A Case Study of Middle School Teachers' Noticing During Modeling with Mathematics Tasks

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Abstract Schoenfeld (Mathematics teacher noticing: Seeing through teachers' eyes. Routledge, New York, pp. 223–238, 2011) wondered about the transferability of teacher noticing across contexts (e.g., different grade levels and task types). This chapter focuses on middle school teachers' noticing during instruction that promoted modeling with mathematics, which is one of eight Standards for Mathematical Practice (SMPs) found in the Common Core State Standards (CCSSI in Common Core State Standards for Mathematics. Author, Washington, DC, 2010). A case study approach was used to explore middle school teachers' noticing during instruction promoting modeling with mathematics. This study focuses on two middle school teachers who enacted modeling-focused lessons. Lessons, videos, and interview data were analyzed using inductive analysis (Hatch in Doing qualitative research in education settings. State University of New York Press, Albany, NY, 2002). We drew two impressions from the data. The first was that teachers' noticing focused on fostering students' use of multiple representations. The second result was that teachers' noticing was framed in ways to assist with making sense of a modeling task or its solution. We connect these results to transferability of teaching noticing, specifically to instruction promoting modeling with mathematics.

Keywords Middle school · Problem-solving · Standards for mathematical practice · Common core state standards-Mathematics · Representations

Teachers manage a number of instructional elements everyday including mathematical tasks, mathematical discourse and interactions during those tasks, and the learning environment (National Council of Teachers of Mathematics [NCTM], 2007). They typically are making choices to attend to certain instructional moments, interpreting those moments, and deciding how to proceed. These

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formative assessments are "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes" (NCTM, 1995, p. 3). Formative assessment supports learning by allowing teachers opportunities to gauge the degree to which students are meeting desired instructional goals and make adjustments (Clark, 2012; William, 2007). Evidence from research on teachers' assessment suggests that teachers ought to pay close attention to students' understanding, or lack thereof, so they can best attend to the students' learning needs and support learning outcomes (Clark, 2012; William, 2007).

Mathematics teachers gather much evidence during instruction; however, the process of what teachers attend to, how they interpret that information, and what they decide to do have not been clear (Jacobs, Lamb, Philipp, 2010; Schack, Fisher, Thomas, Eisenhardt, Tassell, & Yoder, 2013). This process is *teacher noticing*, or put another way "the processes through which teachers manage the 'blooming, buzzing confusion of sensory data' with which they are faced," (Sherin, Jacobs, & Philipp, 2011, p. 5). For example, Jacobs and colleagues (2010) investigated teachers' noticing of children's mathematical thinking as an aim to explore ways in which they might respond to a child's just-in-time thinking. A cross-sectional study of 131 prospective and practicing teachers, all who had differing amounts of teaching experience, was observed. The researchers concluded that there were various forms of teacher noticing, depending on teaching experience. Teaching experience is one of many variables that influence teachers' noticing; others include lesson objectives and grade levels of the teacher (Schack et al., 2013; Sherin et al., 2011; Thomas, Eisenhardt, Fisher, Schack, Tassell, & Yoder, 2015). The focus of this chapter is to explore teacher noticing in a particular context and add to the emergent foundation of knowledge in this area. Ultimately, mathematics education researchers may be able to unpack instructional decisions better through such a focus (Schack et al., 2013; Sherin & Star, 2011; Thomas et al., 2015). We frame our work around mathematics instruction in the Common Core era, specifically focusing on teachers' promotion of the Standards for Mathematical Practice (SMPs; Common Core State Standards Initiative [CCSSI, 2010]).

Related Literature

Standard for Mathematical Practice: Modeling with Mathematics

The SMPs describe a set of mathematical behaviors and habits for students to experience (Table 1).

Table 1
Standards for mathematical practice

Standard for mathematical practice #	Title
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for regularity in repeated reasoning

Teachers in states that have adopted the SMPs are expected to read and understand them, then design instruction promoting them. The SMPs and content standards serve as the expectations of what students should learn and do while engaged in K-12 classroom mathematics teaching (CCSSI, 2010). Modeling with mathematics, the fourth SMP, states

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace… They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. (CCSSI, 2010, p. 7)

Instruction promoting modeling with mathematics can "engage [problem solvers] in a process of interpreting mathematical situations" (Zawojewski, 2010, p. 238). Thus, SMP 4 is operationalized as requiring students to apply real-world knowledge, make assumptions and approximations, and continuously evaluate the reasonableness of a result (Bostic, 2015). Teachers promoting modeling with mathematics (SMP 4) are expected to use mathematical representations appropriate for a lesson, encourage students to use a variety of developmentally and mathematically appropriate models while problem-solving, and continuously remind students to revise their models (Fennell, Kobett, & Wray, 2013). Students' ability to strategically employ multiple representations (e.g., written symbols such as variables, expressions, and equations; tables; diagrams and pictures, concrete manipulatives, and verbal language) during problem-solving is linked to their problem-solving performance (Yee & Bostic, 2014). Broadly speaking, past literature has framed representations as symbolic (i.e., written symbols) and nonsymbolic (i.e., all others) (see Yee & Bostic, 2014 for a review). Translating between representations (e.g., verbal language to a variable) is embodied within this notion because effective and efficient problem-solving often requires navigating between various representations (Yee & Bostic, 2014). It is critical that teachers encourage students to develop flexibility with a variety of representations during problem-solving, which includes tasks promoting modeling with mathematics. The present research provides some insight into what teachers notice during instruction promoting modeling with mathematics and how their instructional decisions aim to benefit students' learning.

Situating the Study in Teacher Noticing

Teacher noticing is a means for teachers to engage in formative assessment practices because "teachers must recognize students' thinking…as it happens and make…instructional choices in response to what they notice" (Luna, Russ, & Colestock, 2009, p. 1). Erickson (2011) argues that teachers tend to engage in noticing as a means to make decisions to benefit students' learning and/or instruction. Teacher noticing includes two key processes: "attending to particular events…[and] making sense of events in an instructional setting" (Sherin et al., 2011, p. 5). Sense making for our study includes interpreting and deciding on a response (Sherin et al., 2011, p. 5).

We approach teacher noticing as a two-stage process: Attending encompasses the first stage then interpreting and deciding characterize the second stage. *Attending* is when the teacher gathers evidence of a student's thinking as it happens in the moment (Jacobs et al., 2010; Schack et al., 2013; Thomas et al., 2015). This may include how a child might behave or how he/she uses specific tools for an activity. Attending leads to sense making, which includes interpreting and deciding. *Interpreting* is when a teacher examines the gathered evidence from the attending phase and "coordinat[es] the observed actions with what is known about…development in a particular area" (Thomas et al., 2015, p. 296). The *deciding* phase is when a teacher collects (considers) the information/observations gathered during the earlier phases and makes an informed decision on how to act (Jacobs et al., 2010; Schack et al., 2013; Thomas et al., 2015). There are multiple decisions a teacher could make using the evidence. For instance, imagine a teacher noticing a small group of students problem-solving and expressing difficulty. The teacher's noticing may lead him/her to re-teach the material in a different manner. Or, the teacher may dismiss the students' difficulties and move on with the lesson. An observer of this situation may wonder what the teacher attended to and how they interpreted the data to make a decision on how to proceed. Moreover, it is uncertain whether teacher noticing might be similar or different across various contexts in the Common Core era. We drew upon this uncertainty as a way to build the foundation of teacher noticing literature.

Schoenfeld (2011) summarizes and pushes the field of teacher noticing forward with a couple thoughts and wonderings. First, teachers need robust pedagogical, mathematical, and mathematics pedagogy knowledge to teach students in the Common Core era because the content and practice standards are not necessarily easy to discern upon inspection (Bostic & Matney, 2014). Second, and germane to this chapter, what does teachers' noticing look like in various contexts? The field of teacher noticing must begin to investigate the transferability of teacher noticing across contexts (e.g., different grade levels, task types, instructional foci; Schoenfeld, 2011). To date, no published study has explored teacher noticing through the lens of instruction promoting the SMPs, much less one or more SMPs. Research on instruction promoting modeling suggests that it is unique from non-modeling instruction (Lesh & Zawojewski, 2007). Moreover, instruction promoting the fourth SMP (modeling with mathematics) tends to appear different from instruction highlighting other SMPs (Bostic, 2015). The present study explores middle school teachers' instruction to better understand teachers' noticing within this context, and respond to Schoenfeld's wondering.

Synthesis

Drawing on a teacher noticing framework, this study takes up Schoenfeld's (2011) wondering about teacher noticing within various contexts (e.g., types of tasks and grade levels). The purpose of this study was to examine how middle school mathematics teachers engage in teacher noticing during instruction that supports modeling with mathematics. Our research question is: What do middle school teachers notice during mathematics instruction that promotes modeling with mathematics?

Method

Methodology

The methodology used for this research is a case study, which "investigates a temporary phenomenon in its real-world context" (Yin, 2014, p. 237). It involves analyzing data from one or more cases to confirm a specific phenomenon happening within those cases (Yin, 2014). This chosen method of research is appropriate as case studies allow researchers the ability to explore a novel phenomenon.

Participants

One male and one female teacher are the focus of the present study. They are identified by pseudonyms, Mr. Brown and Mrs. Zelda. Mr. Brown and Mrs. Zelda were purposefully selected from the larger sample of 38 teachers because they successfully enacted tasks promoting SMP 4 (i.e., modeling with mathematics). Initially, we considered data for this study from 38 middle school teachers (i.e., grades six–eight) who volunteered to take part in yearlong professional development (PD) in Ohio. The aim of the PD was to foster sense making of the Common Core State Standards for Mathematics, particularly the Standards for Mathematical Practice (SMPs).

Mr. Brown was a seventh-grade teacher with 10 years teaching experience. He held a Masters in Education and self-identified himself as Caucasian. Mr. Brown's district is suburban with a low student poverty rate (Ohio Department of Education, 2015). Mrs. Zelda was a seventh-grade teacher with 19 years teaching experience. She earned her Masters in Education and also self-identified herself as Caucasian. Mrs. Zelda's district is a small town with a high student poverty rate (Ohio Department of Education, 2015).

Data Collection

A goal of this study was to closely examine the noticing of middle school teachers who promoted modeling with mathematics during their instruction. To meet that goal, there were two parts to data collection: video and lessons followed by participant interviews.

Videos and lessons. Teachers submitted lesson plans and videos of instruction after experiencing the PD. A team of mathematics education researchers reviewed the lessons for intended foci and later coded videos using a SMP look-for protocol (Fennell et al., 2013). The team examined the lessons and sorted them based on teachers' stated goals and the SMPs they intended to address during instruction. The SMP look-for protocol suggests observable behaviors that a teacher might enact during instruction. There are three statements specific to teachers' promotion of SMP 4: Modeling with mathematics: (1) Use mathematical models appropriate for the focus of the lesson; (2) Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids); (3) Remind students that a mathematical model used to represent a problem's solution is a work in progress, and may be revised as needed (Fennell et al., 2013, p. 12). A randomly selected sample of 20% of the data collected from the PD was coded independently by members of the research team to determine interrater agreement. They agreed 96% of the time, which exceeds the minimum threshold (90%; Ary, Cheser-Jacobs, Sorenson, & Razavieh, 2009).

It was minimally sufficient to say teachers' instruction promoted modeling with mathematics if there was evidence for one of the three statements. Because the nature of this study is geared toward teacher noticing practices within modeling with mathematics instruction, we examined only those who displayed more than minimally sufficient evidence. That is, this purposeful sample is composed of teachers who had two or more indicators for SMP 4 from the SMPs look-for protocol. Mr. Brown and Mrs. Zelda's lessons and videos demonstrated (a) an intentional focus on modeling with mathematics and (b) evidence for at least two of the three statements for modeling with mathematics. These two teachers were then sent requests for follow-up interviews regarding their noticing.

Interviews. The goal of the interview, much like in past teacher noticing research (e.g., Sherin, Russ, & Colestock, 2011) is to make sense of teachers noticing moments through their reflection on unique instructional moments. Each interview lasted approximately 60 minutes and was conducted at the participant's school. The teacher watched his/her instruction on a laptop computer while being filmed by a video recorder. A video camera was placed so that it captured teachers' verbal statements and nonverbal cues during the interview. The interviewer paused the video of the teacher on the laptop for one of two occasions. The first occasion was when a teacher asked the interviewer to stop the video to discuss a specific teacher noticing moment. The second occasion was when the interviewer observed the teacher attending to a unique instructional event during the video of the lesson. For example, the video was paused often when the teacher stepped toward a student who shared a misconception. After a pause for either occasion, the interviewer posed a series of questions with regards to the teacher noticing process of a specific event. The first question was: What made you attend to this specific event? The second prompt was: Describe your thought process during the student–teacher interaction. The third question was: What did you interpret from this event? The fourth and final question was: What did you decide to do after you interpreted what was going on with the situation? Participants were encouraged to share any remaining thoughts at the end of the interview.

Data Analysis

The focus of the analysis was on data collected during the interview. Mr. Brown and Mrs. Zelda's interviews were analyzed using thematic analysis (Hatch, 2002). The goal of thematic analysis is to generate plausible themes based on a plethora of evidence and paucity of counter examples (Hatch, 2002). An analytical approach such as thematic analysis is appropriate in case study work because it allows the researcher to describe and explore a phenomenon of interest (Yin, 2014).

The data analysis was performed in seven steps. First, each interview was viewed in its entirety. During the second step, memos were made when teachers asked to pause the video to discuss their noticing during specific situations. Third, notes were made of specific statements spoken by the teacher during the teaching to find general impressions across the participants. Fourth, general impressions that were common between Mr. Brown and Mrs. Zelda were collapsed into initial, tentative themes. Fifth, evidence supporting (or not supporting) was sought within the interviews, videos of teachers' instruction, and the lessons. Sixth, the interviews were watched a second time to explore the degree to which impressions matched the data. Finally, impressions were synthesized to become themes describing middle grades mathematics teachers' noticing during tasks that promote modeling with mathematics.

Results

We present two themes arising from the data to answer the question: What do middle school teachers notice during mathematics instruction that promotes modeling with mathematics? The first theme was that Mr. Brown and Mrs. Zelda noticed students' struggles with structure found within the tasks. The second theme was that teachers in this case study noticed students' engagement (and struggles with) in translating between representations while problem-solving.

Theme 1: Structure in Mathematics

Each teacher who conducted a modeling with mathematics task shared how students struggled with the inherent mathematical structure within the assigned problem. The Standard of Mathematical Practice (SMP) #7 (CCSSI, 2010) suggests that students should look for patterns and specific structure within a problem. The SMP look-for protocol states three aspects indicative of teachers fostering this SMP.

> (a) Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains. (b) Recognize that the quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson. (c) Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways (e.g. $76 = (7 \times 10) + 6$); discussing types of quadrilaterals, etc. (Fennell et al., 2013, p. 13)

Mr. Brown and Mrs. Zelda engaged students in exploring mathematics to deepen their understanding and promoted modeling with mathematics.

Mr. Brown implemented a modeling with mathematics task focusing on using the slope-intercept equation to understand payment for various jobs. The context of his mathematical task was that an individual made \$10 per hour for their job and received a signing bonus of \$20 (Figure 1). Students determined how much money the individual earned given a number of hours (i.e., represented as x in the equation). The goal was to create a suitable mathematical model for any number of hours and perhaps, be able to transfer this model to situations with different hourly rates and signing bonuses. Students selected and used input/output tables to record data. Most plotted their data from the table onto a coordinate plane. Students sought to analyze the relationship between hours worked and money earned through coordinated efforts with multiple representations.

Description: You are getting your first job delivering papers. The job requires you to deliver papers in town according to the addresses on the list. For performing this job, you will be receiving $10 per hour. Because you have chosen to accept this job, you are receiving a one-time, $20 signing bonus. Your job is to figure out how much money you would make when working "x" amount of hours. Fill in the table below to display your findings. After you fill out the table, graph your findings accordingly.

Input (X)	**Output (Y)**

What is the equation for this task?

How much money do you have when you start?

Figure 1. A portion of the task shared during Mr. Brown's seventh-grade instruction.

The first situation that arose within the lesson where Mr. Brown attended to a student's thinking was approximately 14 minutes into the lesson. One student, instead of leaving the equation in the form y = 10x + 20, added 10x + 20 to make 30x. The misconception that the student had was to collect unlike terms, therefore misunderstanding a key mathematical property: collecting like terms to simplify expressions. Mr. Brown asked to pause the video and talk about attending to students' work on the task. Just a moment before he asked to pause the video, Mr. Brown approached a student and asked him about manipulating an algebraic expression. Mr. Brown said that, "…what this student was doing was he was misunderstanding the equation. What I noticed he was doing was adding 10x + 20 to get 30x thinking he was going to get the same answer as 10x +20." He interpreted that this student and others seemed to misunderstand how to collect terms within an expression. To that end, he decided to manage this misconception by intervening.

> I [Mr. Brown] had to intervene and point out to the student that you can't add unlike terms. So I had to tell him you have to keep it in 10x +20 form. I showed him the difference if I were to keep his answer of 30x versus keeping it as 10x +20. We plugged in values for x and got different results, thus coming to the conclusion that when you add 10x + 20 to equal 30x, it changes the whole problem.

This instance of Mr. Brown's noticing during the task was consistent throughout the interview. He felt that students ought to show flexibility with the mathematical structure found within mathematical models. Mrs. Zelda's lesson coincides with students not understanding the structure within problems shared during a lesson.

Mrs. Zelda conducted a lesson promoting modeling with mathematics that involved students creating equations for a bike tour that includes profit, revenue, and expenses. Bike tours are a common business for a nearby vacation spot where many students' families work and visit. Figure 2 shows the table and description students were given.

Description: Rider Inc. is a business that rents out bikes and camp space. They have some data that shows them the dollar value of a single customer, all the way to 3 customers. The manager has asked you to construct an equation that works for "x" amount of customers. Please finish the table for up to 6 customers, then construct an individual equation for each: profit, revenue, and total expenses that would work for all amounts of customers.

Customers	Revenue	Bike Rental	Food and Camp Costs	Total Expenses	Profit
1	$350	$30	$125		
2	$700	$60	$250		
3	$1050	$90	$375		

Figure 2. The table and description used for Mrs. Zelda's task in her lesson.

Mrs. Zelda pointed out a particular situation within her lesson while watching the video of her lesson. About halfway through the lesson she recalled students started to ask more questions when asked to construct a general equation for profit, total expenses, and revenue. She shared that year after year, students tended to struggle with this because they had a hard time thinking abstractly and generally about equations, as opposed to concrete thinking. Mrs. Zelda paused the video to discuss when she saw students struggling to construct equations from the table. During that moment, she chose to guide students toward constructing an equation, that is, connecting representations (i.e., tables and equations) using mathematical operators. She claimed the students knew how to calculate the total expenses and profit but they had difficulty constructing an equation for the situation. She clarified this further during the interview through a role-play. She role-played the teacher and students' mathematical actions and statements.

Mrs. Zelda: A lot of times especially with linear equations, when they are making equations or putting a situation into an abstract equation, there sometimes is a disconnect. They know what to do, but they can't put letters and numbers and operations together. So I always pull back. … *[Begins role-play]*

Mrs. Zelda: [Teacher]: So if there is one customer and they are going to an amusement park, how much are they going to pay? $40 bucks. What if they bring their boyfriend or girlfriend?'

Mrs. Zelda: [Student] $80 bucks.

Teacher: Ok what if they bring, boyfriend and two other friends? Well then they figure it out. What did you just do?

Student: Well I multiplied.

Teacher: What did you multiply?

Student: I multiplied 40 times however many people were going.

Teacher: Ok, so how can I write that so if I want 120 people going with me?

Student: Well I multiply 40 times 120.

Teacher: What number stayed the same? What number changed? The number that changed, give me a letter that works with that. Whether it's friends, customers, or jellybeans, or whatever. *[Ends role-play]*

Mrs. Zelda: Then they [students] are like "oh." So then what does that tell you when I take 40 times the number of people? The cost of the admission. So sometimes you have to break it down and then go for simple, and then you can get to where they are getting now.

This role-play is evidence that Mrs. Zelda noticed an issue and decided to guide students in their thinking on how to construct an equation. Equations have a unique yet specific mathematical structure that her students seemed to misunderstand. She employed a simple, relatable experience when talking with students about the price to go to an amusement park. Then she built the problem into more complex situations when she saw more students expressing understanding, eventually constructing an equation with a coefficient and variable. She noticed that students were able to carry out procedures to solve the problem but had difficulty expressing the problem situation as an equation using variables. That is, students struggled to move from the situation embedded within the task to creating a table and ultimately to an equation that characterized the structure inherent within the problem.

In conclusion, Mrs. Zelda attended to students struggling with abstracting an equation for the whole problem and constructing the formula using variables. She interpreted that students knew how to find the different values for the table, but could not provide the general equation for profit, revenue, or total expenses. Mrs. Zelda decided to provide her students with a simpler example of going to an amusement park and bringing friends and/or family. This decision guided students to conceiving how they might construct a formula for the particular problem within the lesson. In Mr. Brown's and Mrs. Zelda's cases, it is clear from these teachers' voices that looking for and making use of structure within tasks addressing SMP 4 is important.

Theme 2: Translating Between Representations

A second theme drawn from the interviews was that teachers attended to students' facility translating between representations. As shared in the previous theme, Mr. Brown and Mrs. Zelda attended to situations when students demonstrated difficulty connecting the meaning of situations to terms within equations. They used multiple representations and aimed to assist students to problem-solving using one representation (e.g., graph or table) then translate to another representation (i.e., expression or equation). Concomitantly, teachers felt the need to foster students' sense making of mathematical structure through various ways. Mr. Brown explicitly told and showed students the correct way to manipulate one mathematical structure while Mrs. Zelda scaffolded students' thinking through different examples that might foster greater connections and correct the misconception. Thus, there are natural connections between mathematical structure and representation usage.

Mathematical language (i.e., verbal representations) was a key part of students' struggles with translating between representations and is explored further.

In Mr. Brown's and Mrs. Zelda's lessons, students misunderstood language within the problem or they did not know what certain words meant. For example, Mrs. Zelda shared several thoughts related to the importance of mathematical language during tasks addressing modeling with mathematics. A focus of Mrs. Zelda's lesson was to determine values for variables in an equation using a table. She noticed through some of her questioning that students expressed confused facial gestures when talking about words such as revenue, expenses, and profit. She shared the following during her interview while viewing her lesson and discussing one student–teacher interaction.

> I think they didn't understand the definition of revenue. Therefore with taking the information they had, I guided them through that, and showing them 'this [points to the problem then moves her finger to a specific term] is revenue'. I probably should have said 'What is revenue?' so the definition and vocabulary was there. So I think that's what it was. They didn't get what revenue was. They were looking at profit. They don't know what profit is. They've heard about it, but they don't know what it is, along with expenses and revenue. So it was just a quick lesson to show linear relationships but then they [textbook?] were throwing all this other jargon and vocabulary in. So it sounded like they [students] just didn't understand the vocabulary.

Mrs. Zelda attended to students who seemed confused about the problem and were not progressing in their problem solving. She shared that her interpretation was that they consistently did not understand the problem's language, thus they were uncertain of the problem's goal. This led to her decision to guide students to better understand the problem's language so they might translate verbal representations into symbolic forms (e.g., variables and equations). This happened several times during the lesson and Mrs. Zelda commented on it frequently during the interview. Mr. Brown shared a similar sentiment during his interview when watching his interactions with students.

Mr. Brown expressed that he also tended to focus on mathematical language as a noticing during tasks promoting modeling with mathematics.

> I [Mr. Brown] tend to notice that students have a hard time understanding the variables and constants [in equations]. Like when I use this equation in a story problem context, I find students have a hard time understanding 'mx' means slope times a number. Then they sometimes forget to add the constant at the end, which is what happened in this example. So really the students have trouble understanding how to read the problem.

To summarize, he attended to a student who had a question about completing the problem because of mathematical language written in symbolic terms. He noticed that this student was not adding the constant to each input. Next, he interpreted that this student was confused about what a constant is and what to do with it. He also saw that other students were making the same mistake. After interpreting that students did not know what to do with the constant, or know what it meant within the context of the equation, Mr. Brown decided to address the whole class and remind students to add the constant to each input. Students needed assistance making sense of the mathematical language, as written in abstract terms (i.e., symbolic representation as a variable) rather than in Mrs. Zelda's case that was represented as words (i.e., nonsymbolic representation as a word). This is a struggle for the teacher and students related to representational translations.

Further evidence of Mr. Brown's noticing students' language (verbal representation) came from his response when asked what he finds himself attending to most often when enacting tasks aimed at fostering modeling with mathematics.

> I [Mr. Brown] often find myself having to go over [the whole problem] with the whole class and explain what the problem is asking …. Students tend to forget how to *read* [emphasis added] the problem, so if I find that multiple students aren't getting it, chances are a majority aren't getting it, which means I should address the class as a whole [about the words in problem].

Mr. Brown shared he had attended to enough instructional situations to make sense of the feeling when students may not understand the words within a task associated with SMP 4. Taking instructional time to assist students with making sense of words (i.e., verbal representations) assuredly helped students' understanding of the situational context in the problem, which allowed them to create more appropriate mathematical models and ultimately, generate viable mathematical models (solutions). A related situation arose during Mrs. Zelda's interview.

Mrs. Zelda attended to a frustrated student during her instruction. The student was perplexed by the task and interjected his question while Mrs. Zelda was speaking. Mrs. Zelda shared that her experience was that if this particular student was confused then it was typical for others to feel similarly. Mrs. Zelda's response during the interview highlights her pathway through the noticing framework:

> …he's [the student] forcing me to re-direct and back up and say 'let's look at the revenue, and the expense, and the profit to find a pattern. What's going on?' Then he [the student] took it to "well now we have to look at the pattern." [Mrs. Zelda asks] 'How did you get that pattern? What do you do? Ok do it.' So he, in his mind, was kind of thinking through the process out loud. Which sometimes they have to do because sometimes when going around I see the blank look of "I don't get it." [Mrs. Zelda] 'Ok what don't you get? You have to look at it. What do you know? How do you get the numbers, go back and forth. Then show me what you do next.' And usually if they *verbalize it out loud* emphasis added], it's kind of like a certification that I [the student(s)] am doing it right. And that's what I find a lot of times is they [the students] just need to verbally say it out loud…

Here again, students struggled but sharing ideas aloud and making sense of the words with teacher assistance supported students' problem solving. Mrs. Zelda attended to one student's difficulty with the language in the task so they might translate this word (verbal representation) into a variable (symbolic representation). She perceived this student as a voice for the class hence interpreting that multiple students were also confused. She decided to encourage him and others to use their own words and verbally problem solve. That is, use verbal language as a means to translate between various representations (e.g., symbols and tables) in the task and their problem-solving strategies. It is clear that Mrs. Zelda noticed the representations embedded in tasks promoting modeling with mathematics, especially her students' language. This assisted her students to connect the language in the problem, the words students chose to explain their thinking and/or questions during problem solving, and their symbolic representations.

Discussion

The goal of this research was to identify what middle school teachers notice within instruction promoting modeling with mathematics. Both Mr. Brown's and Mrs. Zelda's instructions were analyzed using a teacher noticing framework to answer the question: What do middle school teachers notice during mathematics instruction that promotes modeling with mathematics?

It was evident that instruction promoting modeling (SMP 4) is qualitatively correlated with a focus on mathematical structure (SMP 7). SMP 7 (Look for and make use of structure) states that students "can step back for an overview and shift perspective" (CCSSI, 2010, p. 8). Relatedly, students engaged in this standard are expected to shift their representational thinking while doing mathematics. They attended to students' struggles with translating between representations, particularly moving from mathematical language embedded in the problem (verbal representation) to symbolic forms. If students do not understand the language embedded within modeling with mathematics tasks then they may not necessarily understand the problem much less be able to solve it (Yee & Bostic, 2014). Tasks fostering modeling with mathematics include several cognitive facets including reading text and other mathematical representations, connecting those representations, and drawing upon them during further problem-solving (Bostic, 2015). It is no surprise that Mr. Brown's and Mrs. Zelda's noticing was focused on ways to encourage students' flexibility with representations during modeling with mathematics instruction as a means to help students arrive at a reasonable result. Instruction promoting modeling with mathematics appears to have a unique facet not raised in prior teacher noticing literature: Mr. Brown and Mrs. Zelda notice how students engage with mathematical structure and translate between representations.

These findings supplement the burgeoning research on teacher noticing with evidence within a specific context. Schoenfeld (2011) called for teacher noticing to address specific contexts, specifically, teacher noticing across instructional contexts fostering modeling with mathematics. We considered numerous variables including years of experience (ten or more), grade levels (middle school), type of task (promoting SMP 4), education completed (M.Ed), and district-level differences (suburban with low poverty rate compared to rural with high poverty rate) and were able to draw out themes across the two cases. Seasoned teachers who are knowledgeable about instruction promoting modeling with mathematics (and other SMPs) are focused on supporting students to look for structure within these tasks and translating between representations. Drawing across these two cases (but not generalizing to the greater population), we conjecture that there are similarities across contexts focused on modeling with mathematics.

Limitations and Future Research

There were a couple limitations to this study. One limitation was that this case study purposefully focused on two teachers' instruction hence results cannot be generalized to the greater population of middle school teachers. Further studies might examine the noticing patterns of more middle school teachers during modeling with mathematics instruction. Given that Mr. Brown and Mrs. Zelda were knowledgeable of the SMPs and instruction promoting them, this begs the question: How does middle school teachers' noticing during modeling with mathematics tasks develop as they gain greater confidence enacting such problems? What differences exist between middle school teachers' noticing during modeling with mathematics instruction and non-modeling with mathematics instruction? Future research may respond to questions like these that build upon this case study. Relatedly we wonder: What do similar (e.g., years of teaching experience and education completed) elementary and high school teachers notice during modeling with mathematics instruction?

A second limitation was that not all the recordings of the two teachers' lessons captured every student interaction. It is possible there may have been situations that were not visible on camera or recalled by Mr. Brown or Mrs. Zelda. Future researchers might consider placing multiple cameras around the room and asking students and teachers to wear microphones to record every student–teacher interaction. Capturing more interactions may allow for deeper exploration into what middle school teachers notice during instruction promoting modeling with mathematics.

Final Thoughts

The goal of this case study was to closely examine two middle school teachers' instruction to understand what they notice during mathematics instruction that promotes modeling with mathematics. Mr. Brown and Mrs. Zelda attended to instructional events most closely associated with mathematical structure and translating between representations. Such a focus is needed during instruction in the Common Core era that includes standards describing mathematical behaviors not typically found in many previous state-level standards.

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