

Hybrid Mean-Variance Mapping Optimization for Economic Dispatch with Multiple Fuels Considering Valve-Point Effects

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Abstract. Many thermal generating units of an electric power system are supplied with multi-fuel sources such as coal, natural gas and oil. These fuels represent irreplaceable natural resources and conservation is used as a way to increase energy efficiency. Economic dispatch (ED) is one of the significance optimization problems in power system operation for fuel cost savings. This paper proposes a new approach which is hybrid variant of mean-variance mapping optimization (MVMO-SH) for solving this problem. The MVMO-SH is the improvement of original mean-variance mapping optimization algorithm (MVMO). This method adopts a swarm scheme of MVMO and incorporates local search and multi-parent crossover strategies to enhance its global search ability and improve solution quality for optimization problems. The proposed MVMO-SH is tested on 10-unit and large-scale systems with multiple fuels and valve-point effects. The obtained results are compared to those from other optimization methods available in the literature. The comparisons show that the proposed method provides higher quality solutions than the others. Therefore, the MVMO-SH is a promising method for solving the complex ED problems in electric power system.

Keywords: Economic dispatch · Multiple fuels · Valve-point effects · Mean-variance mapping optimization · MVMO · MVMO-SH

Nomenclature

N	total number of generating units
F	total operation cost
a_{ik}, b_{ik}, c_{ik}	fuel cost coefficients of generator i
B_{ij}, B_{0i}, B_{00}	total system load demand
P_D	total system load demand
P_i	power output of generator i
$P_{i,max}$	maximum power output of generator i
$P_{i,min}$	minimum power output of generator i

P_L	total transmission loss
K	the penalty factor for the slack unit
P_s	power output of slack unit
n_var	number of variable (generators)
n_par	number of particles
$mode$	variable selection strategy for offspring creation
$iter_{max}$	the maximum number of iterations
Np	number of particles
$archive\ size$	n -best individuals to be stored in the table
Δd_0^{ini}	initial smoothing factor increment
Δd_0^{final}	final smoothing factor increment
g_{p_ini}	max percentage of good particles
g_{p_ini}	min percentage of good particles
m_{ini}	initial number of variables selected for mutation
m_{final}	final number of variables selected for mutation

1 Introduction

Economic dispatch (ED) is determination of optimized real power output from a number of electricity generators needed to meet load requirements at lowest possible cost while satisfying all unit and system constraints. Traditionally, the cost fuel function of each generating unit is presented as the quadratic function approximations [1, 2]. However, in practical power system operation conditions, thermal generating units can be supplied with multiple fuel sources like coal, natural gas and oil. This requires their fuel cost functions to be segmented as piecewise quadratic cost functions where each function reflects the effects of different fuel types. The ED problem has piecewise quadratic cost functions which is a non-convex optimization problem with multiple local optima. This problem is more complicated when the effect of valve point loadings is considered [3]. Therefore, the classical solution methods are difficult to deal with this problem. One approach for solving the problem with such units having multiple fuel options is linearization of the segments and solving them by traditional methods [4]. A better approach is to retain the assumption of piecewise quadratic cost functions and proceed to solve them. A hierarchical approach based on the numerical method (HNUM) has been proposed in [5] as one way to deal with the problem. However, the exponential growing time complexity of the numerical methods is a major problem for large-scale systems, especially for non-convex constraints. More advanced optimization methods based on artificial intelligence concepts have been effectively implemented to the ED problem with MF and VPE such as Enhanced Lagrangian Artificial Neural Network (ELANN) [6], genetic algorithm with multiplier updating (IGA_MU) [7], evolutionary algorithm (EA) [8], new PSO with local random search (NPSO-LRS) [9], an improved quantum-behaved particle swarm optimization (SQPSO) [10], Self-organizing hierarchical particle swarm optimizer (SOH-PSO) [11], and Pseudo-Gradient Based Particle Swarm Optimization Method (PGPSO) [12]. However, the search ability of these methods often

provides near global optimal solution for non-convex optimization problems. The non-convex ED problem is always a challenge for solution methods. Therefore, there is always a need for developing new techniques for solving this problem.

Recently, Mean-variance mapping optimization (MVMO) is developed and introduced by István Erlich [13]. This algorithm possesses conceptual similarities to other known heuristic algorithms in three evolutionary operators including selection, mutation and crossover. However, the special feature of MVMO is the mapping function applied for the mutation based on the mean and variance of n -best population saved in an archive. The original of MVMO utilizes single particle to start the search process. In order to enhance its global searching ability, the search space of MVMO is extended by initializing a set of particles, which formed a swarm variant of MVMO (MVMOS) [14, 15]. The subsequent improvement is hybrid variant of MVMO, referred to as MVMO-SH [16]. It adopts a swarm scheme of MVMO and incorporates local search components. Therefore each particle has its own memory which is represented by the corresponding archive and mapping function. All particles are arranged according to their local best fitness and classified into two groups including good and bad particles. For each good particle, the parent assignment is done by considering the first ranked solution in its particular knowledge archive whereas a multi-parent crossover is used to reorient each bad particle towards different sub-regions of the search space. An interior-point method (IPM) is included for local improvement option. In this paper, the MVMO-SH is proposed as a new method for solving the non-convex ED problem with multiple fuels and valve-point effects. The non-convex and large-scale ED problem is always a challenge for solution methods in terms of optimal solution and computational time. The proposed MVMO-SH is tested on 10-unit and large-scale systems with multiple fuels and valve-point effects. The obtained results have shown that the MVMO-SH method is more effective than many other methods in the literature in terms of optimal solution, especially for large-scale systems. Therefore, the MVMO-SH is a favorable method for solving the non-convex ED problem.

2 Problem Formulation

The main objective of the ED problem with MF and VPE is to minimize total cost of thermal power plants with many different fuels while satisfying equality and inequality constraints. The power system consists of N thermal generating units. Each unit has a fuel cost function, shown as F_i , to generates a power out P_i . The total fuel cost of the system, F_T , is sum of fuel cost of each unit. The optimization problem of the ED is to minimize the total fuel cost F_T , which be written as:

$$\text{Minimize } F_T = \sum_{i=1}^N F_i(P_i) \quad i = 1, 2, 3, \dots, N \quad (1)$$

where the fuel cost function of each generating unit is represented by [12]:

$$F_i(P_i) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_i^2 + |e_{i1} \cdot \sin(f_{i1} \cdot (P_{i1}^{\min} - P_{i1}))|, & \text{for fuel 1, } P_i^{\min} \leq P_i \leq P_{i1} \\ a_{i2} + b_{i2}P_i + c_{i2}P_i^2 + |e_{i2} \cdot \sin(f_{i2} \cdot (P_{i2}^{\min} - P_{i2}))|, & \text{for fuel 2, } P_{i1} \leq P_i \leq P_{i2} \\ \vdots \\ a_{ik} + b_{ik}P_i + c_{ik}P_i^2 + |e_{ik} \cdot \sin(f_{ik} \cdot (P_{ik}^{\min} - P_{ik}))|, & \text{for fuel k, } P_{ik-1} \leq P_i \leq P_i^{\max} \end{cases} \quad (2)$$

The constraints of the ED problem must be satisfied during the optimization process are presented as follows:

1. *Real power balance equation:* The total active power output of generating units must be equal to total active load demand plus power loss:

$$\sum_{i=1}^N P_i = P_D + P_L \quad (3)$$

The power loss P_L is calculated by [1]:

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j + \sum_{i=1}^N B_{0i} P_i + B_{00} \quad (4)$$

2. *Generator capacity limits:* The active power output of generating units must be within the allowed limits:

$$P_{i,\min} \leq P_i \leq P_{i,\max} \quad (5)$$

3 Hybrid Variant of Mean-Variance Mapping Optimization

3.1 Review of Mean - Variance Mapping Optimization (MVMO)

The key feature of MVMO is a special mapping function which applied for mutating the offspring based on mean-variance of the solutions stored in the archive.

The mean \bar{x}_i and variance v_i are calculated as follows [13]:

$$\bar{x}_i = \frac{1}{n} \sum_{j=1}^n x_i(j) \quad (6)$$

$$v_i = \frac{1}{n} \sum_{j=1}^n (x_i(j) - \bar{x}_i)^2 \quad (7)$$

where $j = 1, 2, \dots, n$ (n is population size).

The transformation of x_i^* to x_i via mapping function is depicted as Fig. 1.

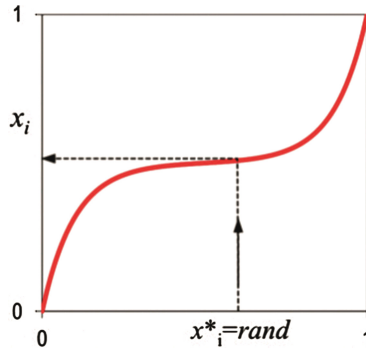


Fig. 1. Variable mapping

The transformation mapping function, h , is calculated by the mean \bar{x} and shape variables s_{i1} and s_{i2} as follows [13]:

$$h(\bar{x}_i, s_{i1}, s_{i2}, x) = \bar{x}_i \cdot (1 - e^{-x \cdot s_{i1}}) + (1 - \bar{x}_i) \cdot e^{-(1-x) \cdot s_{i2}} \quad (8)$$

where

$$s_i = -\ln(v_i) \cdot f_s \quad (9)$$

The scaling factor f_s is a MVMO parameter which allows for controlling the search process during iteration. s_i is the shape variable.

All variables are initialized within the limit range $[0,1]$. The output of mapping function is always inside $[0,1]$. However, the function evaluation is carried out always in the original scales.

3.2 Hybrid Mean-Variance Mapping Optimization (MVMO-SH)

The hybrid variant Mean-variance mapping optimization (MOMO-SH) is a subsequent improvement of the original MVMO. It adopts a swarm scheme of MVMO and incorporates local search and multi-parent crossover strategies to increase the innate power of global searching ability of MVMO. The flowchart of MVMO-SH is depicted in [16].

The variables of optimization problem are recalculated from $[0,1]$ to their original boundaries before fitness evaluation or local search is carried out. An interior-point method (IPM) based strategy is included for local improvement option. The IMP is performed with a probability γ parameter for any child of the population.

The solution archive stores the n -best achieved offspring of each particle in a descending order of fitness. For each particle, its archive is only updated once the new better solution is produced after every step of fitness evaluation or local search. The archive is illustrated in Fig. 2.

PARTICLE Np	Ranking	Fitness	x_1	x_2	---	x_D	
	PARTICLE 2	Ranking	Fitness	x_1	x_2	---	x_D
		PARTICLE 1	Ranking	Fitness	x_1	x_2	---
1 st best	F_1						
2 nd best	F_2						
...							
n	F_A						
Mean \bar{x}_i	---		\bar{x}_1	\bar{x}_2	---	\bar{x}_D	
Shape	---				---		
d-factor	---				---		

Fig. 2. Structure of the solution archive.

At least two fitness functions are independently evaluated by each particle in the early stage of the search process, and those solutions are saved in archive. All particles are ranked based on their local best fitness. Among of them, a set of good particles is determined as in (10), and the remainder is classified as a set of bad particles ($Np-Gp$) [16].

$$Gp = \text{round}(Np.g_p) \tag{10}$$

where

$$g_p = g_{p_ini} + \frac{i}{i_{final}}(g_{p_final} - g_{p_ini}) \tag{11}$$

For each good particle, the first ranked solution in its particular knowledge archive is chosen as the parent for the next offspring. For each bad particle, a multi-parent strategy is used to produce the parent as (12). The detail of parent selection is described in [16].

$$x_k^{parent} = x_{RG}^{best} + \beta(x_{GB}^{best} - x_{LG}^{best}) \tag{12}$$

The x_{RG}^{best} is randomly selected between the x_{GB}^{best} and x_{LG}^{best} , where represent the first and the last global best in the group of good particles, respectively. The factor is a random number which is calculated as in (13). This number is recalculated if any element of the vector x_k^{parent} is outside the [0, 1] bound [16].

$$\beta = 2 \left(\text{rand} + 0.5 \left(\frac{i}{i_{final}} \right)^2 - 0.5 \right) \tag{13}$$

In the selection variables of MVMO-SH, the number m of dimensions to be selected for mutation operation is progressively decreased as follows [16]:

$$m = \text{round}(m_{final} + \text{irand}(m^* - m_{final})) \tag{14}$$

$$m^* = \text{round}\left(m_{ini} - \left(\frac{i}{i_{final}}\right)^2 (m_{ini} - m_{final})\right) \quad (15)$$

4 Implementation of MVMO-SH to ED

4.1 Handling of Constraints

The constraints of the ED problem with MF and VPE include the real power balance constraint (3) and the generator capacity limits constraint (5). To satisfy all these constraints, the slack variable method is used for handling real power balance constraint (3) and the penalty function is used for handling the generator capacity limits constraint (5).

Neglecting the power transmission losses, the real power balance constraint (3) is rewritten by:

$$\sum_{i=1}^N P_i = P_D \quad (16)$$

Slack variable method is used for handling equality constraints in optimization problems where the value of variables is calculated from the others based on the equality constraints. This method is used for calculation of the power output for a slack unit from the power outputs of the remaining units in the system based on the real power balance constraint (3). By using the slack variable method [17], the slack unit can be randomly selected among the available units in the systems to calculate its power output as follows:

$$P_s = P_D - \sum_{\substack{i=1 \\ i \neq s}}^N P_i \quad (17)$$

In this study, the first unit is selected as slack unit for all test system. The power output of the slack unit is then included in the fitness function (18) with high penalty factor for a violation. The fitness function for the proposed MVMO-SH will include the objective function (2) and penalty terms for the slack unit if inequality (5) is violated. The fitness function is as follows:

$$F_T = \sum_{i=1}^N F_i(P_i) + K \times \left[(\max(0, P_s - P_{s,\max}))^2 + (\max(0, P_{s,\min} - P_s))^2 \right] \quad (18)$$

4.2 Implementation of MVMO-SH to ED

The steps of procedure of MVMO-SH for the ED problem are described as follows:

- Step 1:** Setting the parameters for MVMO-SH including $iter_{max}$, Np , n_{par} , $mode$, d_i , Δd_0^{ini} , Δd_0^{final} , $archive\ size$, $f_{s_ini}^*$, $f_{s_final}^*$, $n_randomly$, $n_randomly_min$, $Indep.runs$
Set $i = 0$, i denotes the function evaluation
- Step 2:** Normalize initial variables to the range [0,1]
- ```

for i=1:n_par
 for j=1:n_var
 xx(i,j) = p_min(j) + rand*(p_max(j)-p_min(j));
 end
 x_norm(i,:) = (xx(i,:) - p_min) ./ (p_max(j) - p_min(j));
end
x_normalized = x_norm;

```
- Step 3:** Set  $k = 1$ ,  $k$  denotes particle counters
- Step 4:** If  $rand < \gamma$  parameter, start local search. Otherwise, calculate power output for the slack generator by using (17) to evaluate fitness function in (18)
- Step 5:** Fill/update individual archive
- Step 6:** Classify  $Np$  particles into two groups including a set of  $GP$  good particles and a set of  $(Np-Gp)$  bad particles according to (10)
- Step 7:** If the particles are bad, the parent is produced by using a multi-parent strategy (12). Otherwise, the parent selection is single parent crossover based on local best
- Step 8:** Create offspring generation through three evolutionary operators: selection, mutation and crossover
- Step 9:** If  $k < Np$ , increase  $k = k + 1$  and go to step 4. Otherwise, go to step 10
- Step 10:** Check termination criteria. If stopping criteria is satisfied, stop. Otherwise, go to step 3. The algorithm of the proposed MVMO techniques is terminated when the maximum number of iterations  $iter_{max}$  is reached

## 5 Numerical Results

The proposed MVMO-SH has been tested on 10-unit and large-scale system including 20, 40, 60, 80 and 160 units with multiple fuel and valve-point effects. The convergence of metaheuristic methods may not obtain exactly same solution because these methods initialize variables randomly at each run. Hence, their performances could not be judged by the results of a single run. Many trials should be carried out to reach an impartial conclusion about the performance of the algorithm. Therefore, the implementations of the proposed method are carried out 50 independent trials in this study. The mean cost, max cost, average cost and standard deviation obtained by the proposed method are used to evaluate the robustness characteristic of the proposed method for ED problem. The algorithm of MVMO-SH is run on a Core i5 CPU 3.2 GHz PC with 4 GB of RAM. The implementation of the proposed MVMO-SH is coded in the Matlab R2013a platform.

By experiments, the typical parameters for MVMO-SH are selected as follows:

- $iter_{max}$ : The maximum number of iterations is set to 20000, 40000, 60000, 80000 and 100000 for 10-unit, 20-unit, 40-unit, 80-unit and 160-unit, respectively



- $n\_var$ : number of generators ( $D$ ),  $D = 10$ .
- $n\_par$ : number of particles is set to 50 in all cases.
- $\gamma$ : The probability parameter is set to  $\gamma = 1/100/D$ .
- *archive size*: Size of solution archive is set to 4 in all cases.
- $g_{p\_ini} = 0.7$ ,  $g_{p\_ini} = 0.2$
- *mode*: There are four variable selection strategy for offspring creation. After all simulations, strategy 4 ( $mode = 4$ ) is superior to the other strategy.
- $m_{final} = round(n\_var/2)$ ,  $m_{ini} = 1$
- $\Delta d_0^{ini}$ ,  $\Delta d_0^{final}$ : The range of  $\Delta d_0$  in (14) is [0.01 – 0.4]. By experiments,  $\Delta d_0^{ini}$  and  $\Delta d_0^{final}$  is set to 0.4 and 0.05, respectively.
- $f_{s\_ini}^*$ ,  $f_{s\_final}^*$ :  $f_{s\_ini}^*$  is set to 1 and  $f_{s\_final}^*$  is set to 10.
- *Indep\_run* is set to 3.

### 5.1 10-Unit System

The data of 10-unit test with VPE and MF is given in [7]. This system supplies to the power load demand of 2700 MW with transmission power loss neglected.

The result obtained by the proposed MVMO-SH including power outputs, minimum total cost, average total cost, maximum total cost, standard deviation for this system are shown in Table 1. As seen in Table 1, the difference between the maximum and minimum costs obtained the proposed MVMO-SH is very small and the standard deviation is very low (0.0306). It clearly shows that the performance the proposed MVMO-SH is robust.

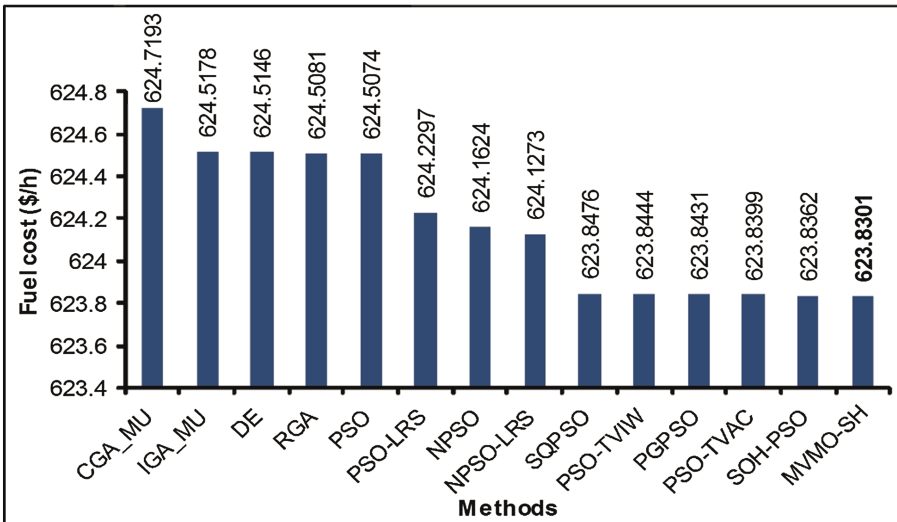
**Table 1.** Results obtained for 10-unit system with MF and VPE for load demands 2700 MW by MVMO-SH

| Unit                      | Fuel $F_i$ | $P_i$ (MW) |
|---------------------------|------------|------------|
| 1                         | 2          | 218.1050   |
| 2                         | 1          | 210.9169   |
| 3                         | 1          | 280.6571   |
| 4                         | 3          | 239.9551   |
| 5                         | 1          | 279.9208   |
| 6                         | 3          | 239.3922   |
| 7                         | 1          | 287.7207   |
| 8                         | 3          | 239.0145   |
| 9                         | 3          | 428.4452   |
| 10                        | 1          | 275.8725   |
| Total power (MW)          |            | 2700.000   |
| Min total cost (\$/h)     |            | 623.8301   |
| Average total cost (\$/h) |            | 623.8997   |
| Max total cost (\$/h)     |            | 623.9619   |
| Standard deviation (\$/h) |            | 0.0306     |

The best total cost and computational time obtained by MVMO-SH are compared to other methods including CGA\_MU, IGA\_MU [7], DE, RGA, PSO [8], PSO-LRS, NPSO, NPSO-LRS [9], SQPSO [10], PSO-TVIW, PSO-TVAC, SOH-PSO [11], and PGPSO [12], which are shown in Table 2. The comparison of best fuel cost is also depicted in Fig. 3. The best total cost obtained by MVMO-SH for this system is less than the other methods as observed from Table 2 and Fig. 3.

**Table 2.** Comparison of best total cost for 10-unit system with MF and VPE

| Method        | Best cost (\$/h) |
|---------------|------------------|
| CGA_MU [7]    | 624.7193         |
| IGA_MU [7]    | 624.5178         |
| DE [8]        | 624.5146         |
| RGA [8]       | 624.5081         |
| PSO [8]       | 624.5074         |
| PSO-LRS [9]   | 624.2297         |
| NPSO [9]      | 624.1624         |
| NPSO-LRS [9]  | 624.1273         |
| SQPSO [10]    | 623.8476         |
| PSO-TVIW [11] | 623.8444         |
| PGPSO [12]    | 623.8431         |
| PSO-TVAC [11] | 623.8399         |
| SOH-PSO [11]  | 623.8362         |
| MVMO-SH       | 623.8301         |



**Fig. 3.** Comparison of best fuel cost for 10-unit test system with VPE and MF

## 5.2 Large-Scale Systems

The large-scale systems are created by duplicating the basic 10-unit system with the load demand of 2700 MW adjusted to the system size proportionally. Table 3 shows the results obtained by the MVMO-SH for these systems including minimum total costs, average total costs, maximum total costs, standard deviations, and computational times. As seen from Table 3, the difference between the maximum and minimum costs obtained the MVMO-SH is small.

**Table 3.** Results for systems with VPE and MF

| No. of units              | 20        | 40        | 80        | 160        |
|---------------------------|-----------|-----------|-----------|------------|
| Min total cost (\$/h)     | 1247.7143 | 2495.6100 | 4991.8861 | 9992.6725  |
| Average total cost (\$/h) | 1247.8435 | 2496.1662 | 4996.3716 | 10002.6651 |
| Max total cost (\$/h)     | 1248.0171 | 2497.7533 | 5006.5585 | 10016.9936 |
| Standard deviation (\$/h) | 0.0863    | 0.5193    | 4.0995    | 6.1357     |
| Average CPU time (s)      | 9.728     | 20.474    | 42.651    | 101.120    |

Table 4 shows the comparison of the average total costs and CPU times between the proposed MVMO-SH and the CGA\_MU, IGA\_MU [5] and PGPSO [10]. The comparison of best fuel cost is also depicted in Fig. 4 for this case. As seen from Table 4 and Fig. 4, in all cases, the MVMO-SH obtains the average total costs less than CGA\_MU, IGA\_MU and PGPSO, especially for the large-scale systems.

**Table 4.** Comparison of average total cost and CPU times for systems with VPE & MF

| Method     | No. of units | Total cost (\$) | CPU time (s) |
|------------|--------------|-----------------|--------------|
| CGA_MU [7] | 20           | 1249.3893       | 80.48        |
|            | 40           | 2500.9220       | 157.39       |
|            | 80           | 5008.1426       | 309.41       |
|            | 160          | 10143.7263      | 621.30       |
| IGA_MU [7] | 20           | 1249.1179       | 21.64        |
|            | 40           | 2499.8243       | 43.71        |
|            | 80           | 5003.8832       | 85.67        |
|            | 160          | 10042.4742      | 174.62       |
| PGPSO [12] | 20           | 1248.9623       | 4.078        |
|            | 40           | 2499.6127       | 18.645       |
|            | 80           | 5003.0250       | 43.191       |
|            | 160          | 10032.4883      | 91.570       |
| MVMO-SH    | 20           | 1247.8435       | 9.728        |
|            | 40           | 2496.1662       | 20.474       |
|            | 80           | 4996.3716       | 42.651       |
|            | 160          | 10002.6651      | 101.120      |

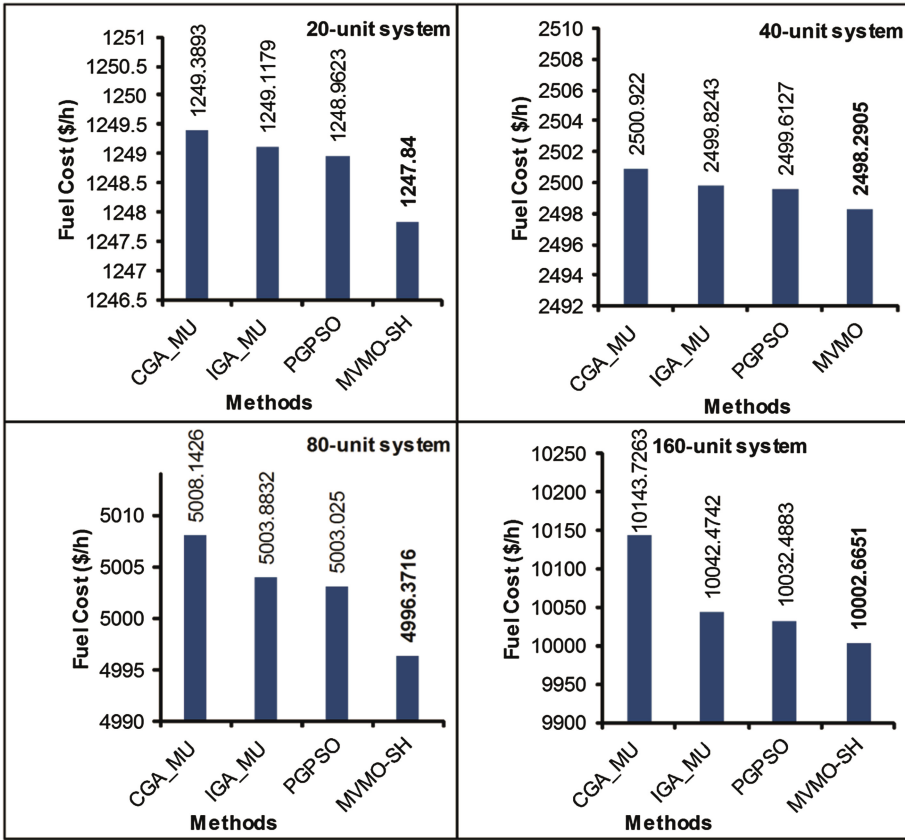


Fig. 4. Comparison of average fuel cost for large-scale test systems with both VPE and MF

## 6 Discussion

The proposed MVMO-SH has been implemented to the non-convex ED problem which takes into account multiple fuels and valve-point effects. From the numerical results, the power outputs obtained by MVMO-SH are between the minimum and maximum generator capacity limits and the total power output of generating units equals to the power load demand. It is indicated that the equality and inequality constraints always satisfy. In addition, the comparisons from Tables 2 and 4 show that the MVMO-SH can obtain better total fuel costs than most of other reported methods, especially for large-scale systems. Consequently, the MVMO methods can obtain very good solution quality for ED problems. However, the computation time is relatively high for large-scale systems. In this study, the proposed algorithm is run 50 independent trials. The mean cost, max cost, average cost and standard deviation obtained by the proposed method to evaluate the robustness characteristic of the proposed method for ED problems. The standard deviation is small. It shows that the performance the proposed MVMO-SH is

robust. The disadvantage of MVMO-SH is computational time. The computation time of the MVMO-SH is relatively high for large-scale systems. Similar to the original MVMO, the number of iterations in MVMO-SH is equivalent to the number of offspring fitness evaluations which is usually time consuming in practical applications. In the future, the proposed method will be further improved for efficient dealing with complex and large-scale optimization problems in power systems.

## 7 Conclusion

The proposed MVMO-SH has been successfully applied to the ED problem with multiple fuels considering valve-point effects. The proposed method is based on the conventional MVMO enhanced with the embedded local search and multi-parent crossover to improve its global search ability and solution quality for optimization problems. The method has been tested on 10-unit system and large-scale systems to demonstrate its effectiveness and efficiency. The numerical results showed that the proposed MVMO-SH has better performance than other optimization techniques exist in the literature in terms of global solution and robustness. Therefore, the proposed MVMO-SH could be favorable for solving other non-convex ED problems.

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