Ear Reader (2014)
15. Negarestani, R.: Torture Concrete. Sequence Press, New York (2014)
16. Badiou, A.: Second Manifesto for Philosophy, Polity (2011)
17. Zalamea, F.: Filosofía Sintética de las Matemáticas Contemporáneas, Editorial Universidad Nacional de Colombia, Bogotá (2009)