Geostatistics on Unstructured Grids, Theoretical Background, and Applications

P. Biver, V. Zaytsev, D. Allard, and H. Wackernagel

Abstract Traditionally, geostatistical simulations are performed on regular grids, in IJK coordinates system, simulating centroids of the cells. This approach (commonly used) has severe drawbacks: the support size effect is not taken into account and some artifacts due to cells distortion may appear. On the other hand, reservoir engineers and hydrogeologists are increasingly referring to new generation of grids to perform dynamic simulation (Voronoï grids, tetrahedral grids, etc.) which require addressing the volume support effect.

In this paper, we present a theoretical framework to simulate variables directly on this new generation of grids, using a depositional coordinates system (UVT) and taking into account the support size effect.

A real field case study is subsequently presented (lithology and petrophysical modeling) to illustrate the possibilities of the new generation of simulation tools. A conclusion is provided and the remaining problems are discussed to propose some guidelines for future works.

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1 Introduction

For more than 40 years, geostatistical estimations and simulations have been performed on regular so-called "sugar-box" grids. This is mainly due to historical reasons. The technology was emerging first from the mining industry, to estimate grades for open pit blocks, and it was a reasonable choice to use such support of information.

These regular grids have been kept for a long time as they allow following stratigraphy in corner point grids geometry commonly used in petroleum industry; they were also a convenient format to optimize algorithms of various kinds (sequential simulations, simulation with fast Fourier transform, multiple-point statistics simulations, etc.).

However, several new grid geometries have emerged in the last decades: tetrahedral meshes in hydrogeology and Voronoi grids with local grid refinements for petroleum industry. These grids are more convenient to solve the physical equations of flow and transport in porous media. Moreover, these grids are emerging in the geo-modeling processes with a relevant formulation of the depositional UVT coordinates system (see Mallet 2004); a dual grid approach was used to address both the dynamic simulation and the geostatistical characterization (flow simulation grid or FSG to solve the physical problem and the geological grid or GG to perform geostatistics).

The dual grid approach has several drawbacks: the geological grid resolution is driven by the smallest cells of the flow simulation grid, and, moreover, an upscaling technique is necessary to transfer information from GG to FSG; this upscaling technique needs to be general enough for the considered topologies.

As a consequence, it was necessary to adapt the geostatistical processes to use directly the flow simulation grids; but, due to the various size and geometry of elementary grid cells, it is mandatory to take into account the support size effect. We present a technique based on the formalism of the discrete Gaussian model (see Emery 2009 and Chilès and Delfiner 2012). A review of other solutions for geostatistical simulations on unstructured grids can be found in Zaytsev et al. 2016. To simulate directly on unstructured reservoir grids, an alternative method proposed by Boucher A. and Grosse H. (2015) will be also commented regarding implementation aspects.

2 Recall of the Discrete Gaussian Model

The presentation of the discrete Gaussian model (DGM) can be found in Chilès and Delfiner (2012). It can be described as follows:

- Each block of the grid v_p is attributed a parameter $r_p \in (0,1)$ which is called the change of support coefficient for this block.
- We work on the Gaussian transform Y of the variable of interest Z.

– We decompose the Gaussian anamorphosis of Z in a basis of normalized Hermite polynomials χ_i $(Y(x))$:

$$Z(x) = \varphi(Y(x)) = \sum_{i=0}^{\infty} \varphi_i \chi_i(Y(x)) \tag{1}$$

– Using the Cartier's relation (Chiles and Delfiner 2012 p. 441), we can derive the block support distribution $Z(v)$ since it can also be represented in the same polynomial basis using the decomposition (1) and the block change of support coefficient r:

$$Z(v) = \varphi_v(Y_v) = \sum_{i=0}^{\infty} \varphi_{i.} r^i \chi_i(Y_v) \tag{2}$$

– By double volumetric integration of the point support covariance $C(x,x')$ over the volume of interest ν, we can derive the change of support coefficients r_p:

$$Var(Z(v_p)) = \frac{1}{|v_p|^2} \int_v \int_v C(x,x') dx\, dx' = \sum_{i=1}^{\infty} \varphi_i^2 r_p^{2i} \tag{3}$$

– Following the same principle, by double volumetric integration of the point support covariance $C(x,x')$ over two different volumes of interest ν_p and ν_q, we can derive the block support covariances:

$$Cov(Z(v_p), Z(v_q)) = \frac{1}{|v_p|\,|v_q|} \int_{v_p} \int_{v_q} C(x,x') dx\, dx' = \sum_{i=1}^{\infty} \varphi_i^2 r_p^i r_q^i \operatorname{cov}(Y_{v_p}, Y_{v_q})^i \tag{4}$$

– Once the change of support is known for each grid cell and covariance is known between each pair of Gaussian random variables characterizing the volumes of each grid cell, we are back to a classical problem of generating a multivariate Gaussian random function with a given covariance matrix which can be solved by classical methods such as SGS.

Different formulations are available for the discrete Gaussian model; in this paper, these aspects will not be addressed; for such discussions, we refer to Zaytsev et al. (2016).

3 Practical Aspects of Applying Discrete Gaussian Model for Geostatistical Simulations on Unstructured Grids

To apply the theory of DGM to unstructured grids, key additional issues need to be addressed.

It is mandatory to integrate efficiently point support covariance (Eqs. 3 and 4) over cells that are not usual octahedrons. This issue is solved with an efficient lumping of each grid cell (see Korenblit and Shmerling 2006) followed by pseudo Monte Carlo integration using Sobol sequence of quasi-random points in the six-dimensional space of integration. This methodology is recommended for high dimension space because of the convergence speed.

Figure 1 illustrates the advantage of Monte Carlo methods for the problem of computing the variance of a block average value. In this figure, several integration methods are compared (subsequent Gauss quadrature integration, approximating the block with regularly spaced points, Monte Carlo integration). Clearly, Monte Carlo and related techniques are much more efficient.

It is also important to have an efficient procedure to navigate in the topology of the grid. The definition of searching neighborhood needs to be addressed in a general and efficient way. We propose to address this issue with a k-d tree efficient search (Bentley 1975). Expressed in the asymptotic notations for comparison of the algorithm performance (Cormen 2009), for a grid of N_b blocks, the k-d tree gives performance $O(\log N_b)$ for the neighborhood search operation, which is much faster than the naïve approach of looking through all the blocks of the grid which is at least $\Omega(N_b)$.

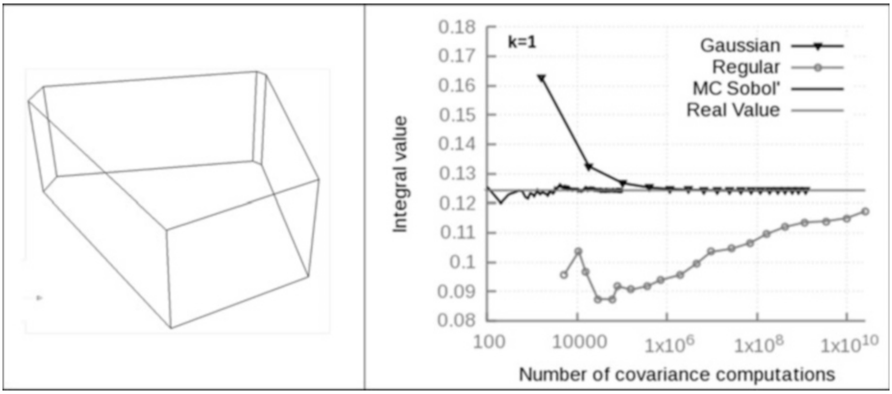

Fig. 1 Comparison of different integration methods to estimate variance of a typical grid cell (Gauss integration, regular spacing, Monte Carlo with Sobol quasi-random sequence)

4 What About Facies Modeling?

In the previous section, we addressed the problem of simulating a continuous variable on an unstructured grid.

Another important problem is to simulate categorical variables on unstructured grids. Following the same general approach, it is important to address the support size effect, at least for sub-seismic heterogeneities.

The problem can be described as follows: let us consider a categorical variable with K states $k = 1,K$. The appropriate way to handle support size effect is to simulate on the unstructured grid a proportion vector $\boldsymbol{p}=(p_1,\ldots,p_K)$ with components summing up to one.

Although using DGM for the problem of simulating categorical variables is possible, the range of applicability of the resulting model is very limited.

In our case study, the problem of facies simulation has been addressed with the method described by Gross and Boucher (2015). In their paper, the authors propose an upscaling-based approach for geostatistical simulations on unstructured grids which enables simulating the block values in a consecutive manner. Although using upscaling, this method does not require creating and storing a refined grid for the entire unstructured model, but only uses discretizations of a limited number of nodes at every step. The algorithm is based on parsimonious simulation of control points inside the cell of the unstructured mesh. The current point is simulated consistently with the other points previously simulated in the cell and the neighboring previously simulated cells; therefore, point-to-block covariances are still needed.

The key issue is to choose efficiently the number and the locations of the control points inside the cell. For this problem, sensitivity tests can be performed similarly than what has done for the integration of Eq. (3). A discretizing set of points can be considered to be good, if it enables to approximate accurately the variance of the block. Precisely, a set of points $\{x_i, i = 1\ldots N\}$ can be used for simulating the value Z(v) if

$$Var\left(\frac{1}{N}\sum_1^N Z(x_i)\right) \approx Var(Z(v)). \tag{5}$$

Our tests indicate that when the Sobol quasi-random sequence of discretizing points is used, a relatively small number of points (between 50 and 100; see Fig. 2) are sufficient to satisfy (5).

5 Application to a Real Field Case

The above-described methods have been applied to field X. The objective is to simulate groundwater flow on a very large grid including regional effects modeled with large grid cells and local details for a zone of interest modeled with much smaller grid cells (100–1,000 times smaller).

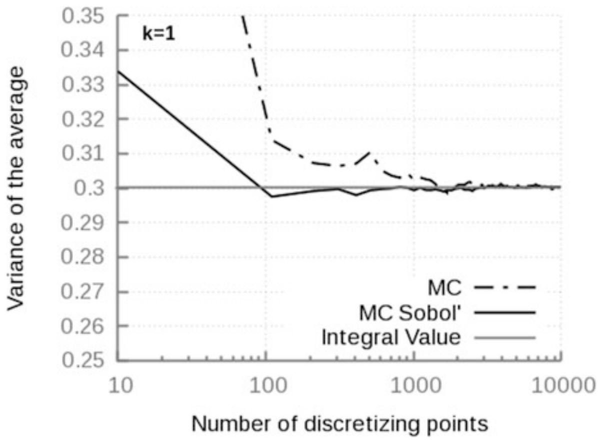

Fig. 2 Approximating the variance of blocks with the variance of a set of discretizing points. Spherical covariance in 3D, the dimensions of the blocks approximately equal to the ranges of the covariance

The area covered by the grid is 70 by 90 km^2. The previous model was built with constant values over very large domain; it was not representative of the variability of facies and porosity that can occur in this domain.

A new facies model (using the method in Gross and Boucher (2015)) was built. Three facies are modeled (shale, shaly sand, and massive sand); the target proportions over the entire grid are 20 % for shales, 45 % for shaly sands, and 35 % for massive sands; the covariance function used for the underlying Gaussian field in the truncated Gaussian simulation is a spherical model with areal ranges of 800 m by 250 m (azimuth of maximum range is 55°); the vertical range is 100 m. The coordinate system is a UVT coordinate system, built from the relevant horizons.

The facies simulation results are illustrated in Fig. 3. The simulated proportions over the grid are clearly illustrative of the support size effect (less variations in large cells, larger variations in small cells).

The important issue of modeling facies and related proportions is illustrated in Fig. 4. The dominant facies (most likely facies regarding proportions) is represented. In the area modeled with large cells, the shale facies is never dominant, and proportional modeling procedure is the only way to keep that facies into account; in the area of local grid refinement, this aspect is less important, and the traditional truncated Gaussian simulation picture is observed.

The porosity model has been built using DGM assumptions. As each facies can occur in each cell, it is therefore important to perform full field porosity model for each facies. Point-scale distributions for each facies are provided in Table 1 (we used beta distributions with p and q referring to the shape parameters); these porosity distributions are clearly different according to facies classification. Point-scale normal score variogram for porosity is a spherical model with areal ranges of 600 m by 200 m (azimuth of maximum range axis is 55°); the vertical range is 80 m.

The porosity model inside each facies is represented in Fig. 5. The support size effect is clearly visible with small variations in large cells and larger variations in smaller cells.

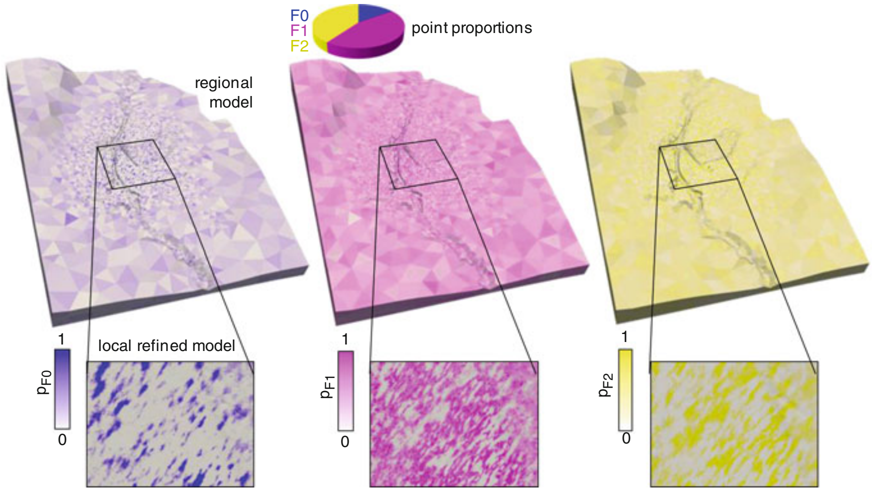

Fig. 3 Facies modeling on field X, proportions maps for silt (*blue*), silty sand (*pink*), and massive sands (*yellow*). The simulated facies proportion map is sharp in the region of the local grid refinement and becomes smoother in the regions of coarse blocks

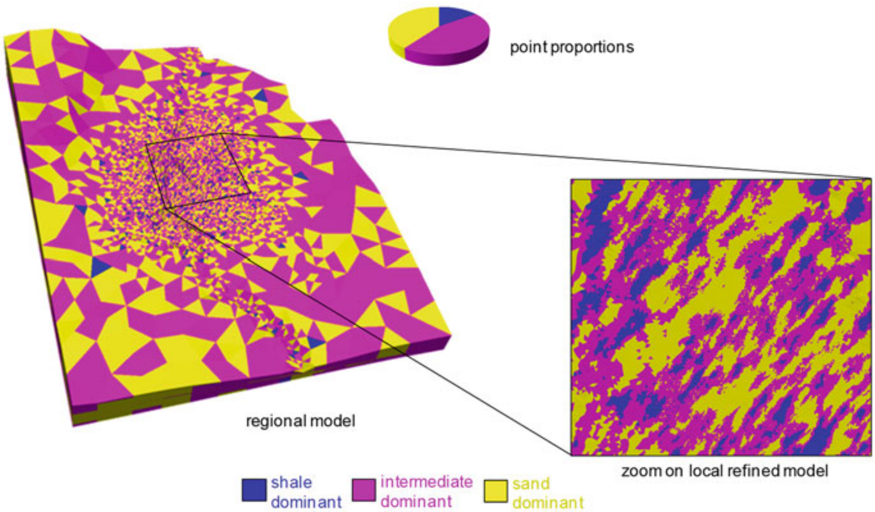

Fig. 4 Facies modeling on field X, dominant facies map

Table 1 Point-scale porosity distributions for field X

Facies	Distribution
Shales	Beta (min = 0, max = 0.4, p = 1, q = 6)
Shaly sands	Beta (min = 0, max = 0.4, p = 3, q = 3)
Sands	Beta (min = 0, max = 0.4, p = 8.5, q = 3.5)

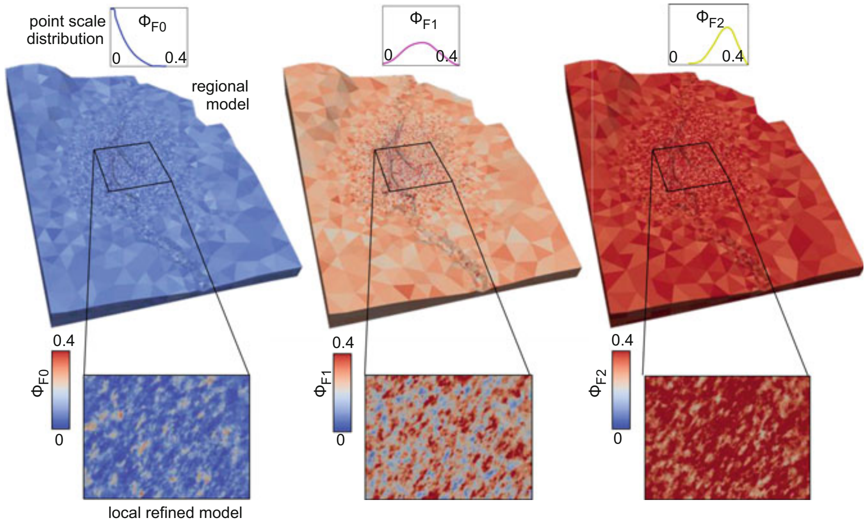

Fig. 5 Full field porosity modeling inside each facies of reservoir X

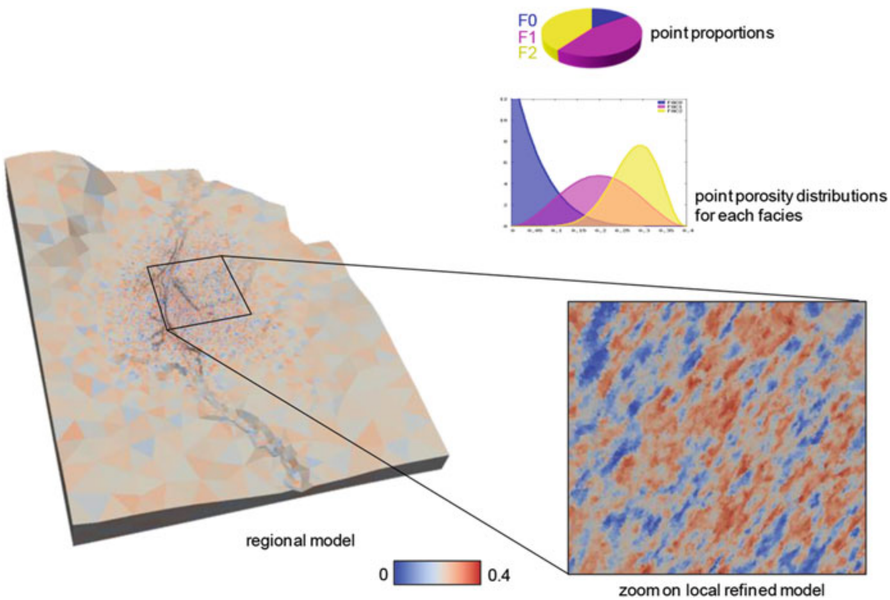

Fig. 6 Full field equivalent porosity inside reservoir X

From these facies porosities, it is possible to derive equivalent porosity for each cell simply by weighting them according to simulated proportions. This result is represented in Fig. 6; the support size effect is still visible.

6 Conclusion and Future Work

In this paper, we have presented a methodology to address geostatistical simulation on unstructured grids. Implementation issues have been discussed, and an illustrative field case example has been performed. We have shown that the world of geostatistics and the world of complex fit-for-purpose gridding can be reconciled.

The proposed workflow can be adapted to co-simulation techniques without major difficulties. A co-simulation approach with DGM on regular grids can be generalized without major modifications for unstructured grids (Emery and Ortiz 2011). In order to use the DGM for co-simulation, a linear model of co-regionalization (LMC) can be used, and conditioning kriging should be substituted with conditioning co-kriging.

However, it is important to notice than we have addressed only the domain of additive variables. An important topic is still to be treated: nonadditive variables such as permeabilities. We envisage treating these variables by using fit-for-purpose transformation like power transform as suggested in Noetinger (1996) and Deutsch (2002); but these transformations need to be calibrated by physical measures and numerical tests. This will be the next challenge of this research.

Other problems would be interesting to investigate: generalization of algorithms to nonstationary cases, addressing facies simulation techniques different from truncated Gaussian simulation.

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