Mathematical Teacher Noticing: The Key to Learning from Lesson Study

Mi Yeon Lee and Ban Heng Choy

Abstract Lesson Study has been adapted by many countries in support of teachers' learning from their practice. However, learning from Lesson Study does not come naturally and it is unclear how teachers can be supported in such learning. Moreover, lesson preparation, a critical component of mathematics teaching, is still largely under-explored in the study of teacher noticing. This chapter presents an analysis of what and how teachers notice when they make instructional decisions during the planning and reviewing stages of Lesson Study. It compares and contrasts two groups of elementary school teachers: one group of pre-service teachers (PSTs) from the United States, and the other group of in-service teachers (ISTs) from Singapore, in terms of what they see and think about their students' mathematical reasoning during Lesson Study. Using a notion of productive noticing, we provide snapshots of mathematics teacher noticing, which highlights the key role noticing plays in learning from Lesson Study, and offer insights as to how teacher noticing can be supported in the context of lesson planning and reflection.

Keywords Noticing · Lesson Study · Lesson planning · Lesson reflection · Teacher education

Introduction

To teach mathematics effectively, teachers should notice and build on student thinking, adjusting their instruction to support their students' learning (National Council of Teachers of Mathematics [NCTM], 2014). Teaching in this manner is

M.Y. Lee (✉)
Arizona State University, Tempe, AZ, USA
e-mail: mlee115@asu.edu

B.H. Choy
National Institute of Education, Nanyang Technological University, Singapore, Singapore
e-mail: banheng.choy@nie.edu.sg

E.O. Schack et al. (eds.), *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks*, Research in Mathematics Education,
DOI 10.1007/978-3-319-46753-5_8

both ambitious and challenging, and requires knowledgeable teachers to enact these productive teaching practices (National Research Council, 2005; Smith & Stein, 2011). In light of this, teacher education researchers suggest that meaningful teacher learning occurs when teachers have opportunities to reflect upon their teaching practice and work in professional communities in order to solve instructional problems related to their teaching practice (Ball & Cohen, 1999; Hiebert, Morris, & Glass, 2003). However, participation in these learning communities alone, while deemed necessary, would be insufficient. Instead, it is crucial that teachers develop a common language to discuss issues with regard to teaching and learning (Bryk, 2009; Loughran, 2009).

Mathematics teacher noticing is one such means to improve teaching expertise because how teachers pay attention to and make sense of what happens in their classroom can influence the quality of mathematics teaching (Jacobs, Lamb, & Philipp, 2010; Sherin, Jacobs, & Philipp, 2011). Despite the growing number of research studies on teacher noticing, many of these studies centred on exploring teacher noticing skills displayed when reviewing their teaching videos (Sherin & van Es, 2009; Star & Strickland, 2008; Stockero, 2008), and only a few studies examined teacher noticing during the lesson preparation (Choy, 2014; Santagata, 2011). For example, some researchers provided teachers with another teacher's instruction video and asked them to describe what they notice in the teaching video (Colestock & Sherin, 2009; Kersting, 2008; Star, Lynch, & Perova, 2011), and others asked teachers to retrospectively recall what they were noticing during their own teaching by watching a video from their own classroom (Ainley & Luntley, 2007). In some cases, researchers asked teachers to watch and discuss excerpts of their teaching video with other teachers as a peer group (Sherin & van Es, 2009). As part of a larger study, Choy (2014) explores what teachers notice during the lesson preparation stage of Lesson Study and extends the realm of the study of noticing to lesson planning.

Lesson Study is a collaborative teacher-inquiry professional development approach that emphasizes reflection on practice and students' thinking (Fernandez & Yoshida, 2004; Stigler & Hiebert, 1999), and can be used to effectively develop teachers' expertise and foster meaningful teacher learning. However, learning from Lesson Study does not come naturally (Takahashi & McDougal, 2016). In this regard, Fernandez, Cannon, and Chokshi (2003) highlight three critical lenses*, that of researcher, curriculum developer,* and *student,*needed to learn from the processes of Lesson Study. Adopting these lenses requires teachers to use varying perspectives to focus their attention on mathematically worthwhile aspects (Schifter, 2001) and interpret students' mathematical ideas in order to make instructional decisions productive for enhancing students' reasoning (Jacobs et al., 2010). Thus, we hypothesize that teacher noticing, which consists of observing, analyzing and responding (Sherin et al., 2011), plays a critical role in teachers adopting these lenses. Furthermore, even though it is important to prepare oneself to notice (Mason, 2002), the role of noticing during lesson preparation has been relatively unexplored.

In this chapter, we will examine mathematics teacher noticing during the planning as well as the review stages of Lesson Study by applying these three critical lenses. The key questions that guided our inquiry are as follows:

1. What do teachers notice when they plan and review lessons during Lesson Study?
2. How do teachers notice what they observe during Lesson Study?
3. How can we support teachers to learn from Lesson Study through a focus on noticing?

Theoretical Framework: Learning from Lesson Study

Critical Lenses for Learning from Lesson Study

Setting the different adaptations of Lesson Study implemented by various countries aside, Lesson Study in essence comprises five essential tasks—(1) developing a research theme; (2) working, discussing and anticipating student thinking through mathematics tasks; (3) developing a shared lesson plan; (4) collecting data during observation of research lesson; and (5) conducting a post-lesson discussion (Lewis, Friedkin, Baker, & Perry, 2011). These five tasks can be applied into three phases of a lesson such as lesson planning (Task 1–3), teaching (Task 4), and lesson reviewing (Task 5). Here, we will focus on teachers' discussion during the planning and reviewing phases of Lesson Study.

The potential of Lesson Study to improve teachers' practice (Fernandez & Yoshida, 2004; Lewis, Perry, & Hurd, 2009; Murata, Bofferding, Pothen, Taylor, & Wischnia, 2012) can only be fully realized when teachers learn how to critically examine their lessons (Takahashi & McDougal, 2016). In this regard, Fernandez et al. (2003) provided three critical lenses that can be applied to examine lessons for the purposes of Lesson Study. The first lens is *the researcher lens* that encourages teachers to see themselves as researchers looking into their problems of practice. Putting on this lens requires teachers to develop the appropriate means to investigate their own research questions, and use evidence to explain the success of their intervention before they apply the findings to other similar contexts (Fernandez et al., 2003). The second lens—*the curriculum developer lens*—focuses teachers' attention on how to sequence activities and connect them to students' learning during the lesson. In this aspect, teachers are concerned with orchestrating students' learning both across and within lessons, bearing in mind the developmental progress of students' thinking. Finally, when teachers attempt to anticipate students' possible solutions to main tasks and consider how to use this knowledge to support students' deep understanding of the content, they are beginning to adopt the *student lens*. Adopting these lenses requires teachers to *notice* mathematically meaningful events in the classroom and adapt their instruction based on students' thinking while providing appropriate curricular materials to support students' learning (Schifter, 2001).

What is Mathematics Teacher Noticing?

Mathematics teacher noticing, a form of professional vision (Goodwin, 1994), can be conceptualized in three different ways (Jacobs et al., 2010; Miller, 2011; Sherin & van Es, 2009): noticing as (1) focusing on what teachers attend to; (2) focusing on teachers' interpretation about what they selectively attend to; and (3) combination of three actions such as attending to, interpreting, and responding to student thinking. In this paper, we adopt the third perspective of noticing, which consists of attending to noteworthy events, interpreting these events, and making instructional decisions based on interpretations of the notable events (Jacobs et al., 2010).

To characterize teacher noticing, two main dimensions of teacher noticing are examined: *what teachers notice* and *how teachers notice* (Sherin & van Es, 2009; van Es, 2011). The first dimension describes both *who* (e.g. whole class, student group, individual student, and the teacher) teachers focus on, and *which topics or issues* (e.g. pedagogical strategies, behaviour, mathematical thinking, and classroom climate) they identify. The second dimension captures how teachers analyse what they notice in terms of *analytic stances* (e.g. descriptive, interpretive, and evaluative) and the *depth of analysis* (e.g. whether to provide few details or ground their comments in evidence) when they make their instructional decisions. These two dimensions are also applicable for researchers seeking to examine teacher noticing during the planning, teaching, and reviewing phases of Lesson Study. Even though van Es developed the framework for learning to notice student mathematical thinking, for our study, the use of her framework is extended to investigate what and how teachers notice during the whole Lesson Study processes (see Table 1).

Noticing as a Way to Put on the Three Critical Lenses

It is "wishful thinking" to expect that "something good will happen" just because one gathers "teachers together to talk about practice" (Bryk, 2009, p. 599). As highlighted, it is crucial that teachers adopt the three critical lenses and focus on student reasoning when reflecting on their teaching in order to learn from Lesson Study. However, applying these critical lenses can be very challenging, and requires teachers to focus their attention on noteworthy aspects of their teaching practice. They need to attend to aspects of student thinking from classroom artifacts; student explanations; and discourses, and interpret them using a mathematical perspective before, during, and after a lesson (Goldsmith & Seago, 2013; Jacobs et al., 2010; Schifter, 2001; Smith & Stein, 2011). In many ways, these characteristics of noticing are similar to the notion of *extended noticing*, as proposed by van Es (2011).

Table 1
A framework for learning during Lesson Study

	What teachers notice	How teachers notice
Level 1 *Baseline*	• Attend to irrelevant details that do not have direct impact on student learning • Attend to whole class environment, behaviour, *generic* content and learning and to teacher pedagogy	• Form general impressions of what occurred • Provide descriptive and evaluative comments • Provide little or no evidence to support analysis
Level 2 *Mixed*	• Primarily attend to teacher pedagogy • Begin to attend to *particular* aspects of mathematical concepts and the difficulties associated with them • Begin to attend to particular students' mathematical thinking and behaviours	• Form general impressions and highlight noteworthy events or details • Provide primarily evaluative with some interpretive comments • Begin to refer to specific events and interactions as evidence
Level 3 *Focused*	• Attend to *particular* aspects of mathematics and relate students' confusion to the teaching approaches • Attend to particular students' mathematical thinking	• Provide interpretive comments • Refer to specific students' difficulties, events and interactions as evidence • Elaborate on these specific students' difficulties, events and interactions
Level 4 *Extended*	• Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking	• Provide interpretive comments • Refer to specific events and interactions as evidence • Elaborate on these specific students' difficulties, events, and interactions • Make connections between events and principles of teaching and learning • On the basis of interpretations, propose alternative pedagogical solutions

Note. Adapted from "A Framework for Learning to Notice Students' Thinking" by van Es (2011, p. 139).

Expert teachers, who are highly proficient in this work, can perceive meaningful patterns from what they see, and connect these observations to what they know, to make productive instructional decisions in the midst of a complex classroom environment (Berliner, 2001). These teachers are more sensitive and attuned to task demands and social contexts, and are better able to call upon different but useful strategies to solve their problems in practice (Berliner, 2001; Mason, 2002). This high level of attention is more active and intentional, rather than passive or

spontaneous (Erickson, 2011; Mason, 2011; Miller, 2011; Sherin et al., 2011), and constantly seeks to use experience as evidence to form new ideas that can inform future practice (Schön, 1991). Hence, we argue that teachers can hone this specialized seeing, sense-making, and decision making by focusing their noticing on mathematically significant aspects of teaching and learning during the processes of Lesson Study. The three critical lenses put forth by Fernandez et al. (2003) will require teachers to notice specifically the mathematical concept, students' difficulty when learning the concept, and whether their teaching approaches address the difficulty.

The Three-Point Framework

These three areas for focusing noticing are similar to what Yang and Ricks (2013) term as the *Three Points*. They detail how Chinese teachers think about the design of a task in a lesson using three focal points: the *Key Point*, the *Difficult Point*, and the *Critical Point* (p. 54). The Key Point refers to the mathematical concept targeted in the lesson, which is sometimes known as the "Big Idea" (Askew, 2013, p. 6). The Difficult Point is the cognitive obstacle or stumbling block that students face when learning the Key Point. This can refer to persistent errors or common misconceptions that are associated with the concepts being taught. By anticipating students' Difficult Point, teachers begin to adopt the three critical lenses and design lessons targeted at the challenging aspects of learning the concept. The Critical Point is then the "heart of the lesson", which highlights the approach that teachers can use to support students in their efforts to overcome the Difficult Point, in order to learn the Key Point (Yang & Ricks, 2012, p. 43).

As an example, to teach fraction–decimal conversion at Grade 4 (age 10), a teacher may identify the key concept as the fact that common fractions and decimal fractions are different representations of the same number (Key Point); highlight students' confusion in terms of their inability to relate fractions with denominators other than 10 to decimals (Difficult Point), that is, they may put 1/5 as 0.15 because the digits "1" and "5" appeared in 1/5; and the proposed Critical Point is to create tasks where students can relate fractions such as 1/5 to fractions with denominators 10, 100, or 1000. This example illustrates how the Three Points can be used to direct teachers' attention to the relationship between specific aspects of the concept (Key Point and Difficult Point) to the design of the task (Critical Point). However, the ability to describe the details of the Three Points is dependent on a good understanding of mathematics as well as the experience in teaching the subject. Hence, this ability has been used as a distinguishing mark between highly and less proficient teachers in China (Yang & Ricks, 2013).

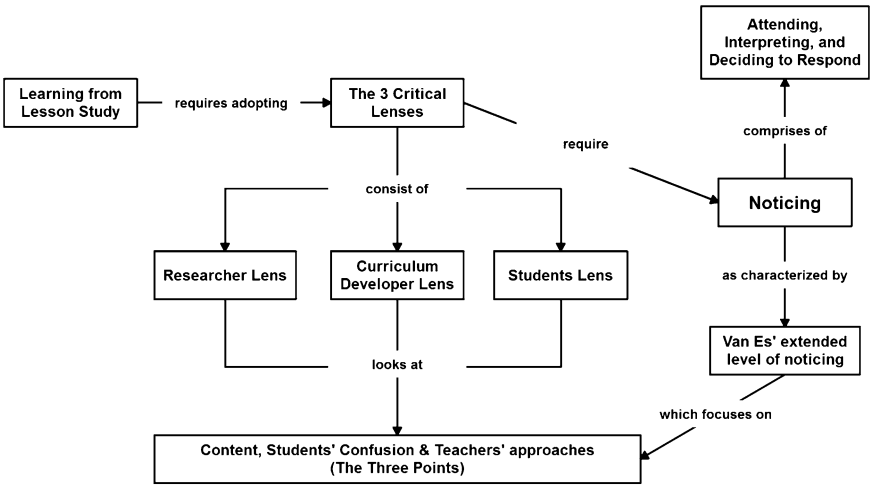

Figure 1. Theoretical framework to learn from Lesson Study.

Together, the Three Points (Yang & Rick, 2013) and van Es' (2011) framework for noticing can provide a useful way to examine what, and how, teachers notice when they proactively adopt the three critical lenses to learn from Lesson Study. In particular, we incorporate the Three Points into van Es′ framework to highlight specifically what teachers notice during Lesson Study. Figure 1 shows the relationships between the different theoretical constructs used in this chapter.

Methodology

Context of the Two Case Studies

Vignettes drawn from two contrasting case studies were explored in this study: pre-service teachers (PSTs) in the United States and in-service teachers (ISTs) in Singapore. These two groups of teachers represent two ends of the teaching experience spectrum with different cultural backgrounds. Even though one may argue that it is unfair to compare PSTs with ISTs from two different countries, we want to highlight that the purpose of this study is *not* to compare them in terms of their noticing expertise. Instead, we want to explore the *common* characteristics of their noticing, which lead to both captured and missed opportunities to learn from the processes of Lesson Study. By selecting these contrasting cases, we believe that the findings have the potential to produce insights about the role of noticing in learning from Lesson Study, particularly when we hypothesized that challenges to noticing productively and the characteristics of more productive noticing might be

similar in these seemingly different cases. This replication logic is an important consideration for case study research (Yin, 2003).

The first Lesson Study group consisted of six elementary PSTs (Amy, Christina, Erin, Hera, Jane, and Mary), a facilitator delegated from the university, and a host teacher invited for this study. The PSTs were engaged in six Lesson Study sessions in a U.S. primary school (aged 6–8) through a weekly three-hour field experience. The PSTs were instructed to teach number sense including counting, addition, and subtraction during the field experience. The vignettes, described in this chapter, were developed from episodes, which occurred at the first and last session respectively. The objectives of the first lesson were to count sets of objects up to 60, and to figure out how many pieces would fit into the large shape by allowing students to cover an area of the large shape using smaller pattern blocks. The objective of the last lesson was to help students generate various strategies to add or subtract two numbers.

In the second case, seven ISTs and a school leader participated in six Lesson Study sessions that explored the teaching of fractions for Primary Two students (aged 7–8) in a Singapore elementary school. Four of the teachers have more than 10 years of teaching experience and the others have at least three years. Two of the more experienced teachers—Zelina (25 years) and Hannah (16 years)—are of particular interest in this chapter. The teachers were part of a larger study on teacher noticing conducted by the second author. However, this chapter reports the initial phase of the study, where the ISTs were not introduced to the notions of teacher noticing. The seven teachers, Hannah; Alice; Heather; Heidi; Jacinda; Sherry; and Zelina, worked together to plan a lesson on ordering *Unit Fractions* for Primary Two students. Vignettes, developed from the planning and review sessions of the Lesson Study, are presented and discussed.

Data Collection

Data for both cases were collected and generated through video or voice recordings. The data from U.S. were collected by video-recoding two Lesson Study discussions at the beginning and the end of a mathematics field experience. In the two videos, the same pair of PSTs co-taught the two mathematics lessons and other PSTs observed their lessons. The data from Singapore were collected through voice recordings of the Lesson Study sessions involving all seven ISTs and their school leader, Jaslyn who participated in the discussions. We watched and listened to the video and voice recordings to mark out segments, which reflected similar levels of noticing according to the adapted van Es' framework. These segments were then transcribed for further analysis without editing any ungrammatical or colloquial language.

Data Analysis

For this study, we characterized teachers' noticing in terms of *what* and *how* they notice. By extending van Es' (2011) framework to Lesson Study (see Table 1), we evaluated *how* teachers notice based on six components: (1) whether the statement was general or specific; (2) whether the statement was descriptive, evaluative, or interpretive; (3) whether the statement was based on evidence; (4) whether the statement elaborated on events and interactions; (5) whether the statement made connections between events and principles of teaching and learning; and (6) whether the statement proposed alternative pedagogical solutions. With regard to *what* teachers notice, we coded what they discussed in terms of the Three Points (Yang & Ricks, 2012). To aid our analysis, we developed a matrix (see Table 2) to examine the three processes of noticing (Jacobs et al., 2010) in relation to the *Three Points* (Yang & Ricks, 2012). We then independently completed the matrix by extracting fragments of transcripts from the selected episodes to uncover the emerging themes.

Table 2
An example of a matrix used in analysis

	Attending to	Making sense of	Deciding to
Key Point			
Difficult Point			
Critical Point			

For each selected segment at the respective noticing levels, we analysed *what* a teacher noticed with regard to the Three Points by deconstructing noticing into the three processes: Attending, interpreting, and deciding how to respond. For example, when teachers noticed at the baseline level, they often only noticed whole class environment, behaviour, or teacher pedagogy, which were not directly related to the Three Points. In such cases, we left the matrix blank and rated "missing the 3 points". In cases where teachers' noticing was more focused, we coded what teachers attended, interpreted, or responded to with regard to the Three Points. When there were discrepancies in analysis, we reconciled our differences by intensively discussing them. We then considered two dimensions of noticing (what and how) from the matrices with our notes and assigned the levels of noticing, assigning a lower level of noticing where the two dimensions misalign. Finally, we wrote vignettes illustrative of noticing at that level.

What and How PSTs and ISTs Notice During Lesson Study?

Our findings indicate that both PSTs and ISTs found it challenging to focus on mathematically significant aspects, such as the Three Points, during initial Lesson Study discussions. Of particular interest in this study, we found that both PSTs and ISTs demonstrated a more focused level of noticing when they noticed aspects related to the Three Points. In this section, we present some representative vignettes of what and how the teachers in our study noticed at the different levels of noticing during the Lesson Study discussions at the planning and reviewing stages.

Teacher Noticing During Lesson Planning

PSTs' focusing on aspects less relevant to mathematics. As Takahashi and McDougal (2016) argue, it is critical for teachers to think about the mathematical content and relate the lesson design to the students' thinking. Therefore, without a focus on the Three Points, teachers are unlikely to gain new understanding of mathematics and teaching. We note that the PSTs tend to focus on less relevant issues when discussing the task, especially during the initial Lesson Study session. For example, the PSTs seemed to focus largely on management and organization issues, instead of lesson content or pedagogical strategies, when examining the textbook during the planning stage. In this excerpt, the facilitator asked the PSTs how they could support students in learning to compare the size of two numbers on a number line. Hera began the discussion with the following idea:

1.	Hera	First, we need to think about how to organize students for this activity.
2.	Erin	Are you thinking of teaching number lines per table or per student?
3.	Host teacher	Or just up on the wall or big group.
4.	Jane	It looks kind of like he [the teacher pictured in the textbook] is teaching the whole class.
5.	Mary	Yeah, I think that's his whole class but I think it'd be nice to follow the small group thing. If not, it would be hard to control students.
6.	Jane	Yeah, I was thinking by table or something. So we would have a number strip per table.

As seen from the exchange, the PSTs decided to use the given activity in the textbook without any reflection and focused mainly on logistical issues during lesson planning. They did not consider whether the activities were appropriate or whether they need additional activities to achieve the Key Point. Also, they neither thought about students' possible Difficult Points in learning the Key Point, nor how to help students overcome the difficulties (Critical Point). Furthermore, the PSTs

did not attend to students' thinking at all in terms of the Key Point, Difficult Point, and Critical Point during the session, and made generic suggestions about the lesson with little or no justification. For instance, when Jane suggested whole class teaching based on the picture in the textbook, Mary suggested group work because of difficulty of managing students, but without any sound pedagogical rationale. Therefore, PSTs generally did not engage in any analysis of the teaching materials and did not provide any interpretative comments with regard to the choice of strategies. In this respect, PSTs showed the *baseline* level of noticing.

ISTs' baseline noticing of the Three Points. It is possible that the PSTs failed to focus on the Three Points because of their lack of teaching experience, however, focusing on the Three Points can be challenging even for the ISTs. Furthermore, the ISTs may focus on the Three Points superficially without noticing specific details. For example, Hannah, an IST, began the initial discussion by sharing the Lesson Study goals on 'Unit Fractions', and suggested that they sharpen their questioning techniques:

> We have picked fractions as the main cause of concern because of the data that we have collected from last year's P2 [Primary 2] cohort teachers saying that the children are still not good in fractions and particularly the basic skills of ordering fractions… also they are having some problems. Because of the data we have collected from item analysis, we then decided to focus on fractions as our area of concern. And also… we also talked about questioning techniques that we have gone through as a school… how we could actually sharpen our questioning techniques to actually help children to learn fractions…

Although Hannah made reference to the concept and confusion targeted in the lesson, she did not elaborate clearly what she meant. Hannah presented the ordering of unit fractions as the Key Point in the lesson. However, she did not articulate the aspect of ordering fractions that was critical for teachers to consider. Instead, she pointed vaguely to "focus on fractions" as the "area of concern". Even though Hannah mentioned that students were "still not good in fractions" based on "evidence" from item analysis, she did not specify what these findings were. These findings would have been useful for teachers to understand students' difficulties with the concept, which could have led to a better design of the lesson.

Moreover, Hannah went on to suggest that teachers focus on their questioning techniques, but she did not link this suggested Critical Point to students' confusion about the topic. Therefore, although Hannah referred to the Three Points, the lack of specific details prevented teachers from pinpointing students' confusion about ordering unit fractions, which could have led to a more targeted approach. Furthermore, Hannah did not offer any evidence to support her analysis. Hence, Hannah's noticing, according to adapted van Es' framework, is at the *baseline* level (van Es, 2011).

Focused noticing of the Three Points. In contrast, during the final Lesson Study discussion, both PSTs and ISTs began to notice at a *focused* level when they adopted the researcher's lens by providing specific details relating to the Three Points. However, here we only illustrate ISTs' case because of page limit. In the following vignette, Hannah was able to attend to a subtle point missed by the other ISTs. In this discussion about the use of examples and non-examples to help

students recap the fractional notation *a*/*b*, the research teacher Zelina wanted to highlight the role of equal partitioning in the fractional notation. She wanted to demonstrate physically an example and a non-example of 1/4. Zelina showed two rectangles—one was divided into four equal parts and the other was not—to demonstrate what she intended to do during the lesson (Figure 2).

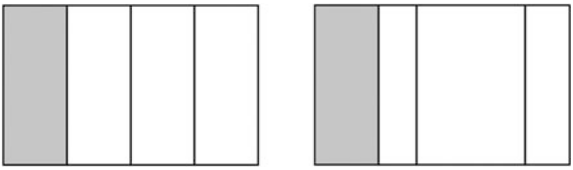

Figure 2. Zelina's representation of an example and non-example of 1/4.

To highlight the importance of equal partitioning in the fractional notation 1/4, Zelina used a detachable piece of the shaded part to show the meaning of 1/4. She removed the first shaded part and compared it to the rest of the parts of the first rectangle to show that they were equal, and hence demonstrating that the shaded part was 1/4. She then took another detachable piece (of the same area) in the second whole, and said that it was not 1/4 of the second whole *because the second whole was not divided equally*. Hannah then raised a point of clarification:

1.	Hannah	If you take the same piece, the same piece is still 1/4 of that whole.
2.	Jaslyn	This is still 1/4 of the whole… this one is not, but no… it's still 1/4 of the whole?
3.	Hannah	Yes. You must take the small one or the big one. It's still 1/4. Because it's equivalent fraction, you can subdivide that…
4.	Zelina	I don't know… make up your mind. Take or don't take?
5.	Hannah	It is still [1/4 of the whole]… you must take something that is not equal to 1/4. Because that is still 1/4 of the whole.
6.	Jacinda	… yes… yes… yes… It's still 1/4.
7.	Zelina	So, take or don't take?
8.	Hannah	You still take. But you must take a smaller or bigger piece. It's the same whole. It's still 1/4, only that we have shifted it in a way…
9.	Zelina	Where? It's not equal, right?
10.	Jaslyn	[Jaslyn shows the piece physically and compares it to the other whole which is not divided up equally] because this piece is still 1/4 of this whole…
11.	Zelina	Oh…I see.

In this episode, Hannah attended specifically to Zelina's statement that the second detachable piece is "not 1/4 because the second whole was not divided equally". This challenged the teachers' notion of equivalent fractions (Lines 2, 4, 6), and

generated a useful point with regard to the choice of example (Line 5). As a result of this specific attention to mathematical details, the teachers became more aware of the subtlety of their own conceptions of fractions, and were more able to see why students might have difficulty with fractions, given that teachers themselves may also sometime struggle with the notion. For example, Jaslyn tried to make sense of what Hannah said by physically manipulating the detachable fractional piece (Lines 2, 10), and she struggled with the concept for a brief moment (Line 2) before she came to the same conclusion as Hannah that "this piece is still 1/4 of this whole" (Line 10). Consequently, Hannah's noticing highlighted Zelina's subtle error to the teachers for discussion, and they were alerted to a possible misconception that might arise as a consequence of overemphasizing the notion of equi-partitioning.

The error involved is not trivial—that the process of dividing a whole into four equal parts gives rise to an object that is 1/4 of the whole and that object can have many different pictorial representations, but it remains 1/4 regardless of any division of the same whole. The error could have occurred because of the partial conception that fractions can only involve equal parts. Unequal partitions can be challenging for students (Schoenfeld & Kilpatrick, 2008) and can be difficult even for some teachers, as suggested in this case. Therefore, as Schoenfeld and Kilpatrick (2008) have emphasized, it is important that teachers are aware of this difficulty and be fluent with the use of different representations of fractions. Hence, Hannah's noticing of Zelina's explanation can be classified as *focused* because she provided interpretative comments about the concept of equi-partitioning and highlighted how Zelina's use of the fractional diagrams might be misleading.

Teacher Noticing During Lesson Reviewing

A critical feature of Lesson Study is teachers reflecting on the lesson to generate new understanding of how students think, and connect this new understanding to broader principles of teaching and learning (Fernandez et al., 2003; Yang & Ricks, 2012). In our study, both PSTs and ISTs often engaged in less-than-effective reflection during initial Lesson Study. In the following vignette, we see that reflecting upon a lesson to gain new insights into teaching and learning was challenging, even for the ISTs.

Not focusing on student thinking. During the initial post-lesson discussion, Zelina's first and only comments were about the clarity of her instructions on the task, and not focused on student thinking. She was pleased that most students were clear about the key task of making comparison statements about fractions except for a few who picked up two equal pieces representing a tenth:

> What I saw was… my instructions were clear enough. I said, 'take out one tenth'. But when I was going around, I realised that some of them took two "tenths" instead of one. Instead of one unit fraction, they took a few more. I think they still have difficulty grasping the greater denominators and smaller fractions. They have some inkling but have not touched down yet… it's not easy… to make the whole.

Even though she gave detailed description of her observations ("… took two tenths instead of one"), she did not seem to attend to details related to the mathematical concept (comparing fractions), students' confusion about the concept (inappropriate ideas related to sizes of numbers), nor how students responded to the lesson approach (the need to reason about the size of fractions). Zelina did not seem to distinguish between what was mathematically relevant and what was not with regard to the lesson, and made general or vague statements about students' thinking. Zelina was aware that her students might not have fully understood the use of denominators to compare unit fractions ("They have some inkling…"), and might have difficulties seeing the relationship between denominators and relative sizes of unit fractions ("they still have difficulty grasping the greater denominators and smaller fractions"). However, she did not give further details on how she came to that conclusion and why that was so. Therefore, while there was evidence that she attended to some aspects of her students' thinking, the lack of detailed connections between what she observed and the 'Three Points' did not help refine ideas about the student' thinking nor the design of the tasks. Hence, Zelina's noticing is at the *baseline* level because she had begun to refer to specific events but did not provide much analysis.

Similarly, the *baseline* level of noticing is demonstrated by the other teachers when they shared their observations. Almost every one referred to an incident where Zelina tried to help her students recall the meaning of numerator and denominator through the use of a song that she composed. Zelina taught two songs in previous lessons to help students remember the definitions of key words such as fractions, numerators, and denominators. Even though the song was never discussed during the meetings, the teachers seemed to be impressed by the use of the song as a mnemonic. For example, Heidi liked how Zelina used songs to help them recall the definition without providing further evidence:

> Actually, I like how she get [sic] them to recall… the numerator and denominator… using a simple song.

Similarly, Jacinda commented that the lesson was good and liked the use of the song to "reinforce" the definitions:

> Overall, I think that her lesson was very good because I can see that her children, even though they are lower ability, they managed to get the concept very well. Like Heidi, I also like the use of the song to reinforce the fractions, the numerators and denominators…

Even though the use of the song might have counted as an instructional strategy, the teachers mostly attended to how the song was "interesting" and "catchy". All the teachers highlighted that the song helped the students remember the terms, but they did not provide any further substantiation, thus noticing at the *baseline* level.

ISTs' focused noticing of the Three Points. To illustrate how noticing directed by the Three Points can promote a *focused* noticing, we examine how Hannah generated useful pedagogical considerations from her detailed observations during the final Lesson Study session. In the following vignette, Hannah described how two students struggled with a question and highlighted that these two students were

still thinking about fractions physically rather than symbolically because they used the aids to help them:

> … [the question] 1/7 is smaller than… he put 1/8. I said look again… then he look [sic] and looked. Although he put there 1/7, they still take the 1/7 fraction disc and put it on top of the representation 1/7. They want to see it … so obviously they are looking at the size, the physical size. So, they put there 1/7 and then put there 1/8, and they put it again … is it smaller, oh, it's swapped. But you can't swap it because it's already written there 1/7. Because it's not an open-ended… 1/7 is written… then they said, 'Oh no, cannot erase…' and then they panicked already… so what to do… Then later, a few minutes later… what can you do … then swapped, swapped, swapped back, but when it's swapped back, it's wrong, wrong, then stack, yeah, it's smaller… then how… then finally [Another student] said, 'take another fraction!'

Hannah's noticing contrasted with that of the other teachers in terms of the level of details given, and more importantly, how she linked her interpretations to specific instances and combined her understanding to generate a useful principle. Hannah felt that not all the students understood, and saw beyond the students' seemingly correct answers during the classroom discussion in her relatively detailed description of a particular student's thinking. She contended that students might not have seen fractions as a representation of a part–whole relationship without the physical manipulative. Moreover, Hannah also noted that the students might have problems seeing how the number of equal pieces needed to make up the whole could have been related to the size of the pieces. Therefore, even though students could have performed the task correctly, or have answered Zelina's questions correctly, they may not necessarily have understood the concept:

> They are able to do but may not be able to relate it back to the whole. Like why is the whole… I think it's logic and we assume that they know… that for the same whole, this one has many pieces and this one has lesser pieces, then this should be a smaller piece. Maybe this logic must come in at another platform… However, the children need some wait time, some thinking time, some verbalisation and articulation among themselves… You might want to hear… are they saying it?

It seemed probable that Hannah did not consider "chorus answers" to be indicative of students' ability to reason about the relative sizes of the unit fractions. Instead, her reflections highlighted the possibility that students may not understand the key idea of the lesson even though they had responded correctly to Zelina's questions. Using what she observed about the two students, Hannah analysed their thinking, and suggested that students need more opportunities to reason amongst themselves. Thus, Hannah's noticing here is at the *focused* level.

PSTs' focused noticing of the Three Points. Similarly, the PSTs were capable of focused noticing when they directed their attention to students' thinking. In a later Lesson Study session, the PSTs provided interpretive comments on students' thinking with detailed examples of students' performances and excerpts from their interactions in which students' thinking was probed. For example, in terms of the Key Point, Mary shared her observation about a student's interesting idea in composing and decomposing numbers for addition by referring to specific events and interactions. That is, during the Lesson Study, Mary demonstrated that she

attended to a student's work on the question asked by the teacher, how 7 + 5 equals 3 + 3 + 3 + 3 as follows:

> Three over three and then a line in the middle and then three over three and then at the bottom he [A student, Adam] had six and six and so I ask him how many do you have all together and he said 66, I was like does 3 + 3 + 3 + 3 = 6 [and] 6 and he was like no and I was like what are you supposed to do, oh six plus six is twelve so he got that concept and then when you [The teacher, Erin] went to how does 7 + 5 equal 3 + 3 + 3 + 3 then he preceded to say ok, you have 3 + 3, which is 6 and he says you take away, you borrow one from the three. ……I don't understand why he put like a one and he was like you take one away from here and he wrote that under or next to his three and he goes ok now you have two and three and that's five and I'm like but six plus five, six plus five is not twelve and he was like no you take the one you borrowed and you add it to the six and that makes seven and you have seven plus five equals twelve.

In this data excerpt, Mary first interpreted that the student tried to solve the given problem in this way: 3 + 3 + 3 + 3 = 6 [and] 6. However, the student's solution did not make sense to Mary and she asked a question for clarification. By attending to the student's explanation, Mary was able to interpret the student's strategy to compose the two 3s and decompose one 3 into 1 and 2 to make 7 + 5. This shows that Mary had initially attended to the student's idea and understood it before she responded in a way that probed the student's idea. By investigating one student's reasoning in detail, Mary demonstrated the three processes of noticing with regard to the Key Point.

Also, in terms of Difficult Point, Mary attended to students' struggle with a question (e.g. if we have 12 kids and 24 cubes, how many cubes would each student get?), which was given after addressing some addition strategies to figure out the total number of students when there are 12 girls and 12 boys in a classroom.

> Well, like it's a lot of memorization because when you did how many, there's twelve of us and you have twenty four blocks how many do each of them get, they were both like add them it was like everybody would get two and then once you broke it down to if there's twenty four kids in the whole class how many will they get so Gabriella's well everybody would get one but she was like if you gave everybody three not everybody would have at least one so like they were going off of that and so then when I was talking to Adam about 24 all together … cause he was confused in the beginning … now there's 24 kids in the classroom and he's like 12 plus 12 is 24 …, he kept saying that, like he knew that was the it … how many will each student get and … he got it but the other two didn't get it but they all understood that 12 plus 12 equals 24.

When Mary changed the question slightly ("how many cubes would each student get if there are 12 kids and 24 cubes?"), another student, Gabriella, did not understand the reasoning behind her own solution although she gave the correct answer to the first question. Mary's response to the student's reasoning demonstrated that she tried to take on the *student lens* by relying on evidence to judge whether students clearly understood the content. Also, Mary highlighted that Adam, a very advanced student in the class, took some time to get the question while two other students did not get it although they understood 12 + 12 = 24.

Mary's remarks showed that she referred to specific evidence and instances of interaction among multiple students and elaborated them in order to provide interpretive comments on students' mathematical thinking, indicative of *focused* noticing. However, Mary did not propose any alternative pedagogical solutions to address students' challenges in learning the Key Point, which would have brought her noticing to the *extended* level.

Discussion

This study supports findings by Fernandez et al. (2003) that adopting the three critical lenses in Lesson Study is not trivial, and extends the findings by Star et al. (2011) to indicate that both experienced and beginning teachers are also not necessarily effective observers of mathematics lessons. More importantly, although Takahashi and McDougal (2016) highlight the key features of Lesson Study that may maximize the impact of Lesson Study, we have demonstrated that *what* and *how* teachers notice is critical for the benefits to be fully realized. Our findings suggest that teachers' higher levels of noticing are usually accompanied by their attention and interpretation of mathematically significant aspects of teaching and learning.

Given the wide spectrum of things to observe, it is not surprising that both groups of teachers may focus on aspects that do little to enhance their understanding of students' thinking. Without an explicit guiding focus, teachers noticed a wide variety of events and details, both relevant and irrelevant to the tasks of Lesson Study (Star et al., 2011). A vague focus, such as student mathematical thinking, also seems to be too broad for teachers to maintain their attention on noteworthy details during Lesson Study. Instead, a sharper set of focal points, such as the *Three Points*, may be more useful for teachers to guide their noticing as suggested by the findings of this research.

This study suggests that Lesson Study can be a possible means to develop noticing expertise. Even without any other professional development activities to hone teachers' noticing, both groups of teachers demonstrated some instances of higher level noticing. However, both groups did not demonstrate an *extended* level of noticing. That is, they did not focus on how their observations and interpretations were related to the instructional decisions that could have potentially enhanced students' mathematical thinking. This highlights that more attention needs to be placed on supporting fruitful teachers' noticing during Lesson Study.

A possible way to do this would be to incorporate the processes of noticing within the Lesson Study protocols. This could come in the form of questions or prompts or lesson plan templates to direct teachers' focusing. Another possible strategy is to use frameworks such as van Es's (2011) framework for noticing student thinking to guide teacher focusing. Our findings also suggest a synergistic relationship between developing teachers' noticing expertise and developing teachers' abilities to adopt the critical three lenses. If teachers noticed

mathematically significant details during Lesson Study, they are more likely to make instructional decisions that promote students' thinking.

On the other hand, Lesson Study, with a special focus on studying lesson materials (Takahashi & McDougal, 2016), can offer opportunities for teachers to develop the eyes to see, the ears to hear, and the mind to think about teaching and learning. More importantly, the evidence from our study reveals the critical role of noticing in learning from Lesson Study. Although our study involved only two small groups of teachers, and the findings are limited by our methodological approach, this research warrants a need to examine, more closely, the role which teacher noticing may play in learning from Lesson Study, as well as how Lesson Study can be used to develop noticing expertise.

References

Ainley, J., & Luntley, M. (2007). The role of attention in expert classroom practice. *Journal of Mathematics Teacher Education, 10,* 3–22.

Askew, M. (2013). Big Ideas in primary mathematics: Issues and directions. *Perspectives in Education, 31*(3), 5–18.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). San Francisco: Jossey-Bass.

Berliner, D. C. (2001). Learning about and learning from expert teachers. *International Journal of Educational Research, 35*(5), 463–482.

Bryk, A. S. (2009). Support a science of performance improvement. *The Phi Delta Kappan, 90*(8), 597–600.

Choy, B. H. (2014). Teachers' productive mathematical noticing during lesson preparation. In C. Nicol, P. Liljedahl, S. Oesterle, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 2, pp. 297–304). Vancouver, Canada: PME.

Colestock, A. A., & Sherin, M. G. (2009). Teachers' sense making strategies while watching video of mathematics instruction. *Journal of Technology and Teacher Education, 17*(1), 7–29.

Erickson, F. (2011). On noticing teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 17–34). New York: Routledge.

Fernandez, C., Cannon, J., & Chokshi, S. (2003). A US–Japan Lesson Study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education, 19*(2), 171–185. doi:10.1016/s0742-051x(02)00102-6

Fernandez, C., & Yoshida, M. (2004). *Lesson Study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, NJ: Lawrence Erlbaum.

Goldsmith, L. T., & Seago, N. (2013). *Examining mathematics practice through classroom artifacts*. Upper Saddle River, New Jersey: Pearson.

Goodwin, C. (1994). Professional vision. *American Anthropologist, 96*(3), 606–633.

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An "experimental" model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 6,* 201–222.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Kersting, N. (2008). Using video clips of mathematics classroom instruction as item prompts to measure teachers' knowledge of teaching mathematics. *Educational and Psychological Measurement, 68*(5), 845–861.

Lewis, C., Friedkin, S., Baker, E., & Perry, R. (2011). Learning from the key tasks of Lesson Study. In O. Zaslavsky & P. Sullivan (Eds.), *Constructing knowledge for teaching secondary mathematics* (pp. 161–176). US: Springer.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: a theoretical model and North American case. *Journal of Mathematics Teacher Education, 12* (4), 285–304. doi:10.1007/s10857-009-9102-7

Loughran, J. (2009). Is teaching a discipline? Implications for teaching and teacher education. *Teachers and Teaching, 15*(2), 189–203. doi:10.1080/13540600902875290

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: RoutledgeFalmer.

Mason, J. (2011). Noticing: Roots and branches. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 35–50). New York: Routledge.

Miller, K. F. (2011). Situation awareness in teaching: What educators can learn from video-based research in other fields. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 51–65). New York: Routledge.

Murata, A., Bofferding, L., Pothen, B. E., Taylor, M. W., & Wischnia, S. (2012). Making connections among student learning, content, and teaching: Teacher talk paths in elementary mathematics Lesson Study. *Journal for Research in Mathematics Education, 43*(5), 616–650.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

National Research Council. (2005).*Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Santagata, R. (2011). A framework for analysing and improving lessons. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 152–168). New York: Routledge.

Schifter, D. (2001). Learning to see the invisible: What skills and knowledge are needed to engage with students' mathematical ideas? In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 109–134). Mahwah, NJ: Lawrence Erlbaum Associate Inc.

Schoenfeld, A. H., & Kilpatrick, J. (2008). Toward a theory of proficiency in teaching mathematics. In T. S. E. Wood & D. V. E. Tirosh (Eds.), *International handbook of mathematics teacher education* (Vol. 2, pp. 1–35). Sense Publishers.

Schön, D. A. (1991). *The reflective practitioner: How professionals think in action*. Great Britain: Ashgate Publishing Limited.

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). Situating the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 1–13). New York: Routledge.

Sherin, M. G., & van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education, 60*(1), 20–37.

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.

Star, J. R., Lynch, K., & Perova, N. (2011). Using video to improve preservice mathematics teachers' abilities to attend to classroom features. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 117–133). New York: Routledge.

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*(2), 107–125. doi:10.1007/s10857-007-9063-7

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

Stockero, S. L. (2008). Using a video-based curriculum to develop a reflective stance in prospective mathematics teachers. *Journal of Mathematics Teacher Education, 11*(5), 373–394. doi:10.1007/s10857-008-9079-7

Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: maximizing the impact of lesson study. *ZDM*. doi:10.1007/s11858-015-0752-x

van Es, E. (2011). A framework for learning to notice students' thinking. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 134–151). New York: Routledge.

Yang, Y., & Ricks, T. E. (2012). How crucial incidents analysis support Chinese Lesson Study. *International Journal for Lesson and Learning Studies, 1*(1), 41–48. doi:10.1108/20468251211179696

Yang, Y., & Ricks, T. E. (2013). Chinese lesson study: Developing classroom instruction through collaborations in school-based teaching research group activities. In Y. Li & R. Huang (Eds.), *How Chinese teach mathematics and improve teaching* (pp. 51–65). New York: Routledge.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks: SAGE Publications.