Predictability and Robustness in the Manipulation of Dynamically Complex Objects

Dagmar Sternad and Christopher J. Hasson

Abstract Manipulation of complex objects and tools is a hallmark of many activities of daily living, but how the human neuromotor control system interacts with such objects is not well understood. Even the seemingly simple task of transporting a cup of coffee without spilling creates complex interaction forces that humans need to compensate for. Predicting the behavior of an underactuated object with nonlinear fluid dynamics based on an internal model appears daunting. Hence, this research tests the hypothesis that humans learn strategies that make interactions predictable and robust to inaccuracies in neural representations of object dynamics. The task of moving a cup of coffee is modeled with a cart-and-pendulum system that is rendered in a virtual environment, where subjects interact with a virtual cup with a rolling ball inside using a robotic manipulandum. To gain insight into human control strategies, we operationalize predictability and robustness to permit quantitative theory-based assessment. Predictability is quantified by the mutual information between the applied force and the object dynamics; robustness is quantified by the energy margin away from failure. Three studies are reviewed that show how with practice subjects develop movement strategies that are predictable and robust. Alternative criteria, common for free movement, such as maximization of smoothness and minimization of force, do not account for the observed data. As manual dexterity is compromised in many individuals with neurological disorders, the experimental paradigm and its analyses are a promising platform to gain insights into neurological diseases, such as dystonia and multiple sclerosis, as well as healthy aging.

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Introduction

Everyday life is full of actions that involve interaction with objects. Grasping and lifting a book involves manipulation of a free rigid object; turning a key in a keyhole involves moving a rigid object against a kinematic constraint. Functional interaction with objects—tool use—is ubiquitous in activities of daily living and the basis for our evolutionary advantage. Tools extend and augment fundamental human capabilities. Surprisingly, how humans interactively control objects or tools is still little understood. Manipulation requires sensing the mechanics and the geometry of the object and adjusting one's movements and forces accordingly to exploit object properties. Manipulation becomes particularly intriguing when the objects have internal degrees of freedom that add complex dynamics to the interaction. An exotic example is cracking a whip, where the flexible whip creates challenging dynamics (infinitely many degrees of freedom) that the hand has to interact with (Bernstein et al. 1958; Goriely and McMillen 2002; Hogan and Sternad 2012). A more mundane example is leading a cup of coffee to one's mouth to drink: the transporting hand applies a force not only to the cup, but also indirectly to the liquid, which in turn acts back onto the hand. These continuous forces require sensitive adjustments to avoid spilling the coffee (Hasson et al. 2012a; Mayer and Krechetnikov 2012; Hasson and Sternad 2014; Sauret et al. 2015). Humans are strikingly adept at interacting with a large variety of such objects, but most studies on object manipulation have been confined to either multi-digit grasping of a static object or grip forces needed for transporting solid objects (Flanagan et al. 1993; Flanagan and Wing 1997; Santello et al. 1998; Gao et al. 2005; Fu and Santello 2014). This chapter will focus on physical interactions with complex objects that are largely unchartered territory in motor neuroscience to date.

Over the last two decades motor neuroscience has made advances in understanding the control of simple movements, for example straight-line reaches in the horizontal plane including adaptation to external force fields or visual perturbations. This research has shed light on significant aspects of adaptation and control, such as error correction mechanisms and internal models (Shadmehr and Mussa-Ivaldi 1994; Scheidt et al. 2001). This paradigm has continued the long tradition of motor neuroscience examining elementary behaviors under strict experimental control. Seminal paradigms range from single-joint wrist movements in primates (Evarts 1968), to the speed-accuracy paradigm (Fitts 1954), to today's center-out reaching task for human and primate studies (Kalaska 2009). While these paradigms render manageable data for analysis and modeling, they are far removed from the richness of everyday actions and interactions. Unfortunately it is difficult, if not impossible, to extrapolate insights to more complex movements. For example, when extending multi-joint movements from 2D to 3D, non-commutative finite rotations introduce entirely new problems (Zatsiorsky 1998; Charles and Hogan 2011). Further and important for this line of study, physical contact with objects introduces bidirectional forces that pose a control challenge that is completely absent in free movements (Hogan 1985). Different from the sequential flow of information processing, physical interactions are fundamentally bidirectional—each system affects the other with mutual causality, an observation first expressed in Newton's third law.

Previous Research on Complex Object Manipulation

Previous research on human control of dynamically complex objects has adopted a variety of theoretical perspectives that, as a whole, still present a rather disconnected picture. One line of studies examined balancing a pole, the classic control theoretical problem of stabilizing an inherently unstable system. Different control mechanisms were proposed, ranging from intermittent to continuous, predictive control, with forward or inverse models (Mehta and Schaal 2002; Gawthrop et al. 2013; Insperger et al. 2013). Nonlinear time-series analysis of the hand trajectory has probed the role of noise and delays to distinguish between continuous versus intermittent control (Cluff et al. 2009; Milton 2011; Milton et al. 2013) or the perceptual information used to stabilize the pole (Foo et al. 2000). Valero-Cuevas and colleagues examined the manual compression of a spring, modeling this dynamical system to include a subcritical pitchfork bifurcation to account for buckling (Venkadesan et al. 2007). Other studies have focused on the role of visual and haptic information to learn complex manipulation (Huang et al. 2002, 2007; Danion et al. 2012). Yet other research examined the displacement of a linear mass-spring object and proposed optimization criteria, such as generalized kinematic smoothness (Dingwell et al. 2004), accuracy and effort (Nagengast et al. 2009), and minimum acceleration with constraints on the center of mass (Leib and Karniel 2012). While interesting insights have been gained, most studies implicitly or explicitly assume that the human has, or has to learn an internal model of the manipulated object. As already hinted above, this may be daunting.

Hypothesis 1: Predictability

When interacting with complex objects, instantaneous action and reaction is critical. Control models for artificial systems have posited internal models and inverse dynamics control plus feedback control, as they are largely devoid of long feedback delays and with relatively low levels of noise (Flanagan et al. 1999, 2003; Kawato 1999; Takahashi et al. 2001). In contrast, in humans feedback-based corrective control is virtually irrelevant due to trans-cortical or trans-cerebellar loop delays on the order of 100 ms or more, which requires exact extrapolation from current state estimates

(Pruszynski et al. 2011). This is difficult as variability and noise in the human system is high, with an approximate precision in timing of 9 ms (Faisal et al. 2008; Cohen and Sternad 2012). Instead, intrinsic musculo-skeletal properties augmented by spinal reflexes deliver essentially instantaneous reaction and can provide stabilization to counteract noise or instability (Colgate and Hogan 1988; Burdet et al. 2001; Franklin et al. 2003; Selen et al. 2009; Lee et al. 2014). While mechanical impedance is essential, dexterous control in the presence of delays nevertheless requires one to anticipate, preempt, and exploit the forces and motions of an object. Yet, prediction for continuous nonlinear objects with chaotic, i.e. unpredictable, behavior is challenging or impossible, even for artificial systems with short delays and low noise. Therefore, rather than expending the neural resources to learn a complex dynamics model, we suggest an alternative hypothesis: humans make the interactions with objects more predictable. This can be achieved by simplifying the interactive dynamics via linearization or avoidance of chaotic regimes.

Hypothesis 2: Robustness

A precise internal dynamic model with complex nonlinear dynamics is difficult, if not impossible to learn. On the other hand, such complex models may not be necessary. For example, humans can proficiently control an automobile without knowing its full dynamical model or even understanding how the various mechanical components of a car work. To cope with such situations, the nervous system should select movement strategies that are robust to modeling errors. The branch of control theory called *robust control* is devoted to solving this problem, i.e. designing controllers that have good performance and stability in spite of modeling errors (Zhou and Doyle 1998). Note that such a controller may not have the same level of performance as one that has access to a perfect dynamics model, but choosing a suboptimal movement strategy, i.e. a "good enough" solution (Loeb 2012) may be an acceptable trade-off for increased robustness to modeling errors. Therefore, we hypothesize that rather than expending the neural resources to learn a complex dynamics model, humans learn a simpler model and select a robust control strategy that offers greater safety margins against failure.

The Model Task: Moving a Cup of Coffee

To test the two hypotheses—humans select movement strategies that make interactions with complex objects predictable and robust—an appropriate test bed is needed. Transporting a cup of coffee is a good candidate as the cup filled with liquid has complex dynamics and there are clear consequences for failure, i.e. spilled coffee. However, transporting a cup with sloshing coffee is a complex problem in fluid dynamics (Mayer and Krechetnikov 2012; Sauret et al. 2015). Hence, the task

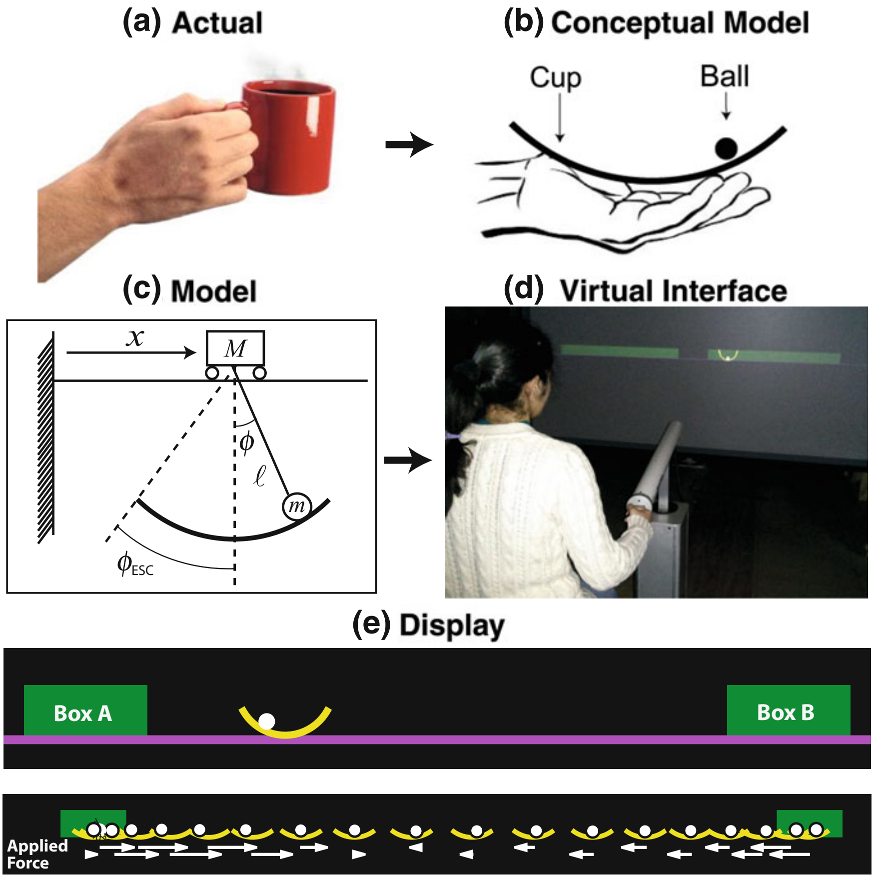

Fig. 1 From the task to the experiment. **a** The actual task. **b** The conceptual model. **c** The cart-and-pendulum model underlying the displayed cup and ball. The cup is the arc of the circular pendulum path, the pendulum bob is the ball. **d** Virtual implementation with robot arm and visual and haptic interface. **e** The display with start and end box targets. The schematic below visualizes the applied force as arrows in accelerating and decelerating directions. Figure modified from Nasseroleslami et al. (2014) with permission under Creative Commons Attribution (CC BY) license

was simplified to that of moving a cup with a ball rolling inside, representing the complex dynamics of the coffee [Fig. 1a, b; (Hasson et al. 2012a)]. Implemented in a virtual environment the cup was visualized as an arc in 2D and modeled as a point mass moving along a horizontal axis. The ball's motion was modeled by a suspended pendulum; the arc of the cup corresponded to the ball's semi-circular path (Fig. 1c). This model system was implemented in a virtual environment, where subjects exert forces on the virtual cup via a robotic manipulandum (Fig. 1d shows the screen display and Fig. 1e (bottom panel) shows a movement of the cup and ball with the applied forces shown at different time points). Importantly, movements of the cup also accelerate the ball, which in turn acts back on the hand. Despite these simplifications, the model system retained essential elements of complexity: it is nonlinear and creates complex interaction forces between hand and object.

In this simplification, the equations of the cup-and-ball system are identical to the well-known cart-and-pendulum problem (Hinrichsen and Pritchard 2005; Ogata 2010). The cup is the cart with a point mass M that moves horizontally; the pendulum comprises a point mass m (the ball) attached to a mass-less rod of length ℓ with one angular degree of freedom θ. Subjects control the ball indirectly by applying forces to the cup, and the ball can "escape", i.e. it can be lost from the cup when the angular distance to the rim is exceeded. The hand moving the cup is represented by an external applied horizontal force F_{A}. The equations of the system dynamics are:

$$(m+M)\ddot{x} = m\ell\left(\ddot{\theta}\cos\theta + \dot{\theta}^2 \sin\theta\right) + F_{\mathrm{A}} \tag{1}$$

$$\ell\ddot{\theta} = \ddot{x}\cos\theta - g\sin\theta$$

where θ, $\dot{\theta}$, and $\ddot{\theta}$ are the ball's angular position, velocity, and acceleration; x, $\dot{x}$ and $\ddot{x}$ are the cart/cup's position, velocity and acceleration; g is gravitational acceleration; damping to pendulum and cart motion can also be added if desired.

To implement this cup-and-ball system in a virtual environment, the cart and the pendulum rod were hidden, but the pendulum bob (the ball) remained visible (Fig. 1e). Subjects manipulate the virtual cup-and-ball system via a robotic arm, which also exerts forces from the virtual object onto the hand [HapticMaster, Motek (van der Linde and Lammertse 2003)]. Using admittance control, the HapticMaster has three controllable degrees of freedom, but was constrained to motion on a horizontal line for the experiments. The pendulum's θ and $\dot{\theta}$ were computed using a 4th-order Runge-Kutta-integrator, and the force of the ball on the cup F_{Ball} was computed based on Eq. 1: $F_{\mathrm{Ball}} = m\ell\left(\ddot{\theta}\cos\theta + \dot{\theta}^2\sin\theta\right)$. This force, combined with any forces exerted by a human F_{A}, accelerated the virtual mass $(m+M)$. The robot motors moved the manipulandum according to $\ddot{x}$ and the visual display was updated. For more details see (Hasson et al. 2012a).

This formalization and its virtual implementation has several advantages. (1) The focus is on the interaction forces between the hand and the object. Confining the physical interaction to a single "interaction port" via the robot handle avoids the complexity of grasp formation (Santello and Soechting 2000; Nowak and Hermsdörfer 2003). (2) Compared to real objects that have dozens of modes, this formalization reduces the object to two modes that facilitate analytical treatment (Hasson et al. 2012b). (3) The virtual implementation enables versatile manipulation of task parameters, including linear and nonlinear aspects. (4) The task involves "skill" and requires practice to arrive at smooth and stable execution. (5) The virtual implementation of the task is equivalent to the dynamic model. Hence, the measured human kinematics and kinetics lends itself to novel mathematical analyses to assess how humans sense and exploit the object's dynamic properties. In sum, the task has manageable but sufficient richness with multiple routes to increment complexity.

Predictable and Unpredictable Interactions—Chaos

Most studies involving object manipulation have used linear systems, such as mass-springs (Dingwell et al. 2002; Svinin et al. 2006; Danion et al. 2012). By definition, such systems display predictable behavior. For example, if one were to oscillate a linear mass spring with the goal of attaining a given oscillation, the execution variables, the amplitude A and frequency f of the cup oscillation relate linearly to the applied forces and the resulting motion of the system: If the system is sinusoidally forced at 1 Hz, it will oscillate sinusoidally at 1 Hz. However, with a nonlinear system, such as the cup-and-ball, this mapping becomes non-trivial: the same forcing input may cause the system to oscillate at an array of frequencies with unpredictable and chaotic behavior.

To illustrate this chaotic behavior in the cup-and-ball system, we applied inverse dynamics to obtain the required force F_A for a given oscillatory cup motion, specified by the scalar execution variables cup amplitude A and cup frequency f, with initial ball angle θ_0 and ball velocity $\dot{\theta}_0$. Shown in Fig. 2 are two simulated examples with the same sinusoidal cup movement x. The only difference is in the initial angle of the ball θ_0, with $\dot{\theta}_0$ set to zero. In one case ($\theta_0 = 1.0\,\text{rad}$), the force required to produce this motion x is periodic and predictable. In the other case ($\theta_0 = 0.4\,\text{rad}$), the force required to produce the same cup motion shows highly irregular fluctuations. To characterize the pattern of force profiles with respect to the cup dynamics, F_A was strobed at every peak of the cup position x. The marginal distributions of strobed force values are plotted as a function of ball angle θ_0 in the bottom panel (Fig. 2). This input-output relation reveals bifurcations with a pattern similar to the period-doubling behavior of chaotic systems. This feature has important implications for control: small changes in initial states can dramatically change the long-term behavior and lead to unpredictable solutions.

Quantifying Predictability

We hypothesize that subjects seek solutions with predictable object behavior (*Hypothesis 1*). To quantitatively test this hypothesis, predictability must be operationalized. One possible measure is the mutual information (MI) between the applied force and the motion of the object, which characterizes the long-term predictability of the object's dynamics (Cover and Thomas 2006; Nasseroleslami et al. 2014). MI is a nonlinear correlation measure defined between two probability density distributions of two random variables and quantifies the information shared

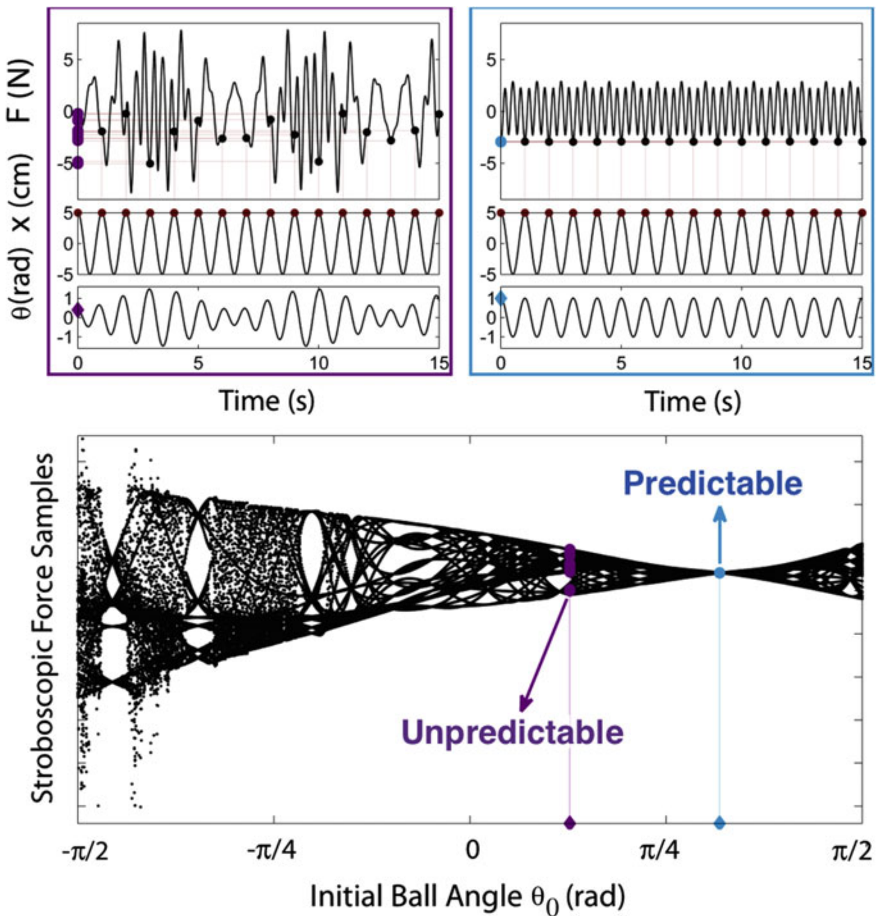

Fig. 2 Simulated force profiles derived from inverse dynamics with specified cup and ball trajectories. The profiles are applied force, cup position, and ball angle (from *top* to *bottom*). The *left panel* was initiated with ball angle θ_0 = 0.4 rad; the right panel with θ_0 = 1.0 rad. The bifurcation diagram below shows the marginal distributions of force values strobed at all peak cup position (see dots in *upper panels*). The two initial ball angles are shown and marked as predictable and unpredictable. The diagram combines 1000 simulations with different initial ball angles θ_0 in the range between $-\pi/2$ to $\pi/2$ rad. The force distributions plotted as a function of ball angle indicate chaotic dynamics. Figure modified from Nasseroleslami et al. (2014) with permission under Creative Commons Attribution (CC BY) license

between the two. MI is calculated between F_A and the phase of the ball φ. This phase was calculated in phase space, spanned by ball angle and velocity:

$$MI(\varphi, F_A) = \iint p(\varphi, F_A) \log_e \frac{p(\varphi, F_A)}{p(\varphi)p(F_A)} \mathrm{d}\varphi \mathrm{d}F_A \tag{2}$$

where $p(.)$ denotes a probability density function. MI can also be calculated for the phase of the cup. MI presents a scalar measure of the performer's strategy that can be calculated for all amplitudes and frequencies of the cup and all initial conditions of the ball. MI can be summarized for each point of the 4D space of execution variables: A,f, θ_0, and $\dot{\theta}_0$.

Experimental Insights

Our recent study provided evidence that subjects increase the predictability of object dynamics with practice and favor predictable solutions over those that minimize expended force and smoothness, criteria that are widely supported criteria for free movements (Nasseroleslami et al. 2014). In this study, subjects (n = 8) oscillated the virtual cup between two targets with a robotic manipulandum, paced by a metronome at 1 Hz for 50 trials, each lasting 45 s. They were free to choose their movement amplitude and relative phase between the ball and cup.

The cup and ball oscillations were analyzed to determine how choices of movement amplitude and relative phase related to three result variables or costs: predictability, exerted force, and movement smoothness (Fig. 3). Figure 3a shows the result space for mutual information; lighter shading indicates that combinations of cup amplitude and ball angle render higher mutual information (higher predictability). The large point indicates the strategy with the highest mutual information. To compare potential alternative explanations, two other result measures, or commonly used costs, were derived for the same model: minimum force and maximum smoothness. The expended force was calculated by the square of the

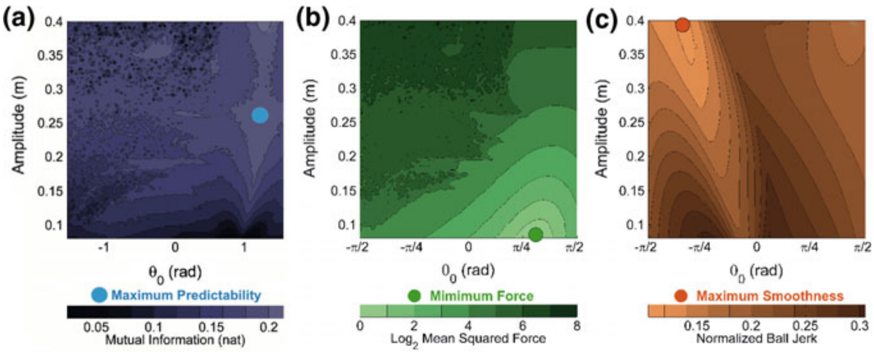

Fig. 3 Result spaces that combine result variables or costs in the space spanned by the execution variables initial ball angle, cup amplitude, frequency (fixed at 1 Hz), and initial ball velocity (set to zero). **a** Mutual information. **b** Mean squared force (log transformed). **c** Mean squared jerk of the ball motion (normalized for amplitude); the large point in each graph indicates the location of maximum cost. Importantly, the minimum/maximum values are located in different parts of the map, providing different predictions. Figure modified from Nasseroleslami et al. (2014) with permission under Creative Commons Attribution (CC BY) license

mean integral over F_A over the course of the trial, mean squared force MSF. Figure 3B shows the resulting pattern of force for the same space of execution variables; lighter shading refers to strategies requiring less force. The point highlights that the minimum force solution is obtained at the smallest allowable cup amplitudes. Lastly, smoothness or jerk was evaluated of the cup trajectory for each of the strategies defined in the execution space. Figure 3c shows smoothness of the ball movements for each strategy, with lighter shades denoting higher smoothness. The point shows that a strategy with high amplitude reaches maximum smoothness or minimum jerk. Importantly, the three maxima lie in different locations of the execution space. Therefore, by looking at which amplitude and relative phase subjects choose, we can infer which of the three costs are most important for subjects' movement control.

Following these simulations, equivalent measures for the execution variables A,f, θ_0, and $\dot{\theta}_0$ had be derived from experimental data. However, the experimental trajectories were not fully determined by the initial values of ball states as variations could be due to online corrective changes. Therefore, to estimate the execution variables from the experimental trajectories, the initial values were extracted at each cycle k (see Fig. 4). Peak excursions of the cup trajectory served as strobe points to estimate A,f, θ_0, $\dot{\theta}_0$ and calculate trial averages $\bar{A}$,$\bar{f}$, $\bar{\theta}_0$, $\bar{\dot{\theta}}_0$ that served as correlates for the variables in the simulations. To exclude transients only the time window after 25 s was considered for analysis. To evaluate *Hypothesis 1*, that subjects seek predictable object interactions, *MI*, and the alternative costs mean squared force *MSF*, and mean squared jerk *MSJ* were calculated for each measured strategy $\bar{A}_k$,$\bar{f}_k$, $\bar{\theta}_k$, $\bar{\dot{\theta}}_k$. Calculation of *MI* followed the same procedure as in the simulated *MI*, except that probability density functions were estimated experimentally (for more details see (Nasseroleslami et al. 2014). To calculate *MSF*, the continuous force profile of each trial was squared and averaged, analogous to the simulated data. *MSJ* was calculated according to the standard equations (Hogan and Sternad 2009).

The main experimental results are summarized in Fig. 5; the mutual information plot is overlaid with contours of selected simulated force values (green). The figure shows how subjects gravitated towards areas with higher *MI*, i.e. strategies with more predictable interactions. In the left panel, each point represents the average strategy for each 45 s trials for all subjects; darker red indicates early practice and lighter red indicates late practice. The right panel shows the same data separated by subject: the red arrows mark how each subject's average strategy changed from early practice (mean of first 5 trials) to late practice (mean of last 5 trials). Both figures clearly show that all subjects increased their movement amplitude, associated with an increase in overall exerted force. The majority of subjects switched from low- to high-predictability regions in the result space. None of the subjects moved toward the minimum force strategy, nor towards a strategy with maximum smoothness. Analysis of *MSF* and *MSJ* over trials shows that indeed exerted force increased and smoothness decreased with practice, counter to findings in free unconstrained movements. Overall, the results rejected the two alternative criteria and were consistent with *Hypothesis 1*.

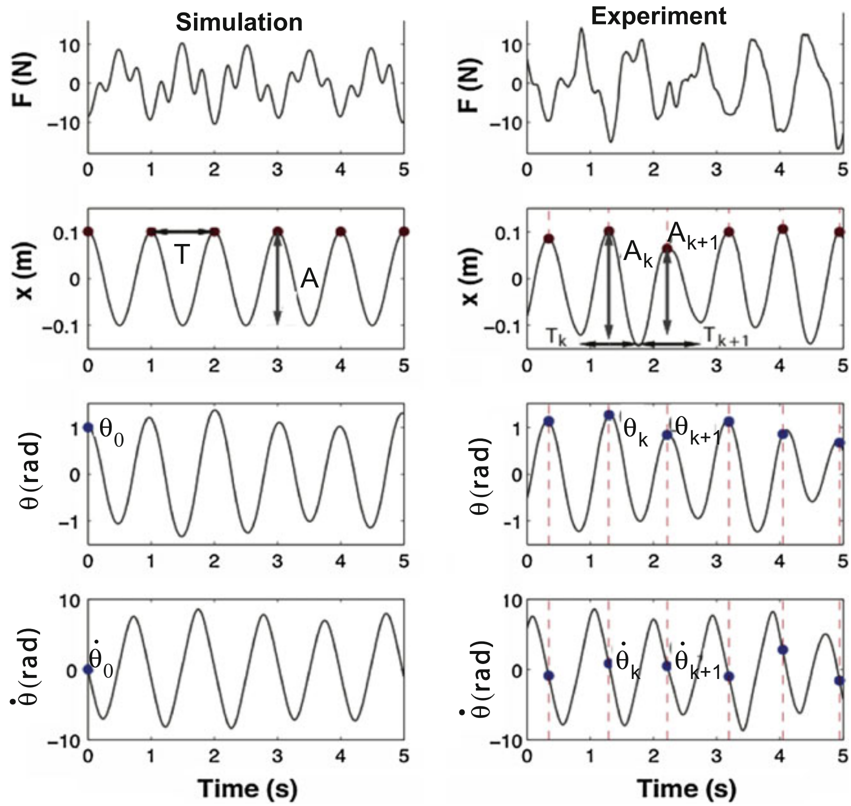

Fig. 4 Exemplary profiles from inverse dynamics simulations and corresponding experimental data for applied force, cup position, ball angle and velocity. Estimates for the execution variables in the data were derived for each cycle as shown and then averaged across the trial to obtain scalar estimates for each trial. Figure modified from Nasseroleslami et al. (2014) with permission under Creative Commons Attribution (CC BY) license

Robust Interactions

The reviewed results suggest that when there is a choice, humans select a strategy that increases the predictability of the human-object interaction. More predictable human-object interactions may lessen the control burden; however, errors in control undoubtedly exist, especially when only rough approximations of internal models of object dynamics are available. Thus, keeping interactions predictable may not be enough—a good strategy should also be robust to control errors. The cup-and-ball task lends itself to experimental investigation of robustness, as there is a well-defined threshold for failure, i.e. the ball escapes the cup—coffee is spilled. Note that in the previous experiment, the ball could not escape, but swung around following the circular path of the pendulum in situations of varying difficulty. By

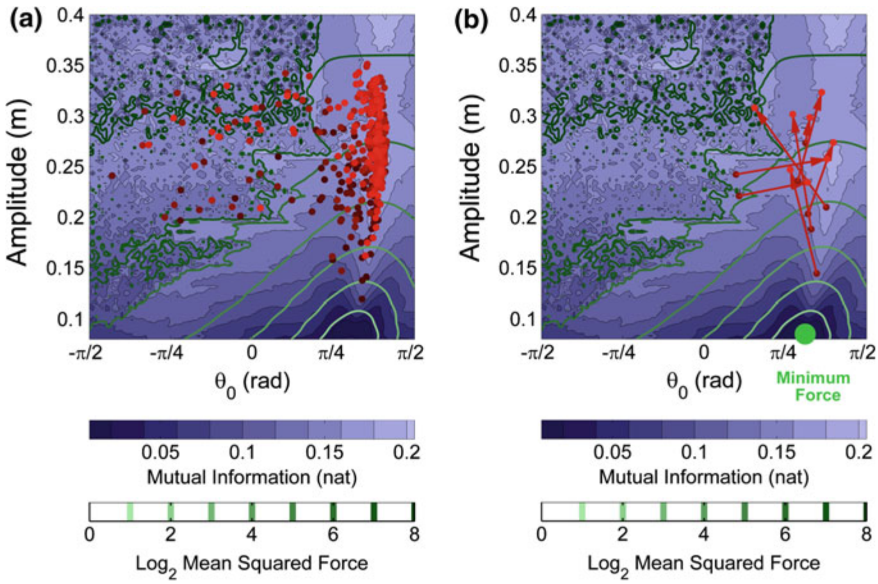

Fig. 5 Main results in the result space for mutual information. The green contours denote different values of mean squared force superimposed onto the same space. **a** Data pooled from all subjects; each point is one trial. Darker red pertains to earlier trials than lighter red. The data show that in the course of practice, subjects shifted their movement strategies to the area of high mutual information (high predictability). **b** Averaged data from eight subjects; each arrow represents one subject, the tail of the arrow is the mean of the first 5 trials, the tip of the arrow is the mean of the last 5 trials. Notice that none of the subjects shifted down towards the area of minimum force, indicated by the green point. Figure modified from Nasseroleslami et al. (2014) with permission under Creative Commons Attribution (CC BY) license

introducing a "rim" and also using a shallower cup, we could probe the use of fragile and robust strategies.

We hypothesized that as a subject learns to manipulate the object, s/he should find strategies that are more robust to failure. In a risky strategy, the ball gets close to the rim of the cup and any small error may lead to loss of the ball. Therefore, a safety margin is critical and might present a sensitive measure distinguishing "fragile" from robust control. We hypothesized that this safety margin should increase with practice (*Hypothesis 2*). Further, we expected that the size of the safety margin depends on the performance variability. Individuals have different degrees of variability and those with more variable movements should seek larger safety margins (*Hypothesis 2a*). Further, if variability decreases with practice, then the safety margin should change accordingly (*Hypothesis 2b*). We will now review two studies that addressed these questions in young and also older healthy adults.

Quantifying Robustness

To test these hypotheses, the safety margin needed to be defined. Safety margins have been most frequently characterized in gait and posture and are typically quantified by the degree of spatial and/or temporal difference between the body center of mass/center of pressure and the base of support boundary (Hof et al. 2005; Hasson et al. 2008). While useful, such measures can be difficult to generalize, because they are specific to upright stance and can depend on the physical attributes of the individual. Therefore, we developed a more general formulation, defined in terms of energy, i.e. an energy margin.

Most objects that we may interact with are initially at rest, and when we pick them up or handle them, we impart energy to them. For example, we push on a shopping cart to start moving it or pick up a cup of coffee to drink. If too much energy is imparted to such objects, an undesirable outcome may occur, such as overturning the shopping cart or spilling the coffee. We define the energy margin *EM* by the difference between the current energy to the energy level that causes failure [see (Hasson et al. 2012a) for more details].

Specifically for the cup-and-ball system, *EM* quantifies how close the total energy of the ball TE_{BALL} is to the energy level that would cause the ball to exceed the rim, i.e. the escape energy E_{ESC}

$$EM = (EM_{\mathrm{ESC}} - TE_{\mathrm{BALL}})/E_{\mathrm{ESC}} \tag{3}$$

EM is normalized to E_{ESC} so that the maximum value is $EM = 1$ (maximum safety). Small values indicate a "dangerous" condition; if *EM* remains below zero the ball will escape from the cup unless a corrective action is taken. TE_{BALL} is given by

$$\begin{aligned} TE_{\mathrm{BALL}} &= KE_{\mathrm{BALL}} + PE_{\mathrm{BALL}} + PSE_{\mathrm{BALL}} \\ &= \left[\frac{1}{2} m\left(\ell\dot{\theta}\right)^2\right] + [mg\ell(1-\cos\theta)] + \left[-m\ddot{x}\ell\sin\theta + m\left|\ddot{x}\right|\ell\right] \end{aligned} \tag{4}$$

where KE_{BALL} is the kinetic energy of the ball, PE_{BALL} is the potential energy of the ball, and PSE_{BALL} is a pseudo-energy because the ball is in an accelerated reference frame relative to the cup. E_{ESC} is defined as

$$E_{\mathrm{ESC}} = mg\ell(1-\cos\theta_{\mathrm{ESC}}) - m\left|\ddot{x}\right|\ell\,\sin\theta_{\mathrm{ESC}} + m\left|\ddot{x}\right|\ell \tag{5}$$

In these equations, there are only three time-varying quantities, the ball angle θ, the ball angular velocity $\dot{\theta}$, and the cup acceleration $\ddot{x}$. These variables are measured and defined as the execution variables, which jointly determine the result variable *EM*. Essentially, *EM* takes the instantaneous state of the cup and ball, which includes inputs from the human hand, and extrapolates to determine whether the ball will escape, given the current value for $\ddot{x}$. At the very next instant in time, a new determination is made based on updated execution values of θ, $\dot{\theta}$, and $\ddot{x}$, and so on for future time points. This analysis approach follows the same logic as for the rhythmic task described above: identify the execution variables that fully determinate the result variable. However, instead of mapping to a predictability measure, *MI* (alternative measures or *MSF* and *MSJ*), the execution variables are mapped into the energy margin *EM*. This same analysis strategy was previously applied to other tasks such as throwing and bouncing a ball (Dijkstra et al. 2004; Cohen and Sternad 2009; Sternad et al. 2014).

For any movement of the cup and ball, the energy margin fluctuates over the time of the movement, as shown in Fig. 6a for an exemplar point-to-point translation of the cup and ball. The normalization of *EM* to E_{ESC} affords an assessment of the risk at any instant during an ongoing movement. When $EM > 0$ and the margin is large, any unexpected disturbance can easily be dealt with or "absorbed". However, when $EM \leq 0$, the ball will escape in a finite "time-to-escape" (red dotted lines in Fig. 6a), unless action is taken to increase the *EM* before the ball reaches the rim. The exemplary profile shows fluctuations that are concurrent, but not coincident with the ball excursions, as the applied force is also important. The same trial can also be plotted as a trajectory in 3D space spanned by the three execution variables θ, $\dot{\theta}$, and $\ddot{x}$. (Figure 6b). The result variable is *EM*. The critical energy E_{ESC} defines a closed two-dimensional manifold two oblique cones joined together;

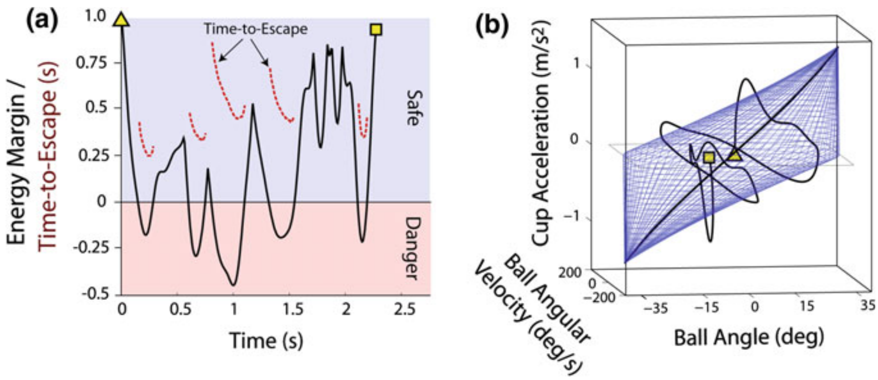

Fig. 6 Exemplary profile of energy margin of one trial during early practice. **a** The energy margin *EM* as a function of time. With the initial high *EM*, the ball is at rest and is unlikely to escape from the cup, even when exposed to a disturbance. However, when the *EM* drops below zero the ball is in a state where it will escape from the cup in a finite time (shown as the red dotted "Time-to-Escape" lines). **b** For the same trial, the three variables that determine *EM*, ball angle and angular velocity and cup acceleration, are shown in a three dimensional execution space. The trial starts in the center (*yellow triangle*) and moves through the space as the trial progresses until the cup is stopped at the spatial target (*yellow square*). The blue mesh represents the escape energy threshold, $EM = 0$; whenever the trajectory breaches the manifold there is danger of ball-escape unless a corrective action is taken. Figure modified from Hasson and Sternad (2014) with permission under Creative Commons Attribution (CC BY) license

see blue mesh. If the trajectory stays within this manifold, $EM > 0$, the time-to-escape is infinite, and the ball is never in danger of escaping. If the manifold is breached, $EM < 0$ and the ball may escape. If the subject applies a corrective action to change $\ddot{x}$ in an appropriate way, then failure may be prevented. However, the available time to make such a correction is finite. If the correction takes too long, the ball will be lost. Note that the time-to-escape is computed at each instant in time, assuming constant $\ddot{x}$, but is then updated at the next instant in time when a new set of execution variables (θ, $\dot{\theta}$, and $\ddot{x}$) is available.

Experimental Insights

A prior study sought to test the hypothesis that humans seek robust movement strategies with appropriate safety margins (Hasson et al. 2012a). Subjects were asked to make a discrete point-to-point translation of the cup, and to complete the movement in a target time of 2 s without losing the ball from the cup. This completion time was comfortable and afforded selection among several strategies. For comparison, a separate group of subjects performed a minimum-time movement, translating the cup as fast as possible over the same distance. Both groups improved their performance, i.e. the timing error and movement time decreased for the target-time and minimum-time groups, respectively. As hypothesized, subjects in the target-time condition increased their energy margin over practice (Fig. 7a). In contrast, the energy margin decreased in the minimum-time task (Fig. 7b). Accordingly, the minimum-time group lost the ball about 10 times as often as the target-time group at the end of practice. These changes in the energy margin typically occurred throughout the entire movement profile, as highlighted by the shading in Fig. 7a, b, although some portions of the movement tended to show larger changes than others. These findings suggest that when urged to move as fast as possible, subjects “live dangerously” and use small energy margins. However, when multiple movement options are available humans prefer those that are more robust to errors in control. This result supported *Hypothesis 2a*.

For a different view on how the energy margin changed with practice, a number of trials from one representative subject are shown in execution space in Fig. 7c. The blue mesh again represents the E_{ESC} manifold; two perspectives on the same data are shown for clarity. Early in practice, the movement trajectories are variable and frequently break through the E_{ESC} manifold by a significant amount, often leading to loss of the ball. This happened mostly near the end of the movement when subjects tried to stop the cup (seen as high cup deceleration). However, after practicing the task, a clear structure becomes visible and the trajectories conform to the E_{ESC} manifold. This “contraction” of the trajectories raises the energy margin, increasing robustness. As long as the trajectory is within the E_{ESC} manifold there is no chance of the ball escaping from the cup. This could be advantageous, as minor

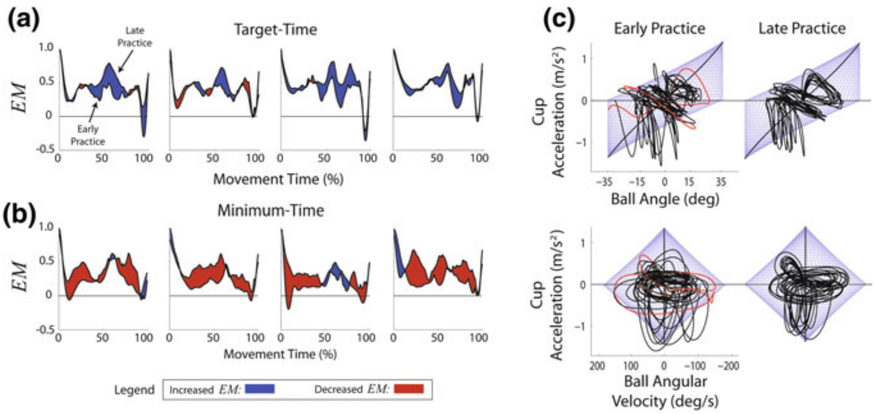

Fig. 7 Changes in the energy margin *EM* with practice. **a** Difference in the energy margins between an early and a late trial in four subjects of the target-time group. **b** Difference in the energy margins between an early and a late trial in four subjects of the minimum-time group. **c** Examples of early and late practice trials for one subject. Trajectories are plotted in execution space, defined by ball angle and velocity and cup acceleration. Two different views of the three-dimensional execution space are shown. Trials in which the ball escaped are shown in red. Note that not all trials that exit the manifold result in failure. Figure modified from Hasson and Sternad (2014) with permission under Creative Commons Attribution (CC BY) license

errors in control would not cause failure, which could free up cognitive resources for higher–level movement planning operations.

Motivated by the robustness hypothesis, we also predicted that the size of the safety margin should depend on subjects' motor variability (*Hypothesis 2a*). This follows previous work suggesting that variability plays a central role in movement control such that the motor system optimizes movements to minimize the effects of variability on task goals (Harris and Wolpert 1998; Trommershäuser et al. 2005; Gepshtein et al. 2007; Cohen and Sternad 2009; Hudson et al. 2010; Sternad et al. 2011; Chu et al. 2013). Specifically, individuals with greater trial-to-trial variability should choose a larger energy margin, and vice versa. To test this hypothesis, the degree of correlation between the energy margin and trial-to-trial variability was assessed for both the target-time and minimum-time tasks. Consistent with *Hypothesis 2a*, results showed a positive correlation, i.e. subjects with high variability at the end of practice also had large safety margins at the end of practice (Fig. 8a). There was no correlation for the target-time task. This could be ascribed to the individual variations in strategies in the target-time group, while subjects in the minimum-time group displayed more similar strategies. When examining potential correlations *across practice* within each individual, there was a significant correlation for the target-time group. Consistent with *Hypothesis 2b*, subjects with large decreases in variability also changed their strategies to smaller energy margins, and vice versa (Fig. 8b). Those subjects who developed a consistent movement pattern may have been more confident in their ability, and therefore did not need large energy margins. Conversely, subjects with greater trial-to-trial variability

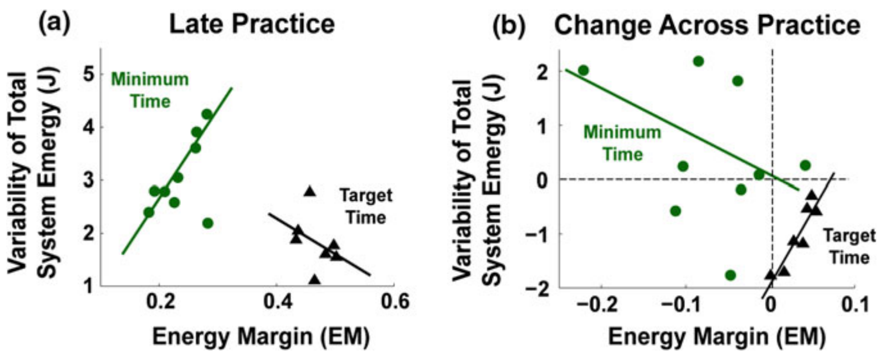

Fig. 8 Changes in energy margin *EM* as a function of trial-to-trial variability and task condition. **a** Correlations between *EM* and trial-to-trial variability of the total ball-and-cup system energy TE_{STD} over the last 30 trials for the Target-Time group (*black triangles*) and the Minimum-Time group (*green circles*). **b** Correlations between the change in *EM* and variability from early (first 30 trials) to late (last 30 trials) practice within each subject. Figure redrawn from Hasson et al. (2012a, 2012b)

chose a larger energy margin to accommodate the greater uncertainty. A connection between variability and safety margins was subsequently demonstrated in other recent studies (Chu et al. 2013, 2016; Hadjiosif and Smith 2015).

Robust control of behavior seems especially essential for individuals with diminished control abilities and who are fragile and prone to injury. One such population is frail older adults who may face catastrophic consequences in the event of an error in movement control, such as a fall. Paradoxically, even though older adults should utilize larger safety margins, in many cases the opposite has been shown. For example, when walking over obstacles or navigating stairs, older adults have smaller foot-obstacle clearances (Begg and Sparrow 2000; McFadyen and Prince 2002; Hamel et al. 2005). During quiet standing their postural sway measures show reduced spatiotemporal margins of stability (Slobounov et al. 1998; Van Wegen et al. 2002). We posited that such "high risk" strategies arise because older adults have more difficulty controlling complex whole body movements. Older adults may strive for high safety margins, but may be unsuccessful due to sensorimotor limitations. To explore this conjecture, we invited older adults to practice the cup and ball task. We tested the hypothesis that older adults have lower energy margins compared to younger adults (*Hypothesis 3a*), but as they learn to control the cup-and-ball dynamics, we expected their energy margins to increase significantly (*Hypothesis 3b*). Support for the latter hypothesis would show that they are indeed striving for larger energy margins as they gain better control of the object dynamics. We therefore asked them to perform the same discrete transport of the cup and ball, emphasizing that they should not lose the ball.

The results showed that with practice, both young and older adults improved their skill in the target-time task (decreased their timing error). Not surprisingly, the younger adults performed better and dropped the ball less often. When comparing

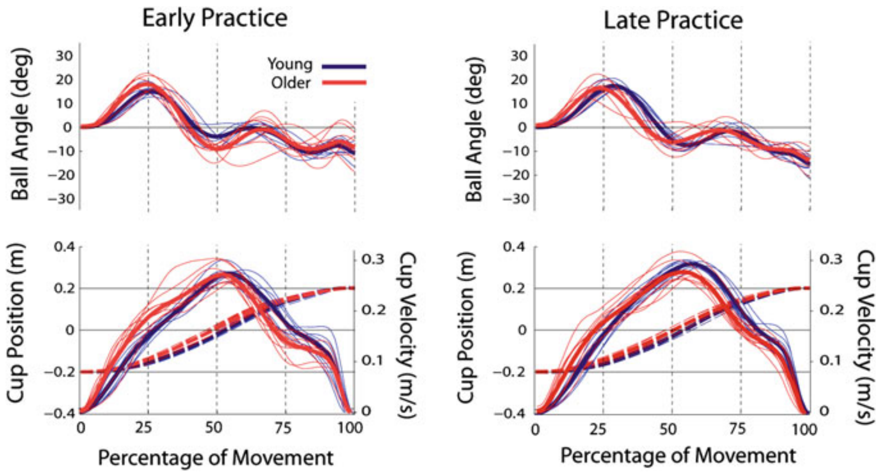

Fig. 9 Cup and ball kinematics in early and late practice for young (*blue*) and older adults (*red*). Individual subjects are shown as *thin lines* and the group means are the *thicker lines*. Variability decreases in both groups, but no other evident differences are discernible. Figure modified from Hasson and Sternad (2014) with permission under Creative Commons Attribution (CC BY) license

the kinematic profiles, i.e. the position and velocity of the cup and ball, only minor differences between the two age groups were discernable (Fig. 9). It was only the energy margins that revealed the differences between the two groups: early in practice, the older adults performed with a significantly lower energy margin (Fig. 10a), supporting *Hypothesis 3a*. Nonetheless, the older adults were able to increase their energy margin with practice, although not to the level of the younger adults (Fig. 10b). This implies that as older adults learned to interact with the cup-and-ball dynamics, they were able to increase the robustness of their movement strategies and, consequently, lost the ball less frequently (Fig. 10c), supporting *Hypothesis 3b*. However, while the energy margins continued to increase in the younger adults, it plateaued in the older adults. This suggests that sensorimotor limitations in older adults limit their ability to keep the cup and ball in a regimen with high safety margins.

A Task-Based Approach for Understanding Human-Object Interactions

How do humans successfully manipulate tools in daily life, an ability that has a long evolutionary history? Manipulation of complex dynamic objects presents daunting challenges, although more for the scientist than the human actor. Extrapolating our current understanding of human control of free movements to those involving object manipulation may not be an incremental process. For example, feedback

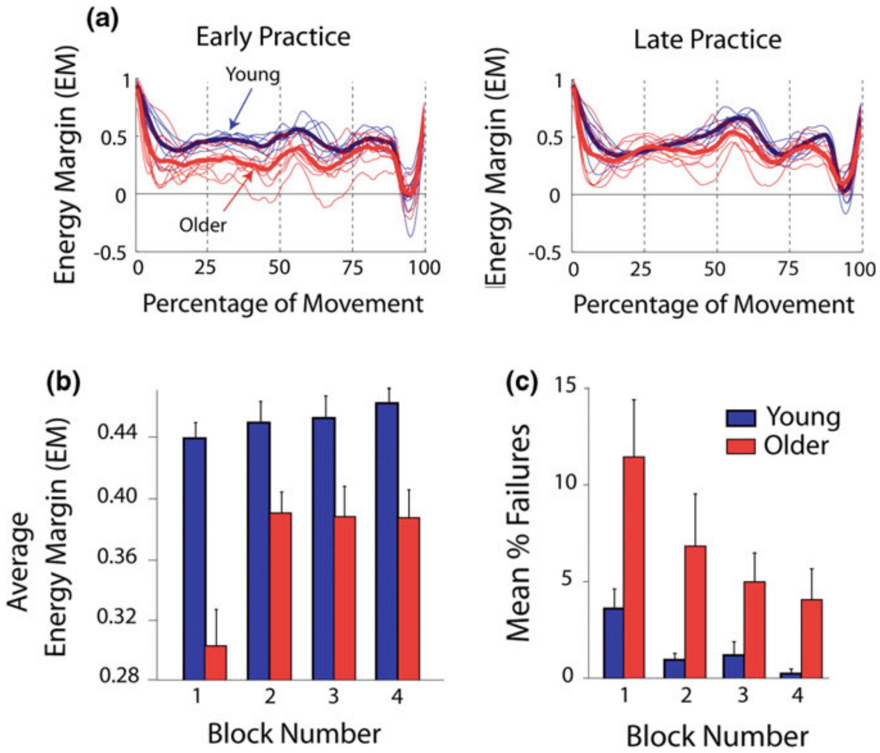

Fig. 10 Changes in the energy margin *EM* and number of ball escapes with practice for young and older adults. **a** The energy margin as a function of normalized movement time in early and late practice: individual subjects are shown as *thin lines* and the group means are the *thicker lines*. Note that in early practice the older subjects had a lower energy margin for most of the movement. **b** Group average *EM* across four blocks of practice. Both young and older adult increase their energy margins, but older adults have significantly smaller energy margins. **c** Group average percentage of trials in which the ball was dropped across four practice blocks. Older adults show visibly more failures than young subjects, but they also improve with practice. Figure modified from Hasson and Sternad (2014) with permission under Creative Commons Attribution (CC BY) license

control based on internal models of the object dynamics appears problematic given the long delays and high levels of noise in the human neuromotor system. Void of knowing the control architecture, we adopted a task-based approach. We analyzed the task and derived the solution space with no assumptions about the human actor and control. Starting with a physical model of the object dynamics and the task, we first identified execution and result variables. Mapping execution to selected result variables rendered a space of solutions. Based on this understanding of the physics, we could formulate quantitative hypotheses about potential strategies and objective functions that humans might use. Implementing the task in an interactive virtual environment we then measured human performance and directly evaluated task performance in the result space. This task-based approach has also been successfully applied in other tasks (Sternad et al. 2014).

Take Home Message

Unlike the body's own limbs, interactions with objects in the external world can be quite unpredictable. This is particularly true for objects with complex dynamics that cannot be directly controlled, such as a cup of coffee or a jostling baby carriage. Using the cup of coffee as a model task, we reviewed studies showing that with practice humans learn to control such objects by making the interaction both predictable and robust. These criteria are important for all individuals, however they may be of special importance to individuals with disabilities, where unpredictable and fragile interactions with the world incur marked effects on the quality of life. For these populations, it would be beneficial to develop interventions that promote predictability and robustness and thereby complement traditional movement criteria from free unconstrained movements such as movement smoothness and economy. The current ecological task may be a first step in this direction.

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