Sound, Pitches and Tuning of a Historic Carillon

Albrecht Schneider and Marc Leman

Abstract The City of Bruges in Flanders owns one of the finest carillons in Europe. Of its 47 bells, 26 are historic specimens, cast by Joris Dumery (Georgius Du Mery) between 1742 and 1748. In 2010/11, the carillon underwent restoration including retuning as necessary. The present article reports the status of the 26 historic carillon bells as recorded by us in the years 1997–2000 prior to restoration. Since the original tuning of the bells has been assumed to be close to quarter-comma meantone temperament, the tuning is investigated both in regard to physical data and scaling (weight, diameter) as well as fundamental frequencies and spectral characteristics of the Dumery bells. Trajectories for the five so-called principal partials hum, prime, tierce, fifth and octave (or nominal) are established to check the smoothness of inner tuning of the 26 bells. From the fundamental frequencies, the tuning of the 26 Dumery bells to a musical scale is derived, and a matrix of fundamental frequencies shows all intervals that can be realized with these bells. A second parameter relevant for the tuning of (swinging and carillon) bells is the so-called strike note, which is first discussed with respect to concepts of pitch perception and then in regard to a possible meantone tuning. Finally, in continuation of previous experiments which demonstrated ambiguity of pitch perception in subjects listening to bell sounds we conducted two small experiments one of which addresses the number of pitches subjects distinguish per bell sound while the other

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explores identification of musical intervals realized with sounds from the historic Dumery bells. Findings are evaluated in regard to perception and musical issues.

1 Introduction

Carillons are peculiar musical instruments in regard to both acoustics and perceptual issues. Historic carillons in Europe typically consist of a set of so-called minor third bells (see below) which are often used to play music that, at least in the past two centuries, increasingly made use of the major scale and tonality. In this respect, the sound structure of minor-third bells, on the one hand, and the tonal and harmonic structure of the music played on carillons, on the other, can lead to perceptual discrepancies (see below, Sect. 4).

The main purpose of our article is to study characteristics of the 26 historic Dumery bells that form the fundamental part of the carillon of Bruges. Therefore, the scaling (weights, diameters) of these bells, their spectral structure as well as their tuning to a musical scale has been investigated in detail. We recorded all 47 bells of the carillon in 1997, 1999, and 2004 prior to a restoration in 2010/11, in which the 26 historic Dumery bells (no. 1–26 of the carillon) were cleaned and their tuning was checked (including some minor revisions as was deemed necessary, see [1]. The other 21 bells which had been cast, in 1968, by The Royal Eijsbouts foundry of Asten, the Netherlands, were replaced by new specimens cast by the same foundry. In this article, we will focus on the historic Dumery bells, leaving aside the recent bells cast by Royal Eijsbouts.

For readers not familiar with carillons, we provide some factual and historical background. Also, fundamentals of bell acoustics are outlined with respect to sound generation in bells as well as radiation of sound from bells. Temporal and spectral parameters are given special attention. In addition to sound analyses, some perceptual issues will be addressed since the tuning of bells has often been discussed in conjunction with concepts of pitch perception. A phenomenon that has been investigated for a long time (probably for centuries) is the so-called ‘strike note’ of bells, a sound perceived by listeners immediately after the bell has been excited by a clapper impact. The strike note is considered a decisive factor in bell tuning and in the formation of the pitch or of several pitches arising from the sound of a bell.

Among previous studies on bells and carillons, some have covered aspects of tuning and also perceptual issues (e.g., [2–5]). One study dealt in detail with sound generation in the bell by the clapper impact as well as with the revoicing of carillon bells, which can become necessary due to the wear and tear of both bells and clappers [6]. A recent study [7] addressed reconstruction of the original tuning of a famous historic carillon that had been cast by the bell founder, Willem Witlockx (of Antwerp), in 1730, for the Royal Palace at Mafra, Portugal (see [8]). Of the original 36 bells, only 12 are extant while the carillon had been restored and expanded later on (the latest revision and expansion was made in 1986 by R. Eijsbouts; the carillon now has 53 bells). Since revoicing and retuning of bells in general involves removal of material from individual bells (even if in small quantity), reconstruction of the original tuning can be quite difficult (see [7]). Luckily, the Dumery bells of Bruges seem to have been left untouched since they were manufactured in the years 1742–1748 [1, 9]. Though the tuning of carillon bells can be affected to some degree by daily use over such a long period, data on bell dimensions and weights (see below) permit to assume that the sounds of the 26 Dumery bells we recorded in the years 1997–2004 still reflect the original tuning closely.

1.1 Some Historical and Factual Background

A carillon is a musical instrument comprising a set of bells tuned to a scale that can be played from a special keyboard or clavier. In addition to the manual, a pedal-board is included in many carillons. Carillons typically cover at least two octaves (23 bells tuned to a chromatic scale) and can have up to 47 bells (four octaves), with the largest instruments comprising 77 bells (Riverside church, New York; see [10], Chap. 11). There are some historic carillons with a smaller ambitus (the world's oldest extant carillon, cast, in 1595/96, by Peter III van den Ghein for the City of Monnickendam, The Netherlands, has 15 carillon bells plus two additional bells used also for a clock). Though carillons nowadays can be found in many places all over the world, much of their history is closely connected with the Low Countries (see [8]). From the historical record we know that carillons including a clavier were introduced in this area, in the 16th century while bell chimes comprising a number of tuned bells that could be activated in some other form by one or several players were in use already in the Middle Ages. In the past, carillons often included bells originally cast as swinging bells to be used in churches or monasteries. As we know from historical sources, the shape of swinging (church) bells has been changed from a more or less cylindrical or beehive design in the Middle Ages to the basically conical structure of bells such as cast by Geert (Gerhardus) de Wou around the year 1500 (for details, see [11]). These bells have a massive sound bow extending the diameter of the nearly cylindrical part (the so-called waist). A bell is closed at the top by a plate that carries the crown with which the bell is fastened to the headstock. At the lower side of the bell wall, the sound bow is continued into the lip or rim which tapers towards to mouth of the bell (see Fig. 1).

The reason for a change in the shape of bells around 1500 apparently was that bell founders attempted to achieve a certain spectral structure in the sound based on the vibration frequencies of several lower partials. The profile of each bell was designed so that the vibration frequencies for the principal partials called hum, prime, tierce, fifth and octave (or nominal) formed ratios like 1:2:2.4:3:4. To be sure, the third partial is a minor third (tierce). The bell following this pattern of partials hence became known as minor third bell (or minor third/octave bell). Once there was a template known for the bell's wall yielding the desired partial structure, one could derive a peel of bells conforming to a musical scale. Basically this was

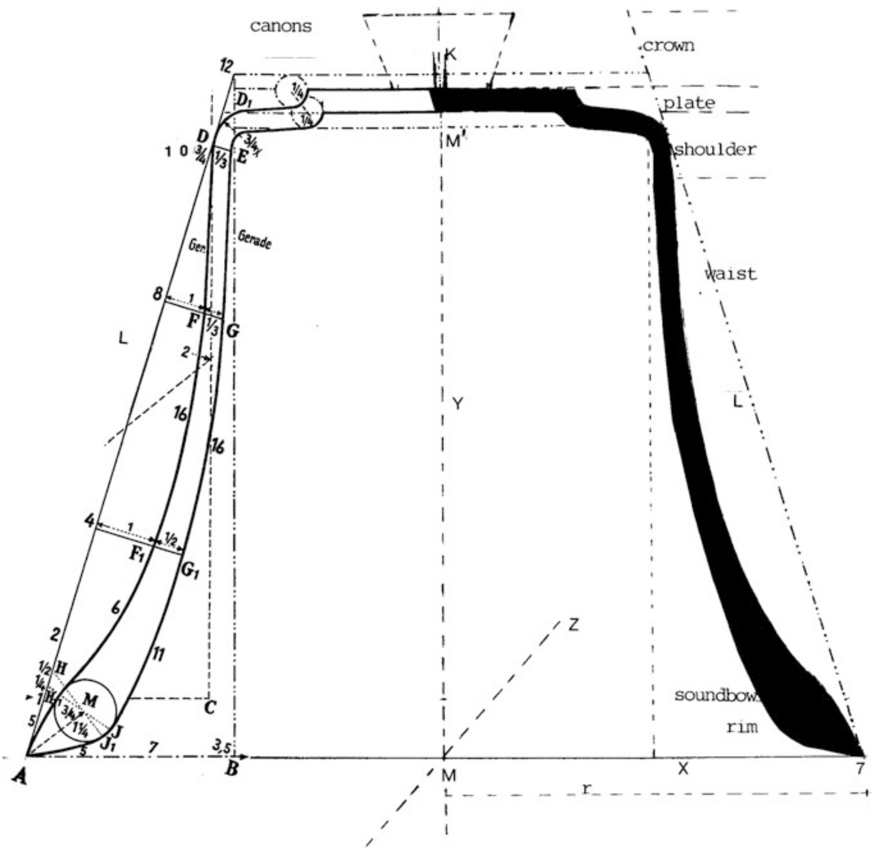


Fig. 1 Bell wall profile, Geert de Wou (adapted from [11])

achieved by scaling (up and down) the overall size of the template as well as the relevant physical parameters (diameter, height, thickness of wall at certain points, weight) associated with the bell shape. However, for reasons of structural stability and also with respect to aspects of sound and tuning there are some limits to such an approach (in particular for small bells; for details, see [11, 12]). Nonetheless, scaling of bells according to a template was and still is a useful approach for designing carillons (see below). One famous bell cast by de Wou, the *Gloriosa* of Erfurt (dating from 1497; see [13, 14]), already shows the pattern of strong spectral components typical of the minor-third/octave bell, that is, the first five major partials exhibit a frequency ratio of close to 1:2:2.4:3:4. In the very large *Gloriosa* bell whose diameter is 257 cm, height ca. 265 cm, weight 11,450 kg, and frequency of the lowest spectral partial is ca. 80 Hz, the actual frequency ratios of the first seven strong components measured from the sound radiated from the bell are close to 1:2.1:2.47:3.04:4.09:5.21:6.1. If the second (and strongest) partial is taken as a reference, the ratios are 0.48:1:1.18:1.45:1.95:2.48:2.91. Hence, this bell and most other specimen of the minor-third type contain a minor third and a fifth within one octave and a major third as well as a fifth in the next upper octave. The presence of a minor and a major third, which form an interval of (close to) a minor ninth has a profound effect on the tone colour or timbre of the bell sound as well as on the pitch or, rather, the pitches attributed by listeners to such sounds. The partial structure of the minor-third swinging bell served as a model for carillon bells, which evolved in the 16th century and were perfected, in the 17th century, by the famous brothers François and Pieter Hemony who worked in the Netherlands since 1641 (see [8]). The Hemony brothers had met with Jacob van Eyck, an outstanding musician and musical scholar, sometime in the 1640s (cf. [15]). Van Eyck lived in Utrecht where he was involved in the development of carillons in many ways, including tuning of bells ([16], 130ff.). It seems that van Eyck was one of the early scientists who explored the phenomenon of resonance. According to a note contained in the diary of Isaac Beeckman (24th of Sept. 1633), van Eyck had told him that he could hear out some of the partials in bells without touching the bell [17]. Apparently, van Eyck used to sing or whistle a tone in order to excite a resonance in a vibrating body like a bell. He observed some of the partials relative to the strike note (Beeckman mentions the "slach", which in van Eyck's scheme is the octave above the fundamental), most of all, the minor third. Van Eyck seems to have understood that the clarity of the bell's partials as well as the pureness of the pitch perceived from a bell sound, depend on the profile (curvature, thickness at certain points) of the bell's wall. The Hemony brothers put these insights to practical use when they cast bells for carillons which, to this day, are regarded unsurpassed in tonal quality. For all these bells the five lower partial frequencies have a ratio of 1:2:2.4:3:4 (or nearly so).

The City of Bruges had a carillon built, in 1675–80, by Melchior de Haze (of Antwerp) that was housed in the large belfry fronting the market place. However, this carillon was destroyed in a fire, in 1741. As a replacement, a new carillon was ordered from Joris Dumery (Georgius Du Mery) who operated a foundry in Bruges. Dumery promised to deliver "flawless" bells with harmonious sounds (cf. [8] and various documents relating to Dumery's carillon in Bruges in [18]). Dumery's fine carillon originally comprised 45 bells of which 26, cast in the years 1742–48, are extant. To this set, 21 new bells were added, in 1969, cast by Royal Eijsbouts of Asten, The Netherlands, which were designed to match the shape and sound structure of the Dumery bells. The carillon has undergone a revision in 1968/69 which, however, left the original Dumery bells untouched as far as retuning is concerned [9]. As mentioned above, the carillon underwent a complete revision in 2010/11, which included restoration (cleaning, retuning) of the Dumery bells and replacement of the 21 bells from 1969 by newly cast specimen that were adapted more closely to the geometry of the original Dumery bells [1].

1.2 Basic Data Concerning the Dumery Bells

Based on data available from two restoration reports [1, 9] and our own measurements, the following table lists the year of manufacture, the musical notes of the 26 Dumery bells according to their fundamental frequencies f_1, the weight (kg) and the diameter (mm) of each bell. The diameter is always taken at the lip or rim of a bell and hence represents its maximum width at the opening.

The weights, diameters and fundamental frequencies of these bells relate clearly to each other as shown in Figs. 2, 3, and 4. Bell founders knew from experience that bells of identical wall profile and material can be scaled according to proportionality rules which state that two fundamental frequencies relate to each other like the inverse ratio of the diameters of the respective bells, that is $f_1/f_2 = d_2/d_1$. This means that, with identical bell shape, halving the diameter means doubling the fundamental frequency of a bell as is evident from Table 1 and Fig. 2.

Likewise, fundamental frequencies can be related to the weights and diameters of bells where the fundamentals of two bells, f_1 and f_2, relate to their mass (weight in kg) and diameter according to

$$\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3 = \left(\frac{f_2}{f_1}\right)^3.$$

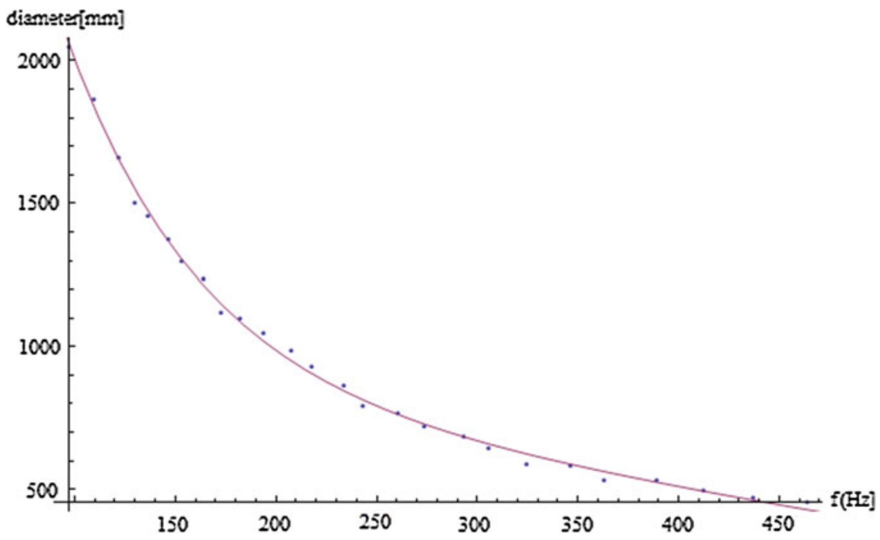


Fig. 2 Diameters and fundamental frequencies of the 26 historic Dumery bells (represented by *dots*)

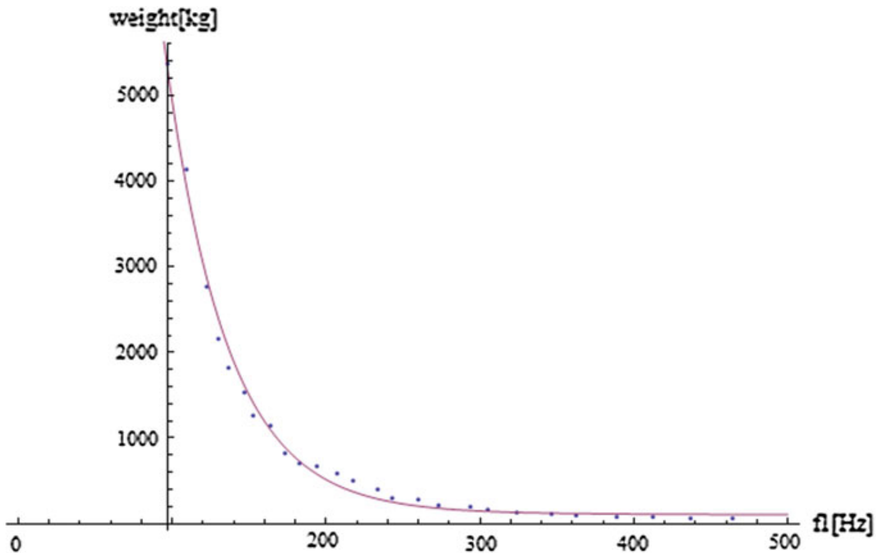


Fig. 3 Weights and fundamental frequencies of 26 historic Dumery bells

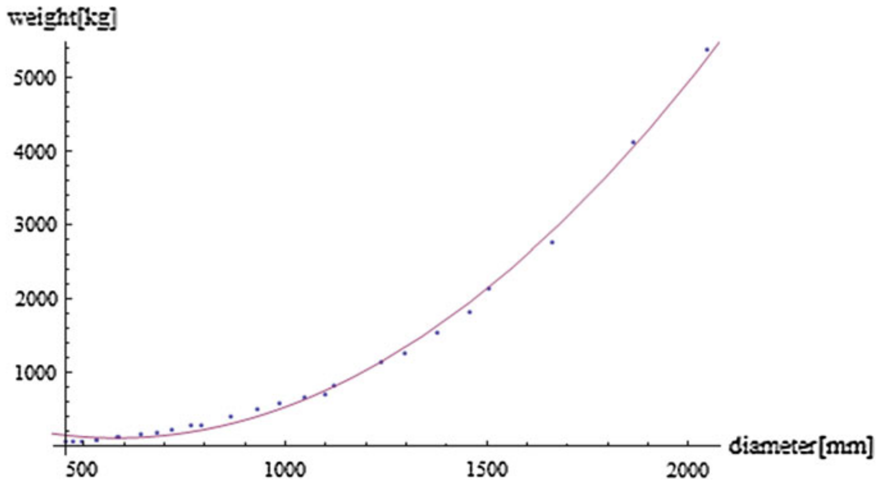


Fig. 4 Weights and diameters of the 26 Dumery carillon bells

This implies that the weight of a bell with a fundamental one octave above that of another bell will be about 1/8 of the bell with the fundamental one octave lower (Table 1 and Fig. 3). Comparing, for example, the G_2 and the G_3 Dumery bells, their weights show a ratio of ca. 8:1. Using the equation above, the cubed ratio of the two diameters yields ca. 7.5 and that of the fundamental frequencies gives 7.86.

Table 1 Basic data, 26 Dumery bells, carillon of Bruges

Bell	Date	Note/Tone	f_1(Hz)	Weight (kg)	Diameter (mm)
1	1744	G_2	97.51	5.378	2047
2	1748	A_2	109.43	4.133	1864
3	1748	B_2	122.05	2.766	1661
4	1743	C_3	129.62	2.153	1503
5	1745	$C\#_3$	136.68	1.825	1457
6	1744	D_3	146.61	1.54	1376
7	1745	Eb_3	153	1.27	1295
8	1744	E_3	164.02	1.145	1236
9	1743	F_3	172.85	830	1120
10	1745	$F\#_3$	182.27	710	1098
11	1745	G_3	193.88	670	1046
12	1745	$G\#_3$	207.51	590	984
13	1745	A_3	217.69	501	930
14	1745	Bb_3	233.84	398	865
15	1743	B_3	243.11	295	793
16	1745	C_4	260.28	281	766
17	1743	$C\#_4$	273.35	222	718
18	1743	D_4	293.56	196	683
19	1745	Eb_4	305.44	165	642
20	1742	E_4	324.18	123	587
21	1743	F_4	346.38	120	583
22	1742	$F\#_4$	362.75	95	533
23	1743	G_4	389.22	81	531
24	1745	$G\#_4$	412.23	77	495
25	1743	A_4	436.65	63	472
26	1743	Bb_4	463.78	61	455

Finally, the weights of bells in a tuned carillon relate to the diameters and the average wall thickness in orderly fashion (see [12, 19]). This can be approximated by the equation $M = c_1hd^2$, where M is the weight (kg), d is the diameter (m), h is the average wall thickness and c_1 is a constant. The data from the 26 Dumery bells again indicate that halving the diameter and doubling the fundamental frequency coincides with a reduction of the weight to about 1/8 to the bell tuned one octave lower (Fig. 4).

Given the fact that the 26 historic bells (Table 1) show an orderly progression in regard to weight, diameter, and fundamental frequency, one can fit a function to the data for each of the parameters whose graph permits a qualitative assessment of the goodness-of-fit (Figs. 2, 3 and 4). The relatively small deviations of the bell parameter values for the 26 bells from the ideal ratios defined by proportionality rules can be taken as a quality mark indicating that Dumery in fact was able to cast carillon bells at a very high level of craftsmanship and precision of tuning.

2 Basics of Bell Acoustics

Bells belong to the class of musical instruments known as idiophones, literally meaning 'self-sounding' instruments. The basic concept is that a solid (such as a bar, plate, or shell) is used as a vibrating body set to vibration by an impulse affected by means of another body (e.g., a mallet or, in the case of the bell, a metal clapper). Examples of idiophones are xylophones (of which different types are found in Africa and in Southeast Asia), metallophones and gong chimes as are central in Javanese and Balinese gamelan music, and carillons comprising a set of tuned bells. In regard to vibration, a bar of quadratic, rectangular, or circular cross section appears as a relatively simple geometry in particular if the diameter of the cross section is small in relation to the length l of such a bar that can be viewed as a one-dimensional continuum (as in the classical Euler-Bernoulli theory). Likewise, a thin flat plate can be viewed as a two-dimensional structure if the thickness h of the plate is very small in relation to its length and width (in a rectangular plate) or its diameter (in a circular plate). Obviously, in practice one often has to deal with geometries that are more complex. For example, bronze plates such as used for Javanese and Balinese metallophones often exhibit a certain thickness and a trapezoid cross section. Moreover, such plates can be curved to some degree. Due to these factors, in plates of the Balinese *gender* modes of vibration can be identified in addition to the standard pattern known from bars and small flat plates (see [20]). In a swinging or carillon bell, the geometry is even more complex due to the variable diameter and thickness of its wall. The bell wall profile plus the shoulder and plate (with the canons attached to it) make up a compound structure that renders calculation of modes of vibration difficult. Before FEM (Finite element method) and BEM (Boundary element method) modelling became available as standard methodology, calculation of shell vibrational modes usually required simplifications, taking characteristics of standard shell models (such as a cylinder or a spherical shell) as a reference (for in-depth treatment of shell vibration theory, see [21–25], [26], Chap. 7, [27, 28]).

For a simplified model of a bell, one may conceive first of a thin circular plate (2D model) and then of a sphere or hemisphere produced from bending a flat circular plate (3D model). If a circular plate free around its circumference is set to vibration, acoustic figures well-known from the seminal work of Chladni [29] can be observed. Such figures result from the node lines between segments of the plate vibrating in opposite phase. For a circular plate, the number of segments representing modes of vibration can be ordered according to nodal meridians (m) and nodal circles (n). Such patterns can be observed (with certain modifications) also in square as well as in rectangular plates (see [19]). Vibration where segments of a plate move inward and outward in general results from flexural or bending waves, which are acoustically the most effective in regard to sound radiation (see below). However, in bars and plates also longitudinal motion occurs inside the structure, as well as torsional vibration due to tangential motion. Since elastic solids exhibit

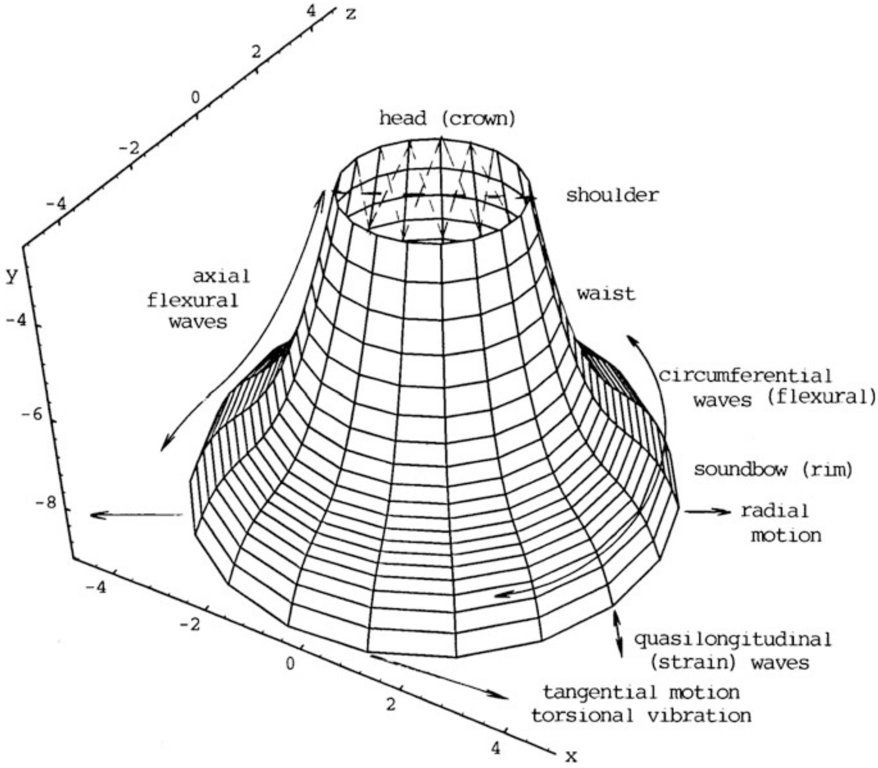


Fig. 5 Types of vibration in a bell-like structure

stress and strain when forced to vibrate, a special type of strain wave labelled quasi-longitudinal is also found.

Taking a sphere or hemisphere as a model, it is straightforward to assume that such a structure, when struck with a mallet at a point on the wall near the edge, may allow flexural motion of its wall around its circumference as well as possibly in axial direction. Furthermore, tangential motion seems feasible. In addition, a structure such as a plate or a hemisphere can exhibit quasi-longitudinal motion within the thickness of a (flat or curved) plate. The respective types of vibration are usually labelled flexural or bending, torsional, and quasi-longitudinal (due to stress/strain), respectively. Taking a structure closer to a real bell as model, the types of motion can be indicated as in Fig. 5.

Wave propagation for flexural motion in such a structure differs from propagation in fluids (such as air) in that shear forces between molecules occur. Consequently, for elastic solids one has to take into account bending stiffness, rotatory inertia and other parameters relating to the geometry as well as to material properties of vibrating structures (see [23, 26–28], [30], Chap. 7, [31]). Wave propagation in solids differs from fluids in that the phase velocity for bending waves is dependent on the frequency of its components. Hence, each component in a wavepacket travels with a particular phase velocity, c_B. While wave propagation in air is constant at $c = \lambda f$ and $\lambda = c/f$, phase velocity for propagation of bending waves in a thin plate of unlimited extension and free around its circumference approximately is $c_B \sim \sqrt{f}$. Phase velocity thus grows in proportion to the root of each frequency component, and is inversely proportional to the wavelength for which $\lambda_B \sim 1/\sqrt{f}$. More precisely, phase velocity in a homogeneous plate is

$$C_B = \sqrt{\omega}\sqrt[4]{B'/m'},$$

where $\omega = 2\pi f$, B' is the bending stiffness, and $m' = \rho h$ is the mass per unit area of the plate. Bending stiffness in a homogeneous plate can be calculated as

$$B' = \frac{E}{1-\mu^2} I',$$

where E is Young's modulus, μ is the Poisson ratio (the value is 0.3), and $I' = h^3/12$ is the geometrical or area moment of inertia. Doubling the thickness h of a plate of given dimensions not only doubles its mass but raises B' by a factor of eight. If thickness h is of significance, bending stiffness has to be calculated as

$$B' = \frac{Eh^3}{12(1-\mu^2)} \quad \left[\mathrm{N\,mm^{-1}}\right];$$

phase velocity then becomes $C_B = \sqrt[4]{\frac{Eh^2}{12(1-\mu^2)\rho}}\sqrt{\omega}$.

In a dispersive medium like a plate, besides phase velocity and group velocity also the wavenumber k_B for bending waves is dependent on frequency:

$$k_B = \sqrt[4]{\omega^2 \frac{m'}{B'}}$$

The equation for bending waves in plates can be written as

$\Delta\Delta\xi + \frac{m'}{B}\frac{\partial^2 \xi}{\partial t^2} = 0$ where $\Delta\Delta\xi \equiv \nabla^4 \xi$; the 4th differential parameter (G. Lamé) is defined as

$$\Delta\Delta\xi = \frac{\partial^4 \xi}{\partial x^4} + 2\frac{\partial^4 \xi}{\partial x^2 \partial y^2} + \frac{\partial^4 \xi}{\partial y^4}$$

The equation of motion for a circular plate with free edge and thickness h can be written in plane polar coordinates (r, φ)

$$\Delta\xi = \frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r}\frac{\partial \xi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \xi}{\partial \varphi^2}$$

where ξ is the displacement of particles and r is the radius of the plate. Solutions are found from linear combinations of Bessel functions $J_m(x)$ and hyperbolic Bessel functions $I_m(x)$. Frequency ratios for bending waves (relative to $m = 2$, $n = 0$ taken as 1.00) in a circular plate are inharmonic (for both $\mu = 0.25$ and $\mu = 0.3$) though some approximately harmonic ratios can also be found.

For a shell of constant radius and thickness, bending stiffness can be calculated as in thick plates; strain stiffness D of a shell is calculated like $D = \frac{Eh}{1-\mu^2}$ [N mm^{-1}]; from B' and D a factor for shells of radius r and thickness h can be calculated, which is

$$\beta = \frac{B'}{D}\frac{1}{r^2} = \frac{h^2}{12r^2}$$

The equation of motion for bending waves in shallow spherical shells can be derived from the equation of motion for flat plates by inserting an additional term (see [19] Chap. 3), which is $\nabla^2 H/R$. Here H is Airy's stress function and R is the radius of the shell's curvature. The example demonstrates that shells of simple geometry can be viewed as derived from flat or curved plates (see also Junger and Feit [26], Chap. 7). Also, several so-called 'membrane' theories of shells (see [21], Chap. II–IV, [23], § 4.3; the name was chosen for some analogies to vibration theory of membranes) assume a very thin, elastic shell in order to simplify parameters relevant for vibration. However, for bending waves in shells of a more complex geometry including thick walls (that is, h is significant relative to the length l of a cylinder or cone), displacement kinematics needs to include shear forces and other factors (see [21], Chap. V, VI, [22], [23], § 8.34). In sum, calculation of modal frequencies in bells is complicated because parameters such as variable diameter, variable wall thickness and, hence, variable bending stiffness, have to be taken into account (see [24, 25]).

From the equations stated above it is evident that propagation of bending waves in solids depends on the geometry and the material parameters of the structure that is vibrating free from outside forces once the impulse force $F(t)$ necessary to excite normal modes of vibration has been applied. Frequency dispersion in solids means that, for bending waves (flexural motion), eigenfrequencies do not form harmonic series but must be determined for certain geometries (such as rectangular or circular plates, cylindrical or conical shells) according to boundary conditions (the most relevant conditions concern free, simply supported or clamped edges) and material parameters (see [27, 31]). Spectra of bars, plates and shells undergoing flexural vibration are essentially inharmonic (for a detailed account including empirical data, see [32]).

2.1 Material and Shape

Bells are made of some alloy (such as bronze, brass, iron or steel) which has characteristic properties relevant for vibration behaviour. In modern Western bells, bronze in general contains about 78–80 % Cu and 20–22 % Sn (plus some small parts of other metals such as Pb, which can be found in a more significant percentage in historic bells). For bell bronze, $E = 9650\ \text{kp/mm}^2 = 94.14$ GPa, $\mu = 0.3$ and density $\rho = 8400\ \text{kg/m}^3$ (for more technical data, see [33]). As to the shape of bells, one can find more or less cylindrical or 'beehive' specimen (as in East Asia) as well as basically conical structures in most Western swinging and carillon bells, which can be derived from a trapezoid representing the cross section of a cone truncated at the peak. Many bell founders used such a frame for drawing a bell's wall profile by means of compasses (see Fig. 1). The wall viewed relative to a symmetry axis located right in the middle of the bell obeys to the principles of a rotational shell with radius r and height l. The basic cylindrical or conical shell in the case of common bells hanging from a support is closed at one end by a stiff and relatively thick plate which, in the case of western swinging and carillon bells, carries canons used to fastening the bell to a headstock (which in turn can be part of a bell frame).

Due to the axisymmetrical shape of the bell mantle, mass distribution ideally is even around the middle axis. However, the geometry of most of the historical swinging bells found in many countries in Europe as well as bells that are used in carillons is more complex in that the radius r, and hence the diameter d of the bell vary along its middle axis $M - M'$(see Fig. 1) as does the thickness h of the bell's wall. Thereby, also bending stiffness B varies with respect to the bell profile. At the so-called soundbow (or ring), stiffness is large consequent to the thickness of the bell's wall at this point whereas at the so-called shoulder of the bell, stiffness again is considerable yet has to be attributed to the shape itself, namely the circular plate adjacent to the shoulder which closes the bell. The plate carries the canons (not detailed in Fig. 1) necessary to fasten a hanging bell to a support. Furthermore, thickness of the wall and stiffness are factors that bear to the internal damping (attributed to stress in the material) of the bell. Because of these features, the vibration theory for this type of compound shell is quite complex (for vibration theory of shells, see [21–23, 25–28]).

2.2 Excitation of Normal Modes and Radiation of Sound

Western swinging and carillon bells are set to vibration by applying a force via a clapper. In carillon bells, the clapper is activated by a player from a clavier from which a transmission leads to the clapper inside the bell. Alternatively, vibration in carillon bells can be excited by mallets in case carillon bells are connected to a clock-work which activates the mallets. While the clapper usually hits the inner wall at the soundbow near the rim, the mallet excites the bell from the outside (at about the same region on the bell's wall). For the clapper hitting the wall, the contact time is quite short at, in general, 0.7–1.5 ms depending on the force applied (and, thus, on the velocity of the clapper motion) as well as on the mass and material of the clapper ball (for data, see [6, 24, 34]). The clapper or hammer transmits a force $F(t)$ via the impulse; the magnitude of the force depends on the impedance, Z, at the bell's surface (in regard to flexural vibrations which are of foremost concern). In earlier experiments it was found that, roughly, the mass of the clapper and the contact time are proportional, that is, doubling the mass of a clapper means approximately doubling the contact time with the bell. This in turn leads to diminishing amplitudes of higher partials in the sound spectrum (cf. Grützmacher et al. [24], 41–43). Impact dynamics in carillon bells investigated in detail [6] revealed that, for an impedance of a bell of $Z = 3 \times 10^4$ kg s^{-1} (a realistic value for a bell of given dimensions), the clapper comes to rest against the bell's wall, and is pushed back immediately by a returning vibromotive pulse. Hence, a very short contact time ensures maximum energy transfer to the bell which, in turn, means that very many eigenmodes are elicited. Since the contact relates closely to the shape of the impulse (duration, height), it is also of influence on spectral energy distribution; short contacts account for 'brighter' sounds because many higher modes are elicited. It has been suggested that re-voicing of carillon bells (whereby the original curvature at the site of the clapper/hammer impact is restored) is suited (a) to increase the impact duration so that the sound becomes more 'mellow' (and less inharmonious), and (b) to make the impact time more dependent on impact velocity whereby 'strong' playing (Dutch: *sterke slag*) makes notes both louder and brighter (the same effect is observed when playing a piano at various dynamic levels). In fact, carillonists can influence the force transmitted by an impulse to a bell by their individual way of playing and can thereby control the dynamics and spectral structure of bell sounds to some degree.

The sound in a bell is produced by vibrations of the bell's wall (including the top plate to some extent) in a pattern of eigenmodes at certain eigenfrequencies. While several lower normal modes can be identified quite easily by detecting the number of nodal meridians and nodal circles (see below), it may be arduous to investigate higher modes due to the small areas involved as vibrating in opposite phase and also because certain modes have nearly identical eigenfrequencies. Due to the bell's axial (rotational) symmetry, modes of flexural vibrations with $m > 2$ are found in doublets (called *Zwillingstöne* in German terminology). This means that such modes occur in nearly degenerate pairs (cf. [35] where the two mode frequencies differ but little. In vibrating systems, two (or more) eigenfunctions and eigenmodes that produce the same eigenfrequency are called degenerate. This condition is often met in quadratic membranes and plates where ω_{mn} can be the same as ω_{nm} (e.g., $\omega_{21} = \omega_{12}$; see [36], 176). However, for the so-called 'breathing mode' of shells ($m = 0$), only one natural frequency obtains (cf. [26], Chap. 7), that is, this mode is a singlet. In case the rotational symmetry of the bell around its middle axis would be perfect, the meridians of one member of the degenerate pair would be found lying exactly on the vibration antinodes of the other.

Historically, the development of the theory of vibration for bells did stem from calculations Leonard Euler provided for vibrating rings. He viewed a bell as a series of *annuli elementares* (simple rings, see [37]). One in fact may break down the bell's mantle into several ring-like segments. For Asian bells close in shape to a 'beehive', conical ring elements were found suited to FEM modelling (cf. Chung and Lee [38]).

Lord Rayleigh [28], Vol. 1, §§ 232–235, on the basis of a cylindrical shell taken as a curved plate, gave a detailed account of the modes of vibration found in a typical bell with (almost perfect) rotational symmetry. He considered flexural vibrations around the bell's circumference (the zero points of which result in nodal meridians at equal distances), and along the bell's axis (the zeros of which result in nodal circles). Nodal meridians and nodal circles divide the surface into segments. The number of nodal meridians and nodal circles defines the number of segments which vibrate opposite in phase to each other (see [24]). Since the corpus of a typical western bell consists of a massive ring near the mouth plus a more or less cylindrical shell (i.e., the waist) added to it (see Figs. 1 and 2), certain modes of vibration appear to be 'ring driven' while others are regarded as being 'shell driven'. It should be mentioned that, in the terminology used by Charnley and Perrin [39] in regard to bells, axial motion is labelled 'meridian-tangential', and tangential is specified as 'ring-tangential'.

Modes of vibration in bells have been analyzed with various methods, and have been described and classified in great detail (cf. [24, 35, 40], [19], Chap. 21.1, [41–45]).

As with other vibrating systems, one has to distinguish between inextensional and extensional modes of vibration (see [28], § 232), the latter involving stretching of the bell's corpus whereas for inextensional modes a neutral ring for each radial plane can be assumed. Though flexural vibrations are most prominent in bells, torsional vibration was also observed [39]. Torsional ('twisting') vibration is well known from rods. In such structures as well as in plates, another type of vibration is found usually labelled 'quasi-longitudinal' (see [30], 78ff.). Quasi-longitudinal waves involve strain/stress of the material along the longitudinal axis as well as contraction of the cross section. In a bar or rod of length l (x-axis) particles thereby are displaced also in the direction of the y- and the z-axis, respectively (in a plate, this motion is almost restricted to the z-dimension); therefore, phase velocity c_D for such waves is considerably smaller than that of pure longitudinal waves. It seems reasonable that quasi-longitudinal (strain) waves might occur also in the wall of bells, in particular in the waist where the wall is relatively thin. There are some indications for such types of vibration in historical bells; in one instance, a mode of vibration was reported having a frequency close to that of the nominal, and being strong enough to interfere with that partial (see [41], p. 2004).

Among the methods that have been employed to investigate normal modes of vibration, and to visualize patterns of vibration in bells, are time-averaged hologram interferograms [44], finite elements (FEM, see [13, 42, 46, 47]), and modal analysis (MA, see [7, 40]). A special study on the mode structure formulated as a dynamic contact problem of bell and clapper was undertaken by Lau et al. [34] with FEM methodology.

Investigations of bells are mostly confined to types of flexural vibration because these cause motion normal to the bell's surface. Since the walls of the bell couple directly to the sound field, motion normal to the surface will radiate most of the sound that becomes audible. Since efficient radiation of bending waves from the vibrating surface of a bell requires that the wave speed C_B must be at least equal to, or greater than, the sound speed in air (340 m/s; see [30], 457ff.), it follows that modes of vibration of higher order can be more prominent in the spectrum than those of lower order.

Radiation of sound from the bell's surface is quite complex due to the different size parts of the wall cover for various modes of vibration, and also due to the curvature of the wall. Consequently, there are different directivity patterns for different normal modes ([47] and additional material presented in a lecture). As to the decay of individual partials, there are several types of damping to be considered. First, damping is small within bell bronze (with a very low damping factor $\delta \approx 0.0004$) in case the material is homogeneous in molecular particle structure and of little porosity (see [33]). Structural damping resulting from the shape of the vibrating body plays a role at the bell's shoulder where the more or less cylindrical or conical part joins the plate at an angle close to 90° (see Fig. 1). Further, there is viscous damping at the boundary between bell and surrounding air as well as acoustical damping within the sound field (as to aspects of sound radiation from bells including damping, see [5, 24], [19], Chap. 21.11, [44, 47]). Internal as well as acoustical damping for at least some of the normal modes in bells seems to be rather small since the decay time in particular of the hum can be very long. For the *Gloriosa* of Erfurt (a huge bell, see above), the sound reportedly was audible for 310 s ([14], 113; in 1985, the *Gloriosa* underwent a major repair, which extended the decay time to ca. 370 s). As a rule of thumb, for large bells a decay time (60 dB from initial level) of the partials dominant in the sound (in particular the prime and the minor third) of $t_d > 10$ s seems reasonable. The hum, though usually relatively weak in amplitude, would be audible much longer..

Note that in carillon bells, a very long decay of partials is not necessarily useful since it could hamper perception of melodic lines (and particularly so when a polyphonic piece of music is played). The overall sound intensity measured from bell no. 2 as radiated in one direction (the site of the microphone used in the recording) shows a decay of more than 40 dB in less than 4 s from maximum (Fig. 6).

The normal modes of vibration (m, n) typical for octave minor third bells, that is, for bells which have the interval of a perfect (or nearly so) octave between hum and prime as well as between prime and nominal, and a strong minor third above the prime (see [41, 48]) are listed in Table 2.

The number of nodal meridians is either given as full meridians (extending over the top of the bell to the opposite side of the wall) or as half meridians ($2m$). Full meridians in Table 2 are given in brackets. In most cases, the partials that have from 4 to 26 nodal meridians but only one nodal circle (or none, as is the case with the

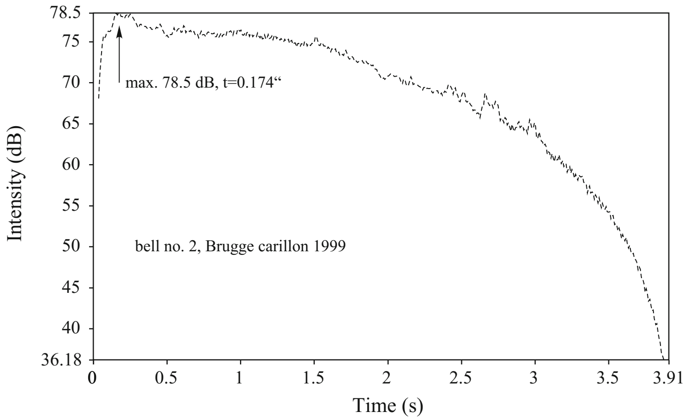


Fig. 6 Sound intensity decay over time, bell no. 2, Brugge carillon

Table 2 Scheme of bell partials and respective modes of vibration

Partial	Frequency ratio to prime	Mode (nodal meridians *m*; nodal circles *n*)
Hum	0.5	4, 0 [2, 0]
Prime [fundamental]	1	4, 1 [2, 1]
Tierce [minor third]	1.2	6, 1 [3, 1]
Quint [fifth]	1.5	6, 1 [3, 1]
Nominal [octave]	2	8, 1 [4, 1]
Tenth [major third]	2.52	8, 1 [4, 1]
Twelfth [fifth]	3	10, 1 [5, 1]
Thirteenth [major sixth]	3.36	10, 1 [5, 1]
Double octave	4	12, 1 [6, 1]
Upper fourth	5.33	14, 1 [7, 1]
Upper major sixth	6.73	16, 1 [8, 1]
Triple octave	8	18, 1 [9, 1]
Minor third	9.5	20, 1 [10, 1]
Fourth	10.68	22, 1 [11, 1]
Fifth	12	24, 1 [12, 1]
Major sixth	13.45	26, 1 [13, 1]

hum) will form the strongest peaks in spectra obtained from actual bell sounds recorded in the free field. The phenomenon that there are pairs of modes that have identical numbers of meridians and circles yet represent different partials (for example, 6, 1 [3, 1], comprising the minor third and the fifth above the fundamental), stems from the fact that the position of the nodal circle on the bell’s surface can vary considerably. The nodal circle for the minor third (6,1) will be found on the waist while the nodal circle for the fifth (6,1) is located near the soundbow of the bell (cf. [19], Chap. 21.1).

Even though the vibrational modes listed in Table 2 in many cases will be the most prominent ones, other modes can be found. For example, the eleventh (ratio to the prime/fundamental: 2.67) has six half-meridians that are equivalent to three full meridians conceived of as extending over the crown of the bell, and two nodal circles ([13, 14]). In measurements based on acoustical excitation of an English church bell, no less than 134 partials representing modes of vibration have been found in the frequency range from 292 Hz (hum note) up to 9300 Hz [42]. Of course, it is not possible to relate all these modes to partials that constitute sections of harmonic series such as will be found in Table 1. In fact, actual bell sounds often contain many more inharmonic components than just the minor third which in many bells is strong in amplitude, and therefore is characteristic of most of our church and carillon bells (see Table 3). The minor third spectral component can interfere with the partial of the major third (or tenth) that is found one octave higher. The spectral composition of minor-third bells, which is inharmonic to some extent, has at times been found unsuited to rendering musical pieces written in a major key. In fact, performance of a piece in a major key played on a typical minor-third carillon may appear ambivalent in regard to perception and musical composition. This issue played a role in the development of major third bells that, to be sure, have a much different geometry and shape of the wall's profile (see [49]).

In case the rotational symmetry of the bell around its middle axis would be perfect, the meridians of one member of a degenerate pair would match the vibration antinodes of the other. Since especially historical swinging bells rarely have been cast to result in perfect mass symmetry, and may exhibit both variations in the thickness along the wall as well as deviations from a perfect ring with respect to the cross section, the two members of a pair have different vibration frequencies. In the spectrum of the bell sound one therefore quite often finds twin peaks representing the two members of a (nearly) degenerate pair. The distance of the peaks increases with the amount of deviation from perfect axial symmetry. Typically, the frequency difference of the two members of such a pair is from less than 1 Hz to a few Hertz (in cents, the difference often is less than 100 cents and sometimes even < 50 cents). Perceptually, the effect can range from a slight shimmering of spectral components (comparable to the 'chorus' effect applied to electric guitars) to audible amplitude modulation (AM) that will be registered as beats or roughness. In case the effect is pronounced, it usually is labelled warble (cf. [50, 35]). There are indications that bell founders of the past deliberately may have allowed small deviations from symmetry; slightly eccentric shapes where degenerate pairs are separated into two independent components possibly were employed to reduce warble. The effect of warble as a source of amplitude modulation is shown in Fig. 7 for bell no. 18 of the Dumery carillon. AM is significant and fairly regular relative to the average decay of the sound level (indicated by the dashed line in Fig. 7).

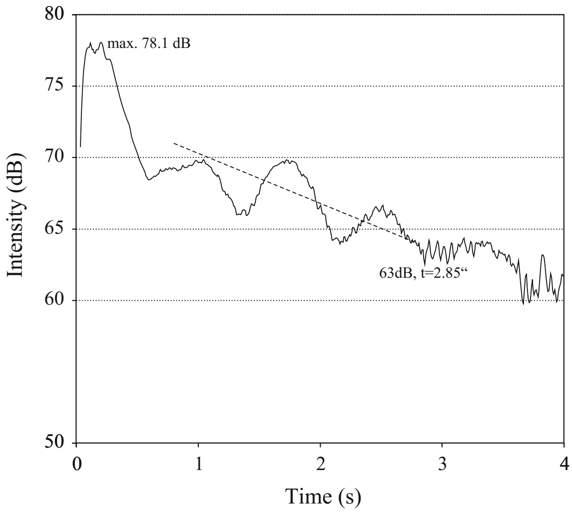
Though spectra of bars, plates and shells are essentially inharmonic, the profile of the typical minor third bell yields spectral components many of which can be assigned to several harmonic series that are, however, incomplete. Also, there are

Table 3 Bruges, bell no. 1, partial structure, main components 0–3 kHz

No.	f[Hz]	A [dB]	ratio f_n/f_1 [ss]	f[Hz]	A [dB]	Ratio f_n/f_1 [ir]	Partial name
1	97.47	53.0	1	97.51	68.3	1	Hum
2				149.95	40.3	1.54	
3	195.69	66.1	2.0	195.74	71.3	2.0	Prime
4	233.48	64.8	2.395	233.49	68.3	2.395	Tierce (minor third)
5	294.65	44.7	3.023	294.94	28.4	3.025	Quint (fifth)
6	391.37	67.7	4.015	391.34	42.2	4.013	Nominal (octave)
7	488.91	40.4	5.016				Tenth (major third)
8	493.50	47.8	5.063	493.54	39.1	5.061	Tenth (major third)
9	515.27	47.6	5.268	515.29	44.2	5.284	
10	525.59	54.7	5.392	526.40	35.1	5.398	
11a	574.09	44.0	5.89				
11b	577.37	46.9	5.923	576.98	23.8	5.917	
12	590.02	60.4	6.053	590.15	44.9	6.052	Twelfth (fifth)
13	628.55	44.1	6.45	628.70	30.8	6.448	Thirteenth (sixth)
14	683.79	34.3	7.015				
15a	819.44	50.1	8.41	819.80	51.9	8.407	'Double octave'?
15b	819.79	50.5	8.41				
16	1073.67	54.1	11.015	1073.63	48.2	11.01	Upper fourth
17a	1091.47	42.0	11.2				
17b	1091.82	42.0	11.2				
18	1197.84	30.6	12.29	1197.96	28.2	12.28	
19	1345.68	46.7	13.8	1345.94	44.8	13.8	Upper major sixth
20	1630.94	35.5	16.73	(a) 1631.14	32.5	16.73	Triple octave
				(b) 1631.54	34.5	16.73	
21	1925.31	38.0	19.75	1925.14	36.1	19.743	Minor third
22	2223.31	27.7	22.81	2223.69	39.0	22.805	Fourth
23	2524.63	19.8	25.9	2524.91	38.8	25.893	Fifth
24	2826.01	28.2	29.0	2826.04	39.8	28.98	

components that have inharmonic frequency relations both with the prime and the hum (either can be taken as fundamental), and among each other. Furthermore, because of the profile necessary to produce the minor third, the octaves even in many swinging bells designed as so-called octave bells are hardly perfect, and rather tend to show some characteristic deviations (cf. [48], also data in [51]).

Fig. 7 AM in the sound of bell no. 18; average decay of level (*dashed line*)



3 Inner Harmony and Tuning

As stated above, bell profiles since about 1500 were designed to produce the first five major partials in frequency ratios as close as possible to 1:2:2.4:3:4, and with partials above this series matching the scheme listed in Table 2 in good agreement. In practice, however, one will always find deviations from these ideal frequency ratios, and one may also encounter spectral components interspersed, as so-called ‘mixture partials’, between the ‘principal partials’ hum, prime, tierce, etc. In order to evaluate sound characteristics of individual swinging and carillon bells, it is always useful to study those individual modes of vibration that can be excited in the bell (what can be done with various methods, see Grützmacher et al. [24], [41, 42]) as well as to identify partials and other components in the sound radiated from a peculiar bell. Table 3 lists frequencies from sound spectral analysis (recordings in 1999 and 2000) as well as from impulse response measurements (recorded in 2014) taken from bell no. 1 of the carillon. Frequency ratios are given for a comparison of the partial structure before and after restoration. The amplitudes also included in this table should be taken as a relative measure only (since depending on the site of recording); however, the readings indicate the relative strength of partials within each set of data (sound spectrum = ss; impulse response = ir).

Data from both measurements in general agree very well (indicating that retuning of this bell, in 2010, has been very slight, if detectable at all). Some of the partials (see nos. 1, 3, 4, 5, 6, 7, 12) are very close to harmonic ratios while some other deviate markedly (e.g., the double octave, which is clearly stretched), and thus create inharmonicity in the spectrum. Moreover, there are some doublets (pairs of closely spaced frequencies such as 11a/11b) which introduce amplitude modulation besides the ‘chorus effect’ they exert on spectral pitches. In the data representing the impulse response only one such doublet is included (at the triple octave). Detecting doublets (which can occur for normal modes $m > 2$) with accelerometers depends largely on where these are put around the circumference of the bell relative to the point of excitation. However, doublets are contained in the sound radiated from the bell where their frequencies can be precisely determined by means of spectral analysis using FFT of sufficient window size plus suitable peak interpolation (if needed). Small differences in frequency between a nearly degenerate pair of eigenmodes indicate that there is a slight deviation in the bell's geometry and mass distribution from a perfect axisymmetrical pattern (see above).

The sound spectra of all 26 Dumery bells were analyzed in detail to identify the range of partials within a band of 0–3 kHz for the larger bells, and up to 5 or 6 kHz for the smaller bells. Taking those partials into account that carry sufficient energy to contribute to the overall sound within the first 1–2 s after excitation of the bell, there are in general some 30–40 spectral components to be considered. See, for example, the spectrum of bell no. 9 (Fig. 8) where the partials up to 5.4 kHz are displayed. Partials 1–5 (hum, prime, tierce, fifth, nominal) are marked with nos. 1–5. The prime (no. 2) in this sound is the strongest component while the fifth (no. 4) is rather weak. One can see several strong partials located between the nominal (octave, no. 5) and the double octave (marked DO).

Among these components, there are often doublets arising from nearly degenerate pairs of eigenmodes. For example, in bell no. 2 (Fig. 9) the hum note contains two closely spaced frequency components (f_{1a} = 109.21 Hz, A_{1a} = 64.1 dB; f_{1b} = 109.45 Hz, A_{1b} = 66.7 dB).

Since the so-called inner harmony of a minor-third bell rests on the first five principal partials whose frequency ratios should be as close as possible to 1:2:2.4:3:4, one can check the overall tuning of a set of carillons by plotting the log frequencies of these five partials as a function of the scale (expressed in HT = half tones). If the bells of the carillon are tuned in the same pattern of just or nearly just intervals, all frequencies representing the same partial would fall on a straight line. Figure 10 shows the frequency trajectories of 26 Dumery bells for the five principal partials hum, prime, tierce, fifth and nominal (or octave). One can see that most of the bell partials in fact match these trajectories fairly well though a few deviations from the template can be observed.

The hum, the prime, the tierce and the nominal (octave) are almost perfect while there is some variation in the trajectory representing the fifth. One should remember, however, that the fifth in general is weaker in amplitude (as well as in SPL) than the often very strong partials prime, tierce, and nominal. Hence, in regard to perception deviations from ideal frequency ratios for the fifth will not count as much as would deviations in these strong partials. To be sure, there is another partial that deviates from ideal frequency ratios in a more or less systematic way: the double octave typically has a frequency ratio of about 8.4 or 8.5 to the hum. For technical reasons (cf. [12, 50, 41]) bell tuning has to seek a compromise between the tuning of the first five partials, on the one hand, and the double octave as well as probably some more higher partials, on the other. To have the frequencies of the low principal partials 1–5 fall into place, the double octave must be stretched. For

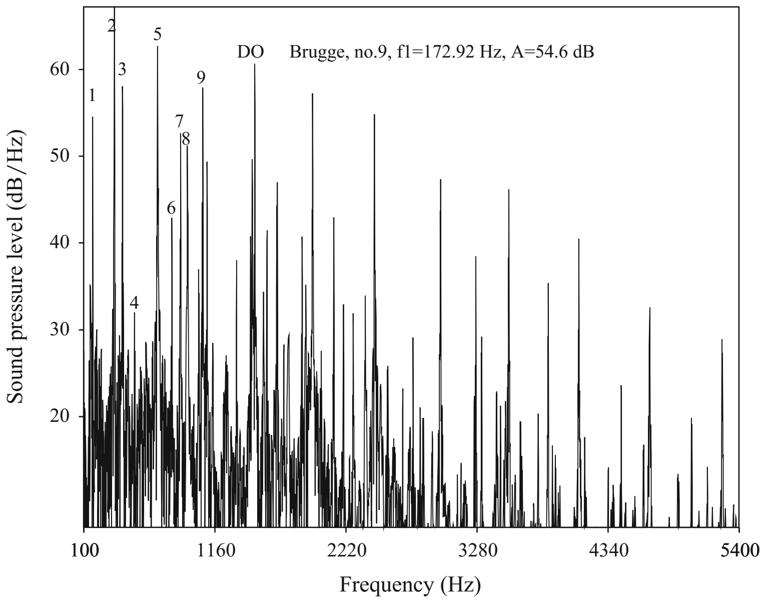


Fig. 8 Bruges, spectrum of bell no. 9. Partials 1–9 marked with nos. 1–9; *DO* double octave

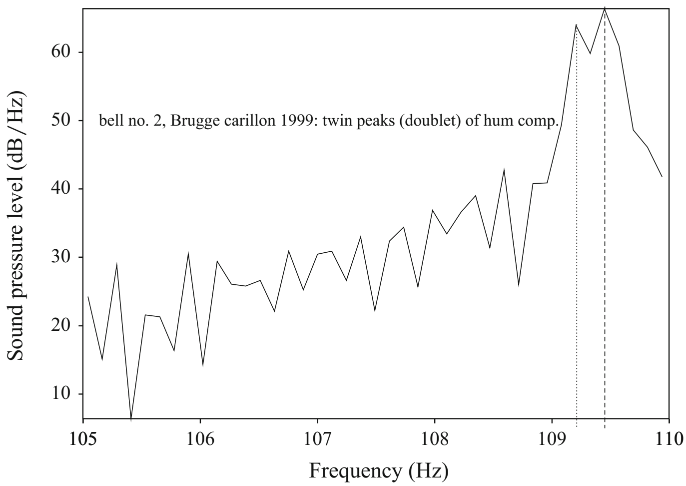


Fig. 9 Doublet, hum partial, two frequency components, bell no. 2

example, in bell no. 2 the frequency ratios are like 1:1.999:4.02:8.41 for the hum, the prime, the nominal, and the double octave. Stretching the double octave coincides with stretching the frequencies of other partials in higher octaves. Note that the double octave usually is a strong partial which, moreover, is in a frequency

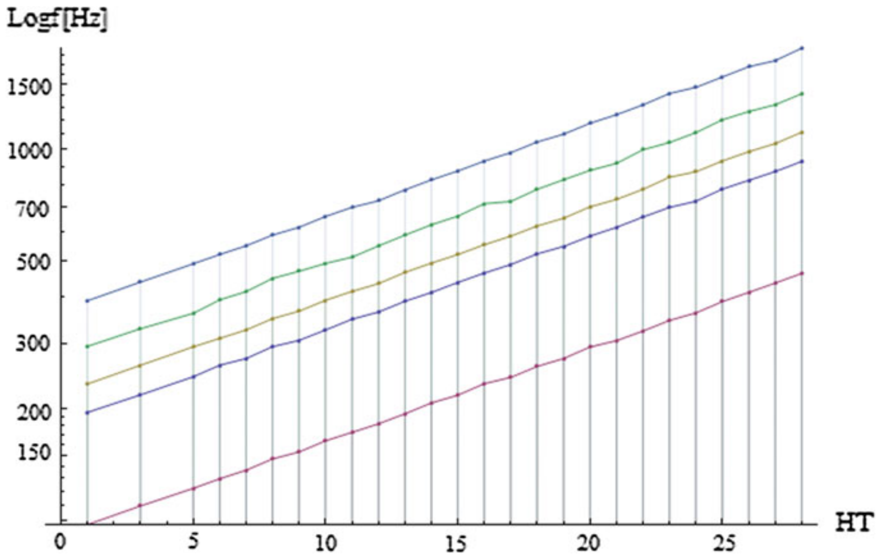


Fig. 10 Trajectories for the principal partials hum, prime, tierce, fifth and nominal (octave), 26 Dumery bells. The abscissa is ordered according to half tone intervals (no. 1 = G_2 bell, no. 3 = A_2 bell, no. 5 = B_2 bell, no. 6 = C_3 bell etc.)

region where the ear is more sensitive than in the low frequency region where the hum and (in large bells) even the prime are found. The stretching of the double octave and the other partials in its vicinity is a source of inharmonicity in the sound. It adds both to the ambiguity of pitch perception and to the sensation of a sometimes shimmering timbre.

Taking the first strong partial (the hum) as the fundamental frequency f_1 of each bell, the tuning can be based on this objective acoustic parameter. If the intervals between the fundamental frequencies are transformed into cents, the tuning can be viewed in musical terms. Table 4 contains all intervals for 26 Dumery bells in a matrix.

It has been speculated at times that the Dumery carillon might have been tuned to a meantone temperament, of which quarter-comma was the most common type (see [52]). A scale of twelve tones would consist of the following intervals:

Tone	1	2	3	4	5	6	7	8	9	10	11	12
Cents	0	75.5	193	310.5	386	503.5	579	696.5	772	889.5	1007	1082.5

Characteristic of this tuning is that it features the pure major third (frequency ratio 5/4) as the basic structural interval while so-called ‘Pythagorean’ tuning is based on a progression in pure fifths (ratio 3/2). Quarter-comma meantone tuning (see also [53]) means that the comma of nearly 22 cent marking the difference

Table 4 Tuning matrix (cents) of 26 Dumery bells based on f_1 measurements

	1 G (G2)	2 A (A2)	3 H (B2)	4 c (C3)	5 cis (C#3)	6 d (D3)	7 es (Eb3)	8 e (E3)	9 f (F3)	10 fis (F#3)	11 g (G3)	12 gis (G#3)	13 a (A3)
1 G (G2)	0												
2 A (A2)	198.6	0											
3 H (B2)	385.5	188.9	0										
4 c (C3)	489.7	293.1	104.2	0									
5 cis (C#3)	581.6	385.0	196.1	91.9	0								
6 d (D3)	703.0	506.35	317.45	213.25	121.4	0							
7 es (Eb3)	776.9	580.23	391.34	287.12	195.25	73.9	0						
8 e (E3)	897.3	700.63	511.7	407.5	315.64	194.27	120.4	0					
9 f (F3)	988.1	791.42	602.5	498.32	406.44	285.1	211.2	90.8	0				
10 fis (F#3)	1080.0	883.3	694.4	590.2	498.31	376.95	303.1	182.7	91.9	0			
11 g (G3)	1186.9	990.2	801.3	697.1	605.2	483.83	409.95	289.56	198.76	106.9	0		
12 gis (G#3)	1306.4	1107.8	918.9	814.7	722.8	601.42	527.54	407.15	316.35	224.48	117.6	0	
13 a (A3)	1389.26	1190.66	1001.75	897.56	805.68	684.31	610.43	490.04	399.24	307.37	200.48	82.9	0
14 b (Bb3)	1513.26	1314.66	1125.75	1021.56	929.68	808.31	734.43	614.04	523.24	431.37	324.48	206.9	124.0
15 h (B3)	1580.66	1382.1	1193.15	1088.96	997.08	875.71	801.83	681.44	590.64	498.77	389.88	274.3	191.4
16 c′ (C4)	1698.76	1500.2	1311.4	1207.06	1115.18	993.81	919.93	799.54	708.74	616.87	507.98	392.2	309.5
17 cis′ (C#4)	1783.76	1585.0	1396.2	1291.86	1199.98	1078.73	1004.73	884.34	793.54	701.67	592.78	477.0	394.3
18 d′ (D4)	1907.26	1708.5	1519.7	1415.36	1322.48	1202.23	1128.23	1007.84	917.04	825.17	716.28	600.5	517.8
19 es′ (Eb4)	1975.76	1777.0	1588.2	1483.86	1390.98	1270.73	1196.73	1076.34	985.54	893.67	784.78	669.0	586.3
20 e′ (E4)	2078.86	1880.1	1691.3	1586.96	1498.08	1373.83	1299.83	1179.44	1088.64	996.77	887.88	772.1	689.4
21 f′ (F4)	2197.06	1998.3	1809.5	1705.16	1612.28	1492.03	1418.03	1297.64	1206.84	1114.97	1006.08	890.3	807.6
22 fis′ (F#4)	2275.76	2077.0	1888.2	1783.86	1690.98	1570.73	1496.73	1376.34	1285.54	1193.67	1084.78	969.0	886.3
23 g′ (G4)	2398.86	2200.1	2011.3	1906.96	1814.08	1693.83	1619.83	1499.44	1408.64	1316.77	1207.88	1092.1	1009.4

(continued)

Table 4 (continued)

	1 G (G2)	2 A (A2)	3 H (B2)	4 c (C3)	5 cis (C#3)	6 d (D3)	7 es (Eb3)	8 e (E3)	9 f (F3)	10 fis (F#3)	11 g (G3)	12 gis (G#3)	13 a (A3)
24 gis′ (G#4)	2498.26	2299.5	2114.3	2006.36	1913.48	1793.23	1719.23	1598.84	1508.04	1416.17	1307.28	1191.5	1108.8
25 a′ (A4)	2597.86	2399.1	2213.9	2105.96	2013.08	1892.83	1818.83	1698.44	1607.64	1515.77	1406.88	1291.1	1208.4
26 b′ (Bb4)	2702.06	2503.3	2318.1	2210.16	2117.28	1997.03	1923.03	1802.64	1711.84	1619.97	1511.08	1395.3	1312.6

Table 5 Scheme of quarter-comma meantone temperament for 12 notes/keys per octave

–2	$f^{\#}$ ….	$c^{\#}$ ….	$g^{\#}$			
	|	|	|			
–1	d ….	a ….	e ….	b ….	$f^{\#}$...	$c^{\#}$
	|	|	|	|	|	|
0	b_b ….	f ….	c ….	g ….	d ….	a
				|	|	|
+1				e_b	b_b	f
	+11	+5.5	0	–5.5	–11 cents deviation from pure fifth	

between the pure major third 5/4 (386 cents) and the Pythagorean major third 81/64 (408 cents) is distributed to intervals of fifths and fourths whereby the fifths are narrowed and the fourths are widened, respectively. Tuning four pairs of just major thirds b^b – d –f#, f – a – c#, c – e – g#, e^b –g – b with c taken as the center, one obtains the following scheme in which just thirds are written in the vertical and connected by the sign | while the narrowed fifths are marked …. The signs 0, −1, −2, +1 denote comma differences of tones/pitches relative to tones in the 0-series of fifths.

If the system would be expanded beyond 12 pitches and tones per octave, continuation would be possible to the right of the bracket (but also on the left side). In ¼-comma meantone temperament that came into use in the 16th century (see [54]), the fifth above *c* is 5.5 cents too narrow while the fifth below *c* (equal to the fourth above *c*) is 5.5 cents wide. The error margin increases by c. 5.5 cents per fifth (a quarter of the 'syntonic comma' of 21.5 cents); 12 keys tuned to this scheme result in the chromatic scale shown above. Characteristic of this scale besides the just major third and a good approximation to the just minor third (ratio 6/5, 316 cents) as well as the just major sixth (5/3, 884 cents) is that it offers two distinct sizes for chromatic and diatonic semitones, whereby both come close to just intonation intervals in which the chromatic semitone (25/24) has only ca. 71 cents while its diatonic counterpart (16/15) is nearly 112 cents wide. Taken together, both semitones add up to a minor whole tone (25/24 * 16/15 = 10/9) while the major whole tone (ratio 9/8, derived from two fifths minus one octave, i.e. 3/2 * 3/2 * 1/2 = 9/8) spans 204 cents. The difference of course is the comma 81/80 = 21.5 cents. The meantone can be found both as the geometric mean between the two whole tone sizes (the exact value being 192.855 cents) and from a progression in narrowed fifths (see Table 5). In fact, the meantone *d* halves the just major third *c* – *e* and is one of the compromises one has to make to obtain as much as 8 just major thirds with only 12 notes and keys (on a keyboard) per octave.

Though the first semitone is missing in the Dumery carillon (as was often the case in historic instruments to cut costs for one expensive large bell that was rarely needed musically), one can see that several of the scale steps as expressed by their f_1 and respective cents approximate the ¼-comma meantone scale fairly well. There is some further evidence in this direction from the tuning of the prime partials (see below). However, there are also some significant deviations from this scale type, making it difficult to decide whether or not meantone temperament was the model for the actual tuning of the carillon. In this respect, one has to take the overall spectral structure of the bell sounds into account. Because of the inherent inharmonicity, it seems problematic to represent each complex sound by f_1 alone (as could be done, at least in principle, with harmonic complexes where $f_n = nf_1$ and where $A_n \sim 1/n$, $n = 1, 2, 3, \ldots$).

It is worth noticing that bell founders and campanologists alike often consider the tuning of a set of bells in terms of their respective 'strike note'. The strike note (from all we know, see Sect. 4) is a subjective virtual pitch difficult to measure in an objective way. Therefore, bell tunings have been given by taking the nominal (the partial situated ideally a perfect octave above the prime) as a reference (hence the name 'nominal'). Similarly, the prime has served as a reference, assuming that the prime is exactly one octave below the nominal. These assumptions may be justified in practice since, in most minor third/octave bells manufactured and tuned with appropriate care, the interval between the nominal and the prime is very close to a perfect octave. However, objective measurements and data from psychoacoustic experiments show that the pitch attributed to the strike note is not always exactly an octave below the pitch of the nominal. Moreover, the pitch of the strike note must not coincide with that of the prime partial (for such data, see e.g. [40, 51, 55, 56]).

4 The Strike Note of Bells and Carillon Tuning

Subjects since long have experienced a certain component in the sound of a bell immediately at the onset that appeared different from partials such as the hum, tierce and nominal both in sound quality and duration. Whereas the low partials, when heard individually, have a rather soft sound quality and decay slowly (what holds true in particular for the hum), there is a component in the sound which most listeners usually describe as sharp and metallic in timbre, and short in duration. Because of these attributes, it was at times believed this component resulted from the clapper impact on the bell's wall, and hence from a metal ball striking a metal surface. Though the contact of clapper and bell may give rise to some transient noise, the contact time in fact is very short (about 1 ms, see above) while the component heard as the so-called strike note seems to last, in most instances, for a fraction of a second so as to yield a more or less clear sensation of a pitch. As can be inferred from psychoacoustic data, a stable sensation of pitch from complex sounds emerges within tens of milliseconds (according to an estimate in de Cheveigné [57, 205], an average value of 66 ms seems reasonable). If the strike note is perceived as an identifiable pitch, this implies relevant sensory information derived from auditory processing of sounds must be present for some time, which can be estimated as covering probably $50 \leq t \leq 200$ ms. A time span of this size of course leaves the bell/clapper contact noise unlikely as a source for the strike note pitch.

4.1 The Strike-Note as a Virtual Pitch

Since the pitch of the strike note subjects perceived could not be attributed to a single spectral partial, the strike note was labelled 'imaginary' to mark the difference from acoustically 'real' partials [58]. Several hypotheses on the nature of the strike note were issued in studies published before ca. 1950 (summarized in [11, 51, 55]), some of which addressed the strike note as a difference tone resulting from a combination of strong partials and as a product of aural distortion. It became clear that the strike note is not a physical component somehow contained in a complex inharmonic sound but a perceptual phenomenon resulting from auditory and possibly neural processing of such complex sounds. In regard to pitch perception, a pitch resulting from any strong spectral component can be labelled 'spectral pitch' while a pitch resulting from a combination of spectral components can been called 'virtual' if it gives rise to a pitch subjects locate at a frequency where no significant spectral energy is found (see [59]). For example, in a harmonic complex such as generated from bowing a string, one can produce one spectral pitch when the fundamental f_1 is by far the strongest partial in a spectrum where partial amplitudes roll off at, e.g., $A = 1/n$, $n =$ harmonic partial no. 1, 2, 3, ... Typically, subjects will hear another spectral pitch one octave above f_1 when the second partial (f_2) is as strong or even stronger in amplitude than f_1. A virtual pitch, on the other hand, results from a combination of several harmonic as well as (depending on conditions) inharmonic spectral components. Subjects in many instances locate a virtual pitch at a frequency where no spectral energy indicative of a partial is found; however, coincidence of a spectral and a virtual pitch is possible (a case relevant for bell sounds, see below).

A reasonable explanation for the strike note understood as a virtual pitch was issued by Schouten [60], also Schouten and t'Hart [61] who argued that several periodicities from a series of harmonic partials combine into a common period, which is enough to produce a pitch percept equivalent to the frequency with which this common period repeats per second. The repetition frequency f_0 of a complex waveshape resulting from the superposition of several consecutive harmonics equals the fundamental frequency f_1 of that harmonic series and gives rise to a pitch corresponding to f_1 even though f_1 might be missing in the signal altogether. For example, harmonic partials 3, 4 and 5 taken with equal amplitudes in either sine or cosine phase combine into their common period $T_0 = 1/f_0$ which equals the period (in ms) corresponding to the fundamental frequency f_1 of the harmonic series chosen. Hence the repetition frequency f_0 derived from the common period is equal to f_1. Since $f_0 = f_1$, the repetition frequency f_0 is substituted for f_1, which is the 'missing fundamental'. The relation sketched here holds even if the harmonics are not linked in a series like, for example, harmonics 3, 5 and 9 which (as harmonic partials in sine phase) combine into a periodic waveshape whose repetition frequency f_0 of course equals f_1 (see Fig. 11).

If musically trained subjects are presented with a complex sound composed from superposition of several (consecutive or non-adjacent) harmonics, they can be

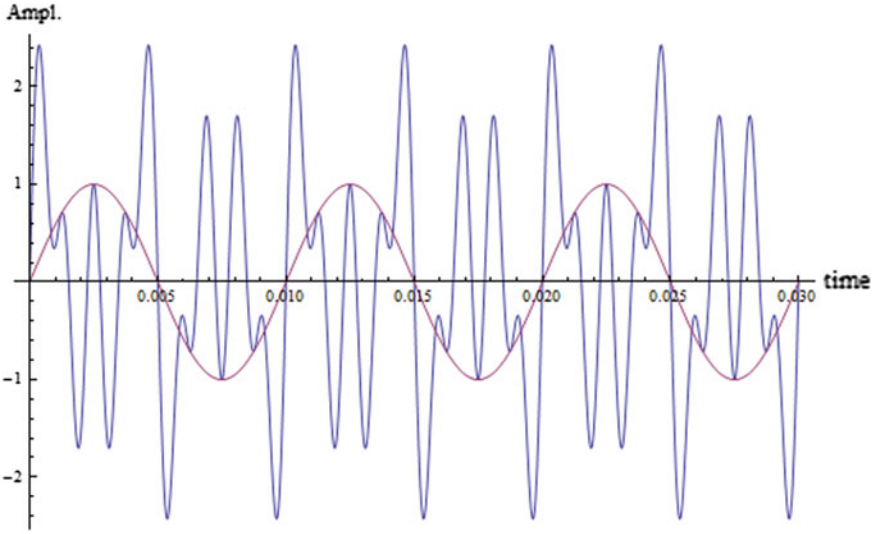


Fig. 11 Periodic waveshape (*blue*) resulting from superposition of harmonics 3, 5 and 9 (300, 500, 900 Hz). Repetition frequency f_0 = 100 Hz (*red*) of the complex waveshape equals f_1 of the harmonic series

expected to sing or hum a pitch they believe represents that complex. Let, for example, the complex comprise three partials of equal amplitude at 700, 900, and 1100 Hz, respectively. If this complex is played back to a subject at ca. 70 dB via loudspeakers for ca. 2 s, and the subject is asked to sing or hum the pitch he or she perceives immediately after the stimulus ends, it is likely that she or he will produce a sound that itself has a f_1 and/or a f_0 at 100 Hz as shown in Fig. 12. To be sure, the stimulus (spectrogram from three harmonic partials) and the response (pitch track from AC analysis) here have been plotted into one graph while the stimulus and the response in fact are two different sounds not overlapping in time.

It is known from experiments that the salience of the pitch corresponding to f_0 of a harmonic complex depends on the harmonic number of the partials as well as on the frequency region in which these partials fall (see [62], [63], Chap. 7). In general, salience of a virtual pitch based on f_0 is greater for complexes comprising low harmonic partials which can be 'resolved' by the cochlear filter bank (preferably, consecutive harmonics such as 3, 4, 5 or 4, 5, 6) and for f_0 falling into a frequency band ranging, roughly, from about 100–500 Hz (which means partial frequencies ranging from about 300 to 3000 Hz). However, it was found that also groups of higher harmonic partials which cannot be resolved aurally into their constituents can give rise to virtual pitches (cf. [64]). Though such groups of higher harmonics establish a common periodicity, the pitch salience for f_0 from such stimuli is significantly lower in comparison to the salience of low spectral pitches and f_0 pitches resulting from low harmonics.

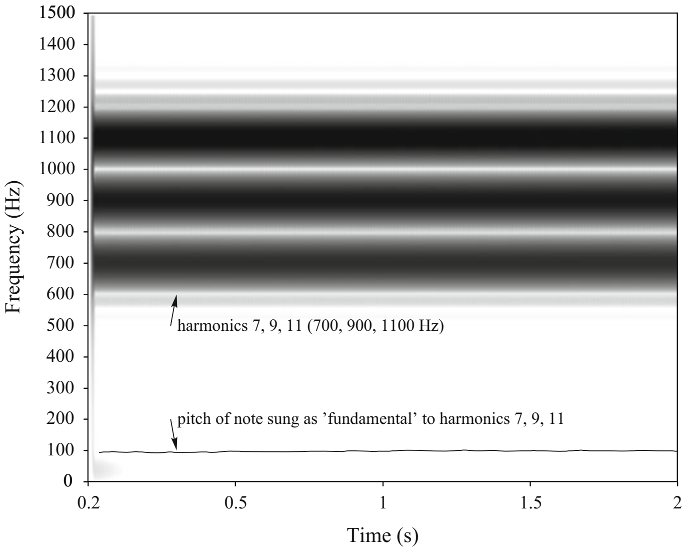


Fig. 12 Spectrogram of a harmonic complex {700, 900, 1100 Hz} and pitch track at about 100 Hz derived from the tone sung by a subject after listening to the complex

A point that needs some comment is the nature of pitch perception based on either f_1 or f_0. In principle, spectral pitches corresponding to pure (sine or cosine) tones of a given frequency result from direct sensory information, namely stimulation of the basilar membrane (BM) at a certain place. The relation of frequency to BM place is known as tonotopic, resulting in a cochleatopic map. Though the area of BM excitation is not indefinitely small even at low sound levels (and broadens significantly with SPL > 40 dB, see [65], there are mechanisms suited to transform the place information into an unambiguous neural signal that preserves the frequency of the stimulus as a basic correlate of 'tone height' (which in turn is a basic constituent of pitch; see [59], Chaps. 9–11, [66]). Since $T = 1/f$, the period corresponding to a certain stimulus frequency is also contained in the neural spike train. In regard to pitch salience, for complex harmonic sounds which include the f_1 partial, and where f_1 often is strong in amplitude, essential sensory information comes from a spatial pattern of cochlear excitation. In addition to place information, a harmonic complex comprising a number of partials including f_1 provides temporal information since the virtual f_0 'fundamental' corresponding to the period of the complex waveshape not only equals but also reinforces the f_1 pitch.

In contrast, for harmonic complexes lacking f_1 (and maybe also other low partials), much of the information attributable to spatial excitation on the BM is missing. Hence, the f_0 pitch must be inferred from temporal information pertaining to the overall periodicity of the stimulus. In certain respects, different mechanisms of excitation and processing which are behind f_1 and f_0 pitch perception can account for differences in pitch salience. While a strong f_1 component in a complex harmonic sound in general gives rise to an immediate sensation of this partial and a clear pitch perception, many harmonic complexes with 'missing fundamental' and concentration of spectral energy towards higher partials (i.e., the centroid moves up) in fact not only lack that particular sensation but also convey a somewhat weaker pitch. In regard to perception (which includes cognitive assessment of what has been perceived), a virtual f_0 'fundamental' appears as being 'implied' by the sensory information derived from peripheral processing of a stimulus lacking f_1 while the f_1 fundamental in a homogeneous harmonic complex (i.e., the series of partials is complete with $A_n \sim 1/n$) seems to result from an immediate and 'automatic' response.

Schouten's concept of 'temporal' pitch (see [60, 62]) based on the common period constituted from several or even many harmonics explains perception of the so-called missing fundamental and served as a hypothesis for the strike note in bell sounds. Schouten [60], also Schouten and 't Hart [61] suggested that partials nos. 5, 7 and 10 of a carillon bell would have frequency ratios close to 2:3:4, and that the strike note close in frequency to the prime (partial no. 2) would thus be perceived as a kind of missing fundamental. In Schouten's scheme (see also Table 2), the relevant partials are the nominal (or octave above the prime), the twelfth, and the double octave, which usually are strong spectral components. With these partials present in the spectrum, and provided they carry sufficient energy, perception of a strike note at about the frequency location of the prime seems feasible. In fact, in a number of experiments the strike note was found to result from a combination of nearly harmonic partials contained in the sound radiated from bells (see [55, 51, 56]). The location of the strike note often (but by no means always) was found close to the prime. For a number of bell sounds, the strike note was either below or (more frequently) above the prime. Also, in the sounds projected from some bells more than one strike note could be identified (a second strike note often appears a fourth or major third higher than the first). The frequency position of a strike note can be shifted by either manipulating the frequencies of relevant components in synthesized bell sounds [55] or in the sound of a real bell [67]. In these experiments, it was found that the octave (nominal), the twelfth and the upper octave partials are the most important contributors to strike note pitch.

Taking the 'strike note' as equivalent to perception of a 'missing fundamental' (see [61]), also a frequency location close to the hum would be possible. To illustrate the case, one may design an 'ideal bell' with partial frequencies like 100, 200, 240, 300, 400, 500, 600, 800, 1200, 1600 Hz and amplitudes similar to those found in the sound of a real bell.

For such a sound synthesized from Fourier components (for simplicity, all in cosine phase, see Fig. 13), the most likely strike note pitch frequency derived from temporal and spectral information would be 100 Hz as was determined with two pitch detection algorithms based on autocorrelation (AC, [68, 69]) and another pitch algorithm based on subharmonic summation (SHS, [70]). Thus, the most likely strike note pitch in this case coincides with the frequency of the lowest spectral component, f_1, which is found as the common denominator fitting best to the partly incomplete and partly not quite harmonic series of partial frequencies listed above. If, however, the sound of a real bell is analyzed which may contain partials with

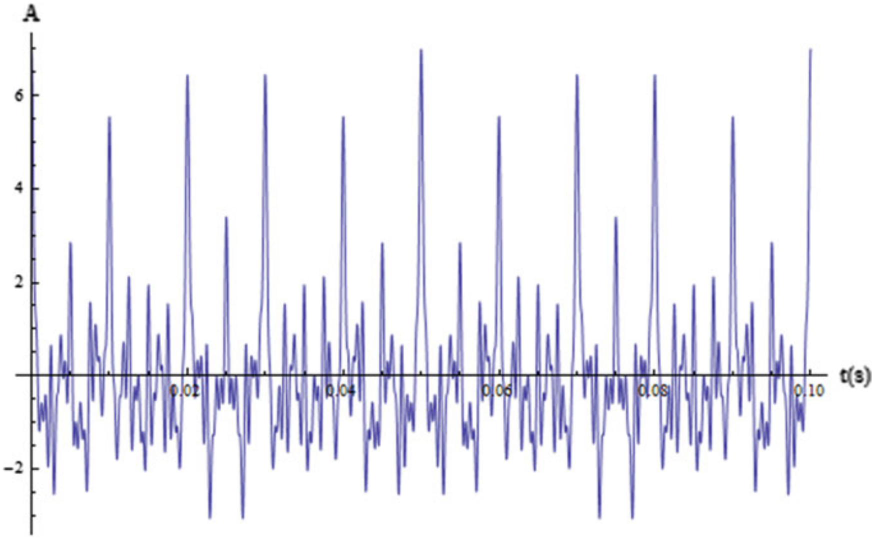


Fig. 13 Synthesized minor-third bell sound, oscillogram for 100 ms. A periodicity at $T = 10$ ms ($f_0 = 100$ Hz) is still clearly visible

frequencies in the same range as those used for the synthesis plus a significant number of more or less inharmonic components interspersed between the 'principal partials' as well as in higher octaves (see Fig. 8), the result may be different. For example, spectral analysis of the sound of bell no. 3 of the Dumery carillon shows that $f_1 = 122.06$ Hz. The pitch detection algorithm based on AC yields ca. 124 Hz measured at both $t = 0.5$ and $t = 1$ s from onset. Thus, the autocorrelation method taken to determine the pitch of the strike note in this case as well as for most of the 26 Dumery bells calculates a frequency close to the fundamental (and in several cases also a frequency close to half of f_1). On the other hand, the subharmonic summation algorithm (SHS) for the same sound finds 246.71 Hz at $t = 0.5''$, 245.5 Hz at $t = 1''$, and 245.31 Hz at $t = 2''$. Hence, the pitch frequency that might represent the strike note in this case is close to, yet not identical with, the frequency of the prime (which in bell no. 3 is a doublet comprising two components at $f_{2a} = 243.75$ Hz, $A_{2a} = 66$ dB and $f_{2b} = 244.01$ Hz, $A_{2b} = 72.1$ dB). Since campanologists tend to locate the strike note at, or close to, the frequency and spectral pitch of the prime, we may accept the result of the SHS measurement as correctly representing the frequency of the strike note pitch. Such a conclusion seems plausible, on the one hand, though it probably simplifies matters, on the other.

4.2 Strike Note, Pitch and Timbre

To understand the issue of strike note pitch as distinct from spectral pitches more closely, one has to take the timbre of sounds and the interdependence of pitch and timbre into account. In much of the relevant literature, the term 'timbre' refers to the spectral composition of a sound including its temporal and dynamic modifications while 'sound colour' rather relates to the shape of the spectral envelope and the position of the spectral centroid (for a detailed discussion of both concepts, see [66]). Hence, the sound of a bell can be described in terms of 'timbre' for the change it shows in spectral energy distribution over time while a sine tone or a harmonic complex in the steady-state can be described in terms of a 'sound colour'. For a sine tone of given frequency and amplitude, there is in general one unambiguous sensation of pitch depending on the stimulus frequency. Note that low to medium sound levels (30–60 dB) exert almost no influence on the pitch (see [71], Chap. 5). There is another attribute of sensation known as tonal brightness which, for pure tones, again depends on stimulus frequency. The 'colour' of simple stimuli such as sine tones does not change significantly within one octave (say, 200–400 Hz) where a smooth increase of relative brightness of the sound with frequency will be observed (the level being held constant). A similar effect can be expected for harmonic complex sounds where all components are coupled in phase and where the amplitudes of partials conform to $A_n = 1/n$, n = harmonic number or roll off at a similar rate per octave (see above). For such sounds, pitch is conveyed unambiguously by both f_1 and f_0 information, and their 'colour' does not change significantly if all partials of a harmonic complex are shifted by a musical interval (say, a major third or even a fifth) up or down while their relative amplitudes are maintained. In effect, this implies a shift of the spectral envelope which causes changes in the relative brightness sensed since the spectral centroid (see [72]) moves up or down in accordance with the shift.

In classical hearing theory [73], the fundamental of a harmonic spectrum was regarded as the main carrier of pitch information (sensation of f_1 at a certain place on the BM), and the energy distribution from the remaining partials as largely determining the sound colour (also labelled 'tone colour'). Hence, pitch as a function of f_1 and sound colour conceived mainly as a function of the shape of the spectral envelope become separable as percepts even if some interaction is taken into account. Of course, the f_1 partial not only defines 'fundamental' spectral pitch but also contributes energy to whatever sound colour. Furthermore, concentration of spectral energy above the f_1 fundamental can provoke a pitch shift (for harmonic spectra, a shift by an octave is a likely case). The Helmholtz model of pitch and sound colour sensation obviously relates to the harmonic line spectrum such as sketched in Fig. 14. For most string and wind instruments (chordophones, aerophones), sounds usually exhibit a strong f_1 partial suited to determine pitch. Amplitudes of higher partials often roll off at a significant rate (e.g., −6 dB/octave) relative to the f_1 amplitude. The number and intensity of the partials above f_1 then would determine the 'colour' of a particular sound.

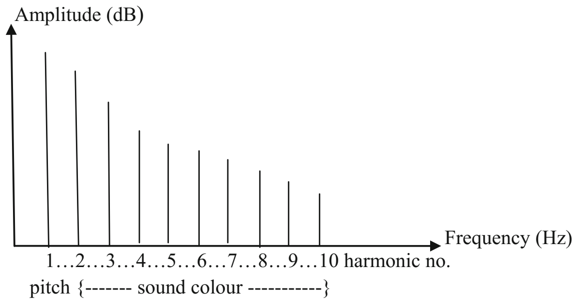


Fig. 14 Harmonic line spectrum as a model of pitch and sound colour sensation

In bell sounds, temporal and spectral structure, in general, are much more complex depending on the regime of vibration (see above). Complexity of sound structure in bells can show strong effects on the sensation and perception of pitch and timbre. The decisive factor for the dynamics and the timbre of carillon bells of course is the magnitude of the impact force with which the clapper excites the bell. In carillons offering a pedal-board (as in Bruges), the player can accelerate the clapper rapidly when kicking one of the pedals, thereby applying a strong impact force to the respective bell. Such a playing technique, which is quite common for bourdon notes that are intended to be perceived as marking the beginning of a phrase, has the effect of eliciting very many eigenmodes in a bell; from the vibration a complex pattern of more or less harmonic partials combined with inharmonic components (not to forget the split of partials into doublets, see above) results in the spectrum. Taking the correspondence between the periodicity of a signal in the time domain and its harmonicity in the frequency domain into account (a correspondence explained by the Wiener-Khintchine theorem; see e.g. [74]), one can measure the degree of periodicity in a signal in the time domain by computing the harmonic-to-noise ratio (HNR, see [68]). The concept rests on viewing a signal as consisting of a number of harmonic partials which correlate among each other while there may be other components (such as noise) uncorrelated with the harmonics. The algorithm treats both parts of the signal in calculating the HNR expressed in dB over time. A strictly periodic signal with no noise can yield high dB ratings (e.g., a sawtooth wave generated with Mathematica® and analyzed with this HNR algorithm reaches some 60–65 dB) while in particular the onset of a bell sound yields quite low dB readings due to spectral inharmonicity even if only a medium impact force is applied to the bell. Later in the sound, periodicity of the signal increases since many of the transient inharmonic partials decay rapidly and then may vanish completely. Consequently, the HNR yields higher dB after about one second of sound has elapsed. To illustrate the case, Fig. 15 shows the HNR (dB) of bells no. 9 and no. 6 from the Dumery carillon.

The rather inharmonic sound structure of bells at the onset and within the first 0.5 s is of consequence also to sensation and perception. First of all, sounds recorded in the vicinity of a bell can reach a fairly high SPL immediately after the clapper impulse is transmitted to the bell and has excited a large number of

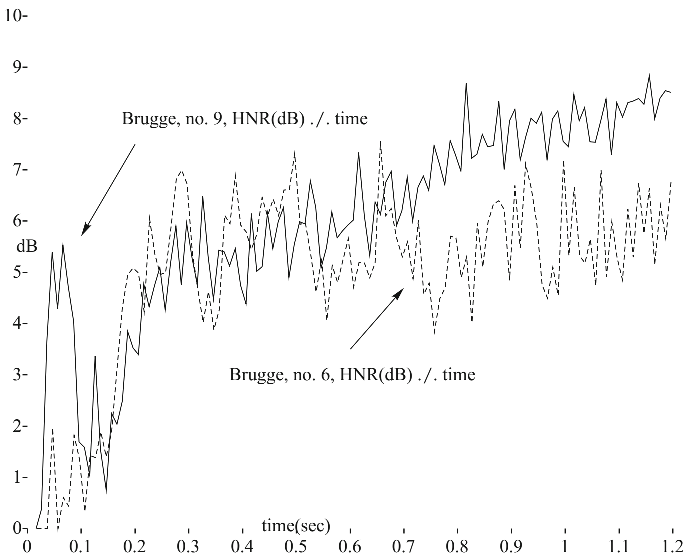


Fig. 15 HNR (dB) over time for sounds from bell no. 9 (*solid line*) and bell no. 6 (*dashed*)

eigenmodes. Second, the broad spectrum covered in the sound by many nearly harmonic as well as inharmonic partials in turn produces broadband excitation on the basilar membrane of the cochlea. The effect of both parameters is evident in Fig. 16 which combines the cochleagram for a sound recorded from bell no. 9 with the plot of the corresponding sound intensity over time (which peaks close to 80 dB).

Sound intensity and wide spectral energy distribution including inharmonic components shortly after onset account for the sensation of a relatively loud sound that to campanologists and musical listeners alike appears metallic and sharp in timbre (see [41, 51, 55]). Sensation of such a timbre may affect pitch perception to some degree since both cannot be neatly separated. In fact, pitch and timbre closely interact in particular in sounds which have inharmonic spectra such as recorded from Javanese and Balinese metallophones or gong chimes (see [32, 75]) where quite often the spectral component corresponding to the lowest mode of vibration is not the strongest in radiated sound level. In addition, sensation of pitch can be blurred from groups of inharmonic components, which are too close in frequency to be 'resolved' on the BM level. Note that such groups of inharmonic components interact so as to give rise to AM and a sensation of roughness and beats. Furthermore, with increasing degree of inharmonicity of the spectrum overall periodicity of the time signal decreases, to the effect that sensation of f_0 pitches is hampered (see [76]). Taking sounds such as recorded from Indonesian gong chimes (e.g., the *bonang* of Java or the *trompong* of Bali), many if not most Western listeners perceive a clangy, metallic sound rather than a musical tone distinct in

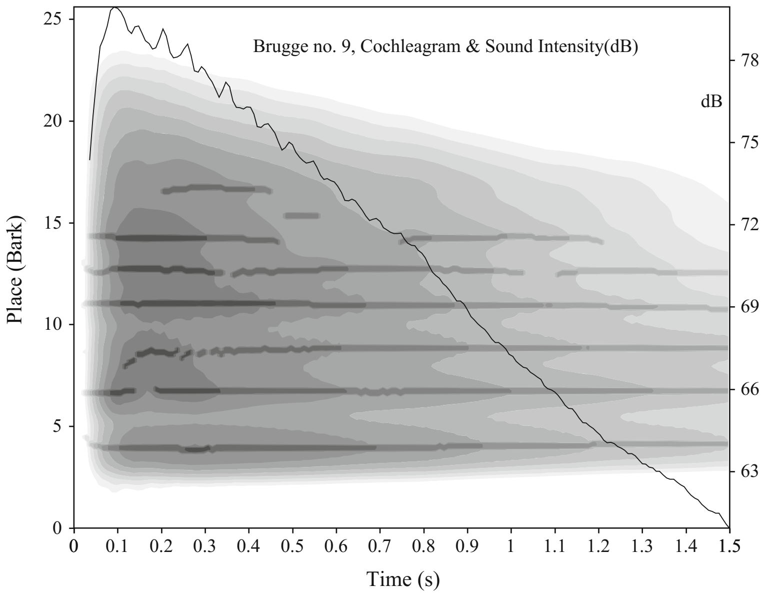


Fig. 16 Cochleagram and sound intensity (dB) as a function of time, bell no. 9

pitch. In the experience of many musically trained subjects (but unfamiliar with these sounds) no clear distinction between pitch and timbre seems possible.

Though many western swinging and carillon bells have spectra which include partials that can be assigned to (one or several) harmonic series, there are still enough inharmonic components to cause ambiguity of pitch perception (see [40, 56]). In general, there seems to be a clear correspondence between spectral inharmonicity and ambiguity of pitches subjects perceive from the sounds of bells. The ambiguity apparently results from a concurrence of several pitches, both spectral and virtual, and also from the interaction of pitch and timbre that seems to affect perceptual analysis of the strike note. While the pattern of partials involved in good minor third/octave bells in general facilitates perceiving a 'main pitch' (which in many cases is the strike note close to the prime), the sharp metallic timbre attributed to the strike note may perhaps diminish the salience of the strike note pitch.

Among the spectral partials of swinging and carillon bells, the minor third because of its level in the spectrum and because of the interval it forms with the prime and the nominal almost always can be identified by listeners as a spectral pitch. In addition, listeners often can hear (and reproduce by humming or singing) a few other partials such as the nominal if bell sounds are presented in isolation. Taken together, the prime and the strike note also form a possible pitch area. Hence, several strong spectral partials plus the strike note may give rise to a sensation of multiple pitches in each complex bell sound. As a hypothesis, one may expect subjects with some musical background to be able to distinguish several spectral

Table 6 Number of pitches perceived per bell, 15 bells, n = 81 subjects

No.	Median	Mean	SD
1	2	2.63	1.07
2	2	2.73	1.09
3	3	2.65	1.07
4	2	2.3	0.86
5	2	2.43	1.14
6	2	2.46	1.17
7	2	2.39	1.04
8	2	2.53	1
9	2	2.44	0.9
10	2	2.28	0.78
11	2	2.2	0.95
12	3	2.99	1.16
13	2	2.48	1.08
14	2	2.23	0.93
15	3	2.65	1.06

partials as well as the strike note that are perceived as separable pitches. To test this hypothesis, we presented the sounds from the first 15 (no. 1–15) of the 26 Dumery bells to a total of 81 subjects, most of them students in musicology at Hamburg in their first year. All sounds were digitally normalized to a level of -6 dB and were played back from hard disc in a class room suited to musical performance and recording via a stereo system comprising high quality loudspeakers at an SPL of ca. 75–80 dB(C) measured 1 m from the source. The level was chosen to offer conditions as one would experience if standing not too far from to the actual bells. Each sound (average duration ca. 4 s) was offered twice, with a break of 1 s in between. Subjects were asked to note the number of pitches perceived per bell sound in a questionnaire. The data for 15 bells (nos. 1–15) are given in Table 6.

Calculated over 15 bell sounds and 81 subjects, the mean is 2.49 and the SD is 1.04. Hence, the average number of pitches subjects perceived from single bell sounds recorded from the historic Dumery carillon in this experiment was from two to three.

To explore the ambiguity of bell sounds in regard to pitch perception further, we conducted an interval identification test with the same 81 subjects. For this task, 20 intervals were formed from the sounds of the bells nos. 1–13 as listed in Table 7. There were 10 intervals in upward direction and 10 intervals in downward direction. The two sounds, A and B, forming an interval were played in succession (A $\rightarrow$ B) where each sound lasted for ca. 3–3.5 s. The gap (silence) between two sounds was 0.5 s. Each pair of sounds representing a certain interval was repeated once after a short break of 0.5 s, thus the sequence was sound A: silence: sound B: silence: sound A: silence: sound B. Subjects were asked to state the size of each interval either with a musical term (e.g., 'major third') or by expressing the interval by the number of semitones it spans (e.g. 8 = minor sixth). In this regard, the exact

Table 7 20 intervals presented with bell sounds as stimuli; no. of hits

Trial no.	Bells no.	Musical interval	Direction	No. hits
1	1 → 4	Fourth	Up	27
2	8 → 6	Major second	Down	19
3	2 → 10	Major sixth	Up	15
4	11 → 4	Fifth	Down	32
5	2 → 5	Major third	Up	26
6	13 → 12	Minor second	Down	50
7	3 → 6	Minor third	Up	41
8	11 → 5	Tritone	Down	5
9	4 → 12	Minor sixth	Up	19
10	13 → 8	Fourth	Down	27
11	7 → 4	Minor third	Down	30
12	1 → 10	Major seventh	Up	19
13	3 → 5	Major second	Up	50
14	12 → 2	Major seventh	Down	18
15	7 → 13	Tritone	Up	14
16	6 → 10	Major third	Up	29
17	2 → 8	Fifth	Up	28
18	3 → 4	Minor second	Up	30
19	13 → 5	Major sixth	Down	11
20	11 → 3	Minor sixth	Down	18

size of the interval relative to a peculiar tuning was not considered (in Pythagorean tuning, the minor sixth has 792.2 cents, in ET12 it has 800 cents, and in just intonation the interval size is 813.7 cents). Hence, the decisions to be made by subjects were between basic musical intervals such as minor second, major second, minor third, major third, etc. Even though subjects were not explicitly asked to mark also the direction of the interval (up, down), most of the subjects did include such information (e.g., by noting “7↑” or “11↓”). The 20 intervals presented as stimuli are listed in Table 7.

Of the responses collected from 81 subjects, 76 lists containing their respective interval ratings were usable. The number of correctly identified intervals (‘hits’) per trial is stated in the last column of Table 7. The sum of correctly identified intervals is 508. Since 76 subjects had to make 76 × 20 = 1520 interval judgements, the number of 508 hits means a fraction of only 33.4 % of the judgements was correct. On average, subjects identified 6–7 intervals out of 20 correctly (median = 6, mean = 6.75, SD = 4.1). The relatively small number of ‘hits’ may reflect a general difficulty for subjects to judge musical intervals played with sounds that for most of the listeners were unfamiliar and, at least for some of the intervals, highly ambiguous if not contradictory (in particular, the major third played with two minor third bells). Inspection of the data reveals that the 76 subjects differed significantly

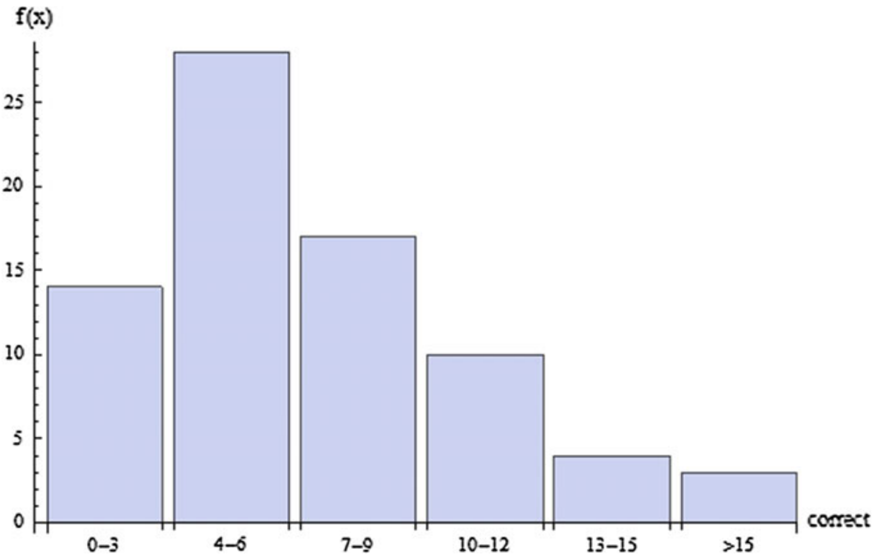


Fig. 17 Frequencies of correctly identified intervals (81 subjects) ordered into 6 classes

in their performance since some achieved a high rate of correct judgements (the maximum was 17 correct out of 20 intervals) while others evidently had great difficulties to identify musical intervals when presented with minor-third bells. However, there was a fairly large number of 'near misses' (judgements falling ± 1 semitone off the correct interval). Further, a number of judgements seemingly reflected confusion errors known also from experiments with so-called 'possessors of absolute pitch'. One such confusion error is that even musically trained subjects at times mistake a pure fifth for a perfect fourth (et vice versa); also, subjects may take a major sixth for a major third, or a minor sixth for a minor third by judging the relative consonance in both pairs of intervals. Ordering the results from 76 subjects into six classes according to the number of hits, the bar chart shown in Fig. 17 can be plotted.

Results from our interval identification task, albeit exploratory in nature, once again underpin pitch ambiguity in carillon bell sounds. The partly inharmonic structure of each minor-third bell sound not only hampers pitch perception but, as a consequence, also interval identification. Since subjects in general perceive more than one pitch from each carillon bell, the difficulty lies in assigning sounds that give rise to several pitches to the steps of a musical scale that in itself is one-dimensional in several respects.

First of all, 'musical pitch' basically is defined by log frequency as note names like A_3 = 220, C_3 = 261.5 or A_4 = 440 Hz imply. Hence, a scale of musical tones in ET12 for one octave (A_3 to A_4) can be plotted as a function of log(*f*) like Fig. 18.

In this scale, the frequencies can either represent sine tones or the f_1 of harmonic complex tones. Conventional (staff) notation represents musical tones

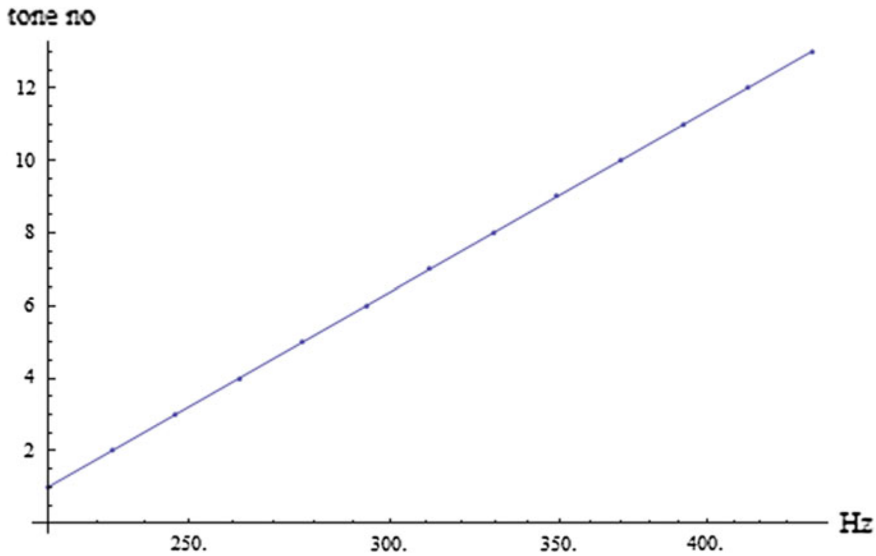


Fig. 18 Musical scale (A_3 to A_4) defined by 13 frequency values (*dots*)

(i.e., harmonic complex tones such as produced by singers or by wind and string instruments) by their respective f_1 while it disregards other spectral partials (as such are believed to be relevant not for pitch but for 'sound colour', see Fig. 14).

Second, the musical scale formed from a number of tones per octave is also conceived as one-dimensional since, in conventional music theory and music psychology, the scale steps are regarded as a category scale which may comprise k pitch categories (depending on musical culture, one finds different values for k). In regard to pitch production and perception, each category can be defined by a center frequency (as are marked by dots in Fig. 18) as well as by boundaries within which sounds with different f_1 shall be taken to represent the 'same' category (a 'pitch category' in certain respects corresponds to a Thurstonian scaling model with mean and variance, respectively; see [66]). Such a scale model works fairly well as long as one is dealing with sounds compliant with the harmonic line spectrum (Fig. 14), meaning (a) the f_1 partial of every harmonic sound (e.g., the musical 'tone' as produced on a chordophone or aerophone) is prominent and suited to determine f_1 pitch, and (b) is close to the (center) frequency defining the respective note and pitch category. Within the limits of each category, a direct relation between f_1 of a harmonic complex sound such as a tone from a wind or string instrument and a certain scale step can be established. For example, if a musical work contains a certain note (say, a G_4) a violinist or singer is expected to produce a harmonic complex tone with f_1 very close to 392 Hz (in ET12). However, a moderately detuned A_4 on a violin with f_1 at 434 Hz is still accepted, by most listeners, as 'representing' the note and the pitch category A_4 though f_1 here is flat by ca. 24 cents (a noticeable deviation yet within the limits of the pitch category).

For many inharmonic sounds, such a simple relation between f_1 component in the spectrum, perceived pitch and musical scale step is not at hand. The task for subjects dealing with sounds from bells (or, even worse, Javanese gong chimes) rather is to make a perceptual evaluation of where a sound that gives rise to several pitches may fit into a one-dimensional musical category scale. If several such inharmonic complex sounds are presented simultaneously and/or in a sequence, subjects have to perform many perceptual analyses and must make decisions as to the presumed pitch structure these sounds might 'represent'. In effect, the multiplicity of pitches induced from several inharmonic sounds increases the perceptual and cognitive workload in particular in a music listening situation where processing needs to be done almost in 'real time'. For actual performances of music on carillons, there is another factor that must be taken into account in regard to perception. Since every bell radiates sound for several seconds after the strike before significant damping takes place (Fig. 6), there is a temporal as well as a spectral overlap of complex inharmonic sounds. Therefore, with two or even three bells played simultaneously, pitch perception and recognition of melodic and harmonic textures can be quite difficult even if the music may be well known from other contexts (see [77]). Though listeners with a musical background can often identify certain songs rendered on a traditional carillon by perceiving their distinctive melodic and rhythmic features, the basic ambiguity of pitch (and also timbre) resulting from the partly inharmonic spectrum as well as from concurrent spectral and virtual pitches remains. Among the truly amazing experiences one may have with carillon music is listening to a piece full with major chords played with a carillon of minor-third bells. Many listeners may experience a certain incongruity or even discrepancy between the sound as sensed and the musical structure perceived as intended. In order to overcome this discrepancy, bells with a major-third spectrum have been designed for carillons (see [49]; swinging bells with a harmonic spectrum have been founded much earlier, see [55]). However, a certain degree of ambiguity may be experienced as appealing to listeners who might esteem a carillon as a musical instrument with a peculiar sound structure. Of course, there is a corpus of (in particular, traditional) music that fits to this sound structure. Various sources of the 17th and 18th centuries, respectively, indicate that music played on Flemish and Dutch carillons included many folk songs and hymns which often were elaborated in a characteristic two-part setting where the melody in the discant was played with ornamentation while the bass line consisted of longer notes (see [78]), thus taking the slow decay of sound level experienced in larger bells into account. In the 19th and 20th centuries, respectively, more of the virtuoso style known from piano and also organ playing was adopted by composers of carillon music as well as by carillonists. Playing music faster on a carillon and producing much more complex sonorities means, however, that the resulting sound mixtures can be so dense with harmonic and inharmonic components that perceptual analysis is very difficult, if possible at all.

Finally, a few tests were run to see which real or virtual component might be dominant in the pitch sensation and perception of musically trained subjects. The background to this issue is that several studies suggest that the acoustically real prime and the virtual strike note are decisive in regard to pitch sensation and perception. The 'nominal' (as the name implies) has also been viewed as the partial that channels pitch perception. It should be noted that, in English campanology, the nominal in fact is the octave above the prime while recently German campanologists have labelled the strike note of a bell as the 'nominal' since the 'nominal pitch' (defining the position of a bell in musical scale) would be that of the strike note [79]. The assumption is that the frequency position of the prime partial is also that of the strike note, whereby the two components together constitute a dominant pitch percept defining the 'nominal' pitch of each bell. However, though this may be the case with many bells, one still should test whether pitch perception is as uniform as suggested.

There are several well-known methods used to test which pitch or which pitches subjects perceive when listening to the sound of bells (or to other sounds which are of interest). One is to make subjects match a sine tone to a test sound so that the two appear equal in pitch (see [40, 56, 59]). Sine tone adjustments can be repeated in case subjects perceive several pitches (one of which may be more prominent than the others and, consequently, be taken as the 'main pitch' evoked from a complex harmonic or inharmonic sound). Another method is to let subjects sing or hum the pitch they believe to have perceived from a test sound presented just before. Again, if several pitches are perceived, subjects may sing or hum tones so as to express these pitches. A third method is to make subjects match a harmonic or inharmonic complex to that of an harmonic or inharmonic test sound so that both appear 'equal' in pitch.

Each method has some advantage as well as possible drawbacks (for a comparison, see [51]). For example, a good reason for matching a sine tone to an inharmonic complex is that the sine tone itself is well-defined in pitch and easy to handle in measurement. However, there is a marked difference between a sine tone of constant 'sound colour' (which appears soft and smooth) and an inharmonic complex which may appear quite rough from AM and harsh in timbre. In particular, the difference between the strike note of bells, judged as metallic in timbre and short in duration, seems significant relative to the steady and soft sine tone. The method of singing what subjects regard as either the main pitch perceived from a bell or at least one of the separable and identifiable pitches is less affected by differences in timbre and has also the advantage that such a response can be uttered immediately after hearing a test sound (that is, by making use of echoic and short-term memory). This method, though, calls for subjects capable of singing a note at a distinct pitch, which in turn may require some musical training. The third method eliminates any significant difference in timbre but introduces unknowns into the experiment since a harmonic or inharmonic complex used for comparison with a test sound may itself give rise to several (virtual as well as spectral) pitches. If in an experiment subjects have to adjust an inharmonic complex (derived from Fourier synthesis as in Fig. 13) to an inharmonic test sound (say, the sound of a carillon bell or a Javanese *bonang* gong) which is likely to produce a multiplicity of pitch sensations, the synthesized complex can also be expected to give rise to more than one (spectral and/or virtual) pitch. Hence, synthesized inharmonic complexes need to be studied with respect to their potential spectral and virtual pitches in beforehand of actual comparison to test sounds.

Taking the pros and cons of the three methods, singing a pitch or several pitches one perceives from listening to a test sound perhaps is a fairly reliable way to explore the issue. In a simple experiment, we asked 8 musically trained subjects (7 male, 1 female) to sing the pitches they perceived from listening to various bell sounds (Dumery bells nos. 1–12). The sounds were presented at low to medium level (ca. 40–50 dB) from a CD system and the responses were digitally recorded on hard disc. Spectral analysis and AC pitch tracking reveal that most responses of the male subjects aimed at producing a sound at a pitch that equals the frequency of the hum partial (rather than the prime or even the nominal). For example, the following correspondences were observed:

Bell no.	f_1 partial (Hz) bell	f_1 of sound sung as equivalent in pitch (Hz)
1	97.5	98.45
3	122	125.1
5	136.7	136.7
7	153	155.8
9	172.85	171.5
10	182.3	184
11	193.9	197

To be sure, f_1 of the vocal responses are averaged over time while the actual utterances showed some fluctuation in pitch (see Fig. 19). Among the responses there was one where a subject did sing a subharmonic (below f_1 of a bell) and very few responses showed subjects aiming at the prime (or, possibly, the strike note); one such response was recorded for bell no. 12 (see Fig. 19) from the female taking part in the experiment.

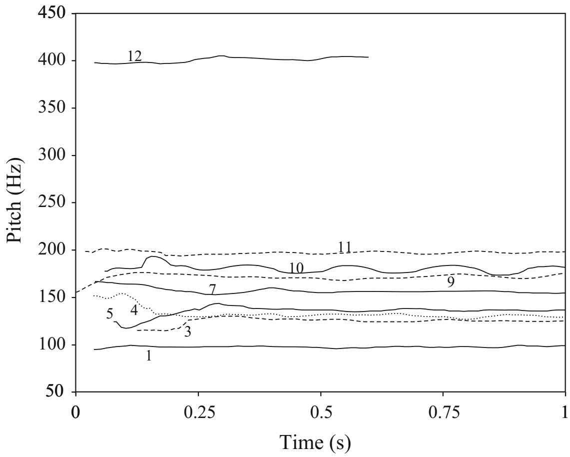


Fig. 19 Pitch tracks from sounds sung by subjects after listening to bells 1, 3, 4, 5, 7, 9, 10, 11, 12 of the Dumery carillon

Though our observations are limited in number, they seem to suggest that musically trained subjects tend to sing a note whose f_1 pitch corresponds to the hum of the bell sound rather than the prime or even the nominal. There are indications that composers like Hector Berlioz and Richard Strauss may also have identified the 'main pitch' of a (swinging) bell with the hum partial (cf. [80]). Further, there are empirical data from pitch perception experiments which corroborate that subjects matched a sine tone to the hum (f_1) of carillon bells [40]. The relative frequency of matches to the hum increased with f_1 of the bells (see [40], Fig. 19). This could be expected, to some degree, since the 16 carillon bells in this experiment were rather small (ranging from G_5 to D_7; f_1 for G5 is ca. 784 Hz). Experiments with sounds from swinging bells [56] showed that most subjects were able to identify the 'pitch category' (in terms of the semitones of a musical scale) of the strike note correctly while, in a number of bells and varying among the subjects, there was uncertainty whether the strike note would be located in the region of the prime or in the region of the hum partial. A possible explanation for this effect could be found in different patterns of spectral energy distribution in the sound recorded from various bells. As has been pointed out (above), spectral energy in sounds varies with the strength of excitation of bells as well as with directivity of radiation. Stronger excitation in general causes a higher spectral centroid and also an increase in spectral inharmonicity of the sound at the onset (see above).

Whether the strike note of a particular bell is perceived at the pitch of the prime or that of the hum (or still at another partial, or even in between partials; see [55, 51, 56]), appears to depend on several physical and also on psychoacoustic parameters. In about 30 % of the 137 (swinging) bells investigated by Terhardt and Seewann [56], more than one strike note could be determined algorithmically as well as in behavioural pitch matching experiments. Perception of more than one strike note for certain swinging bells was also observed in experiments with expert listeners [51, 55].

Taking the concept of Schouten [60], Schouten and t'Hart [61] and later findings one can argue that the strike note is a percept from a selection of strong partials in the bell sound which have nearly harmonic frequency ratios. These components form a complex that produces a virtual f_0 which, in general, induces a pitch close to that of the prime (f_2 in the bell sound) or the hum (f_1 in the bell sound). Hence, in regard to pitches perceived, in most cases $f_0 \simeq f_2$ or $f_0 \simeq f_1$ obtains. However, while the timbre of both the f_2 partial and the f_1 partial, if taken alone, appears rather soft, the timbre of the complex of partials giving rise to f_0 in general is found to be metallic. It is probably this timbral quality which distinguishes the strike note from the spectral pitches subjects identify with some of the more prominent partials, most of all, the prime, the minor third, the hum and the octave above the prime.

4.3 *Tuning of the Dumery Carillon Bells in Regard to Prime (f_2) Frequencies*

Accepting that, for many subjects listening to bells, the main pitch is either located at (or close to) the prime or at the hum, we calculated pitch frequencies with two different algorithms (AC, SHS) for 26 Dumery bell sounds at different time points after onset (t = 0.5, t = 1, and t = 2 s, and in several bell sounds also at t = 0.25″, t = 1.5″ and t = 3″). The data indicate that both algorithms in some sounds determined a pitch frequency close to the prime but in other sounds found a frequency close to the hum as the relevant pitch. Also, frequencies not directly related to either hum or prime as well as subharmonic frequencies (in general, a fraction of the hum frequency) were turned out by both AC and SHS pitch tracking algorithms. To illustrate the case, pitch tracks as calculated for bell no. 17 are shown in Fig. 20.

The AC pitch track (Fig. 20, solid line) starts at about 92 Hz and, after a jump to ca. 110 Hz, falls back to the initial frequency. To be sure, 92 Hz is about 1/3 of the frequency of the hum (273.35 Hz) and close to 1/6 of the prime (548.09 Hz) in this bell. The SHS track (dashed line) at t = 0.5″ and t = 1″ yields ca. 550 Hz and thus is very close to the prime. After a period of transition which reflects the effect of the rather fast decay of higher partials in the sound due to (viscous and acoustic) damping, the SHS pitch at t = 1.5″ and t = 2″ gives ca. 273 Hz, that is, the frequency of the hum note which has a long decay in the sound.

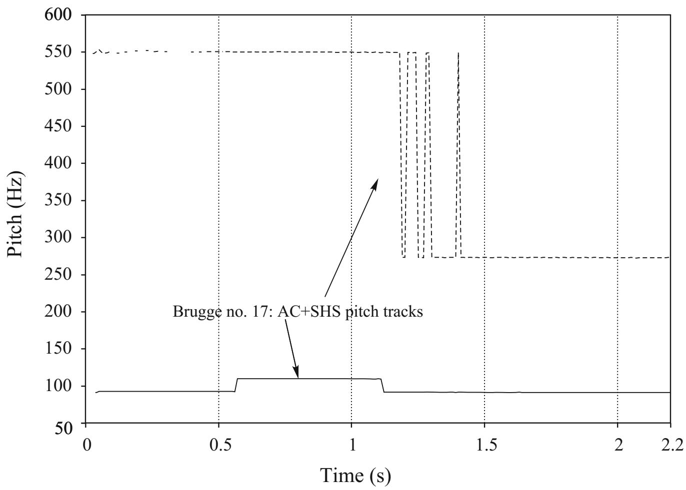


Fig. 20 Pitch tracks calculated for bell no. 17, AC and SHS algorithm

Assuming (a) that either the strike note or another main pitch of a bell can be expressed as a single frequency, and that (b) this frequency in many bell sounds is very close to that of the prime partial, we may devise the tuning of the 26 Dumery bells in terms of the prime frequencies measured from the bell sounds. Since some of the prime partials in fact are doublets, a decision has to be made whether to take the strongest component as relevant for determining the tuning (cf. [51]), or to use the mean of two adjacent frequency components as the respective pitch. In case the two components of a split partial differ significantly in amplitude level (A_1: A_2 > 3 dB), one may take the stronger component as perceptually relevant (the selection being justified by spectral masking). If, however, two components differ but little in level (A_1:A_2 < 1.5 dB), both can be part of the pitch percept (which may be somewhat blurred depending on the frequency distance of the two components as well as on their level relative to other partials). It seems justified, therefore, to calculate the mean of two frequency components for such doublets which have almost equal amplitudes. Hence, we have 26 frequencies for the prime partial to derive the tuning, which is given in cents for the intervals between these frequencies.

In regard to the question addressed above whether or not the Dumery carillon was intended to represent a quarter-comma meantone tuning, the data from Table 8 again offer some clues in this direction. First, there are whole tones smaller in size than 200 cent (and several relatively close to the meantone of 193 cent). Second, the intervals of the major third, the minor sixth and the major sixth in the scale based on the largest bell (G_2) are close to the meantone scale (see above; the major third and the major sixth are also close to just intervals). Third, the fourth is enlarged as in meantone tuning (the fifth G_2–D_3 in the first octave, however, is also enlarged while it should be diminished by about 5–6 cent in meantone tuning). Perhaps the strongest hint to a historical tuning (of which there were very many in use in the 17th and still in the 18th century; see e.g. [52, 54]) is that there are two clearly different semitone intervals, one representing the chromatic semitone (ideally with a frequency ratio of close to 25/24 $\sim$70.6 cent) and another representing the diatonic semitone 16/15 $\sim$111.7 cent. This difference is a characteristic of both meantone tuning and just intonation that has been eliminated in equal temperament (ET12) where all semitones have 100 cent. In sum, the tuning data of the 26 Dumery bells permits us to conclude that the model for the bell tuning might have been quarter-comma meantone temperament or one of the 'well-tempered' tunings in use between ca. 1680 and 1770 while in particular significant differences in the size of semitones in the scale indicate that ET12 cannot be considered as a template for the tuning of this carillon.

Table 8 Tuning of 26 Dumery carillon bells based on prime partial frequencies f_2

Bell	Prime f_2		Cents cum.	Bell	Prime f_2		Cents cum.
1 G_2	195.69		0	14 Bb_3	464.65		1498.1
		188.3				91.3	
2 A_2	218.04		188.3	15 B_3	489.82		1589.4
		194.8				115.3	
3 B_2	244.01		383.1	16 C_4	523.54		1704.7
		124.8				79.3	
4 C_3	262.25		507.9	17 $C\#_4$	548.09		1784.0
		71.7				110.6	
5 $C\#_3$	273.34		579.6	18 D_4	584.26		1894.6
		127.9				95.8	
6 D_3	294.30		707.5	19 Eb_4	617.50		1990.4
		67.4				110.3	
7 Eb_3	305.99		774.9	20 E_4	658.11		2100.7
		110.9				102.6	
8 E_3	326.24		885.8	21 F_4	698.30		2203.3
		116.0				66.5	
9 F_3	348.84		1001.8	22 $F\#_4$	725.65		2269.8
		77.6				130.2	
10 $F\#_3$	364.83		1079.4	23 G4	782.32		2400.0
		116.1				95.1	
11 G_3	390.13		1195.5	24 $G\#_4$	826.49		2495.1
		92.9				94.6	
12 $G\#_3$	411.64		1288.4	25 A_4	872.89		2589.7
		109.3				108.4	
13 A_3	438.46		1397.7	26 Bb_4	929.27		2698.1
		100.4					

5 Conclusion

The 26 Dumery bells (1742–48) from the carillon at Bruges belong to the rather small stock of historic bells from the 18th century that, from all we know, remained unaltered in regard to their shape (relevant for partial frequencies) and tuning to a musical scale until recently. Spectral analyses of the sound of these bells as well as other measurements revealed that each bell conforms closely to the pattern of 'principal partials' (hum, prime, third, fifth, octave, etc.) characteristic of a minor-third bell since about 1500 (G. de Wou). From what is known, Jacob van Eyck by about 1633 had identified at least some of the partials, and the famous Hemony brothers a few years later succeeded in founding carillon bells of highest quality that became kind of a standard in the Low Countries. The Hemony carillon bells are esteemed to this day as model specimens for the minor-third/octave bell.

Though Dumery lived and worked in the 18th century, he apparently was aware of acoustic and technical lore essential for casting high quality carillon bells. As our measurements demonstrate, deviations of the 26 bells from ideal norms and ratios are slight (a fact that is the more surprising if one considers the rather primitive conditions under which the bells were cast at Bruges, in the 1740s).

The tuning of the Dumery bells to a musical scale has at times been interpreted as close to quarter-comma meantone temperament. Our measurements with respect both to hum and prime partial frequencies show that there is some evidence in support of this interpretation; however, some of the data do not conform to this type of temperament. What can be concluded from the hum and prime partial frequencies (which, in most of the 26 bells, are close to forming a perfect octave) is that an unequal temperament is much more likely than ET12 for which actual deviations are too large. Whether the tuning followed one of the 'well-tempered' patterns (such as known from Werckmeister and his contemporaries) or was still aiming for one of the meantone temperaments (besides ¼ comma, there were several variants in practice), is difficult to decide.

Tuning of carillon bells cannot be investigated without regard to partial structure (addressed as 'inner harmony' of bells by campanologists), which in turn brings up the issue of the so-called strike note and its relevance for perception of the bell's pitch or, rather, pitches. While in the literature the position of the strike note as a virtual pitch often is located close to the prime partial frequency, our own measurements and experiments suggest that the hum partial frequency is also a candidate for the main pitch perceived from carillon bell sounds (the 'main pitch' apparently is perceived as 'implied' from quasi-harmonic segments in the spectral structure). As has been found in a number of previous experiments (and is confirmed by our own empirical data), most subjects perceive several spectral and virtual pitches when listening to sounds from minor-third bells. The multiplicity of concurrent pitches is the cause of pitch ambiguity. Moreover, the relative inharmonicity in minor-third bell spectra and the noisy transient sound shortly after the strike of the bell with a clapper hamper precise pitch estimates.

Acknowledgments Our recording of the Dumery carillon was greatly supported by the late Aimé Lombaert (1945–2008), who from 1984 until his untimely death was the chief carillonist (*stadsbeiaardier*) of the City of Bruges. We like to dedicate this study to his memory.

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