Explorations in Keyboard Temperaments. Some Empirical Observations

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Abstract In this article, the topic of tuning and temperament is addressed mainly from an empirical point of view. After furnishing some historical background on tone systems, scales, and tunings (in a review from Greek antiquity to the 18th century), twelve major and twelve minor chords played in two well-known keyboard tunings and temperaments (Werckmeister III, Vallotti) are investigated in regard to acoustical parameters on the basis of sound recordings we made with a Kirckman harpsichord from 1766. Our analysis of major and minor chords employs signal processing methodology, in particular autocorrelation and crosscorrelation from which the harmonics-to-noise ratio (HNR) is computed in the time domain as a measure of the periodicity in a signal. HNR readings vary for different chords relative to the justness of interval ratios and the different degrees to which partial frequencies converge in signals representing several complex harmonic tones such as contained in musical chords. The HNR thus can be taken as an indicator for the relative quality of a particular tuning. In addition, data from two experiments are reported in which listeners judged perceptual qualities as well as the goodness of intonation for various tunings implemented on digital synthesizers or realized by means of a computer. Our study intends to provide empirical data that can help to substantiate discussions of musical tunings and temperaments.

1 Introduction

Tuning of keyboard instruments to certain scale types and temperaments has been an issue for organologists and musicologists since long. In the past decades, surviving instruments have been investigated with the aim to possibly determine and reconstruct their original tuning. This approach proved effective in particular for

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organs of the Baroque era where, for example, the original meantone tunings could be determined from measuring pipe lengths and diameters (see [1]). In regard to harpsichords and clavichords, a number of instruments (primarily from Italy) were found that had more than twelve keys to the octave originally, and were reduced to the conventional format later (see [2]). In addition to surviving instruments and related sources (such as drawings of keyboards), there is of course a huge body of theoretical works in which tone systems, scale types and modes as well as aspects of tuning and intonation are treated from Greek antiquity through the Middle Ages, the Renaissance, and then through modern times up to the present (see, e.g., [3–14]). In addition to theoretical writings, there are of course many musical works which reflect certain ideas about tone systems and modal structures, and which offer also clues in regard to tunings and intonation practice. It is from the analysis of musical works that conclusions may be drawn as to intended tunings (in particular on keyboards; see, e.g., [3, 8, 12, 15–18]).

With an increased interest in organology as well as in historical performance practice of Renaissance and Baroque music in the 20th century, tuning and temperaments gained also practical importance. One outcome of this process was that on a significant number of extant historical organs in Europe their original meantone tuning was reinstalled or that one of the well-tempered tuning systems (such as proposed by Werckmeister [19], Kirnberger [20]) were implemented in a tentative reconstruction of tunings in use before ET12 became the standard (in close connection with the development of the modern piano). Another factor is that harpsichords are now tuned to a rather low pitch (with A_4 often in the range from 385 to 408 Hz) while historical organs are re-tuned to their original pitch, which for many instruments was set by the *Chorton* (church tone, ton de chapelle, etc.) that was in use in a certain region. For instance, in Northern Germany the Chorton in use in the 17th century was about one semitone to two semitones higher than A_4 = 440 Hz (in other regions of Europe, it was almost equal to, or lower than the modern A_4 standard pitch).

With the revival of historical tunings and temperaments, also discussions concerning the merits and shortcomings of particular tunings and temperaments have been revitalized. Readers familiar with historical sources from the 16th, 17th, and 18th century, respectively, will recall that many proposals for temperaments and tunings aimed at providing a tonal basis for harmonic modulation through many keys while pleasing the musical ear (for background information, see e.g. [3, 8, 12, 18]).

2 Just Intervals: Acoustic and Perceptual Aspects

Humans (and apparently also other mammals) perceive two sine tones whose frequency ratio is 2:1 as similar in certain respects. The interval these tones form in music is labelled octave since it comprises, in many musical cultures, a scale of eight tones or notes (in this article, the term *tone* denotes a physical phenomenon while *note* relates to musical notation. Of course, a musical note, say A_3, when played or sung as sound becomes a physical phenomenon as well as a psycho-physiological phenomenon in regard to sensation and perception). Perfect octaves have a distinct quality (which is restricted to the sense of hearing and to auditory perception) since their constituents match in a specific temporal and spectral pattern (see [21]). Likewise, just intervals such as the fifths (3/2) express a clear temporal and spectral structure. For two harmonic complexes each comprising a fundamental frequency, f_1, as well as harmonics f_2, f_3,…, f_n with a spectral envelope where amplitudes decay in a regular pattern like $A_n = 1/n$, the resulting signal is strictly periodic with a period $T = 1/f_0$ as is obvious from Fig. 1. The two fundamental frequencies here are $f_{1a} = 200$ Hz and $f_{1b} = 300$ Hz, and the frequency f_0 (plotted in red) with which the complex waveshape repeats is 100 Hz, that is, $T = 10$ ms.

Strict periodicity in the time domain corresponds to strict spectral harmonicity in the frequency domain. According to theorems developed by Wiener [22] and by Khintchine [23], the power spectrum W(ω) of a stationary time function f(t) equals the Fourier transform of its autocorrelation function φ(τ). Hence, for a periodic signal the autocorrelation function must also be periodic. The theorems of Wiener and Khintchine have been fundamental to the theory of vibration as they relate the concepts of time function and spectrum in regard to periodicity and harmonicity (see [24, 25]). In signals such as musical sound, one can easily see that a periodic vibration pattern observed, for example, from a thin string of a harpsichord, produces a highly harmonic spectrum where $f_n = nf_1$, $n = 1, 2, 3,…, k$, that is, partial

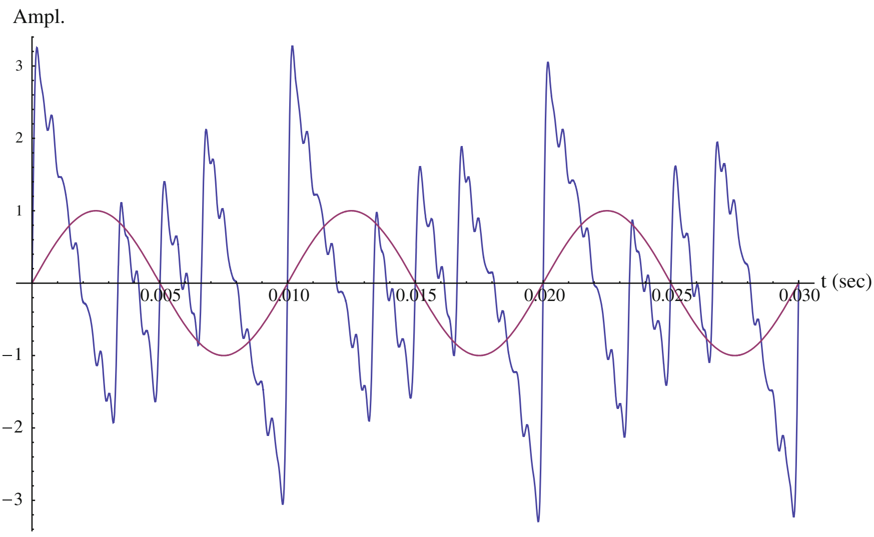

Fig. 1 Perfect fifth, two harmonic complexes, each comprising 10 harmonics, amplitudes $A_n = 1/n$, fundamental frequencies at 200 and 300 Hz, three periods shown. F_0 (repetition frequency of the complex waveshape) marked in *red*

frequencies are in integer frequency ratios (or very nearly so since inharmonicity from stiffness in thin brass strings of a harpsichord is almost negligible, see [26, 27]). The relation between temporal periodicity and spectral harmonicity defined by the theorems of Wiener and Khintchine is of central importance also for auditory perception of pitch and timbre (see [21]). Perceptual salience observed for just intervals can be explained by their high degree of periodicity and harmonicity, respectively, which for pairs of complex harmonic sounds played simultaneously implies a high degree of coinciding partials and, consequently, a low degree of roughness and beats (see [28, 29]). In addition, combination tones as well as perception of the 'missing fundamental' (see [30, 31]) come into play. For the perfect fifth shown in Fig. 1, when set to sound and played with sufficient level, a pitch component at 100 Hz will be clearly audible resulting from both f_0 (which is the repetition frequency of the period that furnishes a 'virtual' fundamental at 100 Hz to f_{1a} = 200 Hz and f_{1b} = 300 Hz) and the difference tone $f_{1b} - f_{1a}$. It is because of these facts which are open to empirical research that humans around the globe opted for musical intervals like the octave, the fifth (and its complementary interval, the fourth) as the most elementary (and most stable) building blocks for tone systems and scales. A good case in point is the scale for anhemitonic pentatony comprising five tones derived from a progression in fifths like c–g–d–a–e → c–d–e–g–a. Anhemitonic pentatony is found in very many music cultures (and may be viewed as a 'near universal' in music).

However, a fundamental problem behind the construction of tone systems and scales is that a finite sequence of just fifths $(3/2)^n$ will not form a cycle (but will take the shape of a spiral instead, see [32]). Taking a series of 12 fifths (e.g., from b_b–f–c–g… to $a^{\#}$), their compound size (which adds up to 8424 cents) overshoots that of seven octaves (8400 cents) by nearly 24 cents. The difference is known as the Pythagorean comma. The mathematical problem stated as $3^n \neq 2^m$ says that powers of one prime number do not equal powers of another prime number. For this reason, also three just major thirds of the ratio 5/4, when added to one interval (e.g., c–e–$g^{\#}$–$b^{\#}$), do not match a full octave 2/1 since their ratio of 125/64 falls short of that interval by about 41 cents (the gap corresponding to an interval of the ratio 128/125 = 1.024; this interval is called, close to classical Greek theory, a diësis). Again, the problem is that a tuning process which involves a series of three just major thirds would not yield an octave since $5^n \neq 2^m$. In effect, a *k*-dimensional tone net or tone lattice results from tone systems based on intervals each of which includes a prime number like 3/2 in the perfect fifth and 5/4 in the just major third. If the tone net represents the perfect fifth on the horizontal axis and the just major third on the vertical as the two fundamental intervals, the tone lattice or tone-net is a plane (as was explored first by Euler, and later by Arthur von Oettingen, Adriaan Fokker, and Martin Vogel). In case the 'natural' seventh 7/4 and thus the prime number 7 is included, the tone net is three-dimensional (see [14]).

A small segment (chosen to avoid double sharps and double flats) from the two-dimensional tone-lattice incorporating perfect fifths and just major thirds would be this:

−2	e	h	$f^{\#}$	$c^{\#}$	$g^{\#}$	$d^{\#}$	$a^{\#}$	$e^{\#}$	$b^{\#}$			
−1	c	g	d	a	e	b	$f^{\#}$	$c^{\#}$	$g^{\#}$	$d^{\#}$	$a^{\#}$	
0	a_b	e_b	b_b	f	**c**	g	d	a	e	b	$f^{\#}$	$c^{\#}$
+1	f_b	c_b	g_b	d_b	a_b	e_b	b_b	f	c	g	d	a

Obviously, there are tones which have the same designation but appear in different rows of the plane. The number −1 indicates that tones in this row are flat by one so-called syntonic comma against the tone of the same name in the basic row (0). The syntonic or 'third' comma (ascribed to the Hellenistic music theorist Didymos) is the difference between two whole tones 9/8 and a just major third like (9/8) * (9/8) * (4/5) = 81/80 = 21.5 cents. For example, the just major third e (5/4) over c (1/1) is one syntonic comma flat against the Pythagorean *ditonos* e (81/64) derived from a progression in perfect fifths c–g–d–a–e. In the scheme of the tone-net sketched above, the tone c in the −1-row is one syntonic comma flat against the c in the 0-row (taken as a centre and marked in bold) while the c in the +1-row is one comma sharp (the tones **c**, c^{-1} and c^{+1} are marked by arrows). To play a chord of c-major in just intonation would require the tones c and g from the 0-row and the tone e from the −1 row (designated e^{-1} or $\underline{e}$). Likewise, a c-minor chord played in just intonation would need the tones c and g from the 0-row and the e_b (designated e_b^{+1} or $\bar{e}_b$) from the +1-row.

Just intonation based on intervals of the perfect fifth and fourth as well as on just major and minor thirds permits to render major and minor chords with a maximum of auditory consonance and thus a minimum of roughness and beats. What is more important, though, is that chord progressions in tonal harmony can be rendered so that truly chromatic and enharmonic textures become audible (and can be appreciated by listeners as complex pitch and interval structures). The cost for this achievement is that, first of all, far more than 12 tones and pitches per octave are required in particular for extended harmonic modulations. Furthermore, a problem can arise if extended modulations lead to chord structures that require tones far away from the centre of the tone-net. In such instances, the pitch level can shift by several commas (see [3, 33, 34]). Of course, in practice one may define a limit from where a 'reset' towards the centre takes place (see [14]). One may also limit the number of just intonation pitches which are implemented, in a fixed tuning on a pipe organ or electronic keyboard instrument, by making a selection of the musically most important tones and intervals. This was the approach chosen by the Norwegian composer and music theorist, Eivind Groven, for a pipe organ which had 36 tones and pitches to the octave, and for an electronic keyboard comprising 43 pitches to the octave (see [35, 36]).

3 Just Intonation and Temperaments: A Brief Historical Review

The theory of just intonation, which has origins in Greek and Hellenistic antiquity (see [14]) stems from both mathematical considerations and empirical observation. In regard to the former, divisions of integer ratios into smaller units were of relevance. Well known are divisions of the tetrachord where the frame of a perfect fourth 4/3 can be divided into three intervals in various ways (yielding either a diatonic, or a chromatic, or an enharmonic tone and interval structure). The so-called Diatonon ascribed to the theorist Didymos (1st century) and the Diatonon syntonon of Claudius Ptolemaios (2nd century) both divide the fourth into a major and minor whole tone, leaving a diatonic semitone as a rational (superparticular) interval: $4/3 = 9/8 \times 10/9 \times 16/15$ (Didymos) and $4/3 = 10/9 \times 9/8 \times 16/15$ (Ptolemy). This division implies the just major third 5/4 since $9/8 * 10/9 = 5/4$. Apparently, the just major third was known to Greek theorists since Archytas of Tarent (4th century B.C.E.). As Ptolemy (ed. Düring 1934, 30f. [37]) asserts, Archytas calculated the diatonon, the chroma, and the enharmonion for a tetrachord, where the enharmonion has these ratios: $5/4 \times 36/35 \times 28/27 = 4/3$. Archytas seems to have been a scholar who, besides being a skilled mathematician, relied on empirical observation (see [38]); it may well be that he tested the intervals he calculated on a kanon or similar stringed instrument by ear.

The point is that Pythagorean tone mathematics (of which Archytas was the most famous representative in the 4th century) was not confined to the prime numbers 2 and 3. It should be added that Pythagorean tuning in perfect fifths produces a number of nearly just major and minor thirds. If we assume Pythagorean tuning was predominant for medieval organs (as can be concluded from treatises on mensuration of organ pipes and sources relating to the construction of early organs, see [10, 12, 13, 39], a chain of twelve pure fifth (e.g., from a_b to $c^{\#}$) would produce the following scale with c taken as the centre (1/1):

c	$c^{\#}$	d	e_b	e	f	$f^{\#}$	g	a_b	a	b_b	b	c′
0	114	204	294	408	498	612	702	792	906	996	1110	1200

In this tuning (given in modern cents rounded to whole numbers) the major thirds $c^{\#}$–f, e–a_b, $f^{\#}$–b_b and b–e_b are almost just at 384 cents; likewise, the minor thirds e_b–$f^{\#}$, a_b–b and b_b–$c^{\#}$ are almost just at 318 cents. If one wants to avoid accidentals for most of the just intervals, an appropriate segment of the chain of fifths has to be selected accordingly (the chain of fifths can be used like a ‘sliding rule’, see [14]). In a Pythagorean tuning based on the scale as indicated above, the major triads e–a_b–b, b–e_b–$f^{\#}$ and $f^{\#}$–b_b–$c^{\#}$ would have perfect fifths and almost just major and minor thirds. Likewise, the minor triads e_b–$f^{\#}$–b_b, a_b–b–e_b and b_b–$c^{\#}$–f have perfect fifths and nearly just minor and major thirds. The problematic triad in major would be the triad $c^{\#}$–f–a_b which offers a nearly just major third but a narrow fifth (at 678 cents), and the corresponding minor triad $c^{\#}$–e–a_b which has the nearly just major third e–a_b but the same narrow fifth $c^{\#}$–a_b.

Though Pythagorean tuning perhaps was suited to late medieval organs still conceived as a so-called Blockwerk (several pipe ranks per scale tone of the keyboard mounted on one undivided wind chest, see [39]), it fell short of providing just intervals needed in musical genres that exposed simultaneous thirds. While the use of just major thirds was apparently common in singing (as several theorists assert), a clear indication for a scale different from Pythagorean lore is found in Ramis de Pareia's *Musica practica* (1482/1901). Ramis de Pareia ([40], part I, Chap. 2) gives a division of the monochord that leads to a scale spanning two octaves. Taking a = 1/1 as the tone corresponding to the full string, a scale a–a″ results

a	b	c′	d′	e′	f′	g′	a′	b′	c″	d″	e″	f″	g″	a″
1/1	8/9	5/6	3/4	2/3	5/8	5/9	1/2	15/32	5/12	3/8	1/3	5/16	5/18	1/4

Ramis de Pareia (40, part I, Chap. 5) expands this diatonic scale to a chromatic one, which represents the following segment of a tone-net (cf. [10, 161]):

		d	a	e	b	$f^{\#}$	$c^{\#}$
a_b	e_b	b_b	f	c	g		

Taking c as the centre, the intervals for the scale would be in modern cents:

c	$c^{\#}$	d	e_b	e	f	$f^{\#}$	g	a_b	a	b_b	b	c′
0	92	182	294	386	498	590	702	792	884	996	1088	1200

This scale has the advantage of including, besides perfect fifths and fourths, four just major thirds, three just minor thirds as well as just major and minor sixths. There are still some Pythagorean intervals (e.g., the minor third c–e_b, the minor sixth c–a_b, the major third e_b–g), however, the just major and minor thirds that could be used for several just major and minor chords (B_b-major, F-major, C-major, d-minor, a-minor, e-minor) would be the main achievement if Ramis' chromatic scale would have been implemented on a keyboard (Ramis addresses the issue of actually tuning his scales in part III, Chaps. 13 and 14 of his treatise). By about 1500, the just major third 5/4 was accepted as a consonance in works on music theory and was used in musical composition. Major chords can be found, for example, ending musical settings assembled in the *Buxheimer Orgelbuch* (ca. 1460-70). The just major third 5/4 has the just minor third 6/5 as a complementary interval within the perfect fifth (5/4 * 6/5 = 3/2), and the two just thirds have the just major sixth (5/3) and the just minor sixth (8/5) as complementary intervals within the octave (5/3 * 6/5 = 2/1; 8/5 * 5/4 = 2/1). Hence, the quest for just major thirds almost automatically involved tuning several intervals to just intonation ratios.

There are musical works from the 15th century onwards proving that major thirds gained importance in keyboard music. The change from textures based on perfect fifths (as are found in the *Estampie* and the *Retrove* of the Robertsbridge manuscript from c. 1325) to a much more frequent use of thirds as in settings of songs (e.g., *Mit ganczem Willen*) in Conrad Paumann's *Fundamentum organisandi* (Nuremberg/Munich 1452) is obvious, and is continued in organ pieces where major thirds are prominent as in *In dulci jubilo*, contained in Fridolin Sicher's organ tabulature (St. Gallen, c. 1512), or in Hans Kotter's Präambulum in fa (tablature, ca. 1520). Tuning organs in major thirds also must have been explored since Schlick [41], an organist experienced in tuning, writes in his treatise on organ builders and organs (1511) that three just major thirds, stacked upon each other, would be good in quality as such, however, would fail to give one octave as the third tone would be too low (in fact, missing the octave by a diesis of 41 cents). Schlick gave a description of a practical tuning process which would result in a temperament similar to what became known later as 1/4-comma meantone temperament. His tuning aims at just major thirds by slightly narrowing the fifths. Basically, tuning four fifths which are somewhat smaller (in regard to fundamental frequencies) than the ratio 3/2 would yield a tone that is close to a ratio 5/4 relative to the first tone, like, for example,

$\underline{e}$ ←

c .... g .... d .... a .... → e

Taking the difference between the fourth fifth e and the just major third $\underline{e}$ (e^{-1}), which is the syntonic comma of 21.5 cents, it has been equally distributed to the four fifths which are thus narrowed each by c. 5.5 to c. 696.5 cents. The tone d would be the mean (193 cents) between c and e. In a tentative reconstruction of Schlick's meantone temperament [42, 26–29], the scale he tuned would be close to these cents:

c	$c^{\#}$	d	e_b	e	f	$f^{\#}$	g	$g^{\#}$	a	b_b	b	c′
0	76	193	310	386	503	579	696.6	793	890	1007	1083	1200

This temperament offers no less than seven just major thirds and works fairly well for a number of major and minor chords (C, D, E_b, F, G, A, B_b-major; a, b, c, d, e, $f^{\#}$, g-minor) which are in the center of harmonic keys in use at that time. There are some intervals which are problematic in regard to roughness (e.g., the thirds $c^{\#}$–f, $f^{\#}$–b_b, b–e_b are c. 427 cents wide, the fifth $c^{\#}$–$g^{\#}$ has c. 717 cents).

Indications for a temperament that features major thirds and accepts narrowed fifths are found in various sources after 1500 (cf. [8, 12]). However, the tuning instructions that appear as an appendix to the important organ tablature of Johannes of Lublin (Jan z Lublina, c. 1540) still feature perfect fifths with only the two fifths f–c and g–d narrowed, and several major thirds clearly sharpened; the two fifths have been tentatively estimated at being narrowed by 1/3 of a syntonic comma or ca. 7 cents (see [43]). One reason to keep closer to a Pythagorean type of tuning perhaps was the system of authentic and plagal modes (as elaborated in [44]) and the modal structure in particular of church music; many sources indicate a rather gradual development from medieval psalmody and modal scale concepts to modern tonality as is evident also in secular works for organ (see [45]).

In a treatise of Pietro Aaron (or Aron) the issue of temperament (labelled 'participatione') is addressed where the major third c–e shall be tuned sonorous and just (*sonora et giusta*, [46], cap. XLI). Though *giusta* could be taken to mean 'correct' as well as 'just', *sonora* suggests this major third should be in just frequency ratio (or very nearly so) in order to avoid beats and roughness (as one will experience with Pythagorean major thirds 81/64). If the just major third had become the decisive interval in regard to tuning, perception, and composition of musical works, the system that could provide for a maximum of eight just major thirds contained in a scale of but twelve tones is what we know as 1/4-comma meantone 'temperament'; the term 'temperament' is not quite correct since there are eight just major thirds at the core of the system (while the technical term 'meantone' is from the 19th century and reflects the division of the major third in two equal whole tones). Eight just major thirds are at hand if four pairs of just thirds (b_b–d–$f^{\#}$, f–a–$c^{\#}$, c–e–$g^{\#}$, e_b–g–b) are tuned like

Scheme of quarter-comma meantone temperament for 12 keys

-2	$f^{\#}$ ….	$c^{\#}$ ….	$g^{\#}$		
	|	|	|		
-1	d ….	a ….	e ….	b	
	|	|	|	|	
0	b_b ….	f ….	c ….	g	
				|	
+1				e_b	
	+11	+5.5	0	-5.5	cent deviation from pure fifth

The just major thirds are in the vertical in this lattice and connected by the sign |. The fifths narrowed by one or two quarters of a syntonic comma are in horizontal direction and connected by…. in this scheme. In 1/4-comma meantone tuning the most problematic interval is $g^{\#}$–e_b, which has 738.5 cents and can hardly be used as a fifth. This was the prize to be paid for the sweetness of so many just thirds and sixths. The scale corresponding to the scheme shown above is

c	$c^{\#}$	d	e_b	e	f	$f^{\#}$	g	$g^{\#}$	a	b_b	b	c′
0	75.5	193	310.5	386	503.5	579	696.5	772	889.5	1007	1082.5	1200

In this meantone scale, d is right in the middle between c and e (consequent to dividing the major third by half). In a straightforward mathematical treatment, the division of the major third can be done like $\sqrt{\frac{5}{4}}$, which yields 1.11803 = 193.2 cents. This, however, is a modern way of calculation that was not feasible by about 1520. In retrospect, the division of the rational interval of the just major third 5/4 into two whole tones 9/8 and 10/9 (which are also rational superparticular intervals, see [10]) as anticipated in the tetrachord divisions of Didymos and Ptolemy (see above) seems of importance since it permitted to build a just diatonic scale suited to intonation of harmonic major.

	9/8	10/9	16/15	9/8	10/9	9/8	16/15
c	d	$\underline{e}$	f	g	$\underline{a}$	$\underline{b}$	c’
1/1	9/8	5/4	4/3	3/2	5/3	15/8	2/1

Arranged as a small segment of the tone-net, the tones will form a structure like

-1	[$\underline{d}$]	$\underline{a}$	$\underline{e}$	$\underline{b}$	
0		f	c	g	d

The tones of this diatonic scale suffice to create three major chords (C, F, G) forming a harmonic cadence. There are also three minor triads (a, e, and d). However, for the minor triad $\underline{d}$–$\underline{a}$–f, the tone $\underline{d}$ (in brackets) is not available from this scale. The fifth d–$\underline{a}$ (of the ratio 40/27 = 680.5 cents) included in the diatonic scale is narrowed by one comma.

Probably the first theorist who understood the dilemma of tuning just fifths and major thirds in regard to building a chromatic scale with a rather small number of scale steps was Fogliano. He (1529, fol. xxxv) presented a chromatic scale which doubles the tones d and b_b; as he uses both the diatonic (16/15) and the chromatic (25/24) semitone as well as the syntonic comma 81/80, the following scale results:

c	$c^{\#}$	$\underline{d}$	d	e_b	$\underline{e}$	f	$f^{\#}$	g	$g^{\#}$	$\underline{a}$	b_b	$\overline{b}_b$	$\underline{b}$	c′
1/1	25/24	10/9	9/8	6/5	5/4	4/3	25/18	3/2	25/16	5/3	16/9	9/5	15/8	2/1

Ordered into a tone-net, the fourteen tones per octave result in this structure:

-2	$f^{\#}$	$c^{\#}$	$g^{\#}$		
-1	d	a	e	b	
0	b_b	f	c	g	d
+1				e_b	b_b

The expansion of the chromatic scale from twelve to but fourteen tones per octave yields no less than eight just major chords and seven just minor chords. In addition, it offers chromatic and diatonic semitones (for example, $\underline{e}$–e_b, c–$c^{\#}$, f–$f^{\#}$, g–$g^{\#}$ = 25/24 = 70.7 cents; $\underline{d}$–$c^{\#}$, $\underline{e}$–f, b_b–$\underline{a}$ = 16/15 = 111.7 cents) as well as a semitone 135/128 = 92.2 cents (the step b_b–$\underline{b}$) and a semitone 27/25 = 133.2 cents ($f^{\#-2}$–g). In addition, there is the fourth $\underline{d}$–g and the fourth f–$\bar{b}_b$ which have a ratio of 27/20 = 519.6 cents. The cost for realizing Fogliano's scale in a keyboard instrument would be adding two extra keys and strings or pipes per octave. Fogliano did not see this as practical and considered tuning a tone halfway between the two doubled tones (d, d^{-1} and b_b, b_b^{+1}) instead. In a modern approach, the geometric mean of the whole tones 10/9 and 9/8 would be calculated like $(\sqrt{5})/2 = 1.11803$, which equals 193.2 cents, the size of the meantone. Though Fogliano's 14-tone scale apparently was a construct devised to solve a problem in music theory, it was well within the possibilities of instrument building in the 16th and 17th centuries when indeed a considerable number of organs and harpsichords had one or several split keys (see [1, 3, 12, 47]).

Meantone tunings (of which several varieties were in use) featuring just major thirds basically face the same problem one experiences with Pythagorean chains of perfect fifths: they do not easily lead to a cycle within an octave that comprises no more than twelve tones and keys. Therefore, adding at least one or two tones and keys per octave was inevitable if the so-called "wolf" $g^{\#}$–e_b was to be eliminated. The most common solutions were split keys for $d^{\#}/e_b$ and $g^{\#}/a_b$ as well as, in a smaller number of instruments, $a^{\#}/b_b$. Expanding meantone tuning even further, split keys for all accidentals were implemented (=17 tones/keys per octave). Adding two more tones and keys for $e^{\#}/f_b$ and $b^{\#}/c_b$ results in a 19-tone cembalo cromatico (as shown in [48, 141]) and described, for the cembalo universale owned by Karel Luyton at Prague, in Praetorius' treatise ([49, T. II, 63ff.]) on music and musical instruments.

Assuming that a maximum of just major thirds was the main purpose for developing extended meantone tunings, the scheme of 19 tones implemented in the cembalo cromatico and, by comparison, the 24 tones/keys per octave Zarlino had on his enharmonic harpsichord built in 1548 (cf. [3, 17ff.]) can be shown in a tone lattice like

+4					$g^{\#\#}$	$d^{\#\#}$	$a^{\#\#}$	
+3	$e^{\#}$	$b^{\#}$			$e^{\#}$	$b^{\#}$	$f^{\#\#}$	$c^{\#\#}$
+2	$c^{\#}$	$g^{\#}$	$d^{\#}$	$a^{\#}$	$c^{\#}$	$g^{\#}$	$d^{\#}$	$a^{\#}$
+1	a	e	b	$f^{\#}$	a	e	b	$f^{\#}$
0	f	c	g	d	f	c	g	d
-1	d_b	a_b	e_b	b_b	d_b	a_b	e_b	b_b
-2				g_b				g_b

For an arrangement of keys and pitches shown in the tone lattice on the left, the term 'cembalo cromatico' seems not quite correct since there are intervals smaller than the chromatic semitone 25/24 (70.7 cents). Hence, enharmonic melodic phrases could be realized making use of the difference in pitch between the sharps and the flats (e.g., d_b–$c^{\#}$ etc.) as well as between $b^{\#}$ and c, $e^{\#}$ and f. Zarlino's instrument in fact was suited to playing enharmonic intervals and melodic phrases. Vicentino [50] had the number of tones and pitches on his keyboard expanded to 31 to the octave (a similar instrument was built, in 1606, by Vitus de Trasuntino; for his 'Clavemusicum omnitonum', see [3, 25f.]).

Zarlino, in Part II of the *Istitutioni*, offers an in-depth elaboration of tetrachord divisions, scales and aspects of tuning strings on a monochord in which he refers to Greek writers, in particular to Ptolemy and his tetrachord divisions; the diatonon syntonon was of special interest to Zarlino because of the just major third and its division into two whole tones of different ratio and interval size. In regard to tuning keyed instruments (such as a gravecembalo), Zarlino distinguishes between 'natural' intervals and temperaments (*temperamento o participatione*). After briefly mentioning his own instrument (a *clavocembalo* built, in 1548, by Domenico da Pesare; see Istituioni 1558, 140/41) suited to realize chromatic and enharmonic harmony, Zarlino [51] refers to yet another instrument of which more would be said in his *Demostrationi harmoniche* (published in 1571 but apparently written at the same time as his first book). In this book, however, there is again only a brief passage (on p. "212", which is the wrongly numbered p. 221) while a full description is found in Zarlino's *Sopplimenti musicali* (1588, cap. XI). Zarlino discusses a tone system (systema massimo arteficiale del naturale ò syntono diatonico) which comprises 33 tones and pitches in a two-octave range (A–a–aa). He takes whole numbers (a method known since Aristides Quintilianus and Boethius, see [10, 147] which can be taken to represent the distances between tones on a string of a monochord. Transferred into modern cents, the structure of his scale may be shown here only for the lower octave:

A 112 B_b 21.5 B_b 71 B 112 c 70 $c^{\#}$ 113 d 21.5 d 90 e_b 21.5 e_b 71 e 112 f 71 $f^{\#}$ 21.5 $f^{\#}$ 112 g 71 $g^{\#}$ 112 a

Relative to c, the lattice of tones would contain

-2		$f^{\#}$	$c^{\#}$	$g^{\#}$		
-1		d	a	e	b	$f^{\#}$
0	e_b	b_b	f	c	g	d
+1					e_b	b_b

Implemented on a keyboard (as shown by [52, 156]), these 16 tones and pitches per octave would offer a range of eight just major and seven minor triads. It seems the solution proposed by Fogliano [53] in regard to a doubled d and a doubled b_b thereby was used for the tuning of an advanced keyboard instrument (though the doubled e_b in Zarlino's scale hardly offers any benefit – while a $d^{\#\ -2}$ or an a_{b+1} tuned instead would have).

Though Zarlino discusses Greek scales in his treatises extensively, it was not his intention to revive the music of the 'antichi' in the sense of using chromatic or even enharmonic scale models based on tetrachords. Rather, his goal was to explore intervals and chords in regard to just intonation as is obvious from his own compositions, in particular the *Modulationes sex vocum* [54], a collection of motets published in 1566 (a critical edition by Collins Judd and Schiltz was published in 2015; a recording by the ensemble Singer Pur of Munich was issued in 2013). These works are in the tradition of Adrian Willaert and the vocal polyphony for which Venice was famous. Zarlino composed motets rich in harmony based on the just thirds he had justified, in the *Istitutioni*, with his concept of the *senario*. In this respect, his approach was different from that of some contemporaries, among them Nicola Vicentino, who apparently had a more experimental attitude towards the use of chromatic and enharmonic intervals in musical settings. A good example is the small madrigal ('madrigaletto') *Dolce mio ben*, of which Vicentino (1555, cap. LII) offers three versions, one in the diatonic genus, one in the chromatic, and one in the enharmonic (for a detailed analysis including sound examples of the different versions, see Cordes [33]). For the rehearsals with his students as well as for demonstrations, Vicentino used an archicembalo that had 31 tones to the octave. The exact tuning of the instrument has been a matter of debate since the two schemes Vicentino offers for tuning allow for some interpretation (see [12, 390ff.]). However, a model where just thirds are piled up in the vertical in six or seven rows (cf. [3, 25]) seems plausible since it can be taken as a further extension of the meantone tuning beyond the 19-tone cembalo cromatico (see above). If Vicentino's division of the whole tone into five parts is taken as meaning interval steps of equal size, a regular temperament could be assumed where the diesis of 38.71 cents is the basic unit. Multiples of this unit result in the chromatic and in the diatonic semitone

(77.5 and 116.1 cents, respectively), the minor and the major third (at 310 and 387 cents, respectively), and the minor and major sixth (at 813 and 890 cents, respectively). This system, described much later (1661/1691) with mathematical background by Christiaan Huygens (see [6, 7]), offers a range of nearly just intervals (including a 'natural' seventh close to the ratio 7/4 at 968 cents) but maintains the slightly narrowed fifths and slightly widened fourths as well as the whole tone (at 193.55 cents) halfway between 10/9 and 9/8. In this respect, Huygens' cycle is an expansion of the meantone system (with a number of additional tones and pitches that were of little use in Baroque music but became a means for contemporary music in the 20th century, see [7]). Implementing 31 tones and pitches to the octave on a keyboard is a demanding task for both the instrument builder (skilfully mastered by Trasuntino and other artisans) and the musician who must adapt to a keyboard with at least three rows of keys. Vicentino was not the only enharmonic experimentalist. There were more instruments with more than 19 keys to the octave in use (see [3, 47]). A late specimen of a sophisticated keyboard with 31 keys to the octave is a Hammerclavier built by Johann Jakob Könnicke, in 1796 (see the photo in [3, 465]). The keys are ordered in a very intelligent fashion, which makes playing certain chord patterns fairly easy (a description of the arrangement of keys and pitches is given by Vogel [14], 304-08 and pp. 319-23 in the English edition of 1993). The 31-tone pipe organ which was built in the Netherlands in 1945 also offers a special keyboard designed by Fokker (see photos in [7]) which permits to play sequences of major or minor chords by shifting the hands in diagonals without changing the fingering.

Of course, raising the number of pitches and keys per octave in a regular division improves the approximations to just intonation pitches. While a division of the octave into 31 dieses of 38.71 cents each is sufficient to produce nearly just thirds and sixths as well as the 'natural' seventh, just fifths and fourths require a division of the octave into 53 equal parts of 22.64 cents each. Evidently, the unit here is a 'comma' (close in size to the syntonic comma of 21.5 cents), the multiples of which will give suited musical intervals (e.g., the sum of 17 commas yields a major third of 385 cents, 22 commas make up a perfect fourth of 498 cents, the sum of 31 commas gives a perfect fifth of 702 cents, etc.). The division of the octave into 53 equal steps, which seems to have been calculated by the mathematician Nicolaus Mercator by about 1660 (he first calculated a 'comma' corresponding to a division of the octave into 55 equal parts), found renewed interest in the 19th century (see [3]). There are more such equal divisions (e.g., 72 pitches and scale steps to the octave), some of which have been used in composition and in the performance of microtonal works by making use of electronic keyboard instruments (see [55]). However, in a historical perspective, mechanical instruments were difficult (and costly) to build with more than 12–14 keys per octave. Even though, the chromatic and enharmonic keyboard instruments that were built, in particular in Italy in the 16th and 17th centuries (for a survey, see [2, 3, 47]), respectively, greatly supported musical practice which saw a range of highly chromatic works for keyboards written by, among others, Merulo, Mayone, and M. Rossi.

Even the 'standard' 1/4-comma meantone tuning confined to 12 keys and pitches to the octave supports chromatic expression to some degree since it offers a diatonic (117.5 cents) and a chromatic semitone (75.5 cents) which are audibly distinct. Progressions in semitones as are found frequently in keyboard works of the 17th century (written by, among others, Sweelinck, Bull, Philips, Schildt, Froberger), when played on harpsichords and organs tuned to 1/4-comma or one of the meantone varieties, are of interest to listeners who may recognize different interval sizes. Keys available in a common meantone tuning with good sound quality typically span from $E_{\flat}$-major to A-major (that is, from three flats to three sharps). There are works in E-major like the Praeludium in E from Dietrich Buxtehude (BuxW 141) which can also be played on an organ in meantone tuning (with cautious registration in regard to the use of mixture stops and still accepting a few relatively harsh sonorities), and even many of Bach's organ works can be played on an organ tuned to 1/4-comma meantone though there are some parts in a number of works that sound quite harsh in this temperament (cf. [18]). A scale of but 12 pitches to the octave for a number of Bach's organ and harpsichord works seems insufficient since, for example, in the Fantasie und Fuge in g-minor (BWV 542), for the harmonic modulation found in measures 31–38 of the Fantasie, one would need a total of about 25 different pitches and tones if this part would be played in just intonation, that is, with perfect fifths and fourths as well as with just major and minor thirds. Of course, many works for keyboards of the 17th and early 18th century were far less bold in their harmonic structure, and restricted to those keys and chords which turn out to be pleasing in their sound in meantone tuning. To be sure, the meantone concept was developed with the major third as the basic structural interval in mind, and in regard to the 'sweetness' of simultaneous thirds and sixths it could offer to the player and listener alike. It was for this effect that various composers adapted Dowland's *Lachrimae* to versions for keyboard instruments.

One has to remember that compositional practice in the 17th century and even in the first half of the 18th century still included the regular use of modal scales and melodic patterns while chord progressions were formed in simple or extended cadences that established the concept of major and minor tonalities, respectively (elements fundamental to this new concept were discussed, for example by Rameau in his books on music theory of the 1720s and 1730s, see [56, 57]). A harmonic tonality typically involves a centre expressed by a major or a minor chord in a certain key from which one can modulate into adjacent or more distant keys. The 'distance' thereby in general is conceived in terms of fifths, and the geometric structure to represent keys is known as the 'cycle of fifths'. In 'western' music theory, ideas on such a cycle were issued before 1700 (for example, by A. Kircher). A more formal discussion on the relationship of tones was offered by Johann Heinichen who, in 1711, published a 'musical cycle' that shows the tones and keys actually used at his time plus a few more distant tones and keys that were conceivable yet not practical. A revised version of the Musicalischer Circul was published by Heinichen [58]. Heinichen ([59, 261ff.]) explains that the use of tones and keys in practice could go as far as B-major on the side of the sharps around the circle, and to b_b-minor on the side of the flats, both taken as 'extremes'. This would mean twenty out of twenty-four major and minor chords and keys were in use. In Johann Fischer's *Ariadne Musica* (1702, 1710) there are twenty tonalities, ten major (A_b, E_b, B_b, F, C, G, D, A, E, B), nine minor (f, c, g, d, a, e, b, $f^{\#}$, $c^{\#}$), and e-Phrygian. However, the e-minor is conceived as e-'Dorian', and there are more modal remnants in Fischer's cycle (see [45]). Heinichen [59] warned that the use of the most distant keys and chords in his cycle would be of no avail. One possible interpretation of his statement could be that these distant keys are too remote in regard to forming meaningful sequences of keys and chord progressions relative to a well-established tonal centre (which he identifies as C-major). Another aspect possibly included in Heinichen's discussion is that of tunings and temperaments. Though it is relatively certain that 1/4-comma meantone tuning remained the standard in many areas of Europe well into the 18th century (see [1]), it is also known from various sources that organ builders and organists experimented with temperaments where the size of fifths and thirds varied in such a way that certain keys were quite smooth in regard to roughness and beats (the 'good keys') while others were more harsh in particular when chords were played with a registration that involved mixture stops (which to this day are tuned in just intervals). In the period from c. 1680 to c. 1770 various 'well-tempered' tunings were proposed and/or explored in practice (see [8, Chap. 7]). Werckmeister offered several tunings of which Werckmeister III (sometimes also counted as no. IV) became well-known as "the" Werckmeister tuning model (see [19, 60]). The concept of this tuning was a closed circle of fifths, which means that several or all fifths need to be narrowed in order to distribute the 'overshoot' (see above) of a Pythagorean comma (ca. 24 cents). In Werckmeister III there are four fifths (c–g, g–d, d–a, and b–$f^{\#}$) which are narrowed by a quarter of the Pythagorean comma ([4, 161]; [60]). The following scale results (rounded to full cents):

c	$c^{\#}$	d	e_b	e	f	$f^{\#}$	g	$g^{\#}$	a	b_b	b	c′
0	90	192	294	390	498	588	696	792	888	996	1092	1200

In this scale, the d is still a meantone, c–e_b comes as a Pythagorean minor third, and the fifth c–g is of nearly the same size as the tempered fifth in 1/4-comma meantone whereas the fourth c–f here is perfect, and the b_b is slightly flattened and the b sharpened in comparison to 1/4-comma meantone. The major third is still quite good though a C-major chord suffers from the third being slightly too wide and the fifth being narrowed, the interval between them a minor third of 306 cents. The third f–a (390 cents) is good and g–b (396 cents) acceptable, however, the thirds $c^{\#}$–f, $f^{\#}$–b_b and $g^{\#}$–c are Pythagorean (408 cents). The major thirds e_b–g, e–$g^{\#}$ and a–$c^{\#}$ all have 402 cents. In Werckmeister III chords in the center (C-major, F-major, G-major, D-major) appear quite fair relative to just intonation intervals while triads in keys with more accidentals are less satisfactory. In this respect, major and minor chords in various keys can be distinguished by their sonorous quality (for data, see below) while there is no obvious discordance in Werckmeister III like the 'wolf' in 1/4-comma meantone. Thus, Werckmeister III would support modulation through a wider range of keys as is suggested by Heinichen [58, 59]. It is a common feature of 'well-tempered' tuning models discussed or empirically tested that they seek to allow modulation through most or even all (commonly accepted) major and minor keys while maintaining some musical and perceptual discriminability between different keys.

The important achievement of the 1/4-comma meantone tuning had been a maximum of eight major thirds out of a scale comprising, in its basic form, only twelve tones and pitches to the octave, at the cost of the 'wolf fifth' as well as some other relatively poor intervals. The 'well-tempered' tunings could remedy the obvious defects of 1/4-comma meantone yet had to sacrifice the just major thirds to some extent. In sum, one can see that the improvement of the fifths in 'well-tempered' systems as well as the possibility for harmonic modulation through many keys was kind of an intermediate solution between the Pythagorean approach (just fifths and fourths plus a few nearly just thirds) and the meantone concept (numerous just thirds and sixths, tempered fifths and fourths). 'Well-tempering' in many instances was derived from the experience of tuning keyboards as apparently was the case with J.S. Bach who tuned his own instruments (there are legions of interpretations what 'well-tempered' may have been for Bach and his 'Well-Tempered Clavier', see [8, 15–17, 61]). Some of the more theoretical approaches (e.g. [19]) to finding the 'very best temperament' still made use of geometrical tools such as dividing strings on a monochord into sections, or tried to calculate equal temperaments from a basically geometric perspective (as did Neidhardt in a number of studies, see [8, 264ff.]). Of course, there were also attempts at finding a circular equal temperament in an algebraic calculation. The means for such calculations included logarithms which had been developed already in the 16th century. However, sources indicate that Juan Caramuel Lobkowitz in about 1647 was the first to suggest logarithms to base 2 as a measure suited to calculate and represent musical intervals (see [3, 282ff.]). The mathematicians Isaac Newton and Leonhard Euler also contributed to such calculations. In the 19th century, another measure was proposed by the French acoustician, Felix Savart, which defines 1 octave = 1000 $\log_2$ (=301.03 Savart, see [62, 3f.]). Further, the physicist Arthur von Oettingen calculated intervals as milli-octaves, mo (cf. [14, 111]). The mo, which is 1/1000 of an octave, can be expressed like

$$1\,\text{mo} = \sqrt[1000]{2} = 2^{1/1000} = 1.000934.$$

Thus, a sine tone differing from a standard (say, A_4 = 440 Hz) by 1 mo, would have a frequency of 440 × 1.000934 = 440.4109 Hz. The pure fifth (3/2) has 585 mo, the just major third has 322 mo, the just minor third 263 mo. The advantage of the mo is that just intervals result in whole numbers.

Since the octave typically comprises twelve semi-tones (of equal or unequal size), a division of the octave into 1200 basic units rather than 1000 mo seemed appropriate. Alexander J. Ellis suggested the modern cent as $\sqrt[1200]{2} = 2^{1/1200}$, whereby 1 cent = 1.00058. This unit is convenient for expressing musical intervals in ET12 where 1 semitone = 100 cents, meaning all intervals in ET12 are multiples of 100 cent. However, their frequency ratios are complicated consequent to the tempering which, in ET12, defines the fifth as 1:1.498307 (=700 cents) and the frequency ratio of the major third as 1:1.259921 (=400 cents). Representing two sine tones (in this article, a tone is considered as a physical phenomenon notwithstanding its musical functions) each by a single frequency, f_1 and f_2, the interval they form can be expressed as the ratio f_2:f_1 and the interval can be calculated in cents like 1200 $\log_2 (f_2/f_1)$. For example, taking two sine tones of 200 and 300 Hz, respectively, the pure (or just) fifth thereby can be calculated like

$$1200 \log_2(300/200) = 701.955 \, \text{cents}.$$

The difference between structurally important intervals in just intonation and ET12 is this:

Interval	Just	ET12
Fifth	702	700
Fourth	498	500
Major third	386	400
Minor third	316	300
Major sixth	884	900
Minor sixth	814	800
Minor seventh	969	1000

The largest deviation from a just interval thus is about 16 cents, with the exception of the minor seventh. If one accepts that, for example in a dominant seventh chord, the seventh should be of the ratio 7/4 (see [14]), corresponding to the 'natural seventh' (the seventh harmonic in a harmonic partial structure), the deviation in ET12 from the just interval is more than 31 cents.

The quest for ET12 can be viewed as a solution to the prime number discrepancy stated as $3^n \neq 2^m$ and $5^n \neq 2^m$. In order to derive cyclic scales closed within each octave, some adjustment of the size of intervals is necessary (cf. [11]). This led to concepts of regular as well as irregular temperaments (meanings of the Latin noun *temperamentum* include 'the right measure'). A regular temperament does not imply that all scale steps are of the same size (see [8]). However, a regular temperament can be established by dividing the octave into *k* equal parts. With a division into twelve parts, ET12 can be realized as a tuning (the term tuning rather denotes the actual process of pitch adjustment than the calculation of pitch frequencies or pitch ratios). In ET12, the deviations from just intonation are small for fifths and fourths yet considerable for thirds and sixths, putting ET12 relatively close to Pythagorean tuning. If one prefers a temperament and tuning that offers nearly just thirds and sixths as well as 'natural sevenths', ET31 would be the choice. In case fifths and fourths as well as thirds and sixths should be close to just intonation pitches, ET53 seems the best solution. If the prime number 7 is taken into account in addition to the prime numbers 2, 3 and 5 for the generation of musical intervals, a division of the octave into 171 small steps gives the best approximation to just pitch ratios (see [14]).

The choice for a particular regular or irregular temperament may be guided by certain criteria such as the maximum deviation from just intervals one is willing to accept in terms of cents (for mathematical models and calculations of scale models and tunings, see e.g. [63, 11, 17, 9]), or the amount of roughness and beats one may allow in simultaneous intervals and chords (see [29]). ET12 can be regarded a good compromise since it offers (1) a closed cycle of tones per octave as well as (2) usability of twelve major and twelve minor keys. In the 17th and well into the 18th century, exact calculation of ET12 pitch ratios was a problem, and actually tuning an organ to ET12 was difficult because ET12 involves irrational pitch ratios, on the one hand, and quite irregular beat frequencies, on the other (the German term *gleichschwebende Temperatur* for ET12 is misleading. While the size of semitones in ET12 is fixed, beat frequencies vary for the eleven intervals within different octaves).

The mathematical solution for ET12 nowadays is straightforward by solving the equation (in the syntax of Mathematica©)

$\text{Solve}\,[x^{12} = 2^7, x]//\text{N}$ or solving the equation $\text{Solve}\,[(x/2)^\wedge 12 - (2/1)^\wedge 7 == 0, x]//\text{N}$

For x, a set of solutions is obtained which includes $x \rightarrow 1.49831$, meaning the size of the fifths must be narrowed from a ratio of 3:2 (or 1.5:1) to 1.49831:1 to make 12 fifths equal 7 octaves. In fact, the number 1.49831 indicates an interval size of 700 cents (the fifths in ET12) which results from distributing the overshoot of 24 cents equally to 12 fifths. Likewise, the frequency ratio for the semitone in ET12 can be found from the equation

$$\text{Solve}\,[2^7 + x^{12} == 0, x]//\text{N}$$

where the set of solutions includes $x \rightarrow 1.05946$, which equals $\sqrt[12]{2}$ = 100 cents. From here, finding the major third in ET12 is easy since $\left(\sqrt[12]{2}\right)^4$ = 1.25991 ≈ 400 cents.

Though ET12 appears as an elegant solution in that it distributes the Pythagorean comma equally to twelve fifths, it met considerable resistance in the 18$^{\text{th}}$ century because it practically eliminated musical and perceptual differences between keys. As an alternative, various temperaments were explored which affect the basic intervals (semitones, tones, fifths, fourths, thirds, sixths) to different degrees (see [8]). Solutions depend on decisions one makes in order to keeping certain intervals close to just frequency ratios while others then will deviate a bit more from just ratios. Such decisions in general have effects on the musical keys and chord textures that sound smoothly within a given temperature and tuning. Deviations from just intervals must be small enough to avoid whatever perceptions of mistuning of certain scale steps and intervals. Among the temperaments that met this requirement is 1/6-comma meantone, where the fifths are narrowed by 1/6 of a syntonic comma (3.6 cents, see [64, 456]) to c. 698 cents, to the effect that most of the major thirds (c–e, e–$g^{\#}$, f–a, a–$c^{\#}$, g–b, d–$f^{\#}$, e_b–g, b_b–d) are close to 393 cents. The 'wolf' between $g^{\#}$ and e_b is not eliminated but is reduced to 718 cents. There are three rather problematic major thirds ($f^{\#}$–b_b, $c^{\#}$–f, $g^{\#}$–c) which have c. 413 cents, and, correspondingly, there are three problematic minor thirds (b_b–$c^{\#}$, f–$g^{\#}$, e_b–$f^{\#}$) which are significantly narrow.

Among the temperaments that have gained importance is one attributed to the Italian composer, organist and theorist, Francesco Antonio Vallotti (a part of his work was published in 1779, while the part containing his concept of temperament was left in manuscript and published only in 1950; see [8, 306]). A very similar temperament was devised by the English scientist, Thomas Young (who actually proposed two temperaments in 1800). The basic idea in Vallotti's temperament is to tune six fifths f–c–g–d–a–e–h so that each fifth is narrowed by 1/6 of a comma (to 698 cents), and to tune another six fifth in just frequency ratios. Correctly notated, these intervals would be f–b_b–e_b–a_b–d_b–g_b–c_b, however, usually the tones are given as f–b_b–e_b–a_b–d_b–g_b–b or, if tuning in upward direction is chosen, as b–$f^{\#}$–$c^{\#}$–$g^{\#}$–e_b–b_b–f in order to underline the circular character of this temperament. The scale then has these tones and intervals (rounded to full cents):

c	$c^{\#}$	d	e_b	e	f	$f^{\#}$	g	$g^{\#}$	a	b_b	b	c′
0	94	196	298	392	502	592	698	796	894	1000	1090	1200

In this temperament, there are major thirds of different size. Major thirds in the middle of the tonal area (f–a, c–e, g–b) have 392 cents, and b_b–d and d–$f^{\#}$ have 396 cents. The thirds e_b–g and a–$c^{\#}$ have 400 cents, e–$g^{\#}$ and $g^{\#}$–c have 404 cents, and b–e_b, $f^{\#}$–b_b, and $c^{\#}$–f have 408 cents, respectively. Hence, there is a gradation in the thirds from those relatively close to the just ratio to thirds close to ET12, and further on to a few major thirds which equal the Pythagorean ditonos. Correspondingly, there is a number of Pythagorean minor thirds of 294 cents (e.g., e_b–$f^{\#}$, $g^{\#}$–b_b) while the minor thirds closest to just ratios are a–c, e–g, and b–d, each of 306 cents. The remaining minor thirds are in between (see [64, 457]). The obvious advantage of the Vallotti temperament is that no 'wolf' interval is encountered, and that modulation through all major and minor tonalities seems possible, though with increasing deviations from just tuning towards the periphery. The gradation of intervals and chords in regard to roughness and sensory consonance can help to differentiate between keys and tonalities, and may be appreciated by listeners. It is in fact interesting to listen to Beethoven's piano works when performed in temperaments and tunings such as that proposed by Vallotti (and, with small variations, by Thomas Young).

4 Empirical Investigation of Temperaments and Tunings

In a number of studies, deviations of intervals in various temperaments from just ratios have been calculated [8, 11, 17, 63–65]. Some investigations also include calculations made from the scores of musical works where the occurrence of certain intervals and chords has been considered for such calculations. When synthesizers and digital signal processing methodology became available to sound and music research, investigations could be expanded from scores to recordings of music, and different tunings could be studied by manipulating sound parameters (see, e.g., [3, 18]).

The present study makes use of signal processing methodology in that the periodicity and harmonicity of major and minor chords is measured in the time domain using autocorrelation (AC) and crosscorrelation (CC) tools developed by Boersma [66]. These tools measure the harmonics-to-noise ratio (HNR) for a given time signal $x(t)$ which is expressed in dB. The sensitivity of the tools depends on jitter in time signals and hence on the frequency and energy distribution of spectral components as well as on temporal factors. In this respect, the dB readings allow a relative scaling of signals in regard to their periodicity. The maximum that we attained with a perfect major chord composed of three harmonic complexes each comprising ten harmonics locked in zero phase with attenuation of amplitudes like $A_n = 1/n$ was ≥ 60 dB.

In a previous study of 1/4-comma meantone tuned with precision on a historical organ built by Arp Schnitger (Hollern, Northern Germany, 1688), a clear gradation for twelve major and twelve minor chords was found [67]. Concerning major chords, there is a grading from very good (C, D) to good (G, A, E, E_b, F), while B and $G^\#$ appear as less acceptable, and $C^\#$ and $F^\#$ are problematic given their low HNR readings. Likewise, for minor chords, a-minor and d-minor are best, followed by $f^\#$-minor and e-minor while c-minor and $c^\#$-minor gave low readings.

In this study, data for two 'well-tempered' systems will be presented, namely Werckmeister III and Vallotti as tuned on a harpsichord. For our investigation, Werckmeister III and Vallotti as well as 1/4-comma meantone and some other systems were tuned on a historical Jacob Kirckman harpsichord (London c. 1766) from the collection of the second author (see [68, no. 60, 216–223]). This instrument is of interest for some extraordinary mechanical and acoustical features (see [69]). For the recordings, only one 8' stop was used and the strings of all other stops were dampened with cloth. The recordings were made with a single condenser mic (Neumann TLM 170) placed ca. 40 cm over the strings. The sound was recorded on DAT at 48 kHz/16 bit. The tuning was done relative to A_4 = 408 Hz, which is a common pitch for historical harpsichords. For the tuning, a precision digital device (TLA CTS 5-PE) was used which reads fundamental frequencies of sounds radiated from the instrument. The tuning was checked by means of spectral analysis and f_0 tracking of sounds recorded from single complex harmonic tones.

The point where the actual plucking takes place divides each string of a harpsichord into two parts from where waves propagate into opposite direction. Because partials which have a node at or near the plucking point cannot be excited to undergo vibration (as was reported by Thomas Young, in 1800), the amplitude spectrum shows characteristic troughs and dips defined by L/l (L = string length, l = plucking point measured from bridge; see [26, 27]). Since certain partials are weak or even cancelled out, the spectrum for the sound from each string becomes more or less cyclic as is shown in Fig. 2 (where a formant filter envelope is included that also shows the peaks and dips in spectral energy distribution).

The dips in spectral energy found in the sound of a single string are levelled out to some extent when several strings are played simultaneously in a chord, and partial frequencies of several tones coincide, as can be expected in particular if several tones of the chord are doubled at the octave as was the case in our experiment. The major and minor chords played on the Kirckman comprised five notes and tones each, for example, C-major consists of c_2, g_2, c_3, e_3, g_3, while for $C^{\#}$-major the notes are simply shifted in parallel by one semitone upward, for D-major by a whole tone, etc. The recordings were actually done twice, one run starting at A (because it serves as referent also for the historical tunings in the electronic tuner we used), the other at C. Because of the large number of partials contained already in the sound of individual strings, the spectrum for each chord is rather dense. Figure 3 shows the spectrum for the $C^{\#}$-major chord where the fundamental $c_2^{\#}$ is at 63.99 Hz, and significant spectral energy is found up to 6 kHz (all amplitudes are given relative to 0 dbfs).

Looking closer into the spectrum of the $C^{\#}$-major chord reveals several partials from different tones of the chord differ slightly in their respective frequency. While coincidence of harmonic partials from tones in a chord since long has been recognized as a factor relevant for sensory consonance [70], small divergence in frequency of such partials (each of them carrying sufficient energy) gives rise to

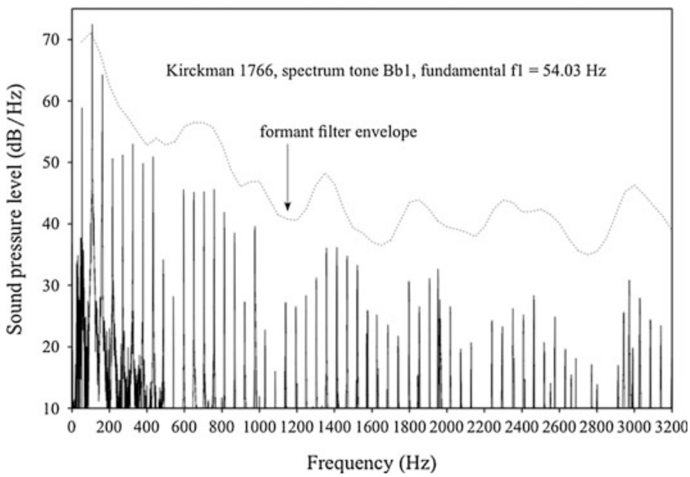

Fig. 2 Kirckman 1766, tone/string Bb_1, sound spectrum, f_1 = 54.03 Hz

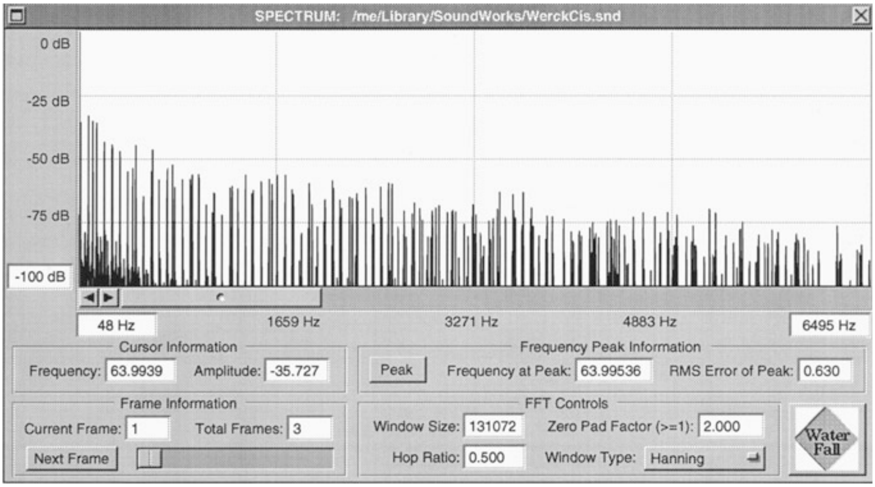

Fig. 3 Spectrum, Kirckman 1766, C$^{\#}$-major chord, Werckmeister III

auditory roughness (see [28, 29]). A clear sign of spectral inharmonicity is amplitude modulation (AM) visible in the temporal envelope of partials (see Figs. 6 and 7) as well as in the envelope of the complex signal representing a chord.

The HNR readings for major and minor chords in Werckmeister III are listed in Table 1. For each chord, decibels represent the means for HNR averaged over two seconds of sound from the onset and the standard deviation (SD) for the same segment. Taking the two first seconds of each chord seems sufficient since, due to the plucking mechanism of strings on a harpsichord, the sound level reaches maximum typically within c. 100–150 ms and then decays smoothly. For the C$^{\#}$-major chord shown in Fig. 4, the decay after two seconds is c. 12 dB from maximum.

Table 1 HNR data, Werckmeister III

Chord	dB (mean)	dB (SD)	Chord	dB (mean)	dB (SD)
C-major	14.87	3.39	c-minor	11.09	2.06
C$^{\#}$-major	10.42	2.65	c$^{\#}$-minor	7.46	1.22
D-major	11.9	2.39	d-minor	11.91	2.38
E_b-major	10.61	2.7	e_b-minor	6.0	2.65
E-Major	11.92	2.22	e-minor	4.02	1.48
F-Major	18.85	3.53	f-minor	9.01	2.59
F$^{\#}$-major	10.29	2.73	f$^{\#}$-minor	6.59	1.99
G-Major	13.74	2.57	g-minor	7.57	1.45
A_b-major	12.25	3.02	a_b-minor	8.45	2.22
A-major	15.29	3.77	a-minor	4.85	1.22
B_b-major	11.94	2.52	b_b-minor	6.63	2.46
B-major	10.14	2.49	b-minor	7.57	1.22

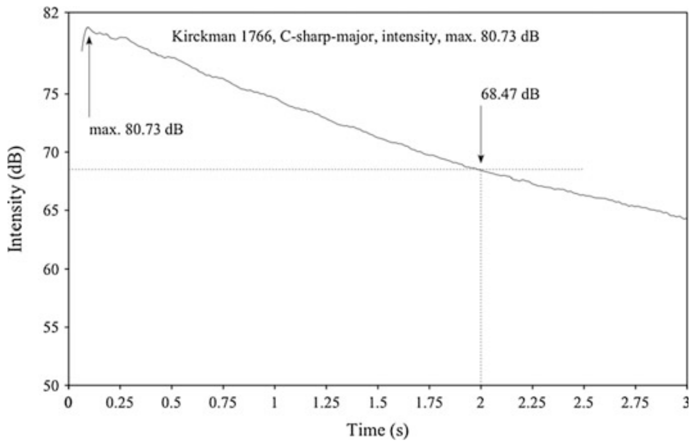

Fig. 4 Kirckman 1766, C$^{\#}$-major, intensity (dB) over time for the first 3 s

The data for the Vallotti tuning are given in Table 2.

An inspection of the data reveals, first of all, a significant difference between HNR for major and for minor chords that are due to differences in their harmonic structure. Such differences were observed also for major and minor chords in 1/4-comma meantone (see [67]). Another factor that seems of interest is the relatively large standard deviation calculated from the HNR data for various major and minor chords. In this context it should be recalled that measurements of sound signals give high readings for HNR with low SD if a signal is strictly periodic in the time domain (Fig. 1), which implies it is strictly harmonic in the spectral domain. Since in both Werckmeister and Vallotti intervals in major and minor chords deviate to some extent from just ratios, interference between pairs or groups of

Table 2 HNR data, Vallotti tuning

Chord	dB (mean)	dB (SD)	Chord	dB (mean)	dB (SD)
C-major	15.99	5.18	c-minor	10.2	1.47
C$^{\#}$-major	11.46	3.42	c$^{\#}$-minor	9.11	2.82
D-major	13.53	3.36	d-minor	3.92	1.65
E_b-major	9.8	2.21	e_b-minor	3.94	2.27
E-Major	13.23	2.65	e-minor	3.29	0.82
F-Major	14.78	3.29	f-minor	6.32	2.15
F$^{\#}$-major	11.45	3.53	f$^{\#}$-minor	5.32	1.55
G-Major	16.27	4.53	g-minor	7.75	0.94
A_b-major	12.19	2.65	a_b-minor	9.05	2.79
A-major	14.24	3.4	a-minor	11.25	2.32
B_b-major	16.17	3.43	b_b-minor	6.87	1.19
B-major	11.51	1.47	b-minor	8.35	1.11

partials takes place which results in amplitude modulation (AM) as well as in a certain amount of auditory roughness. A simple method suited to check AM in the complex time signal for each individual chord is measuring the intensity of the sound as a function of time. If AM is present in the signal, the decay curve will show many small fluctuations as are visible in Fig. 5 for the e_b-minor chord in Werckmeister III and the d-minor chord in Vallotti. The modulation frequency and the depth of AM permit a rough assessment of the spectral inharmonicity and the quality of tuning for a certain chord. While the curve of intensity decay is smooth for chords in just tuning (or nearly so), AM increases with deviations from just pitch ratios as well as with spectral inharmonicity corresponding to such deviations.

A signal processing approach suited to investigate AM of individual partials of a complex tone or of a chord comprising several harmonic complexes is the phase vocoder which can be viewed as a filter bank that can be tuned so that the base frequency of the filter bank equals the fundamental of a harmonic complex. For the present study, the sndan software [71, 72] was used which includes tools suited to analyzing AM as well as spectral inharmonicity in harmonic complexes. One of the tools is a 3D-plot of the amplitudes of harmonic partials over time where the temporal envelope for individual partials can be displayed so that AM or other processes become visible. Figure 6 shows partials no. 1–20 from tones in the B-major chord played in Werckmeister III. Figure 7 shows partials 1–20 from the tones in the B-major chord in the Vallotti tuning. B-major is one of the more problematic chords in both tunings (with relatively low HNR readings, see Tables 1 and 2). As is obvious from the graphics displayed, there is considerably AM in both chords. The cause of AM is that, while in just intonation partials from several tones of a major chord played like c_2, g_2, c_3, e_3, g_3 would coincide, in temperaments such as Werckmeister III or Vallotti (or ET12, for that matter) partial frequencies deviate

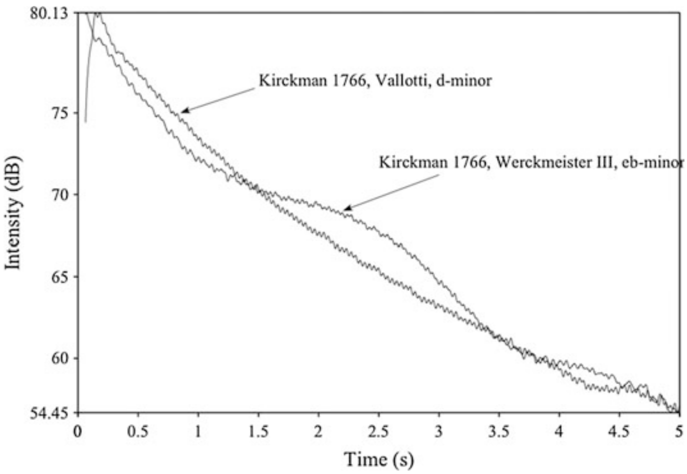

Fig. 5 Decay curves for the e_b-minor chord in Werckmeister III and the d-minor chord in Vallotti show many small fluctuations resulting from harmonic partials undergoing AM

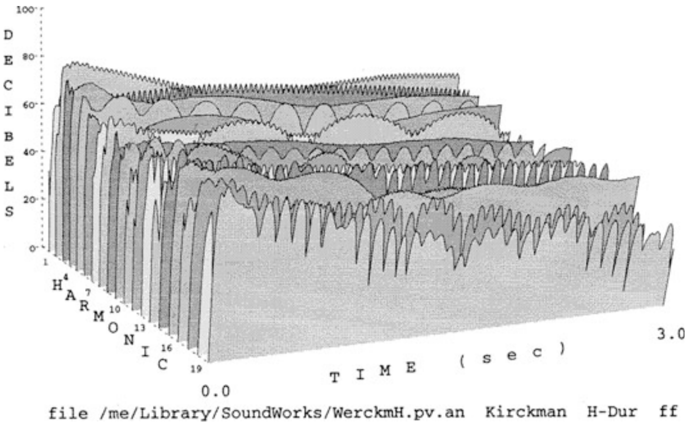

Fig. 6 Werckmeister III, B-major chord, partials 1–20, AM pattern

to some degree from each other (depending on the temperament chosen and the chord that is played). Deviations between pairs or groups of partials can be precisely determined in spectral analysis with appropriate FFT-settings (since $df = fs/N$, where df is the difference limen for two frequency components to be separated, fs is the sampling frequency of the signal, and N is the length of the FFT transform or 'window'). For short FFT windows (e.g., 1024 or 2048 samples per frame), separation is not possible, to the effect that two closely spaced spectral components interact so as to exhibit AM in harmonic plots (see Figs. 6 and 7).

Tools available in sndan furthermore permit to measure the deviation of individual partials from harmonic frequencies as well as to compute such deviation for the weighted average of a number of partials. The results are available as lists

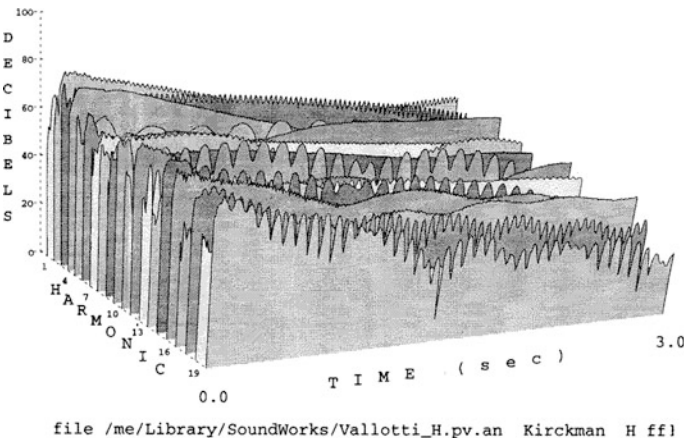

Fig. 7 Vallotti, B-major chord, partials 1–20, AM pattern

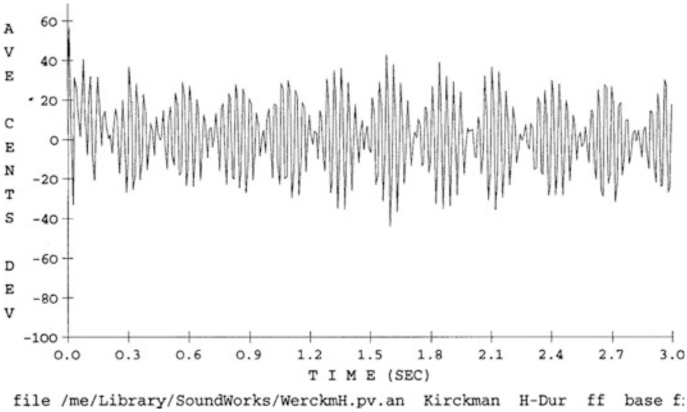

Fig. 8 Werckmeister, B-major chord, weighted average deviation, partials 1–5, cents

(including some statistics) and can be used for a quantitative assessment. Results can also be displayed as a graphic. For example, Fig. 8 shows the weighted average for partials 1–5 of the B-major chord in Werckmeister III and Fig. 9 the same measurement in Vallotti as recorded from the Kirckman.

Deviation at the onset of each sound (0–100 ms) results from the plucking of strings and is found in all tunings. As Figs. 8 and 9 demonstrate, the B-major chord in Werckmeister III shows smaller deviations on average over the first three seconds of recorded sound than the same chord tuned to Vallotti. To be sure, for the given chord structure, computation of the weighted average for the first five harmonic partials already captures four pairs of corresponding partials, one of which relates to the fifth, and another to the major third in the chord, respectively. Hence, deviations

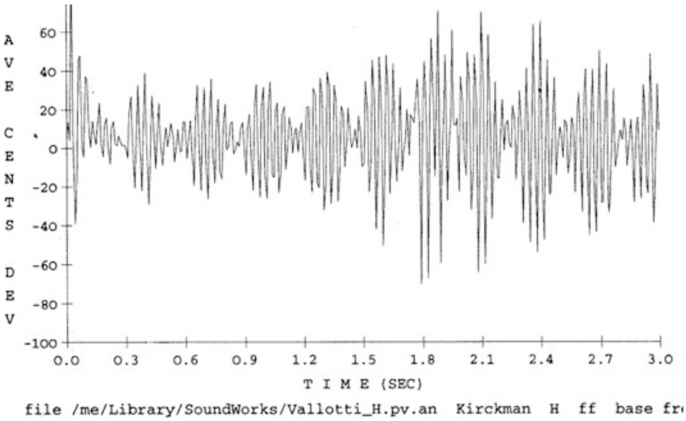

Fig. 9 Vallotti, B-major chord, weighted average deviation, partials 1-, cents

in these two intervals from just ratios account for the computation of the weighted average.

The data from various types of signal analyses may be used for a comparison including statistics. A comparison of the HNR data (Tables 1 and 2) for Werckmeister III and Vallotti might be tempting yet is not easy since the means only indicate the average level (in dB) for certain chords computed from data representing sound segments of a given length. The SD computed for the same data block is considerable for most of the chords, indicating they undergo significant change over time. The HNR in fact goes up with time, for many natural sounds generated by means of an initial impact causing energy transfer into a vibrating system (such as a string that is plucked or a membrane that is struck), because the dissipation of energy due to radiation of sound leads to the rather fast damping of higher partials, meaning the number of partials that can cause inharmonicity (or jitter) in a complex sound such as a chord played on a harpsichord tuned to some temperament diminishes with time. Hence, a certain amount of the variance expressed as SD for each sound of a major and minor chord in our study is attributable to damping out of higher partials due to sound radiation and energy consumption, which lets the HNR rise with time as is shown for three major chords ($F^{\#}$, A_b, A in Werckmeister III) in Fig. 10.

Given these conditions, a weighting of the means of the HNR data by their respective SDs, which can be done by calculating the coefficient of variation (CV) as a statistical parameter, will not be much of help for sounds recorded from plucked strings (while it is a different matter with steady-state sounds recorded from organ pipes). Of course a percentage of the variance in our HNR data results from

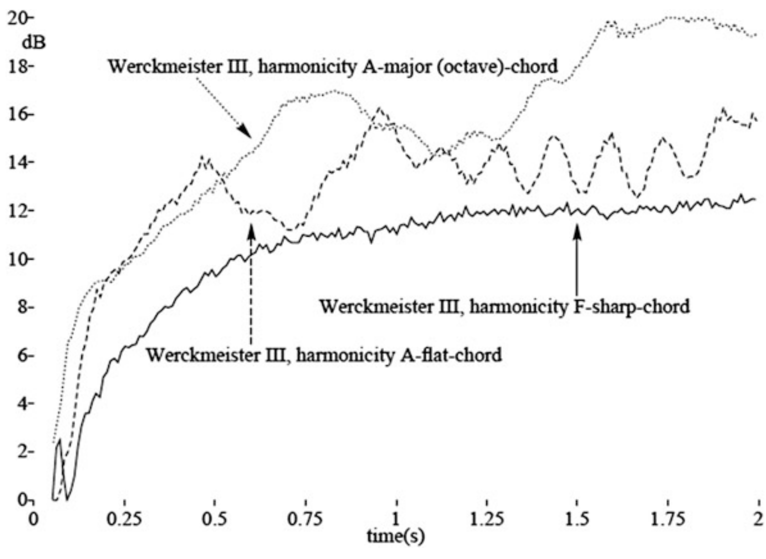

Fig. 10 Evolution of HNR over time for three major chords in Werckmeister III

the relative inharmonicity of partials in chords due to different tunings and, correspondingly, different deviations (in cents) of tones in a given major or minor chord from just intervals. However, the acoustical factor of damping which reduces jitter and "smoothens out" spectral structure in sounds from harpsichord strings also is relevant for explaining the considerably large SD in our HNR data. If one would assume that the damping is more or less the same for chords played in various temperaments (what in fact is not quite true, see for example the decay curves displayed in Fig. 5), one might compare only the means computed for each chord in individual tunings as well as taking the sums of HNR readings for all major and minor chords. In so doing, a small but recognizable advantage of Vallotti (Major chords, $\sum = 160.22$ dB, minor chords $\sum = 93.72$ dB) over Werckmeister III (Major chords $\sum = 152.22$ dB, minor chords $\sum = 91.15$ dB) may be seen. However, comparison in pairs of corresponding major as well as minor chords shows Werckmeister prevails in some of the keys, and Vallotti in others. This is what one would expect from temperaments that, with only 12 tones and pitches to the octave, cannot but seek to install a compromise tuning suited to perform music in all common major and minor keys without producing too much of auditory roughness or even audible mistuning of intervals and chords.

5 Perceptual and Aesthetic Aspects

Keyboard temperaments and tunings have been an issue since the early Renaissance in Europe when the medieval practice of 'Pythagorean' tuning did no longer fit the interval and chord structure developed in a growing number of musical works. Many historical sources on organology, tuning and temperament from the 16th, 17th, and 18th century, respectively, clearly indicate that musicians and also listeners were sensitive to beats and roughness arising from poor tunings. One effect reported quite often was that chords played on keyboard instruments tuned to some temperament did not fit well to melodic lines and polyphonic textures coming from singers, and would interfere in particular with brass instruments most of which were without valves, slides, or keys and thus producing only natural tones of the harmonic series. For example, Mattheson [73, 143–149] who supported equal temperament for keyboard instruments because he saw its advantages (most of all, modulation through all keys), argued that all semitones in ET12 would sound out of tune if compared to the actual intonation of singers, and in particular if compared to the pitches of the trumpet and similar instruments. In an interesting rational discussion of the pros and cons of equal temperament and tuning, he said introducing ET12 in church music would meet grave resistance, first of all, for the sheer number of organs that would need to be retuned (from meantone to ET12), second, in regard of the costs this would generate for each parish. As a third reason, Mattheson pointed to the organ-builders who he said were stubborn and unwilling to let theorists teach them how to tune an organ. A fourth factor according to Mattheson [73, 144] would be the singers and instrumentalists who, after the introduction of

ET12 as a standard tuning, without doubt would sing and play out of tune for some time to come. Finally, as a fifth factor, he mentions the listeners "who didn't yet temper their ears according to the numbers" (such as had been published at the time for ET12). In what looks like an early contribution to the nature vs. nurture debates so common in theories of perception later, Mattheson believed that adaption to equal temperament on the side of listeners would be possible "since habit is the other nature also in this matter".

In several experiments we conducted in the past, samples of subjects were asked to judge harmonic cadences and chord progressions or excerpts from polyphonic pieces of music played in various tunings in regard to perceptual qualities and aesthetic appreciation (see [74–76]). Most of the subjects had musical training, though on different levels of expertise ranging from elementary music education to music academy training as singer, instrumentalist, or conductor. The experiments used various temperaments and tunings (ET12, Vallotti/Young, 1/4-comma meantone, Werckmeister III, Pythagorean, Kirnberger III, as well as selections of just intonation pitches from a 2D tone lattice). Also included in some experiments was the effect of a transposition of a piece from one key into another while the 1/4-comma meantone tuning remained unchanged. Furthermore, the general sensitivity of subjects for defects in tuning was checked by shifting a melodic line 50 cents up or down in pitch while the harmonic accompaniment was left unchanged (cf. [74]). Experimental data subjected to statistical analysis demonstrate that subjects in general are capable of distinguishing temperaments and tunings which they evaluate in regard to perceptual qualities such as consonance, on the one hand, and auditory roughness, on the other. In several of our experiments, subjects were asked to evaluate items also in regard to correctness of musical syntax. Furthermore, in some experiments subjects rated their aesthetic appreciation of musical excerpts played in different tunings. In the following, some of the aforementioned aspects will be addressed with reference to hitherto unpublished data from previous experiments.

In psychoacoustics, it is a common investigation making subjects judge optimal interval sizes. There are several experimental procedures for such tests, for example, one may use two signal generators one of which delivers a signal at a fixed frequency (if the signal is a sine tone) or fundamental frequency (if the signal is a harmonic complex), while the output of the second generator is varied in frequency either by the experimenter or by the subject so that the subject perceives the musical interval (say, a major third) as 'just' or 'perfect'. Typically, musically trained subjects are capable of matching two signals so that their frequencies are in small integer ratios (or nearly so, see, e.g. [77]). The sensory factor most relevant for these judgements is that auditory beats and roughness disappear if two periodic signals are in harmonic frequency ratios for musical intervals such as the fifth, fourth, or major third. Even in successive intervals (tone A followed by tone B) small integer ratios are prevalent as can be tested for the octave; the opinion according to which octaves must be "stretched" to appear as correct in regard to an optimum interval size was not confirmed in a series of experiments we conducted using a standard experimental setup (offered in [78]; see [79, 482–484; 21]).

The general sensitivity of subjects for melodic phrases and/or sequences of chords where some or all tones are out-of-tune can be checked in experiments with variables relating to sensory consonance and dissonance, respectively. In one experiment (2006), 50 students of the University of Hamburg were asked to rate the consonance and dissonance they perceived with musical stimuli on scales ranging from 1 (low) to 7 (high). The variables in this experiment were (1) consonance, (2) dissonance, (3) goodness of intonation (German: Intonationsgüte), and (4) aesthetic overall impression. Measures 1–8 from Bach's Invention no. 1 (C-Major, BWV 772) served as a musical stimulus, from which several variants were produced with different tunings and sounds. Version 1 has the sound of a harpsichord tuned to ET12. The sound comes from FM synthesis (Yamaha TX 81 Z) and appears realistic in regard to temporal and spectral features. Version 2 has the same sound but employs Vallotti/Young tuning. Version 3 again is in ET12 but has a special sound synthesized from components spaced in octaves and played from a hardware sampler (SE synthesis on an EMAX II stereo). Version 4 has the harpsichord sound and ET12 tuning yet with a stretch of 50 cent between notes of the voices in the two-part invention. Hence all simultaneous intervals are too wide by a margin of 50 cents (a quarter of a whole tone in ET12). Version 5 was based on a selection of 12 pitches from a 2D lattice (tone net) comprising fifths and major thirds in just intonation, played with the harpsichord sound used also in versions 1, 2, and 4. Subjects were asked to rate both consonance and dissonance as two variables, which not only generates additional data but permits a more precise assessment of the perceptions subjects have from the stimuli. Of course, these two variables interrelate closely (meaning high ratings for consonance should go along with low ratings for dissonance, and vice versa). The descriptive statistics for the five versions are listed in Table 3.

Without going into a detailed analysis of the data at this place, one can see that version 2 in Vallotti/Young received best ratings on three of the four variables, and that version 4 was perceived as clearly out-of-tune by the subjects in the sample as is evident from low ratings for consonance and for goodness of intonation as well as for overall aesthetic impression while ratings for dissonance are much higher in this version than in any other. We may conclude from these figures that detuning tones in simultaneous intervals in a musical setting by as much as 50 cents will have strong perceptual and aesthetic effects on listeners. However, ratings for version 1 in ET12 and for version 2 (Vallotti/Young) differ not significantly for the variables based on sensory qualities (consonance, dissonance) while the difference for goodness of intonation is more marked, and that for aesthetic overall impression

Table 3 Means, SDs for Invention no. 1, 5 versions (2006, n = 50 subjects)

Variable	Version 1	Version 2	Version 3	Version 4	Version 5
Consonance	5.73, 1.22	5.81, 1.1	5.46, 1.22	2.56, 1.75	5.0, 1.4
Dissonance	1.96, 1.18	1.91, 1.06	2.29, 1.22	5.35, 1.9	2.83, 1.46
Intonation goodness	5.20, 1.59	5.43, 1.37	4.78, 1.57	2.24, 1.8	4.24, 1.39
Aesthetic impression	2.74, 1.41	3.04, 1.38	3.36, 1.41	1.44, 0.73	2.60, 1.24

still somewhat greater as is indicated by median values (ET12 = 2, Vallotti/Young = 3; this difference though is not large enough to yield significant results in an U-test where z = 1.146 (1.960, p = 0.05). From the results of this experiment one may conclude that musical excerpts played in either ET12 or in Vallotti/Young differ not significantly in regard to perceptual effects and aesthetic appreciation even though Vallotti/Young, in a direct comparison such as performed in this experiment, prevails. The somewhat higher ratings for Vallotti/Young as are reflected in the judgements could be attributed to the slightly better figures this tuning achieves in an overall assessment of deviations from just intonation (cf. [63–65]).

Another comparative evaluation can be made from data of an experiment (2001) in which a sample of 44 subjects (all students in their first or second semester) listened to measures 1–15 from J.S. Bach's Sonata in E_b-Major (BWV 552) in five different tunings, namely (1) Vallotti/Young, (2) 1/4-comma meantone tuned from c as base note, (3) ET12, (4) 1/4-comma meantone with the scale based on e_b, (5) Kirnberger III. The five musical excerpts were performed with a synthesized pipe organ sound (TX 81 Z) and were judged on the dimensions (a) consonance, (b) roughness, and (c) goodness of intonation (German: Intonationsgüte). The design can be stated as one factor (tunings) with 5 conditions. The descriptive statistics for the data are summarized in Table 4.

Table 4 Judgement of different tunings, n = 44 subjects, Hamburg 2001

Tuning	Data file no.	Mean	SD	Median	Range	CV (%)
(1) Vallotti/Young						
Consonance	1	4.55	1.21	4.5	2–7	26.6
Roughness	2	3.36	1.51	3	1–7	44.92
Intonation	3	3.98	1.23	4	2–7	30.91
(2) Meantone (c)						
Consonance	4	4.05	1.38	4	2–6	34.12
Roughness	5	3.77	1.63	3.5	1–7	43.11
Intonation	6	3.16	1.45	3	1–7	45.77
(3) ET12						
Consonance	7	4.96	1.14	5	2–7	23.02
Roughness	8	3.32	1.44	3	1–6	43.49
Intonation	9	4.34	1.35	4	1–7	31.0
(4) Meantone (e_b)						
Consonance	10	4.96	1.43	5	2–7	28.86
Roughness	11	3.55	1.56	3	1–7	44.06
Intonation	12	4.21	1.61	4	1–7	38.24
(5) Kirnberger III						
Consonance	13	4.8	1.46	5	1–7	30.36
Roughness	14	3.8	1.62	3	1–7	42.74
Intonation	15	4.1	1.76	4	1–7	42.84

Data files were checked in groups representing the three variables for homogeneity of variances with Bartlett-tests. Since variances are sufficiently homogeneous, an ANOVA conducted for consonance (data files 1, 4, 7, 10, 13) yields $F = 3.628$ ($F[0.01] = 3.418$), which is very significant, but for roughness (data files 2, 5, 8, 11, 14) ANOVA is not significant at $F = 0.901$. ANOVA for goodness of intonation (data files 3, 6, 9, 12, 15) yields $F = 4.301$, which again is very significant. Following the ANOVA, a multiple-mean test (Scheffé) was conducted for each variable, which yields significant contrasts between data files for consonance (D 4/7 and D 4/10, $p < 0.025$), and between data files for the goodness of intonation (D 6/12, $p < 0.05$, D 6/9, $p < 0.01$). Hence, the meantone scales tuned from either c or e_b make a perceptual difference for a piece of music in the key of E_b. For the data files representing five tunings, also a MANOVA can be computed, taking consonance, roughness and goodness of intonation as three variables dependent on the factor (tunings). MANOVA yields Wilks-$\lambda = 0.57$, $F = 8.11$ ($F[0.001] = 3.113$), which underpins the tunings differ in regard to perceptual and musical effects. However, an inspection of the descriptive statistics shows that the SD and the range for all variables is large (as is the CV for variables in several tunings), indicating that subjects in the sample varied markedly in their individual judgements. There are several possible explanations for these figures, one of which is that the subjects in the sample were young students not experienced with different tunings and apparently quite uncertain in their judgments. Furthermore, the sample is not homogeneous and in fact contains several groups of subjects that differ in regard to their musical ability, education, and preferences. This holds true even for students in musicology where, besides individuals with a conventional ‘classical’ music training, today one finds a growing number of young people who come from jazz, rock/pop, electronic music genres or (depending on their family history as migrants) various non-western music cultures. In particular students with a musical background predominantly in rock/pop/electronic genres are used to ET12 as this is the standard tuning not only on keyboards but also on fretted string instruments (guitars, bass). In addition, these subjects are used to sounds that are heavily processed with effect units, many of which involve temporal and spectral modulation (see [80–82]). For example, spectral envelope and energy distribution can be modulated with a bandpass filter where the center frequency and the bandwidth vary with time. In a phaser circuit, a low-frequency oscillator (LFO) controls the amount of time by which a signal is delayed relative to the dry input signal. Adding the delayed signal to the dry signal, constructive and destructive interference results depending on the phase angle of the delayed signal relative to the dry one. Since the delay is varied with time, the phase angle between the two signals also varies periodically as does the spectral energy distribution, giving sounds processed in this way a “breathy” timbral quality. Some effects such as chorus and flanger can be set so that the pitch of a sound is modulated periodically up and down the fundamental frequency of the input signal. Effect units which modulate spectral and temporal characteristics are employed not the least to give synthesized or sampled sounds played on a keyboard instrument in ET12 a lively quality. Also, effects such as chorus and delay effects are used to double and ‘broaden’ pitches for singers who thus seem to possess of a ‘big voice’. Singing with a chorus effect (instead of precise intonation) has become ‘industrial standard’ in many pop productions. Apparently, young students already in the 1990s were so used to sounds undergoing permanent modulation that, in some of our experiments, they rated harmonic major cadences or musical pieces played with a synthesized pipe organ or similar harmonic complex sound and tuned to ET12 higher with respect to ‘pleasantness’ than any tuning that produced a high degree of sensory consonance yet seemed static in regard to both pitch and spectrum. For instance, among the music examples we employed in several experiments was a polyphonic setting of the chorale *Wie schön' leucht uns der Morgenstern* (BWV 763), of which Reinier Plomp [31] provides an excerpt (measures 1–6) in two versions, one in just intonation, and another in ET12. The version in just intonation offers a high degree of consonance (for the coincidence of many partials and the lack of beats and roughness) but is a bit sharp in timbre (again, for the coincidence of many partials which brings the spectral centroid up to higher frequencies). In contrast, the version in ET12 is less transparent yet may appear “warm” in sound quality because of the interference of partials as well as the various modulation products which become audible over time. The response of young students (with a major in systematic musicology or in other subjects such as sociology, media science, etc.), and in particular of those with a background in pop/rock/electronic music typically was that they rather preferred the ET12 version as this apparently was close to the sound quality they had experienced in music genres of their choice. Given these changes in listening attitudes and preferences, Mattheson (1731, see above) perhaps was right when he argued that “habit is the other nature”.

6 Conclusion

A review of historical sources (e.g., [4, 8, 12]) shows tuning and temperaments was a major issue in music theory and organology in the time from, roughly, 1400 to 1900. Besides more theoretical elaborations, there are many sources which clearly indicate musicians and instrument builders experimented with different tunings to find solutions for a discrepancy that stems from the nature of musical intervals being governed by different prime numbers (2, 3, 5). In addition to the octave, the perfect fifth and the fourth that had ranked as ‘symphonous’ in medieval music theory (see [10]), the just major third and other harmonic intervals were included into composition and performance from c. 1450–1500 on, forcing music theorists and instrument builders to deal with this new situation. Accepting the just major third as a fundamental interval bears implications both in regard of the division of the fifth into major and minor third as well as the division of the major third into greater and lesser whole tone. Extending the range of usable harmonic intervals to the just major and minor third and the just major and minor sixth means appropriate tones or ‘pitches’ in a scale must be available if the same intervals shall be played, on a keyboard instrument, in various keys. With only 12 tones and pitches to the octave, one can tune these so that *some* major and minor chords are in just intonation, that is, their fifths are perfect and their major and/or minor thirds are in just frequency ratios (the equivalent to string sections on a monochord, as shown by Ramis de Pareia in [40]). Just intonation plus capability to modulate among several keys (both sharps and flats) inevitably leads to more than 12 tones and pitches to the octave. If one wants to build a keyboard instrument in just intonation, the number of tones and pitches required per octave and the actual selection of pitches and tones in a tuning depends on the range of keys that shall be covered and the degree of justness that is deemed appropriate (see [35]).

A temperament in certain respects is a means to reduce the larger number of pitches that would be needed, in just intonation (perfect fifths and fourths, just thirds and sixths), to a number considerably smaller but still sufficient to realize pitches so that they are relatively close to the just intonation pitches they 'represent'. Let m be the large number of tones and pitches in a 2D-lattice of just intonation, and n the number of pitches and tones available from a certain temperament, where $m \gg n$. Since technology and playability impose restrictions on the design of conventional keyboard instruments, the problem here is to find the smallest number n suited to 'represent' as many of the tones and pitches m as are deemed necessary by a composer (taking the notation in individual works of music as a source for analysis). For example, the Duetto I (BWV 802) from the 3rd part of Bach's *Clavier-Übung*, even though it is only a two-part musical setting, has no less than 17 different notes in the score (supervised for print by Bach himself), which express musically distinct intervals as intended by the composer. If the Duetto I were to be played in a tuning suited to keep different intervals distinct as simultaneous sonorities, more than twelve tones and pitches to the octave will be needed. In other advanced organ and harpsichord works of Bach like the Fantasie und Fuge in g-minor (BWV 542) or the Chromatische Fantasie und Fuge in d-minor (BWV 903), the number of pitches and tones found in the notation as well as by an analysis of the harmonic structure is considerably higher than 17.

There have been various attempts at finding an optimal relation for $\{m, n\}$, one of which is a division of the octave into equal parts, where n can be any whole number such as 72 (see [55]) or 31 (the Cycle Harmonique of Huygens from 1661/1691, see [6, 187ff.]). If the aim is to realize all thirds (5/4) and fifths (3/2) with very good approximations, $n = 53$ will be chosen (as calculated first by Mercator and Holder and advocated also by Helmholtz and Bosanquet, in the 19th century). Though some keyboard instruments have been manufactured with 53 pitches and keys to the octave, mainly for experimental purposes, it is not a very practical solution for a harpsichord maker (and less so for an organ builder). In case one wants just thirds and sixths and is willing to accept the somewhat narrowed fifths and widened fourths of Huygens' cycle, then $n = 31$ will do. This is a solution that has been implemented (either as an equal temperament, or with some variation in interval size, see [3, 7, 33]) on several keyboard instruments from the 16th century to our times (the 31-tone pipe organ conceived by Fokker was installed in 1950, and in the early 1970s, an electronic keyboard with the Huygens-Fokker-tuning was built for Webster College in St. Louis). A division of the octave into 19 equal parts has also been discussed at times, but the benefit from $n = 19$ in regard to approximating just intonation pitches will be quite small. Finally, confining the number of tones and pitches per octave to $n = 12$, several models for tuning a temperament are feasible (cf. [11, 17]), one of which is ET12, another is Werckmeister III, and still another is the scale and tuning model devised by Vallotti and by Young. The 1/4-comma meantone model, which in general is regarded as a temperament, in fact is a mixture of just intonation intervals (taking the eight just major thirds and seven just minor thirds that can be realized in this tuning, see above) and a tempering of the fifths and fourths as well as a division of the major third into two meantones of equal size.

The pros and cons of various temperaments have been discussed extensively in works on tuning and temperament, often relying on personal experience of musicians and theorists as well as on reports from organ builders or music experts that were called to examine new organs (like J.S. Bach). Though such reports are valuable as historical sources, an objective assessment of tunings and temperaments by means of computing deviations from just intonation intervals allows quantifying the goodness-of-fit of various temperaments (cf. [63, 65]). In addition, examination of tunings on the basis of actual sound recordings of real or synthesized instruments subjected to signal analysis seems necessary since the quality of a tuning for musicians and listeners depends on factors such as periodicity in the time domain and spectral harmonicity of partials in the frequency domain (cf. [18, 28, 29, 67]). In the present study, these parameters have been investigated, to some extent, for the Werckmeister III and the Vallotti temperament tuned on a historical Kirckman harpsichord by computing the HNR for signals recorded for major and minor chords played in the aforenamed temperaments. Furthermore, we reported empirical data from some experiments where subjects were asked to rate musical excerpts played in several tunings such as 1/4-comma meantone, Werckmeister III, Vallotti/Young, and ET12. The data suggest that subjects with some musical training can distinguish between different tuning models in case their differences are large enough to have effects for perception that can be measured on psychoacoustic variables (consonance, dissonance, roughness). The effects are less marked, though, for aesthetic appreciation where in particular young students nowadays seem to prefer ET12 because of their intensive exposure to sounds from electronic keyboards including digital audio effects employed for spectral and pitch modulation.

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